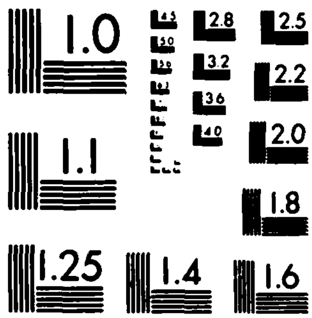
AD-A128 501 A COLLISION RESOLUTION PROTOCOL WITH LIMITED CHANNEL SENSING - FINITELY M..(U) CONNECTICUT UNIV STORRS DEPT OF ELECTRICAL ENGINEERING AND CO.. 1/1

UNCLASSIFIED P PAPANTONI-KAZAKOS ET AL. FEB 83 F/G 17/2 NL

END
DATE
FILMED
DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963-A

Department of

Electrical Engineering and Computer Science

A COLLISION RESOLUTION PROTOCOL WITH
LIMITED CHANNEL SENSING - FINITELY MANY USERS

by

P. Papantoni-Kazakos
Glenn D. Marcus
Michael Georgiopoulos

Technical Report TR-83-2

February 1983

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFOSR-TR- 83-0419	2. GOVT ACCESSION NO. AD-A128 301	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) A COLLISION RESOLUTION PROTOCOL WITH LIMITED CHANNEL SENSING - FINITELY MANY USERS		5. TYPE OF REPORT & PERIOD COVERED TECHNICAL
		6. PERFORMING ORG. REPORT NUMBER TR-83-2
7. AUTHOR(s) P. Papantoni-Kazakos, Glenn D. Marcus, and Michael Georgiopoulos		8. CONTRACT OR GRANT NUMBER(s) AFOSR-78-3695
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Electrical Engineering and Computer Science, University of Connecticut, Storrs CT 06268		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS PE61102F; 2304/A5
11. CONTROLLING OFFICE NAME AND ADDRESS Mathematical & Information Sciences Directorate Air Force Office of Scientific Research Bolling AFB DC 20332		12. REPORT DATE FEB 83
		13. NUMBER OF PAGES 42
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In this paper, the authors consider the random-accessing of a single slotted channel by a finite number of independent, data transmitting bursty users. They adopt the assumption that each user monitors the channel only while he is blocked. They also assume that the channel outcomes (visible to each user) are ternary. That is, each channel slot is perceived as either empty or successfully busy, or as a collision slot. The authors disregard propagation delays.

For the above model, the authors propose and analyze a collision (CONTINUED)

DD FORM 1 JAN 73 1473

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

ITEM #20, CONTINUED: resolution protocol (CRLS) with tree search characteristics. For identical users with binomial transmission processes, they find lower bounds on the CRLS throughput, and they compute upper bounds on the induced delays in transmission. The authors compare their results with those induced by the dynamic tree protocol of Capetanakis; where the feedback sensing is continuous in the latter. The CRLS performs surprisingly well. For asymptotically many users, its throughput is higher than the throughput of the nondynamic tree protocol of Capetanakis, and less than 7 percent lower than the throughput of the dynamic form of the latter. The CRLS also compares well in terms of delays, and it is robust in the presence of channel errors.

A COLLISION RESOLUTION PROTOCOL WITH LIMITED CHANNEL SENSING - FINITELY MANY USERS

P. Papantoni-Kazakos, Glenn D. Marcus, and Michael Georgiopoulos
University of Connecticut
Storrs, Connecticut 06268

Abstract

In this paper, the authors ~~we~~ consider the random-accessing of a single slotted channel by a finite number of independent, data transmitting bursty users. They ~~We~~ adopt the assumption that each user monitors the channel only while he is blocked. they ~~We~~ also assume that the channel outcomes (visible to each user) are ternary. That is, each channel slot is perceived as either empty or successfully busy, or as a collision slot. ~~We disregard~~ Propagation delays are disregarded.

For the above model, the authors ~~we~~ propose and analyze a collision resolution protocol (CRLS) with tree search characteristics. For identical users with binomial transmission processes, they ~~we~~ find lower bounds on the CRLS throughput, and we compute upper bounds on the induced delays in transmission. Their results are compared ~~We compare our results~~ with those induced by the dynamic tree protocol of Capetanakis; where the feedback sensing is continuous in the latter. The CRLS performs surprisingly well. For asymptotically many users, its throughput is higher than the throughput of the nondynamic tree protocol of Capetanakis, and less than 7 percent lower than the throughput of the dynamic form of the latter. The CRLS also compares well in terms of delays, and it is robust in the presence of channel errors.

This work was supported jointly by the National Science Foundation under the Grant ECS-8119885, and the Air Force Office of Scientific Research under Grant AFOSR-78-3695. Part of the work presented at the 21st CDC, Dec., 1982.

I. INTRODUCTION

We consider a number of data-transmitting bursty users who request access to a single network resource. We assume that the users do not communicate with each other directly, and that their data are formatted into packets of identical length. Such a user model arises, for example, when a number of computer terminals access a single host computer. Let us further assume that the network resource is a single transmission channel whose time is slotted. The length of each channel time slot is equal to one packet. Also, each user can attempt transmission of a packet, starting only at the beginning of a slot.

Given the general model above, a variety of transmission protocols can be devised depending on the specific characteristics of the user and channel models. Such characteristics include finite number of well-identified users versus an asymptotically large number of ill-specified users, as well as various levels of feedback information provided to the users by the channel.

For an asymptotically large number of users, researchers have considered a variety of feedback information levels. The slotted ALOHA protocol [1,2,3] assumes that each user tunes to the feedback broadcasting only during those channel slots which correspond to his own transmission attempts. Tsybakov and Vvedenskaya [14] proposed and analyzed a collision resolution protocol, for the case where each user inspects the feedback continuously only while there is an unsuccessfully transmitted packet in his buffer. The protocols developed by Capetanakis [5] and Tsybakov and Mikhailov [7] assume that all users are constantly inspecting the feedback broadcasting. Finally, the same assumption is adopted in [8] and [9], where it is also assumed that collision multiplicities are included in the feedback information. As the level of feedback information inspected by each user increases, the protocol throughput increases also.

When a *finite number of well-identified* users is involved, the protocol designer has considerably increased flexibility. Here, either collision-permitting or collision-free transmission protocols may be adopted. The collision-free protocols involve some scheduling technique implemented either by some centralized capacity allocation mechanism (as in [12]), or through system state information transmitted within certain dedicated channel slots or mini-slots (as in [10], [11], [13]). The implied assumption here is that each user constantly inspects the scheduling feedback information. The existing collision-permitting protocols, universally assume that each user inspects the feedback broadcasting constantly (for every channel slot at all times). The Hayes [4] and Capetanakis [6] protocols are based on a binary tree search. The Hluckyj and Gallager protocol [15] is based on sequential user subgrouping. The protocols in [4], [6] and [15] are collision resolution protocols; they suspend new transmissions until a collision is resolved.

In the present paper, we assume finitely many users, and we adopt similar user and channel models as in [4,6,15]. In contrast to those models as well as to the collision-free models, however, we assume that each user inspects the broadcast feedback only while he is blocked. By blocking we mean the existence of some unsuccessfully transmitted packet in the user's buffer. Our assumption is the same as in [14], and it eliminates the often undesirable requirement that all users monitor the channel constantly, even when empty. For the above model, we propose and analyze a collision resolution protocol. We name this protocol Collision Resolution with Limited Sensing (CRLS).

The organization of the paper is as follows.

In section II, we describe the CRLS protocol, and we study its essential properties. In section III, we study the performance characteristics of the CRLS, for users with binomial transmission processes. We also find lower bounds on the throughput of the protocol, then. In section IV, we present and discuss numerical results. In section V, we draw some conclusions.

II. THE CRLS PROTOCOL

As in [6], we assume that 2^n identical, independent, and packet-transmitting users share a single slotted channel. If a single packet is transmitted within a slot, it is received correctly. If at least two packets are simultaneously transmitted, a collision occurs, all information contained in the collided packets is lost, and retransmission is necessary. The ternary outcome of each slot (empty, busy with one packet, collision) is broadcasted to all users without propagation delays.

Let the system start operating at time zero, and let time be measured in slot units. Initially, each user does not inspect the feedback, and he is free to transmit a packet in any slot. Let some user transmit his first packet at time t. He then inspects the feedback that corresponds to slot t. If he sees success, he stops inspecting the feedback until his next transmission. If, instead, he sees a collision, he initiates the CRLS protocol for collision resolution, while inspecting the feedback continuously. *The user perceives the collision as resolved*, as soon as his collided packet is successfully transmitted. He then stops inspecting the feedback, until his next transmission. Transmission of new arrivals is not attempted by the user, until his own collision has been resolved.

In this section, we will describe the CRLS protocol and we will analyze its operational characteristics. Since those aspects are independent of the packet-generating process per user, we will make no assumptions on the latter at this point. We will need such assumptions, only when we study the expected delays induced by the protocol, and its stability properties.

A. The CRLS General Operation

Given 2^n users, consider the binary tree with 2^n leaves. The tree has n+1 levels of depth, numbered from 0 to n. Depths 0 and n correspond respectively to the root and the leaves of the tree. In general, there exist 2^i nodes at depth i. Each of these nodes is the root of a binary subtree with 2^{n-i} leaves. Each tree node beyond depth 0 is identified by a binary codeword, where the codeword of each node at depth $i : i \geq 1$ contains i bits. Consider the depths of the binary tree evolving sequentially from left to right (as in figure II.1), and let some node at depth $i : 1 \leq i \leq n-1$ be identified by the binary codeword $x_1x_2...x_i$. Then, the two nodes at depth i+1 that branch off node $x_1x_2...x_i$ are identified by the codewords $x_1...x_i0$ and $x_1...x_i1$; node $x_1...x_i1$ lies under node $x_1...x_i0$. The two nodes at depth 1 that branch off the tree root R are identified by the length one codewords 0 and 1, where node 1 lies under node 0. It is clear from the above that each binary codeword $x_1...x_i : 1 \leq i \leq n$ identifies a single tree node at depth i; thus, it also identifies the unique path that connects this node with the tree root. In particular, each one of the 2^n distinct binary codewords of length n identifies a single tree leaf. Consider now a one-to-one correspondence between the 2^n users and the 2^n binary codewords of length n. Then, each encoded user is uniquely identified by a single binary codeword of length n. But, as we saw above, each such codeword also identifies a single leaf on the (n+1) - depth binary tree. Thus, there exists a one-to-one correspondence between those tree leaves and the 2^n encoded users.

Let the 2^n users be encoded by binary sequences of length n, and let each user have in its memory a reproduction of the (n+1) - depth binary tree. Each user considers himself placed on the tree leaf whose codeword coincides with his own. If we number the 2^n users from 1 to 2^n, the codeword of the ith user is the length n binary representation of the number i-1. From now on we will identify some user either by his number or by his codeword.

Let t_o be some time instant such that all collisions involving packets from user i have been previously resolved. Let user i transmit again at time t, where $t \geq t_o$ and the user did not transmit in the time interval (t_o, t). Let there be a collision in slot t. User i observes this collision and starts the motions for

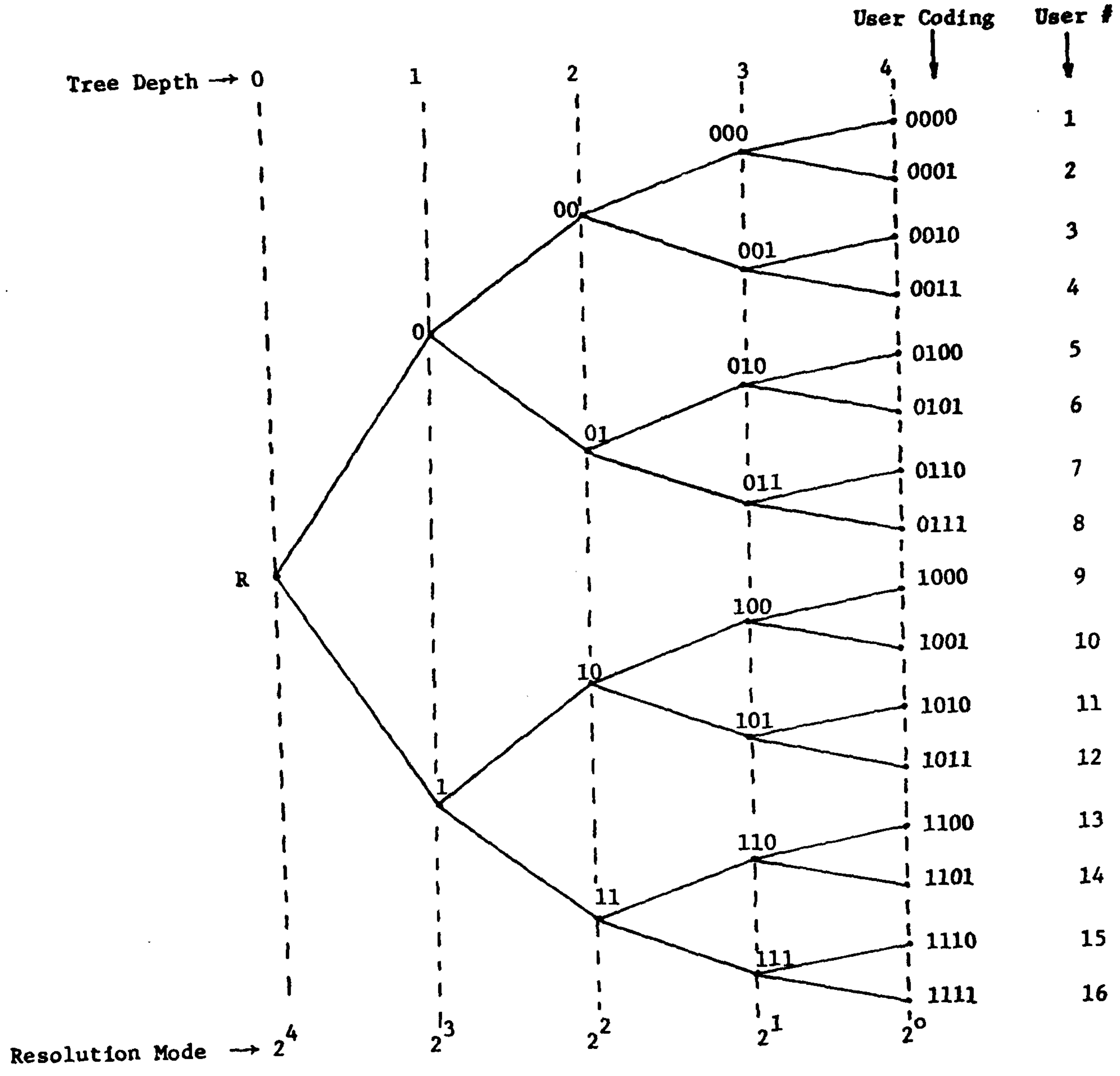


Figure II.1

The Binary Tree for 2^4 Users

its resolution. In the process, he uses the (n+1) - depth binary tree in his memory, inspecting continuously the feedback. He traces sequentially neighboring tree nodes, in an order dictated by the feedback. At the same time, he imagines himself placed at different tree nodes depending on the relationship between the current node in the trace and the user's codeword. The user transmits, when the node he has placed himself at coincides with the trace node. To explain the collision resolution process coherently, we need to discriminate between node tracing and user node self-placing. To do that, we first present the following definition.

Definition 1

At some point in time, user i is blocked if he is in the process of resolving a collision. The blocked user i is in resolution mode 2^k ; $0 \leq k \leq n$ if he imagines himself placed at a node that lies at the tree depth n-k. If $k < n$, and if $x_1 x_2 \ldots x_n$ is the user's codeword, the node's codeword is $x_1 \ldots x_{n-k}$. The user is in resolution mode 2^k and active (2^k_a) if he is in resolution mode 2^k and transmits. He is in resolution mode 2^k and withholding (2^k_w) if he is in resolution mode 2^k and does not transmit.

The blocked user i is in state $(2^k_a, y_1 \ldots y_\ell)$ or $(2^k_w, y_1 \ldots y_\ell)$; $\ell \leq n$ if he is in resolution mode 2^k, his mode tracing has reached the tree node whose binary codeword is $y_1 \ldots y_\ell$, and he is respectively active or withholding.

Let $x_1 \ldots x_n$ be the codeword of user i. Let the user be unblocked at time t-1, and let him transmit and be blocked at time t. Then, the CRLS protocol performed by the user is described by the following statements.

1. At time t, the user imagines himself placed at the root of the tree and transmitting. Observing collision, he also starts his node-tracing at the tree root. Thus, at time t user i is in state $(2^n_a, R)$.

2. Let at time t_1 user i be blocked and in resolution mode 2^k; $0 \leq k \leq n$. Then, by definition 1, the user has placed himself at node $x_1 \ldots x_{n-k}$. If he is in resolution mode 2_a^k, he transmits in slot t_1. If he is in resolution mode 2_w^k, he does not. He can not be in resolution mode 2_a^k, unless his node-tracing has reached node $x_1 \ldots x_{n-k}$.

Let the user be in resolution mode 2_a^k at time t_1. Then,

i) If he observes success in slot t_1, he becomes unblocked.

ii) If he observes collision in slot t_1, he moves at time t_1+1 to resolution mode:

2_a^{k-1} ; if $x_{n-k+1} = 0$, and $k \geq 1$

2_w^{k-1} ; if $x_{n-k+1} = 1$, and $k \geq 1$

2_a^{n-1} ; if $x_1 = 0$, and $k = 0$

2_w^{n-1} ; if $x_1 = 1$, and $k = 0$

Let the user be in resolution mode 2_w^k at time t_1. Then,

i) If he observes a collision in slot t_1, he moves at time t_1+1 to resolution mode:

2_a^{n-1} ; if $k = 0$, and $x_1 = 0$

2_w^{n-1} ; if $k = 0$, and $x_1 = 1$

Otherwise, he remains in resolution mode 2_w^k.

ii) If he observes an either empty or a successful slot t_1, he moves at time t_1+1 to resolution mode:

2_a^{k-1} ; if his node-tracing reaches node $x_1 \ldots x_{n-k+1}$ at time t_1+1.

Otherwise, he remains in resolution mode 2_w^k.

3. Let at time t_1 user i be blocked. Let $y_o y_1 \ldots y_\ell$; $0 \leq \ell \leq n$ be the node reached by the user's node-tracing at time t_1; where if $\ell = 0$ the node is the tree root. Then,

i) If the user observes a collision at t_1, he moves at time t_1+1 his node-tracing to node:

$y_1 \ldots y_\ell 0$; if $1 \leq \ell \leq n-1$

0 ; if either $\ell = 0$ or $\ell = n$

ii) If the user is in withholding resolution mode at time t_1, and observes either empty or successful slot t_1, he moves at time t_1+1 his node-tracing to node:

$y_1 \ldots y_{m-1} 1$; $1 \leq m \leq \ell$

; where $m : y_j = 1$; $m+1 \leq j \leq \ell$ and $y_m = 0$, for $\ell \geq 1$ and $y_1 \ldots y_\ell$ such that not all bits y_j are equal to one.

iii) If the user is in active resolution mode at time t_1, and observes success, he becomes unblocked.

Statements 1 to 3 above, basically describe a tree search as in [6]. The difference here is that this search is not performed simultaneously be all users. Thus, it is possible that when some user's search reaches a leaf node, a collision occurs. Then, as statements 1 to 3 indicate, the user interprets this collision as a root collision, and he reinitiates his tree search. The evolution of the protocol is perhaps exhibited better by the state transitions in time. Those transitions are dictated by statements 1 to 3, and they are presented by the statements below.

a. Let user $i : x_1 \ldots x_n$ be unblocked at time t-1, and become blocked at time t. Then, at time t he is in state $(2_a^n, R)$.

b. Observing collision at the tree root, the user moves his node-tracing to node 0, at time t+1. He also imagines himself placed in resolution mode 2^{n-1}. If his codeword is such that $x_1 = 0$, he transmits. If, instead, $x_1 = 1$, he withholds. Thus, at time t+1, the user is in state:

$$(2_a^{n-1}, x_1) \quad ; \quad \text{if} \quad x_1 = 0$$

$$(2_w^{n-1}, 0) \quad ; \quad \text{if} \quad x_1 = 1$$

If he is in state $(2_a^{n-1}, 0)$, he has placed himself at node 0. So, if then he also observes success at time t+1, he becomes unblocked.

c. In general, if at some time instant t_1 user i is still blocked, his node-tracing has reached node $y_1 \ldots y_\ell$, and the user transmits, then he has also placed himself at node $y_1 \ldots y_\ell$. Therefore, as stated in definition 1 and as dictated by statements 1 to 3, we have then that $y_1 \ldots y_\ell = x_1 \ldots x_\ell$. Thus, the user is in state $(2_a^{n-\ell}, y_1 \ldots y_\ell) = (2_a^{n-\ell}, x_1 \ldots x_\ell)$ at time t_1. Then,

i) If the user observes success at t_1, he becomes unblocked.

ii) If $\ell < n$ and the user observes collision at time t_1, he moves his node-tracing to node $x_1 \ldots x_\ell 0$, and places himself in resolution mode $2^{n-\ell-1}$. If his codeword is such that $x_{\ell+1} = 0$, he also transmits. If, instead, $x_{\ell+1} = 1$, he withholds. Thus, at time t_1+1, user i is in state:

$$(2_a^{n-\ell-1}, x_1 \ldots x_\ell \, x_{\ell+1}) \quad ; \quad \text{if } 1 \leq \ell < n, \text{ and } x_{\ell+1} = 0$$

$(2_w^{n-\ell-1}, x_1 \ldots x_\ell\ 0)$; if $1 \le \ell < n$, and $x_{\ell+1} = 1$

iii) If user i is in state $(2_a^o, x_1 \ldots x_n)$ at time t_1, he has reached his own tree leaf. If he then observes collision, he starts his tree search again, considering the collision occurred at the tree root. Thus, at time t_1, he renames the state $(2_a^o, x_1 \ldots x_n)$ as $(2_a^n, R)$. At time t_1+1, he then proceeds as in statement b.

d. Let at time t_1 user i be blocked and in resolution mode $2_w^k : 0 \le k \le n-1$. Then, he has placed himself at node $x_1 \ldots x_{n-k}$. That implies that node $x_1 \ldots x_{n-k-1}$ has been simultaneously visited by both the user-placement and the node-tracing processes; where for $k = n-1$, node $x_1 \ldots x_{n-k-1}$ is the tree root. Furthermore, at the time $t_o < t_1$ of that visit, user i experienced collision, and then he moved to resolution mode 2_w^k, where he remained until time t_1. By statement 2, it is also implied then that $x_{n-k} = 1$. Also, due to statement 3, all the nodes traced by the user in the time interval $[t_o+1, t_1]$ branch off node $x_1 \ldots x_{n-k-1}\ 0$. Thus, if at time t_1 user i is in state $(2_w^k, y_1 \ldots y_\ell)$; $0 \le k \le n-1$, then $\ell > n-k$, the user's codeword is such that $x_{n-k} = 1$, and $y_1 \ldots y_{n-k} = x_1 \ldots x_{n-k-1}\ 0$. Therefore, at time t_1 the user is then in state $(2_w^k, x_1 \ldots x_{n-k-1}\ 0 \ldots y_\ell)$.

Then,

i) If the user observes either a successful or an empty slot t_1, he moves at time t_1+1 to state:

$$(2_a^k, x_1 \ldots x_{n-k-1}\ x_{n-k}) \ ; \text{ if: } \begin{cases} \text{either } \ell = n-k \\ \text{or } k > 0,\ \ell > n-k, \text{ and } y_j = 1 \ ;\ n-k+1 \le j \end{cases}$$

$$(2_w^k, x_1 \ldots x_{n-k-1}\ 0 \ldots y_{m-1}\ 1) \ ; \text{ if } k > 1,\ \ell > n-k+1,\ m \le \ell-1,\ y_m = 0, \text{ and } y_j = 1 \ ;\ m+1 \le j \le \ell.$$

$$(2_w^k, x_1 \ldots x_{n-k-1}\ 01) \ ; \text{ if } k > 0,\ \ell = n-k+1, \text{ and } y_\ell = 0$$

; where $x_{n-k} = 1$

ii) If the user observes a collision at time t_1, he moves at time t_1+1 to state:

$$(2_w^k, x_1 \ldots x_{n-k-1}\ 0 \ldots y_\ell\ 0) \ ; \text{ if } \ell < n \text{ (implying } k > 0)$$

$$(2_a^n, x_1) \ ; \text{ if } \ell = n, \text{ and } x_1 = 0$$

$$(2_w^n, 0) \ ; \text{ if } \ell = n, \text{ and } x_1 = 1$$

If $\ell = n$, the user observes a collision at a leaf node of his node-tracing; thus he reinitializes his tree search.

In figure II.2, we exhibit the state transitions explained by statements a to d. In figure II.3 we present two examples, regarding the evolution of the state transitions in time. In the first example, we assume that the total number of users is 8, and only users 5, 6, 4 and 8 transmit at various times. We assume that users 1, 2, 3, and 7 remain unblocked throughout the observed time period. In the second example, we assume 16 users totally. Again, we consider a time period throughout which only users 4, 5, 6, and 8 transmit at various times. In both examples, we present the evolution of the protocol from the time instant when some users become blocked, to the time instant when all the involved users are unblocked.

To this point, we have described the operation of the CRLS protocol, and we have presented and analyzed the state transitions that the protocol induces. In the remaining part of this section, we will study some of its properties that are essential for performance evaluation.

B. Properties of the CRLS

As we explained previously, the CRLS tree search is not performed simultaneously by all the 2^n users. Indeed, each user initiates his own CRLS when he becomes blocked. Therefore, at some point in time, there may be some blocked users at various levels of their tree search, and some unblocked users. The unblocked users do not inspect the feedback. Among the blocked users, there will be some in active resolution mode and some in withholding resolution mode. Any possible collision will be caused by the active users, and will be observed by all the blocked users. Among the users who are blocked and in withholding mode, we will single out those who are either in state $(2^k_w, x_1 \ldots x_{n-k-1}\ 0)$; $0 \leq k \leq n-1$, or in some state $(2^k_w, x_1 \ldots x_{n-k-1}\ 01 \ldots 1)$; $1 \leq k \leq n-1$, where $x_1 \ldots x_{n-k-1}\ 1$ are the first n-k bits of the user's codeword. To discriminate between those users and the remaining blocked and withholding users, we will denote the resolution mode of the first $2^k_{w_e}$. As shown in figure II.2, if a user is in resolution mode

State at t	Slot t Outcome	State at t+1
$(2_a^n, R)$	s	unblocked
	c	$(2_a^{n-1}, x_1)$; if $x_1 = 0$ $(2_w^{n-1}, 0)$; if $x_1 = 1$
$(2_a^{n-\ell}, x_1\ldots x_\ell)$; $1 \le \ell \le n$	s	unblocked
	c	$(2_a^{n-\ell-1}, x_1\ldots x_\ell\, x_{\ell+1})$; if $\ell < n$, $x_{\ell+1} = 0$ $(2_w^{n-\ell-1}, x_1\ldots x_\ell\, 0)$; if $\ell < n$, $x_{\ell+1} = 1$ $(2_a^{n-1}, x_1)$; if $\ell = n$, $x_1 = 0$ $(2_w^{n-1}, 0)$; if $\ell = n$, $x_1 = 1$
$(2_w^k, x_1\ldots x_{n-k-1}0\ldots y_\ell)$; $0 \le k < n$; $n \ge \ell \ge n-k$; $x_{n-k} = 1$	s or e	$(2_a^k, x_1\ldots x_{n-k-1}\, x_{n-k})$; if $\begin{cases}\text{either } \ell = n-1 \\ \text{or } k > 0,\ \ell > n-k,\ y_j = 1 \\ n-k+1 \le j \le \ell\end{cases}$ $(2_w^k, x_1\ldots x_{n-k-1}\, 0\ldots y_m 1)$; if $k > 1$, $\ell > n-k+1$, $m \le \ell-1$, $y_m = 0$, $y_j = 1$; $m + 1 \le j \le \ell$ $(2_w^k, x_1\ldots x_{n-k-1} 01)$; if $k > 0$, $\ell = n-k+1$, $y_\ell = 0$
	c	$(2_w^k, x_1\ldots x_{n-k-1}0\ldots y_\ell 0)$; if $\ell < n$ $(k > 0)$ $(2_a^n, x_1)$; if $\ell = n$, $x_1 = 0$ $(2_w^n, 0)$; if $\ell = n$, $x_1 = 1$

Code

s : success
e : empty
c : collision

Figure II.2

State Transitions for Blocked at Time t User $x_1\ldots x_n$.

User	Slot outcome, → Time												
	c	c	s(4)	s(8)	c	c	c	e	c	c	s(5)	s(6)	s(8)
4:011	U	$(2_a^3,R)$	$(2_a^2,0)$	U	U	U	U	U	U	U	U	U	U
5:100	$(2_a^3,R)$	$(2_w^2,0)$	$(2_w^2,00)$	$(2_w^2,01)$	$(2_a^2,1)$	$(2_a^1,10)$	$(2_a^o,100)$	$(2_w^2,0)$	$(2_a^2,1)$	$(2_a^1,10)$	$(2_a^o,100)$	U	U
6:101	$(2_a^3,R)$	$(2_w^2,0)$	$(2_w^2,00)$	$(2_w^2,01)$	$(2_a^2,1)$	$(2_a^1,10)$	$(2_w^o,100)$	$(2_w^2,0)$	$(2_a^2,1)$	$(2_a^1,10)$	$(2_w^o,100)$	$(2_a^o,101)$	U
8:111	U	$(2_a^3,R)$	$(2_w^2,0)$	$(2_a^2,1)$	U	U	$(2_a^3,R)$	$(2_w^2,0)$	$(2_a^2,1)$	$(2_w^1,10)$	$(2_w^1,100)$	$(2_w^1,101)$	$(2_a^1,11)$

Example 1 : $2^3 = 8$ Users

User	Slot outcome, → Time								
	c	c	c	s(4)	s(8)	c	c	s(5)	s(6)
4:0011	U	$(2_a^4,R)$	$(2_a^3,0)$	$(2_a^3,00)$	U	U	U	U	U
5:0100	$(2_a^4,R)$	$(2_a^3,0)$	$(2_w^2,00)$	$(2_w^2,000)$	$(2_w^2,001)$	$(2_a^2,01)$	$(2_a^1,010)$	$(2_a^o,0100)$	U
6:0101	$(2_a^4,R)$	$(2_a^3,0)$	$(2_w^2,00)$	$(2_w^2,000)$	$(2_w^2,001)$	$(2_a^2,01)$	$(2_a^1,010)$	$(2_w^o,0100)$	$(2^o,0101)$
8:0111	U	$(2_a^4,R)$	$(2_a^3,0)$	$(2_w^2,00)$	$(2_a^3,01)$	U	U	U	U

Example 2 : $2^4 = 16$ Users

Code

s(i) : successful slot, busy with transmission from user i
e : empty slot
c : collision slot
U : unblocked user

Figure II.3

Evolution of State Transitions in Time: Two Examples

$2^k_{w_e}$ and observes an either empty or a successful slot, he moves to resolution mode 2^k_a. Let us denote,

$(\{2^k_a, N_k\};\ 0 \le k \le n,\ \{2^k_{w_e}, P_k\};\ 0 \le k \le n-1, [M])$: The event that at some point in time there are M unblocked users, N_k; $0 \le k \le n$ users in resolution mode 2^k_a, and P_k; $0 \le k \le n-1$ users in resolution mode $2^k_{w_e}$. (1)

The above event implies that there are $2^n - M - \sum_{k=0}^{n} N_k - \sum_{k=0}^{n-1} P_k$ users who are are blocked and withholding, without being in resolution mode $2^k_{w_e}$, for some k. If some user $x_1 \ldots x_n$ is in either one of the resolution modes 2^k_a and $2^k_{w_e}$, then he has placed himself at node $x_1 \ldots x_{n-k}$; where if k=n this node is the tree root R. Any user $x_1 \ldots x_n$ who is in resolution mode $2^k_{w_e}$ is such that $x_{n-k} = 1$. As shown by the state transitions in figure II.2, a user is in resolution mode 2^n_a only when he first transmits from a previously unblocked state. Also, if a user is in resolution modes 2^o_a or $2^o_{w_e}$ and observes collision, he interprets this collision as occurred at the tree root, and moves to resolution mode 2^{n-1}. We now present a proposition whose proof is in appendix A.

Proposition 1

Given 2^n users, consider the event $(\{2^k_a, N_k\};\ 0 \le k \le n,\ \{2^k_{w_e}, P_k\}\ ;\ 0 \le k \le n-1, [M])$ at some point in time. Then, for every $k \le n-1$ such that $N_k > 0$, the first n-k codeword bits of the N_k users are identical; thus the N_k users branch off the same tree node at depth n-k. The same is true for the P_k users, if $P_k > 0$. Therefore, $N_k \le 2^k$, and $P_k \le 2^k$.

Let us now suppose that at some time instant t, the event $(\{2^k_a, N_k\};\ 0 \le k \le n,\ \{2^k_{w_e}, P_k\};\ 0 \le k \le n-1, [M])$ occurs. For some k such that $0 < k \le n$, let the codewords of the N_k users have the common prefix $x_1(k) \ldots x_{n-k}(k)$; where for k=n this prefix is the tree root. Among those users, let us have N_{k1} with common codeword prefix $x_1(k)\ldots x_{n-k}(k)0$; let us have N_{k2} with common codeword prefix $x_1(k)\ldots x_{n-k}(k)1$. For some k such that $0 \leq k \leq n-1$, let the codewords of the P_k users have the common prefix $y_1(k)\ldots y_{n-k-1}(k)1$. Then, $y_1(k)\ldots y_{n-k-1}(k)1$ can not be identical to $x_1(k)\ldots x_{n-k}(k)$. If $P_o + N_o = 2$, then $y_1(0)\ldots y_{n-1}(0)$ and $x_1(0)\ldots x_{n-1}(0)$ are identical. This last statement evolves from proposition 1, and it is proved in appendix A. If $\sum_{k=0}^{n} N_k = 0$, slot t is empty. If $\sum_{k=0}^{n} N_k = 1$, slot t is a successful slot, and then the one active user becomes unblocked. If $\sum_{k=0}^{n} N_k \geq 2$, slot t is a collision slot. The outcome of slot t is observed by all, but the M unblocked users. Let now $\sum_{k=0}^{n} N_k \geq 2$. Then, the events evolving at times t+1 and t+2 have been derived in the proof of proposition 1, and they are shown in figure II.4. We observe that the event at t+2 depends at most on the events at the time instants t and t+1. Let both slots t+1 and t+2 be noncollision slots. It is then clear from the transitions in figure II.4 that the original collision at t has been resolved. That is, the $\sum_{k=0}^{n} N_k$ users (who are then exactly two) are both unblocked at t+2. Furthermore, if at time t the two users were in resolution modes $2_a^{k_1}$ and $2_a^{k_2}$ respectively, and the one in resolution mode $2_a^{k_1}$ became unblocked at t+1, then the prefices of the users' codewords are necessarily $x_1(k_1)\ldots x_{n-k_1}(k_1)0$ and $x_1(k_2)\ldots x_{n-k_2}(k_2)1$. Another important observation from figure II.4 is that if t+1 is a collision slot, then the event at t+2 does not include any of the users who at t were in some resolution mode $2_{w_e}^k$; $0 \leq k \leq n-1$. Finally, we observe that independently of the slot t+1 outcome, the only user who being in withholding resolution mode at t, may be active at either t+1 or t+2 is he who at t was in resolution mode $2_{w_e}^o$.

The above observations lead to some simplifications regarding the events in (1) and their transitions in time. To show that, let us first denote,

Slot t

Event : $(\{2^k_a, N_k\};\ 0 \le k \le n,\ \{2^k_{w_e}, P_k\};\ 0 \le k \le n-1,\ [M])\ ;\ \sum_{k=0}^{n} N_k \ge 2$

Result : Collision

Slot t+1

Event : $(\{2^n_a, N^1_n\}, \{2^{n-1}_a, N_{n1}+[1-x_1](P_o+N_o)\}, \{2^{k-1}_a, N_{k1}\};\ 1 \le k \le n-1,$

$\{2^{n-1}_{w_e}, N_{n2}+x_1(P_o+N_o)\}, \{2^{k-1}_{w_e}, N_{k2}\};\ 1 \le k \le n-1, [M-N^1_n])$

$$\text{Result} : \begin{cases} \text{collision; if } N^1_n + [1-x_1](P_o+N_o) + \sum_{k=1}^{n} N_{k1} \ge 2 \\ \text{emptiness or success; otherwise} \end{cases}$$

Slot t+2

$$\text{Event} : \begin{cases} (\{2^n_a, N^2_n\}, \{2^{n-1}_a, N^1_{n1}+[1-x_1(1)]N_1\}, \{2^{k-1}_a, N_{k+1,11}\};\ 1 \le k \le n-1, \\ \{2^{n-1}_{w_e}, N^1_{n2}+x_1(1)N_1\}, \{2^{k-1}_{w_e}, N_{k+1,12}\};\ 1 \le k \le n-1,\ [M-N^1_n-N^2_n]) \\ ;\ \text{if } N^1_n + [1-x_1](P_o+N_o) + \sum_{k=1}^{n} N_{k1} \ge 2 \\ (\{2^n_a, N^2_n\}, \{2^{n-1}_a, N^1_{n2}+x_1(P_o+N_o)\}, \{2^{k-1}_a, N_{k2}\};\ 1 \le k \le n-1, \{2^k_{w_e}, P_k\}; \\ 1 \le k \le n-1,\ [M-N^2_n]) \\ ;\ \text{if } N^1_n + [1-x_1](P_o+N_o) + \sum_{k=1}^{n} N_{k1} \le 1 \end{cases}$$

Code:

x_1 : 1st codeword bit $x_1(0)$ of the P_o+N_o users (if any).

N^j_n : # of previously unblocked users who transmitted at t+j

$x_1(1)$: 1st codeword bit of the N_1 users (if any).

N_{k11}, N_{k12} : those of the N_{k1} users (if any) whose (n-k+2)th codeword bit is 0 and 1 respectively.

Figure II.4

Transitions of the Events in (1)

$(\{2_a^k, N_k\};\ 1 \le k \le n, [M])$: The event such that at some point in time there are M unblocked users, N_k; $1 \le k \le n$ users in resolution mode 2_a^k, and no users in resolution mode 2^o.

(2)

The above event is a simplification of the event in (1), where in the latter $N_o = P_o = 0$. Indeed, the event in (2) does not include then the users who are in resolution modes $2_{w_e}^k$; $1 \le k \le n-1$. We observe that the event in (2) will occur at some point in time. If t_o is the time when the system first starts operating, an event as in (2) will occur before any blocked users have reached the resolution mode 2^o. Let now the event $(\{2_a^k, N_k\};\ 1 \le k \le n, [M])$ occur at time t, and let $\sum_{k=1}^{n} N_k$ be more than one. Then, t is a collision slot. It is then clear from figure II.4 that at t+1 the users N_{k1}; $1 \le k \le n$ transmit. If success or emptiness, those users become unblocked at t+1. If, instead, slot t+1 is a collision slot, then the event at t+1 is readjusted to include the N_1 users in resolution mode 2_a^n; thus the resulting event includes then no users who are in resolution mode 2^o. This is clear from the transitions in figure II.4. Therefore, if we generalize the event in (2), to include users in resolution mode 2_a^o, the event at t+1 is initially (from figure II.4) $(\{2_a^n, N_n^1\}, \{2_a^{k-1}, N_{k1}\};\ 1 \le k \le n, [M-N_n^1])$; where N_n^1 the number of previously unblocked users who transmitted at t+1. However, if t+1 is a noncollision slot, this last event is adjusted to empty. If, instead, t+1 is a collision slot, the event is adjusted to $(\{2_a^n, N_n^1+N_1\}, \{2_a^{k-1}, N_{k1}\};\ 2 \le k \le n, [M-N_n^1])$. The latter has no users in resolution mode 2^o, and it is as in (2). The procedure explained above is exhibited in figure II.5. From that figure, it is clear that any collision event is as in (2).

Event at t	Initial Generalized Event at t+1	Adjusted Event at t+1
$(\{2_a^k, N_k\};\ 1\le k\le n, [M])$; $\sum_{k=1}^{n} N_k \ge 2$	$(\{2_a^n, N_n^1\}, \{2_a^{k-1}, N_{k1}\}; 1\le k\le n, [M-N_n^1])$	empty ; if $N_n^1 + \sum_{k=1}^{n} N_{k1} \le 1$
		$(\{2_a^n, N_n^1+N_1\}, \{2_a^{k-1}, N_{k1}\};\ 2 \le k \le$ $[M-N_n^1])$; if $N_n^1 + \sum_{k=1}^{n} N_{k1} \ge 2$

Code

As in figure II.4

Figure II.5

Transitions of the Events in (2)

We now present a proposition whose proof is in appendix A.

Proposition 2

Let at some time instant t the collision event $(\{2_a^k, N_k\};\ 1 \le k \le n,\ [M])$ occur. Let for some k : $1 \le k \le n$ be N_{k1} users with codeword prefix $x_1(k)\ldots x_{n-k}(k)0$, and N_{k2} users with codeword prefix $x_1(k)\ldots x_{n-k}(k)1$. Then, none of the $\sum_{k=1}^{n} N_{k2}$ users and the users that are in withholding mode at time t become unblocked, before all the $\sum_{k=1}^{n} N_{k1}$ users do, and before all the users who become blocked in the mean time transmit successfully.

Let us now denote,

$L(\{2_a^k, N_k\};\ 1\le k\le n, [M])$: The expected number of slots needed for the resolution of the collision represented by the event $(\{2_a^k, N_k\};\ 1\le k\le n, [M])$ in (2), just after this collision has been observed.

(3)

If the event $(\{2_a^k, N_k\};\ 1 \le k \le n, [M])$ is not a collision event, then $L(\{2_a^k, N_k\};\ 1 \le k \le n, [M])$ is zero; where then $\sum_{k=1}^{n} N_k \le 1$. From the transitions in figure II.5, and due to the conclusions in proposition 2, we obtain in a straightforward manner recursions for the expected value in (3). Those recursions are given by expression (4) below. If t_o is the time instant when the event $(\{2_a^k, N_k\};\ 1 \le k \le n, [M])$ occurred, the parameter m in (4) denotes the number of previously unblocked users who transmitted at time t_o+1. If $m + \sum_{k=1}^{n} N_{k1} \le 1$, s denotes the number of previously unblocked users who transmit at t_o+2. If $m + \sum_{k=1}^{n} N_{k1} \ge 2$, and t is the time instant when the event $(\{2_a^n, m+N_1\}, \{2_a^{k-1}, N_{k1}\};\ 2 \le k \le n,\ [M-m])$ is resolved, the parameter s denotes the number of previously unblocked users who transmit at t+1. We observe that at time t+1 the $N_1 + \sum_{k=2}^{n} N_{k1}$ users have been added then to the users who are unblocked. This is due to proposition 2.

$$
\begin{array}{l}
L(\{2_a^k, N_k\}; 1 \le k \le n, [M]) = \\
\quad ;\ \sum_{k=1}^{n} N_k \ge 2
\end{array}
\begin{cases}
2 + L(\{2_a^n, s+N_{12}\}, \{2_a^{k-1}, N_{k2}\};\ 2 \le k \le n, [M + \sum_{k=1}^{n} N_{k1} - s]) \\
\qquad ;\ \text{if } m + \sum_{k=1}^{n} N_{k1} \le 1 \\[2ex]
2 + L(\{2_a^n, m+N_1\}, \{2_a^{k-1}, N_{k1}\};\ 2<k<n,\ [M-m]) \\
\quad + L(\{2_a^n, s\}, \{2_a^{k-1}, N_{k2}\};\ 2 \le k \le n, [M+N_1 + \sum_{k=2}^{n} N_{k1} - s]) \\
\qquad ;\ \text{if } m + \sum_{k=1}^{n} N_{k1} \ge 2
\end{cases}
\tag{4}
$$

Let us now consider the N_k users who are in resolution mode 2_a^k. Those users have a common codeword prefix $x_1(k)\ldots x_{n-k}(k)$, and they are at most 2^k. Let us define by $P(N_{k1}/N_k)$, the probability that given the number N_k, there are N_{k1} users whose codeword prefix is $x_1(k)\ldots x_{n-k}(k)0$. Then, N_{k2} will be equal to $N_k - N_{k1}$, and clearly,

$$P(N_{k1}/N_k) = \frac{\binom{2^{k-1}}{N_{k1}}\binom{2^{k-1}}{N_k - N_{k1}}}{\binom{2^k}{N_k}}\ ;\ \max\ (0, N_k - 2^{k-1}) \leq N_{k1} \leq \min\ (2^{k-1}, N_k) \qquad (5)$$

At this point, we conclude section II. In the next section, we will adopt certain assumptions on the transmission process per unblocked user, and we will subsequently study the performance of the CRLS protocol.

III. THE CRLS PERFORMANCE

As we saw in section II, if at some time instant t the event $(\{2^k_a, N_k\}; 1 \le k \le n, [M])$ occurs, then M denotes the number of users who are unblocked at t. To this point, we made no assumptions as to the transmission characteristics of those users. Here, we will assume, as in [6], that an unblocked user transmits with probability q per slot. Thus, if at some point in time the number of unblocked users is M, then the probability Q(m,M) that m users will transmit is given by the following expression.

$$Q(m,M) = \binom{M}{m} q^m (1-q)^{M-m} \; ; \; 0 \le m \le M \tag{6}$$

The implication behind the expression in (6) is that the unblocked users transmit independently. This is consistent with the independence assumption made at the beginning of this paper. Using expressions (4), (5), and (6), we can now express an equation for the expected value $L(\{2^k_a, N_k\}; 1 \le k \le n, [M])$. This equation is given by expression (7), and it relates the expected values of different events as in (2). It is not hard to see that the expression in (7) actually determines a system of linear equations, whose solution is the set of expected values as in (3). This set is determined by all the possibilities regarding the N_k ; $1 \le k \le n$ and M values. In general, it is possible that for some q values the linear system determined by (7) may have an either unbounded or negative solution, or both. A negative solution is, of course, unacceptable here. An unbounded solution translates to infinitely long transmission delays; thus it means instability. Therefore, we will search for those q values that provide a nonnegative and bounded solution for the linear system in (7). We observe that at least one such value exists; it corresponds to q = 0. Also, the probabilities Q(m,M) appear as coefficients in the linear system, and they are monotonically increasing with q, for q values such that $2^n q < 1$. It is thus easy to see that if there exists some positive value q_o such that $2^n q_o < 1$, and such that it provides a bounded and nonegative solution for the system in (7), then every $q : q < q_o$ will also provide such a solution.

$$L(\{2_a^k,N_k\};1\le k\le n,[M]) = 2 + \sum_{\substack{\max[0,N_k-2^{k-1}]\le m_k\le\min[2^{k-1},N_k] \\ ;\ 1\le k\le n}} P(\{N_k,m_k\};1\le k\le n) \Bigg\{U\Big(1-\sum_{k=1}^{n} m_k\Big)\Bigg[\sum_{m=0}^{1-\sum_{k=1}^{n} m_k} Q(m,M)\Bigg]$$

$$\cdot\sum_{s=0}^{M+\sum_{k=1}^{n} m_k} Q\Big(s,M+\sum_{k=1}^{n} m_k\Big)\cdot L\Big(\{2_a^n,s+N_1-m_1\},\{2_a^{k-1},N_k-m_k\};\ 2\le k\le n,[M+\sum_{k=1}^{n} m_k-s]\Big)$$

$$+\ U\Big(M-\max[0,2-\sum_{k=1}^{n} m_k]\Big)\sum_{m=\max\ [0,2-\sum_{k=1}^{n} m_k]}^{M} Q(m,M)\cdot L\Big(\{2_a^n,m+N_1\},\{2_a^{k-1},m_k\};\ 2\le k\le n,[M-m]\Big)$$

$$+\ U\Big(M-\max[0,2-\sum_{k=1}^{n} m_k]\Big)\Bigg[\sum_{m=\max[0,2-\sum_{k=1}^{n} m_k]}^{M} Q(m,M)\Bigg]\cdot$$

$$\cdot\sum_{s=0}^{M+N_1+\sum_{k=2}^{n} m_k} Q\Big(s,M+N_1+\sum_{k=2}^{n} m_k\Big)\cdot L\Big(\{2_a^n,s\},\{2_a^{k-1},N_k-m_k\};\ 2\le k\le n,[M+N_1+\sum_{k=2}^{n} m_k-s]\Big)\Bigg\} \quad (7)$$

; where $Q(m,M)$ is given by (6), $0 \le N_k \le 2^k$; $1 \le k \le n$,

$0 \le M \le 2^n - \sum_{k=1}^{n} N_k$, $\sum_{k=1}^{n} N_k \ge 2$, $L(\{2_a^k,N_k\};\ 1 \le k \le n,\ [M]) = 0$; if $\sum_{k=1}^{n} N_k \le 1$, and:

$$U(x) \triangleq \begin{cases} 1 ; & x \ge 0 \\ 0 ; & x < 0 \end{cases}$$

$$P(\{N_k, m_k\};\ 1 \le k \le n) \triangleq \prod_{k=1}^{n} \frac{\binom{2^{k-1}}{m_k}\binom{2^{k-1}}{N_k - m_k}}{\binom{2^k}{N_k}} \tag{8}$$

Definition 2

Given 2^n users, the throughput $2^n q_n$ of the CRLS protocol is such that every q less than q_n provides a bounded and nonnegative solution for the linear system in (7), and no q value larger than q_n does.

For given q value, the solution of the linear system in (7) can be obtained numerically. The throughput $2^n q_n$ can be also found numerically through the trials of different q values. We will present such numerical results in section IV. At this point, we will search for a lower bound on the throughput. An upper bound is provided by the throughput of the dynamic protocol in [6].

The system in (7) has the form $x_k = \sum_j a_{kj}\, x_j + 2$; varying k, where $0 \le a_{kj} < \infty$; $\forall\, k, j$, and where the unknowns x_k represent the expected values $L(\{2_a^k, N_k\};\ 1 \le k \le n, [M])$, for various choices of the N_k ; $1 \le k \le n$ and M values. We now present a lemma, that is basically theorem 2 in [14], and whose easy proof is included in this last reference.

Lemma 1

Let $\{b_j\}$ be a set of positive and bounded numbers such that $b_k > \sum_j a_{kj}\, b_j + 2$; $\forall\, k$. Then, the linear system $x_k = \sum_j a_{kj}\, x_j + 2$ has a nonnegative and bounded solution.

Lemma 1 provides a sufficient condition for the existence of a nonnegative and bounded solution for the linear system $x_k = \sum_j a_{kj}\, x_j + 2$; varying k. We notice that in our case the coefficients $\{a_{kj}\}$ are functions of the selected q value. Thus, if we select a set $\{b_j\}$ of positive and bounded numbers, and we require that it satisfy the condition in the lemma for the linear system in (7), then any restrictions that may result regarding the set $\{a_{kj}\}$ of coefficients, will reflect restrictions on the q values. Furthermore, since the lemma provides a sufficient but not necessary condition, the restrictions on the q values will represent then lower bounds on the throughput. Let us now consider the numbers $\alpha \sum_{k=1}^{n} N_k - c$, for $\sum_{k=1}^{n} N_k \geq 2$; where $\alpha \geq c$, $\alpha > 0$, and $c > 0$. For 2^n users, we then have that $0 < 2\alpha - c \leq \alpha \sum_{k=1}^{n} N_k - c \leq \alpha\, 2^n - c < \infty$; $\forall \sum_{k=1}^{n} N_k \geq 2$. Thus, the above numbers comprise a legitimate set $\{b_j\}$. In the right hand part of the equation in (7), we substitute now each expected value $L(\{2_a^k, P_k\};\ 1 \leq k \leq n,\ [T])$ with $\sum_{k=1}^{n} P_k \geq 2$, by the number $\alpha \sum_{k=1}^{n} P_k - c$. Then, we require that the resulting expression be less than $\alpha \sum_{k=1}^{n} N_k - c$. We impose this requirement for every $\sum_{k=1}^{n} N_k$ value that is larger than one, and for every M. As a result, we obtain a q_n^* number, such that the system in (7) has a nonnegative and bounded solution for at least every q less than q_n^*. We include our derivations in appendix B. Here, we present the results.

Let us define the following *functions*.

$$G_o(2^n,m,q) = 1-2^{-m-1}q[2^n+m]-2^{-2^n}\left\{m[1+q-q2^n]+q2^n+q2^{2n}+(1-q)^{2^n}\right\};\ m = 0,1,2 \tag{9}$$

$$G_1(2^n-1,m,q) = 1-q-2^{-m-1}q[2^n+m+1]-2^{-2^n+1}\left\{1+q(m^2-3)-q^2(m-1)-2^n q(m-1)(2-q) +2^{2n}+(1-q)^{2^n}\right\};\ m = 0,1,2 \tag{10}$$

$$G_2(2,m,q) = 1-2^{n+1}q+q(4-m)-q(2-m)2^{-m-1}-2^{-2}\left\{2(2-m)q+(1+q^2)(1-q)^{2^n-2}\right\};\ m = 0,1,2 \tag{11}$$

It has been shown in appendix B, that all the above functions have unique positive zeros for $m = 0,1,2$. Let us denote by q_{jm} ; $j = 0,1,2$; $m = 0,1,2$ the zero of the function $G_j(\cdot,m,q)$. Let us also denote $q_n^* = \min(q_{jm}\ ;\ j = 0,1,2\ ;\ m = 0,1,2)$. Then, we can express the following theorem, whose proof is in appendix B.

<u>Theorem 1</u>

Given 2^n users, the number $2^n q_n^*$ is a lower bound on the throughput $2^n q_n$.

Given n, the lower bound $2^n q_n^*$ can be obtained numerically. We include such numerical results in section IV.

Given 2^n users, let us now suppose that at some time instant t all the users are unblocked. Such a time instant exists. If the system starts operating at t_o, one such time instant is $t_o - 1$. Let now N users transmit at t+1. Then, the event (as in (2)) at t+1 will be $(\{2^n_a, N\}, [2^n - N])$; where $2^n - N$ users are unblocked. If each unblocked user transmits with probability q, the above event will occur with probability $\binom{2^n}{N} q^N (1-q)^{2^n - N}$. If N = 1, the corresponding user will be unblocked again at t+1. If $N \geq 2$, a collision will occur; the expected number of slots for its resolution will be $1 + L(\{2^n_a, N\}, [2^n - N])$. Let us define.

$D(2^n, q)$: The expected number of slots for the resolution of some collision at t+1, given 2^n users, given that at t all users are unblocked, and given that each unblocked user transmits with probability q. (12)

The quantity $D(2^n, q)$ above is parallel to the expected delay E{delay} in [6], and it is clearly given by the following expression.

$$D(2^n, q) = 1 + \sum_{N=2}^{2^n} \binom{2^n}{N} q^N (1-q)^{2^n - N} L(\{2^n_a, N\}, [2^n - N]) \tag{13}$$

The expected values $L(\{2^n_a, N\}, [2^n - N])$ in (13) are given by the solution of the linear system in (7). In section IV, we will present some numerically computer values of $D(2^n, q)$, for various n and q choices.

For 2^n users, the stability region of the CRLS consists of those $2^n q$ values that provide a bounded and nonnegative solution for the linear system in (7). For such $2^n q$ values, the lengths $L(\{2^n_a, N\}, [2^n - N])$; $2 \leq N \leq 2^n$ will be finite; thus collisions that are signified by events $(\{2^n_a,N\},[2^n-N])$ will end in finite time. Furthermore, due to proposition 2, when such collisions end, all the users will be unblocked. In conclusion, the quantity $D(2^n,q)$ in (13) provides the expected length of independent episodes, and $D(2^n,q)-1$ bounds from above the expected per packet delays (measured in slots).

IV. NUMERICAL RESULTS

We first found numerically the unique real roots of the polynomials $G_0(2^n,m,q)$, $G_1(2^n,m,q)$, and $G_2(2,m,q)$ in (9), (10), and (11) respectively, for the computation of the lower bound in theorem 1. We found that this lower bound is provided in all cases by the unique real root of the function $G_2(2,m,q)$, at m=0 and m=2. We list our results in table IV.1, together with the lower bound of the CRLS for asymptotically many users. The latter is derived from a modification of the protocol in [14]*;where the modification consists of eliminating the skip step, when a collision is followed by an either successful or empty slot. The lower bounds in table IV.1 are such that, for every 2^nq value in $[0,2^nq_n^*]$ (for 2^n users), bounded delays are induced. Therefore, for 2^n users, the CRLS is stable at least in the 2^nq region $[0,2^nq_n^*]$. We notice from [16] that for asymptotically many users the minimum stability region for the CRLS is [0,.36], while the same region for the dynamic protocol in [6] is [0,.429]. Thus, the gain as one moves from the CRLS to the dynamic protocol in [6] is less than seven percent. This is significant, since the former implies limited feedback sensing, in contrast to the latter. In fact, the CRLS stability region [0,.36] is larger than that of the nondynamic tree protocol in [6] and [5]. The latter is [0,.346].

# of Users	$2^nq_n^*$
2^3;n=3	.47192
2^4;n=4	.44336
2^5;n=5	.43008
2^6;n=6	.42368
$n\to\infty$	.36

Table IV.1
Lower Bounds on the CRLS Throughput

* The lower bound of the protocol in [14] is .384.

We should emphasize here that the performance of the CRLS should be compared with the performance of the dynamic, rather than the nondynamic tree protocol in [6]. Indeed, the CRLS has implicit dynamic characteristics, due to the fact that unblocked users may enter the system without delay. For the same reason, the expected number $D(2^n,q)-1$ in (13) represents an upper bound on the delays induced by the CRLS, and it is comparable to E{delay} in [6], rather than to the expected number of algorithmic steps for the resolution of an initial collision. We computed the numbers $D(2^n,q)-1$ numerically from the linear system in (7), for various n and q values. We plot our results in figure IV.1, together with the expected delays of the nondynamic and the dynamic tree protocols in [6], for 2^6 users (the delays in figure 4.1 in [6] are normalized by the probability of zero arrivals; ours are not). In table IV.2, we list the $D(2^n,q)-1$ numbers, for various 2^nq and n values. From figure IV.1, we observe that for 2^6 users, the delays induced by the CRLS are lower than those induced by the nondynamic tree in [6]. That was expected, since for asymptotically many users the stability region of the CRLS is larger than the stability region of the nondynamic tree. For every n, the throughput of the CRLS corresponds to this 2^nq value, where the $D(2^n,q)-1$ curve approaches asymptotically large numbe[rs]. Comparing these values with the numbers in table IV.1, we observe that the lower bounds $2^nq_n^*$ approach the throughput, as n increases. This is so, because the bounds we used in the proof of theorem 1 become tighter as n increases.

2^nq	⟶	.1	.2	.3	.4	.47	.5	.6	.7	.75
$D(2^n,q)-1$	n=3	.0174	.0923	.2870	.7389	...	1.7543	4.019	9.0944	13.6820
	n=4	.0208	...	.3856	...	...	3.3333	15.759	61.2730	...
	n=5	.0233	...	.5382	...	...	15.9070	159.490	...	...
	n=6	...	.1470	.6559	...	39.762	101.320	...	...	...

Table IV.2

Upper Bounds-CRLS Transmission Delays

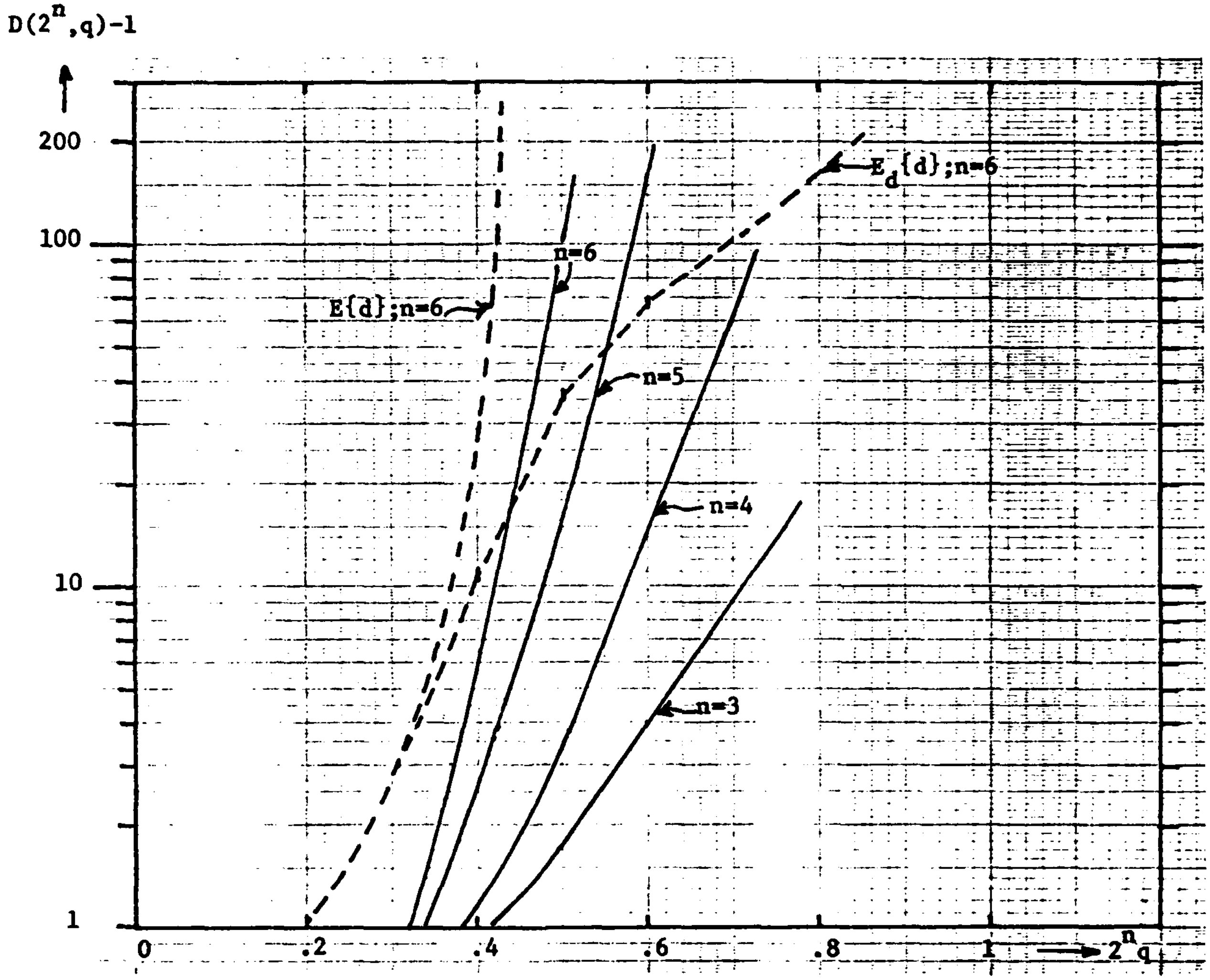


Figure IV.1

Upper Bounds on Transmission Delays for the CRLS

Code

E{d} : E{delay} in [6], nondynamic tree
E_d {d}: E{delay} in [6], dynamic tree

Solid Lines: CRLS performance
Dashed Lines: Tree protocols from [6]

The definition of throughput we have used corresponds to the transmission rate of the system, rather than the arrival rate. This was also done in [6], and there is a reason for it. Indeed, this measure of throughput is indicative of the change that occurs (performancewise) as the number of users in the system increases. In fact, for asymptotically many users, the throughput that corresponds to the transmission rate is identical to the throughput that corresponds to the arrival rate. Then, each successful transmission basically signifies a single bursty user, who then leaves the system. For finite number of users, it is necessary, however, that any conclusions on the transmission rate be translated to possible restrictions on the arrival rate. Consider 2^n users, let the probability of transmission per unblocked user be q, and let the transmission rate $2^n q$ be within the stability region of the CRLS. Let the arrival process of each of the 2^n identical users be Bernoulli with parameter p. Then, the arrival rate of the system is $2^n p$, and it is maintained if it is the same with the transmission rate $2^n q$. It is not maintained, and monotonically increasing accumulations in the buffers of the users occur, if $p>q$. The arrival rate is the same with the transmission rate, if at most one new arrival per user occurs (on the average), from a time instant when the user becomes blocked to the time his blocked packet is successfully transmitted. Since the number $D(2^n,q)-1$ in (13) is an upper bound to all such blocked-unblocked time intervals, a sufficient condition for the satisfaction of the above property is given by the following expression.

$$p[D(2^n,q)-1]=q[D(2^n,q)-1] \leq 1 \;;p=q \tag{14}$$

The condition in (14) provides a lower bound on the admissible arrival rates;that is the arrival rates that are maintained by the CRLS. For 2^n users, let us denote this lower bound $2^n q^*_{an}$, and let $2^n q_{an}$ be the true maximum arrival rate that is maintained by the protocol. Then, $2^n q^*_{an} < 2^n q_{an} < 2^n q_n$; where $2^n q_n$ is the throughput in definition 2, and where at least all the arrival rates in $[0,2^n q^*_{an}]$ are maintained. In table IV.3, we list numerically computed values of the quantities $2^n q^*_{an}$; where for $n\to\infty$, $2^n q^*_n=2^n q^*_{an}$ $=.36$. In the same table, we also list the highest maintainable input rates T_n and T^d_n

from the nondynamic and the dynamic tree protocols in [6] respectively, for $n=6$ and $n\to\infty$. For $n=6$, the quantities T_6 and T_6^d have been computed from the condition $qE\{delay\} \leq 1$, as imposed on both the nondynamic and the dynamic tree protocols. Comparing the results in tables IV.1 and IV.3, we observe that as n increases the bounds $2^n q_n^*$ and $2^n q_{an}^*$ approach each other. This was expected because both the computed bounds are more accurate for large number of users, and because as n increases the throughput and the maintainable arrival rate approach each other. Finally, we observe from table IV.3, that for 2^6 users the lower bound $2^6 q_{a6}^*$ is larger than T_6 and smaller than T_6^d, as expected.

# of Users	$2^n q_{an}^*$	T_n	T_n^d
2^3;n=3	.725	...	...
2^4;n=4	.630	...	...
2^5;n=5	.560	...	...
2^6;n=6	.515	.430	.675
$n\to\infty$	.36	.346	.429

Table IV.3

Lower Bounds on the Maintainable by the CRLS Input Rates

We will conclude this section by pointing out that the CRLS does not utilize the distinction between empty and successfully busy slots, in contrast to the protocol in [14] and [17]. This characteristic makes it robust in the presence of channel errors that prevent such distinction; it also makes the CRLS usable in Spread Spectrum, where due to the low power signals the above distinction is not feasible.

V. CONCLUSIONS

In this paper, we proposed and analyzed a collision resolution protocol (CRLS), for Bernoulli independent users and limited feedback sensing. Our protocol is different than the protocol in [14] and [17] (for finitely many users in the second), since it eliminates the skip step in the latter, when a collision is followed by a noncollision slot. This elimination makes the CRLS robust in the presence of channel errors. Indeed, although we have assumed here that the transmission channel is errorless, it can be easily shown that when single direction channel errors occur, the CRLS eventually correc them, at the expense of additional delays. The protocol in [14] and [17] does not; it leads to deadlocks,instead. We should point out that the protocols in [6] are also robus in the presence of channel errors, and so are the protocols in [9] and [16]. The latter is basically the CRLS, for asymptotically many users.

The CRLS has dynamic characteristics, as the dynamic tree protocol of Capetanakis in [6]. For that reason, it performs better than the nondynamic tree protocol in [6], despite the fact that full feedback sensing is used in the latter. In addition, the CRLS performance is not much worse than that of the dynamic tree protocol in [6]. Asympt tically (for infinitely many users), its throughput is less than seven percent lower than the throughput of the dynamic tree protocol. For any number of users, the delays induced by the CRLS are comparable to those induced by the dynamic protocol in [6],and they are lower than those induced by the nondynamic protocol. The CRLS is also easy to implement, it operates in a distributed fashion, and it eliminates the undesirable requirement that users sense the feedback constantly even if unblocked or empty.

The dynamic characteristics of the CRLS may be further enhanced if a dynamic tree search as in [6] is employed. Then, a newly blocked user initiates his tree search at a depth K higher than the root depth, he still interprets a leaf collision as a root colli sion, and he visits the tree nodes at depth K sequentially, from top to bottom (fig.II. The resulting protocol will be again robust in the presence of channel errors, and it i expected to have throughput higher than that of the dynamic tree protocol in [6]. We ar currently working on this protocol.

References

[1] N. Abramson, "The ALOHA System-Another Alternative for Computer Communications," AFIPS Conf. Proc., Fall Joint Comp. Conf. Vol. 37, pp. 281-285, 1970.

[2] L. G. Roberts, "ALOHA Packet System with and without Slots and Capture," ASS Note 8, ARPA Network Info. Ctr., Stanford Res. Inst., Stanford, CA., June 1972.

[3] L. Kleinrock and F. A. Tobagi, "Packet Switching in Radio Channels: Part I - Carrier Sense Multiple-Access Modes and Their Throughput-Delay Characteristics," IEEE Trans. Comm., Vol. COM-23, pp. 1400-1416, Dec. 1975.

[4] J. F. Hayes, "An Adaptive Technique for Local Distribution," IEEE Trans. Comm., Vol. COM-26, pp. 1178-1186, August 1978.

[5] J. I. Capetanakis, "Tree Algorithms for Packet Broadcast Channels," IEEE Trans. Inf. Th., Vol. IT-25, pp. 505-515, Sept. 1979.

[6] J. I. Capetanakis, "Gen ralized TDMA: The Multi-Accessing Tree Protocol," IEEE Trans. Comm., Vol. COM-27, pp. 1476-1484, Oct. 1979.

[7] B. S. Tsybakov and V. A. Mikhailov, "Free Synchronous Access of Packets to a Broadcast Channel with Feedback," Probl. Peredachi Inf., Vol. 14, No. 4, pp. 32-59, 1978.

[8] B. S. Tsybakov, "Resolution of a Conflict of Known Multiplicity", Probl. Peredachi Inf., Vol. 16, No. 2, pp. 69-82, 1980.

[9] L. Georgiadis and P. Papantoni-Kazakos, "A Collision Resolution Protocol for Random Access Channels with Energy Detectors," IEEE Trans. Comm., Vol. COM-30, pp. 2413-2420, Nov. 1982.

[10] L. W. Hansen and M. Schwartz, "An Assigned-Slot Listen-Before Transmission Protocol for A Multiaccess Data Channel," IEEE Trans. Comm., Vol. COM-27, pp. 846-857, June 1979.

[11] L. Kleinrock and M. Scholl, "Packet Switching in Radio Channels: New Conflict-Free Multiple Access Schemes for a Small Number of Data Users," Proceedings ICC Conf., June 1977, pp. 22.1-105-22.1-111.

[12] J. W. Mark, "Global Scheduling Approach to Conflict-Free Multiaccess Via a Data Bus," IEEE Trans. Comm., Vol. COM-26, pp. 1342-1352.

[13] G. D. Marcus and P. Papantoni-Kazakos, "Dynamic Scheduling Protocols for a Multiple-Access Channel," IEEE Trans. Comm., submitted. Also, part of work in Proceedings, 1981 ICC, Denver, June,

[14] B. S. Tsybakov and N. D. Vvedenskaya, "Random Multiple-Access Stack Algorithm," Probl. Peredachi Inf., Vol. 16, No. 3, pp. 80-94, 1980.

[15] M. G. Hinchyj and R. Gallager, "Multiaccess of a slotted channel by Finitely Many Users," Proceedings, NTC' 81, New Orleans, Nov., pp. D4.2.1-D4.2.7.

[16] L. Merakos, " Limited Sensing Random Multiple Access Using Binary Feedback," 1983 Johns Hopkins Conference on Information Sciences and Systems.

[17] P.Papantoni-Kazakos and G.D.Marcus, "Random Access with Limited Sensing," 21st IEEE Conference on Decision and Control,Dec. 1982.

APPENDIX A

Proof of Proposition 1

We will prove the proposition by induction in time. Let us denote by t_o a time instant such that at time t_o-1 all the 2^n users are unblocked, and at t_o some $N_{t_o} \geq 2$ users become blocked. Then, all the N_{t_o} users initiate simultaneously the CRLS, they are all placed at node R, and the event at time t_o is $(\{2_a^n, N_{t_o}\}, [2^n - N_{t_o}])$. Among the N_{t_o} users, let there be N_{t_o1} with first codeword prefix 0, and N_{t_o2} with first codeword prefix 1. Then, at time t_o+1 the N_{t_o1} users move to resolution mode 2_a^{n-1}, and place themselves at node 0. At the same time, the N_{t_o2} users move to resolution mode $2_{w_e}^{n-1}$ and place themselves at node 1. Also, some previously unblocked users may transmit at time t_o+1. If the number of those users is N_{t_o+1}, they will all be placed at the node R. Thus, the event at time t_o+1 is $(\{2_a^n, N_{t_o+1}\}, \{2_a^{n-1}, N_{t_o1}\}, \{2_{w_e}^{n-1}, N_{t_o2}\}, [2^n - N_{t_o} - N_{t_o+1}])$. Therefore, the proposition clearly holds at times t_o and t_o+1.

Let now t be some time instant beyond t_o+1, such that there is no time instant in $[t_o, t]$ at which all users are unblocked. Let us accept that the proposition holds at all time instants in $[t_o, t]$. Let the event at time t be $(\{2_a^k, N_k\}; 0 \leq k \leq n, \{2_{w_e}^k, P_k\}; 0 \leq k \leq n-1, [M])$. Given some k less than n, let the N_k users be placed at node $y_1(k) \ldots y_{n-k}(k)$; let the P_k users be placed at node $x_1(k) \ldots x_{n-k-1}(k)\ 1$. The N_n users are placed at the node R. There are three possibilities regarding slot t. It will be either empty, or successful, or a collision slot. Let t be a collision slot. For given $k > 0$, let there be N_{k1} users with codeword prefices $y_1(k) \ldots y_{n-k}(k)\ 0$, and N_{k2} users with codeword prefices $y_1(k) \ldots y_{n-k}(k)\ 1$; where $N_{k1} + N_{k2} = N_k$. Then, at time t+1, the N_{k1} users place themselves at node $y_1(k) \ldots y_{n-k}(k)\ 0$, and move to resolution mode 2_a^{k-1}. At the same time, the N_{k2} users place themselves at node $y_1(k) \ldots y_{n-k}(k)\ 1$, and move to state $(2_{w_e}^{k-1}, y_1(k) \ldots y_{n-k}(k)\ 0)$. The user (if any) who is in resolution mode 2_a^o at time t, moves at time t+1 either to resolution mode 2_a^{n-1} and to node 0; if the first bit in his codeword is 0, or to state $(2_{w_e}^{n-1},0)$ and to node 1; if the first bit in his codeword is 1. At the same time, the P_k; $0 < k < n$ users remain at node $x_1(k)\ldots x_{n-k-1}(k)\ 1$, and move to state $(2_w^k,\ x_1(k)\ldots x_{n-k-1}(k)\ 01\ldots10)$. The user (if any) in resolution mode $2_{w_e}^o$, moves to state $(2_a^{n-1},0)$; if his first codeword bit is 0. He moves to state $(2_{w_e}^{n-1},0)$, otherwise. It is thus clear that the proposition holds at time t+1, if t is a collision slot. Also, since we have accepted the proposition in $[t_o,t]$, it is also clear that $x_1(0)\ldots x_{n-1}(0) = y_1(0)\ldots y_{n-1}(0)$; thus the first n-1 bits of users P_o and N_o are identical. This last assumption clearly holds at time t+1, and it implies that user's N_o codeword is $x_1(0)\ldots x_{n-1}(0)\ 0$, while user's P_o codeword is $x_1(0)\ldots x_{n-1}(0)\ 1$. The event at time t+1 becomes:

$$(\{2_a^n,N\},\{2_a^{n-1},N_{n1}+P_o+N_o\},\{2_a^{k-1},N_{k1}\};\ 1\le k\le n-1,\{2_{w_e}^{k-1},N_{k2}\};\ 1\le k\le n,[M-N])$$

; if the first codeword bit of users P_o and N_o is 0, and N previously unblocked users transmit at t+1.

$$(\{2_a^n,N\},\{2_a^{k-1},N_{k1}\};\ 1\le k\le n,\{2_{w_e}^{n-1},N_{n2}+P_o+N_o\},\{2_{w_e}^{k-1},N_{k2}\};\ 1\le k\le n-1,[M-N])$$

; if the first codeword bit of users P_o and N_o is 1, and N previously unblocked users transmit at t+1.

Let now t be a either empty or successful slot. Then, as can be easily concluded from figure II.2, the users who at t were in resolution mode $2_{w_e}^k$ will move at time t+1 to resolution mode 2_a^k. In fact, only those plus any previously unblocked and now transmitting users will be the only active users at time t+1. It is also clear from the state transitions in figure II.2, that those users who at time t were in either state $(2_w^k,x_1\ldots x_{n-k-1}00)$; $1 \le k \le n-1$ or in some state $(2_w^k,x_1\ldots x_{n-k-1}01\ldots10)$; $2 \le k \le n-1$, will move at t+1 to states $(2_{w_e}^k,x_1\ldots x_{n-k-1}01)$; $1 \le k \le n-1$ and $(2_{w_e}^k,x_1\ldots x_{n-k-1}01\ldots11)$; $2 \le k \le n-1$ respectively. Furthermore, there are no other users who at time t+1 will be in resolution mode 2^k_w; $0 \leq k \leq n-1$. But it is then clear (from the transitions in figure II.2) that the users who at t are in states $(2^k_w, x_1 \ldots x_{n-k-1}00)$; $1 \leq k \leq n-1$, $(2^k_w, x_1 \ldots x_{n-k-1}01 \ldots 10)$; $2 \leq k \leq n-1$ must have been at t-1 in states $(2^k_{w_e}, x_1 \ldots x_{n-k-1}0)$; $1 \leq k \leq n-1$, $(2^k_{w_e}, x_1 \ldots x_{n-k-1}01 \ldots 1)$; $2 \leq k \leq n-1$ respectively, and slot t-1 must have been a collision slot. The proposition clearly holds now at time t+1, if t is an either empty or a successful slot. Furthermore, if the event at time t-1 is $(\{2^k_a, N^{(1)}_k\};\ 0 \leq k \leq n, \{2^k_{w_e}, P^{(1)}_k\};\ 0 \leq k \leq n-1, [M^{(1)}])$, we will find the event at t+1, distinguishing between different cases.

Case: slots t-1 and t both either empty or successful

event at t+1:

$(\{2^n_a, N\}, \{2^k_a, P_k\};\ 0 \leq k \leq n-1, [M-N])$; if N previously unblocked users transmitted at t+1

Case: slot t-1 collision slot, slot t either empty or successful

event at t+1:

$(\{2^n_a, N\}, \{2^k_a, P_k\};\ 0 \leq k \leq n-1, \{2^k_{w_e}, P^{(1)}_k\};\ 1 \leq k \leq n-1, [M-N])$; if N previously unblocked users transmitted at t+1

Proof of Proposition 2

We will prove the proposition by inverse induction in time.

The proposition clearly hodls if the event is such that there are only two active users, one with codeword prefix $x_1(k_1) \ldots x_{n-k_1}(k_1)0$, and one with codeword prefix $x_1(k_2) \ldots x_{n-k_2}(k_2)1$; for some k_1, k_2, and if at the time the first transmits no previously unblocked user transmits also.

Let now the collision event $(\{2^k_a, N_k\};\ 1 \leq k \leq n, [M])$ occur at time t. If $N^1_n + \sum_{k=1}^{n} N_{k1} \leq 1$, the $\sum_{k=1}^{n} N_{k1}$ users and the newly blocked at t+1 users N^1_n become unblocked at t+1, and the proposition holds. Let, instead, $N^1_n + \sum_{k=1}^{n} N_{k1} \geq 2$. Then the event at t+1 becomes $(\{2^n_a, N^1_n + N_1\}, \{2^{k-1}_a, N_{k1}\};\ 2 \leq k \leq n, [M-N^1_n])$, and at the same time the $\sum_{k=1}^{n} N_{k2}$ users move to withholding mode. Let the proposition hold from time t+1 and on. Then, the $N_n^1+N_1+\sum_{k=2}^{n} N_{k1}$ users will be unblocked before the $\sum_{k=1}^{n} N_{k2}$ users are, and before any additional users that are in withholding mode at t+1 transmit successfully. But the latter (if any) were already in withholding mode at t. Also, among the N_1 users, the one with codeword prefix $x_1(1)\ldots x_{n-1}(1)1$ will not be unblocked before $x_1(1)\ldots x_{n-1}(1)0$ is. This is so, because both the N_1 users (if they are two) perform simultaneous tree search, starting at t+2.

The proof of the proposition is now complete.

APPENDIX B

Proof of Theorem 1

We select arbitrary positive numbers α, c such that $\alpha \geq c$. Then, in the right hand part of the equation in (7), we substitute each $L(\{2^k_a, Q_k\};\ 1 \leq k \leq n, [S])$ with $\sum_{k=1}^{n} Q_k \geq 2$, by $\alpha \sum_{k=1}^{n} Q_k - c$. We require that the resulting expression is strictly less than $\alpha \sum_{k=1}^{n} N_k - c$. As a result, we obtain, in a straightforward fashion, three inequalities that correspond to $M = 0$, $M = 1$, $M \geq 2$, in the expected value $L(\{2^k_a, N_k\};\ 1 \leq k \leq n, [M])$ in (7). To express those inequalities in a simple form, we first define:

$$P_o \triangleq P(\{N_k, 0\};\ 1 \leq k \leq n)$$

$$P_1 \triangleq \sum_{\{m_k\}\ :\ \sum_{k=1}^{n} m_k = 1} P(\{N_k, m_k\};\ 1 \leq k \leq n)$$

$$P_{11} \triangleq \sum_{\{m_k\}\ :\ \sum_{k=2}^{n} m_k = 1,\ m_1 = 0} P(\{N_k, m_k\};\ 1 \leq k \leq n)$$

$$T_i \triangleq \sum_{\{m_k\}\ :\ \sum_{k=1}^{n} m_k \geq 2,\ \sum_{k=2}^{n} m_k = i} P(\{N_k, m_k\};\ 1 \leq k \leq n) \tag{B.1}$$

$$P_T \triangleq \sum_{2-m_1 \leq i \leq \sum_{k=2}^{n} N_k - 2} T_i$$

$$E_T = \sum_{2-m_1 \leq i \leq \sum_{k=2}^{n} N_k - 2} i\, T_i$$

; where $P(\{N_k, m_k\};\ 1 \le k \le n)$ is given by expression (8).

To simplify our notation further, we denote $\sum_{k=1}^{n} N_k$ by s. Then, from the expressions in (B.1) we clearly have:

$$P_o + P_1 + P_T + T_{s-N_1-1} + T_{s-N_1} = 1 \tag{B.2}$$

We now define:

$$\begin{aligned}
f_o(q) &\triangleq -qE_T + P_1(1-q) - P_T\, qN_1 - T_{s-N_1-1}[q(s-1) - (1-q)^{s-1}] - T_{s-N_1} q\, s[1 - (1-q)^{s-1}] \\
g_o(q) &\triangleq 1 - P_o - P_1 - T_{s-N_1-1}(1-q)^{s-1} - T_{s-N_1}(1-q)^{s-1}\,(1-q+qs) \\
f_1(q) &\triangleq -qE_T + P_{11}(1-2q) - P_o q - P_1[4q + q^2(N_1-1)-1] - P_T q(N_1+2) - \\
&\quad - T_{s-N_1-1}[q(s+1) - (1-q)^s] - T_{s-N_1}[q(s+2) - N_1 - (s+1)q(1-q)^s] \\
g_1(q) &\triangleq P_1 q + P_T + T_{s-N_1-1}[1-(1-q)^s] + T_{s-N_1}[1-(1-q)^{s+1}-(s+1)q(1-q)^s] \\
f_2(q,M) &\triangleq -2qM - qN_1 - qE_T + P_o q(1-q)^{M-1}[N_1(1-q)+M(1+qN_1)] + P_1(1+qN_1-q)(1-q)^M \\
&\quad -P_{11}[q-(1-q)^M] - T_{s-N_1-1}[q(s-N_1-1) - (1-q)^{M+s-1}] \\
&\quad + T_{s-N_1}[(1+q)N_1 + (M+s)q(1-q)^{M+s-1}] \\
g_2(q,M) &\triangleq 1 - P_o(1-q)^{M-1}(1-q+Mq) - P_1(1-q)^M - T_{s-N_1-1}(1-q)^{M+s-1} \\
&\quad - T_{s-N_1}(1-q)^{M+s-1}\,[1-q+(M+s)q]
\end{aligned} \tag{B.3}$$

It can be easily shown that the functions in (B.3) have the following properties. At $q = 0$, they are all positive. The functions $f_o(q)$, $f_1(q)$, and $f_2(q,M)$ are monotonically decreasing with increasing q, becoming negative at $q = 1$. The functions $g_o(q)$, $g_1(q)$, and $g_2(q,M)$ are monotonically increasing with increasing q. The three inequalities we mentioned in the first paragraph of this proof are as follows.

$$\alpha\ f_o(q) > -\ c\ g_o(q) \quad ;\ \text{for } M = 0 \tag{B.4}$$

$$\alpha\ f_1(q) > -\ c\ g_1(q) \quad ;\ \text{for } M = 1 \tag{B.5}$$

$$\alpha\ f_2(q,M) > -\ c\ g_2(q,M) \quad ;\ \text{for } M \geq 2 \tag{B.6}$$

We are seeking those q values that satisfy all the three inequalities above. Let q_0^o, q_1^o, q_2^o be the zeros of the functions $f_o(q)$, $f_1(q)$ and $f_2(q,M)$ respectively. Then, since $\alpha > 0$, $c > 0$, and $g_o(q) > 0$; $\forall q$, $g_1(q) > 0$; $\forall q$, $g_2(q,M) > 0$; $\forall q$, $M \geq 2$, inequalities (B.4), (B.5), and (B.6) are clearly satisfied for $q \leq q_0^o$, $q \leq q_1^o$, and $q \leq q_2^o$ respectively. We will now examine the possibility of having q values beyond the above regions, that still satisfy the inequalities in (B.4), (B.5), and (B.6). We rewrite then the inequalities as follows.

$$\begin{aligned} c[g_o(q) + \tfrac{\alpha}{c}\ f_o(q)] &> 0;\ \text{for } q > q_0^o,\ \tfrac{\alpha}{c} \geq 1,\ c > 0 \\ c[g_1(q) + \tfrac{\alpha}{c}\ f_1(q)] &> 0;\ \text{for } q > q_1^o,\ \tfrac{\alpha}{c} \geq 1,\ c > 0 \\ c[g_2(q,M) + \tfrac{\alpha}{c}\ f_2(q,M)] &> 0;\ \text{for } q > q_2^o,\ \tfrac{\alpha}{c} \geq 1,\ c > 0 \end{aligned} \tag{B.7}$$

Within the considered q regions above, the functions $f_o(q)$, $f_1(q)$, and $f_2(q,M)$ are negative. It is then clear, that the highest q values that satisfy the inequalities in (B.7) correspond to $\alpha, c : \frac{\alpha}{c} = 1$. As a conclusion from all the above, we are finally searching for those q values that satisfy all the three inequalities below.

$$F_o \triangleq g_o(q) + f_o(q) > 0 \tag{B.8}$$

$$F_1 \triangleq g_1(q) + f_1(q) > 0 \tag{B.9}$$

$$F_2(q,M) \triangleq g_2(q,M) + f_2(q,M) > 0;\ \forall\ M \geq 2 \tag{B.10}$$

It is easily varified that the functions $F_o(q)$, $F_1(q)$, and $F_2(q,M)$ above are all positive at $q = 0$, and are all monotonically decreasing with increasing q.

They also become negative at $q = 1$. Thus, they have unique zeros. Those zeros will be functions of the parameters in (B.1); thus they will be functions of the set $\{N_k;\ 1 \le k \le n\}$. Furthermore, the zero that corresponds to the function $F_2(q,M)$, will be also a function of M. We are searching for the minimum among all those zeros, so that the inequalities in (B.8), (B.9), and (B.10) are satisfied for every set $\{N_k;\ 1 \le k \le n\}$, and every M value. In terms of M, it can be easily shown that the function $F_2(q,M)$ is minimized at $M = 2^n - s$; for every q, and every $\{N_k;\ 1 \le k \le n\}$. Thus, we first substitute M by $2^n - s$, in expression (B.10). Then, using expressions (B.2) and (B.3), we find easily that for given set $\{N_k;\ 1 \le k \le n\}$, the following expressions hold.

$$F_o(q) = 1 - q\,E_T - P_o - P_1 q - P_T q N_1 - T_{s-N_1-1}\, q(s-1) - T_{s-N_1}\left[qs + (1-q)^s\right] \tag{B.11}$$

$$\begin{aligned} F_1(q) > F_1^{(1)}(q) \triangleq{}& 1-q-q\,E_T - P_{11}\, 2q - P_o - P_1\left[1+q^2 N_1 - (1-q)^2\right] - P_T q\,(N_1+1) \\ & - T_{s-N_1-1}\, qs - T_{s-N_1}\left[q(s+1) + (1-q)^{s+1}\right] \end{aligned} \tag{B.12}$$

$$\begin{aligned} F_2(q,2^n-s) > F_2^{(1)}(q,s) \triangleq{}& 1 - 2q(2^n-s) - q\,E_T - qN_1 - P_o(1-q)^{2^n-s} - P_1 q(1-q)^{2^n-s} - P_{11} q \\ & - T_{s-N_1-1}\, q(s-1) - T_{s-N_1}(1-q)^{2^n} \end{aligned} \tag{B.13}$$

It is easily seen that the functions $F_1^{(1)}(q)$ and $F_2^{(1)}(q,s)$, in (B.12) and (B.13) respectively, are both positive at $q = 0$, they are monotonically decreasing with increasing q, and they are negative at $q = 1$. Thus, these functions have unique zeros. Furthermore, every q value below these zeros satisfies the inequalities (B.9) and (B.10). So, to this point, we have reduced the problem to searching for the zeros of the functions $F_o(q)$, $F_1^{(1)}(q)$, and $F_2^{(1)}(q,s)$; for given set $\{N_k;\ 1 \le k \le n\}$ Then, we will search for the minimum among those zeros. To accomplish that, we instead search for lower bounds on the functions $F_o(q)$, $F_1^{(1)}(q)$, and $F_2^{(1)}(q,s)$ first.

We want those bounds to be independent of the set $\{N_k;\ 1 \le k \le n\}$, but functions of $s = \sum_{k=1}^{n} N_k$. We use some easy upper bounds on the parameters in (B.1). Those bounds are given below.

$$P_o < 2^{-s} \qquad T_{s-N_1-1} + T_{s-N_1} + P_T < 2^{-s} \sum_{i=0}^{s-N_1} \binom{s-N_1}{i} = 2^{-N_1}$$

$$P_1 < s\ 2^{-s}$$

$$T_i < \binom{s-N_1}{i} 2^{-s};\ i \ge 0 \qquad E_T < 2^{-s} \sum_{i=0}^{s-N_1-2} i \binom{s-N_1}{i} < (s-N_1)\ 2^{-N_1-1} \qquad \text{(B.14)}$$

$$P_{11} < (s-N_1)\ 2^{-s}$$

If we substitute the parameters P_o, P_1, T_i, P_{11}, P_T, E_T be their bounds in (B.14), we obtain from expressions $F_o(q)$, $F_1^{(1)}(q)$, and $F_2^{(1)}(q,s)$ the following inequalities:

$$F_o(q) > G_o(s,N_1,q) \triangleq 1 - 2^{-N_1-1} q(s+N_1) - 2^{-s}[N_1(1+q-qs)+qs+qs^2+(1-q)^s] \qquad \text{(B.15)}$$

$$F_1^{(1)}(q) > G_1(s,N_1,q) \triangleq 1 - q - 2^{-N_1-1} q(N_1+s+2) - 2^{-s}[1+qN_1(N_1-2)+sq(4+qN_1-q-2N_1) + qs^2 + (1-q)^{s+1}] \qquad \text{(B.16)}$$

$$F_2^{(1)}(q,s) > G_2(s,N_1,q) \triangleq 1 - 2q(2^n-s) - qN_1 - q(s-N_1)2^{-N_1-1} - 2^{-s}[(1-q)^{2^n-s} + sq(1-q)^{2^n-s} + (s-N_1)q + (s-N_1)(s-1)q + (1-q)^{2^n}] \qquad \text{(B.17)}$$

The functions $G_j(s,N_1,q)$; $j = 0,1,2$ in (B.15), (B.16), and (B.17) can be easily shown to be positive at $q = 0$, negative at $q = 1$, and monotonically decreasing with increasing q; for all allowed N_1 and s. Also, the functions $G_o(s,N_1,q)$ and $G_1(s,N_1,q)$ are monotonically decreasing with increasing s, for all allowed N_1 and q; where $N_1 = 0,1,2$, and where (for 2^n users) $s \le 2^n$ for the function $G_o(s,N_1,q)$, and $s \le 2^n-1$ for the function $G_1(s,N_1,q)$. The function $G_2(s,N_1,q)$ is monotonically increasing with increasing s, for all allowed N_1 and q; where $N_1 = 0,1,2$ and where (for 2^n users) $2 \leq s \leq 2^n-2$. Due to the monotonicity of the above functions with respect to s, it is now clear that to obtain sufficient conditions for the existence of a nonnegative and bounded solution for the system in (7), we should search for the zeros of the functions $G_o(2^n,N_1,q)$; $N_1 = 0,1,2$, $G_1(2^n-1,N_1,q)$; $N_1 = 0,1,2$, and $G_2(2,N_1,q)$; $N_1 = 0,1,2$. Among those zeros the smallest q_n^* provides a q region $[0,q_n^*]$ within which the system in (7) has necessarily a nonnegative and bounded solution.

The proof of the theorem is now complete.