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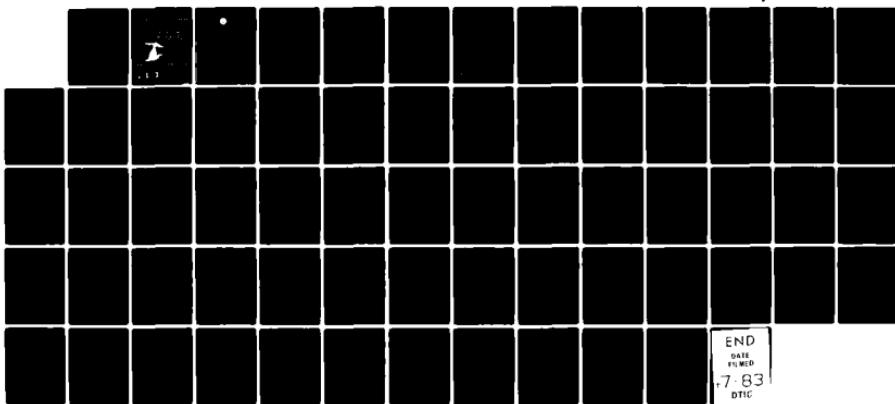
A PROGRAM TO COMPUTE VERTICAL ELECTRIC ELF FIELDS IN A
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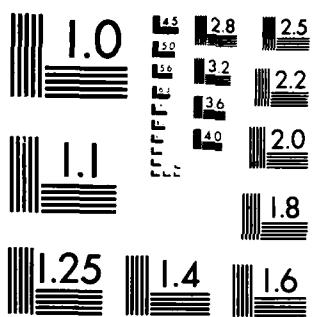


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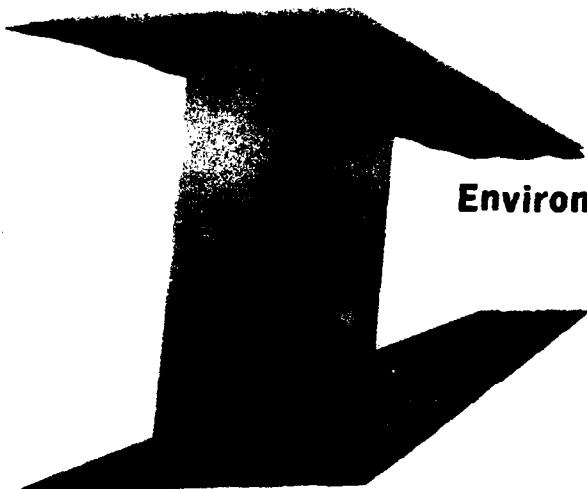
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Technical Report 851

A PROGRAM TO COMPUTE VERTICAL ELECTRIC ELF FIELDS IN A LATERALLY INHOMOGENEOUS EARTH-IONOSPHERE WAVEGUIDE



Environmental Sciences Department

JA Ferguson
LR Hitney
and
RA Pappert

1 December 1982

Prepared for
Defense Nuclear Agency

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San Diego, California 92152

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I. INTRODUCTION

Since an ELF signal from a remote transmitter is received over a range of azimuth angles, lateral ionospheric gradients produced by sporadic E layering or nuclear depressions can produce significant effects on propagation in the lower ELF band. This is because the Fresnel zone size can be large as compared with the distance over which the ionosphere changes significantly in the lateral direction. Although a number of workers have addressed the question of off-path effects (Wait¹, Galejs², Greifinger and Greifinger³, Field^{4,5}, Field and Joiner^{6,7}, Pappert⁸) no formulation exists which can fully account for the propagation effects produced by a localized disturbance with simultaneous allowance for vertical inhomogeneity, lateral inhomogeneity, and anisotropy in a spherical geometry. It has been common practice to estimate the effects of lateral gradients by using a simple surface propagation model introduced by Wait and more fully developed by the Greifingers and Field. The formulation reduces the problem to an integral equation description of propagation along the earth's surface. The theory is predicated on the palatable assumption that the field can be separated into lateral and height dependent functions when the lateral ionospheric gradients are considerably smaller than the vertical gradients. When applying the method to nocturnal environments additional assumptions are made. Among these is the omission of nonreciprocal effects. This is well justified in the ambient case⁹ as well as for daytime and depressed ionospheres. However, it is known that under sporadic E layering considerable mixing between TE and TM wave can occur¹⁰. Thus, when the surface propagation model is applied to sporadic E environments the scattered TE component is neglected. The validity conditions

for the formulation are probably best satisfied under conditions of either natural or man-made depressed ionospheres.

The purpose of this report is to document a computer program, based on the surface propagation model, which is useful to the user community for estimating the effects of localized ionospheric disturbances on propagation of the vertical electric field component E_z at lower ELF frequencies. A moments method¹¹ serves as the basis for the solution of the integral equation. Though the method is powerful, in the present program, practical storage requirements restrict application to disturbances which effectively vanish outside a rectangle of several megameters on one side. It is hoped that limitation will be relaxed in future work. The program allows in some measure, for modeling of rectangular, circular, and elliptical disturbance shapes. The lateral propagation function, W , defined as the ratio of the disturbed laterally dependent part of the vertical field component, E_z , to the undisturbed field component is calculated as is the absolute value of the total E_z field component in the disturbed guide. The latter calculation allows in approximate manner for guide height effects via WKB formalism as applied to waveguide propagation.

The program requires eigenangle inputs for both the ambient and disturbed regions of the guide as well as end-on horizontal dipole excitation factors for the vertical E_z field component. These must be supplied from a waveguide program such as that of reference 12.

II. SUMMARY OF EQUATIONS

Subject to the assumption that the vertical, E_z , field component can be separated into lateral and height dependent functions, the lateral dependence $\psi(x,y)$ is given by⁵

$$\psi(x,y) = \psi^i(x,y) - \frac{ik^2}{4} \int_{-\infty}^{\infty} \int dx' dy' (S^2(x',y') - S_0^2) G(|\vec{r}-\vec{r}'|) \psi(x',y') \quad (1)$$

where S is the sine of the eigenangle for the disturbed guide, S_0 the sine for the unperturbed guide, k the free space wave number, and the superscript i signifies the unperturbed incident field. The Green's function $G(|\vec{r}-\vec{r}'|)$ is given by

$$G(|\vec{r}-\vec{r}'|) = \sqrt{\frac{|\vec{r}-\vec{r}'|}{a_e \sin(\frac{|\vec{r}-\vec{r}'|}{a_e})}} H_0^{(2)}\{kS_0 |\vec{r}-\vec{r}'|\} \quad (2)$$

where

$$\vec{r} = xi + yj \text{ and } \vec{r}' = x'i + y'j. \quad (3)$$

Here a_e is the earth's radius and the x , y , z are the Cartesian coordinates. z is measured vertically upwards into the ionosphere and x is measured horizontally with $x-z$ the plane of incidence. Unit vectors in the x and y directions are denoted by i and j . The square root factor in equation (2) has been introduced to allow for the geometric spreading appropriate to a spherical geometry. Beyond that, x , y and x' , y' are taken to be rectangular coordinates in a flat earth geometry. The quantity $H_p^{(2)}$ is the Hankel

coordinates in a flat earth geometry. The quantity $H_p^{(2)}$ is the Hankel function of order p of the second kind. The unperturbed lateral dependent function, ψ^i , is taken to be appropriate for the vertical electric field, E_z , launched by the end-on component of a ground based horizontal electric dipole source oriented in the x direction and given by

$$\psi^i = B \sqrt{\frac{|\vec{r}|}{a_e \sin(\frac{|\vec{r}|}{a_e})}} H_1^{(2)}\{ks_0 |\vec{r}|\} \frac{x}{|\vec{r}|} \quad (4)$$

where B is a constant. Again the square root factor allows for the spreading appropriate to a spherical geometry. The discrete analog of equation (1) for field points within the disturbed region consists of the $N \times N$ system of linear equations:

$$\sum_{n=1}^N A_{mn} \psi_n^i = \psi_m^i ; m = 1, 2, \dots, N \quad (5)$$

Allowance is made for modeling rectangular, circular, and elliptical disturbances by subdividing them into square mesh cells. It is common practice in such circumstances to simplify the integrations involving cylindrical functions by approximating each square cell by a circle of equal area. It is also common practice to take the electric field to be constant over the area of a cell. However, larger cells can be tolerated if the electric field is allowed to vary over the area of a cell. At least two methods of allowing for this variation have been described in the literature¹³. In this work the method called 'plane wave correction' is used. The method assumes isotropic propagation within and between each cell and that should be a reasonable approximation in the present problem since anisotropy of propagation at ELF

frequencies is quite small. Translated into the notation of this work, the results of reference 13 yield for the A-matrix elements:

$$A_{mm} = 1 + \left(\frac{s_m^2}{s_0^2} - 1 \right) \left[\left(\frac{s_m^2}{s_0^2} + 1 \right) + \frac{i\pi}{2} ks_0 a \left(1 - \frac{1}{4} (ks_m a)^2 \right) H_1^{(2)}(ks_0 a) \right. \\ \left. + \frac{i\pi}{4} (ks_m a)^2 H_2^{(2)}(ks_0 a) \right] \quad (6)$$

$$A_{mn} = \frac{i\pi ks_0 a}{2} \left(\frac{s_n^2}{s_0^2} - 1 \right) G(|\vec{r}_m - \vec{r}_n|) \left[\left(1 - \frac{1}{4} (ks_n a)^2 \right) J_1(ks_0 a) \right. \\ \left. + \frac{1}{2} \left(\frac{s_n^2}{s_0^2} \right) ks_0 a J_2(ks_0 a) \right]; m \neq n \quad (7)$$

The Green's function, G , is given by equation (2) with \vec{r}_m replacing \vec{r} and \vec{r}_n replacing \vec{r}' . Also, J_p is the Bessel function of order p of the first kind. In terms of the ψ_n determined by equation (5), and the A_{mn} given by equation (7), the field at a point, \vec{r}_m , exterior to the disturbed region is given by:

$$\psi(\vec{r}_m) = \psi^i(\vec{r}_m) - \sum_{n=1}^N A_{mn} \psi_n \quad (8)$$

Equations (5) through (8) are used in the present program to determine the dB value,

$$W = 20 \log_{10} \{ \psi(x, y) / \psi^i(x, y) \}, \quad (9)$$

of the disturbed lateral function relative to its undisturbed value.

Another output of the program makes allowance for dependence of the vertical electric field on height of the guide via the approximate WKB formalism. For a laterally homogeneous guide the E_z field generated by the end-on component of a horizontal dipole may be expressed as

$$E_z \sim -iQ \frac{s^{3/2}}{\frac{\partial F}{\partial \theta}} \frac{(1 - {}_1 R_{11} \bar{R}_1)}{{}_{||} \bar{R}_{||}} (1 + {}_{||} \bar{R}_{||}) \{C(1 - {}_{||} \bar{R}_{||})\} \Psi(x, y) \quad (10)$$

where F is the modal function and $\partial F / \partial \theta$ its derivative evaluated at the eigen-angle. S and C are the sine and cosine of the eigenangle. ${}_1 \bar{R}_1$ and ${}_{||} \bar{R}_{||}$ are TE and TM Fresnel reflection coefficients referenced to the ground and ${}_1 R_1$ is the ionospheric TE reflection coefficient referenced to the ground. In the absence of anisotropy the quantity $(1 - {}_1 R_{11} \bar{R}_1)$ would cancel an identical term occurring in the $(\partial F / \partial \theta)$. The Fresnel coefficient ${}_{||} \bar{R}_{||}$ is

$${}_{||} \bar{R}_{||} = (N_G^2 C - \sqrt{N_G^2 - S^2}) / (N_G^2 C + \sqrt{N_G^2 - S^2}) \quad (11)$$

where N_G is the complex refractive index of the ground. Because the magnitude of N_G is much greater than unity in the lower ELF band good approximations are:

$$1 + {}_{||} \bar{R}_{||} \approx 2 \quad \text{and} \quad C(1 - {}_{||} \bar{R}_{||}) \approx 2/N_G \quad (12)$$

Thus, within the spirit of the WKB approximation the E_z field becomes

$$E_z \approx -4Qi \left[\frac{s^{3/2}}{\frac{\partial F}{\partial \theta}} (1 - R_{11} \bar{R}_1) \right]_r^{1/2} \left[\frac{s^{3/2}}{\frac{\partial F}{\partial \theta}} (1 - R_{11} \bar{R}_1) \right]_t^{1/2} \frac{1}{N_G t} \Psi(x, y) \quad (13)$$

where Q is a constant dependent upon dipole moment and frequency and the subscripts r and t stand for receiver and transmitter. That is, the first term in parenthesis is evaluated at the receiver while the second term in parenthesis and the factor N_G^{-1} are evaluated at the transmitter.

The factor B in equation (4) is taken to be

$$B = \sqrt{\frac{\pi k s_0^a e}{2}} e^{-(3/4)\pi i} \quad (14)$$

To express the vertical electric field, E_z , as given by equation (13) in microvolts/m, the factor Q has the value

$$Q = 2.849 \times 10^{-3} f_{kHz}^{3/2} (Idl) \quad (15)$$

where f_{kHz} is the frequency in kHz and Idl is the current moment in ampere meters.

III. GEOMETRICAL MODELING OF THE DISTURBANCE

As described in the introduction, the program can be used to model square, rectangular, circular, and elliptical disturbances. In all cases the disturbance is defined to be symmetrical about both its x and y axes. The special case of a rectangular disturbance is required to be uniformly disturbed. Let x_0 and y_0 denote the coordinates of the center of the disturbance with respect to the location of the transmitter. Let L_x and L_y be the size of the disturbance along the x and y axis. The disturbance is overlaid by a grid which has n_x squares along the x axis and n_y squares along the y axis. The choice of L_x , L_y , n_x , and n_y must be such that $L_x/n_x = L_y/n_y$.

Since we assume that the disturbance is symmetrical about the x and y axis, we only need to specify the waveguide eigen solution parameters along the x axis. Let us denote these solutions by S_i . We take $i = 0$ to denote the ambient or undisturbed values. The remaining S_i (S_1 through S_N) is assumed to be uniformly distributed along the x axis from x_0 to $x_0 + \frac{1}{2}L_x$. Note that the value of n_x is not related to that of N . If $N = 1$, then a uniform disturbance will be assumed. This is required for the rectangular disturbance. The remaining problem is to fill the disturbance grid with interpolated values of S .

Let us now consider a single subsquare within a square disturbance. Let the coordinates of the center of this subsquare be x and y . The smaller value of $|x - x_0|$ and $|y - y_0|$ is used to interpolate a value of S from the input list of S_i . This results in the disturbance grid being filled as illustrated in figure 1. In the figure the similarly shaded regions would all have the same value of S .

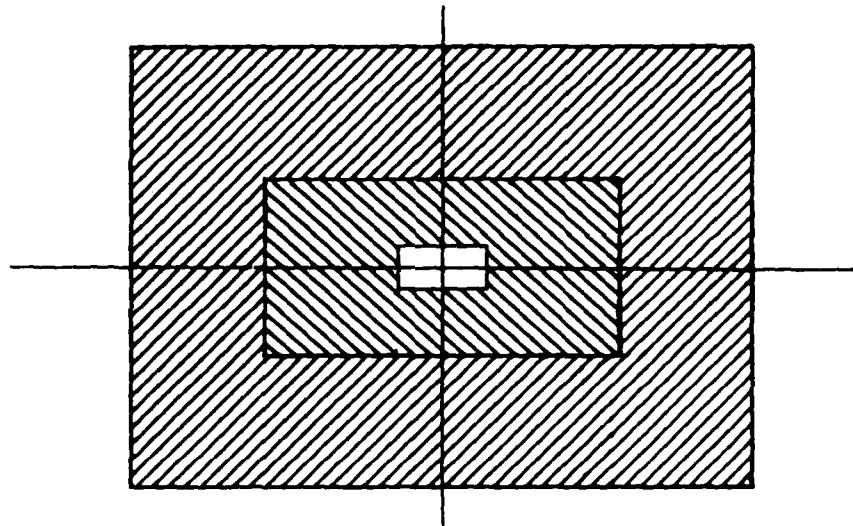


Figure 1. Diagram illustrating the distribution of S within a rectangular disturbance.
The similarly shaded regions have the same value of S .

Let us consider an elliptical disturbance with an axial ratio $R = L_y/L_x$. The center of a given subsquare is at x and y . We can define an ellipse which is concentric with that of the outer edge of the disturbance. This ellipse intersects the x axis at a distance from x_0 defined by

$$A = \{(x-x_0)^2 + [(y-y_0)/R]^2\}^{1/2} \quad (16)$$

The above expression can be used to calculate the value of A for each corner of the subsquare, say A_i where $i = 1, 2, 3$ or 4 . If the smallest of the A_i is greater than L_x , then the subsquare is entirely outside the disturbance and S_0 is used in the grid at that subsquare. If the largest of the A_i is less than L_x , then the subsquare is entirely inside the disturbance and the value of S for that subsquare is interpolated from the input list of S_i . The remaining case is that of a subsquare on the edge of the disturbance. In this case the subsquare is subdivided into 16 smaller squares. Let the coordinates of the

center of each of these smaller squares be x_m and y_m . For each of these squares we calculate A_m using equation (16). The value of S for the subsquare is given by

$$S = \frac{nS_N - (16-n)S_0}{16} \quad (17)$$

where n is the number of subsquares which are within the ellipse. A circular disturbance is treated the same way as an elliptical one with $R = 1$.

IV. DESCRIPTION OF INPUT

All input to the sporadic-E program is given in a data deck on the standard input unit. A listing of sample input showing the data deck setup is shown in figure 2.

There are two parts to the input. This first part is read in by means of a FORTRAN NAMELIST input format. The first card of each set of input must contain '&DATUM' in columns 2-7. This is followed by at least one blank and then data items separated by commas. The data items have the following forms:

'variable name' = constant,

or

'array name' = set of constants (all separated by commas).

The data list is terminated by '&END'. All cards must have a blank in column 1.

The second part of the input follows the NAMELIST. The first card for this part is an identification card. It contains up to 80 columns of alphanumeric information and is used to label the output plots. Following the identification card a series of punched cards (obtained from the programs described in reference 14 with NPUNCH = 1) is input for each mode.

The first card gives the values of R, FREQ, AZIM, CODIP, MAGFLD, SIGMA and EPSR. Next there are two cards per mode. The first of these contains the complex eigenangle at the ground and values for the complex quantities T1 and T2. The second card contains the eigenangle at the ground (duplicate input) and values for T3 and T4. The quantities T1, T2, T3, and T4 are defined in reference 14.

```

1      &DATAUM
2      DM1H=150.,DMAX=15000.,DELD=150.
3
4      YMAX=0.,DELY=0.,
5      IFLAG=2,IGRID=0,
6      XC=2000.,YC=0.,
7      NUNX=4,NUNY=8,
8      SIZEX=500.,SIZEY=1000.,
9      IPLOT=1,
10     X1N=5.,Y1N=8.,
11     EXTIC=3000.,WXTIC=3000.,WYTIC=1.,EYTIC=5.,
12     WIN=-6.,WMAX=2.,
13     EMIN=5.,EMAX=45.,
14     REND
15     SAMPLE-X-VARIATION_PLOT
16     R .000 F 0.0750 A 105.000 C 13.000 M 0.4100-004 S 3.200-004 E 10.
17     1 83.98519-24.959091 7.15494419-002 2.04701227+000-3.39161358-002 1.5526
18     2 -83.488519-34.969091-
19
20     R .000 F 0.0750 A 105.000 C 13.000 M 0.4100-004 S 3.200-004 E 10.
21     1 59.39295-65.52164-3.75059881-001-1.76435304-000-1.40913780+000-3.52235
22     2 50.39295-55.552161
23
24

```

Figure 2. Sample data input for the computer program.

The following variables and arrays may be specified in the NAMELIST input:

DM dipole moment in ampere meters.

DMIN the minimum range in kilometers at which fields are calculated, printed, and plotted for IFLAG = 2.

DMAX the maximum range in kilometers at which fields are calculated, printed, and plotted for IFLAG = 2.

DELD the increment, in kilometers at which fields are calculated, printed, and plotted for IFLAG = 2.

YMAX the maximum off axis value, in kilometers, at which fields are calculated, printed, and plotted for IFLAG = 1.

DELY the increment in kilometers, at which fields are calculated printed and plotted for IFLAG = 1.

IFLAG for IFLAG = 1 the program calculates fields as a function of y - the distance is fixed at DMIN and the disturbance moves in the y direction.
for IFLAG = 2 the program calculates fields as a function of distance with the disturbance fixed.

IGRID IGRID = 0 indicates that the disturbance is either square or rectangular.
IGRID = 1 indicates that the disturbance is either circular or elliptical.

X0,Y0 coordinates in kilometers at center of disturbance. Y0 is also the initial value at which fields are calculated, printed, and plotted for IFLAG = 1.

NUMX, NUMY the disturbance is divided into NUMX grids in the x- direction and NUMY grids in the y-direction (these are n_x , n_y in the text).

SIZEX SIZEY defines the physical size of the disturbance. It is SIZEX kilometers by SIZEY kilometers.

IPLOT a flag controlling whether or not plots are generated. If IPLOT = 0 no plots are generated. If IPLOT = 1 two plots are generated. For IFLAG=2 the first plot is WMAG(DB) vs X(KM), equation (9). The second plot consists of two curves: EZUMAG (DB), equations (13) and (4), is a solid curve and EZPMAG (DB), equations (13) and (8), is a dashed curve. For IFLAG=1 WMAG, EZUMAG, and EZPMAG are plotted with respect to y for a fixed value of x.

XLNG the length in inches for the x-axis and y-axis for the field plots.
YLNG
WMIN the minimum and maximum values, in dB, desired on y-axis for the WMAG plot.
WMAX
EMIN the minimum and maximum values in DB desired on y-axis for the EZREL plot.
EMAX
XYIC the units per tic mark along the x-axis and y-axis for both plots.
YTIC

Initial values of the namelist variables are presented in table 1.

TABLE 1
Namelist variables and initial values.

<u>NAME</u>	<u>INITIAL VALUE</u>	<u>UNITS</u>
DM	6.75E6	Ampere meters
DMIN	25.0	Kilometers
DMAX	1000.0	Kilometers
DELD	25.0	Kilometers
YMAX	5000.0	Kilometers
DELY	25.0	Kilometers
IFLAG	2	
IGRID	0	
X0	0.0	Kilometers
Y0	0.0	Kilometers
NUM	0	
NUMY	0	
SIZEX	1000.0	Kilometers
SIZEY	1000.0	Kilometers
IPILOT	0	
XLNG	5.0	Inches
YLNG	6.0	Inches
WMIN	-6.0	Decibels
WMAX	2.0	Decibels
EMIN	-60	Decibels
EMAX	2.0	Decibels
XTIC	200.0	Kilometers
YTIC	0.2	Decibels

V. DESCRIPTION OF OUTPUT

A listing of sample output is shown in figure 3. The resulting plots are found in figures 4 through 7. Figures 4 and 5 are the output from IFLAG = 2. Figures 6 and 7 are the output from IFLAG = 1. The first section of output is an echoing of the namelist input variables. This is followed by a schematic drawing of the disturbed region showing grid numbers and the coordinates of the center of the corner meshes. The next section shows additional input parameters: the identification label that will be on the plots, the frequency, conductivity, dielectric function, and the complex eigenangle and excitation factor for each region. YMID represents the y-value coordinate of the midpoint of the disturbance.

In the printout, the tables for WI, EZ0, and EZS, are the magnitudes of the quantities as given by equations 9, 10, and 13. These quantities are computed at the midpoints of each grid square.

The last table in the printout lists the following quantities at the given distance from the transmitter along the x-axis at y=0:

EZREL,EZANG	magnitude(dB) and phase angle (radians) of equation 8
WMAG,WANG	magnitude(dB) and phase angle (radians) of equation 9
EZUMAG,EZUANG	magnitude(dB) and phase angle (radians) of equation 10
EZPMAG,EZPANG	magnitude(dB) and phase angle (radians) of equation 13

```

$DATUM
DN = .6750000+007,DMIN = .1500000+003,DMAX = .1500000+005,DELD = .0000000
IFLIG = 2,IGRID = 0,XO = .2000000+004,YO = .0000000+004,NUMX =
SIZEY = .5000000+003,SIZEZ = .1000000+004,IPLOT =
NMAX = .2000000+001,EMIN = .5000000+001,EMAX = .4500000+002,EXTC = .3000000+004,EYTC = .5000000+001,WXTIC =
WYTIC = .1000000+001,SENDO = .0000000+000,DELY = .0000000

```

18

COORDINATES AT CENTER OF MESH NUMBER	1 ARE : X = 1812.50 Y = 437.50
COORDINATES AT CENTER OF MESH NUMBER	4 ARE : X = 2187.50 Y = 437.50
COORDINATES AT CENTER OF MESH NUMBER	29 ARE : X = 1812.50 Y = -437.50
COORDINATES AT CENTER OF MESH NUMBER	32 ARE : X = 2187.50 Y = -437.50

Figure 3. Sample printed output generated by the first case called out in the sample data.

SAMPLE	X-VARIATION PLOT	FREQ = .750-001 SIGMA = .320-003 EPSR= 10.00	XTRA
GRID	THETA		
0	83.98519 -34.96909	-8.960726708-001	-9.687796474+000
1	59.39295 -65.55216	-2.835879266+000	-1.158006728+001
YMID = .00			
	EZO		
1.51+000	1.45+000	1.39+000	1.34+000
1.55+000	1.48+000	1.42+000	1.36+000
1.57+000	1.50+000	1.44+000	1.38+000
1.59+000	1.51+000	1.45+000	1.38+000
1.59+000	1.51+000	1.45+000	1.38+000
1.57+000	1.50+000	1.44+000	1.38+000
1.55+000	1.48+000	1.42+000	1.36+000
1.51+000	1.45+000	1.39+000	1.34+000
	EZS		
1.13+000	1.03+000	9.84-001	9.64-001
1.11+000	1.02+000	9.83-001	9.73-001
1.10+000	1.01+000	9.80-001	9.78-001
1.09+000	1.00+000	9.78-001	9.80-001
1.09+000	1.00+000	9.78-001	9.80-001
1.10+000	1.01+000	9.80-001	9.78-001
1.11+000	1.02+000	9.83-001	9.73-001
1.13+000	1.03+000	9.84-001	9.64-001
	WI		
-2.5496	-2.9501	-3.0140	-2.8561
-2.8676	-3.2269	-3.1821	-2.9128
-3.1044	-3.4413	-3.3171	-2.9656
-3.2293	-3.5572	-3.3921	-2.9972
-3.2293	-3.5572	-3.3921	-2.9972
-3.1044	-3.4413	-3.3171	-2.9656
-2.8676	-3.2269	-3.1821	-2.9128
-2.5496	-2.9501	-3.0140	-2.8561
	DIST		
0	150.0	-1.18907-001	-1.18907-001
0	300.0	-1.33171-001	-1.33172-001
0	450.0	1.36797-003	1.36797-003
0	600.0	2.62275-001	2.62075-001
0	750.0	5.64055-001	5.64055-001
0	900.0	7.95926-001	7.95926-001
0	1050.0	8.52173-001	8.52191-002
0	1200.0	6.44159-001	1.11490-001
0	1350.0	1.01138-001	1.65756-001
0	1500.0	-8.21781-001	1.89704-001
0	1650.0	-2.06253+000	1.44464-001
0	1800.0	-3.15872+000	-1.40568-002
0	1950.0	-3.54067+000	-1.98468-001
0	2100.0	-3.27354+000	-3.16967-001
0	2250.0	-2.81518+000	5.93701+000
0	2400.0	-2.49567+000	5.96869+000
0	2550.0	-2.27920+000	5.99499+000
0	2700.0	-2.12389+000	6.01585+000
	WAG		
19	WANG		
	EZREL		
	EZANG		
	EZUMAG		
	EZUANG		
	EZPANG		
	EZPMAG		

Figure 3. Continued.

-0	2850.0	-2.000008+000	6.03249+000	-2.000808+000	2.89084+000	2.39415+001	2.57626-002
-0	3000.0	-1.91842+000	6.08574+000	-1.91842+000	2.90419+000	2.35886+001	2.65373+000
-0	3150.0	-1.84848+000	6.05670+000	-1.84848+000	2.91511+000	2.32482+001	2.38814+000
-0	3300.0	-1.78124+000	6.05580+000	-1.78824+000	2.92420+000	2.29192+001	2.12045+000
-0	3450.0	-1.73929+000	6.07347-7.000	-1.73929+000	2.9318R+000	2.20066+001	2.11310+001
-0	3600.0	-1.69769+000	6.08000+000	-1.69769+000	2.93845+000	2.22916+001	2.09613+001
-0	3750.0	-1.66143+000	6.08574+000	-1.66143+000	2.94415+000	2.20915+001	2.05939+001
-0	3900.0	-1.63055+000	6.09C72+000	-1.63055+000	2.94913+000	2.16996+000	2.03297+001
-0	4050.0	-1.60298+000	6.09512+000	-1.60298+000	2.95353+000	2.14152+001	2.00690+001
-0	4200.0	-1.57945+000	6.09904+000	-1.57845+000	2.95744+000	2.11379+001	1.98122+001
-0	4350.0	-1.55646+000	6.10255+000	-1.55646+000	2.96096+000	2.08674+001	1.93109+001
-0	4500.0	-1.53658+000	6.10573+000	-1.53658+000	2.96313+000	2.06030+001	1.86736+000
-0	4650.0	-1.51855+000	6.10861+000	-1.51855+000	2.96702+000	2.03446+001	1.80665+001
-0	4800.0	-1.50198+000	6.11125+000	-1.50198+000	2.96965+000	2.00918+001	1.76724+000
-0	4950.0	-1.48683+000	6.11366+000	-1.48683+000	2.97207+000	1.98442+001	1.72922+001
-0	5100.0	-1.47280+000	6.11590+000	-1.47280+000	2.97430+000	1.96018+001	1.68736+000
-0	5250.0	-1.45936+000	6.11796+000	-1.45936+000	2.97637+000	1.94363+000	1.64771+000
-0	5400.0	-1.44771+000	6.11988+000	-1.44771+000	2.97829+000	1.91309+000	1.58886+000
-0	5550.0	-1.43633+000	6.12165+000	-1.43633+000	2.9806+000	1.89018+001	1.51026+000
-0	5700.0	-1.42603+000	6.12333+000	-1.42603+000	2.98173+000	1.86762+001	1.43177+000
-0	5850.0	-1.41604+000	6.12490+000	-1.41604+000	2.98250+000	1.84551+001	1.40312+000
-0	6000.0	-1.40507+000	6.12638+000	-1.40607+000	2.98479+000	1.82404+001	1.373643+000
-0	6150.0	-1.39690+000	6.12774+000	-1.39690+000	2.98615+000	1.80269+001	1.345906+000
-0	6300.0	-1.38865+000	6.12902+000	-1.38865+000	2.98743+000	1.78157+001	1.3149+000
-0	6450.0	-1.38083+000	6.13030+000	-1.38083+000	2.98871+000	1.76099+001	1.29032+000
-0	6600.0	-1.37023+000	6.13052+000	-1.37023+000	2.9892+000	1.74982+000	1.26664+000
-0	6750.0	-1.36380+000	6.13266+000	-1.36380+000	2.99097+000	1.72116+001	1.24804+000
-0	6900.0	-1.35635+000	6.13375+000	-1.35635+000	2.99215+000	1.70193+001	1.2028+000
-0	7050.0	-1.35141+000	6.13486+000	-1.35141+000	2.99327+000	1.68923+001	1.16427+000
-0	7200.0	-1.34180+000	6.13486+000	-1.34480+000	2.99427+000	1.66099+001	1.15158+000
-0	7350.0	-1.33843+000	6.13683+000	-1.33843+000	2.99523+000	1.64468+001	1.23786+000
-0	7500.0	-1.33225+000	6.13775+000	-1.33235+000	2.99616+000	1.62631+001	1.20874+000
-0	7650.0	-1.32540+000	6.13865+000	-1.32640+000	2.99705+000	1.60822+001	1.18979+000
-0	7800.0	-1.32064+000	6.13952+000	-1.32064+000	2.99793+000	1.59041+001	1.14076+000
-0	7950.0	-1.31509+000	6.14025+000	-1.31510+000	2.99837+000	1.57983+001	1.13171+000
-0	8100.0	-1.30934+000	6.14117+000	-1.30953+000	2.99950+000	1.55561+001	1.47322+000
-0	8250.0	-1.30415+000	6.14196+000	-1.30415+000	2.000337+000	1.53461+001	2.8546+000
-0	8400.0	-1.29707+000	6.14276+000	-1.29707+000	2.00116+000	1.52183+001	2.57356+000
-0	8550.0	-1.29108+000	6.13439+000	-1.29483+000	2.00190+000	1.50540+001	2.29446+000
-0	8700.0	-1.28151+000	6.14215+000	-1.28961+000	2.00260+000	1.48918+001	1.41076+000
-0	8850.0	-1.28456+000	6.14496+000	-1.28456+000	2.00337+000	1.47322+000	1.42466+000
-0	9000.0	-1.28031+000	6.14560+000	-1.28031+000	2.00400+000	1.45361+001	2.8522+000
-0	9150.0	-1.27437+000	6.14628+000	-1.27478+000	2.00469+000	1.44220+001	2.29562+000
-0	9300.0	-1.26333+000	6.14622+000	-1.26393+000	2.00532+000	1.42680+001	2.01534+000
-0	9450.0	-1.26364+000	6.14757+000	-1.26535+000	2.00598+000	1.41182+001	1.37599+001
-0	9600.0	-1.26084+000	6.1496+000	-1.26084+000	2.00661+000	1.40760+001	1.36022+001
-0	9750.0	-1.25337+000	6.14884+000	-1.25637+000	2.00724+000	1.38705+001	1.34626+000
-0	9900.0	-1.25196+000	6.14946+000	-1.25196+000	2.00786+000	1.36834+001	1.32946+000
-0	10050.0	-1.24758+000	6.15007+000	-1.24758+000	2.00847+000	1.35434+001	1.31453+000
-0	10200.0	-1.24324+000	6.15067+000	-1.24324+000	2.00908+000	1.34057+001	1.29980+001
-0	10350.0	-1.24034+000	6.15127+000	-1.23833+000	2.00967+000	1.32705+001	1.28748+000
-0	10500.0	-1.23843+000	6.15186+000	-1.23464+000	2.01026+000	1.31317+001	1.26779+000
-0	10750.0	-1.23038+000	6.15244+000	-1.23038+000	2.01085+000	1.30073+001	1.25696+000
-0	10800.0	-1.22613+000	6.15302+000	-1.22613+000	2.01143+000	1.28794+001	1.24315+000
-0	10950.0	-1.22189+000	6.15359+000	-1.22189+000	2.01200+000	1.27539+001	1.16532+000
-0	11100.0	-1.21766+000	6.15417+000	-1.21766+000	2.01257+000	1.26309+001	1.14320+000
-0	11250.0	-1.21342+000	6.15473+000	-1.21342+000	2.01314+000	1.25103+001	1.12969+001

Figure 3. Continued.

.0	1400.0	-1.20119+000	6.15530+000	-1.20918+000	3.01371+000	1.23423+000	2.99327+000	1.11131+000	6.00698+000
.0	11550.0	-1.20494+000	6.15587+000	-1.20427+000	3.01427+000	1.22768+001	2.71397+000	1.10719+001	5.72824+000
.0	11700.0	-1.20069+000	6.15643+000	-1.20068+000	3.01484+000	1.21639+001	2.43466+000	1.09632+001	5.44950+000
.0	11850.0	-1.19641+000	6.15699+000	-1.19641+000	3.01540+000	1.20536+001	2.15535+000	1.08572+001	5.17075+000
.0	12000.0	-1.19211+000	6.15756+000	-1.19211+000	3.01595+000	1.19459+001	1.87603+000	1.07538+001	4.89199+000
.0	12150.0	-1.18778+000	6.15812+000	-1.18778+000	3.01653+000	1.18408+001	1.59670+000	1.06531+001	4.61323+000
.0	12300.0	-1.18342+000	6.15869+000	-1.18342+000	3.01709+000	1.17385+001	1.31737+000	1.05551+001	4.33417+000
.0	12450.0	-1.17403+000	6.15926+000	-1.17904+000	3.01766+000	1.16389+001	1.03804+000	1.04599+001	4.05570+000
.0	12600.0	-1.17459+000	6.15983+000	-1.17459+000	3.01823+000	1.15422+001	7.58698-001	1.03676+001	3.77693+000
.0	12750.0	-1.17011+000	6.16040+000	-1.17011+000	3.01881+000	1.14482+001	4.79354-001	1.02781+001	3.49816+000
.0	12900.0	-1.16557+000	6.16098+000	-1.16557+000	3.01939+000	1.13572+001	2.00006-001	1.01917+001	3.21939+000
.0	13050.0	-1.16047+000	6.16157+000	-1.16097+000	3.01997+000	1.12692+001	6.20384+000	1.01083+001	2.94063+000
.0	13200.0	-1.15630+000	6.16216+000	-1.15630+000	3.02056+000	1.11843+001	5.92448+000	1.00280+001	2.66186+000
.0	13350.0	-1.15157+000	6.16275+000	-1.15157+000	3.02116+000	1.11025+001	5.64512+000	9.95089+000	2.38310+000
.0	13500.0	-1.14675+000	6.16336+000	-1.14675+000	3.02176+000	1.10239+001	5.36576+000	9.87710+000	2.10434+000
.0	13650.0	-1.14185+000	6.16397+000	-1.14185+000	3.02237+000	1.09486+001	5.08639+000	9.80671+000	1.82558+000
.0	13800.0	-1.13685+000	6.16459+000	-1.13685+000	3.02300+000	1.06767+001	4.80702+000	9.73982+000	1.54683+000
.0	13950.0	-1.13176+000	6.16522+000	-1.13176+000	3.02363+000	1.08083+001	4.52765+000	9.67655+000	1.26809+000
.0	14100.0	-1.12655+000	6.16586+000	-1.12655+000	3.02427+000	1.07436+001	4.24827+000	9.61703+000	9.89533-001
.0	14250.0	-1.12122+000	6.16651+000	-1.12122+000	3.02492+000	1.06826+001	3.96889+000	9.56138+000	7.10626-001
.0	14400.0	-1.11576+000	6.16718+000	-1.11576+000	3.02559+000	1.06255+001	3.68951+000	9.50976+000	4.31909-001
.0	14550.0	-1.11016+000	6.16786+000	-1.11016+000	3.02627+000	1.05725+001	3.41012+000	9.46232+000	1.53204-001
.0	14700.0	-1.10441+000	6.16856+000	-1.10441+000	3.02697+000	1.05237+001	3.13073+000	9.41925+000	6.15770+000
.0	14850.0	-1.09850+000	6.16928+000	-1.09850+000	3.02768+000	1.04792+001	2.85134+000	9.38074+000	5.87902+000
.0	15000.0	-1.09240+000	6.17001+000	-1.09240+000	3.02842+000	1.04394+001	2.57194+000	9.34699+000	5.60036+000

PLOTTING COMMENCING
.....
DISSPLA VERSION 9.0
NO. OF FIRST PLOT 1
PLOT ID. READS
PLOT 1 11.03.23 THUR 23 SEP, 1982
JOB=SPORAD ,
DISSPLA 9.0

21

PLOT NO. 1 WITH THE TITLE
HAS BEEN COMPLETED.

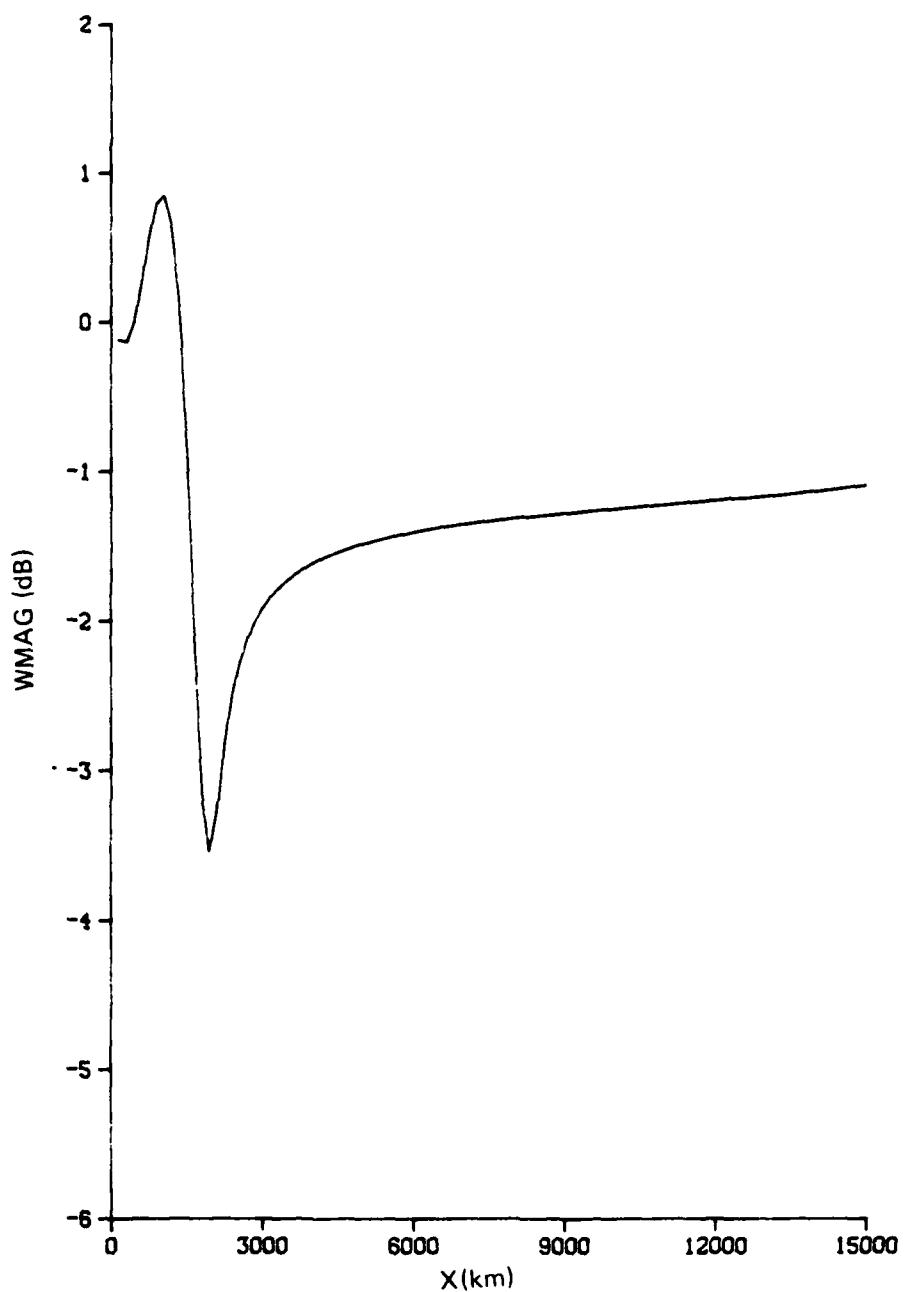
PLOT ID. READS
PLOT 1 11.03.23 THUR 23 SEP, 1982
JOB=SPORAD ,
DISSPLA 9.0

DATA FOR PLOT

NO. OF CURVES DRAWN 1

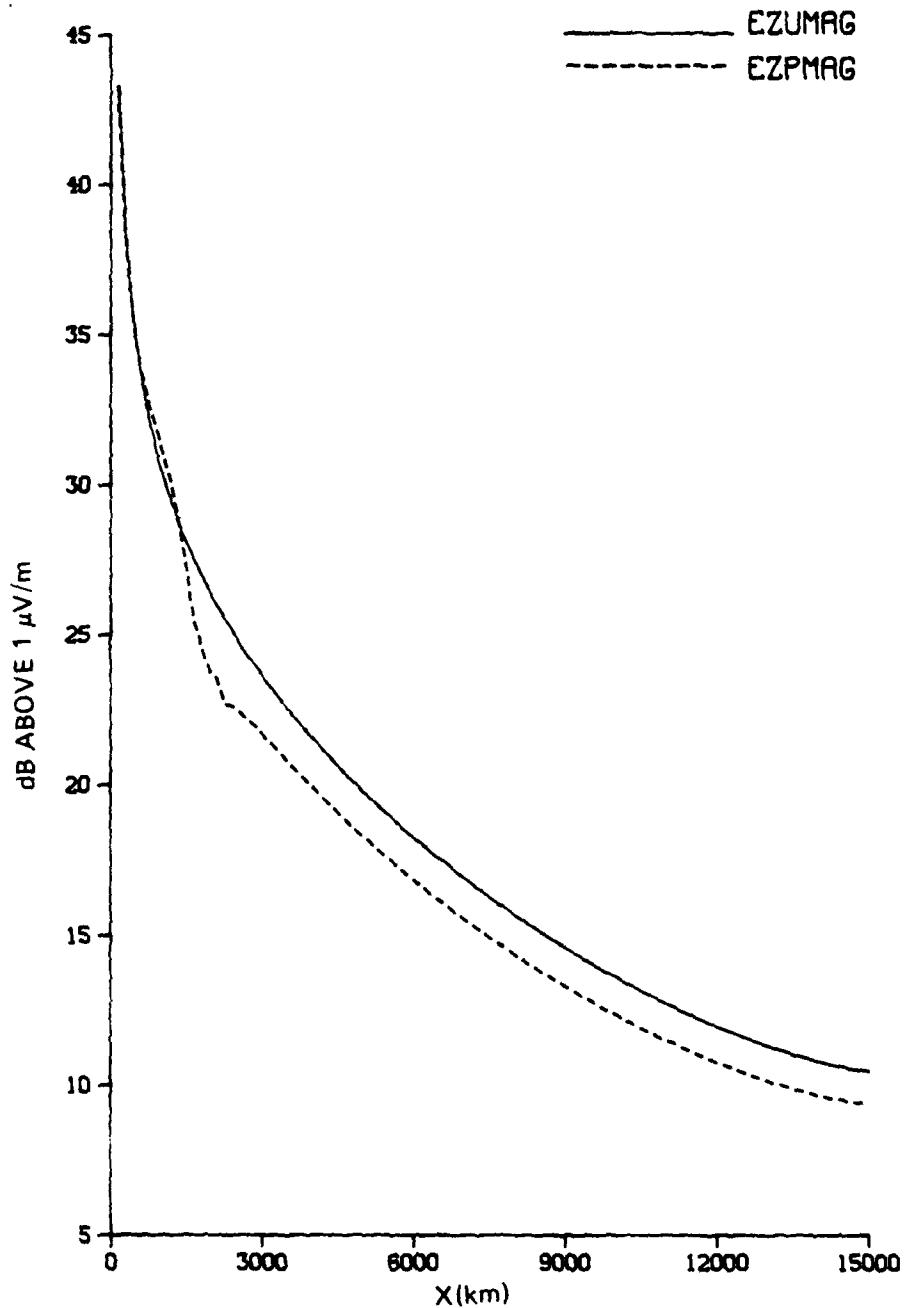
HORIZ. AXIS LENGTH 5.0 INS.
VERT. AXIS LENGTH 8.0 INS.

Figure 3. Continued.



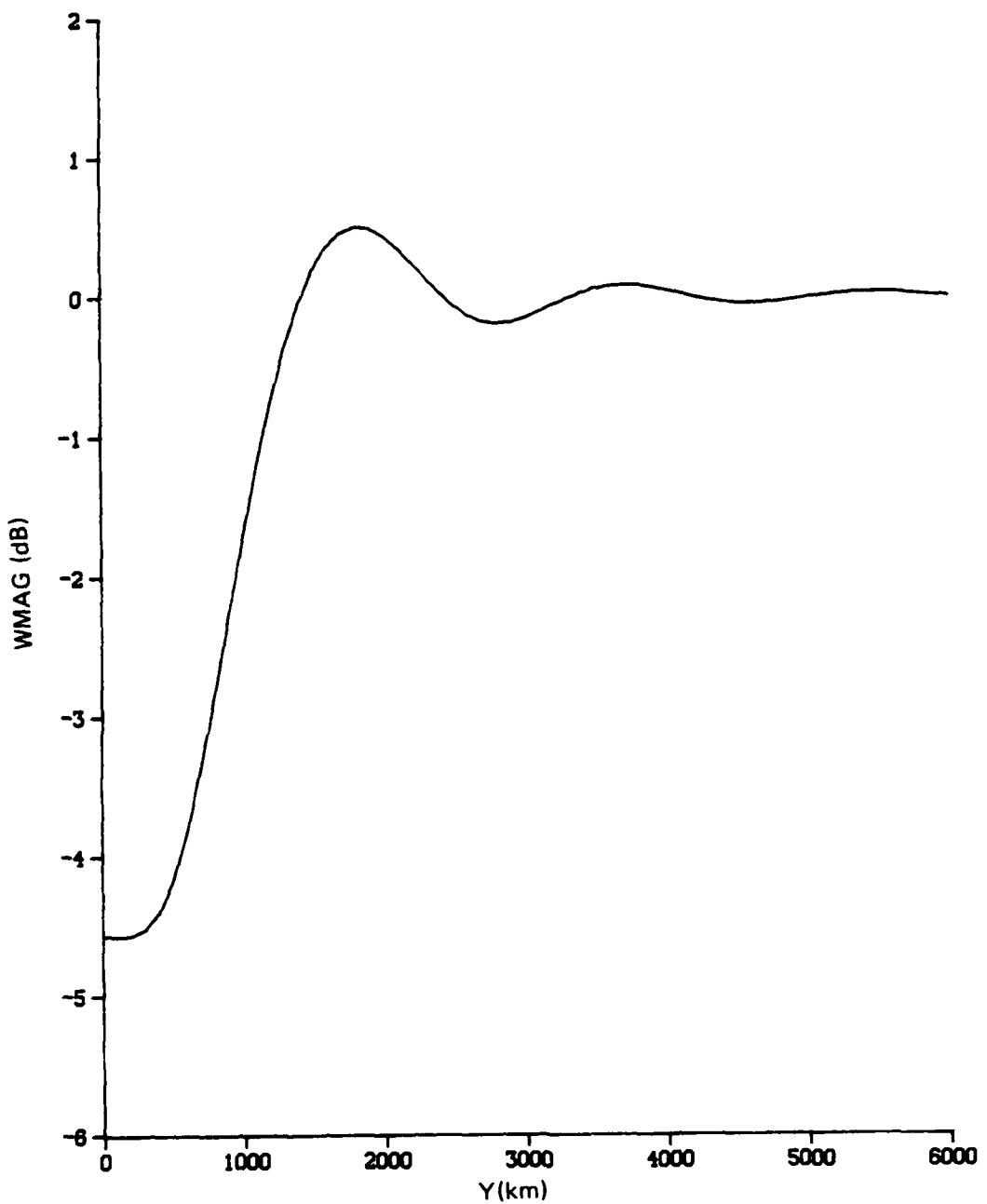
SAMPLE X-VARIATION PLOT
SQUARE OR RECTANGULAR DISTURBANCE
FREQ = 0.075
SIZEX= 500.0 X0= 2000.0 NUMX= 4
SIZEY= 1000.0 Y0= 0.0 NUMY= 8

Figure 4. Sample x-variation plot for WMAG assuming a rectangular disturbance.



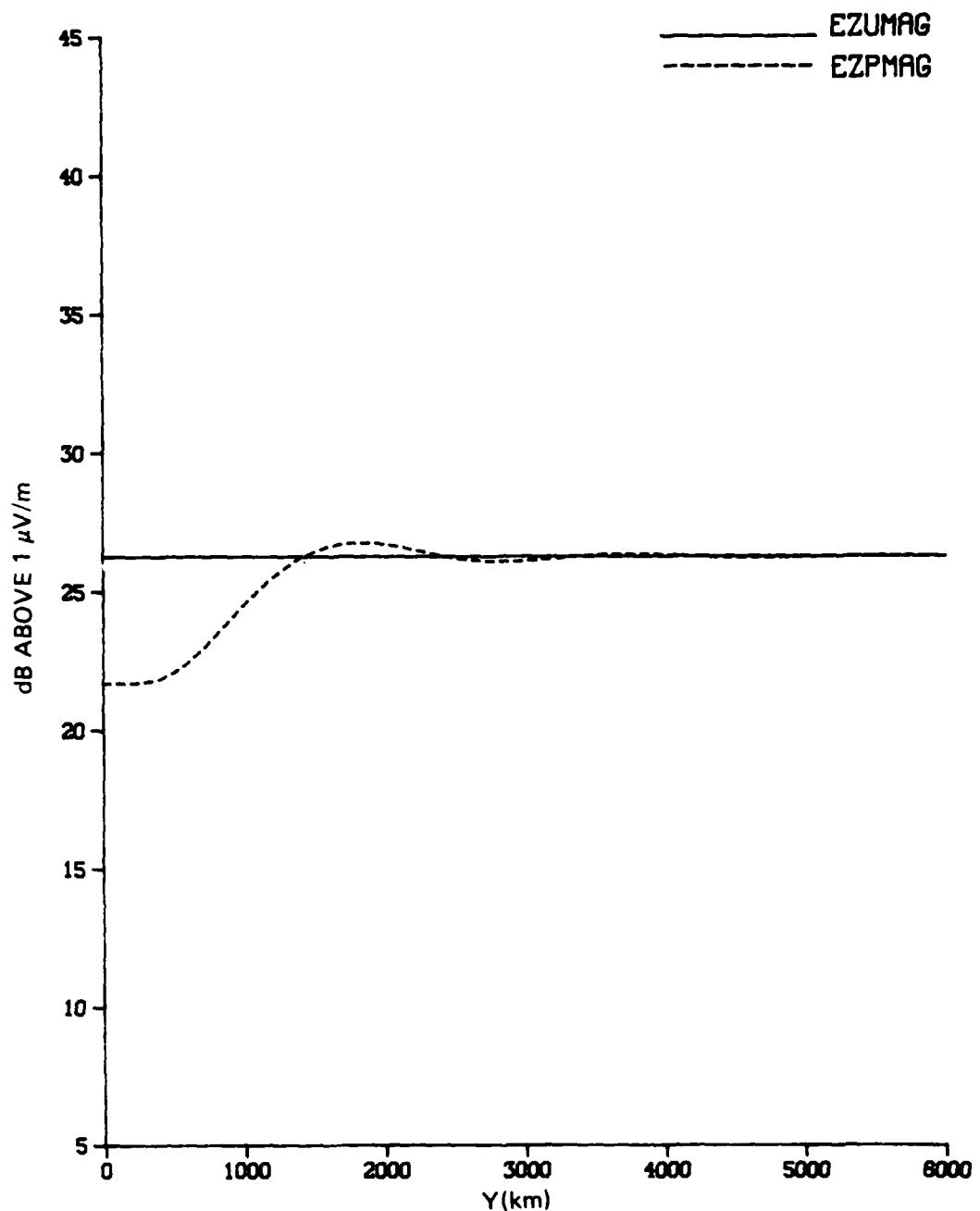
SAMPLE X-VARIATION PLOT
 SQUARE OR RECTANGULAR DISTURBANCE
 FREQ - 0.075
 SIZEX- 500.0 X0- 2000.0 NUMX- 4
 SIZEY- 1000.0 Y0- 0.0 NUMY- 8

Figure 5. Sample x-variation plot for EZUMAG and EZPMAG assuming a rectangular disturbance.



SAMPLE Y-VARIATION PLOT
SQUARE OR RECTANGULAR DISTURBANCE
FREQ - 0.075 DMIN - 2000.0
SIZEX- 1000.0 XO- 1000.0 NUMX- 5
SIZEY- 1000.0 YO- 0.0 NUMY- 5

Figure 6. Sample y-variation plot for WMAG assuming a rectangular disturbance.



SAMPLE Y-VARIATION PLOT
 SQUARE OR RECTANGULAR DISTURBANCE
 FREQ - 0.075 DMIN - 2000.0
 SIZEX- 1000.0 XO- 1000.0 NUMX- 5
 SIZEY- 1000.0 YO- 0.0 NUMY- 5

Figure 7. Sample y-variation plot for EZUMAG and EZPMAG assuming a rectangular disturbance.

VI. PROGRAM CHECKS

Several program checks have been made of flat earth geometry cases which can be solved in terms of well known functions. In each case the disturbance is azimuthally symmetric with the transmitter located at the origin.

The first case considered is that of a uniform circular disturbance for which

$$\begin{aligned} s^2 &= s_p^2 \quad \text{for } r < r_0 \\ s^2 &= s_0^2 \quad \text{for } r > r_0 \end{aligned} \tag{18}$$

For this case the solution for the ratio of the disturbed lateral function, Ψ , to the undisturbed, Ψ^1 is for $r < r_0$

$$\frac{\Psi}{\Psi^1} = \frac{s_p}{s_0 H_1^{(2)}(ks_0 r)} \left\{ \begin{array}{l} H_1^{(2)}(ks_p r) - \\ \frac{(H_1^{(2)}(ks_p r_0) h(ks_0 r_0) - h(ks_p r_0) H_1^{(2)}(ks_0 r_0))}{(J_1(ks_p r_0) h(ks_0 r_0) - j(ks_p r_0) H_1^{(2)}(ks_0 r_0))} J_1(ksr) \end{array} \right\} \tag{19}$$

and for $r > r_0$

$$\frac{\Psi}{\Psi^1} = \frac{s_p}{s_0} \left\{ \frac{H_1^{(2)}(ks_p r_0) j(ks_p r_0) - h(ks_p r_0) J_1(ks_p r_0)}{H_1^{(2)}(ks_0 r_0) j(ks_p r_0) - h(ks_0 r_0) J_1(ks_p r_0)} \right\} \tag{20}$$

In these equations

$$h(x) = \frac{x}{2} [H_0^{(2)}(x) - H_2^{(2)}(x)] \quad (21)$$

$$j(x) = \frac{x}{2} [J_0(x) - J_2(x)] \quad (22)$$

It is clear from equation (20) that ψ/ψ^i is constant for $r > r_0$ as expected. Figure 8 shows the results calculated by using equation (19) along with the moment method results. The radius r_0 is 500 km. The unperturbed eigenangle is $(83.985^\circ, -34.909^\circ)$ or equivalently $S_0 = (1.185, -0.0681i)$ and the perturbed eigenangle is $(59.393^\circ, -65.552^\circ)$ or equivalently $S_p = (1.488, -0.718i)$. S_0 is appropriate to a nighttime ambient ionosphere at 75 Hz, whereas, S_p is appropriate to a nighttime ionosphere with a sporadic E layer⁹. The 13×13 mesh results give agreement to within a few hundredths of a dB of the exact results. In the moments method, the lateral function is calculated at the center points of each square mesh and the lateral function is linearly interpolated between those points. Because of approximations made in the slab containing the transmitter, the first meaningful data point obtained from the moment method is the first point to fall in a slab adjacent to that containing the transmitter. This explains the starting ranges for the moment method results.

A second check case considered is that of a circular disturbance which is uniform out to a radius r_0 , then falls off as $1/r^2$ between r_0 and r_1 , and is equal to S_0 beyond r_1 . The mathematical description of S is

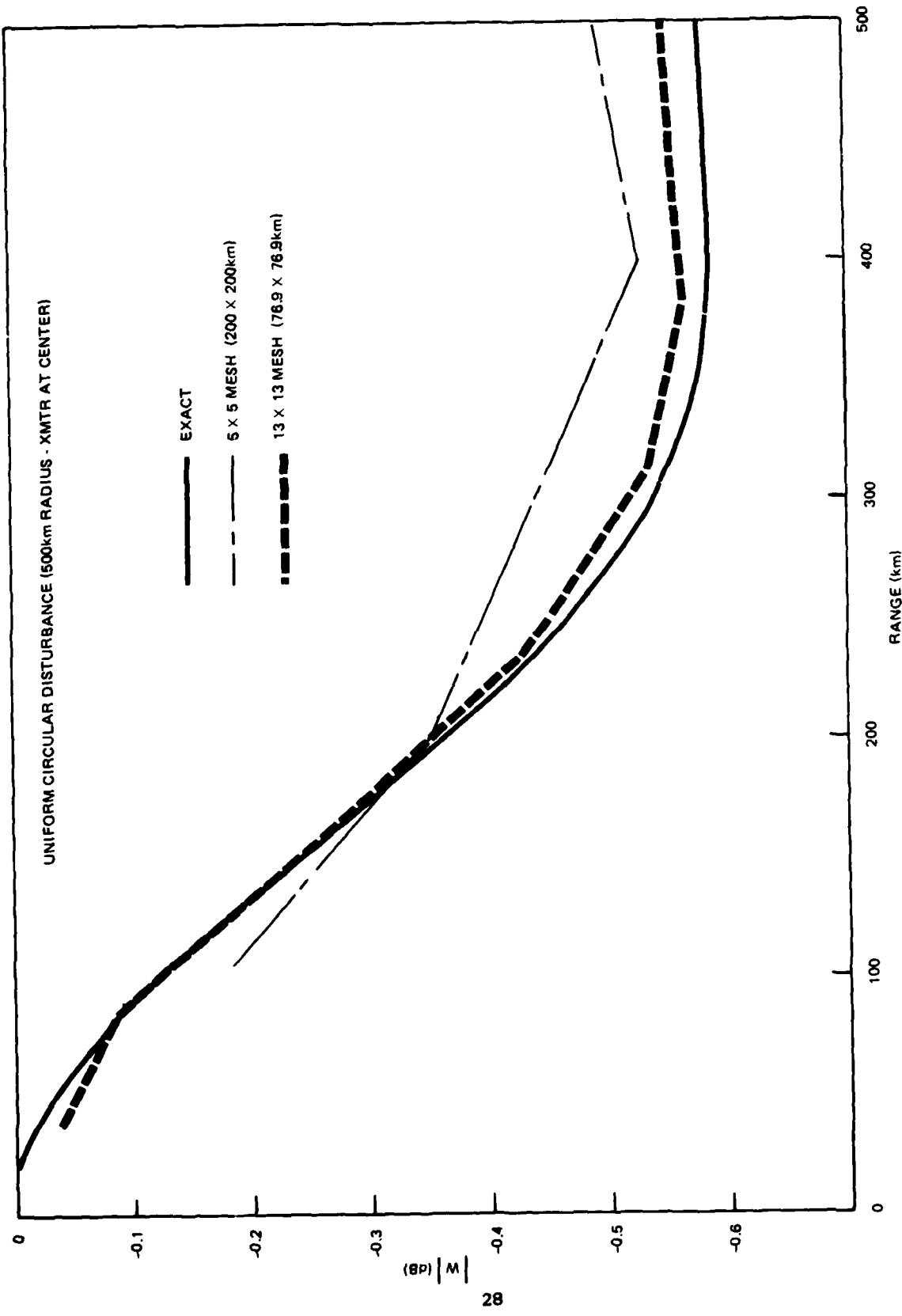


Figure 8. Comparison between analytic solution and the computer program output for problem 1: a uniform circular disturbance.

$$\begin{aligned}
 s^2 &= s_p^2 && ; r < r_0 \\
 s^2 &= s_0^2 + (s_p^2 - s_0^2) \frac{r_0^2}{r^2} && ; r_0 < r < r_1 \\
 s^2 &= s_0^2 && ; r > r_1
 \end{aligned} \tag{23}$$

When the transmitter is at the center of the disturbance the solution for the ratio of the disturbed lateral function, ψ , to the undisturbed, ψ^i , is for
 $r < r_0$

$$\frac{\psi}{\psi^i} = \frac{1}{s_0 H_1^{(2)}(ks_0 r)} [sH_1^{(2)}(ks_p r) + aJ_1(ks_p r)] ; r < r_0 \tag{24}$$

$$\frac{\psi}{\psi^i} = \frac{1}{s_0 H_1^{(2)}(ks_0 r)} [bJ_v(ks_0 r) + dJ_{-v}(ks_0 r)] ; r_0 < r < r_1 \tag{25}$$

$$\frac{\psi}{\psi^i} = \frac{T}{s_0} ; r < r_1 \tag{26}$$

In these equations

$$v = [1 - (s_p^2 - s_0^2) (kr_0)^2]^{1/2} \tag{27}$$

$$a = \frac{1}{\Delta} \left| \begin{array}{cccc} sH_1^{(2)}(ks_p r_0) & J_v(ks_0 r_0) & J_{-v}(ks_0 r_0) & 0 \\ sH_1^{(2)}(ks_p r_0) & J_v(ks_0 r_0) & J_{-v}(ks_0 r_0) & 0 \\ 0 & J_v(ks_0 r_1) & J_{-v}(ks_0 r_1) & -H_1^{(2)}(ks_0 r_1) \\ 0 & J_v(ks_0 r_1) & J_{-v}(ks_0 r_1) & -h_1(ks_0 r_1) \end{array} \right| \tag{28}$$

$$b = \frac{1}{\Delta} \begin{vmatrix} -j_1(k s_p r_0) & sh_1^{(2)}(k s_p r_0) & j_{-v}(k s_0 r_0) & 0 \\ -j_1(k s_p r_0) & sh_1(k s_p r_0) & j_{-v}(k s_0 r_0) & 0 \\ 0 & 0 & j_{-v}(k s_0 r_1) & -h_1^{(2)}(k s_0 r_1) \\ 0 & 0 & j_{-v}(k s_0 r_1) & -h_1(k s_0 r_1) \end{vmatrix} \quad (29)$$

$$d = \frac{1}{\Delta} \begin{vmatrix} -j_1(k s_p r_0) & j_v(k s_0 r_0) & sh_1^{(2)}(k s_p r_0) & 0 \\ -j_1(k s_p r_0) & j_v(k s_0 r_0) & sh_1(k s_p r_0) & 0 \\ 0 & j_v(k s_0 r_1) & 0 & -h_1^{(2)}(k s_0 r_1) \\ 0 & j_v(k s_0 r_1) & 0 & -h_1(k s_0 r_1) \end{vmatrix} \quad (30)$$

$$T = \frac{1}{\Delta} \begin{vmatrix} -j_1(k s_p r_0) & j_v(k s_0 r_0) & j_{-v}(k s_0 r_0) & -sh_1^{(2)}(k s_p r_0) \\ -j_1(k s_p r_0) & j_v(k s_0 r_0) & j_{-v}(k s_0 r_0) & sh_1(k s_p r_0) \\ 0 & j_v(k s_0 r_1) & j_{-v}(k s_0 r_1) & 0 \\ 0 & j_v(k s_0 r_1) & j_{-v}(k s_0 r_1) & -h_1(k s_0 r_1) \end{vmatrix} \quad (31)$$

$$\Delta = \begin{vmatrix} -j_1(k s_p r_0) & j_v(k s_0 r_0) & j_v(k s_0 r_0) & 0 \\ -j_1(k s_p r_0) & j_v(k s_0 r_0) & j_{-v}(k s_0 r_0) & 0 \\ 0 & j_v(k s_0 r_1) & j_{-v}(k s_0 r_1) & -h_1^{(2)}(k s_0 r_1) \\ 0 & j_v(k s_0 r_1) & j_{-v}(k s_0 r_1) & -h_1(k s_0 r_1) \end{vmatrix} \quad (32)$$

$$j_q(x) = \frac{x}{2} [j_{q-1}(x) - j_{q+1}(x)] \quad (33)$$

$$h_q(k) = \frac{x}{2} [H_{q-1}^{(2)}(x) - H_{q+1}^{(2)}(x)] \quad (34)$$

Again it is clear from equation (26) that ψ/ψ^1 is constant for $r > r_1$. Figure 9 shows results for $r_0 = 300$ km, $r_1 = 1000$ km and the same S_0 and S_p used for the results of figure 8. The moment method results are for a 10×10 mesh and the results beyond 100 km agree with the exact values to better than a tenth of a dB.

A third check based on the model described by equations (23) is shown in figure 10. S_0 and S_p are assigned the same values as for figure 8; however, r_0 has been taken equal to 588 km and $r_1 = 2000$ km. The moment method result is for a 17×17 mesh which is the largest the program can handle because of storage limitations. The mesh size in this instance is approximately 235 km. In this connection Hagmann et al¹³, state that the mesh size must be less than $155 \lambda_0$ where λ_0 is the unperturbed wavelength. In the present case of 75 Hz, $\lambda_0 \approx 4000/S_0r \approx 3375$ km. This would give a maximum cell size of ≈ 523 km. It has been our experience that approaching this limit leads to substantial error and it would probably be best at 75 Hz not to exceed mesh sizes of several hundred kilometers. This would limit the linear dimension of the disturbance to something on the order of 5000 km at 75 Hz. Figure 10 shows the agreement between the exact calculation and the moment method to be within a few tenths of a dB. It is also very likely true that the cases checked are some of the most difficult for the moment method to handle because the gradient of the incident field is largest close to the transmitter. Thus, it would be expected that for disturbances remote from the transmitter better accuracy would be obtained for the same mesh size.

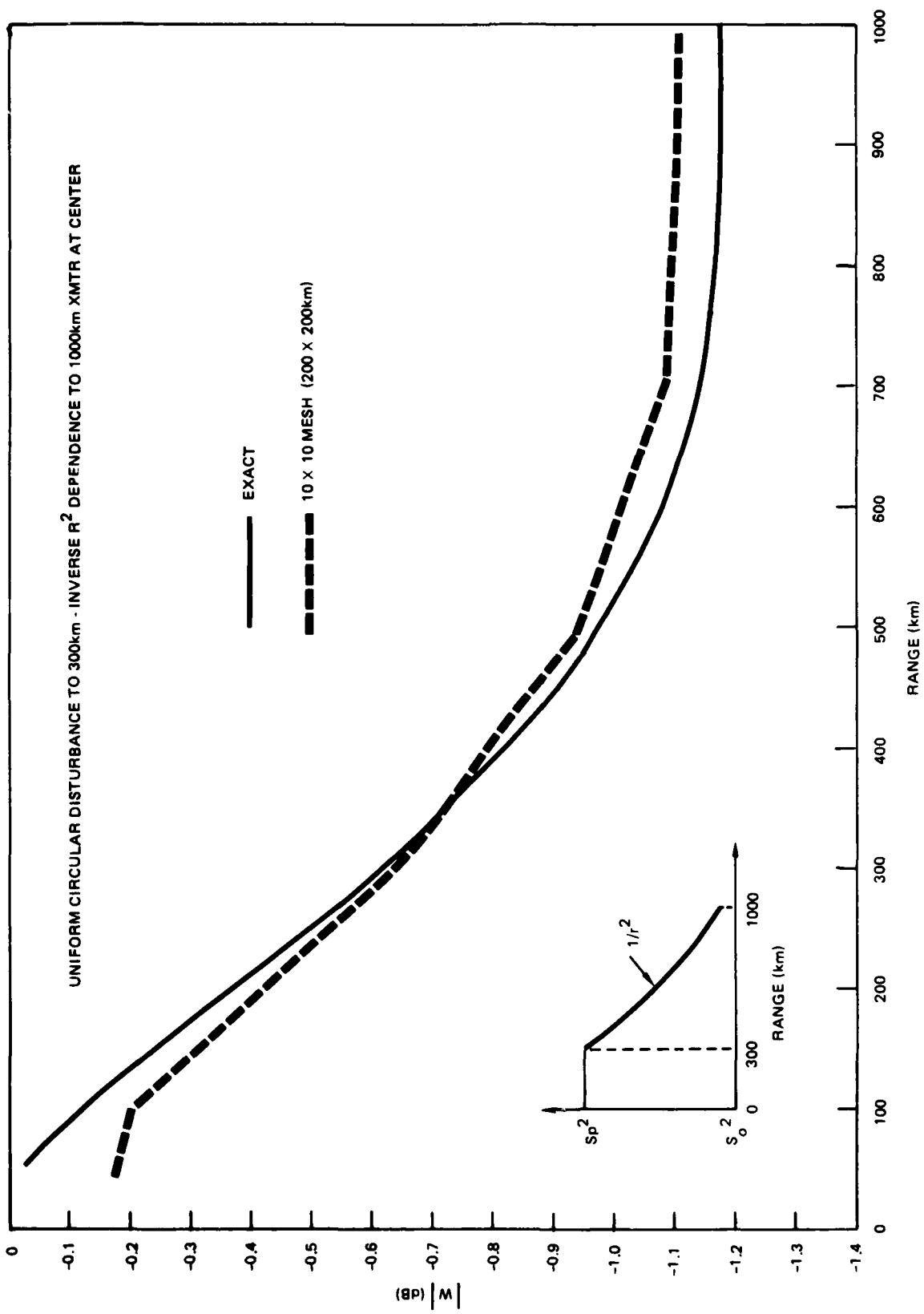


Figure 9. Comparison between analytic solution and the computer program output for problem 2: a uniform circular disturbance to r_0 with an inverse r^2 dependence of S_p between r_0 and r_1 using a 10×10 mesh.

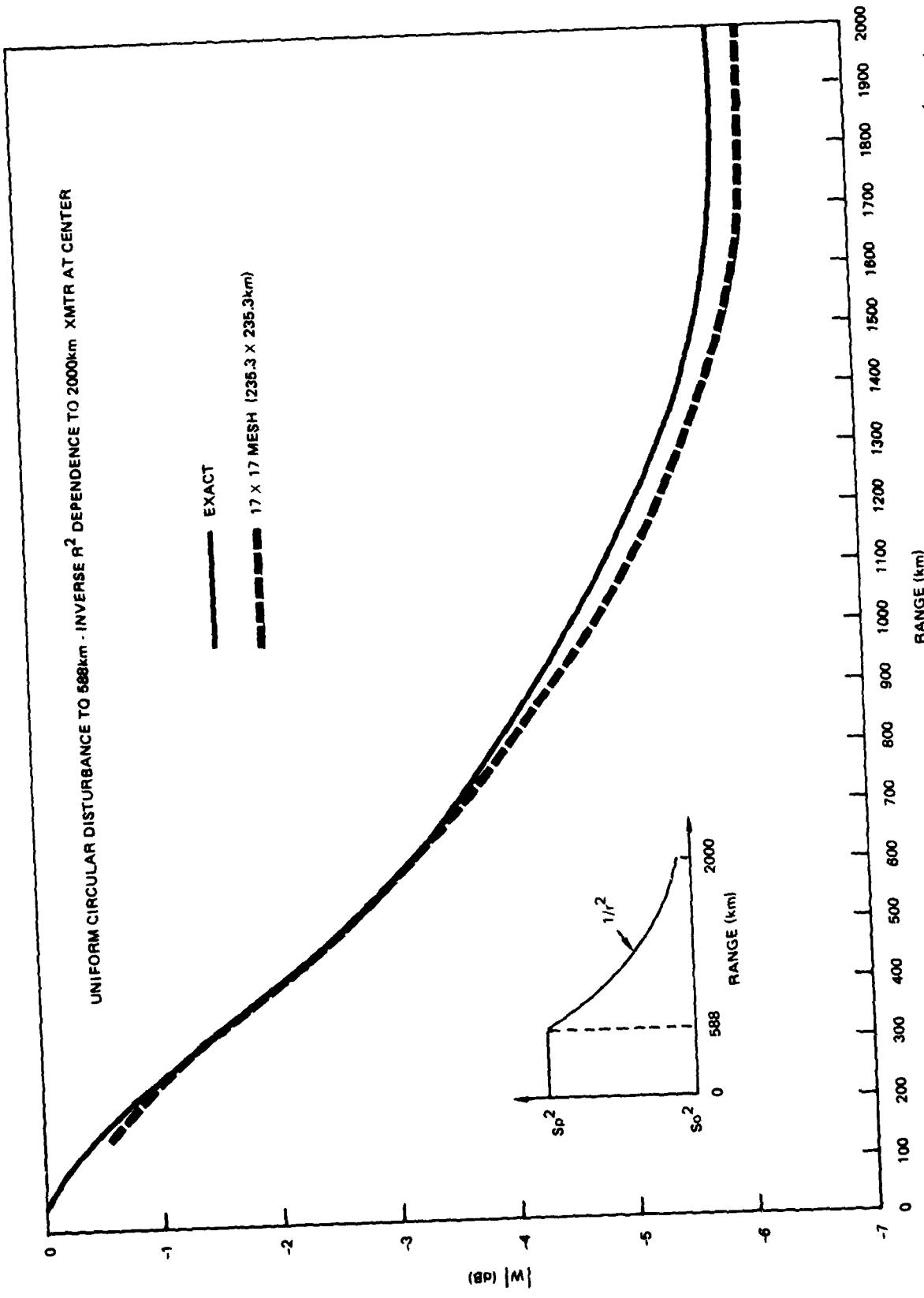


Figure 10. Comparison between analytic solution and the computer program output for problem 3: a uniform circular disturbance to r_0 with an inverse r^2 dependence of S_p between r_0 and r_1 using a 17×17 mesh.

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VIII. APPENDIX: PROGRAM LISTING

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@FTN,USFO    SPORAD.MAIN
FTN 10R1A 09/22/82-11:32(9,10)
1. C
2. C     SPORADIC-E PROGRAM FOR ELF
3. C
4. C     IMPLICIT COMPLEX (A-H,O-Z)
5. C
6. C     PARAMETER MAXNX=12,NDIM=100,NRPTS=500
7. C
8. C     MAXNX IS THE MAX NUMBER OF DISTURBED EIGENS WHICH CAN BE INPUT
9. C     NDIM IS THE MAX VALUE OF NUMX*NUMY
10. C    NRPTS IS THE MAX NUMBER OF POINTS TO PLOT
11. C
12. C     COMPLEX IM/(0.E0,1.E0)/,J1,J2,NGY,IM,K50,NGSQ,
13. $     A(NDIM,NDIM),
14. $     EZ0(NDIM),EZS(NDIM),SD(NDIM),XTRD(NDIM),WI(NDIM),
15. $     SI(MAXNX),XTRI(MAXNX)
16. C
17. C     REAL XM,IM,XGSQ,AI,SIZEY,XDINC,X0,Y0,YOVERX,DELTAX,
18. $     DELTAY,X1,Y1,XINC,YINC,EPSR,RHOMN,EZREL,EZANG,KA,DELY,
19. $     YMAX,F,WMAG,WANG,YMID,OMEGA,RADIUS,WAVENO,FREQ,SIGMA,
20. $     ERH,DIST,DMIN,DM,DELD,DMAX,HXINC,HYINC,RG,AIMAX,DST,
21. $     RGMAX,RGMIN,EZUMAG,EZUANG,EZPMAG,EZPANG,PI,DTR,EPSO,
22. $     XJ(4),YG(4),
23. $     X(NDIM),Y(NDIM),CLNQ(NDIM),XX(NDIM),YY(NDIM),
24. $     XI(MAXNX),YI(MAXNX),
25. $     XPLLOT(NRPTS),Y1PLOT(NRPTS),Y2PLOT(NRPTS),Y3PLOT(NRPTS),
26. $     EXTIC,EYTIC,WXTIC,WYTIC,XMAX,XLNG,YLNG,WMIN,WMAX,EMIN,
27. $     EMAX
28. C
29. C     INTEGER IROW(NDIM)
30. C
31. C     CHARACTER*1 LABEL0/'?'/
32. C     CHARACTER*4 ID(20)
33. C     CHARACTER*5 LABEL1/' ?'/, LABEL2/' ____ ?'/, LABEL3/' ____ '
34. C     CHARACTER*8 KLABEL
35. C     CHARACTER*36 LABEL
36. C
37. C     COMMON/ONE/H12,H22,J1,J2,S0,ARG0,KA,K50
38. C     COMMON/LABELS/ID,LABEL,XLABEL
39. C     COMMON/INPUT/FREQ,DMIN,DMAX,SIZEY,SIZEY,X0,Y0,EXTIC,EYTIC,WXTIC,
40. $     WYTIC,XLNG,YLNG,EMIN,EMAX,WMIN,WMAX,XMAX,NUMX,NUMY,
41. $     IFLAG
42. C     COMMON/PLDATA/XPLOT,Y1PLOT,Y2PLOT,Y3PLOT,NPTS
43. C
44. C     DM - DIPOLE MOMENT IN AMPERE-METERS
45. C     DMIN - MINIMUM RANGE (KM) FOR DISTANCE VARIATION
46. C     DMAX - MAXIMUM RANGE (KM) FOR DISTANCE VARIATION
47. C     DELD - INCREMENT (KM) FOR DISTANCE VARIATION
48. C     YMAX - MAXIMUM OFF-AXIS VALUE (KM) FOR Y VARIATION
49. C     DELY - INCREMENT (KM) FOR Y VARIATION
50. C     IFLAG=1 Y VARIATION
51. C     IFLAG=2 DIST VARIATION
52. C     IGRID = 0 SQUARE OR RECTANGULAR DISTURBANCE
53. C     IGRID = 1 CIRCULAR OR ELLIPTICAL DISTURBANCE
54. C     X0,Y0 - COORDINATES (KM) AT CENTER OF DISTURBANCE
55. C     Y0 IS ALSO INITIAL VALUE FOR Y VARIATION

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56. C      NUMX,NUMY - THE DISTURBANCE IS DIVIDED INTO NUMX GRIDS IN THE
57. C      X DIRECTION AND NUMY GRIDS IN THE Y DIRECTION
58. C      SIZEX,SIZEY - DEFINES THE PHYSICAL SIZE OF THE DISTURBANCE - IT IS
59. C      SIZEX KM BY SIZEY KM
60. C      IPLOT=0  DON'T PLOT
61. C      IPLOT=1  PLOT
62. C      XLNG,YLNG - LENGTH (INCHES) OF X-AXIS AND Y-AXIS RESPECTIVELY
63. C      WMIN,WMAX - MINIMUM AND MAXIMUM (DB) FOR Y-AXIS OF WMAG PLOT
64. C      EMIN,EMAX - MINIMUM AND MAXIMUM (DB) FOR Y-AXIS OF EZ PLOT
65. C      EXTIC,EYTIC - UNITS PER TIC MARK ALONG X AND Y AXIS RESPECTIVELY
66. C      FOR EZ PLOTS
67. C      WXTIC,WYTIC - UNITS PER TIC MARK ALONG X AND Y AXIS RESPECTIVELY
68. C      FOR WMAG PLOTS
69. C
70. C      NAMELIST/DATUM/DM,DMIN,DMAX,DELD,YMAX,DELY,IFLAG,IGRID,
71. $          X0,Y0,NUMX,NUMY,SIZEX,SIZEY,IPLOT,XLNG,YLNG,
72. $          WMIN,WMAX,EMIN,EMAX,EXTIC,EYTIC,WXTIC,WYTIC
73. C
74. DATA IPLOT/0/
75. DATA PI/3.14159265358979E0/,DTR/.017453292E0/,EPS0/B.85434E-12/
76. DATA XLNG/5./,YLNG/6./,WMIN/-6./,WMAX/2./,EMIN/-6./,EMAX/2./
77. DATA EXTIC/200./,EYTIC/5./,SIZEX/1.E3/,SIZEY/1.E3/
78. DATA WXTIC/200./,WYTIC/1./
79. DATA DMIN/25.E0/,DMAX/1000.E0/,DELD/25.E0/
80. DATA YMAX/500.E0/,DELY/25.E0/,X0/0.E0/,Y0/0.E0/
81. DATA DM/6.75E6/
82. DATA IFLAG/2/,IGRID/0/
83. DATA CONST/(-.707106781186548E0,-.707106781186548E0)/
84. C
85. C
86. DEFINE CAPG2(ARG) = PI/(2.0E0* SIN(ARG/6371.0E0))
87. C
88. PRINT 915
89. READ(5,DATUM)
90. WRITE(6,DATUM)
91. C
92. NU = NUMX*NUMY
93. IF(NU .GT. NDIM)
94. $  THEN
1 95.     PRINT 971
1 96.     STOP
1
97. ENDIF
98. NPTS = 0
99. DIST = DMIN
100. YMID = Y0
101. C
102. C      SET UP DISTURBED REGION GRID
103. XINC = SIZEX/NUMX
104. HXINC = .5*XINC
105. YINC = SIZEY/NUMY
106. HYINC = .5*YINC
107. C
108. C      GRID ELEMENTS MUST BE SQUARE
109. IF(XINC .NE. YINC)
110. $  THEN
1 111.     PRINT 976
1 112.     STOP

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1      113.      ENDIF
1      114.      YOVERX = SIZEY/SIZEX
1      115.      X1 = X0-0.5E0*(SIZEX-XINC)
1      116.      Y1 = Y0+0.5E0*(SIZEY-YINC)
1      117.      RADIUS = SQRT((XINC*YINC)/PI)
1      118.      XX(1) = X1
1      119.      DO 12 I=2,NUMX
1      120.      12 XX(I) = XX(I-1)+XINC
1      121.      YY(1) = Y1
1      122.      DO 13 J=2,NUMY
1      123.      13 YY(J) = YY(J-1)-YINC
1      124.      M = 0
1      125.      DO 15 J=1,NUMY
1      126.      DO 15 I=1,NUMX
2      127.      M = M+1
2      128.      X(M) = XX(I)
2      129.      Y(M) = YY(J)
2      130.      15 CONTINUE
2      131.      C
2      132.      C      PRINT SCHEMATIC OF DISTURBED REGION GRID
1      133.      IF(NUMY .GT. 10) PRINT 915
1      134.      PRINT 920
1      135.      PRINT 930,(LABEL3,J=1,NUMX)
1      136.      DO 18 K=1,NUMY
1      137.      PRINT 910,LABEL0,(LABEL1,J=1,NUMX)
1      138.      PRINT 940,LABEL0,((K-1)*NUMX+J,LABEL0,J=1,NUMX)
1      139.      PRINT 910,LABEL0,(LABEL1,J=1,NUMX)
1      140.      PRINT 910,LABEL0,(LABEL2,J=1,NUMX)
1      141.      18 CONTINUE
1      142.      C
1      143.      C      PRINT COORDINATES OF GRID
1      144.      PRINT 920
1      145.      N = 1
1      146.      WRITE(6,100) N,X(N),Y(N)
1      147.      WRITE(6,100) NUMX,X(NUMX),Y(NUMX)
1      148.      N = NU-NUMX+1
1      149.      WRITE(6,100) N,X(N),Y(N)
1      150.      WRITE(6,100) NU,X(NU),Y(NU)
151.      C
152.      C      READ GRID DATA
153.      C
154.      C      MODE PARAMETERS ARE FROM USING NPUNCH=1 IN
155.      C      THE WAVEGUID OR MODESRCH COMPUTER PROGRAMS
156.      C      READ 1000,10
157.      C      PRINT 1001,10
158.      C
159.      C      AMBIENT
160.      C      READ 1010,FREQ,SIGMA,EPSR
161.      C      PRINT 1011,FREQ,SIGMA,EPSR
162.      C      OMEGA=6.28318530717959E3*FREQ
163.      C      WAVENO = .020958445E0*FREQ
164.      C      NGSQ = SIGMA/(IM*OMEGA*EPS0)+EPSR
165.      C      NGXTM = C SQRT(NGSQ)
166.      C      IF(DREAL(NGXTM) .LT. 0.0E0) NGXTM=-NGXTM
167.      C      EX0 = 1.0E0/NGXTM
168.      C      ECNST=.03248E6*DM*EX0*WAVENO**2/(5.0E3* SQRT(FREQ))
169.      C      N = 0

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170.      PRINT 1009
171.      READ 1020,THETA0,TMP1
172.      THTAO = THETA0*DTR
173.      S0 = C SIN(THTAO)
174.      CSQ = (1.E0,0.E0)-S0**2
175.      C = C SQRT(CSQ)
176.      SQROOT = C SQRT(NGSQ-CSQ)
177.      XTRA0 = -4.0*TMP1*S0
178.      PRINT 1021,N,THETA0,XTRA0
179.      KA = WAVENO*RADIUS
180.      KSO = WAVENO*S0
181.      ARGO = KSO*RADIUS
182.      CALL CBESJY(ARGO,1,BJ,BY,0,0)
183.      H12 = BJ- IM*BY
184.      J1 = BJ
185.      CALL CBESJY(ARGO,2,BJ,BY,0,0)
186.      H22 = BJ-IM*BY
187.      J2 = BJ
188.      C
189.      C DISTURBED
190.      25 READ 1010,F
191.      IF(F .NE. 0.)
192.      $ THEN
1 193.      N = N+1
1 194.      IF(N .EQ. MAXNX)
1 195.      $ THEN
2 196.          PRINT 974
2 197.          STOP
2 198.      ENDIF
1 199.      READ 1020,THETAP,TMP1
1 200.      THTAP = THETAP*DTR
1 201.      SP = C SIN(THTAP)
1 202.*     XTRAP=-4.0*TMP1*SP
1 203.      PRINT 1021,N,THETAP,XTRAP
1 204.      SI(N) = SP
1 205.      XTRI(N) = XTRAP
1 206.      GO TO 25
1 207.      ELSE
1 208.          NMAX = N
1 209.      END IF
1 210.      C
211.      IF(NMAX .EQ. 1)
212.      $ THEN
213.          C UNIFORM DISTURBANCE
1 214.          NMAX = 2
1 215.          SI(2) = SP
1 216.          XTRI(2) = XTRAP
1 217.      ENDIF
1 218.      C
1 219.      SET UP INTERPOLATION ARRAYS
220.      XDINC = 0.5E0*SIZEX/(NMAX-1)
221.      DO 33 N=1,NMAX
1 222.      XI(N) = XDINC*(N-1)
1 223.      33 YI(N) = YOVERX*XI(N)
1 224.      C
1 225.      FILL GRID OF SD AND XRD
1 226.      IF(IGRID .EQ. 0)

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227.      $ THEN
228.      C          SQUARE OR RECTANGLE
229.      C
230.      C          A RECTANGLE CAN ONLY BE UNIFORMLY DISTURBED
231.      C          A SQUARE CAN HAVE A NON-UNIFORM DISTURBANCE
1       232.      C          LABEL = 'SQUARE OR RECTANGULAR DISTURBANCE '
1       233.      DO 39 M=1,NU
2       234.      XM = ABS(X(M)-X0)
2       235.      YM = ABS(Y(M)-Y0)
2       236.      IF(XM .GT. .5E0*SIZEX .OR. YM .GT. .5E0*SIZYE)
2       237.      $ THEN
3       238.          SD(M) = S0
3       239.          XTRD(M) = XTRA0
3       240.      ELSE
3       241.          I = 1
3       242.          K = 1
3       243.      C          35
3       244.          IF(XM .LE. XI(I+1)) GO TO 37
3       245.          I = I+1
3       246.          GO TO 35
3       247.      37          IF(YM .LE. YI(K+1)) GO TO 38
3       248.          K = K+1
3       249.          GO TO 37
3       250.      38          IF(I .GT. K)
3       251.      $ THEN
4       252.          SLOPE = (XM-XI(I))/(XI(I+1)-XI(I))
4       253.          SD(M) = SI(I)+(SI(I+1)-SI(I))*SLOPE
4       254.          XTRD(M) = XTRI(I)+(XTRI(I+1)-XTRI(I))*SLOPE
4       255.      ELSE
4       256.          SLOPE = (YM-YI(K))/(YI(K+1)-YI(K))
4       257.          SD(M) = SI(K)+(SI(K+1)-SI(K))*SLOPE
4       258.          XTRD(M) = XTRI(K)+(XTRI(K+1)-XTRI(K))*SLOPE
4       259.      ENDIF
3       260.      ENDIF
3       261.      C          CONTINUE
2       262.      39
2       263.      C
1       264.      ELSE
1       265.      C          ELLIPSE OR CIRCLE
1       266.      LABEL = 'CIRCULAR OR ELLIPTICAL DISTURBANCE '
1       267.      AIMAX = XI(NMAX)
1       268.      DO 59 M=1,NU
1       269.      C          COORDS OF CENTER OF GRID RELATIVE TO CENTER OF DISTURBANCE
2       270.      XM = X(M)-X0
2       271.      YM = Y(M)-Y0
2       272.      C          COORDS OF CORNERS OF GRID
2       273.      XG(1) = XM-HXINC
2       274.      XG(2) = XM+HXINC
2       275.      XG(3) = XG(1)
2       276.      XG(4) = XG(2)
2       277.      YG(1) = YM-HYINC
2       278.      YG(2) = YG(1)
2       279.      YG(3) = YM+HYINC
2       280.      YG(4) = YG(3)
2       281.      RGMAX = -1.0E6
2       282.      RGMIN = 1.0E6
2       283.      DU 45 I=1,4

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2     284. C      RANGE OF EACH CORNER FROM CENTER OF DISTURBANCE
3     285.          RG = SQRT(XG(I)**2+(YG(I)/YOVERX)**2)
3     286.          RGMAX = AMAX1(RGMAX,RG)
3     287.    45      RGMIN = AMIN1(RGMIN,RG)
2     288.          IF(RGMIN .GE. AIMAX)
2     289.      $      THEN
2     290. C      GRID IS COMPLETELY OUTSIDE OF DISTURBANCE
3     291.          SD(M) = S0
3     292.          XTRD(M) = XTRA0
3     293.          ELSE
3     294.          IF(RGMAX .LE. AIMAX)
3     295.      $      THEN
3     296. C      GRID IS COMPLETELY INSIDE OF DISTURBANCE
4     297.          I = 1
4     298.          AI = SQRT(XM**2+(YM/YOVERX)**2)
4     299.    50      IF(AI .LE. XI(I+1)) GO TO 51
4     300.          I = I+1
4     301.          GO TO 50
4     302.    51      SLOPE = (AI-XI(I))/(XI(I+1)-XI(I))
4     303.          SD(M) = SI(I)+(SI(I+1)-SI(I))*SLOPE
4     304.          XTRD(M) = XTRI(I)+(XTRI(I+1)-XTRI(I))*SLOPE
4     305.          ELSE
4     306. C      GRID IS ON BORDER OF THE DISTURBANCE
4     307. C      DIVIDE GRID INTO 16 SUBSQUARES
4     308.          DELTAX = XINC/8.
4     309.          DELTAY = YINC/8.
4     310.          XG(1) = (XM-DELTAX*3.)*2
4     311.          XG(2) = (XM-DELTAX )**2
4     312.          XG(3) = (XM+DELTAX )**2
4     313.          XG(4) = (XM+DELTAX*3.)*2
4     314.          YG(1) = ((YM+DELTAY*3.)/YOVERX)**2
4     315.          YG(2) = ((YM+DELTAY )/YOVERX)**2
4     316.          YG(3) = ((YM-DELTAY )/YOVERX)**2
4     317.          YG(4) = ((YM-DELTAY*3.)/YOVERX)**2
4     318. C      COUNT NUMBER OF SUBSQUS WITHIN DISTURBANCE
4     319.          N = 0
4     320.          DO 55 I=1,4
4     321.          XGSQ = XG(I)
4     322.          DO 55 J=1,4
4     323.          RG = SQRT(XGSQ+YG(J))
4     324.    55      IF(RG .LE. AIMAX) N=N+1
4     325.          SLOPE = N/16.
4     326.          SD(M) = S0+(SI(NMAX)-S0)*SLOPE
4     327.          XTRD(M) = XTRA0+(XTRI(NMAX)-XTRA0)*SLOPE
4     328.          ENDIF
3     329.          ENDIF
2     330.    59      CONTINUE
1     331.          ENDIF
1     332. C      SET UP A
1     333. C      DO 55 M=1,NU
1     334.          DO 65 N=1,NU
1     335.          A(M,N) = CAPA(SD(N),SQRT((X(M)-X(N))**2+(Y(M)-Y(N))**2))
2     336.    65
2     337. C      DO 79 M= 1,NU
1     338.    70      MGRID = 0
1     339.          DO 79 M= 1,NU
1     340.          NGRID = 0

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```

1      341.      XM = X(M)
1      342.      YM = Y(M)
1      343.      IF(XM+ HXINC .LT. 0.0E0 .OR. XM- HXINC .GE. 0.0E0 .OR.
1      344.      $ YM+ HYINC .LT. 0.0E0 .OR. YM- HYINC .GE. 0.0E0) GO TO 75
1      345.      C      XMTR IS INSIDE GRID
1      346.      C      GRID POINT IS DISTURBED
1      347.      XG(1) = XM-.25*XINC
1      348.      XG(3) = XG(1)
1      349.      XG(2) = XM+.25*XINC
1      350.      XG(4) = XG(2)
1      351.      YG(1) = YM+.25*YINC
1      352.      YG(2) = YG(1)
1      353.      YG(3) = YM-.25*YINC
1      354.      YG(4) = YG(3)
1      355.      MGRID = M
1      356.      PRINT 955,M
1      357.      NQUAD = 0
1      358.      SUM = 0.0E0
1      359.      71      NQUAD = NQUAD+1
1      360.      IF(XG(NQUAD)+.125*XINC .GE. 0.0E0 .AND.
1      361.      $ XG(NQUAD)-.125*XINC .LT. 0.0E0 .AND.
1      362.      $ YG(NQUAD)+.125*YINC .GE. 0.0E0 .AND.
1      363.      $ YG(NQUAD)-.125*YINC .LT. 0.0E0) GO TO 71
1      364.      NGRID = NGRID+1
1      365.      XM = XG(NQUAD)
1      366.      YM = YG(NQUAD)
1      367.      75      RHOIN = SQRT(XM**2+YM**2)
1      368.      ARGU = K50*RHOIN
1      369.      CALL CBESJY(ARGU,1,BJ,BY,0,0)
1      370.      H12 = BJ-IM+BY
1      371.      IF(XM .EQ. 0.0E0) XM=1.0E-6
1      372.      EZ0(M) = C SQRT(ARGU*CAPG2(RHOIN))+CONST*H12*XM/RHOIN
1      373.      IF(NGRID .EQ. 0) GO TO 78
1      374.      SUM = SUM+EZ0(M)
1      375.      IF(NQUAD .LT. 4) GO TO 71
1      376.      EZ0(M) = SUM/NGRID
1      377.      78      IF(C ABS(EZ0(M)) .LE. 1.0E-6) EZ0(M)=1.0E-21
1      378.      79      CONTINUE
1      379.      C
1      380.      C      PRINT TABLE OF EZ0
1      381.      WRITE (6,110) YMID
1      382.      WRITE (6,113)
1      383.      DO 81 I=1,NUMY
1      384.      WRITE(6,112) (C ABS(EZ0(NUMX*(I-1)+J)),J=1,NUMX)
1      385.      81      CONTINUE
1      386.      C
1      387.      C      SOLVE FOR SCATTERED FIELDS
1      388.      CALL CLINEQ(A,EZ0,EZS,IROW,CLNQ,NU,NDIM,NPTS,ERR)
1      389.      C
1      390.      /      PRINT TABLE OF EZS
1      391.      WRITE (6,111)
1      392.      DO 83 I=1,NUMY
1      393.      WRITE(6,112) (C ABS(EZS(NUMX*(I-1)+J)),J=1,NUMX)
1      394.      83      CONTINUE
1      395.      C
1      396.      DO 85 M=1,NU
1      397.      RATIO = EZS(M)/EZ0(M)

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1      398.      IF(MGRID .EQ. M) RATIO=1.0
1      399.      WI(M) = C LOG(RATIO)
1      400.      85      CONTINUE
1      401.      C
1      402.      C      PRINT TABLE OF WI
1      403.      WRITE(6,102)
1      404.      DO 87 I=1,NUMY
1      405.      WRITE(6,103) (REAL(WI(NUMX*(I-1)+J))*8.686,J=1,NUMX)
1      406.      87      CONTINUE
1      407.      C
1      408.      WRITE (6,950)
1      409.      DIST = DMIN
1      410.      90      NPTS = NPTS+1
1      411.      IF(NPTS .GT. NRPTS)
1      412.      $      THEN
1      413.          PRINT 973
1      414.          STOP
1      415.      ENDIF
1      416.      C
1      417.      ARGU = KSO*DIST
1      418.      CALL CBESUY(ARGU,1,BJ,BY,0,0)
1      419.      H12 = BJ-IM*BY
1      420.      EZU = C SQRT(ARGU+CAPG2(DIST))*CONST*H12
1      421.      IF(XX(1)-HXINC .GE. DIST .OR. XX(NUMX)+HXINC .LE. DIST .OR.
1      422.      $      YY(1)+HYINC .LE. 0.E0 .OR. YY(NUMY)-HYINC .GE. 0.E0)
1      423.      $      THEN
1      424.      C      RCVR IS OUTSIDE OF DISTURBED AREA
1      425.          XTRA2 = XTRA0
1      426.          SUM = (0.0E0,0.0E0)
1      427.          DO 95 N=1,NU
1      428.          RHOMN = SQRT((DIST-X(N))**2+Y(N)**2)
1      429.          SUM = SUM+CAPA(SD(N),RHOMN)*EZS(N)
2      430.      95      CONTINUE
1      431.          EZ = EZU-SUM
1      432.          RATIO = EZ/EZU
1      433.          WMAG = 20.0E0+ ALOG10(C ABS(RATIO))
1      434.          WANG = C ANG(RATIO)
1      435.      ELSE
1      436.      C      RCVR IS IN DISTURBED AREA
1      437.          I = 1
1      438.          DO 96 J=1,NUMX
2      439.          IF(XX(J)-HXINC .LT. DIST .AND. DIST .LE. XX(J)+HXINC)
2      440.          $      GO TO 97
2      441.          I = I+1
2      442.      96      CONTINUE
1      443.      97      K = I
1      444.          DO 98 J=1,NUMY
2      445.          IF(YY(J)-HYINC .LT. 0.E0 .AND. 0.E0 .LE. YY(J)+HYINC)
2      446.          $      GO TO 99
2      447.          K = K+1
2      448.      98      CONTINUE
1      449.      99      M = (K-1)*NUMX+I
1      450.          XTRA2 = XTRD(M)
1      451.          EZ = EZU
1      452.          IF(M .EQ. MGRID) GO TO 130
1      453.          IF(I .GT. 1) GO TO 120
1      454.          N1 = M

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1   455.      N2 = M+1
1   456.      IF(K .GT. 1) GO TO 106
1   457. 105    N3 = N1+NUMX
1   458.      N4 = N3+1
1   459.      GO TO 125
1   460. 106    IF(K .EQ. NUMY) GO TO 108
1   461. 107    IF(YY(K ) .GE. 0.) GO TO 109
1   462. 108    N3 = N1-NUMX
1   463.      N4 = N3+1
1   464.      GO TO 125
1   465. 109    N3 = N1+NUMX
1   466.      N4 = N3+1
1   467.      GO TO 125
1   468. 120    IF(I .LT. NUMX) GO TO 122
1   469.      N1 = M-1
1   470.      N2 = M
1   471. 121    IF(K .EQ. 1) GO TO 105
1   472.      IF(K .EQ. NUMY) GO TO 108
1   473.      GO TO 107
1   474. 122    IF(XX(I) .LE. DIST) GO TO 123
1   475.      N1 = M-1
1   476.      N2 = M
1   477.      GO TO 121
1   478. 123    N1 = M
1   479.      N2 = M+1
1   480.      GO TO 121
1   481. 125    SLOPE = -Y(N1)/(Y(N3)-Y(N1))
1   482.      W1 = WI(N1)+(WI(N3)-WI(N1))*SLOPE
1   483.      W2 = WI(N2)+(WI(N4)-WI(N2))*SLOPE
1   484.      IF(I .EQ. NUMX .AND. DIST .GT. X(N2) .OR.
1   485.      I .EQ. 1 .AND. DIST .LT. X(N1))
1   486.      THEN
2   487.          IF(I .EQ. 1)
2   488.              THEN
3   489.                  DST = X0-.5*SIZEX
3   490.                  DELTAX = HXINC
3   491.                  W2 = W1
3   492.                  N2 = N1
3   493.          ELSE
3   494.              DST = X0+.5*SIZEX
3   495.              DELTAX = -HXINC
3   496.          ENDIF
2   497.          ARGU = KSO*DST
2   498.          CALL CBESJY(ARGU,1,BJ,BY,0,0)
2   499.          H12 = BJ-IM*BY
2   500.          EZ1 = C SQRT(ARGU*CAPG2(DST))*CONST*H12
2   501.          SUM = (0.0E0,0.0E0)
2   502.          DO 126 N=1,NU
3   503.          RHOMN = SQRT((DST -X(N))**2+Y(N)**2)
3   504.          SUM = SUM+CAPA(SD(N),RHOMN)*EZS(N)
3   505. 126    CONTINUE
2   506.          W1 = CLOG((1.0,0.0)-SUM/EZ1)
2   507.          RATIO = W1+(W2-W1)*(DIST-DST)/DELTAX
2   508.          ELSE
2   509.              RATIO = W1+(W2-W1)*((DIST-X(N1))/XINC)
2   510.          ENDIF
1   511.          WMAG = REAL(RATIO)*8.686

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1      512.          WANG = AIMAG(RATIO)
1      513.          EZ = C EXP(RATIO)*EZU
1      514.          ENDIF
1      515.          C
1      516.          C
1      517.          130 IF(MGRID .EQ. 0)
1      518.          $ THEN
1      519.          C           KMTR IS OUTSIDE OF DISTURBED AREA
1      520.          XTRA1 = XTRA0
1      521.          ELSE
1      522.          C           XMTR IS IN DISTURBED AREA
1      523.          XTRA1 = XTRD(MGRID)
1      524.          ENDIF
1      525.          C
1      526.          EZUABS = ECONST*EZU*XTRA0
1      527.          EZPABS = ECONST*EZ*C SQRT(XTRA1*XTRA2)
1      528.          RATIO = EZPABS/EZUABS
1      529.          EZREL = 20.0E0 ALOG10(C ABS(RATIO))
1      530.          EZANG = C ANG(RATIO)
1      531.          EZUMAG = 20.0E0 ALOG10(C ABS(EZUABS))
1      532.          EZUANG = C ANG(EZUABS)
1      533.          EZPMAG = 20.0E0 ALOG10(C ABS(EZPABS))
1      534.          EZPANG = C ANG(EZPABS)
1      535.          WRITE (6,960) YMID,DIST,WMAG,WANG,EZREL,EZANG,
1      536.          $           EZUMAG,EZUANG,EZPMAG,EZPANG
1      537.          Y1PLOT(NPTS) = WMAG
1      538.          Y2PLOT(NPTS) = EZUMAG
1      539.          Y3PLOT(NPTS) = EZPMAG
1      540.          IF(IFLAG .EQ. 2) GO TO 140
1      541.          C
1      542.          C           Y VARIATION
1      543.          DO 131 N = 1,NU
1      544.          131 Y(N) = Y(N)+DELY
1      545.          DO 132 J=1,NUMY
1      546.          132 YY(J) = YY(J)+DELY
1      547.          XPLOT(NPTS) = YMID
1      548.          XLABEL = ' Y(KM) '
1      549.          YMID = YMID+DELY
1      550.          IF(YMID .LE. YMAX) GO TO 70
1      551.          XMAX = YMAX
1      552.          IF(IPLOT .EQ. 1) GO TO 800
1      553.          STOP
1      554.          C
1      555.          C           DISTANCE VARIATION
1      556.          140 XPLOT(NPTS) = DIST
1      557.          XLABEL = ' X(KM) '
1      558.          DIST = DIST + DELD
1      559.          IF(DIST .LE. DMAX) GO TO 90
1      560.          XMAX = DMAX
1      561.          IF(IPLOT .EQ. 1) GO TO 800
1      562.          STOP
1      563.          C
1      564.          C           PLOTTING
1      565.          800 CALL DBPLDT
1      566.          C
1      567.          100 FORMAT(' COORDINATES AT CENTER OF MESH NUMBER ',I3,' ARE: X=',,
1      568.          $           F8.2,' Y=',F8.2)

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569. 102 FORMAT(/,10X,'WI')
570. 103 FORMAT(1X,13F10.4)
571. 110 FORMAT(1H0,'YMID = ',F12.2)
572. 111 FORMAT(/,10X,'Ezs')
573. 112 FORMAT((1X,(1P13E10.2)))
574. 113 FORMAT(/,10X,'Ez0')
575. 910 FORMAT(20X,A1,13A5)
576. 915 FORMAT('1')
577. 920 FORMAT('0')
578. 930 FORMAT(21X,13A5)
579. 940 FORMAT(20X,A1,13(I3,1X,A1))
580. 950 FORMAT(/,4X,'YMID',5X,'DIST',6X,'WMAG ',8X,'WANG ',8X,'EZREL',
581.      $ 8X,'EZANG',8X,'EZUMAG',8X,'EZUANG',8X,'EZPMAG',8X,'EZPANG')
582. 955 FORMAT('0XMTR IS INSIDE GRID ',I3)
583. 960 FORMAT(2(2X,F7.1),8(2X,1PE12.5))
584. 971 FORMAT(' NU = NUMX*NUMY IS GREATER THAN PARAMETER VARIABLE NDIM')
585. 972 FORMAT(' NMAX DOES NOT EQUAL NU OR 1')
586. 973 FORMAT(' NUMBER OF POINTS PLOTTED IS GREATER THAN PARAMETER VARIAB
587.      $ LE NRPTS')
588. 974 FORMAT(' N IS GREATER THAN PARAMETER VARIABLE MAXNX')
589. 976 FORMAT(' XINC MUST EQUAL YINC')
590. 1000 FORMAT(20A4)
591. 1001 FORMAT('1',20A4)
592. 1009 FORMAT(2X,'GRID',6X,'THETA',25X,'XTRA')
593. 1010 FORMAT(10X,E8.0,34X,E10.0,2X,E5.0)
594. 1011 FORMAT(' FREQ = ',E10.3,' SIGMA = ',E10.3,' EPSR=',F7.2)
595. 1020 FORMAT(1X,2I9.0,1X,2E15.0//)
596. 1021 FORMAT(1X,13.2F10.5,2X,1P2E18.9)
597. END

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END FTN 2678 IBANK 22972 DBANK 2068 COMMON

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1      SUBROUTINE DBPLOT
2      PARAMETER NRPTS=500
3      CHARACTER*8 XLABEL
4      CHARACTER*36 LABEL
5      CHARACTER*4 ID(20)
6      DIMENSION XPLOT(NRPTS),Y1PLOT(NRPTS),Y2PLOT(NRPTS),Y3PLOT(NRPTS)
7      COMMON/LABELS/ID,LABEL,XLABEL
8      COMMON/INPUT/FREQ,DMIN,DMAX,SIZEX,SIZEY,X0,Y0,EXTIC,EYTIC,WXTIC,
9      WYTIC,XLNG,YLNG,EMIN,EMAX,WMIN,WMAX,XMAX,NUMX,NUMY,
10     $ IFLAG
11     $ COMMON/PLDATA/XPLOT,Y1PLOT,Y2PLOT,Y3PLOT,NPTS
12     CALL BGNPL(1)
13     CALL PHYSOR(2.0,2.0)
14     CALL TITLE(' ',1,XLABEL,8,'WMAG(DB)',8,XLNG,YLNG)
15     CALL INTAXS
16     CALL YAXANG(0.0)
17     CALL GRAF(0.,WXTIC,XMAX,WMIN,WYTIC,WMAX)
18     CALL CURVE(XPLOT,Y1PLOT,NPTS,0)
19     CALL MESSAG(ID,80,0.0,-0.8)
20     CALL MESSAG(LABEL,36,0.0,-1.0)
21     CALL MESSAG('FREQ = ',7,0.0,-1.2)
22     CALL REALNO( FREQ,3.0,7,-1.2)
23     IF(IFLAG .EQ. 1) CALL MESSAG('DMIN = ',6,2.0,-1.2)
24     IF(IFLAG .EQ. 1) CALL REALNO( DMIN,1.2,7,-1.2)
25     CALL MESSAG('SIZEX=          X0=          NUMX= ',32,0.0,-1.4)
26     CALL REALNO( SIZEX,1.0,7,-1.4)
27     CALL REALNO( X0,1.2,0,-1.4)
28     CALL INTNO(NUMX,3.7,-1.4)
29     CALL MESSAG('SIZEY=          Y0=          NUMY= ',32,0.0,-1.6)
30     CALL REALNO( SIZEY,1.0,7,-1.6)
31     CALL REALNO( Y0,1.2,0,-1.6)
32     CALL INTNO(NUMY,3.7,-1.6)
33     CALL ENDPL(1)
34     CALL BGNPL(2)
35     CALL PHYSOR(2.0,2.0)
36     CALL TITLE(' ',1,XLABEL,8,'DB ABOVE 1 MICROVOLT/METER',26,
37     $ XLNG,YLNG)
38     CALL INTAXS
39     CALL YAXANG(0.0)
40     CALL GRAF(0.,EXTIC,XMAX,EMIN,EYTIC,EMAX)
41     CALL CURVE(XPLOT,Y2PLOT,NPTS,0)
42     CALL DASH
43     CALL CURVE(XPLOT,Y3PLOT,NPTS,0)
44     CALL RESET('DASH')
45     CALL STRPT(XLNG-2.0,YLNG)
46     CALL CONNPT(XLNG-0.9,YLNG)
47     CALL MESSAG('          EZUMAG',16,XLNG-2.0,YLNG)
48     CALL MESSAG('----- EZPMAG',16,XLNG-2.0,YLNG-0.3)
49     CALL MESSAG(ID,80,0.0,-0.8)
50     CALL MESSAG(LABEL,36,0.0,-1.0)
51     CALL MESSAG('FREQ = ',7,0.0,-1.2)
52     CALL REALNO( FREQ,3.0,7,-1.2)
53     IF(IFLAG .EQ. 1) CALL MESSAG('DMIN = ',6,2.0,-1.2)
54     IF(IFLAG .EQ. 1) CALL REALNO( DMIN,1.2,7,-1.2)

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```
55      CALL MESSAG('SIZEX=          X0=      NUMX=  ',32,0.0,-1.4)
56      CALL REALNO( SIZEX,1,0.7,-1.4)
57      CALL REALNO( X0,1,2.0,-1.4)
58      CALL INTNO(NUMX,3.7,-1.4)
59      CALL MESSAG('SIZEY=          Y0=      NUMY=  ',32,0.0,-1.6)
60      CALL REALNO( SIZEY,1,0.7,-1.6)
61      CALL REALNO( Y0,1,2.0,-1.6)
62      CALL INTNO(NUMY,3.7,-1.6)
63      CALL ENDPL(2)
64      RETURN
65      END
```

```

1      SUBROUTINE CBESJY(Z,K,BJ,BY,KIND,NPRINT)
2      IMPLICIT COMPLEX      (A-H,O-Z)
3      REAL    DGMSUM,DG1,DG2,PI/3.14159265358979E0/,RRT,
4      $      EULER/0.577215664901533E0/
5
C THIS SUBROUTINE CALCULATES BESSEL FUNCTIONS OF THE FIRST KIND(JN) AND
6  C OF THE SECOND KIND(YN). N IS THE ORDER AND IS ALLOWED TO BE ANY
7  C POSITIVE INTEGER.
8  C THE ARGUMENT MUST BE DECLARED COMPLEX IN THE CALLING ROUTINE.
9  C IF A REAL ARGUMENT IS DESIRED JUST SET THE IMAGINARY PART TO ZERO.
10 C FOR THE ARGUMENT Z=RHO*C EXP(I*PHI) THE EMPLOYED EQUATIONS ARE VALID
11 C AS FOLLOWS:
12 C     IF 0<RHO<13 THEN PHI=ANY VALUE
13 C     IF RHO=13 OR 13<RHO<INFINITY THEN PHI=PI OR -PI<PHI<PI
14 C
15 C FOR RHO=0 J0 AND J1 ARE SET TO THEIR CORRECT VALUES.
16 C HOWEVER, Y0 AND Y1 ARE NOT CALCULATED SINCE THEY APPROACH INFINITY
17 C IN THE REAL AND/OR IMAGINARY PART, THEREFORE AN ERROR MESSAGE IS
18 C PRINTED.
19 C
20 C
21 C
22 C Z IS THE ARGUMENT.
23 C K IS THE ORDER.
24 C BJ IS THE BESSEL FUNCTION OF THE FIRST KIND.
25 C BY IS THE BESSEL FUNCTION OF THE SECOND KIND.
26 C KIND=1 CAUSES CALCULATION OF THE BESSEL FUNCTION OF THE FIRST KIND
27 C ONLY.
28 C ANY OTHER VALUE OF KIND CAUSES CALCULATIONS TO BE DONE FOR BESSEL
29 C FUNCTIONS OFF BOTH THE FIRST AND SECOND KIND.
30 C NPRINT=0 CAUSES NO DEBUG PRINTOUT.
31 C NPRINT=1 CAUSES DEBUG PRINTOUT.
32 C
33 C THE CALLING STATEMENT MUST HAVE DECLARED THE PARAMETERS CORRESPONDING
34 C TO Z, BJ, AND BY AS COMPLEX .
35 C
36 C
37      IF(C ABS(Z) .NE. 0.E0) GO TO 7
38      BJ=(0.0E0,0.0E0)
39      IF(K .EQ. 0) BJ=(1.0E0,0.0E0)
40      IF(KIND .NE. 1 .AND. K .EQ. 0) PRINT 400
41      400 FORMAT(1H0,'*** Y NOT CALCULATED FOR ARGUMENT OF MAGNITUDE 0'//)
42      RETURN
43      7 IF(C ABS(Z) .LT. 13.E0) GO TO 10
44 C
45 C
46 C ASSYMPTOTIC EXPANSION
47 C
48      RHO=8.*Z
49      MU=4*K**2
50      RT=C SQRT(2./(PI*Z))
51      RRT=RT
52      IF(RRT .LT. 0.) RT=-RT
53      P=0.
54 C DO LOOP FOR CALCULATING P

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55      DO 1 N=1,30
56      M=N-1
57      MM=2*M
58      IF(N .EQ. 1) GO TO 2
59      TERM=(-1)*(MU-(2*MM-3)**2)*(MU-(2*MM-1)**2)/(MM*(MM-1)*RHO**2)*
60      $TERM
61      IF(NPRINT .EQ. 1) PRINT 100,TERM
62      100 FORMAT(1X,'TERM=' ,2E30.15)
63      P=P+TERM
64      IF(C ABS(TERM) .GE. C ABS(TERMS) .OR. C ABS(TERM) .LE. 1.E-17) GO
65      $TO 3
66      TERMS=TERM
67      GO TO 1
68      2 TERM=1.E0
69      P=TERM
70      IF(NPRINT .EQ. 1) PRINT 100,TERM
71      TERMS=TERM
72      1 CONTINUE
73      3 CONTINUE
74      IF(NPRINT .EQ. 1) PRINT 200,P
75      200 FORMAT(1H0,'P=' ,2E30.15)
76      Q=0.
77      C DO LOOP FOR CALCULATING Q
78      DO 4 N=1,30
79      M=N-1
80      MM=2*M
81      MMM=2*M+1
82      IF(N .EQ. 1) GO TO 5
83      TERM=(-1)*(MU-(2*MM-1)**2)*(MU-(2*MM+1)**2)/(MMM*(MMM-1)*RHO**2)*
84      $TERM
85      IF(NPRINT .EQ. 1) PRINT 100,TERM
86      Q=Q+TERM
87      IF(C ABS(TERM) .GE. C ABS(TERMS) .OR. C ABS(TERM) .LE. 1.E-17) GO
88      $TO 6
89      TERMS=TERM
90      GO TO 4
91      5 TERM=(MU-1)/RHO
92      Q=TERM
93      IF(NPRINT .EQ. 1) PRINT 100,TERM
94      TERMS=TERM
95      4 CONTINUE
96      6 CONTINUE
97      IF(NPRINT .EQ. 1) PRINT 300,Q
98      300 FORMAT(1H0,'Q=' ,2E30.15)
99      BJ=RT*C COS(Z-K*PI/2.-PI/4.)*P-RT*C SIN(Z-K*PI/2.-PI/4.)*Q
100     IF(KIND .EQ. 1) GO TO 8
101     BY=RT*C SIN(Z-K*PI/2.-PI/4.)*P+RT*C COS(Z-K*PI/2.-PI/4.)*Q
102     8 RETURN
103     C
104     C
105     C POWER SERIES EXPANSION
106     C
107     10 NTERMS=35
108     KFAC=1
109     IF(K .LE. 1) GO TO 30
110     DO 20 N=2,K
111     20 KFAC=KFAC*N

```

```

112      30 TERM=(Z/2.)**K/KFAC
113      BJ=TERM
114      IF(KIND .EQ. 1) GO TO 91
115      DG1=0.
116      DG2=0.
117      IF(K .EQ. 0) GO TO 80
118      DO 60 N=1,K
119      60 DG2=DG2+1./N
120      80 TSUM3=-TERM*DGT2
121      SUMT3=TSUM3
122      91 DO 40 M=1,NTERMS
123      TERM=TERM*(Z/2.)**2*(-1)/((K+M)*M)
124      BJ=BJ+TERM
125      IF(KIND .EQ. 1) GO TO 92
126      DG1=DG1+1.E0/M
127      DG2=DG2+1.E0/(M+K)
128      DGMSUM=DG1+DG2
129      TSUM3=-TERM*DGMSUM
130      SUMT3=SUMT3+TSUM3
131      92 IF(C ABS(TERM) .LE. 1.E-17) GO TO 50
132      40 CONTINUE
133      50 IF(KIND .EQ. 1) GO TO 93
134      TERM3=SUMT3/PI
135      TERM1=(2./PI)*BJ*(EULER+C LOG(Z/2.))
136      SUMT2=(0.,0.)
137      IF(K .EQ. 0) GO TO 120
138      KM1FAC=KFAC/K
139      TSUM2=KM1FAC*(Z/2.)**(-K)
140      SUMT2=TSUM2
141      IF(K .EQ. 1) GO TO 120
142      KM1=K-1
143      DO 130 M=1,KM1
144      KMM=K-M
145      TSUM2=TSUM2/(KMM*M)*(Z/2.)**2
146      130 SUMT2=SUMT2+TSUM2
147      120 TERM2=-SUMT2/PI
148      BY=TERM1+TERM2+TERM3
149      93 RETURN
150      END

```

```

1      SUBROUTINE CLIN EQ (A,B,X,IROW,Q,N,NDIM,IFLAG,ERR)
2
3      C CLIN EQ USES L-U DECOMPOSITION TO
4      C FIND THE TRIANGULAR MATRICES L, U
5      C SUCH THAT L * U = A. L AND U ARE
6      C STORED IN A. THIS FORM IS USED WITH
7      C BACK-SUBSTITUTION TO FIND THE SOLN
8      C X OF A * X = L * U * X = B.
9      C N IS THE NUMBER OF EQUATIONS AND
10     C NDIM IS THE DIMENSION OF ALL ARRAYS
11     C IN THE PARAMETER LIST.
12
13     C IF IFLAG = 0, L, U, AND X ARE
14     C COMPUTED.
15     C IF IFLAG IS NON-ZERO, IT IS ASSUMED
16     C THAT L AND U HAVE BEEN COMPUTED IN
17     C A PREVIOUS CALL AND ARE STILL STORED
18     C IN A. THUS ONLY X IS COMPUTED.
19     C ERR IS THE ESTIMATED RELATIVE
20     C ERROR OF THE SOLUTION VECTOR.
21
22     COMPLEX   A, B, X, T
23     REAL      ERR
24     DIMENSION A(NDIM,NDIM),B(NDIM),X(NDIM)
25     DIMENSION IROW(NDIM),Q(NDIM)
26     DATA EPS /1.0E-15/
27
28     C
29     IF (N.GT.NDIM) GO TO 900
30     IF (IFLAG.NE.0) GO TO 600
31     DO 050 I = 1,N
32     Q(I) = 0.0
33     DO 040 J = 1,N
34     QQ = C ABS (A(I,J))
35     040 IF (Q(I).LT.QQ) Q(I) = QQ
36     IF (Q(I).EQ.0.0) GO TO 901
37     050 CONTINUE
38     ERR = EPS
39     PPIV = 0.0
40     DO 100 I = 1,N
41     100 IROW(I) = I
42
43     C
44     DO 500 L = 1,N
45     PIVOT = 0.0
46     K = L - 1
47     DO 240 I = L,N
48     IF (K.LT.1) GO TO 230
49     DO 220 J = 1,K
50     220 A(I,L) = A(I,L) - A(J,L) * A(I,J)
51     230 F = C ABS (A(I,L)) / Q(I)
52     IF (PIVOT.GT.F) GO TO 240
53     PIVOT = F
54     NPIVOT = I
55     240 CONTINUE

```

```

55      IF (PIVOT.EQ.0.0) GO TO 901
56      IF (PPIV.LE.PIVOT) GO TO 250
57      ERR = ERR * PPIV / PIVOT
58      IF (ERR.GE.1.0) GO TO 901
59 250  PPIV = PIVOT
60      IF (NPIVOT.EQ.L) GO TO 280
61      Q(NPIVOT) = Q(L)
62      J = IROW(L)
63      IROW(L) = IROW(NPIVOT)
64      IROW(NPIVOT) = J
65      DO 260 I = 1,N
66      T = A(L,I)
67      A(L,I) = A(NPIVOT,I)
68      A(NPIVOT,I) = T
69 260  CONTINUE
70      IF (L.EQ.N) GO TO 500
71      T = (1.0E0,0.0E0) / A(L,L)
72      K = L + 1
73      M = L - 1
74      DO 450 I = K,N
75      IF (M.I.T.1) GO TO 400
76      DO 350 J = 1,M
77      350 A(L,I) = A(L,I) - A(L,J) * A(J,I)
78      400 A(L,I) = T * A(L,I)
79      450 CONTINUE
80      500 CONTINUE
81      IF (ERR.GT.1.0E-5) PRINT 998, ERR
82
C
83
84      600 DO 620 I = 2,N
85      620 X(I) = (0.0E0,0.0E0)
86      J = IROW(1)
87      X(1) = B(J) / A(1,1)
88      DO 700 I = 2,N
89      J = IROW(I)
90      K = I - 1
91      DO 650 L = 1,K
92      X(I) = X(I) + A(I,L) * X(L)
93      X(I) = (B(J) - X(I)) / A(I,I)
94      700 CONTINUE
95      K = N - 1
96      DO 800 I = 1,K
97      J = N - I
98      M = J + 1
99      DO 800 L = M,N
100     X(J) = X(J) - X(L) * A(J,L)
101
102     800 CONTINUE
103     RETURN
104
C
105     900 PRINT 999
106     ERR = 1.0
107     RETURN
108     901 PRINT 997
109     ERR = 1.0
110     RETURN
111     997 FORMAT ('1ERROR IN CLIN EQ, MATRIX IS SINGULAR')
112     998 FORMAT (' CAUTION-' ,

```

```
112      S ' CLIN EQ HAS DECOMPOSED AN ILL-CONDITIONED MATRIX.',/
113      S ' RESULTS WILL HAVE RELATIVE ERROR =',E11.2)
114      999 FORMAT ('1ERROR IN CLIN EQ, MATRIX SIZE GREATER THAN NDIM')
115      END
```

```
1      FUNCTION C ANG(ARG)
2      IMPLICIT REAL (A-H,O-Z)
3      COMPLEX ARG,MINUSI/(0.0E0,-1.0E0)/
4      ARGR=ARG
5      ARGI=MINUSI*ARG
6      C ANG= ATAN2(ARGI,ARGR)
7      IF(ARGI .LT. 0.) C ANG=C ANG+6.2831853072E00
8      RETURN
9      END
```

```

1      COMPLEX FUNCTION CAPA(SP,RHO)
2      IMPLICIT COMPLEX (A-H,D-Z)
3      COMPLEX IM/(0.E0,1.E0)/,J1,J2
4      COMPLEX KSO
5      REAL KA,RHO
6      COMMON/DNE/H12,H22,J1,J2,S0,ARG0,KA,KSO
7      DEFINE CAPG1(ARG) = SQRT(ARG/(6371.0E0* SIN(ARG/6371.0E0)))
8      ARGP = KA*SP
9      TERM1 = (0.0E0,0.785398163397448E0)*((SP/S0)**2-1.0E0)
10     TERM2 = 2.0E0*ARG0*(1.0E0-.25E0*ARGP**2)
11     IF(RHO .EQ. 0.0) THEN
12         CAPA = (SP/S0)**4+TERM1*TERM2*H12+ARGP**2*H22)
13     ELSE
14         COFAMN = TERM1*(TERM2*J1+ARGP**2*J2)
15         ARGU = KSO*RHO
16         CALL CBESJY(ARGU,0,BJ,BY,0)
17         H02 = BJ-IM*BY
18         CAPA = COFAMN*H02*CAPG1(RHO)
19     END IF
20     RETURN
21     END

```

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