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Parallel computation; matrix inversion; p-adic approximation.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The Newton-Schultz iterative scheme is reformulated in an algebraic setting to compute the exact inverse of a matrix (or the solution of a linear system of equations) over the ring of integers, with a high order of convergence, by using a finite segment p-adic representation of a rational. This method is divergence-free; it starts with the inverse of a given matrix over a finite field (called the priming step) and then iterates successively to construct, in parallel, the p-adic approximants (Hensel Codes) of the rational elements of the inverse matrix. The p-adic approximant is then converted back (CONTINUED)

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ITEM #20, CONTINUED: to the equivalent rational using the extended Euclidean algorithm.

The method involves only parallel matrix multiplications and complementations and has a quadratic convergence rate. Extension to achieve higher order convergence is straightforward if parallel matrix arithmetic facilities for higher precision operands (in a prime base system) are available.

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ERROR-FREE PARALLEL HIGH-ORDER CONVERGENT ITERATIVE MATRIX INVERSION BASED ON p-ADIC APPROXIMATION

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ABSTRACT

The Newton-Schultz iterative scheme is reformulated in an algebraic setting to compute the exact inverse of a matrix (or the solution of a linear system of equations) over the ring of integers, with a high order of convergence, by using a finite segment p-adic representation of a rational. This method is divergence-free; it starts with the inverse of a given matrix over a finite field (called the priming step) and then iterates successively to construct, in parallel, the p-adic approximants (Hensel Codes) of the rational elements of the inverse matrix. The p-adic approximant is then converted back to the equivalent rational using the extended Euclidean algorithm.

The method involves only parallel matrix multiplications and complementations and has a quadratic convergence rate. Extension to achieve higher order convergence is straightforward if parallel matrix arithmetic facilities for higher precision operands (in a prime base system) are available.

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1. Introduction

Error-free direct methods for the inversion of numerical and polynomial matrices are available in the literature [1] [2]. In this paper we describe a parallel error-free high-order convergent matrix inversion method for matrices over integers, based on the Newton-Schultz iterative scheme [3] [4] and the p-adic approximation [5-9]. Some of the important aspects of this scheme are:

(i) Inversion of matrices over p-adic fields, analogously to inverting or reciprocating the numbers, without any convergence problem.

(ii) The exact and simultaneous determination of the rational elements of the inverse matrix in p-adic digit parallel fashion with a quadratic or higher rate.

(iii) Easy realization of the scheme and its variants (higher-order convergent extensions) by parallel matrix multiplications.

This paper is organized in seven sections. In the second section we outline the principle of the Newton-Schultz scheme for reciprocating numbers. The third section describes the reformulation of the Newton-Schultz scheme in an algebraic setting to compute the p-adic approximant to the inverse of a matrix over the ring of integers. In the fourth section we describe the extended Euclidean algorithm that converts a given p-adic approximant over a range of rationals into an equivalent rational. The fifth section contains an example.

In Section 6 we briefly deal with the solution of a linear system of equations, having a linear convergence rate. Several remarks pertaining to possible extensions and generalizations are provided in the last section.

2. The principle

Let $f(x)$ be a real function of the real variable x and $x=\alpha$ be a root of $f(x)=0$. We assume that:

(a) $f(x)$, $f'(x)$ and $f''(x)$ are continuous in a neighborhood $[a,b]$ of $x=\alpha$; (b) $x=\alpha$ is an isolated root in $[a,b]$; (c) $f'(x)$ and $f''(x)$ do not vanish in $[a,b]$. The search for the root $x=\alpha$ entails finding the root of the equation

$$x = x - \frac{f(x)}{f'(x)} \equiv \phi(x) .$$

Since $\phi'(\alpha)=0$ there exists a neighborhood of $x=\alpha$ such that the sequence $\{x_i\}_{i\geq 0}$ defined by

$$x_n = x_{n-1} - f(x_{n-1})/f'(x_{n-1}) \quad (n=1,2,\ldots) \tag{1}$$

converges to $x=\alpha$ if the first approximation $x=x_0$ lies in this neighborhood. Applied to the function $f(x)=1/x-a$ (1) gives the Newton-Schultz scheme

$$x_n = x_{n-1}(2-ax_{n-1}) \tag{2}$$

The sequence (2) converges to a^{-1}. The matrix inversion algorithm to be described in the next section is by analogy based on the sequence of iterates defined by (2) [3] [4].

3. The Newton-Schultz method

Let $A=[a_{ij}]$ be a matrix over the ring of integers Z and p a prime such that det A mod $p \neq 0$. (The reason for this will become clear later.) The algorithm first constructs A^{-1} mod p and using this in the Newton-Schultz recurrence obtains a segmented p-adic representation of the inverse matrix [5-9].

Theorem 1: There exists a matrix sequence $\{B_{2^i}\}_{i \geq 0}$ such that $AB_{2^i} \bmod p^{2^i} = I$ for all $i \geq 0$, where A is the matrix to be inverted and I is the identity matrix; B_{2^i} is the inverse of A(in Z) mod p^{2^i} (or B_{2^i} is the p-adic approximant of A^{-1}).

Proof: We show the sequence $\{B_{2^i}\}_{i \geq 0}$ can be generated recursively and then prove by induction that it has the property stated, namely, $AB_{2^i} \bmod p^{2^i} = I$. The first member of the sequence B_1 is obtained in a priming step by solving

$$AB_1 \bmod p = I$$

by Gaussian elimination or some other method. It amounts to finding the inverse of A in Z mod p. Then in a powering step we use the recurrence relation

$$B_{2^i} = B_{2^{i-1}} (2I - AB_{2^{i-1}}) \bmod p^{2^i} \quad (i \geq 1) \qquad (3)$$

to construct the successive iterates.

To see that the theorem holds let $AB_{2^i} \bmod p^{2^i} = I$ be true for $i = n-1 (n \geq 1)$; then, by (3)

$$(AB_{2^n}) \bmod p^{2^n} = AB_{2^{n-1}}(2I - AB_{2^{n-1}}) \bmod p^{2^n}$$

Since $AB_{2^{n-1}} \bmod p^{2^{n-1}} = I$ by the induction hypothesis, we have

$$AB_{2^{n-1}} = I + p^{2^{n-1}} E_{n-1},$$

where E_{n-1} is the error matrix. Thus we can write

$$AB_{2^n} \bmod p^{2^n} = (I + p^{2^{n-1}} E_{n-1})(I - p^{2^{n-1}} E_{n-1}) \bmod p^{2^n}.$$

Since by construction the theorem holds for $n=0$, it is true for all $n\geq 0$ by induction.

Our algorithm first obtains B_{2^k} by iterating k times, where k is the minimum integer satisfying the inequality

$$\sqrt{\frac{p^{2^k}-1}{2}} \geq \prod_{i=1}^{n} \left(\sum_{j=1}^{n} a_{ij}^2\right)^{1/2} \tag{4}$$

This inequality ensures that the largest element of the inverse matrix lies within the range of the segmented p-adic representation of the corresponding rational [5] [8].

Let N denote a positive integer satisfying the inequality

$$N \leq \sqrt{\frac{p^{2^k}-1}{2}} \tag{5}$$

We define a finite subset F_N of the rational numbers Q as the set

$$F_N = \{\alpha = \frac{c}{d};\ 0\leq|c|\leq N \text{ and } 0\leq|d|\leq N\}$$

We call the set F_N the order N Farey fractions, or simply Farey rationals of order N.

If p and k are properly chosen to satisfy (4) then the rationals F_N which are mapped onto their segmented p-adic representations in B_{2^k} can be uniquely recovered using an algorithm which is based on the extended Euclidean algorithm for finding the greatest common divisor of two integers [10] [11].

Let a/b and w be the ij-th entry of A^{-1} and B_2k respectively. Then

$$ab^{-1} \bmod p^{2^k} = w \tag{6}$$

since b^{-1} exists mod p^{2^k}, due to the fact that det A mod $p \neq 0$.

In the following section we describe how to recover a/b given w, provided (4) is satisfied. This algorithm filters out a very small subset of rationals among which the desired rational belonging to F_N occurs. We will call the function that computes a/b given w, the EUCLID; thus EUCLID$(w)=a/b$.

Remark

The number k determined from (4) is generally larger than desired; so to iterate k times entails much superfluous computation. A practical method of avoiding this would be to compute EUCLID (B_2k) and EUCLID(B_2k+1) starting with some reasonable k and stop as soon as they are equal. This would unambiguously determine the inverse.

4. Computation of Farey rationals using the Euclidean algorithm

The Euclidean algorithm [11] constructs three pairs of numbers (u_i,u_i'), (a_i,b_i), (t_i,t_i') for each $i=0,1,2,\ldots,k$ starting with $u_0=p^r, u_0'=0$, $a_0=w, b_0=1$ and ending when $t_i=0$, as illustrated in Table 1; here the symbol [ ] denotes the lower integral part.

Note that the q_i's here correspond to the continued fraction expansion [5] [12] of p^r/w.

It can easily be shown that the pairs (a_i,b_i) in Table 1 satisfy the following conditions [10] [11]:

Cross-product rule:

$$|a_i \cdot b_{i+1}| + |a_{i+1} \cdot b_i| = p^r \leq 2N^2+1$$

Monotonicity:

$$|a_{i+1}| \leq |a_i|, \text{ with } a_0=w, a_k=1 \tag{8}$$

$$|b_{i+1}| \geq |b_i| \text{ with } b_0=1, b_k=w^{-1} \bmod p^r \tag{9}$$

where w is such that $\gcd(w,p^r)=1$ and w^{-1} denotes the multiplicative inverse of w mod p^r.

It is now necessary to show that (i) there exists a pair (a_j,b_j) in Table 1 which satisfies the condition of a Farey rational F_N (Section 3), and (ii), such a pair is unique in the sense that there exists no other pair belonging to F_N.

To prove this, we use the fact that a_i (starting with $a_o=w$) successively decreases to 1; and b_i (starting with $b_o=1$) successively increases to w^{-1} when $\gcd(w,p^r) = 1$.

Let us assume that for some j, b_j has already increased from 1 to $|N'|$ with $|N'| \leq |N|$ and is close to $|N|$, and the corresponding a_j has already decreased from w to $|N''|$ where $|N''| > |N|$ and is close to $|N|$. Then using (7) we can prove that the succeeding pair (a_{j+1}, b_{j+1}) will have to be in F_N or in other words a pair of the form (a_{j+1}, b_{j+1}) with $|a_{j+1}| \le N$ and $|b_{j+1}| > N$ which skips a Farey rational belonging to F_N cannot exist.

For if $|a_j| \ge N+1$ and $|b_j| \le N$ and $|a_{j+1}| \le N$ and $|b_{j+1}| \ge N+1$, we have $|a_{j+1} \cdot b_j| \le N^2$. Using this in (7) we obtain $|a_j \cdot b_{j+1}| \ge N^2+1$. But we have $|a_j| \ge N+1$. Therefore $|b_{j+1}| \le (N^2+1)/(N+1) = [N]$. Hence our assumption $|b_{j+1}| > N$ is false.

We will now show that there is only one such rational belonging to F_N. In other words, we will show that if for some j, (a_j/b_j) belongs to F_N then (a_{j+1}/b_{j+1}) cannot be in F_N. Note that the cross-product is maximum when

$$|a_j| = N, \ |b_j| = N - 1$$

$$|a_{j+1}| = N-1, \ |b_{j+1}| = N.$$

In such a case

$$|a_j \cdot b_{j+1}| + |b_j \cdot a_{j+1}| = (N-1)^2 + N^2 < 2N^2 + 1$$

would still be short of satisfying (7). Notice that for any other choice of a_j, b_j, a_{j+1}, b_{j+1} the condition (7) would be more severely violated. Also when $|a_j| = |b_j| = N$, it is not possible for $|a_{j+1}| = N$, since a_{j+1} would become zero by the algorithm in Table 1.

Thus a p-adic approximant (Hensel Code [5]) with the weight w corresponds to the rational a_j/b_j belonging to F_N and the conversion is complete.

Remarks

(i) The class of rationals generated by the above algorithm may contain a rational (in non-reduced form) whose reduced form is in F_N; but this is an invalid choice. (See example.)

(ii) If $\gcd(w, p^r) \neq 1$, the factor is taken out and the result adjusted suitably.

Example

Let $p=5$, $r=4$, and $w=448$. Hence $N \leq 17$. We now show in Table 2 the computations corresponding to Table 1 of the algorithm. The Farey rational is 11/7 (and not 5/60).

5. <u>Matrix-inversion example</u>

$$\text{Let } A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 4 \\ 0 & 1 & -2 \end{bmatrix}$$

Let p = 3:

$$\|[A]\|_3 = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 2 & 2 \end{bmatrix} \quad (\text{mod } 3)$$

$$B_2 = \begin{bmatrix} 1 & 0 & 1 \\ 6 & 7 & 2 \\ 3 & 8 & 5 \end{bmatrix} \quad (\text{mod } 3^2=9)$$

$$B_4 = \begin{bmatrix} 1 & 0 & 1 \\ 60 & 61 & 20 \\ 30 & 71 & 50 \end{bmatrix} \quad (\text{mod } 3^4=81)$$

$$B_8 = \begin{bmatrix} 1 & 0 & 1 \\ 4920 & 4921 & 1640 \\ 2460 & 5741 & 4100 \end{bmatrix} \quad (\text{mod } 3^8=6561)$$

$$B_{16} = \begin{bmatrix} 1 & 0 & 1 \\ 32285040 & 32285041 & 10761680 \\ 16142520 & 37665881 & 26904200 \end{bmatrix} \quad (\text{mod } 3^{16}=43046721)$$

We find that

$$\mathrm{EUCLID}(B_{16}) = \begin{bmatrix} 1 & 0 & 1 \\ -3/4 & 1/4 & -1/4 \\ -3/8 & 1/8 & -5/8 \end{bmatrix} = \mathrm{EUCLID}(B_8) = A^{-1}$$

Note that the inverse matrix elements are simultaneously determined in p-adic digit parallel fashion with a quadratic rate of convergence.

6. Solution of a system of linear equations by linear convergence

We now briefly consider the problem of determining the solution to a system of linear equations iteratively.

Let $Ax=b$ be a system of linear equations such that det A mod $p \neq 0$, p being a prime. Let $A=A_1$ mod p and $b_1=b$ mod p. We first solve $A_1\ x^{(1)}=b_1$ mod p by Gaussian elimination (say) and thereafter use the iterative scheme

$$x^{(k+1)} = (p\ A_1^{-1}\ M\ x^{(k)} + A_1^{-1}\ b) \bmod p^{k+1} (k=1,2,\ldots)$$

where $A=A_1-p\ M$ and M is the error matrix. We can easily show by induction that

$$(Ax^{(k)} - b) \bmod p^k = 0.$$

Then, our algorithm is formally:

Step 1 Solve $A_1\ x^{(1)}=b_1$ mod p.

Step 2 Use $x^{(k+1)}=(p\ A_1^{-1}\ M\ x^{(k)} + A_1^{-1}\ b) \bmod p^{k+1}$ to obtain the next iterate.

Step 3 If EUCLID(x^k) = EUCLID(x^{k+1}) stop; else go to 2.

Remark

Note that this scheme for the solution of linear equations has only a linear order convergence. However, it has the advantage of using only matrix-vector multiplications unlike the Newton iterative scheme where matrix-matrix multiplications are involved.

7. Concluding remarks

(i) The scheme of formula (3) gives rise to quadratic convergence. It is possible to use schemes having higher-order convergence. The following scheme, for example,

$$B_{3^n} = B_{3^{n-1}} (I + (I - B_{3^{n-1}}) (2I - AB_{3^{n-1}})) \bmod p^{3^n} \quad (10)$$

has cubic convergence.

(ii) We have assumed throughout that det A mod p $\neq 0$, but in actual computation we cannot assume this a priori. We can keep choosing one prime after another until we succeed; but this is very expensive computationally. It would be better to use the method of rank 1 update, which is as follows:

We apply our algorithm to A+V instead of A where

$$V = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} [b_1, b_2, \ldots, b_n] = ab^t \quad \text{is arbitrarily chosen.} \quad (11)$$

Finally, we use the formula

$$A^{-1} = (A+V)^{-1} + \frac{(A+V)^{-1} \quad V(A+V)^{-1}}{1-b^t \ (A+V)^{-1} \ a} \quad (12)$$

to retrieve the actual inverse. This method always succeeds except when $A^{-1}=0$ over Z and A mod p=0.

(iii) It is possible to extend the scope of our algorithm for the determination of the g-inverse of a singular matrix.

(iv) The algorithm determines all the elements of the inverse matrix simultaneosly in p-adic digit parallel fashion with a quadratic or higher-order convergence rate [13].

(v) In solving a system of linear equations, we note that we have split the matrix A in a very special way, namely, $A=A_1 - p M$. We could try splitting it as in the Jacobi, Gauss-Seidel or SOR method [3]; but unfortunately, the convergence in our sense is not realizable in these cases.

(vi) We can invert polynomial matrices whose elements are in Z [2] by constructing the inverses of the matrices Z[x] mod p_i for several primes p_i and then using the Chinese Remainder Theorem to construct the actual inverse [1].

i	(u_i, u'_i)	(a_i, b_i)	q_i	(t_i, t'_i)
0	$(p^r, 0)$	$(w, 1)$	$[u_o/w]$	$(u_o - a_o q_o, u'_o - b_o q_o)$
1	$(w, 1)$	(t_o, t'_o)	$[u_1/a_1]$	$(u_1 - a_1 q_1, u'_1 - b_1 q_1)$
2	(t_o, t'_o)	(t_1, t'_1)	$[u_2/a_2]$	$(u_2 - a_2 q_2, u'_2 - b_1 q_2)$
.	..	..	..	. . . .
k	(u_k, u'_k)	$(1, w^{-1})$	$[u_k/a_k]$	$(0, (-1)^{k+1} p^r)$

Table 1

Euclidean Algorithm

i	(u_i, u'_i)	(a_i, b_i)	q_i	(t_i, t'_i)
0	(625, 0)	(448, 1)	1	(177, -1)
1	(448, 1)	(177, -1)	2	(94, 3)
2	(177, -1	(94, 3)	1	(83, -4)
3	(94, 3)	(83, -4)	1	(11, 7)
4	(83, -4)	(11, 7)	7	(6, -53)
5	(11, 7)	(6, -53)	1	(5, 60)
6	(6, -53)	(5, 60)	1	(1, -113)
7	(5, 60)	(1, -113)	5	(0, 625)

Table 2

Example of Euclidean algorithm

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