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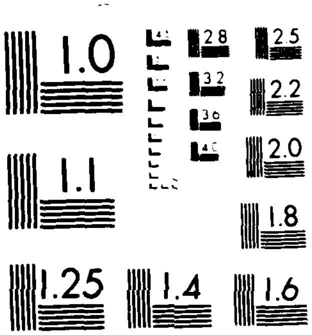
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EFFECTS OF ATMOSPHERIC
INHOMOGENEITY ON
LONG RANGE ION BEAM PROPAGATION

-- FINAL REPORT --

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A procedure has been developed to determine the particle energy and current for long range, high power flux ion beams. The smallest practical energy for a proton beam was found to be 1 GeV for beam propogation between the position of a high altitude aircraft and 100 km. This beam required a current of about 500 A to supress extensive radial expansion. A theory cont

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of radial beam expansion over decades of inhomogeneity is proposed. It defines a scattering quantity W in the high density segment of the beam trajectory. W is related to the relative amount of radial spread due to scattering occurring within a betatron wavelength. W >> 1 and W << 1 correspond to the well known free expansion and Nordsieck regimes, respectively. When W ≈ 1 the character of beam expansion has distinct features to which we assign the name moderate scattering regime. The properties of the charge neutralization channel required for effective beam propogation are considered. It is shown that neutralizing electrons may respond to the bare ion beam charge differently at different locations on the trajectory. This has important implications in the creation of such channels. The effects of an imperfect neutralization channel on beam aiming is discussed.

SUMMARY

A procedure has been developed to determine the particle energy and current for long range, high power flux ion beams. Once the tolerable energy loss fraction is specified the energy is determined by the standard Bethe stopping power formula and by limiting the effect of orbit curvature due to the Larmor effect. The larger of these two energies is then used as input for the solution of the envelope differential equation, which contains the effects of two competing mechanisms that determine the rate of radial beam spread: particle scattering off the ambient atmosphere and collective self-pinch. This equation was solved numerically over many decades of atmospheric densities, characterizing the long range trajectories. The smallest practical energy for a proton beam was found to be 1 GeV for beam propagation between the position of a high altitude aircraft and 100 km. This beam required a current of about 500 A to suppress extensive radial expansion.

A theory of radial beam expansion over decades of inhomogeneity is proposed. It defines a scattering quantity W in the high density segment of the beam trajectory. W is related to the relative amount of radial spread due to scattering occurring within a betatron wavelength. $W \gg 1$ and $W \ll 1$ correspond to the well known free expansion and Nordsieck regimes, respectively. When $W \simeq 1$ the character of beam expansion has distinct features to which we assign the name moderate scattering regime.

The properties of the charge neutralization channel required for effective beam propagation are considered. It is shown that neutralizing electrons may respond to the bare ion beam charge differently at different locations on the trajectory. This has important implications in the creation of such channels. The effects of an imperfect neutralization channel on beam aiming is discussed.

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Section 1

INTRODUCTION

The purpose of this report is to specify ion beam energies and currents that permit narrow beam propagation over many scale heights of the atmosphere. The beam width is kept narrow by the pinch effect of the self magnetic field which counteracts the beam spread tendency resulting from collisions of beam particles with the ambient environment. When we assume charge neutralization of the beam there will be no repulsive electrostatic force that increases the beam diameter.

Ion beams are one component of the particle beam triad that forms a part of the discipline of directed energy systems. This includes electrons and neutral nucleii as well as ions. High energy ions lose a negligible amount of radiation energy when they interact with target nucleii. This means that the effective energy stopping range of ions through Coulomb collisions increases with increasing energy, so that ions can propagate over large distances, whereas the range for high energy electrons is independent of energy. Thus for example, that for sea level number densities high energy electrons can propagate only a few kilometers before losing a substantial fraction of their initial energy.

The range of a neutralized charge particle beam is limited also by bending of the beam in the ambient magnetic field. The effective range is a field averaged Larmor radius. Because of the large mass of an ion compared to an electron, this range for ions is much greater than for electrons.

At very high altitudes neither ion nor electron beams will be very effective because of the difficulty in creating charge neutralization. Intense charged particle beams require neutralizing channels created by pre-beam conditioning of the ambient atmosphere. At high altitudes where the ambient density is too low, such neutralization becomes impossible. In this region a beam of neutral particles may be feasible. The major requirement is that the atmosphere be tenuous enough for the neutral beam that electron stripping is no problem. Stripping of electrons would result in a charged ion beam leading again to unacceptable beam spread due to the unneutralized electrostatic force.

The above discussion suggests that ions are an attractive candidate for long range beam particle systems, where at least part of the trajectory is at relatively low altitude. For such conditions electrons would lose too much energy through bremsstrahlung radiation and neutral beam propagation is impractical. Typically such ion beam propagation would occur over many atmospheric scale heights, so that we must take into account the inhomogeneities of the ambient environment. The presence of inhomogeneity is a crucial factor in a program to quantify beam and neutralizing of channel characteristics associated with ion beam trajectories of interest.

The organization of this report is as follows. In Section 2 the beam particle energies and beam currents required to give minimal beam spread over selected trajectories are obtained. The trajectories are discussed in Section 2.1. The energy requirements for beam along the chosen trajectories are found in Section 2.2 after consideration of stopping power and Larmor radius effects. For given trajectories and energies the currents required to ensure minimal beam spread are found in Section 2.3 where the spatial envelope equation is solved. In Section 3 the results are interpreted. In Section 4 the effect of the channel electron density upon the time needed for beam neutralization is considered. In Section 5 the effects of incomplete neutralization of the beam from the channel electrons on deviations from the single particle ion trajectory are considered.

Section 2
DETERMINATION OF BEAM PARAMETERS

The questions to be answered here concern the beam particle energy and beam current required to satisfy minimal requirements of propagation. We have taken these to be :

1) energy losses of order 20%
2) effective Larmor radius longer than beam range
3) beam spread over trajectory limited to a few times the initial diameter of 50 cm.

The first requirement guarantees that most of the initial beam energy reaches the target. The second requirement ensures that the beam can reach any target position within a given range. If the ion has a small Larmor radius it is restricted to propagate long distances essentially along the field line, and we consider such a trajectory as unacceptable exclusive. The third requirement is needed in order that the beam size not exceed that of the target, so that the entire beam actually hits the target.

All computations reported here are for proton beams. Such a limitation is not at all fundamental but merely represents a convenient restriction to the parameter space for beam propagation calculations over inhomogeneous atmospheres. In addition to the obvious extension of the parameter space engendered by consideration of the heavy ions, the problem of electron stripping of heavy ions in the ambient environment becomes important. Choice of the proton species avoids this difficulty.

2.1 Mission Requirements

We have carried out calculations for several positions of the beam source and the target. We take the point of view that any potential target can also be a potential source of ion beams. The positions of interest are sea level, 50,000 feet (15.2 km), 100 km, 400 km, and 32,000 km. These altitudes correspond respectively to the locations of a myriad of ground based facilities, large aircraft, missiles, low earth orbit satellites, and geosynchronous orbit satellites. For our purposes we assume that any of the altitudes represent the position of either a beam source or a target. The atmospheric number densities for these altitudes are given in Table 1. It is clear that all permutations of different altitudes imply trajectories through substantially varying number densities. This is the primary justification for this study.

We take a simple set of trajectories between source and target positions and assume that the beam particles move in straight line orbits normal to the earth's surface. Such trajectories give minimum path lengths for the source-to-target engagement.

2.2 Beam Energy Requirements

For an ion beam the principle source of energy loss is the elastic Coulomb scattering with nucleii in the atmosphere. The classical Bethe stopping power formula is given by:

$$-\frac{dE}{ds} = \frac{4\pi z^2 e^4 Zn}{m_e v^2} \left(\ln \frac{2m_e v^2}{I} + \ln \frac{1}{1-\beta^2} - \beta^2 \right) \tag{1}$$

Altitude (km)	Atmospheric Number Density (cm^{-3})
Ground	$2.5 \cdot 10^{19}$
15.2	$3.9 \cdot 10^{18}$
100	$1.2 \cdot 10^{13}$
400	$3.5 \cdot 10^{8}$
32,000	94.

Table 1

Molecular Number Densities
in the Atmosphere at Altitudes of Interest

h (km)	F
0	2.14
15.24	$2.48 \cdot 10^{1}$
100	$8.58 \cdot 10^{6}$
400	$1.29 \cdot 10^{12}$

Table 2

Estimated Maximum Values of F as Determined from (3)
to Limit Energy Loss to 20% of Value of Initial Energy.

where I is the mean excitation energy for the target material; for air we have taken[1] I = 80.5 eV. m_e is the electron mass, z the charge number of the incident ion, v the incident ion velocity, $\beta = v/c$, Z is the atomic number of the target nucleus, and n is the target nucleus number density. It is convenient to write (1) in the form:

$$-\frac{dE}{E} = ds\,\frac{4\pi e^4 z^2 n\cdot Z}{m_e c^2 m_i c^2}\,F(\gamma) \qquad (2)$$

$$\text{where } F(\gamma) = \left(\ln\frac{2m_e v^2}{I} + \ln\frac{1}{1-\beta^2} - \beta^2\right)/\gamma - 1)\,\beta^2$$

and γ is the relativistic factor. In an inhomogeneous atmosphere most of the incident particle energy loss will occur within a scale height, where the background density is largest for the beam trajectory interest. We can assume that the atmospheric properties vary little over the scale height. Furthermore if the energy loss is small compared to the initial energy, a requirement for beam effectiveness, then γ changes little in the integration over the trajectory. Under these conditions the integration of (2) becomes simple. If we now specialize to a proton beam and limit the energy loss to 20%, then the requirement on F is:

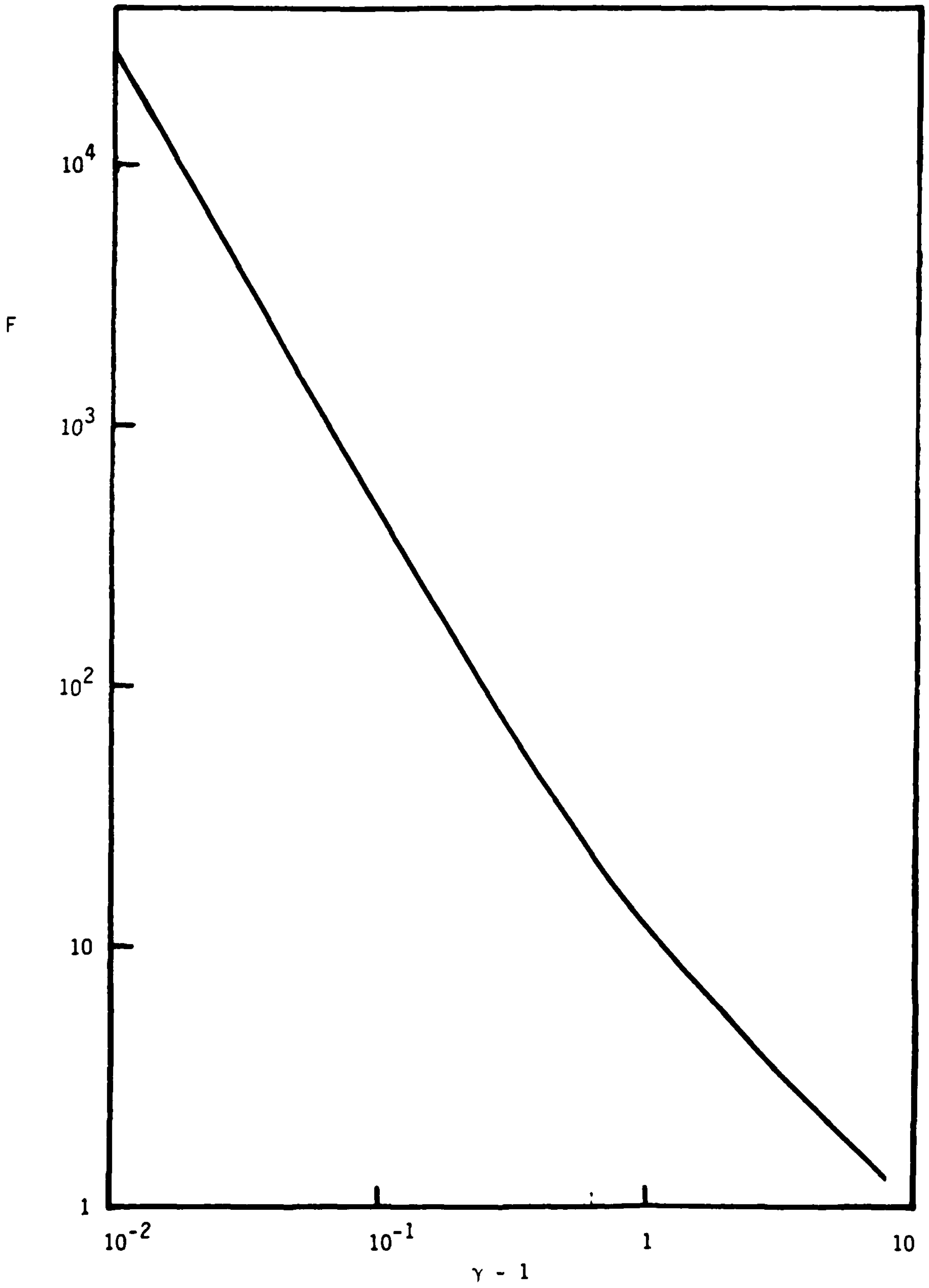
$$F \leqq \frac{3.7\cdot 10^{26}}{L_s n Z} \qquad (3)$$

where L_s is the scale height and n is the density at the location on the trajectory where the atmospheric density is large. For purposes of evaluating (3) these quantities are in cm and cm^{-3}, respectively. On each of the possible trajectories the position of maximum atmospheric density is either ground, 50,000 ft, 100 km, or 400 km. The required value of F from (3) is tabulated in Table 2. Values for scale height as well as number density and effective atomic number were obtained from the U.S. Standard Atmosphere, 1976[2] and from Jacchia.[3] To find the corresponding γ and hence the incident proton energy, we have constructed the plot of F as γ-1 in Figure 1. We draw the conclusion that a 5 GeV proton can propagate through the complete atmosphere without appreciable energy loss. If the highest density along a trajectory is less than the value at sea level, then the required proton energy will be reduced accordingly.

The particle also must have sufficient energy so that the Larmor radius is not small compared to the distance between source and target. It is therefore clear that the larger of the two energies required to satisfy the energy loss and larmor radius criteria should be selected as the actual beam energy. Here the Larmor radius is taken to be the ion velocity divided by the Larmor angular frequency (that is, the velocity and magnetic field directions are assumed perpendicular to one another). For relativistic particles the Larmor radius then can be written as:

$$r_L = \beta c^2 \gamma m_i / zeB \qquad (4)$$

Figure 1. Plot of F from Equation (2) as a function of $\gamma - 1$.



For relativistic particles γ increases linearly with increasing kinetic energy so that the Larmor radius can always be made larger than the length of the particle trajectory. The earths' magnetic field in general varies over the trajectory, so it is useful to define an averaged Larmor radius:

$$\bar{r}_L = \beta c^2 \gamma m_i / ze\bar{B}$$

where

$$\bar{B} = \int B ds / |s_0 - s_f|$$

and the integral is carried out over the trajectory; s_0 and s_f are the source and target positions respectively. We use the dipole approximation for the earth's field. At the equator $B = B_0 (r_E/r)^3$ where r is the distance from the earth's center, r_E is the earth's radius, and B_0 is the magnetic field at the earth's surface. The condition that Larmor effects are not deleterious is that

$$\frac{\bar{r}_L}{|s_0 - s_f|} \equiv \frac{2\beta c^2 \gamma m_i s_f^2 s_0^2}{ze|s_0^2 - s_f^2| B_0 r_E^3} \qquad (5)$$

be sufficiently large. The value of the ratio (5) considered to be satisfactory from the point of view of aiming is somewhat arbitrary. Clearly the above rate must exceed $(2\pi)^{-1}$ or else the beam won't attain its expected range.

In order to find a satisfactory large beam energy so that Coulomb losses and deleterious Larmor effects are negligible it is useful to plot $\frac{\Delta E}{E}$ and $\frac{|s_0 - s_f|}{\bar{r}_L}$ as functions of the proton energy. Such a plot for beam particle excursions between 50,000 ft and 100 km is given in Figure 2. Note that if we assume (conservatively) that the Larmor radius ratio $\frac{|s_0 - s_f|}{\bar{r}_L}$ is as small as $\frac{1}{2}$, then the particle energy need be 1 GeV or greater. Note that this energy also satisfies the minimum requirement on acceptable energy loss ratio since only about 10% of the initial beam energy is lost over the trajectory at this energy.

A summary of the energies satisfying both the energy loss and Larmor curvature criteria is given in Table 3. For short trajectories with sea level as an end point (i.e. either as the beam source or target), the Coulomb energy loss criterion determines the particle energy. When both trajectory end points are at higher altitudes the energy needed to avoid large fractional energy loss is reduced. For longer trajectories the Larmor criterion becomes dominant in determing the maximum acceptable particle energy. The lowest acceptable energy for all the trajectories considered was 1 GeV for an orbit extending between 50,000 ft and 100 km. As shown in

Table 3

PROTON BEAM PARTICLE ENERGIES FOR VARIOUS MISSIONS

MISSION SOURCE/TARGET POSITIONS	ENERGY (GEV)
GROUND-50,000 FT	5
GROUND-100 KM	5
GROUND-400 KM	10
GROUND-GEO	65
50,000 FT-100 KM	1
50,000 FT-400 KM	10
50,000 FT-GEO	65
100 KM-400KM	5
100 KM-GEO	65
400 KM-GEO	65

Figure 2 both the Coulomb energy loss and Larmor criteria for this particular trajectory share equal importance in determining the minimum particle energy. Note that for any trajectories extending out to the geosynchronous altitude (32,000 km) the minimum particle energy is always driven by the Larmor criterion. It should be emphasized that the corresponding energy value 65 GeV is obtained from the

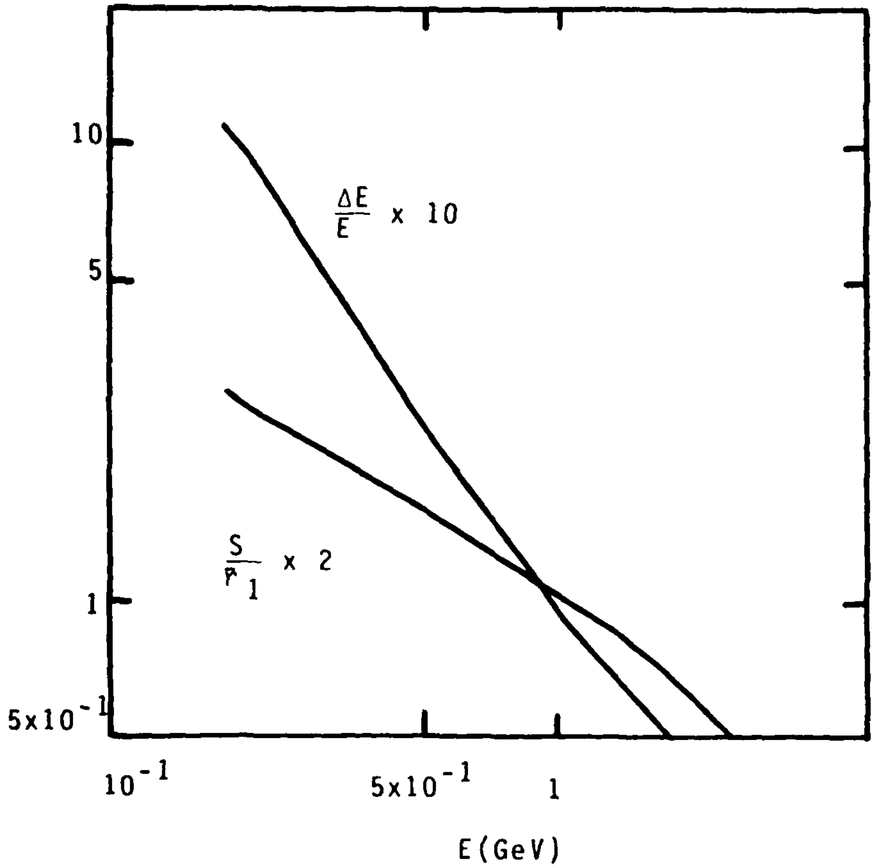
requirement that $\frac{s}{\bar{r}_L} \lesssim \frac{1}{2}$. Relaxation of this requirement, which ensures a very small curvature in trajectory, would lead to an appreciable reduction in the minimum particle energy. For purposes of this work, however, we base our subsequent discussions on the energies indicated in Table 3.

2.3 Beam Current Requirements

The previous analysis has involved a study of single particle interactions with the atmosphere and the ambient magnetic field. Directed energy systems require high power fluxes incident on target. In the case charged particle beam a high particle density would give rise to beam spread through electrostatic repulsive forces. In order to prevent this prior to beam injection the atmosphere must not be conditioned to form a neutralizing channel.

Even with charge neutralization the beam can spread due to small angle scattering interactions with the atmosphere. If, however, the beam is charge neutralized but not current neutralized the self-magnetic field can pinch the beam particles toward axis and thus counteract the effects of scattering.

Figure 2



VARIATION OF ENERGY LOSS RATIO AND PATH LENGTH TO LARMOR RADIUS RATIO AS A FUNCTION OF BEAM ENERGY. PATH IS ALONG THE EARTH SURFACE NORMAL FORM 50,000 FT TO 100 KM.

The embodiment of the physics of the above competing processes is the so-called envelope equation,[4] a differential equation for the effective beam radius. We solve this equation for the particle energies derived in the previous section for the various missions. We are interested in determing the minimum beam current required to restrain the beam from appreciable increases in its diameter. It should be pointed out that the minimum current is not necessarily the optimum one. The particulars of a mission will require a certain power flux, a quantity that involves the beam current and particle energy as well as the beam diameter. We make no attempt here to characterize the power flux requirements. The differential envelope equation in space coordinates reads:

$$\frac{d^2R}{ds^2} + \frac{U}{(\beta c)^2 R} - \frac{\varepsilon_0^2}{(\beta c\gamma)^2 R^3} \approx \frac{1}{R^3} \int ds\, R^2 \frac{|\delta\theta|^2}{\Delta\ell} \qquad (6)$$

U has dimensions of a velocity squared and is associated with the collective self-pinch for a fully neutralized system:

$$U = (\beta c)^2\, I_b/I_A$$

where I_b is the beam current and I_A is the Alfven current:

$$I_A = \beta\gamma m_i c^3/ze$$

The factor $\varepsilon_o^2 \equiv \gamma^2 R_o^2 \, [V_o^2 - (dR/ds)_o \, \beta c]$

is the initial square of the emittance and V_o is the initial random beam particle velocity perpendicular to the direction of motion. Derivation of (6) holds only if there is a negligible directed energy loss, that is if $d\gamma/ds$ 0.

$|\delta\theta|^2/\Delta\ell$ is the mean angular scatter per unit length of the incident ions off of the atmospheric nucleii. We use the Rossi-Greisen multiple scattering formulation:[5]

$$\frac{|\delta\theta|^2}{\Delta\ell} = \frac{z^2}{X_o} \left(\frac{E_s}{\gamma m_i c^2 \beta^2}\right)^2$$

where E_s is a characteristic scattering energy:

$$E_s = m_e c^2 \, (4\pi/\alpha)^{1/2} = 21.2 \text{ MeV}$$

and X_o is the classical high energy electron radiation loss length,

$$\frac{1}{X_o} = 4\alpha \, \frac{n}{A} \, z^2 \gamma_o^2 \, \ln \, (183 \, Z^{-1/3})$$

where A is the target nucleus atomic mass, and r_o is the classical electron radius.

Equation (6) has been solved numerically for source target locations at the altitudes pointed out previously. For simplicity the beam direction has been taken as perpendicular to the earth's surface. Furthermore the beam particle orbits are taken to be linear. This assumption is reasonable since the particle energies have been chosen to be sufficiently large that the Larmor orbit curvature effect is small. At the source position the beam is assumed to correspond to an equilibrium configuration in the absence of all scattering. This means that the second and third terms (on the left hand side of (6) cancel one another and that (dR/ds)o = 0. In all cases the initial radius was taken as 25 cm. Since γ is fixed once the energy is specified, the only parameter in (6) left to fix is the beam current I_b. Trial values of I_b were chosen, and (6) was solved numerically for the radius as a function of position along the trajectory of interest. A "best" current was considered to be a value that resulted in a beam spread on target of between 50 cm and 100 cm radius. From (6) it is clear that increasing the current beyond its optimal value would reduce the beam spread. Conversely decreasing the current would enhance the spread.

2.3.1 Characteristics of the Envelope Equation

There are two well known regimes for beam spreading described by the envelope equation. The first or "free expansion" regime is obtained by neglecting the second and third terms on the left hand side of (6) with respect to the scattering integral on the right hand side. This assumption means that self pinch effects are unimportant. The second or "Nordsieck" regime is obtained mathematically by neglecting the second derivative term in (6). This regime corresponds to a beam with a strong self-pinch, so that the beam spread due to scattering is constrained. In the Nordsieck limit (6) can be solved. There results

$$\ln\left(\frac{R}{R_0}\right)^2 = \frac{I_A}{Ib}\int_{s_0}^{s} ds\ |\delta\theta|^2/\Delta\ell \qquad (7)$$

We can use (7) to calculate the magnitude of second derivative term that has been neglected. Neglect of the term implies that

$$\left(\frac{|\delta\theta|^2}{\Delta\ell}\frac{R}{2}\right) << \left(\frac{I_b}{I_A}\right)^3 \qquad (8)$$

The condition (8) is a caution against inappropriate use of the Nordsieck equation. Although (7) was derived on the basis of retention of the self-pinch term, it shows that the beam spreads exponentially with the rate of angular scatter. Moreover since normally $I_A >> I_b$, the exponent apparently can be large. This would suggest paradoxically that the Nordsieck equation could predict an extremely rapid beam expansion. The condition (8) tells us that such a conclusion is not correct. In fact, the most rapid rate of expansion for a neutralized beam is the free expansion discussed above.

It is of interest to investigate the behavior of the envelope equation in the absence of scattering. In this limit we have from (6)

$$\frac{d^2R}{ds^2} + \frac{U}{(\beta c)^2 R} - \frac{\epsilon_o^2}{\beta^2 c^2 \gamma^2 R^3} = 0 \qquad (9)$$

Equating the second and third terms of (9) gives an expression for an equilibrium radius R_o. If we look for small perturbations in R around R_o, we can linearize (9). Oscillating solutions result. The interpretation is simple. The beam envelope oscillates because of the effects of the self-pinch and the restoring thermal force. The wavenumber of the oscillations is easily shown to be

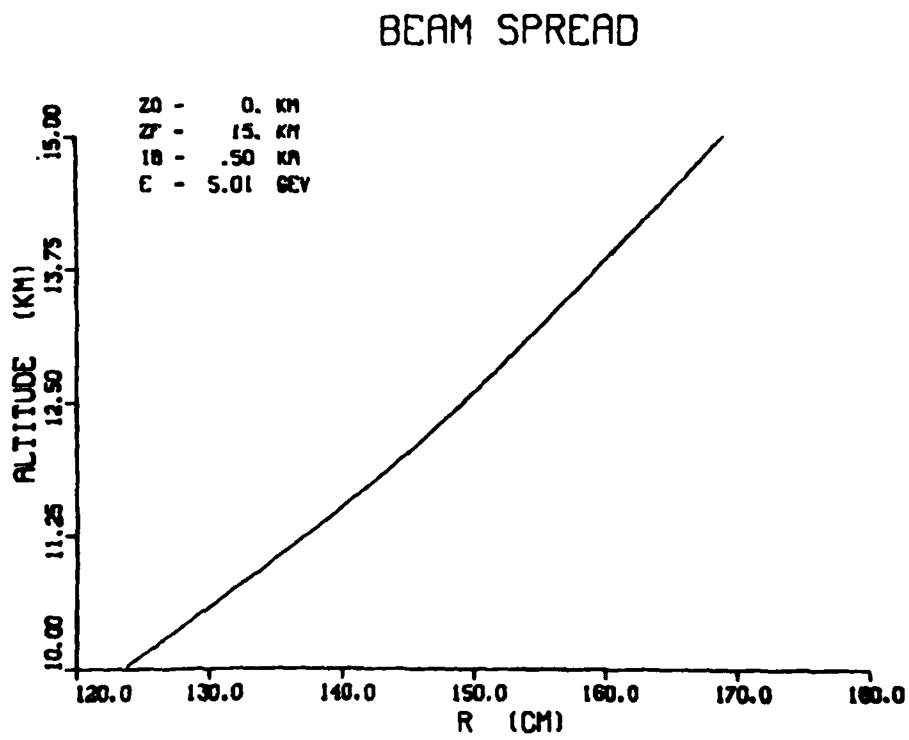
$$k_o = \sqrt{2U}/\beta c R_o$$

where $\lambda_o = 2\pi/k_o$ is the wavelength.

2.3.2 Results of Computations

Figures 3-8 show results of computations for the beam spread as a function of altitude for specified beam parameters and trajectories. Figure 3 shows the behavior of a 500 A, 5 GeV beam originating from the earth's surface in an altitude range from 10-15 km. Note how much the beam has spread from its initial 25 cm radius. Figure 4 shows the beam spread in the same altitude range when the beam current has been increased ot 1000 A. Not only is the beam

Figure 3

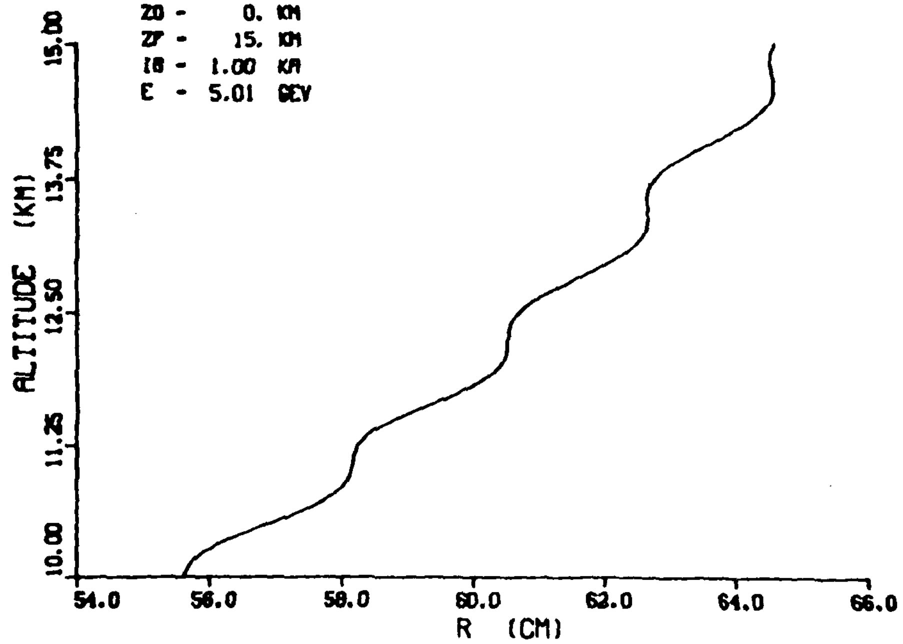


BEAM SPREAD

BEAM ENVELOPE BEHAVIOR FOR PROPAGATION FROM THE EARTH'S SURFACE TO 50,000 FT AND 100 KM. NOTE ABSENCE OF OSCILLATORY MOTION AS BEAM SPREADS.

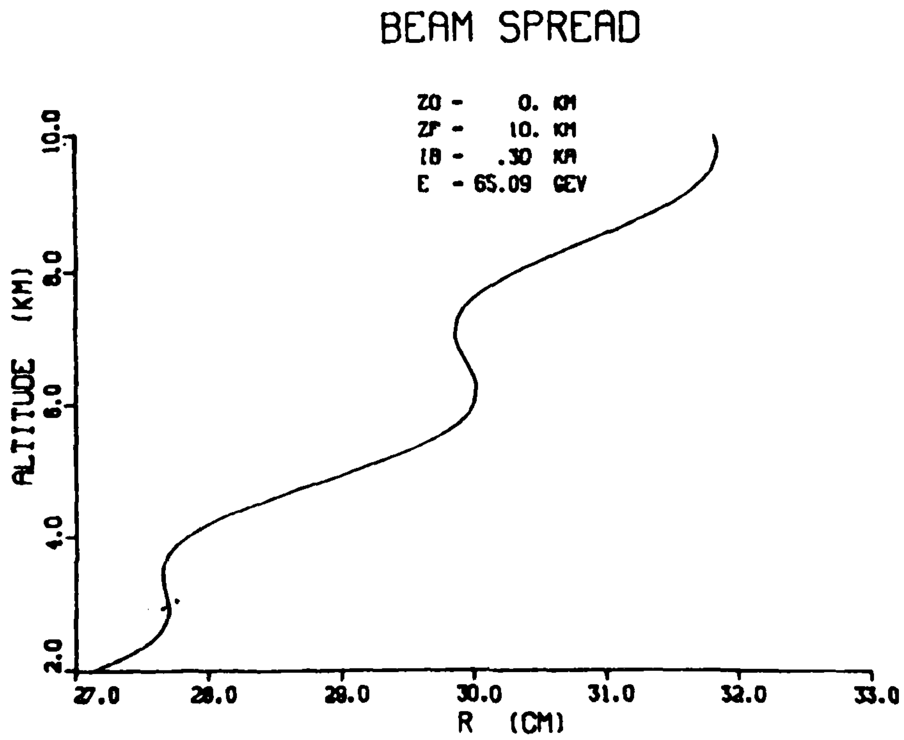
Figure 4

BEAM SPREAD



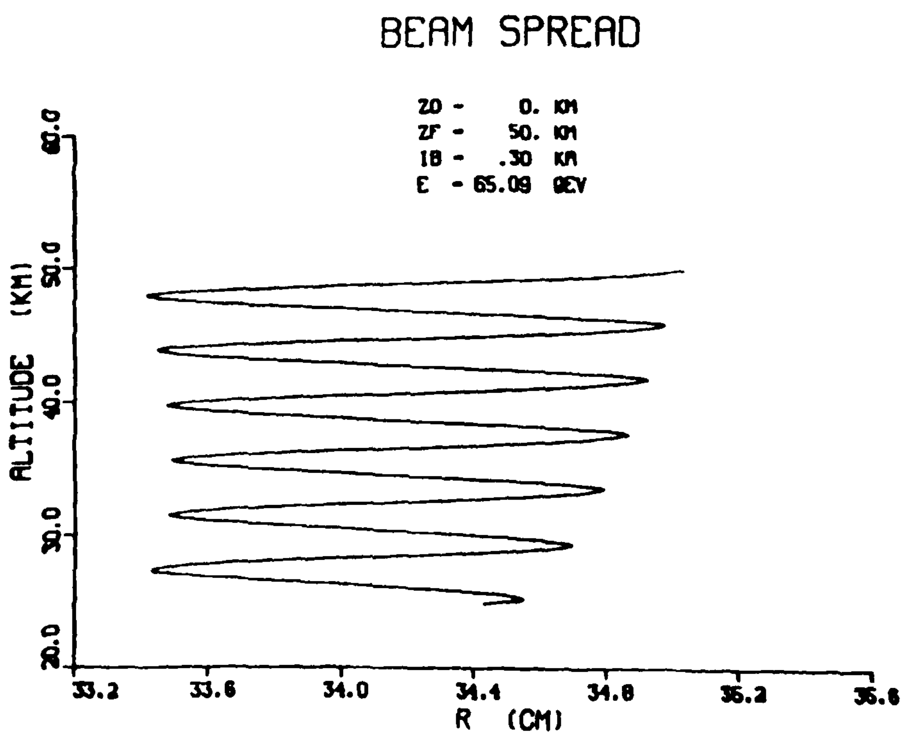
BEAM ENVELOPE BEHAVIOR FOR PROPAGATION FROM THE EARTH'S SURFACE TO 50,000 FT AND 100 KM, WHERE THE CURRENT IS DOUBLE THE VALUE IN THE PREVIOUS FIGURE. NOTE THE HIGHER SELF-FIELD GIVES RISE TO OSCILLATIONS.

Figure 5



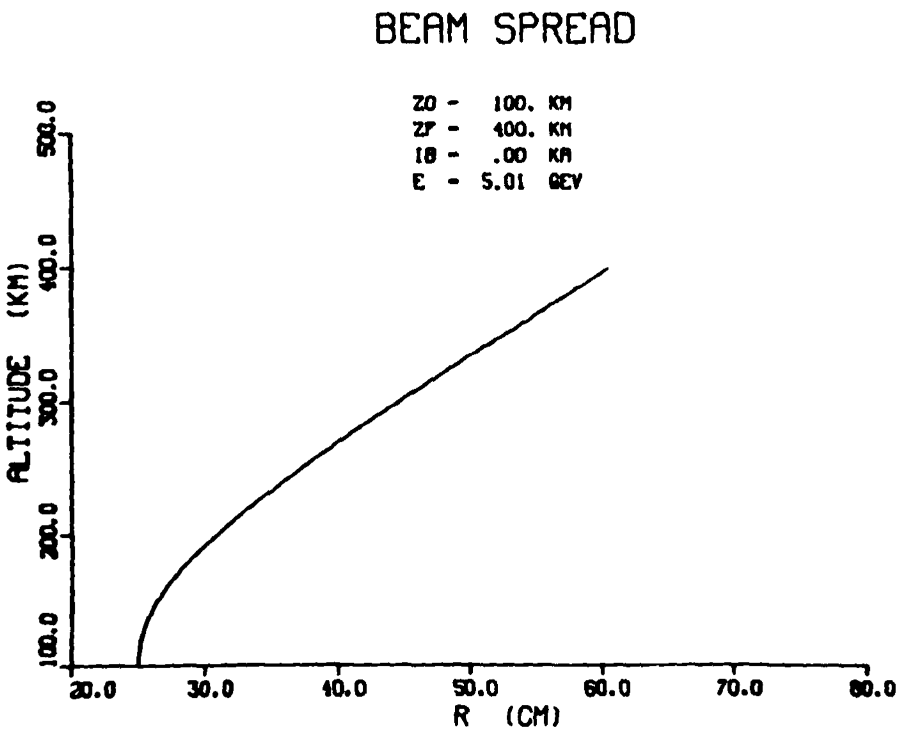
ENVELOPE BEHAVIOR NEAR THE SOURCE FOR A BEAM PROPAGATING FROM EARTH'S SURFACE TO GEOSYNCHRONOUS ORBIT. NOTE THE OSCILLATION SUPERPOSED ON THE SPREADING ENVELOPE.

Figure 6



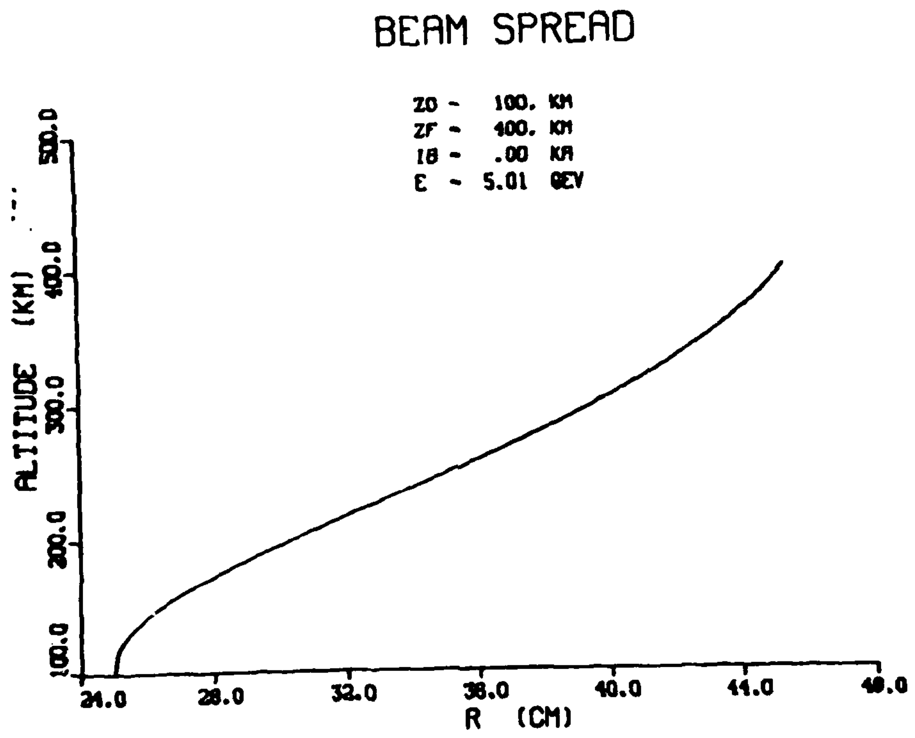
THE SAME BEAM AS IN THE PREVIOUS FIGURE SHOWN AT A HIGHER ALTITUDE. NOTE THAT THE ENVELOPE MOTION IS ALMOST PURELY OSCILLATORY. THE SPREAD AT THIS CURRENT (300 A) IS SMALL.

Figure 7



BEAM RADIUS AS A FUNCTION OF PATH LENGTH FOR 5 GEV, 10^{-4} BEAM PROPAGATING BETWEEN 100 AND 400 KM. THE PLOT IS TYPICAL OF THE STRONG SCATTER CASE WHERE THE EXPANSION VELOCITY GOES TO A CONSTANT IN THE NON-SCATTER REGION.

Figure 8



SAME CONDITIONS AS PREVIOUS FIGURE EXCEPT THAT CURRENT IS 10^{-3} A. THE CORRESPONDING CURVE ILLUSTRATES A PROMINENT CHARACTERISTIC OF MODERATE SCATTERING - A DECREASE OF THE BEAM SPREAD RATE IN THE NON-SCATTER REGION.

spread diminished, but the increased current gives rise to the betatron oscillations characteristic of the self-pinching force. In Figure 5 we note that betatron oscillations occur near the earth's surface for relatively small currents if the energy is large (65 GeV). Figure 6 shows the same beam at higher altitudes, where the altitude scale has been expanded. The expansion of the beam within the more tenuous atmosphere has nearly stopped, while the betatron oscillations are evident.

Figures 7 and 8 show spread behavior for a beam propagating from 100-400 km. From 300-400 km, Figure 7 shows that the radius of the 5 GeV very low current (10^{-4}A) beam changes linearly with distance. This is characteristic of the behavior of a beam that has scattered strongly off of a relatively dense atmosphere and spreads linearly with distance in a tenuous environment. This contrasts with the behavior of the beam with a current ten times as large, which is shown in Figure 8. Here it is evident that the beam radius does not increase uniformly in that part of the trajectory where the atmosphere is particularly tenuous. It is tempting to interpret this behavior as simply a manifestation of a very long wavelength betatron oscillation. Certainly the betatron wavelength for such low current beams is of order 100's of kilometers, and this tends to support the oscillation argument. We propose, however, that this behavior is indicative of a distinct beam propagation regime intermediate between that of the Nordsieck and free expansion regimes. We refer to this regime as the moderate scattering regime and will discuss some of its characteristics in the next section.

Finally, we present the results of the beam currents required to limit beam spread for various missions in Table 4. What

Table 4

SUMMARY OF CURRENTS
NEEDED TO LIMIT BEAM
SPREAD FOR SELECTED TRAJECTORIES

MISSION	ENERGY (GeV)	CURRENT (A)
GROUND-50,000 FT	5	1000
GROUND-100 KM	5	1000
GROUND-GEO	65	100
50,000 FT-100 KM	1	500
50,000 FT-400 KM	10	50
100 KM-400 KM	5	10^{-4}

is interesting is that once the energy is large enough to minimize the Coulomb stopping power effect and the path deviation due to Larmor motion, only modest currents are required to keep the beam from spreading out to an unacceptably large diameter. In fact several trajectories to geosynchronous altitude required sub-amp currents since the beam energy was so high (65 GeV). For the most part these results are not included in the Table because the corresponding delivered power was relatively small. The currents presented in Table 4 are rounded off values corresponding to terminal beam radii of less than a meter. The values of current obtained for particular trajectory end points are relatively insensitive to beam direction, whether it is being sent upward or downward.

Section 3
INTERPRETATION OF RESULTS

In the study of beam propagation in inhomogeneous atmospheres it is convenient to characterize the scattering history of a beam by its behavior in a tenuous region where scattering is insignificant. This kind of propagation is very realistic. It corresponds simply to a beam propagating upward from the dense to the tenuous atmosphere (see Figure 9). Moreover characterization of the scattering history of the beam by its behavior in a non-scattering region is a diagnostic approach with powerful precedents. The effects of particle interactions with an environment as usually ascertained in a region where the particle trajectory is "free."

Our numerical results indicate that there is a useful dimensionless parameter characterizing the strength of the beam interaction with the atmosphere. This quantity is related to the fractional beam spread that occurs over one betatron wavelength, i.e.

$$\frac{\Delta R}{R} = \left[\frac{2\pi}{(kR)^3} \; R \, \frac{|\delta\theta|^2}{\Delta\ell}\right]^{1/2}$$

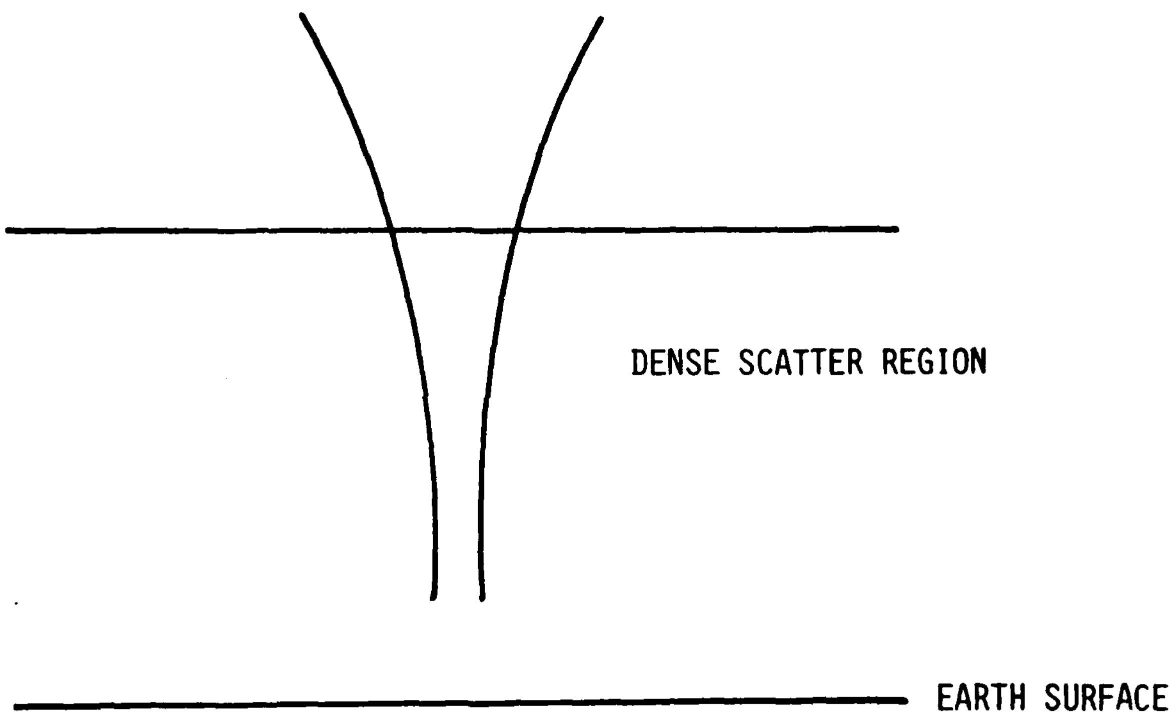
Define

$$W \equiv R \, \frac{|\delta\theta|^2}{\Delta\ell} / (kR)^3$$

where k is the "instantaneous" wave number

$$k = \sqrt{2U}/\beta c R$$

Figure 9



TYPICAL BEAM PROPAGATION UPWARD FROM EARTH SURFACE. BEAM SPREAD BEHAVIOR IN THE NON-SCATTER REGION IS USED TO CHARACTERIZE BEAM-ATMOSPHERE INTERACTION IN THE SCATTER REGION.

When $W >> 1$ the scattering of beam particles off the atmosphere is strong. For $W << 1$ it is weak. Note that the weak scattering condition is consistent with the condition (8) demanded for validity of the Nordsieck equation. In fact it is easily shown that Equation (8) is equivalent to $W^2 << 1$.

Our numerical results tend to confirm that if a beam moving from a dense scatter region into a tenuous non-scatter region is characterized by $W >> 1$ in the dense region, then the beam will expand uniformly in the tenuous region. When $W << 1$ in the dense region the beam is in the Nordsieck regime and will not expand in the tenuous region. When $W \gtrsim 1$ *the beam may be considered to have* interacted moderately with the dense atmosphere. In this case, as indicated in Figure 8, the expansion is not as rapid as a free expansion.

We now offer evidence that the moderate scattering regime is qualitatively different than either the weak or the strong scattering regimes. We may rewrite the envelope equation as

$$\frac{d^2R}{ds^2} = \frac{U}{R(\beta c)^2} \times [(\frac{R_{eq}}{R})^2 - 1]$$

where

$$R_{eq}{}^2 \equiv \frac{\varepsilon_o{}^2 + (\beta c \gamma)^2 \int ds\ R^2\ |\delta\theta|^2/\Delta\ell}{\gamma^2 U}$$

is the equilibrium radius. In the non-scatter region R_{eq} is a constant. for a beam that has undergone weak scatter, then in the non-scatter region $R \gtrsim R_{eq}$. For strong scatter U is small but non-zero and $R \ll R_{eq}$, that is, R approaches R_{eq} from below. The moderate scattering beam, on the other hand, is characterized by a value $R > R_{eq}$ in the tenuous non-scatter region. In our calculations we have found that $\frac{R - R_{eq}}{R_{eq}} \sim 0(1)$ so that the beam behavior does not correspond to long wavelength linearized oscillations. Note that for $R > R_{eq}$, $\frac{d^2R}{ds^2} < 0$, which shape was seen in Figure 8.

Section 4

CHANNEL NEUTRALIZATION

In the previous sections we have determined the currents and energies required for ideal beams to propagate without appreciable beam spread and path deviation caused by the ambient magnetic field. The only source of beam spread was assumed to be beam particle interaction with the ambient nucleii, i.e. no electrostatic spreading was considered. The requisite for this assumption is that the ion beam has been neutralized. Neutralization of the initially bare ion beam comes from the ambient charged particles in the vicinity of the beam. There may or may not be sufficient charged particles in the natural environment to effect neutralization. In the latter case the requisite charge must be created by active techniques.

In this section we inquire into the neutralization process. In particular, we estimate the relation between the charge density of the neutralizing background and the time required for neutralization to take place once the charge of the bare ion beam shows up in the environment. The neutralization is assumed due to mobile electrons, so that neutralizing electrons move from the beam edge toward the beam axis. The calculation is straightforward. Electrons respond to the radial electrostatic field resulting from the bare ions. At the edge of the beam the radial field is

$$E = 4\pi zen_b R/2$$

where n_b is the ion beam number density, z is the charge number of the ions, and R is the beam radius. Ambient electrons conduct into the beam with flux

$$j = \sigma E$$

where

$$\sigma = \frac{n_e e^2}{m_e} \; \frac{\nu}{(\nu^2 + \omega_{ce}{}^2)}$$

n_e is the ambient electron density. ν is the electron neutral collision frequency, and ω_{ce} is the cyclotron frequency corresponding to the beam self magnetic field, which dominates the ambient field. Neutralization will be essentially complete when the number of ambient electrons entering the beam channel becomes equal to the ion beam charge density. It follows directly that this occurs in a time Δt :

$$\Delta t = \frac{m_e(\nu^2 + \omega_{ce}{}^2)}{4\pi n_e e^2 \nu} \tag{10}$$

For a uniform ion beam channel

$$\omega_{ce} = \frac{2eI_b}{m_e Rc^2}$$

where I_b is the beam current.

At altitudes where the background is dominated by N_2, the collision frequency is estimated to be[6]

$$\nu = \sigma n v_e = 2 \cdot 10^{-8} n$$

where n is the ambient number density. This expression corresponds to an ambient temperature of 0.2 eV.

The expression (10) for the time increment Δt required to neutralize the beam shows a significant effect for propagation through the inhomogeneous atmosphere. Not only does the required time increment vary inversely with the ambient charge density, but it depends on the magnitude of the ambient electron neutral particle collision frequency relative to the electron cyclotron frequency due to the beam magnetic field. To be specific, let us consider the neutralization of the beam propagating between sea level and 100 km. The results of the envelope equation calculations, summarized in Table 4, show that the requisite beam current is 1000 A. We find an electron cyclotron frequency at the beam edge $\omega_{ce} = 7 \cdot 10^7$. The electron neutral collision frequency varies with altitude. Using the number densities at sea level and 100 km we obtain different collision frequencies for each altitude, viz. $\nu_{sl} = 4 \cdot 10^{11}$ and $\nu_{100} = 2.4 \cdot 10^5$. Thus at sea level $\nu \gg \omega_{ce}$ whereas at 100 km $\omega_{ce} \gg \nu$. This means that the neutralization time increment at sea level $(\Delta t)_{sl} = 1.57\ 10^2/n_e$ while at 100 km $(\Delta t)_{100} = 6.48/n_e$. The result is somewhat discouraging from an engineering point of view.

If one wishes the neutralization time to be a fixed value for all altitudes, then the channel electron density at 100 km need be 25 times less than the corresponding number at sea level. However, the density of ambient neutrals, from which the charged particle channel is created, differs by more than 6 orders of magnitude between the altitudes. Thus the degree of ionization must be much larger at 100 km than at sea level.

The physics that explains this behavior is well known. In a weak magnetic field the conductivity increases with decreasing collision frequency, that is the electrons respond to a field by a rapid free streaming response. When the magnetic field is strong the electrons are inhibited from moving across field lines. Only the occurrence of collisions allows electrons to respond to the electric field by being "jolted" onto other magnetic field lines.

Section 5

BEAM AIMING DIFFICULTIES DUE TO INCOMPLETE NEUTRALIZATION

The analysis so far has considered an ion beam that is completely charge neutralized but is not at all current neutralized. This scenario implies that electrons move radially inward from the ionized channel through which the beam propagates and short out the electric fields due to the initial charge imbalance. Because the electrons move radially they do not affect the magnitude of the ion current along axis. The computations carried out in the previous section indicate that charge neutralization may be incomplete. This would result in enhanced beam spread. But it seems likely that beam ions themselves would compensate for the lack of charge neutrality by pulling electrons within the ionized channel axially. This neutralizing process might reduce radial spreading and perhaps not reduce the ion current appreciably, but it could have serious consequences with respect to aiming. In this section we discuss certain aspects of this problem.

Most work on self-pinching charged particle beams has neglected polarization fields. The beam particles travel in the usual single particle orbits, that is, once the ambient magnetic field lines are specified the particle motion along and curvature drift across the field line can be determined. Thus, knowledge of the ambient field structure serves as the basis of beam aiming. Even in the case uncertainties may exist, because fluctuations of the fields can occur. It is obvious that small field changes can cause large differences in the trajectories of beams traveling hundreds or even thousands of kilometers. We believe that the current neutralization channel itself may be an important reason for beam aiming uncertainties.

To the degree that electrons are co-moving with the ions, the beam may be thought to possess some properties of a plasmoid. A plasmoid is a neutralized system propagating in a near vacuum where all the neutralizing electrons co-move with the ions. It is well-known[7] that for dense plasmoids (in particular, where the ion plasma frequency squared is much larger than the ion cyclotron frequency squared quantity corresponding to the external magnetic field), the plasmoid moves in nearly straight line orbits across field lines instead of following the field. This happens because the ions and co-moving electrons have opposing directions of gyration about a magnetic field line. The resulting separation of charges gives rise to the polarization fields. We expect in our case that most of the polarizaiton field will be shorted out by the electrons in the charge neutralization channel, but that this process won't be complete. Thus, by means of this small collective effect, the beam will not quite follow a simple particle orbit. This situation is similar to the low density plasmoid case where the dielectric permeability is near unity.

We have not yet done a quantitative study of the effects on beam aiming. Our main point is that the requirement for a charge neutralization channel to prevent electrostatic beam spread can give rise to a reservoir of electrons that, under plausible conditions, can be pulled along by the beam ions and cause a collective perturbation to the single particle orbit. We believe that such an effect could be quite important in the determination of the true beam trajectory.

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