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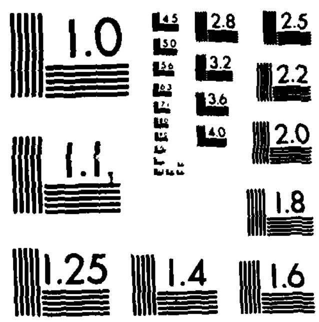

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STABILITY OF COMPRESSIBLE
WAKE AND JET FLOWS

G. R. Verma
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W. L. Hankey

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In this report the stability of compressible inviscid jets and wakes has been investigated for various wave numbers and Mach numbers for different velocity profiles.

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FOREWARD

This report is the result of work carried on in Computational Aerodynamics Group, Flight Dynamics Laboratory, Wright Patterson Air Force Base by Dr. G.R. Verma, Dr. W.L. Hankey and Mr. S.J. Scherr, from June 1, 1982 to August 17, 1982. During this period Dr. Verma's work was supported by a grant from Air Force Office of Scientific Research (Grant # AFOSR 82-0130). Additional support was provided under project 2307N436. The authors would like to thank the Air Force Systems Command, Air Force Office of Scientific Research and Wright Patterson Air Force Base for providing resources for the senior author to spend the summer of 1982 at WPAFB.

SECTION I

Introduction

In Ref [1] and [2] the stability of the lower branch solution of the Falkner-Skan similar boundary layer equations was investigated. These velocity profiles possess <u>one inflection point</u> and give rise to the "Rayleigh Instability". The analysis of this instability proved extremely useful in interpreting self excited oscillation occuring in cavities, over spike tipped bodies and in inlets (Ref. [3, 4, 5, 6, 7, 8, 9]).

Other classes of self-excited oscillations have been observed in jets (e.g. edge tones) and in the wakes of bluff bodies (e.g. periodic shedding of vortices behind cylinders). The velocity profile for this class of flows possess two inflection points which give rise to two different modes of instability (Ref. [10, 11]). To assist in the interpretation of these observed instabilites it was felt useful to further investigate the stability features of compressible wake and jet profiles. For this reason eigenvalue solutions for a series of typical profiles were computed for the following types,

(a) Symmetric jet $U = \text{sech}^2\, y$

(b). Symmetric wake $U = \text{sech}^2\, y$

(c) Anit-symmetric (Combined wake and jet) $U = \frac{3}{2}\sqrt{3}\, \text{sech}^2\, y\, \tanh y$

(d) Asymmetric jet

$$U = \begin{cases} 0 & -\infty<y<-2.5 \\ .23529(y+2.5)^2 & -2.5<y<-.8 \\ 1-.5\,y^2 & -.8<y<1.25 \\ 1.7857(y-1.6)^2 & 1.25<y<1.6 \\ 0 & 1.6<y<\infty \end{cases}$$

(e) Asymmetric wake $U = -U$ (of case d)

The results of the stability analysis are compiled and catologed to permit our conclusions regarding the behavior of these flows.

Objective

The objective was to determine the amplification factor, disturbance propagation speed and wave number for typical velocity profiles with two or three inflection points at various Mach numbers. It was anticipated that some overall characteristics for wake/jet flows could be deduced from these series of calculations.

SECTION II

Governing Equations

In this report we study the stability of compressible wakes and jets in two dimensional flows. Let u represent the velocity component in the x direction and v the velocity component in the y direction. p, ρ and T are pressure, density and temperature respectively.

The basic equaitons are

$$\frac{1}{\rho}\left[\frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial x} + v\frac{\partial\rho}{\partial y}\right] + \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0 \tag{2.1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} \tag{2.2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} \tag{2.3}$$

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} = \frac{\partial p}{\rho}\left[\frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial x} + v\frac{\partial\rho}{\partial y}\right] \tag{2.4}$$

Eliminating ρ between equations (2.1) and (2.4) we obtain

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + \gamma p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0 \tag{2.5}$$

Equations (2.2), (2.3) and (2.5) have a steady state solution

$$u = \bar{u}(y),\ v = 0,\ p = \bar{p} = \text{constant} \tag{2.6}$$

we assume the time depndent perturbed flow as [12,13]

$$u = \bar{u}(y) + u'(x,y,t) \tag{2.7}$$

$$v = v'(x,y,t) \tag{2.8}$$

$$p = \bar{p} + p'(x,y,t) \tag{2.9}$$

Substituting these values of u, v and p in equations (2.2), (2.3) and (2.5); and retaining only linear terms in u', v' and p' we obtain

$$\frac{\partial u'}{\partial t} + \bar{u}\,\frac{\partial u'}{\partial x} + v'\,\frac{\partial \bar{u}}{\partial y} + \frac{1}{\bar{\rho}}\,\frac{\partial p'}{\partial x} = 0 \qquad (2.10)$$

$$\frac{\partial v'}{\partial t} + \bar{u}\,\frac{\partial v'}{\partial x} + \frac{1}{\bar{\rho}}\,\frac{\partial p'}{\partial y} = 0 \qquad (2.11)$$

$$\frac{\partial p'}{\partial t} + \bar{u}\,\frac{\partial p'}{\partial x} + \gamma\bar{p}\,\left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y}\right) = 0 \qquad (2.12)$$

We seek the periodic solutions of the form

$$u' = \hat{u}(y)\; e^{i\alpha(x-ct)} \qquad (2.13)$$

$$v' = \hat{v}(y)\; e^{i\alpha(x-ct)} \qquad (2.14)$$

$$p' = \hat{p}(y)\; e^{i\alpha(x-ct)} \qquad (2.15)$$

where $\hat{u}$, $\hat{v}$, $\hat{p}$ are complex, c is a complex constant and α is a real constant.

Substituting (2.13), (2.14) and (2.15) into equations (2.10), (2.11) and (2.12) we obtain

$$i\alpha(\bar{u} - c)\,\hat{u} + u_y\,\hat{v} = -i\alpha\,\frac{\hat{p}}{\bar{\rho}} \qquad (2.16)$$

$$i\alpha(\bar{u} - c)\,\hat{v} = -\frac{1}{\bar{\rho}}\,\hat{p}_y \qquad (2.17)$$

$$i\alpha(\bar{u} - c)\,\hat{p} = -\gamma\bar{p}\,(i\alpha\hat{u} + \hat{v}_y) \qquad (2.18)$$

We eliminate $\hat{p}$ and $\hat{u}$ from the above equations, use the relation $\bar{p} = \bar{\rho}\,R\,\bar{T}$ and obtain

$$\left[\frac{(\bar{u} - c)\,\hat{v}_y - \bar{u}_y\,\hat{v}}{\gamma\,R\,\bar{T} - (\bar{u} - c)^2}\right]_y = \frac{\alpha^2(\bar{u} - c)\,u}{\gamma\,R\,\bar{T}} \qquad (2.19)$$

Now using

$$\gamma\,R\,\bar{T} = \frac{1 + .2\,M_\infty^2\,(1 - \bar{u}^2)}{M_\infty^2} \qquad (2.20)$$

and doing some calculations we obtain

$$\left[\frac{(\bar{u} - c)\,\hat{v}_y - \bar{u}_y\,\hat{v}}{(1+.2M^2) - M_o^2\,(\bar{u} - c)^2}\right]_y = \alpha^2\,(1 + .2M^2)\,(\bar{u} - c)\,\hat{v} \qquad (2.21)$$

where

$$M_o^2 = \frac{M_\infty^2}{1+.2\,M_\infty^2} \qquad (2.22)$$

and

$$M^2 = \frac{M_\infty^2\,\bar{u}^2}{1+.2\,M_\infty^2(1-\bar{u}^2)} \qquad (2.23)$$

If we write

$(1 + .2M^2)^{-1} - M_o^2\,(\bar{u} - c) = g$, and replace $\hat{v}$ by ϕ and $\bar{u}$ by U in equation (2.21) we obtain

$$\left[\frac{(U - c)\,\phi_y - \bar{U}_y\,\phi}{g}\right]_y = \alpha^2(1 + .2M^2)\,(U - c)\phi \qquad (2.24)$$

For boundary conditions we assume that for unbounded flows the initial disturbances die down at far from the disturbances. Therefore we get

$$\phi\,(-\infty) = 0,\ \phi(\infty) = 0 \qquad (2.25)$$

For fined wave numbers (α = constant) equations (2.24) and (2.25) is an eigenvalue problem. ϕ is eigenfunction and c is eigenvalue.

We solve this eigenvalue problem for the following velocity profiles

$U(y) = \operatorname{sech}^2 y$, symmetric jet (2.26)

$U(y) = -\operatorname{sech}^2 y$, symmetric wake (2.27)

$U(y) = \frac{3}{2}\sqrt{3}\,\operatorname{sech}^2 y\,\tanh y$, anti-symmetric (combined wake and jet) (2.28)

$$U(y) = \begin{cases} 0 & -\infty < y < -2.5 \\ .23529(y+2.5)^2 & -2.5 < y < -.8 \\ 1 - .5y^2 & -.8 < y < 1.25 \\ 1.7857(y-1.6)^2 & 1.25 < y < 1.6 \\ 0 & 1.5 < y < \infty \end{cases}$$

asymmetric jet (2.29)

$U(y) = -U(y)$ of (2.29)

Asymmetric wake (2.30)

SECTION III

Numerical Procedure

Eigenvalues of ϕ were determined by a shooting method [1]: starting with boundary conditions at y_{min}, integrating over the range of y, and comparing the result with the outer boundary condition, namely $\phi = 0$ at y_{max}. The process involved minimization of the error caused by the deviation. This was chosen to be the square of the norm of ϕ, $|\phi|^2 = \phi^2 + \phi_i^2$. The integration was done using a fourth-order Runge-Kutta method.

Boundary conditions at y_{min} were determined by observing the behavior of (2.24) as $y \to -\infty$. The equation reduces to

$$\phi_{yy} = \alpha^2 \phi \tag{3.1}$$

Since we desire $\phi(-\infty) = 0$, we choose

$$\phi(y_{min}) = e^{|\alpha| y_{min}}, \qquad \phi'(y_{min}) = |\alpha| e^{|\alpha| y_{min}} \tag{3.2}$$

as our boundary conditions.

The method of finding eigenvalues utilized the same minimization routine as in previous investigations [1,2]. The user provides a storing guess, for c in the case, and the routine begins by searching along a constant line of c_i with increasing steps until the error begins to increase. It then uses the last three calculated values to determine a parabola, with the c_r value at the vertex used as the new approximation. Then this value of c_r is held constant and a search along a line of changing c_i is carried out. After a new relative minimum is found, the quadratic approximation is used to determine a new value for c_i. The third step involves searching the line connecting the original guess and the new point in the same manner. If the error is not less than a preset limit, here 10^{-6}, the routine starts again with the latest value used in place of the original guess.

SECTION IV

RESULTS

The eigen value problem represented by (2.24) and (2.25) was solved numerically for the velocity profiles given by (2.26), (2.27), (2.28), (2.29) and (2.30). The results are tabulated for a wide range of wave numbers (α) and Mach numbers (M_∞). The instability characteristics for a sy metric jet, asymmetric jet and anti-symmetric jet are given in tables (1a) (3h). For $M_\infty = 0$ there values agree with those given in [10 and 11].

The velocity profiles are plotted in Figures (1 - 3). Tr ies of α, versus c_i, and α versus c_r and c_i versus c_r are plotted in Figures (4 - 19). ϕ, $\hat{u}$ and $\hat{p}$ are plotted for some special values of M_∞ α and c in Figures 20 a to 21 c, and mangitudes and phases of ϕ, $\hat{u}$ and $\hat{p}$ are plotted in Figures 22a to 22 c, and 28 a to 28 c.

Solutions were obtained with convergence error criteria of at least 10^{-6} for all cases.

SECTION V

Summary

The stability of compressible inviscid jets and wakes has been investigated by utilizing the linearized equations resulting from a small perturbation analysis. The resulting eigenvalue problems were solved numerically for various wave numbers (2) and Mach numbers (M_∞) for different velocity profiles. In the cases of symmetric jets and wakes and that of asymmetric jets and wakes we found two propagation modes corresponding to two inflection points. The sinuous mode for even eigenfunctions and varicose mode for odd eigenfunctions.

In varicose modes the magnitude of amplification decreased as Mach number (M_∞) increased and the flow became completely stable at $M_\infty = 2$. In sinuous modes the amplification did decrease a little with the increase of Mach number but we did not find any upper limit in Mach number above which the flow was completely stable.

In the case of anti-symmetric profile there are three modes corresponding to the three inflection points. Two propagating modes, one propagating to the right and the other propagating to the left; and one standing mode. The magnitude of amplification for propagating modes decreased as the Mach number increased, and completely died down at Mach number of 1.5. On the other hand, we could not find an upper limit of Mach number for the standing mode above which the flow was completely stable. The authors believe that these results will be useful for analyzing aerodynamic instabilities encountered in wakes and jets.

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INSTABILITY CHARACTERISTICS FOR THE SYMMETRIC JET $u=\mathrm{sech}^2 y$

TABLE 1.

1a) $M_\infty=0.0$, SINUOUS MODE

α	c_r	c_i
.1	.061256	.119380
.2	.137549	.205118
.3	.207237	.241188
.4	.266554	.249623
.5	.316088	.244302
.6	.357248	.231763
.7	.392290	.215421
.8	.422860	.197142
.9	.450120	.177992
1.0	.474924	.158612
1.1	.497882	.139405
1.2	.519408	.120635
1.3	.539851	.102479
1.4	.559444	.085063
1.5	.578370	.065476
1.6	.596770	.052789
1.7	.614738	.038056
1.8	.632352	.024322
1.9	.649655	.011026
2.0	.666667	$.282007(10)^{-11}$

1b) M_∞=1.0, SINUOUS MODE

α	c_r	c_i
.1	.067994	.128268
.2	.156481	.216384
.3	.233560	.245433
.4	.296861	.246748
.5	.346972	.236112
.6	.387995	.219914
.7	.422773	.201173
.8	.453158	.181434
.9	.480444	.161569
1.0	.505465	.142098
1.1	.528781	.123328
1.2	.550770	.105484
1.3	.571686	.088689
1.4	.591693	.073020
1.5	.610908	.058512
1.6	.629391	.045172
1.7	.647185	.032987
1.8	.664327	.021922
1.9	.680823	.011935
2.0	.696684	.022972

1c) M_∞=2.0, SINUOUS MODE

α	c_r	c_i
.1	.086251	.149243
.2	.203675	.235800
.3	.293797	.247300
.4	.359668	.234900
.5	.410110	.214300
.6	.451142	.191700
.7	.486228	.169100
.8	.517361	.147500
.9	.545685	.127500
1.0	.571836	.109323
1.1	.596155	.093041
1.2	.618823	.078622
1.3	.639958	.065944
1.4	.659655	.054849
1.5	.678011	.045186
1.6	.695111	.036856
1.7	.710884	.029902
1.8	.724660	.023967
1.9	.737094	.017381
2.0	.749815	.011191
2.1	.761940	.006051

1d) M_∞=3.0, SINUOUS MODE

α	c_r	c_i
.1	.112176	.172854
.2	.259379	.247000
.3	.355744	.237003
.4	.422588	.214158
.5	.474158	.186233
.6	.517294	.159762
.7	.555257	.136500
.8	.589379	.117193
.9	.619734	.102317
1.0	.645257	.091204
1.1	.665674	.081286
1.2	.683340	.070539
1.3	.700638	.059729
1.4	.717718	.050427
1.5	.733757	.042849
1.6	.748628	.036578
1.7	.762489	.031398
1.8	.775722	.027867
1.9	.785647	.027098
2.0	.791696	.022900

1 e) M_∞=4.0, SINUOUS MODE

α	c_r	c_i
.1	.143092	.193015
.2	.311005	.249100
.3	.409095	.225500
.4	.477423	.191509
.5	.532312	.159900
.6	.579928	.135133
.7	.620580	.119050
.8	.650447	.109428
.9	.671343	.098700
1.0	.689858	.085042
1.1	.709335	.071677
1.2	.728751	.061123
1.3	.746833	.053174
1.4	.763496	.047336
1.5	.778596	.044463
1.6	.788371	.043617
1.7	.795081	.038781
1.8	.804006	.032273
1.9	.813824	.027343
2.0	.823122	.023604

1f) M_∞=0.0, VARICOSE MODE

α	c_r	c_i
.05	.862061	.030296
.10	.867554	.108700
.20	.796327	.121800
.30	.759672	.114815
.40	.733435	.102812
.50	.713113	.088200
.60	.697180	.071556
.70	.684964	.053968
.80	.676071	.035862
.90	.670036	.017700
1.00	.666667	.000000

1g) M_∞=1.0, VARICOSE MODE

α	c_r	c_i
.050	.814895	.011590
.100	.834826	.151395
.200	.765617	.075559
.300	.737390	.056628
.400	.721227	.039567
.500	.710643	.022751
.525	.708624	.018530
.550	.706811	.014300
.575	.705210	.010064
.600	.703712	.005774
.625	.702614	.001584
.650	.701492	.000049

1h) M_∞=2.0, VARICOSE MODE

α	c_r	c_i
.54	.988650	.000784
.55	.980628	.001722
.56	.973019	.002772
.57	.965803	.003849
.58	.958940	.004919
.59	.952416	.005956
.60	.946200	.006928
.61	.940272	.007830
.62	.934607	.008647
.63	.929187	.009374
.64	.923988	.010011
.65	.918995	.010557
.66	.914191	.011011
.67	.909559	.011378
.68	.905084	.011660

INSTABILITY CHARACTERISTICS FOR

THE ANTISYMMETRIC JET $U = \frac{3}{2}\sqrt{3}\ \mathrm{sech}^2 y \tanh y$

TABLE 2.

2a) M_∞=0.0, PROPAGATING MODE

α	c_r	c_i
.05	.920689	.090656
.10	.879349	.116531
.20	.820211	.136200
.30	.771224	.138417
.40	.730500	.130034
.50	.699088	.115459
.60	.675954	.098057
.70	.659475	.079866
.80	.648131	.061943
.90	.640730	.044786
1.00	.636391	.028606
1.10	.634469	.013483

2b) M_∞=1.0, PROPAGATING MODE

α	c_r	c_i
.10	.824508	.095961
.15	.790995	.094567
.20	.766441	.090119
.25	.746227	.083887
.30	.728650	.075980
.35	.713325	.066800
.40	.700178	.056500
.45	.689115	.045423
.50	.680052	.034040
.60	.667009	.011367
.65	.662642	.000350

2c) $M_\infty=1.2$, PROPAGATING MODE

α	c_r	c_i
.10	.808530	.007919
.15	.768575	.067700
.20	.746423	.060019
.25	.728869	.051536
.30	.713971	.041967
.35	.701157	.031200
.40	.6[illegible]0315	.019372
.45	.681388	.006778
.47	.678352	.001596

2d) M_∞=0, STANDING MODE

α	c_r	c_i
.1	0.0	.231871
.2	0.0	.351572
.3	0.0	.421749
.4	0.0	.467320
.5	0.0	.495630
.6	0.0	.509976
.7	0.0	.512729
.8	0.0	.506042
.9	0.0	.491820
1.0	0.0	.471650
1.1	0.0	.446790
1.2	0.0	.418208
1.3	0.0	.386635
1.4	0.0	.352607
1.5	0.0	.316510
1.6	0.0	.278608
1.7	0.0	.239067
1.8	0.0	.197972
1.9	0.0	.155345
2.0	0.0	.111149
2.1	0.0	.065246

2e) M_∞=1.0, STANDING MODE

α	c_r	c_i
.15	0.0	.325303
.20	0.0	.376416
.30	0.0	.434305
.40	0.0	.454416
.50	0.0	.450267
.60	0.0	.429563
.70	0.0	.396980
.80	0.0	.355153
.90	0.0	.305120
1.00	0.0	.246296
1.10	0.0	.175520
1.20	0.0	.081841
1.23	0.0	.043534

2f) M_∞=1.4, STANDING MODE

α	c_r	c_i
.1	0.0	.267995
.2	0.0	.395734
.3	0.0	.441200
.4	0.0	.441578
.5	0.0	.414618
.6	0.0	.368707
.7	0.0	.306499
.8	0.0	.224023
.9	0.0	.093603

2g) M_∞=2.0, STANDING MODE

α	c_r	c_i
.15	0.0	.381035
.20	0.0	.425659
.30	0.0	.444403
.40	0.0	.409147
.50	0.0	.339340
.60	0.0	.229747
.65	0.0	.137767

INSTABILITY CHARACTERISTICS FOR THE ASYMMETRIC JET

TABLE 3.

3a) M_∞=0.0, SINUOUS MODE

α	c_r	c_i
.1	.022716	.034685
.2	.057622	.075168
.3	.096191	.106869
.4	.134367	.126540
.5	.169211	.135130
.6	.199485	.134785
.7	.223771	.127902
.8	.241919	.116944
.9	.254235	.104212
1.0	.261494	.091611
1.1	.265033	.080302
1.2	.266183	.070682
1.3	.265879	.062672
1.4	.264760	.056016
1.5	.263194	.050437
1.6	.261399	.045706
1.7	.259499	.041642
1.8	.257556	.038108
1.9	.255643	.035004
2.0	.253755	.032250
2.5	.245207	.022082
3.0	.238341	.015553
3.5	.233025	.011077
4.0	.229011	.007910

3b) M_∞=1.0, SINUOUS MODE

α	c_r	c_i
.1	.025250	.038126
.2	.066401	.083135
.3	.114810	.116629
.4	.163526	.132723
.5	.206953	.132538
.6	.240919	.119500
.7	.263146	.098537
.8	.271909	.076103
.9	.271495	.059204
1.0	.268354	.048282
1.1	.264800	.041100
1.2	.261773	.036078
1.3	.259106	.032331
1.4	.256705	.029392
1.5	.254633	.026992
1.6	.252696	.024966
1.7	.250912	.023213
1.8	.249280	.021664
1.9	.247718	.020275
2.0	.246241	.019014
2.5	.239841	.014013
3.0	.234696	.010404
3.5	.230608	.007697
4.0	.227434	.005654

3c) M_∞=2.0, SINUOUS MODE

α	c_r	c_i
.2	.092437	.102199
.3	.169565	.130870
.4	.242842	.123709
.5	.315747	.081390
.6	.405591	.064530
.7	.470175	.050658
.8	.525611	.040951
.9	.571746	.036654
1.0	.607611	.035926
1.1	.633812	.035146
1.2	.652273	.035042
1.3	.665219	.031993
1.4	.674078	.027458
1.5	.679530	.022257
1.6	.682270	.017361
1.7	.683250	.013417
1.8	.683357	.010509
1.9	.683125	.008409
2.0	.682793	.006866
2.5	.681431	.003002

3d) M_∞=3.0, SINUOUS MODE

α	c_r	c_i
.1	.041923	.058484
.2	.134351	.120942
.3	.250400	.125258
.4	.364384	.112313
.5	.441116	.093284
.6	.508862	.070646
.7	.568281	.057581
.8	.613491	.053023
.9	.645471	.049900
1.0	.668287	.045140
1.1	.684931	.037703
1.2	.696740	.026809
1.3	.700800	.010626
1.4	.692436	.002767
1.5	.688292	.001691
1.6	.686144	.001325
1.7	.684820	.001130
1.8	.683920	.001000
1.9	.683265	.000899
2.0	.682766	.000814

3e) M_∞=4.0, SINUOUS MODE

α	c_r	c_i
.1	.054213	.071056
.2	.185692	.129058
.3	.343024	.122561
.4	.440501	.105400
.5	.523520	.078015
.6	.590604	.067213
.7	.636500	.061840
.8	.668800	.055668
.9	.692983	.046829
1.0	.713555	.034536
1.1	.736208	.020525
1.2	.760259	.011891
1.3	.780733	.007100
1.4	.798010	.003777
1.5	.812881	.001170

3f) $M_\infty=0$, VARICOSE MODE

α	c_r	c_i
.1	.791496	.176117
.2	.694068	.187158
.3	.634863	.173131
.4	.596700	.152443
.5	.572033	.130738
.6	.556473	.110405
.7	.548758	.092825
.8	.548309	.078848
.9	.553977	.068563
1.0	.564245	.061258
1.1	.577329	.055838
1.2	.591419	.051351
1.3	.605205	.047201
1.4	.617916	.043095
1.5	.629175	.038880
1.6	.638852	.034763
1.7	.646961	.030638
1.8	.653605	.026674
1.9	.658945	.022975
2.0	.663174	.019621
2.5	.673978	.008493
3.0	.677488	.003768
3.5	.678856	.001753
4.0	.679452	.000846

3g) M_∞=1.0, VARICOSE MODE

α	c_r	c_i
.000	.999458	.000934
.050	.823680	.124270
.100	.749102	.137267
.200	.666039	.122735
.300	.621290	.093539
.400	.596116	.060028
.500	.583795	.024059
.525	.582989	.014683
.550	.582512	.005117
.575	.583089	$.602216(10)^{-5}$

3h) M_∞=1.3, VARICOSE MODE

α	c_r	c_i
.01	.888249	.045163
.05	.793824	.096293
.10	.727671	.103920
.15	.685409	.095711
.20	.656887	.080776
.30	.628048	.041152
.35	.628602	.016927
.40	.634545	.005976

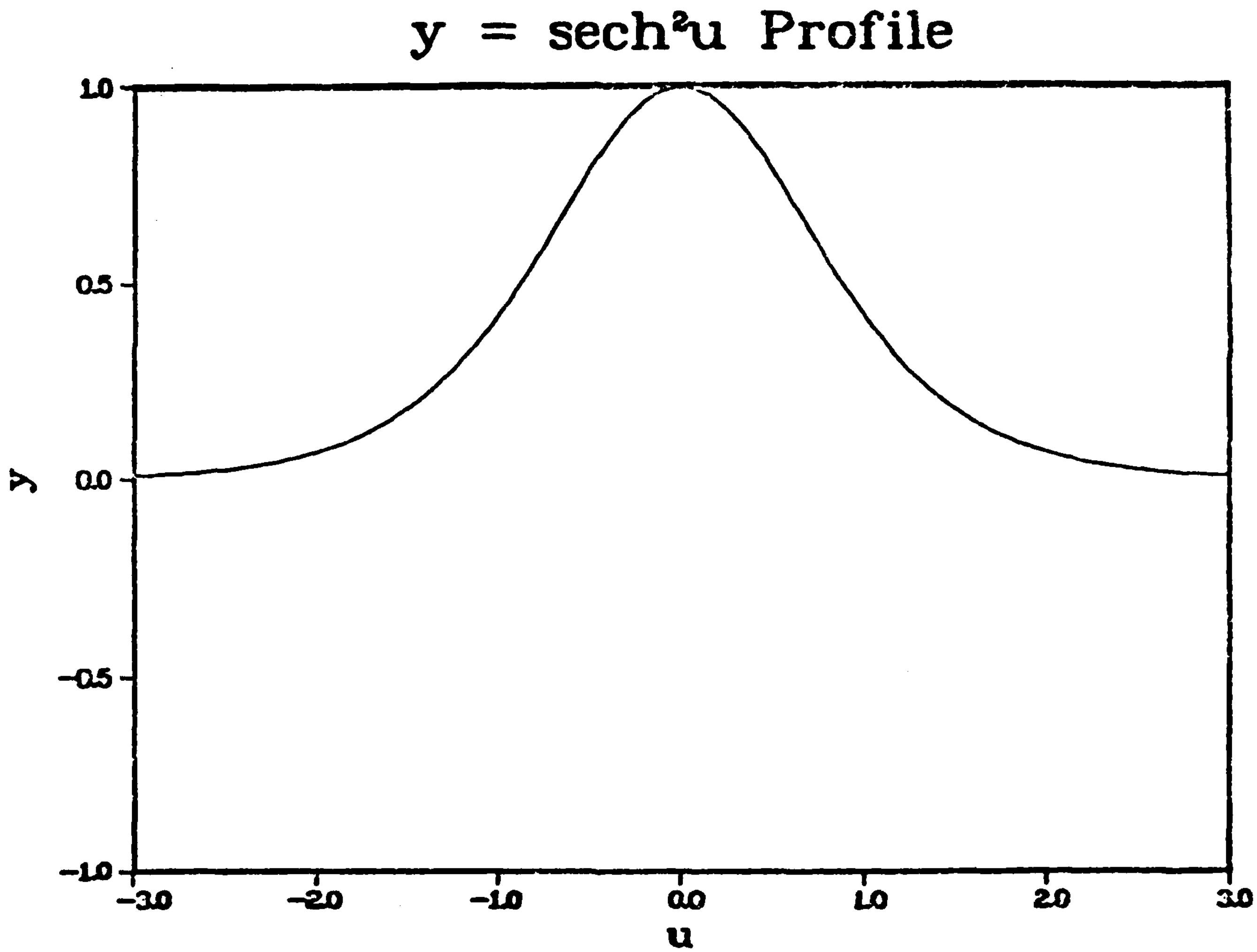


Figure 1 Symmetric Profile

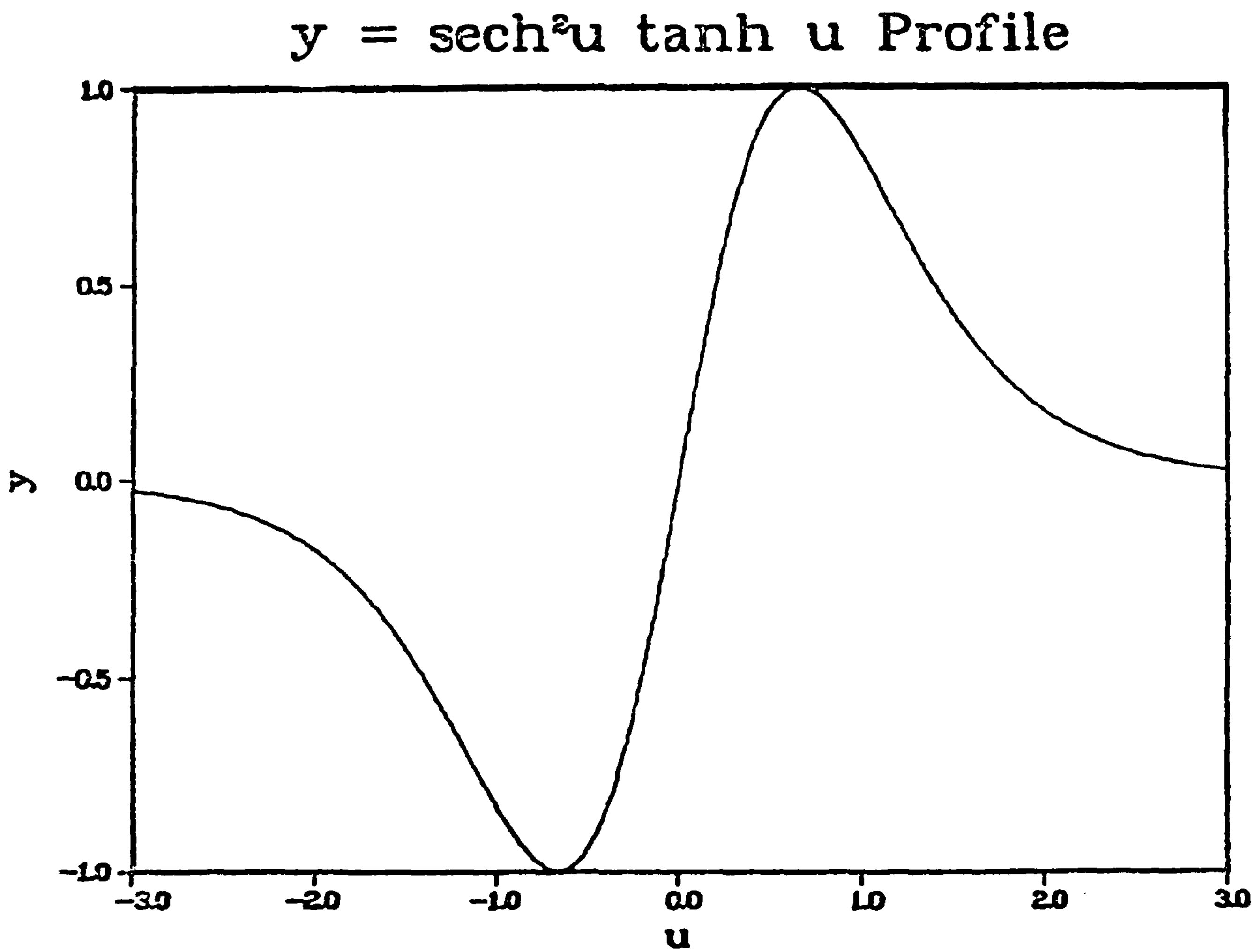


Figure 2 Antisymmetric Profile

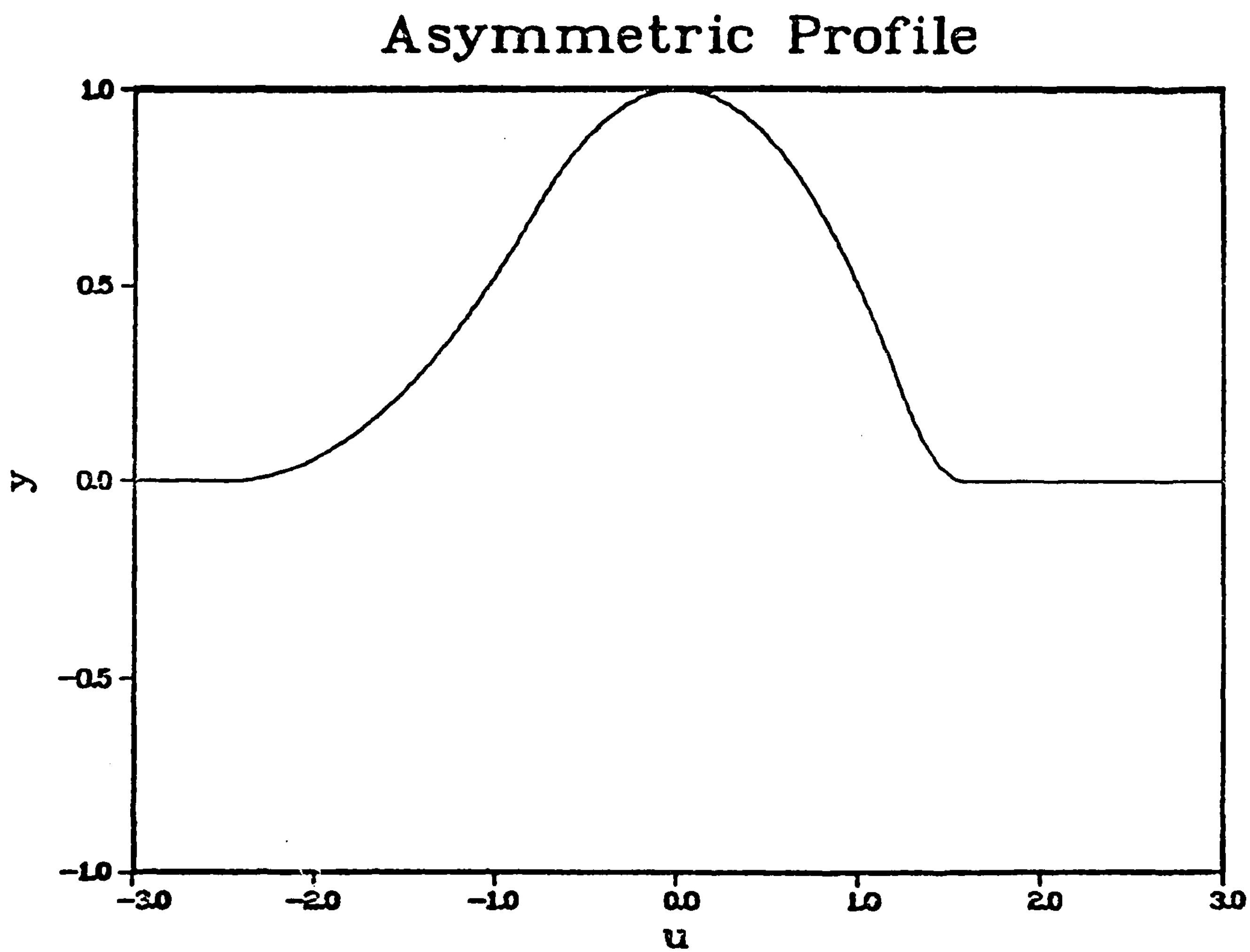


Figure 3 Asymmetric Profile

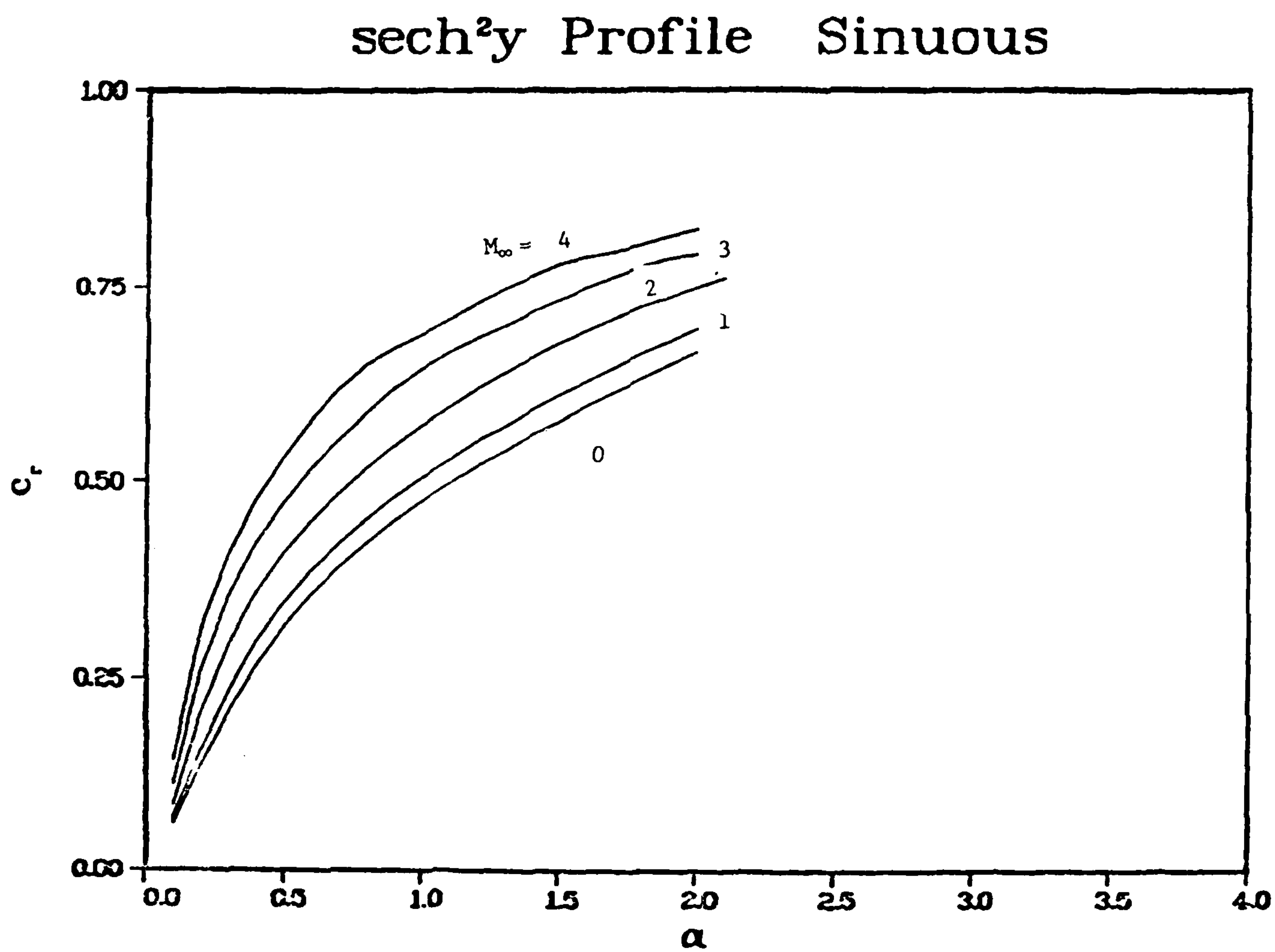


Figure 4 α vs. c_r

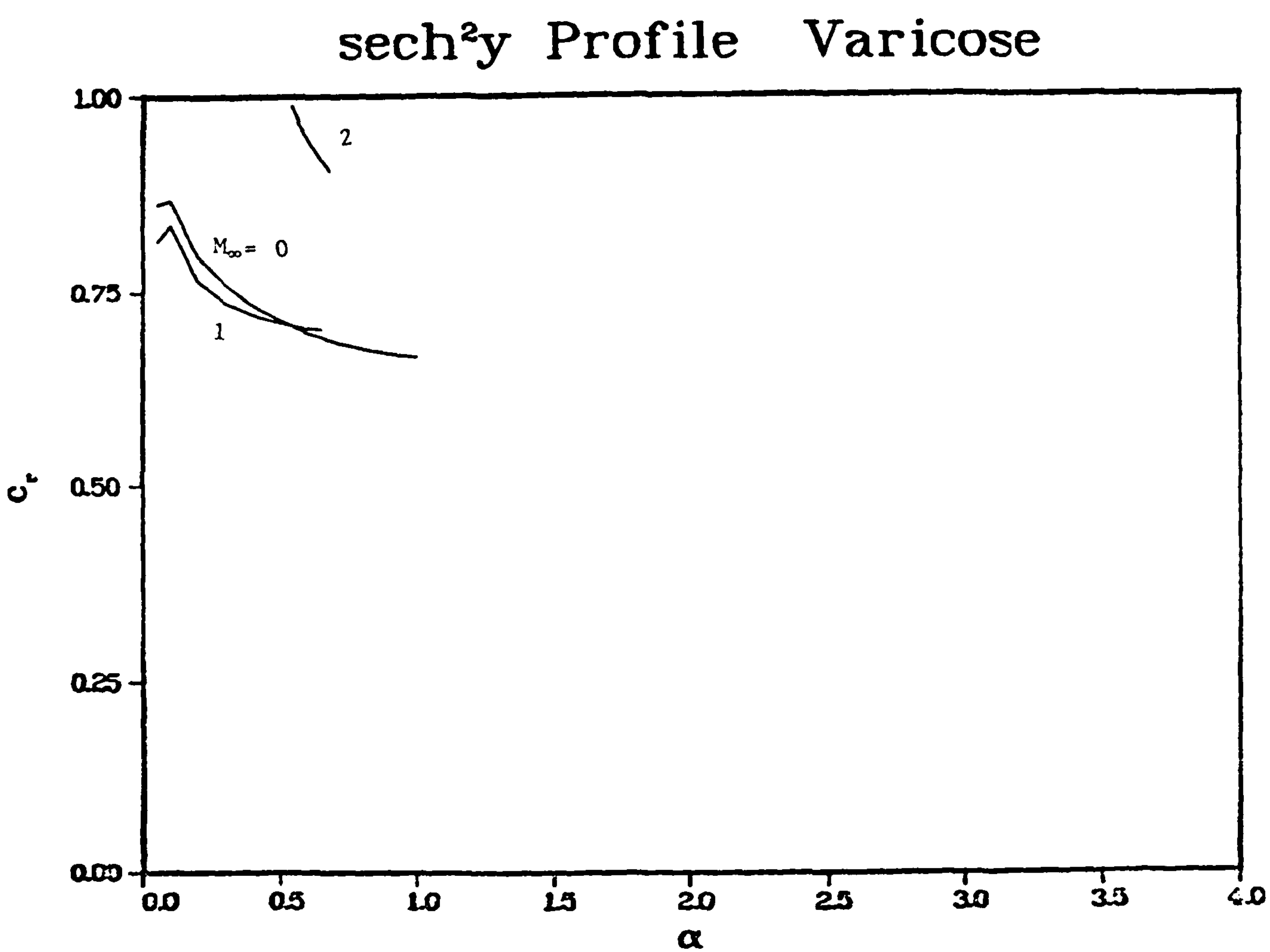


Figure 5 α vs. c_r

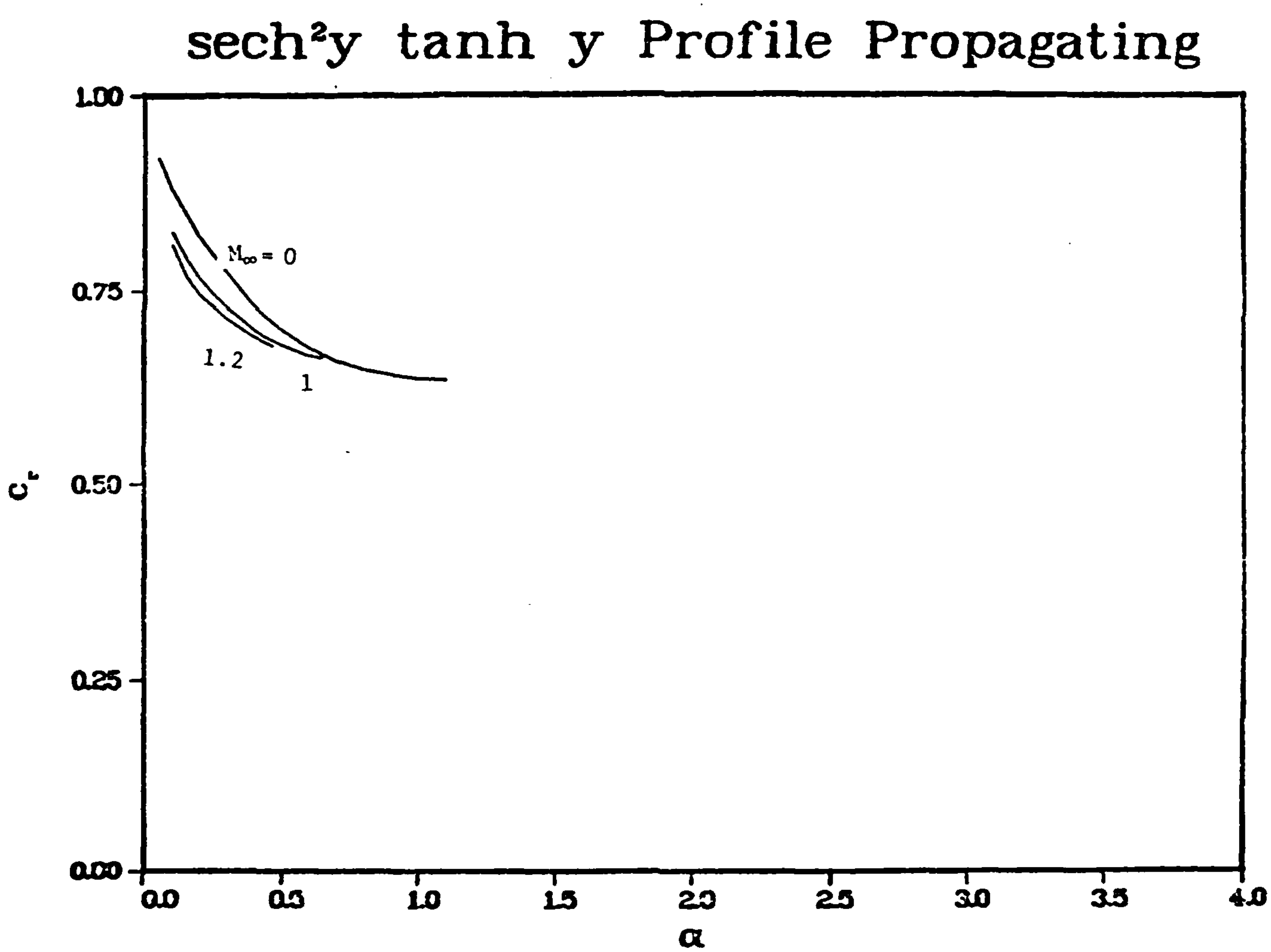


Figure 6 α vs. c_r

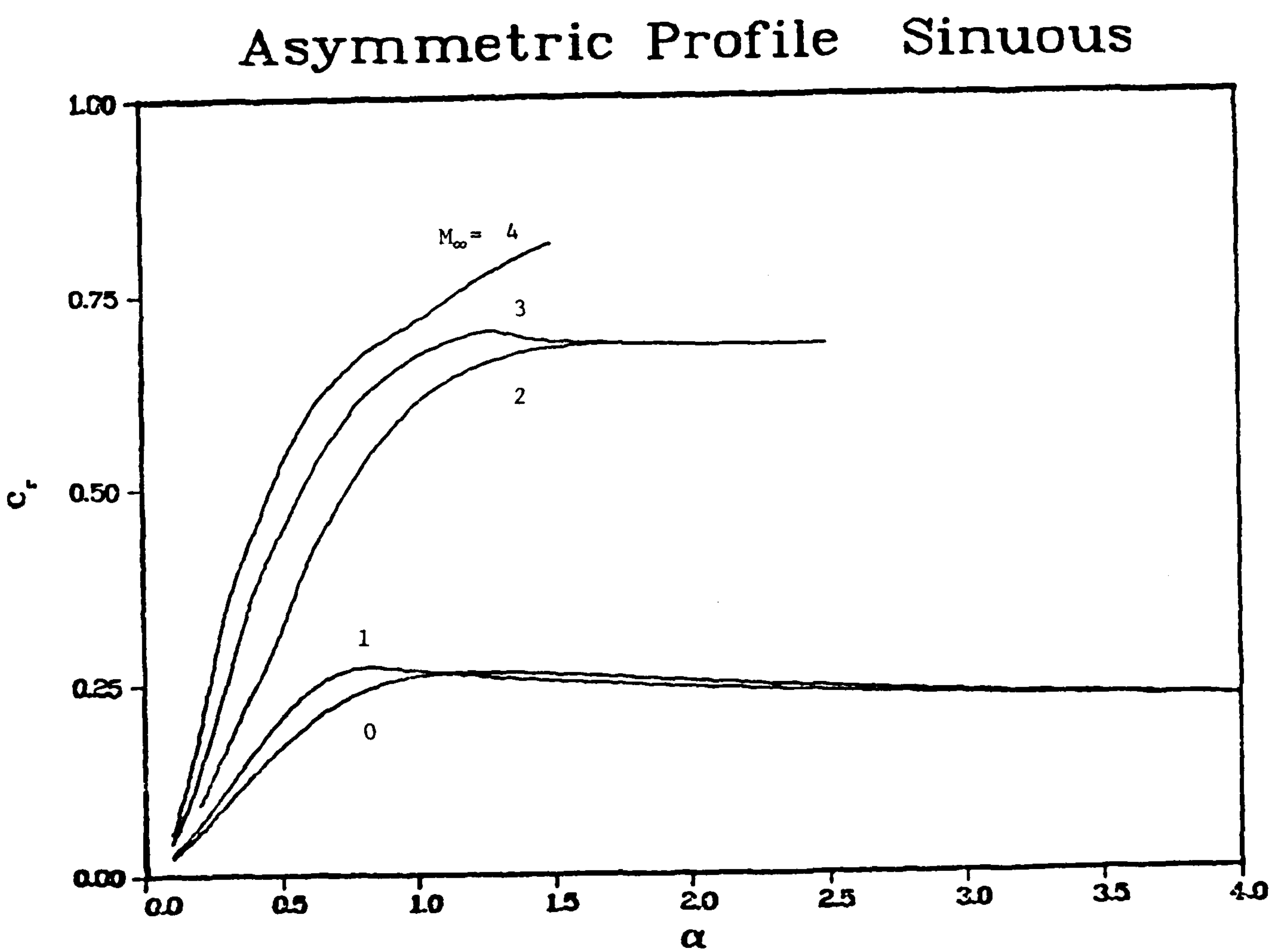


Figure 7 α vs. c_r

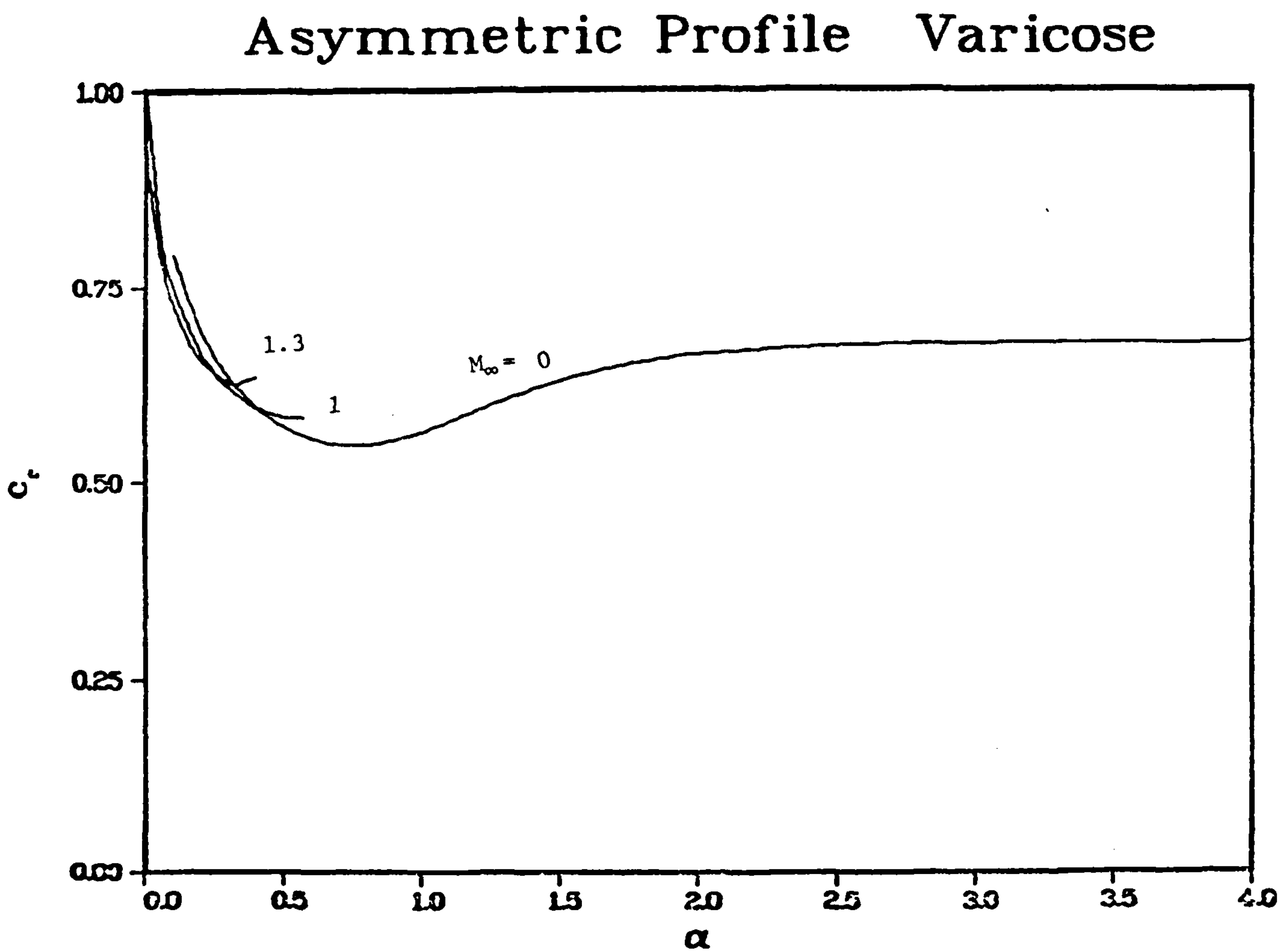


Figure 8 α vs. c_r

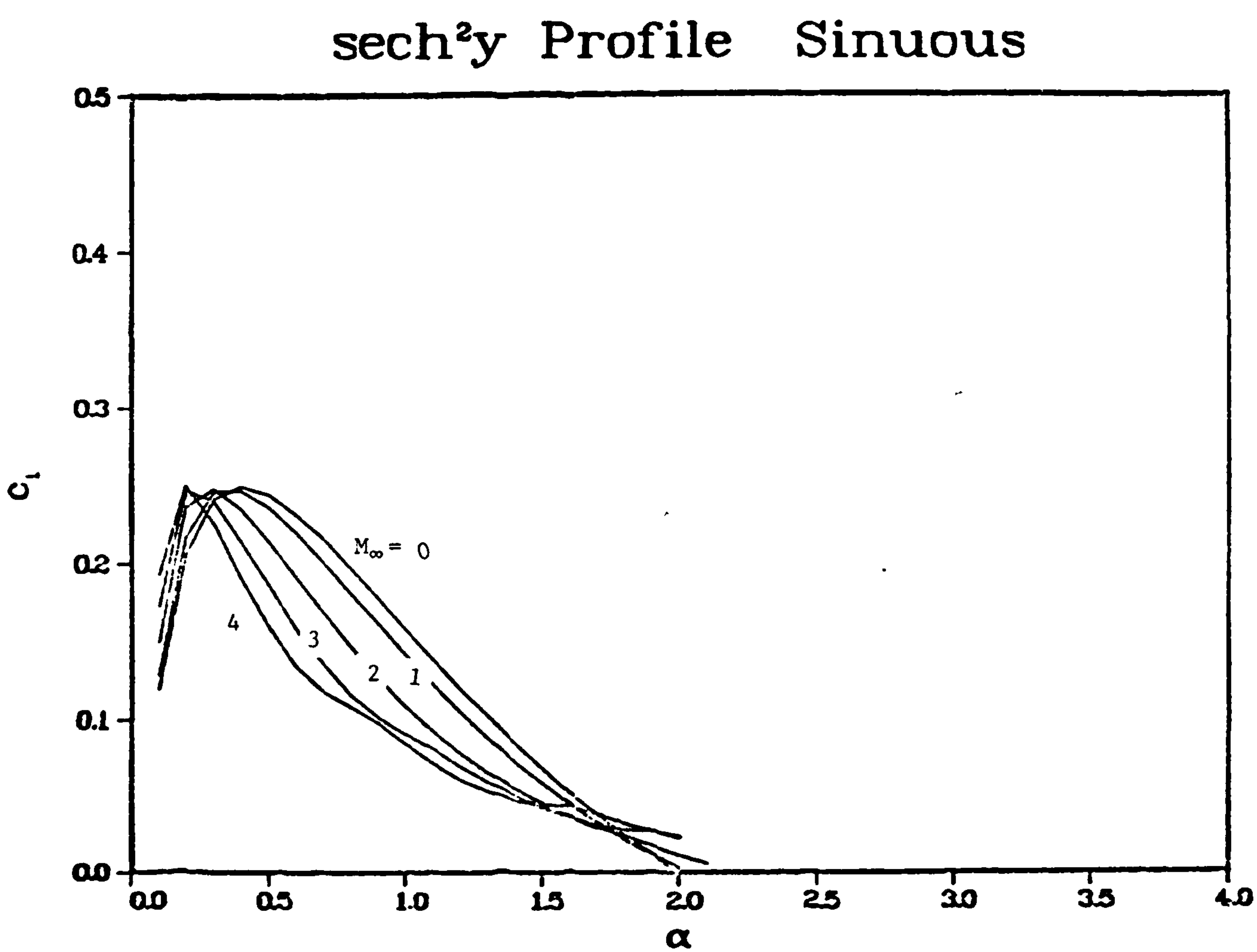


Figure 9 α vs. c_i

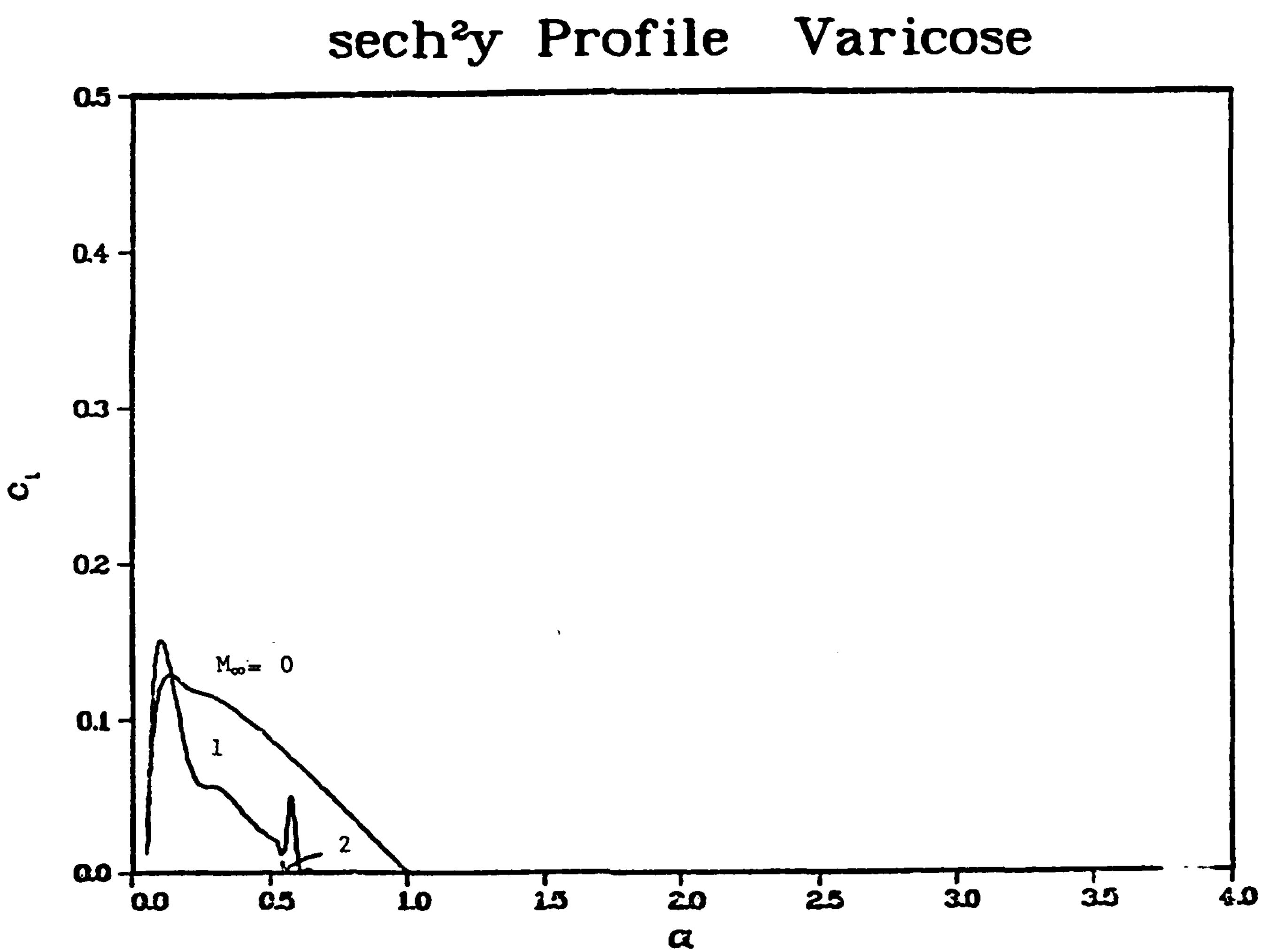


Figure 10 α vs. c_i

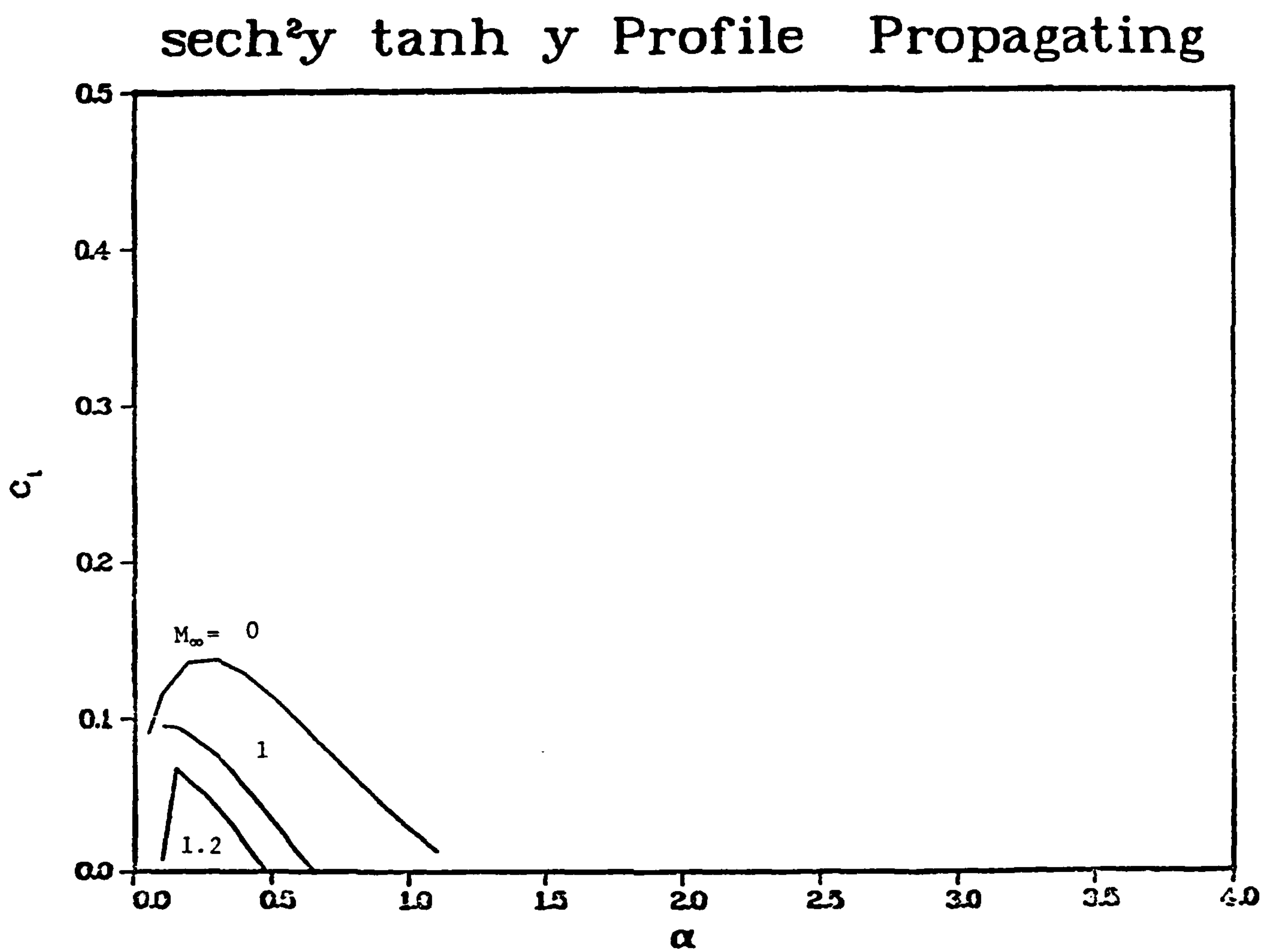


Figure 11 α vs. c_i

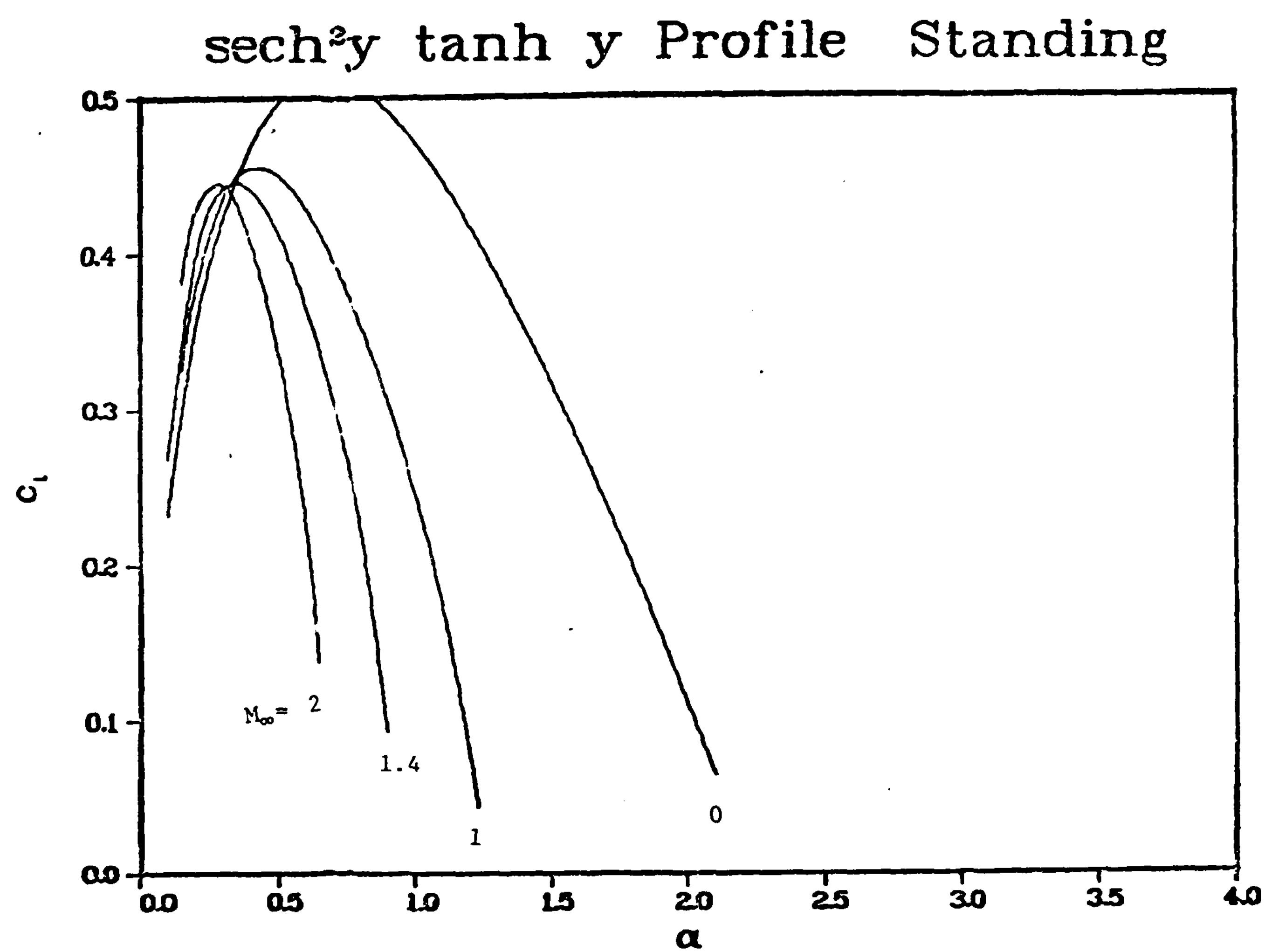


Figure 12 α vs. c_i

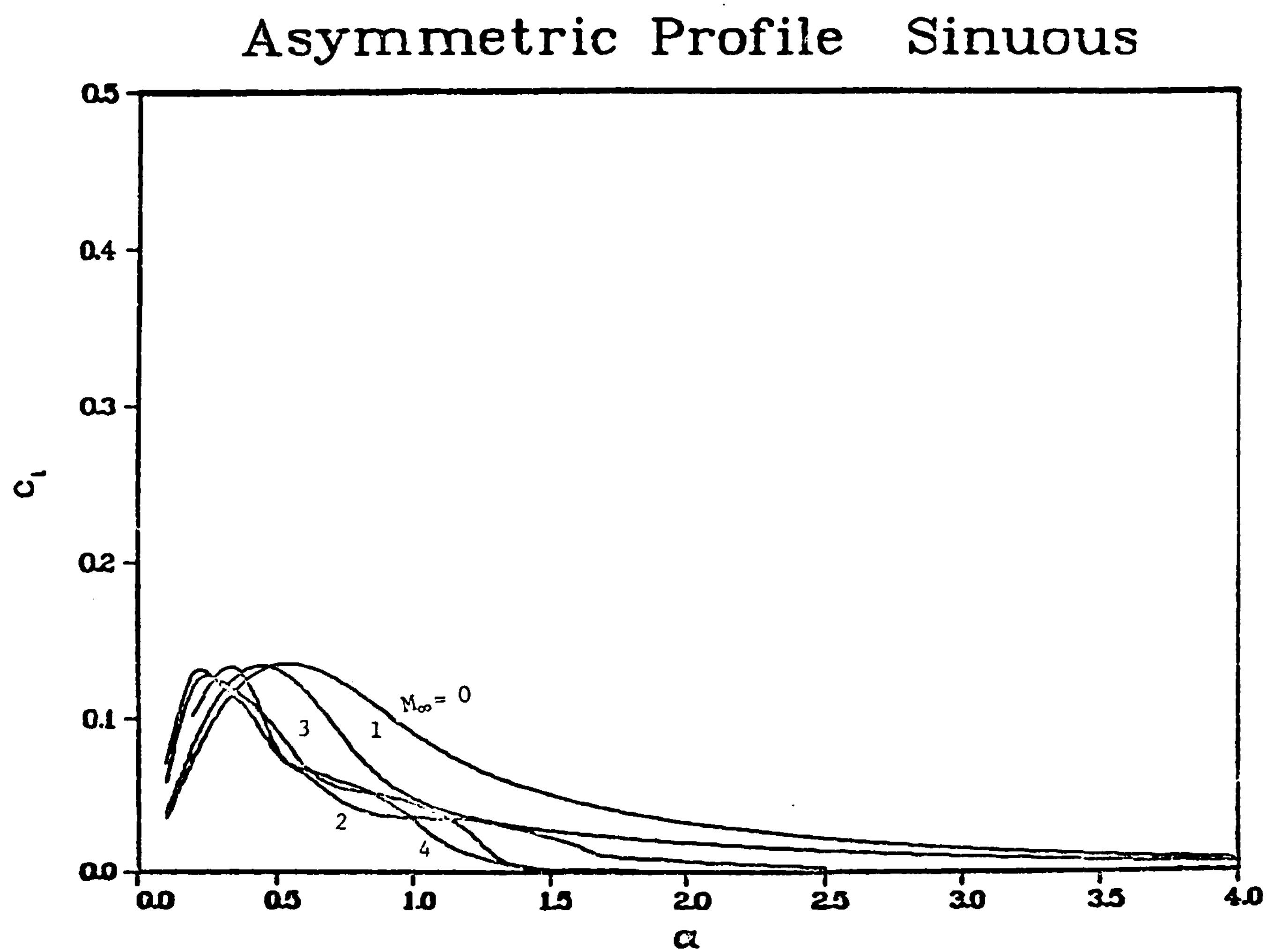


Figure 13 α vs. c_l

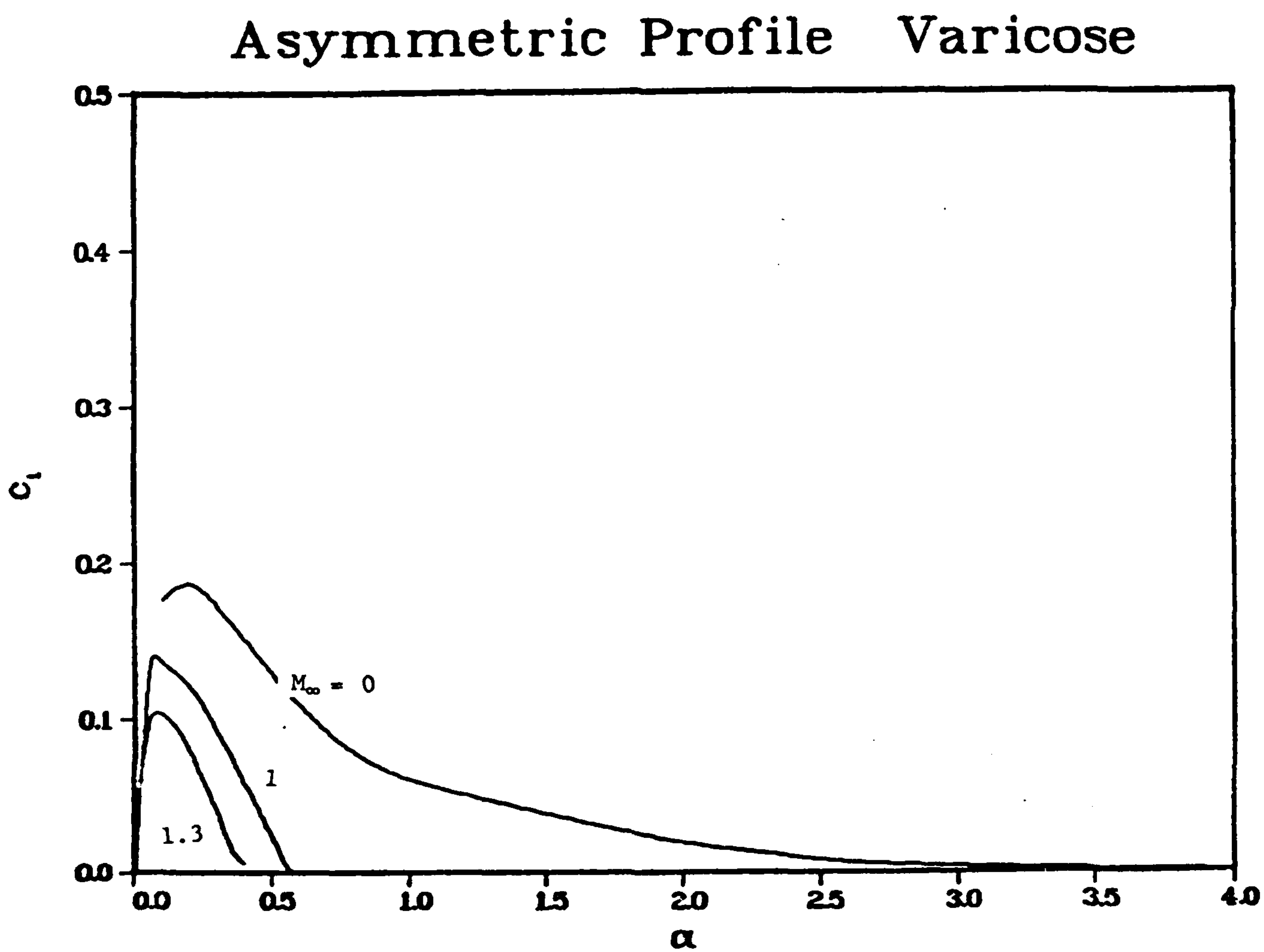


Figure 14 α vs. c_i

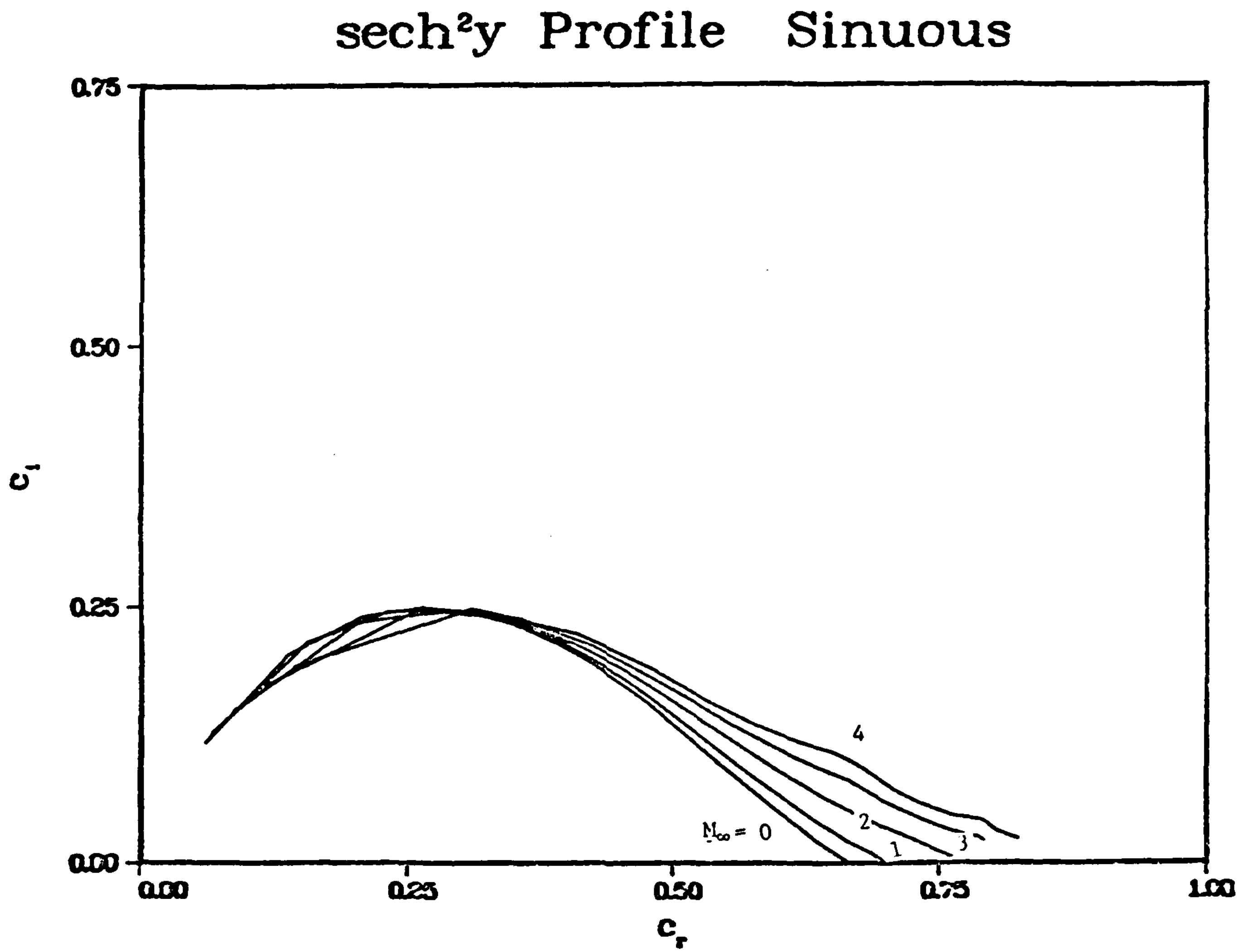


Figure 15 c_r vs. c_i

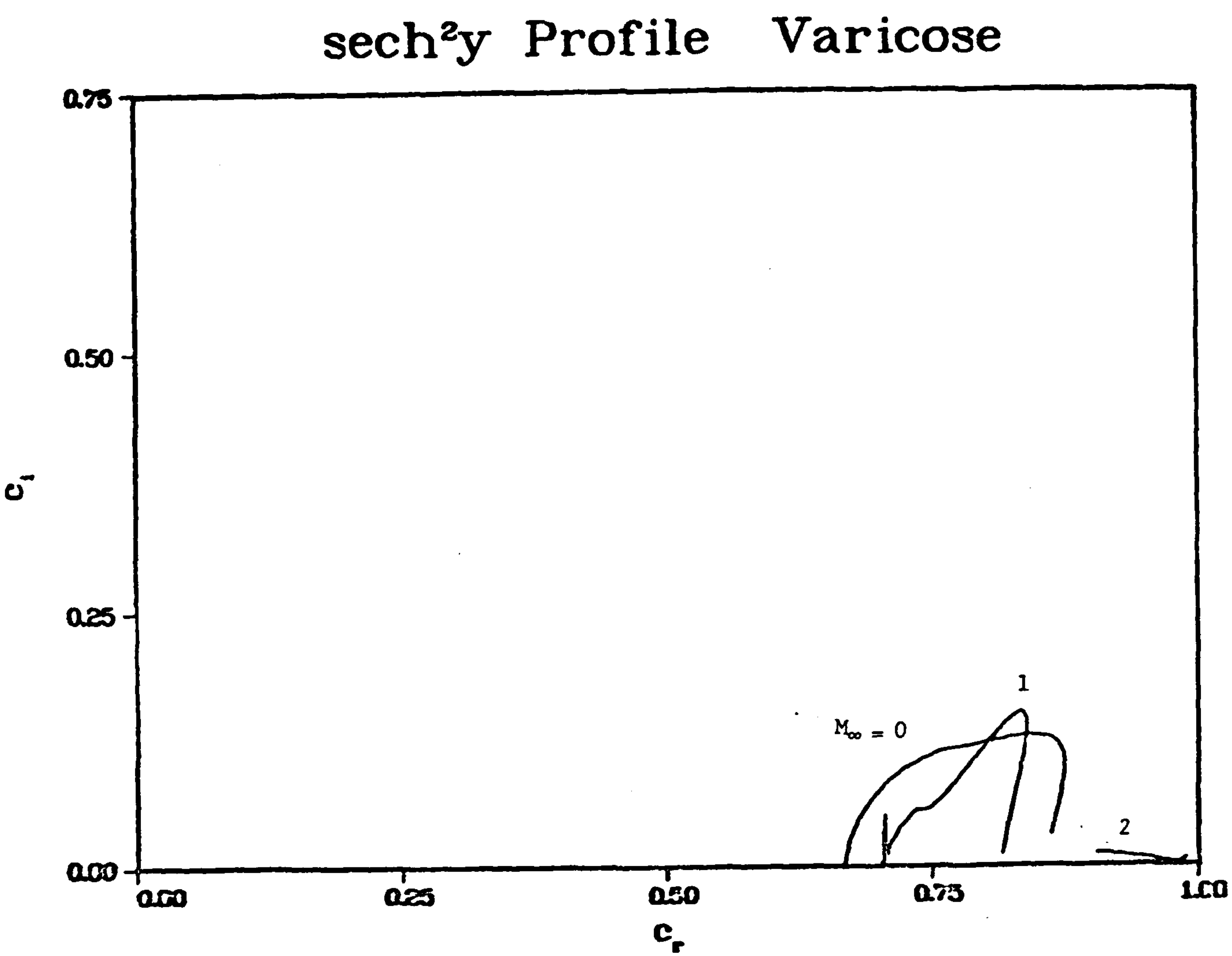


Figure 16 c_r vs. c_i

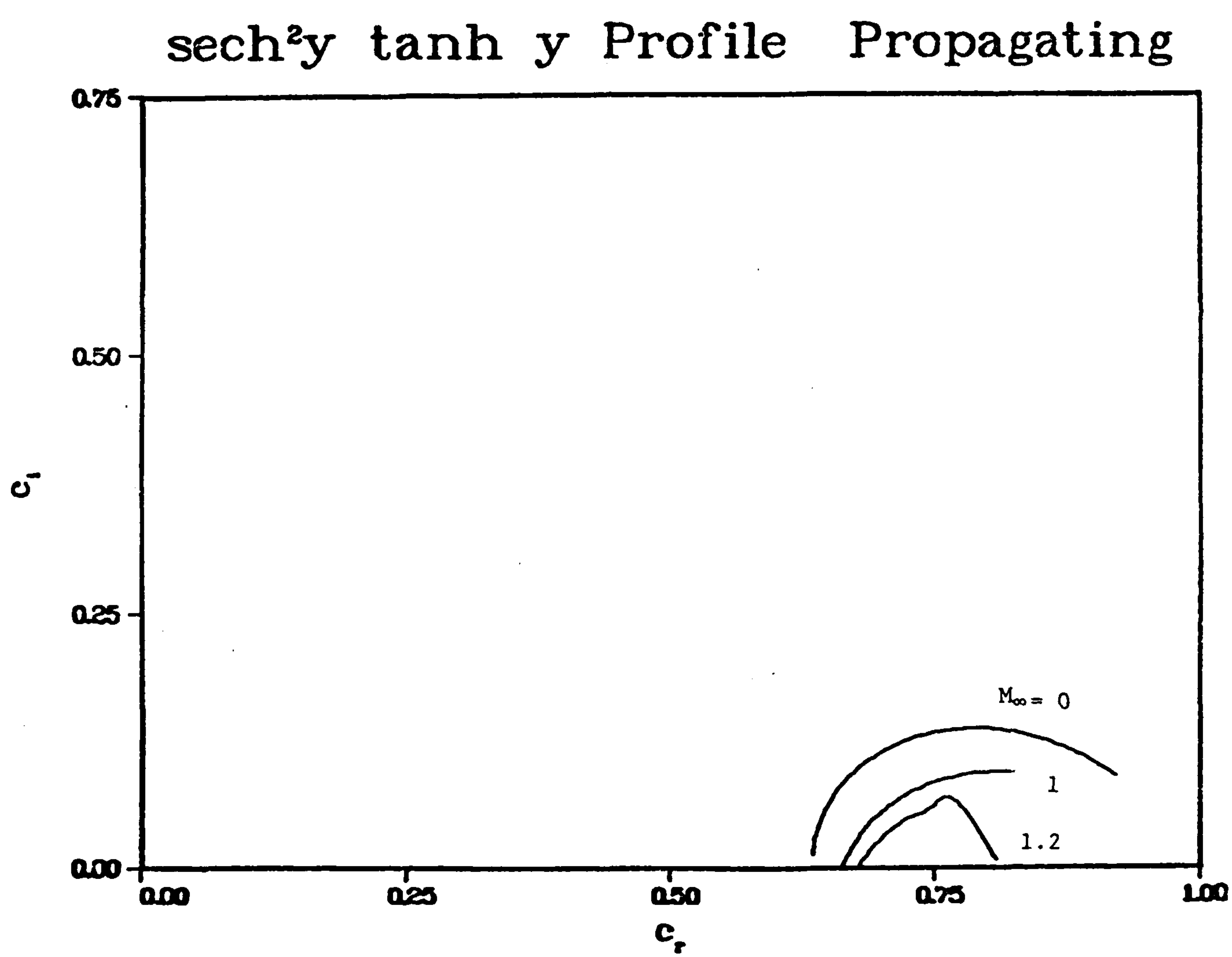


Figure 17 c_r vs. c_i

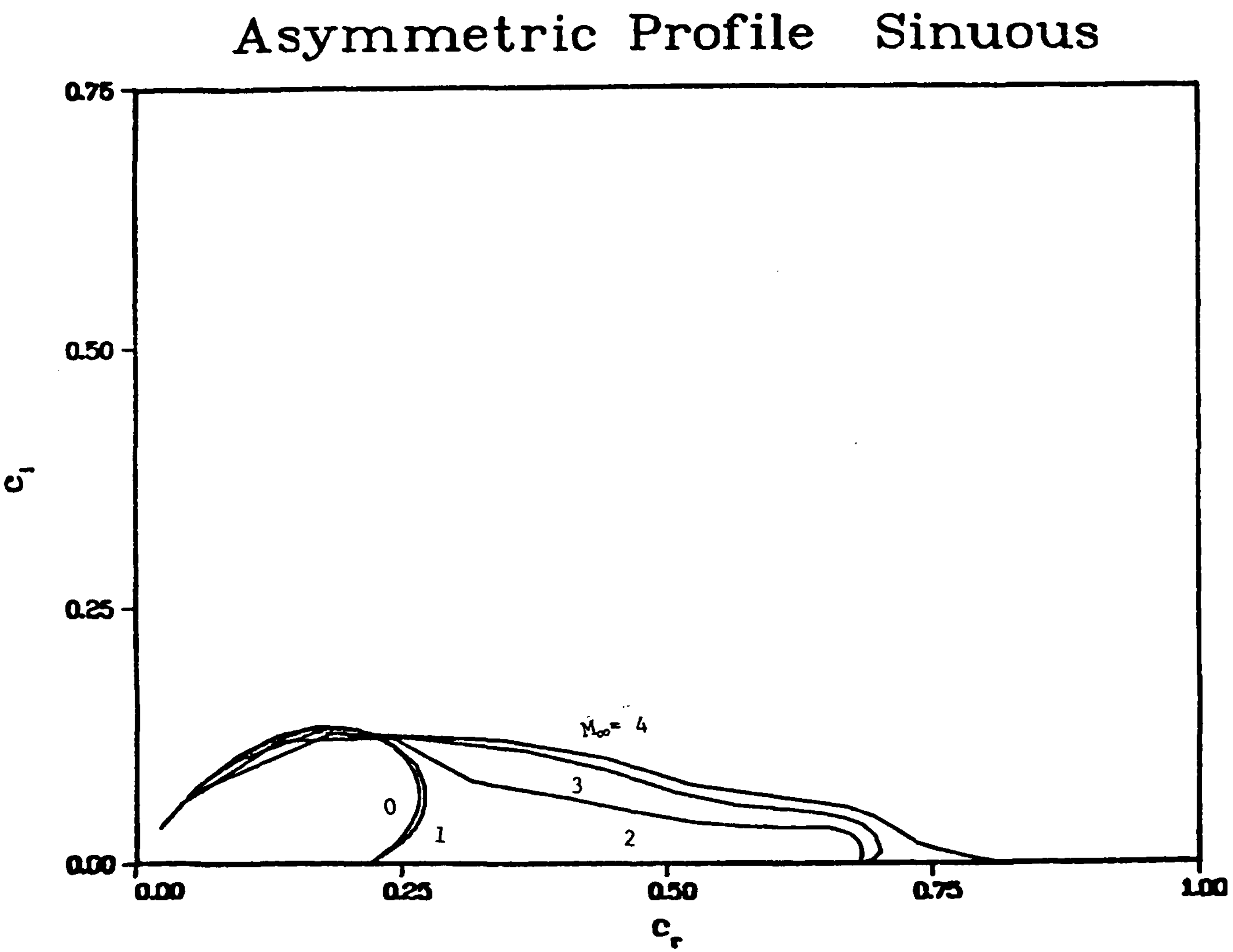


Figure 18 c_r vs. c_i

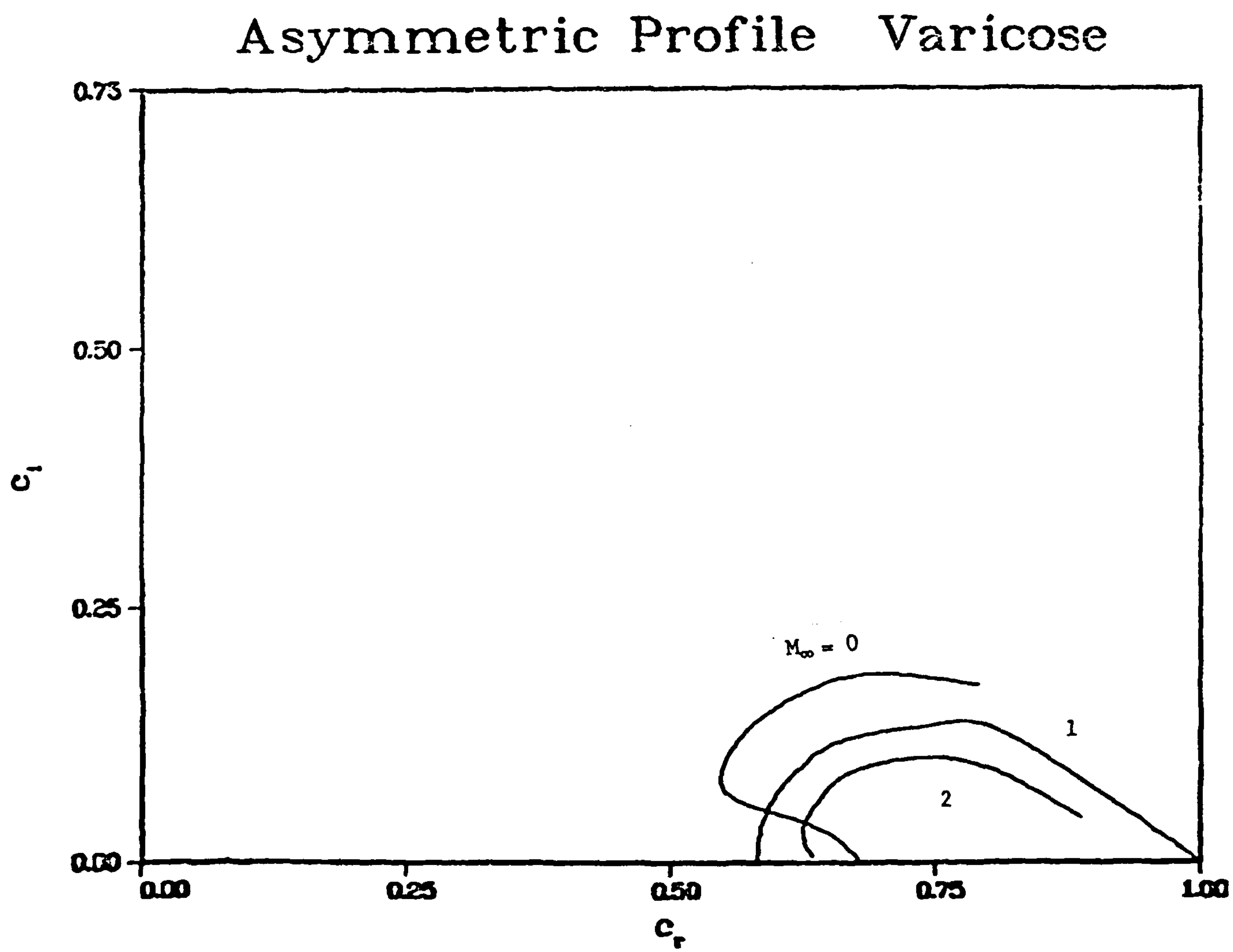


Figure 19 c_r vs. c_i

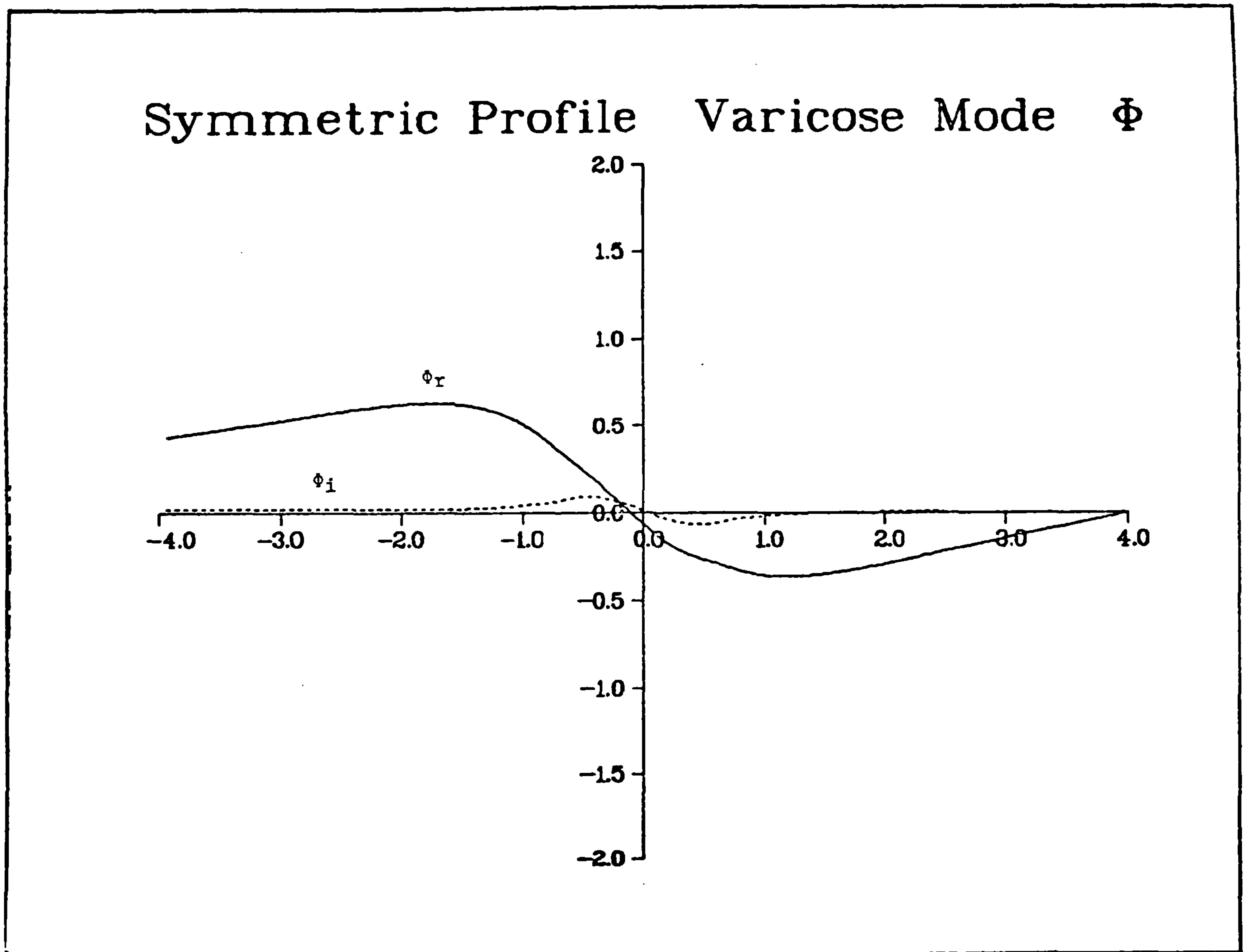
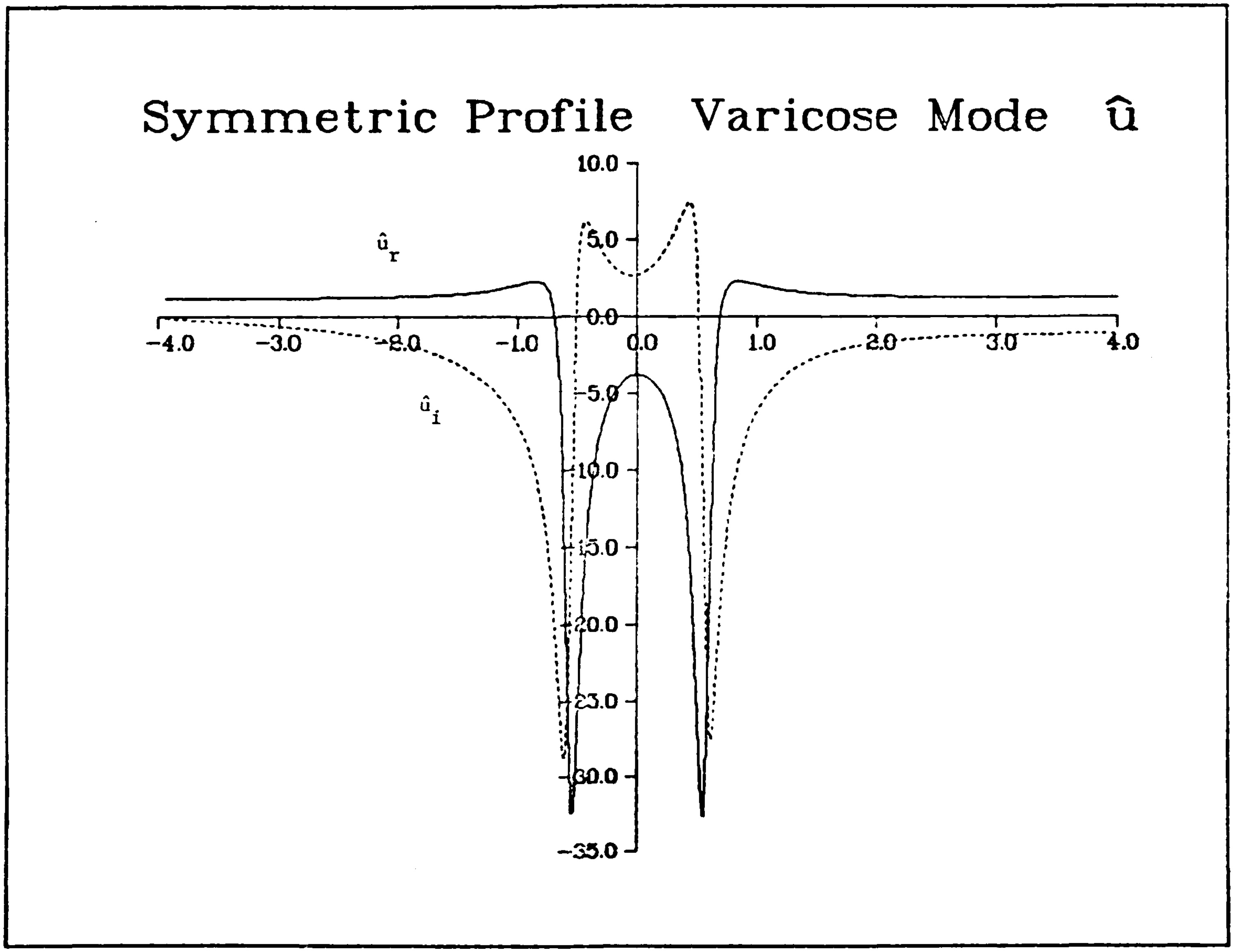


Figure 20a $M_\infty = 1.0$ $\alpha = .30$ $c = .737390+.0566281$



Symmetric Profile Varicose Mode û
$\hat{u}_r$
$\hat{u}_i$
10.0
5.0
0.0
-5.0
-10.0
-15.0
-20.0
-25.0
-30.0
-35.0
-4.0
-3.0
-2.0
-1.0
0.0
1.0
2.0
3.0
4.0

Figure 20b

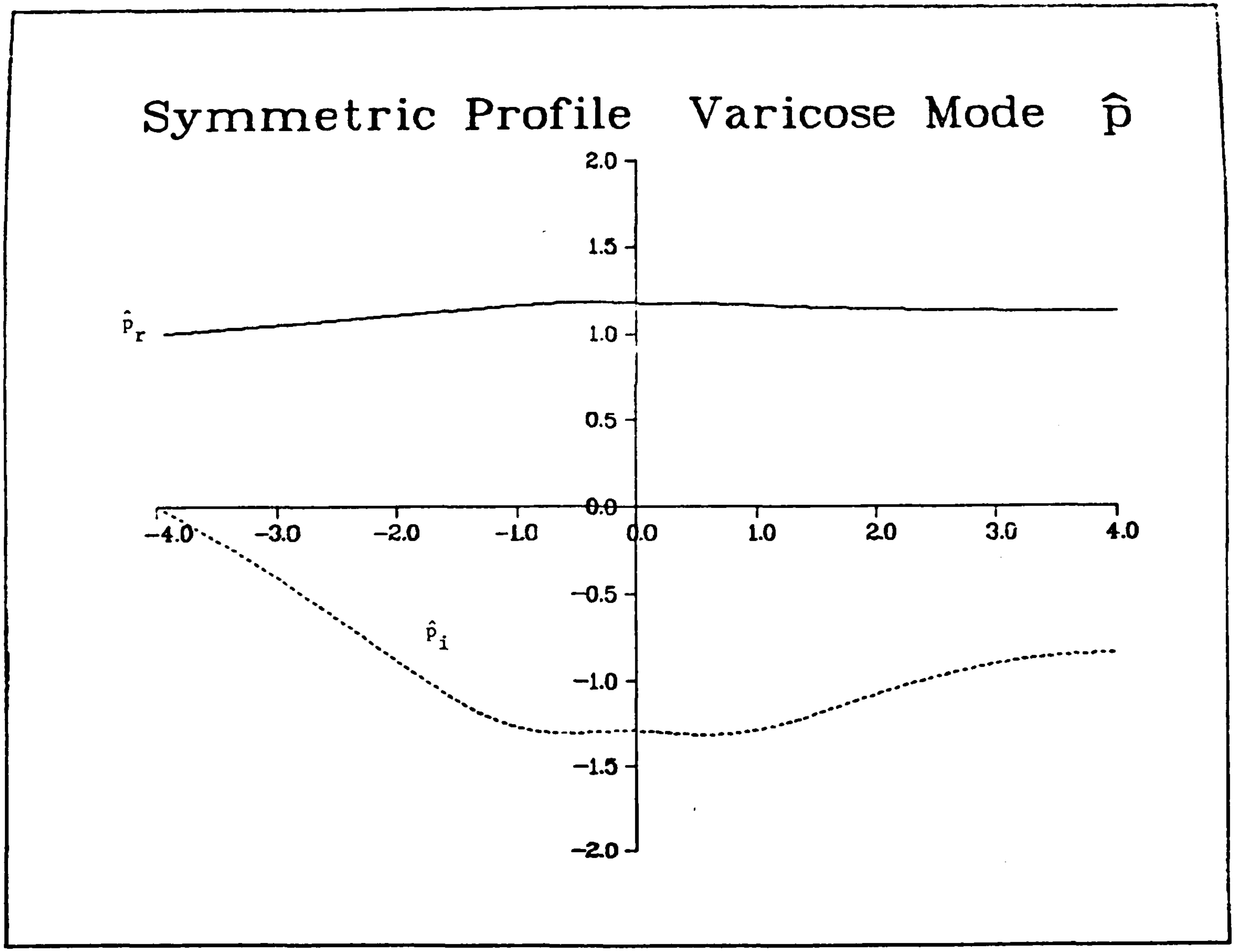


Figure 20c

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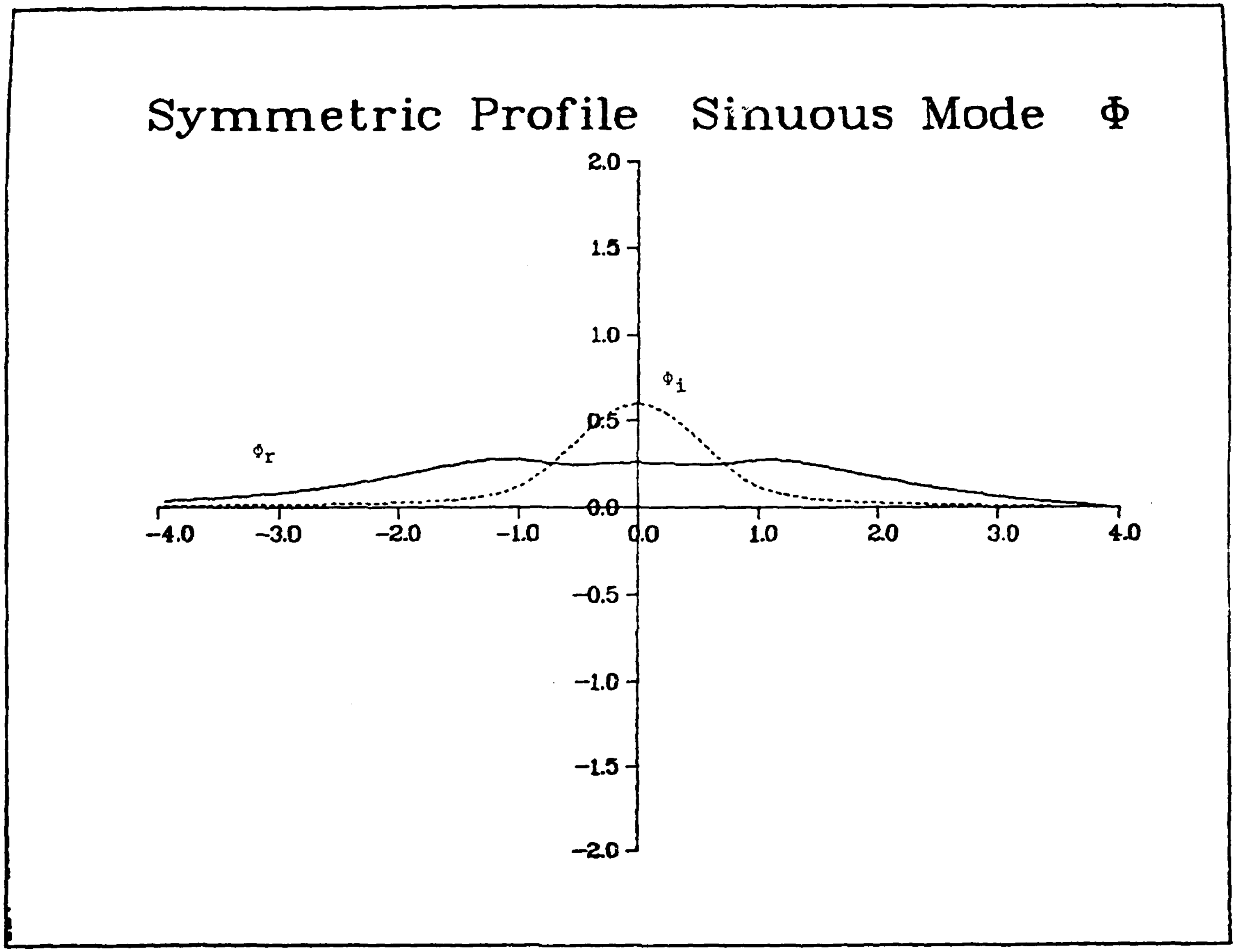


Figure 21a M_∞ = 1.0 α = .90 c = .480444+.161569i

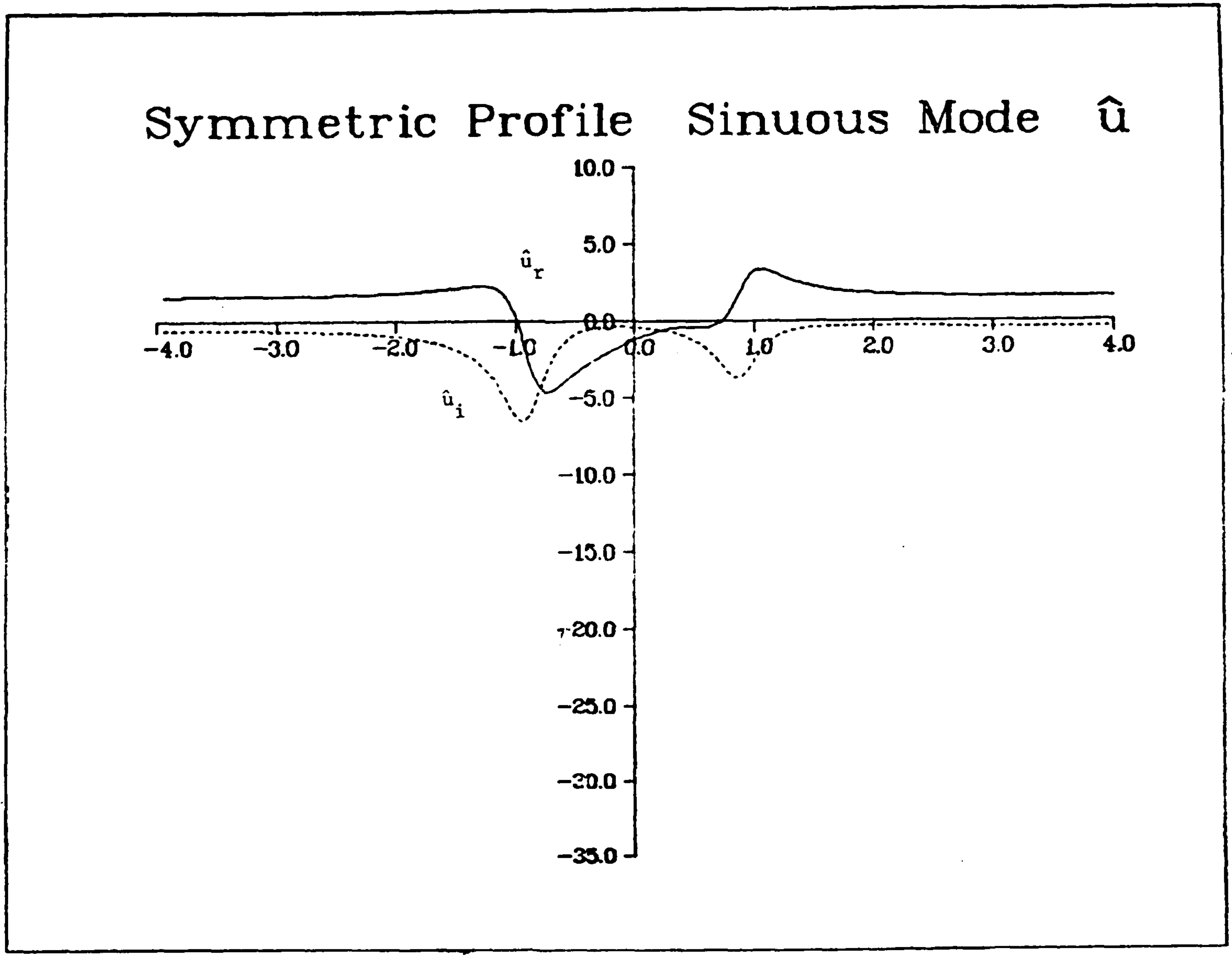

Symmetric Profile Sinuous Mode û
10.0
5.0
0.0
−5.0
−10.0
−15.0
−20.0
−25.0
−30.0
−35.0
−4.0
−3.0
−2.0
−1.0
0.0
1.0
2.0
3.0
4.0
$\hat{u}_r$
$\hat{u}_i$

Figure 21b

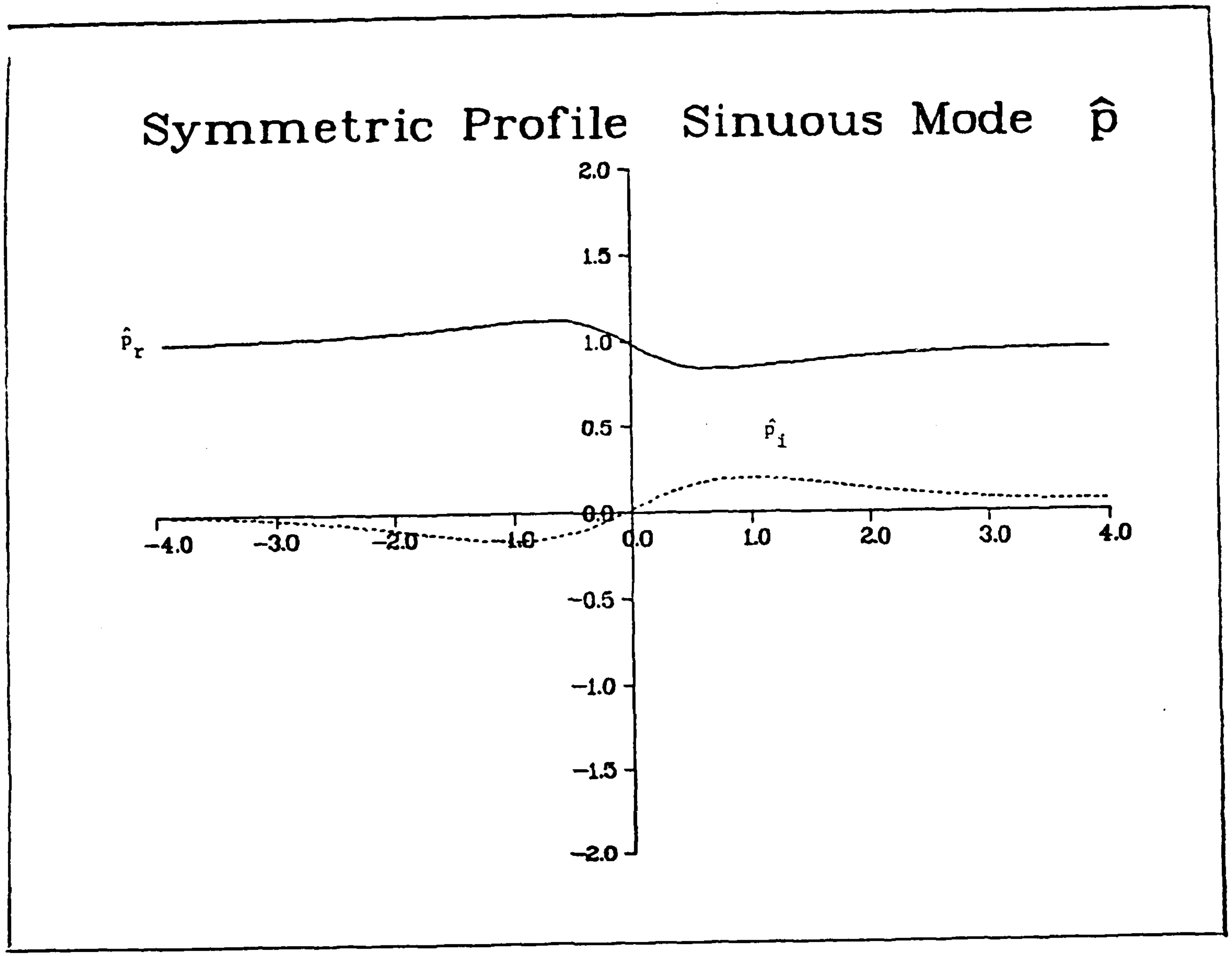

Symmetric Profile Sinuous Mode $\hat{p}$
$\hat{p}_r$
$\hat{p}_i$
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−4.0
−3.0
−2.0
−1.0
0.0
1.0
2.0
3.0
4.0

Figure 21c

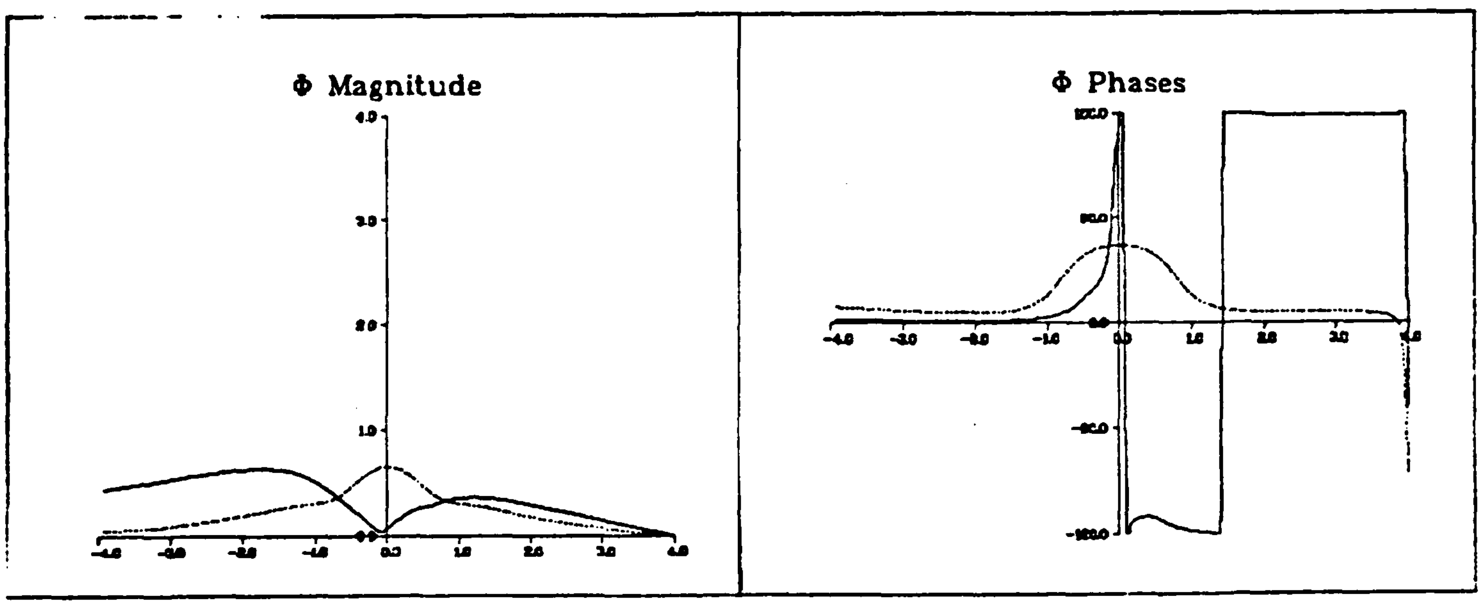


Figure 22a

Varicose Mode

Solid lines
$M_\infty = 1.0$
$\alpha = .30$
$c_r = .737390$
$c_i = .056628$

Sinuous Mode

Dotted lines
$M_\infty = 1.0$
$\alpha = .90$
$c_r = .480444$
$c_i = .161569$

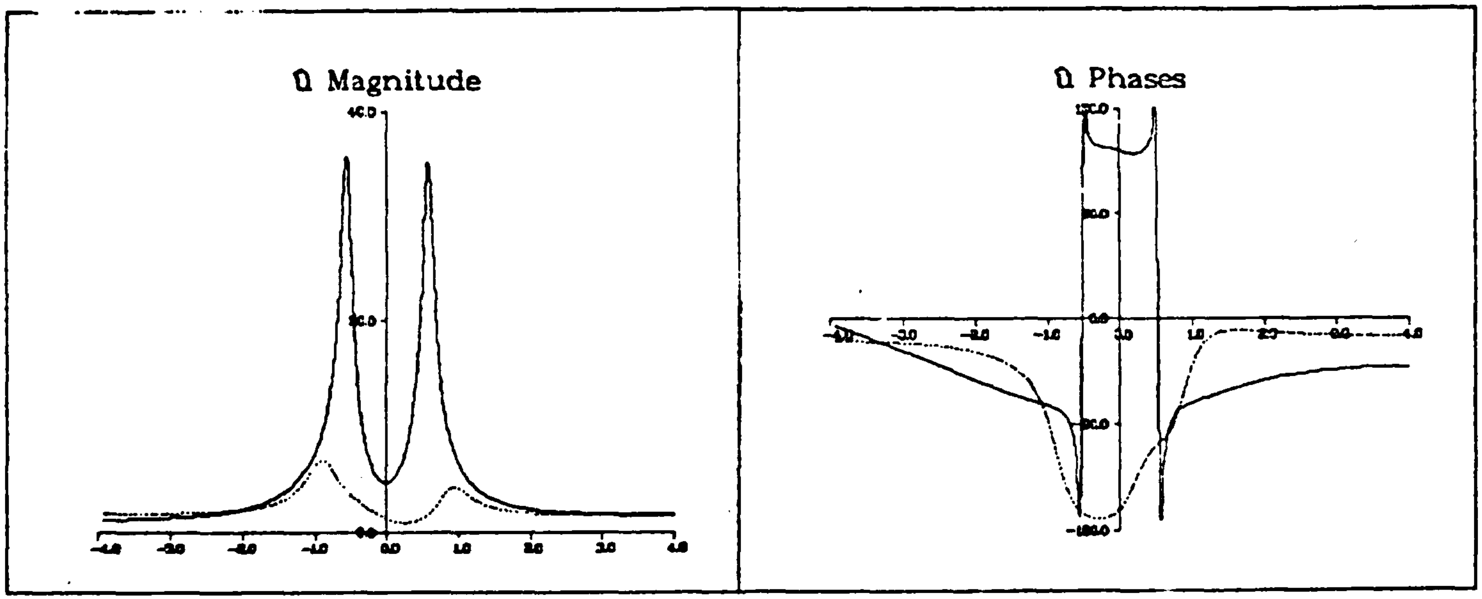

Ω Magnitude
40.0
20.0
0.0
-4.0
-3.0
-2.0
-1.0
0.0
1.0
2.0
3.0
4.0
Ω Phases
180.0
90.0
0.0
-90.0
-180.0

Figure 22b

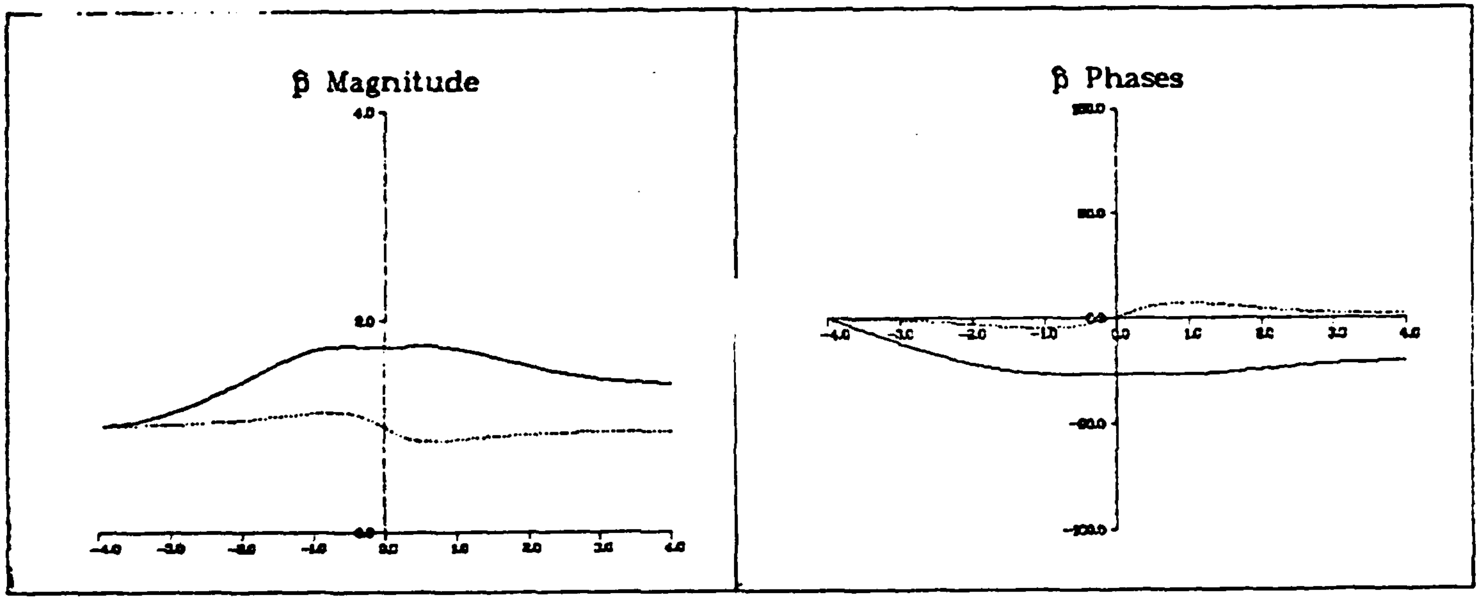


Figure 22c

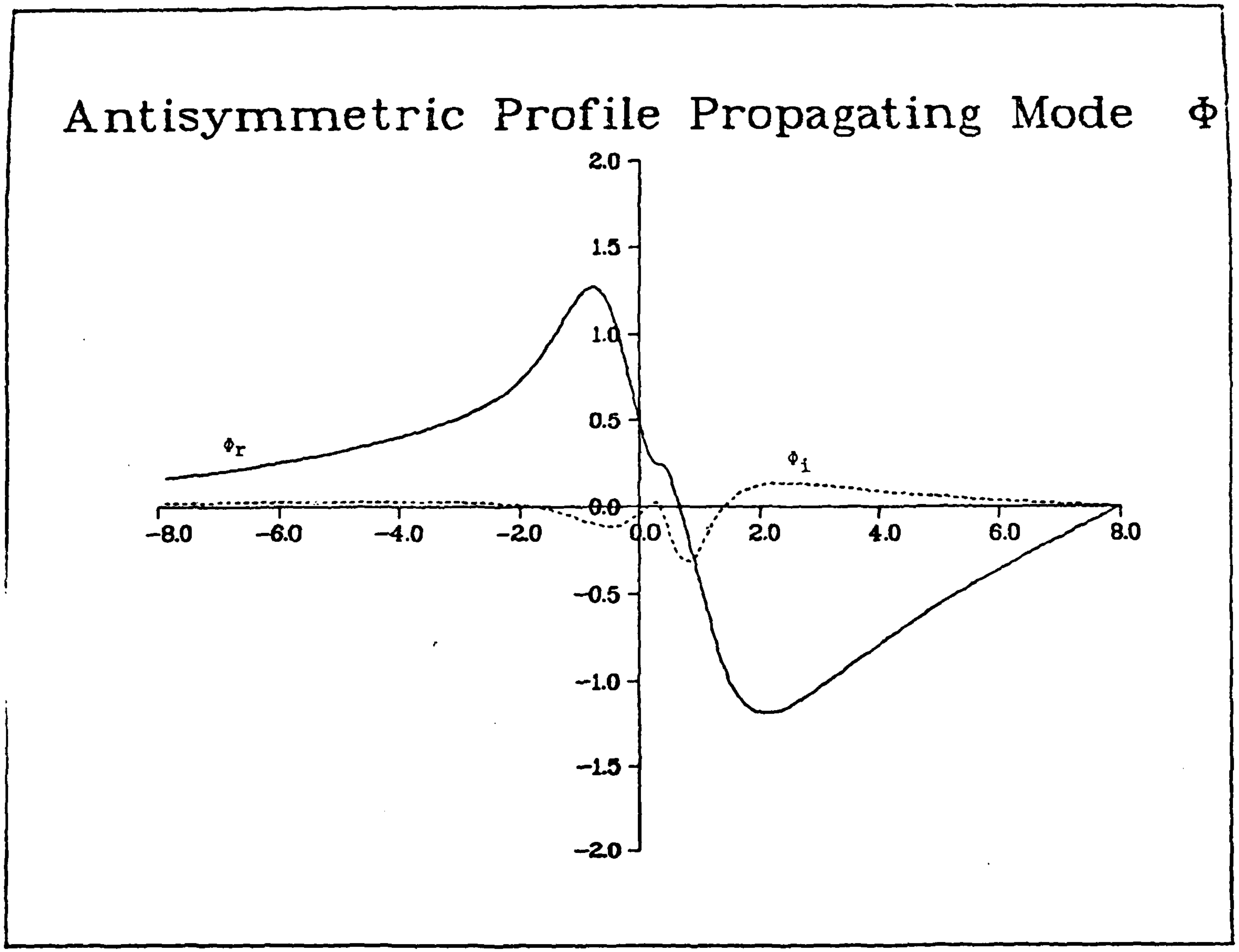


Figure 23a $M_\infty = 1.0$ $\alpha = .30$ $c = .728650+.075980i$

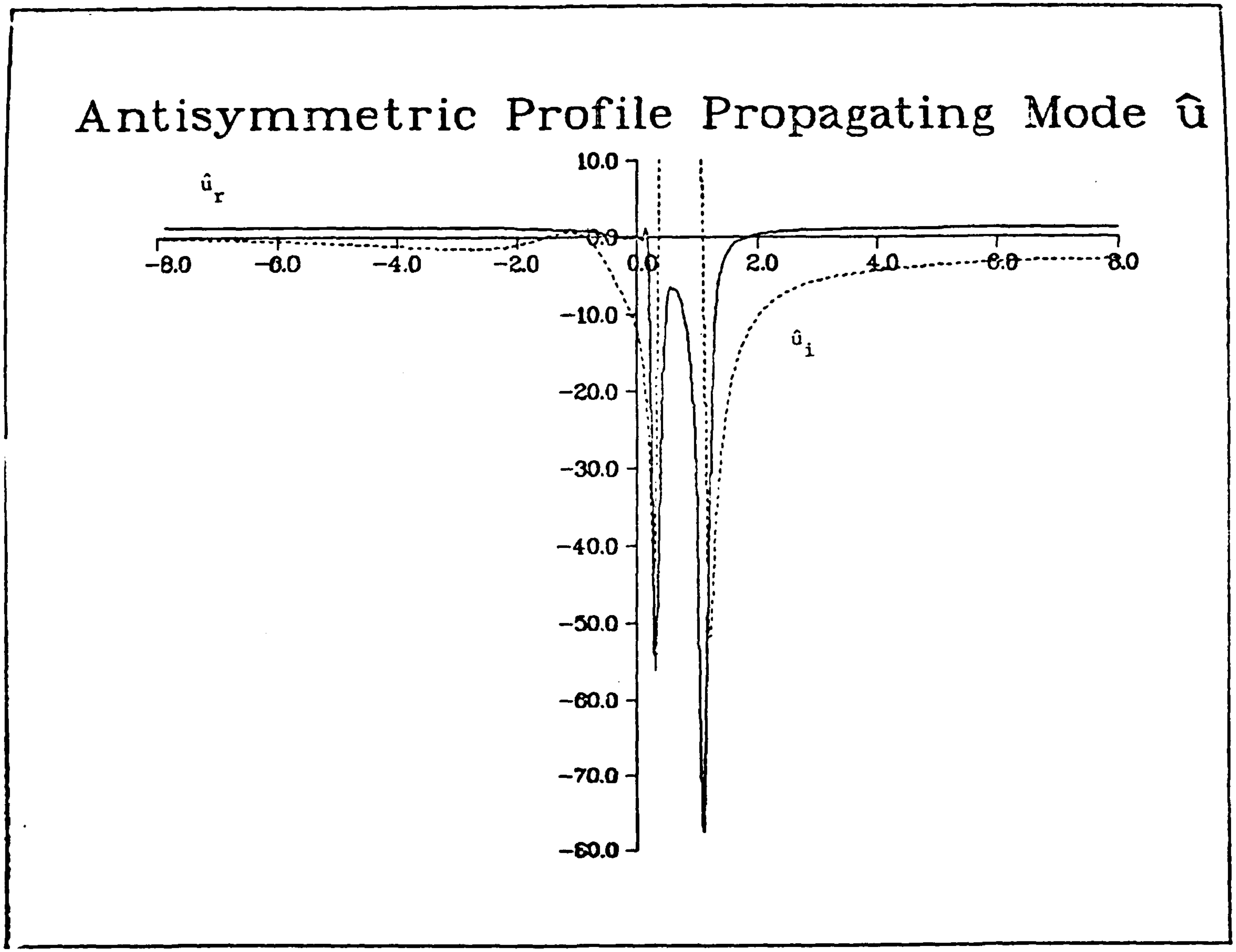
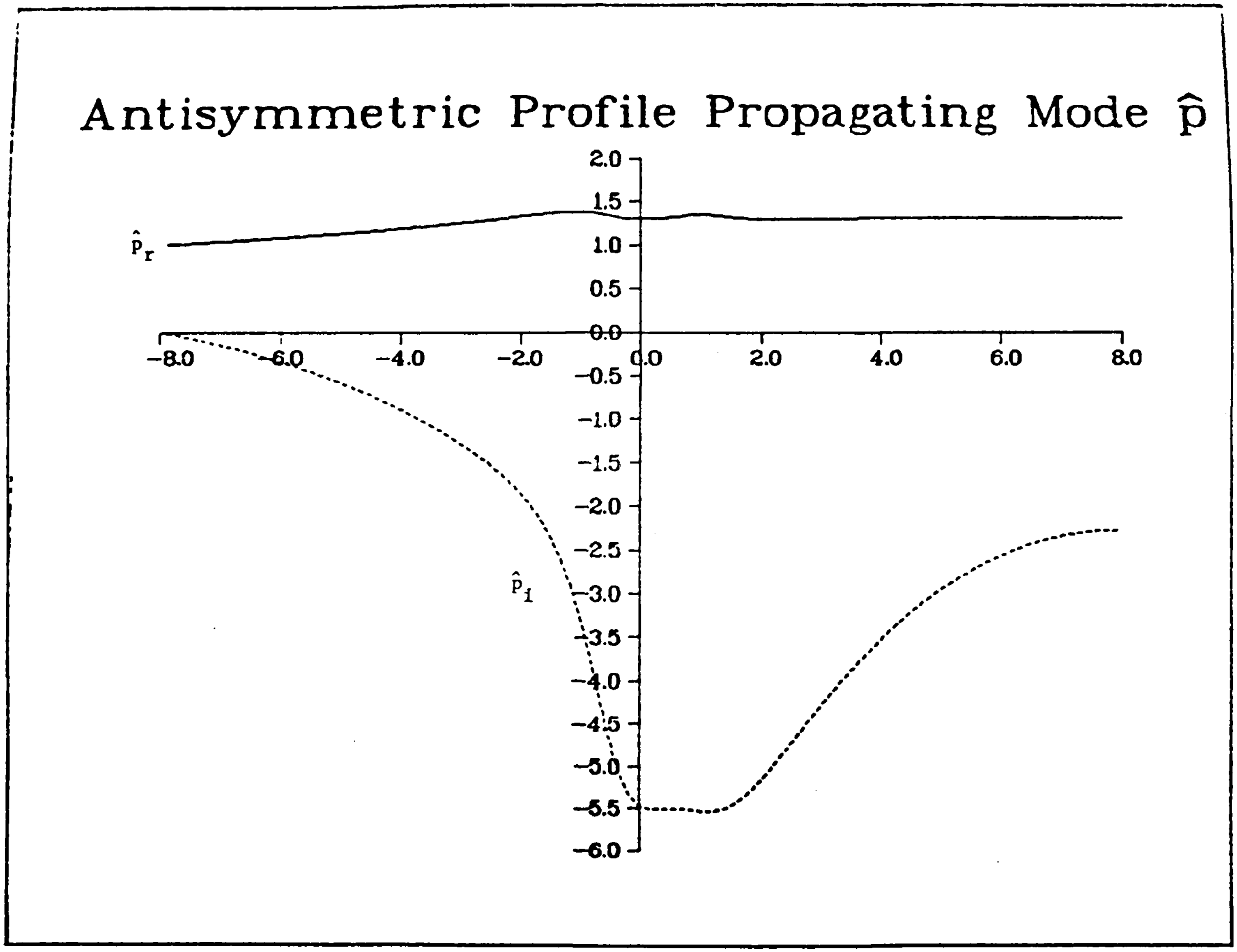

Antisymmetric Profile Propagating Mode û
10.0
0.0
-10.0
-20.0
-30.0
-40.0
-50.0
-60.0
-70.0
-80.0
-8.0
-6.0
-4.0
-2.0
0.0
2.0
4.0
6.0
8.0
û r
û i

Figure 23b



Antisymmetric Profile Propagating Mode $\hat{p}$
$\hat{p}_r$
$\hat{p}_i$
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
-3.0
-3.5
-4.0
-4.5
-5.0
-5.5
-6.0
-8.0
-6.0
-4.0
-2.0
0.0
2.0
4.0
6.0
8.0

Figure 23c

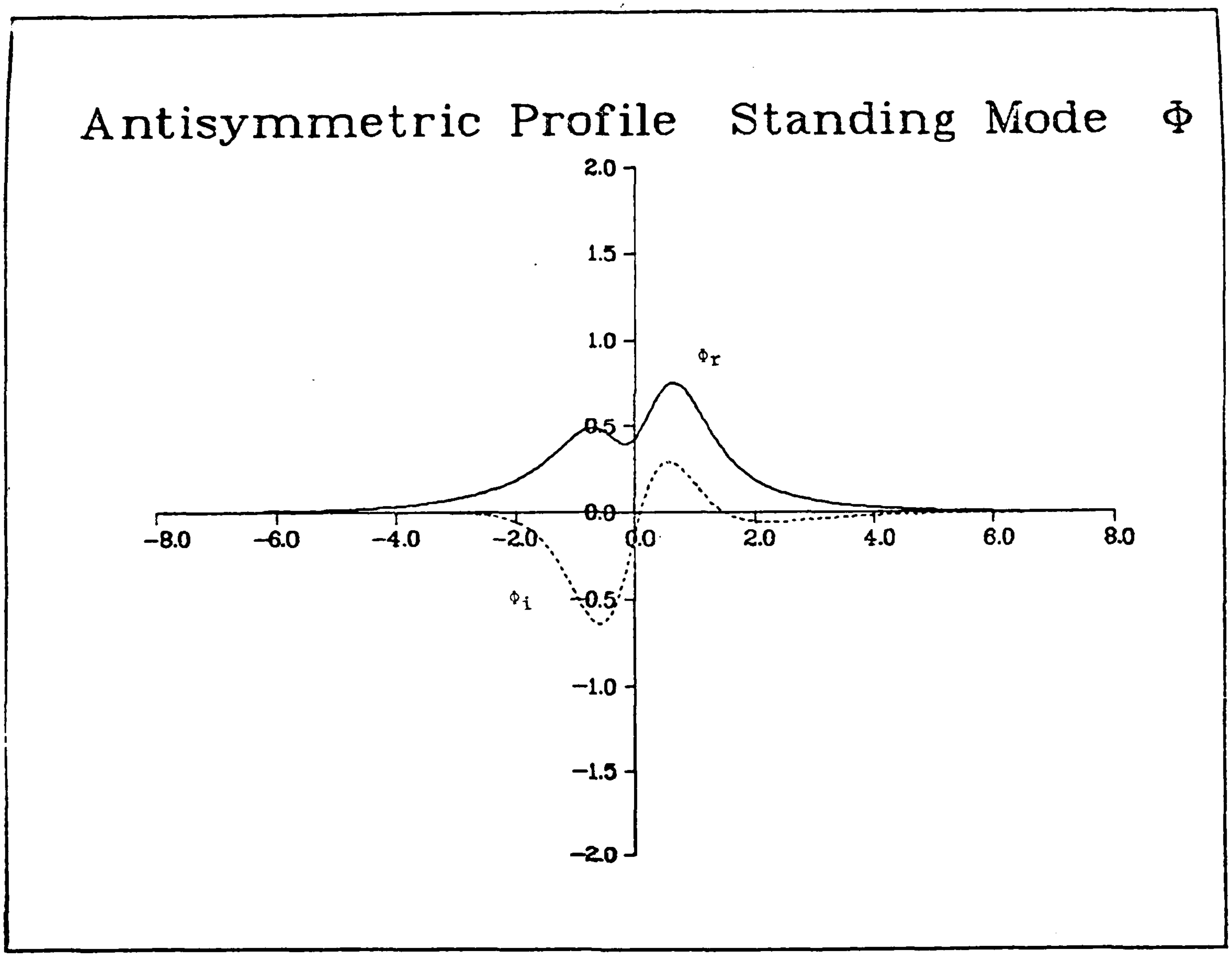


Figure 24a $M_\infty = 1.0$ $\alpha = .80$ $c = .355153i$

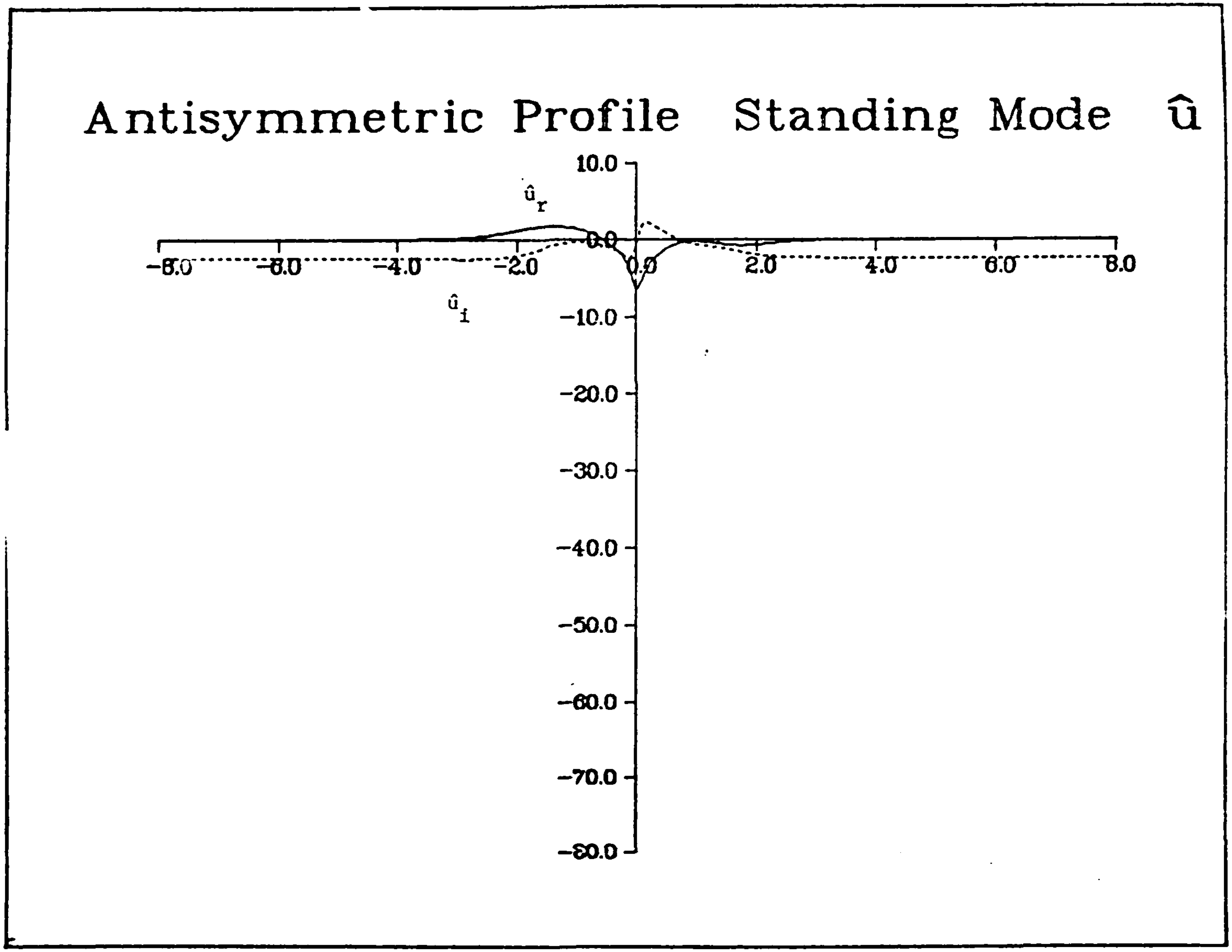

Antisymmetric Profile Standing Mode $\hat{u}$
$\hat{u}_r$
$\hat{u}_i$
10.0
0.0
-10.0
-20.0
-30.0
-40.0
-50.0
-60.0
-70.0
-80.0
-8.0
-6.0
-4.0
-2.0
0.0
2.0
4.0
6.0
8.0

Figure 24b

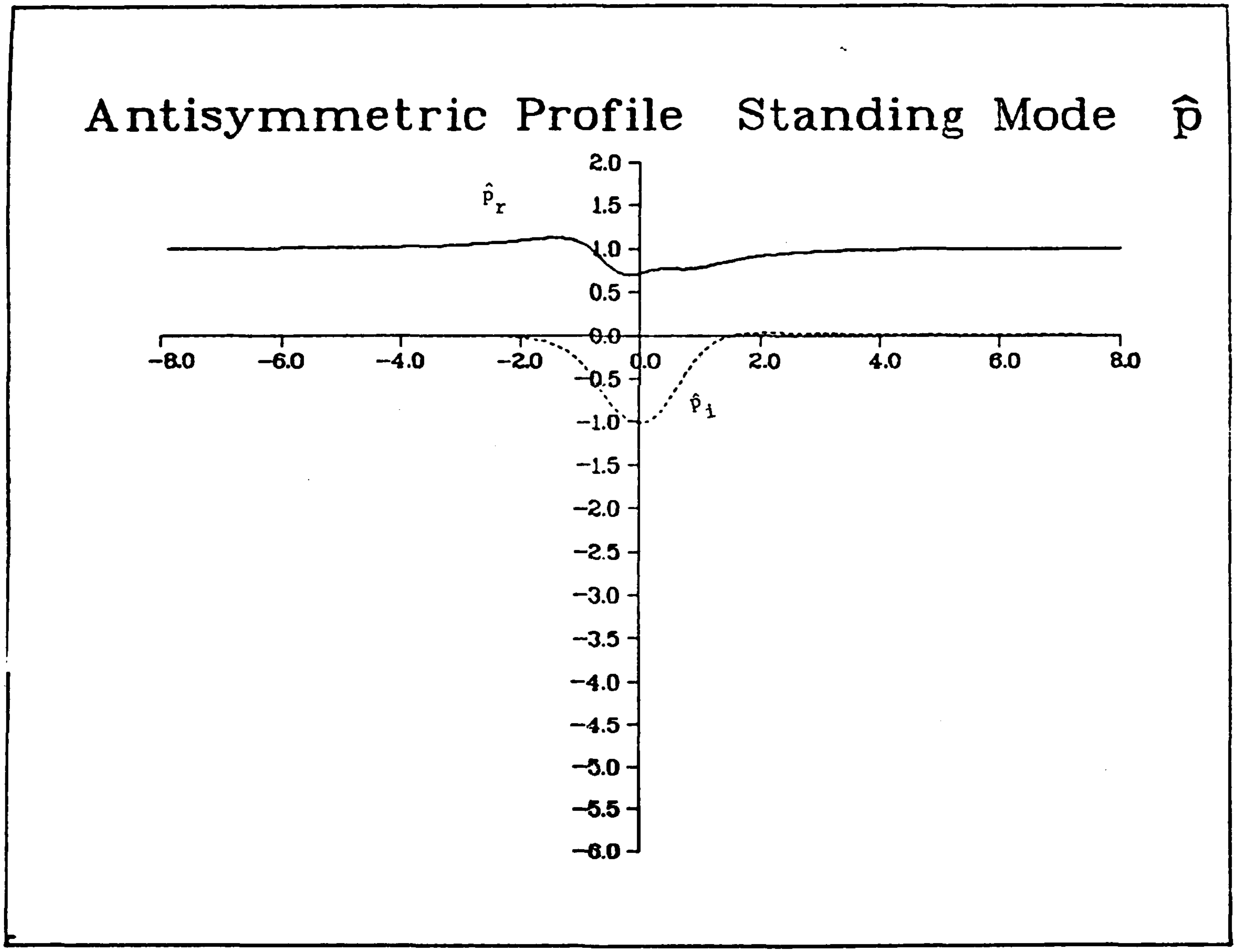

Antisymmetric Profile Standing Mode $\hat{p}$
$\hat{p}_r$
$\hat{p}_i$
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
-3.0
-3.5
-4.0
-4.5
-5.0
-5.5
-6.0
-8.0
-6.0
-4.0
-2.0
0.0
2.0
4.0
6.0
8.0

Figure 24c

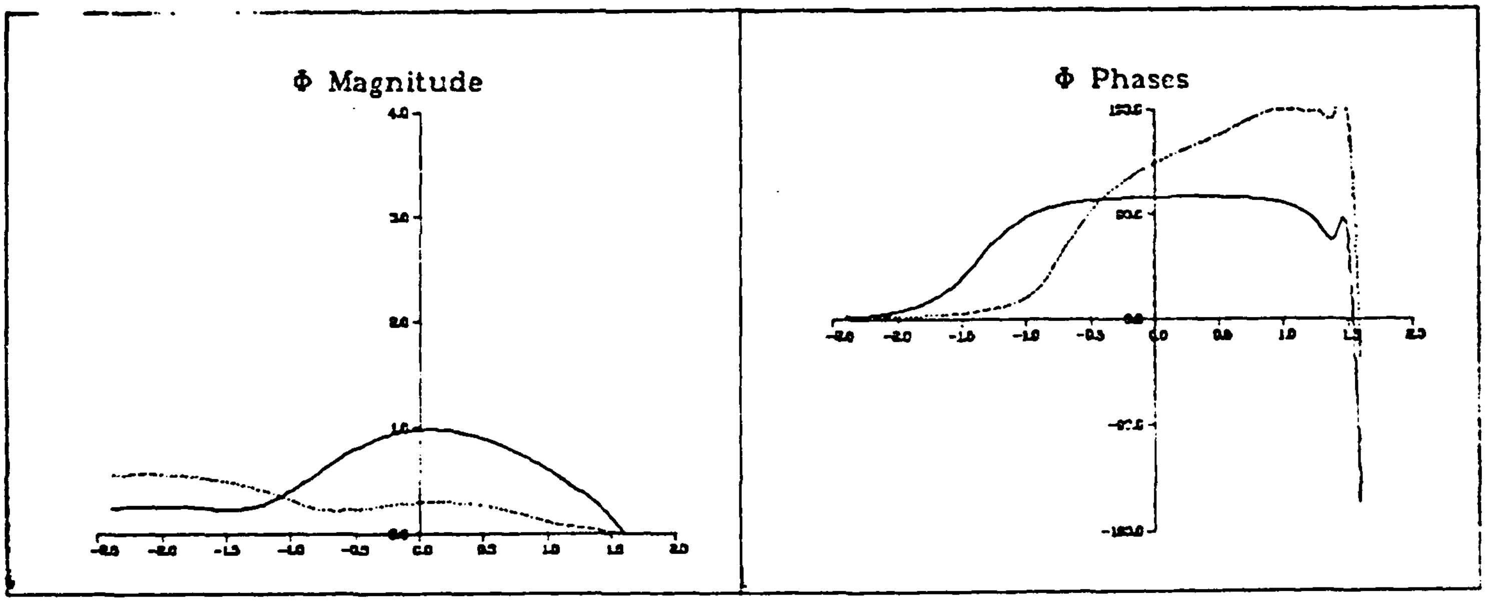


Figure 25c

<u>Propagating Mode</u>

Solid lines
$M_\infty = 1.0$
$\alpha = .30$
$c_r = .728650$
$c_i = .075980$

<u>Standing Mode</u>

Dotted lines
$M_\infty = 1.0$
$\alpha = .80$
$c_r = 0.0$
$c_i = .355153$

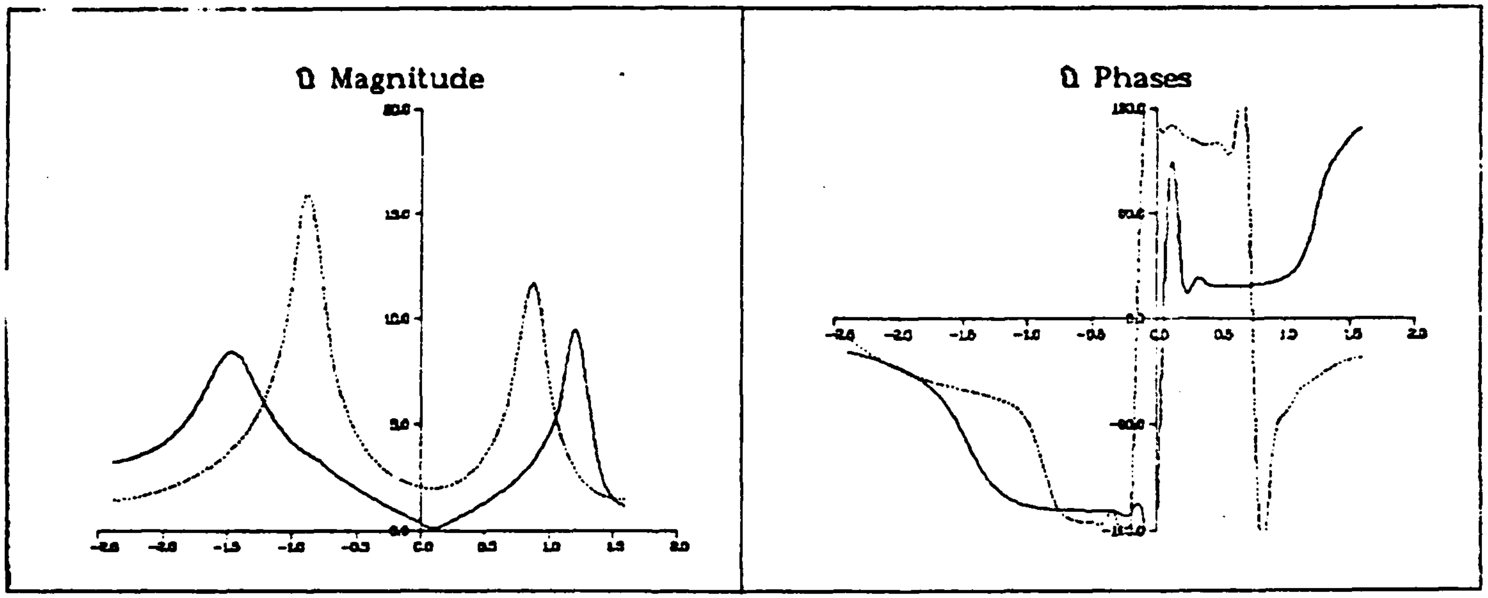


Figure 25b

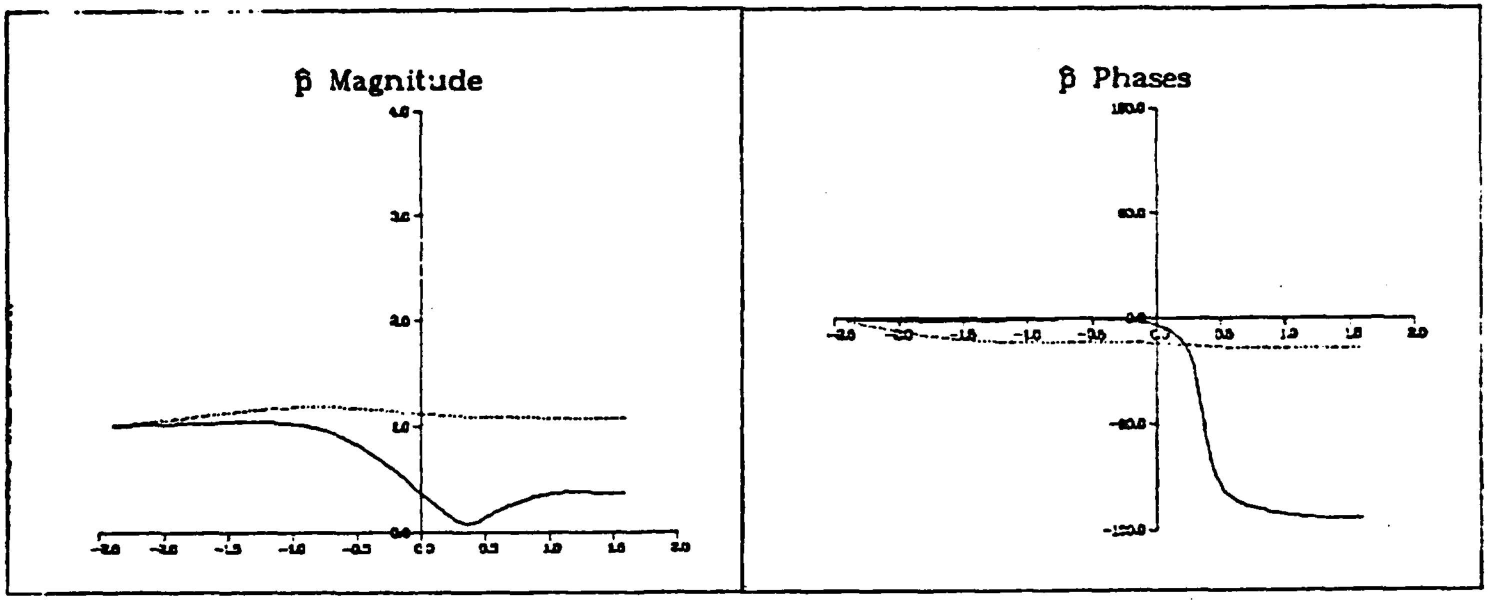


Figure 25c

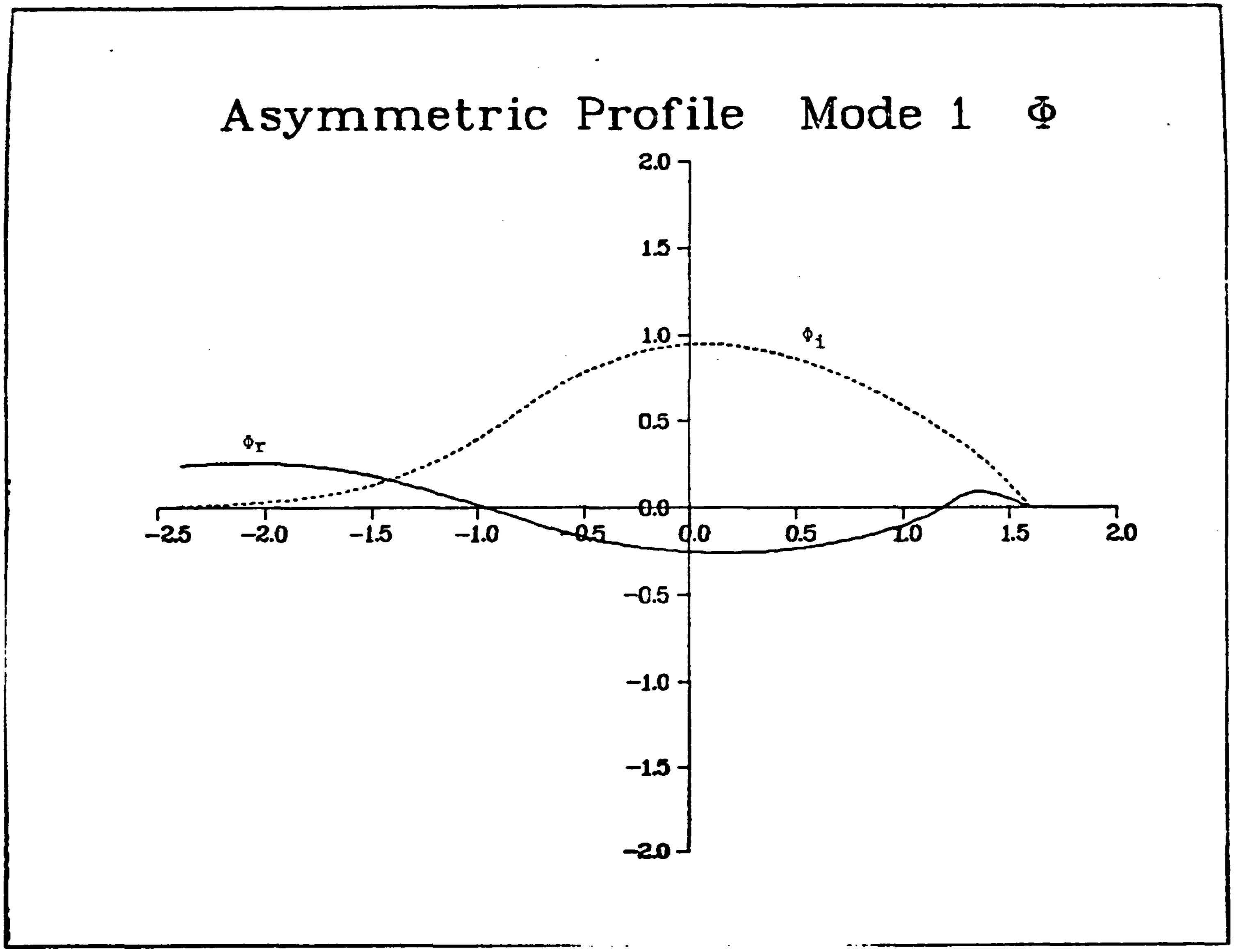


Figure 26a M_∞ = 1.0 α = .60 c = .2409919+.1195001i

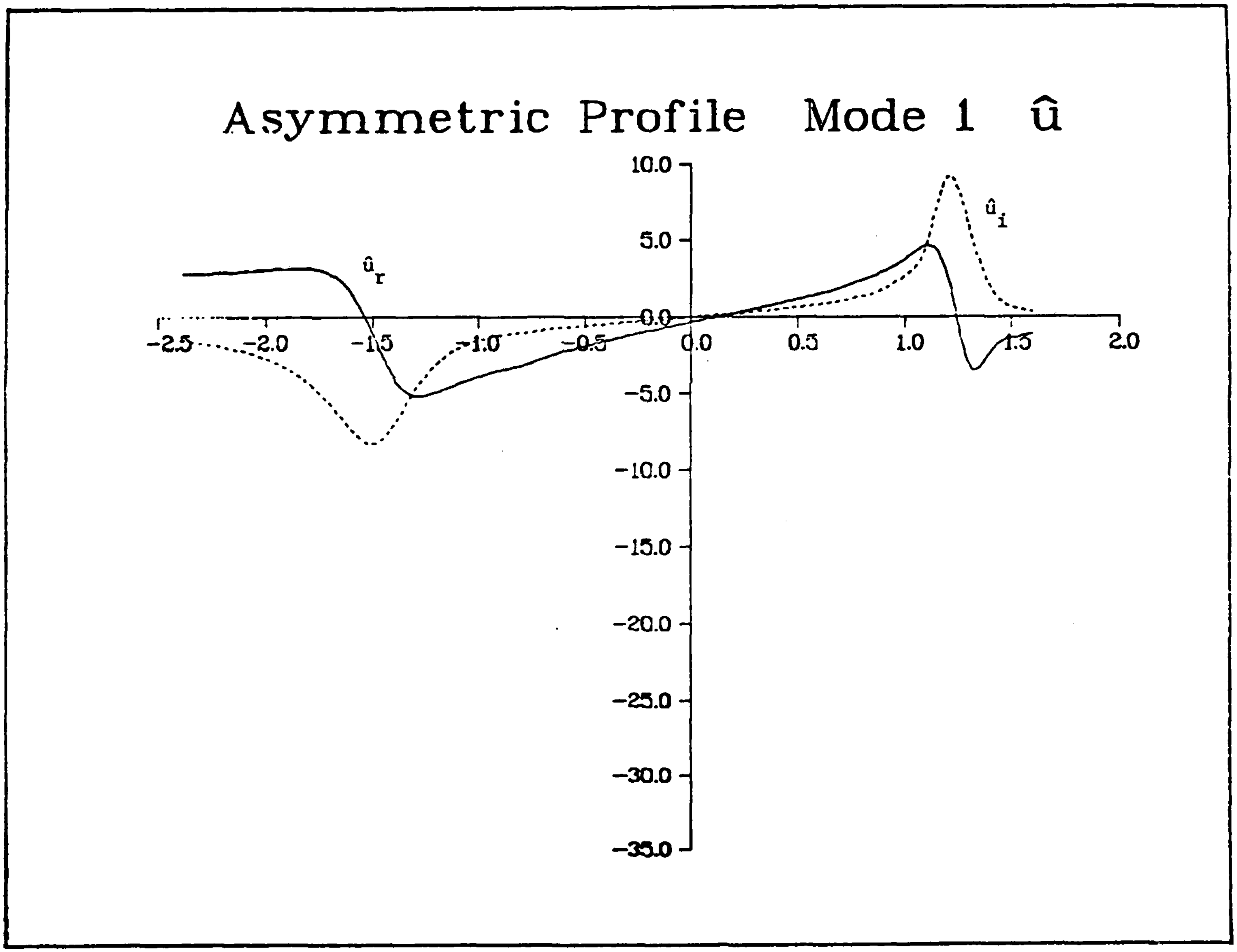
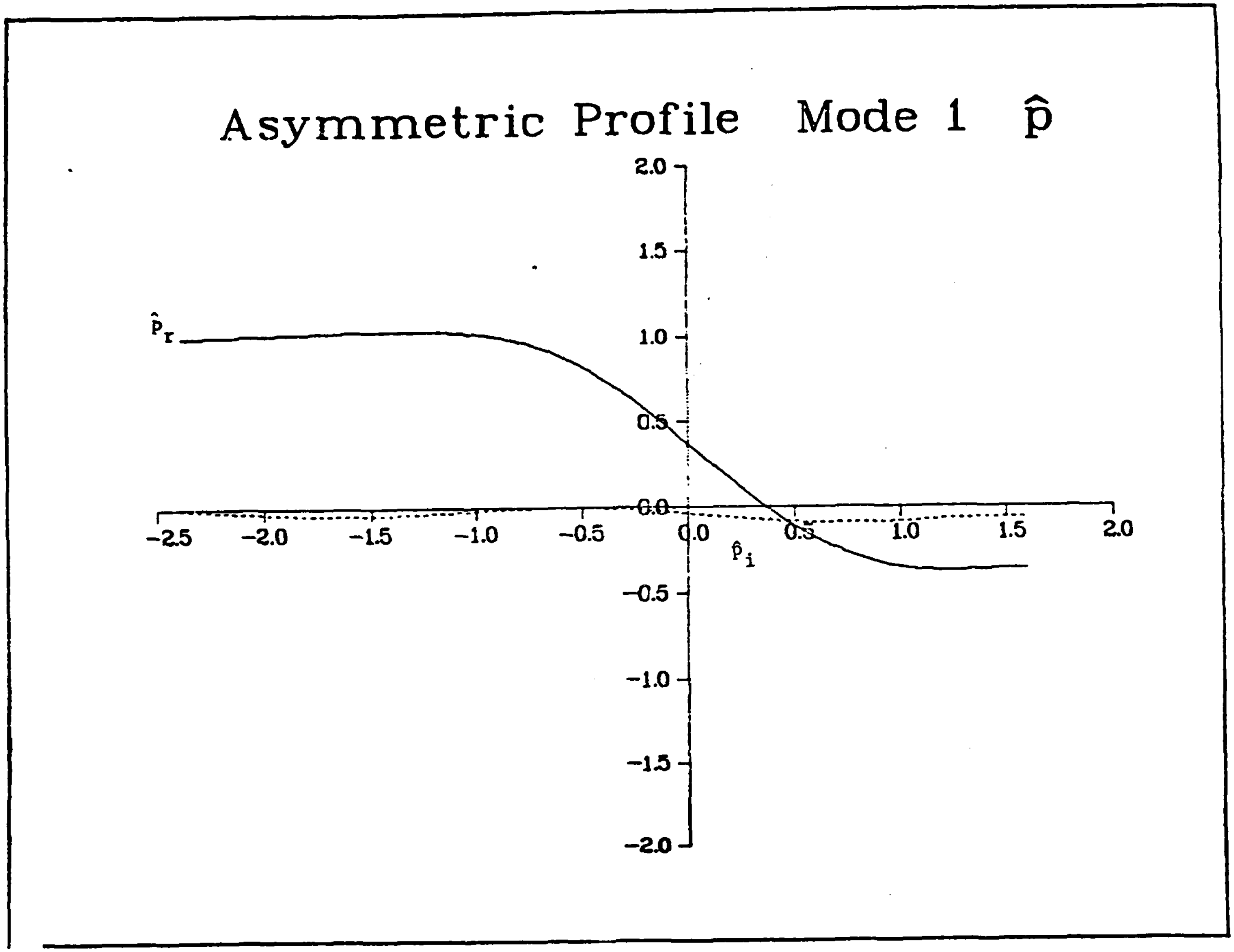

Asymmetric Profile Mode 1 $\hat{u}$
$\hat{u}_r$
$\hat{u}_i$
10.0
5.0
0.0
-5.0
-10.0
-15.0
-20.0
-25.0
-30.0
-35.0
-2.5
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0

Figure 26b



Asymmetric Profile Mode 1 $\hat{p}$
$\hat{p}_r$
$\hat{p}_i$
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5

Figure 26c

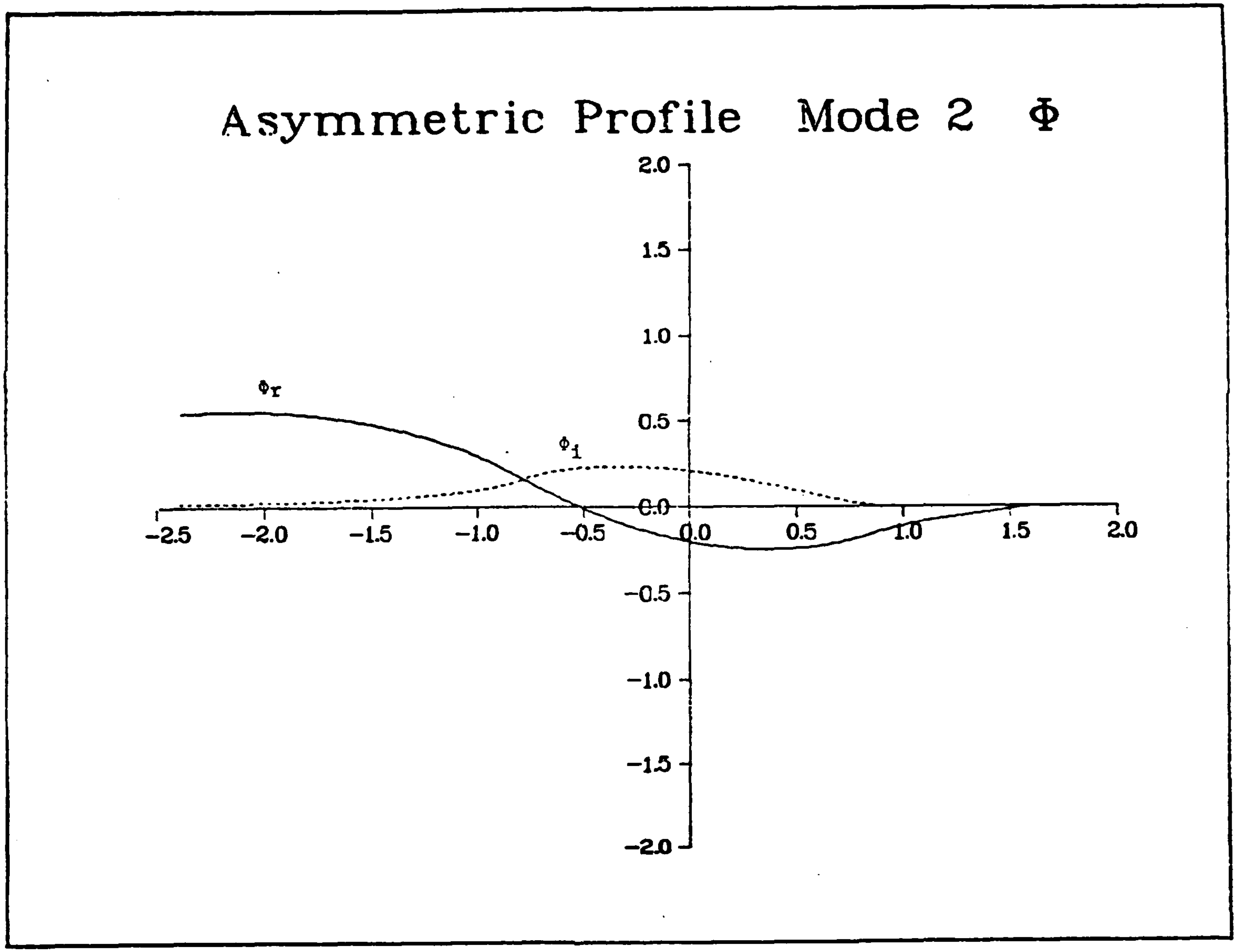


Figure 27a $M_\infty = 1.0$ $\alpha = .30$ $c = .621290+.093539i$

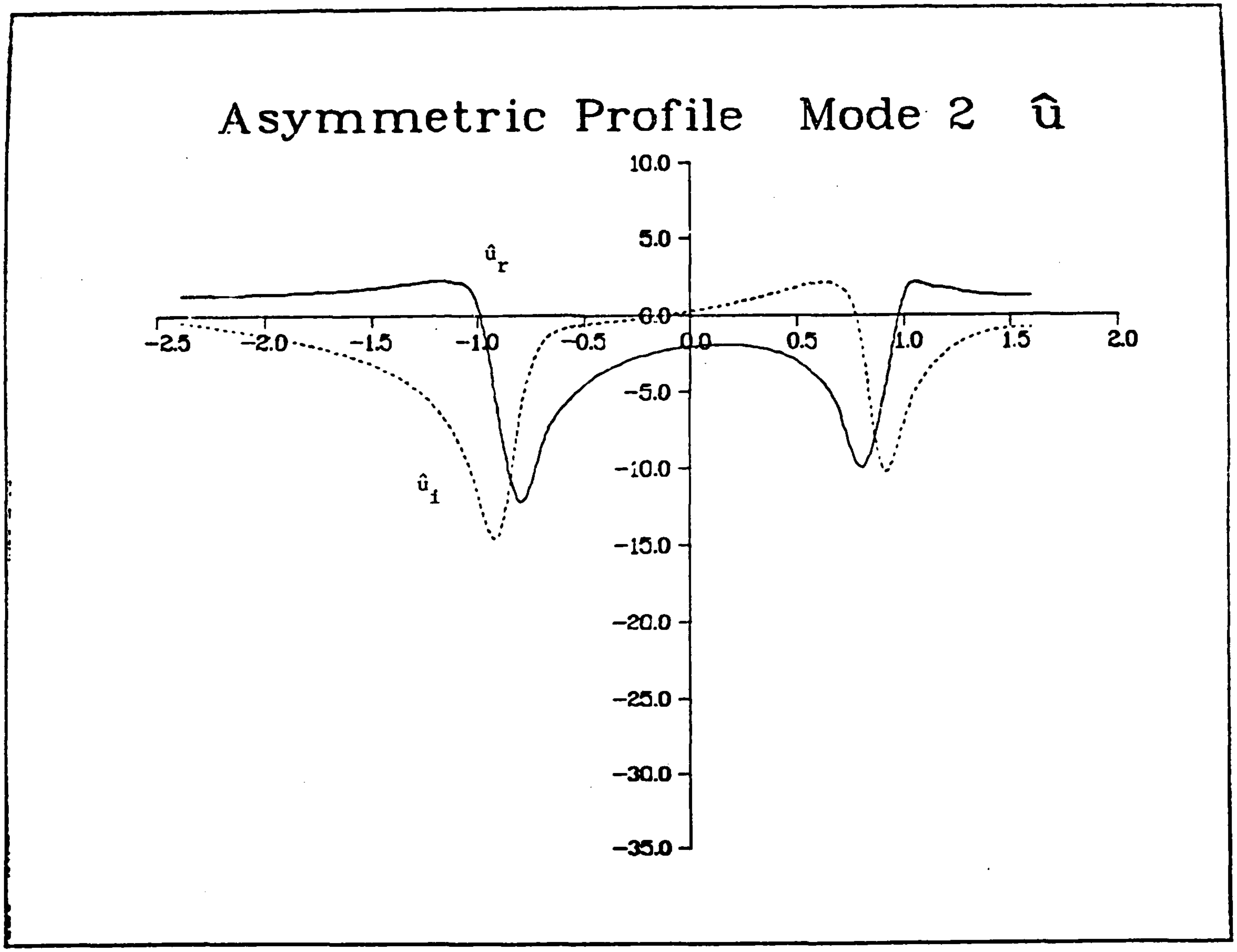

Asymmetric Profile Mode 2 û
10.0
5.0
0.0
-5.0
-10.0
-15.0
-20.0
-25.0
-30.0
-35.0
-2.5
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
$\hat{u}_r$
$\hat{u}_i$

Figure 27b

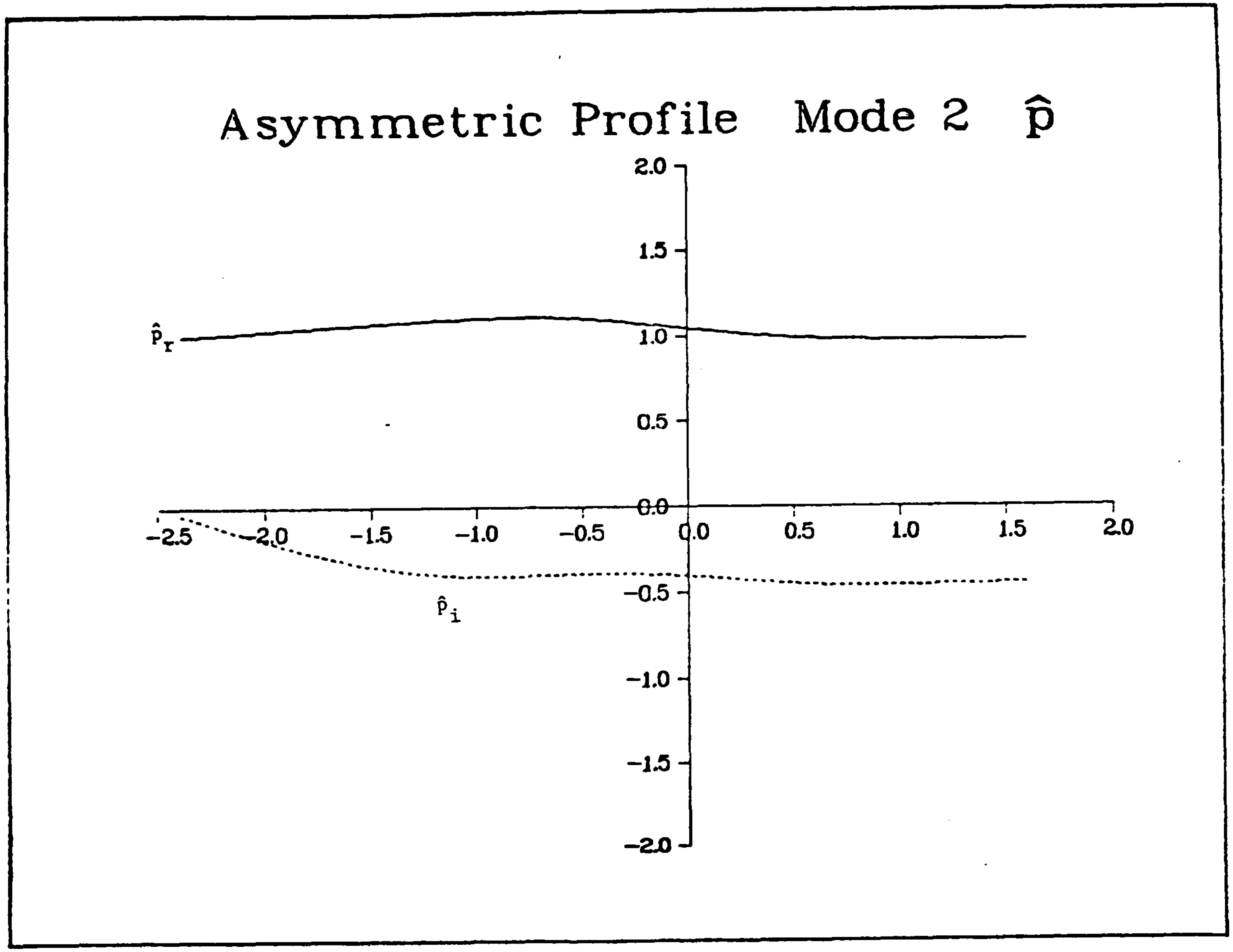

Asymmetric Profile Mode 2 $\hat{p}$
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
$\hat{p}_r$
$\hat{p}_i$

Figure 27c

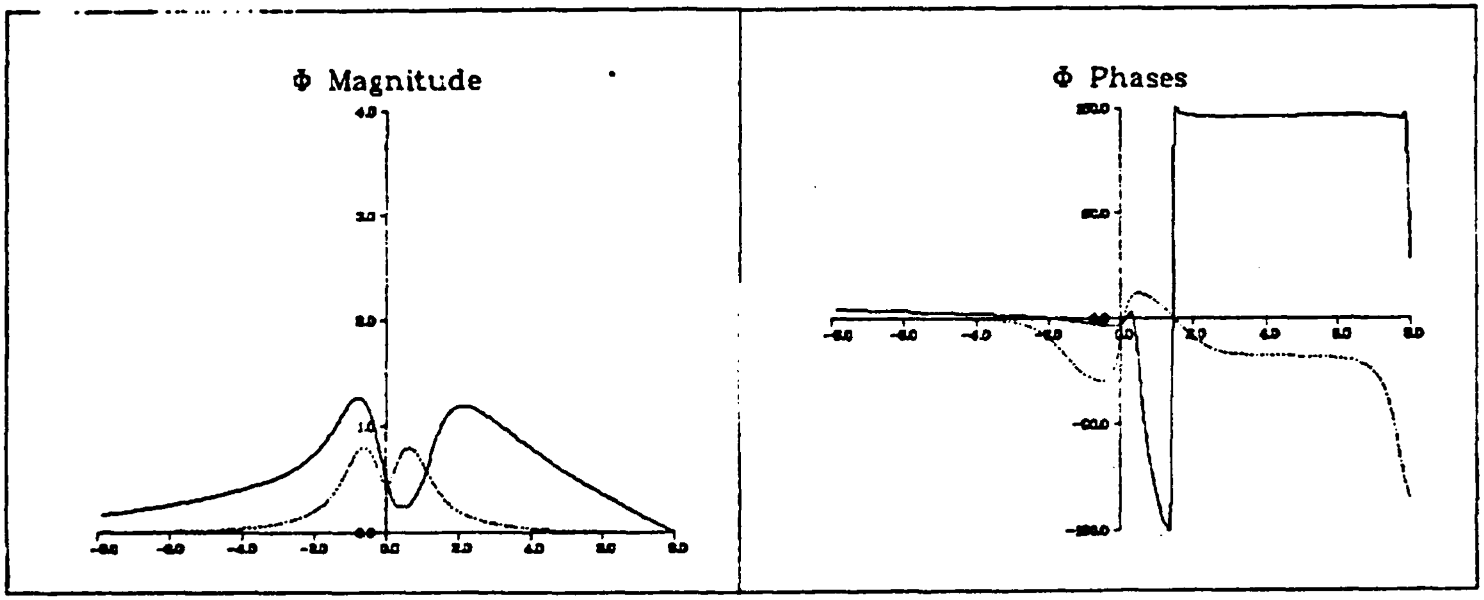


Figure 28a

<u>Mode I</u>

Solid lines
$M_\infty = 1.0$
$\alpha = .60$
$c_r = .240919$
$c_i = .119500$

<u>Mode II</u>

Dotted lines
$M_\infty = 1.0$
$\alpha = .30$
$c_r = .621290$
$c_i = .093539$

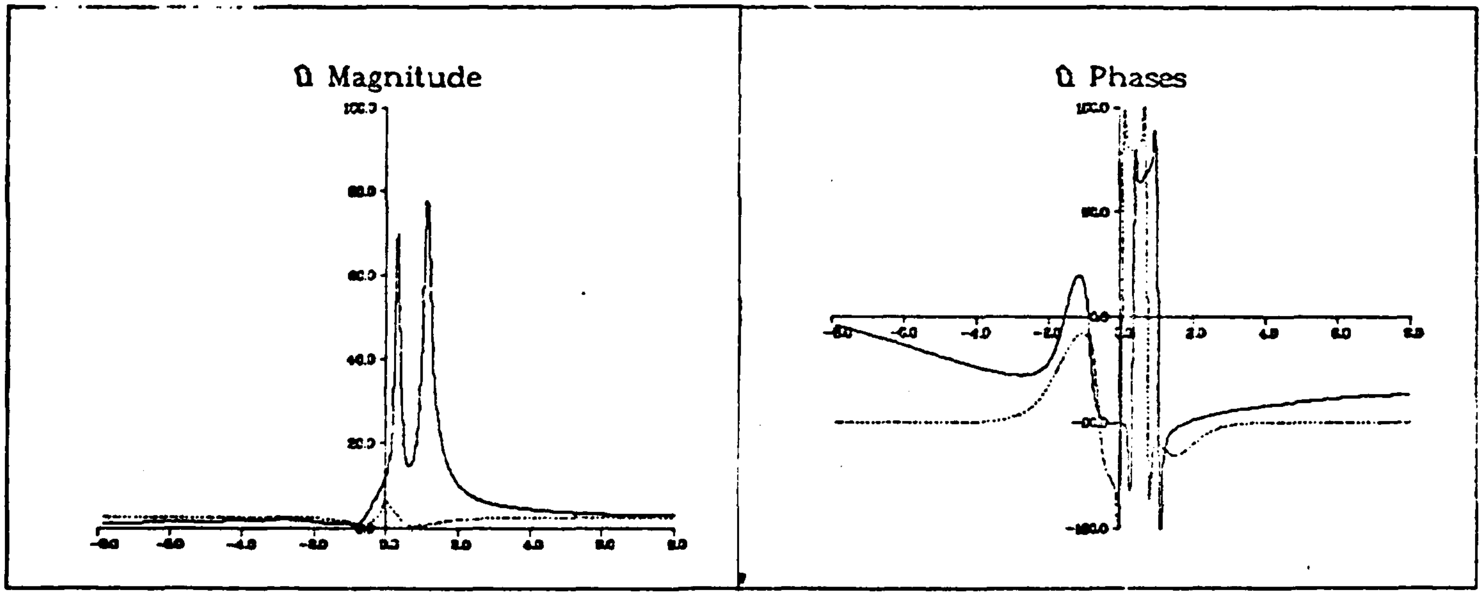


Figure 28b

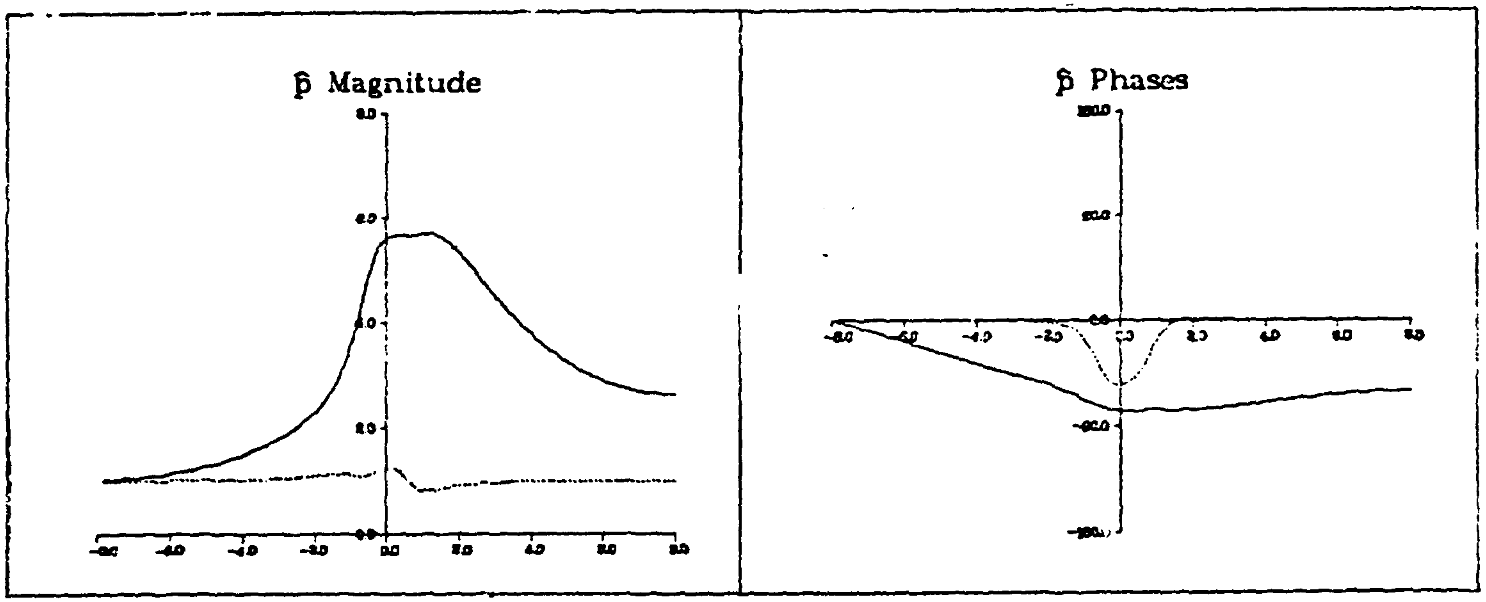


Figure 28c

D
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