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An extension of Kendall's concordance test where ties are allowed

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Probability tables are given that will allow the user to test the hypothesis of random ranking of M items by N judges. The tables pertain to a limited number of values of N and M and, unlike Kendall's concordance tests, ties are allowed.

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PREFACE

This study was conducted under DA Project 4A762707A855, Task B, Work Unit 0026, "Topographic Mapping Technology".

The study was done during the summer of 1982 under the supervision of Mr. D.E. Howell, Chief, Information Sciences Division; and Mr. L. A. Gambino, Director, Computer Sciences Laboratory.

COL Edward K. Wintz, CE, was Commander and Director and Mr. Robert P. Macchia was Technical Director of the Engineer Topographic Laboratories during the study and report preparation.

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AN EXTENSION OF KENDALL'S CONCORDANCE TEST WHERE TIES ARE ALLOWED

INTRODUCTION

Kendall's measure of concordance and associated probability tables establishes critical regions for testing the null hypothesis of random rankings of M items by N judges.[1] The M items are ranked according to an agreed upon criterion, such as beauty and cost effectiveness, or to the efficacy of a compression scheme, which was the basis for this Research Note. The concordance probability values are approximated by the F-distribution and the tests require that each judge produce a valid ranking of the M item, i.e. ties among the items are not allowed. The purpose of this work was to develop exact probability tables for a limited number of values of the parameters N and M where, in fact, ties were allowed.

BACKGROUND

The purpose of the original study[2] was to determine which of several compression techniques was best in the sense that it produced the most acceptable digital image when the compressed image was decoded and displayed on a TV at 8 bits. Five compression techniques labeled C2 through C6 were evaluated along with the uncompressed image labeled C1. The compression techniques as well as the image types are not relevant to this study. In the original study, the null hypothesis of no difference in the effects of compression was tested by the chi-square test.

Each test image was compressed, then decompressed, and finally stored in DIAL as an 8-bit image. Six compressions of each image were stored in DIAL for subsequent viewing and comparison. There were 45 such images used in the experiment. Each interpreter (there were 12) was required to choose either the right or the left image when a pair was displayed side by side on a TV display. Thus, each interpreter made $C_{6,2} = 15$ comparisons of the six processing schemes (five compressions and one original) for each of the 45 images. The pairs to be compared as well as the image type were presented in a random manner to the interpreter. The

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[1] M. Hammerton, Statistics for the Human Sciences, London and New York; Longman, 1975.

[2] SPIRIT II: Special Imagery Recognition and Interpretation Tests, Final Technical Report, Prepared for U.S. Army Engineer Topographic Laboratories, Fort Belvoir, VA. Prepared by Autometric, Incorporated, Falls Church, VA, June 1982.

rank score for each processing scheme was the totality of the choices over image and over interpreter. Thus, there were 15*45*12 = 8100 choices made, and under the null hypothesis, the expected value for each processing scheme was 8100/6 = 1350. The chi-square test is valid for this experiment since the expected value of each of the six cells is well above 5.

Today, because of relatively low cost CPU time, there is little reason why experimental results should be tested with approximate probability distributions.[3] It would appear that although the chi-square test was a valid test, it provided very little information, especially when the large number of tests is considered. Kendall's concordance test, which is more nearly appropriate, enables the null hypothesis of random ranking of the six processing schemes by the 12 interpreters to be tested. However, Kendall's measure of concordance cannot be used since the theory does not allow ties, and the likelihood of ties is high in the experimental procedure described above (see appendix A). The purpose of this study is to demonstrate how exact probability values that pertain to the experiment at hand can be calculated and used. It should be noted that the derived probability tables (see appendix B) are general, and they can be applied to a variety of similar experiments.

NUMERICAL TESTS

The history of the comparison tests was reorganized into a 3-dimensional array and stored on disc for subsequent analysis. The first dimension of the array specified the L = 1, 45 images; the second dimension specified the K = 1,12 interpreters; and the third dimension defined the I = 1,36 values associated with the $(L,K)^{th}$ image-interpreter event. The values that pertain to the 6 - x 6-score matrix are given in table 10.

[3]B. Efron, "Computers and the Theory of Statistics: Thinking the Unthinkable," SIAM Review, Vol. 21, No. 4, October 1979.

TABLE 1. SCORE MATRIX

1	v_{12}	v_{13}	v_{14}	v_{15}	v_{16}
v_{21}	1	v_{23}	v_{24}	v_{25}	v_{26}
v_{31}	v_{32}	1	v_{34}	v_{35}	v_{36}
v_{41}	v_{42}	v_{43}	1	v_{45}	v_{46}
v_{51}	v_{52}	v_{53}	v_{54}	1	v_{56}
v_{61}	v_{62}	v_{63}	v_{64}	v_{65}	1

If $V_{IJ} = 1$, then the I^{th} compression was judged to be superior to the J^{th} compression. Note that if $V_{IJ} = 1$, then $V_{JI} = 0$. The ones along the diagonals are added to each score so that scores will range from 1 to 6. The I^{th} score is determined by summing the values in the I^{th} row.

Two statistical tests were conducted using the image comparison histories described above. The first test was organized to determine whether the rankings of the compression schemes were random. The second experiment was developed to test whether there was a difference between the rankings as determined by beginners when compared to the rankings as determined by experts. Six of the 12 interpreters were regarded as experts, and 6 were regarded as beginners, or non-experts. The non-experts were given the same background information as the experts.

Forty-five independent rank tests were performed by 12 image interpreters. The 45 tests pertain to the 45 images that were subjected to six (one baseline and five compression schemes) digital processing exercises. When all six compressions were evaluated, the average ranking over all 45 images by the 12 interpreters turned out to be the following:

C1 : 1.49
C2 : 4.18
C3 : 5.96
C4 : 3.02
C5 : 3.21
C6 : 3.12

The smallest J-statistic was 1206 (image #21), which is beyond all of the entries for N = 12 in table B8. The hypothesis of a random ranking of the six compression schemes by the 12 observers is totally unacceptable.

The interpreters were extremely consistent in ranking C3 last, and to a lesser degree, they were consistent in ranking C1 (baseline) first, and C2 fourth. Note that C4, C5, and C6 seem to be tied. The comparison data was then processed to determine whether the hypothesis of random ranking was tenable when five compression schemes were evaluated. This was done three times, where in turn, C1, C2, and C3 were eliminated from consideration. The averaged rankings turned out to be the following:

	C1 out	C2 out	C3 out
C1 :		1.40	1.49
C2 :	3.27		4.16
C3 :	4.96	4.99	
C4 :	2.16	2.78	3.01
C5 :	2.35	2.96	3.22
C6 :	2.25	2.87	3.12

The smallest J-statistic when C1 was removed was 364 (image #11), and when C2 was removed, the smallest J-statistic was 774 (image #21). Both of these values are beyond all of the entries for N = 12 in table B7. The smallest J-statistic when C3 was removed was 126 (image #21), and from table B7 the statistic is significant at the 0.926 probability level. Although this calculation does not demonstrate randomness in the rankings, it does indicate a trend toward randomness when the highly consistent ranking of C3 being last is removed from consideration. Several other image comparisons provided J-statistics that were not beyond the tabulated entries. For example, the next smallest J-statistic was 238 (images #33 and #36), where from table B7 the significance level is 0.998.

The next step was to remove the three possible pairs of C1, C2, and C3 to determine whether the hypothesis of random ranking was tenable when four compression schemes were evaluated. The averaged rankings turned out to be the following:

	C1 and C2 out	C1 and C3 out	C2 and C3 out
C1 :			1.40
C2 :		3.25	
C3 :	3.99		
C4 :	1.92	2.15	2.78
C5 :	2.09	2.35	2.95
C6 :	2.00	2.25	2.87

The smallest J-statistic when C1 and C2 were removed was 350 (image #13). This value is beyond all of the entries for N = 12 in table B6. The smallest J-statistic when C1 and C3 were removed was 6, which is at the 0.107 significance level. In fact, 20 of the J-statistics were at or below 90, which is at the 0.950 significance level. The smallest J-statistic when C2 and C3 were removed was 54, which is at the 0.804 significance level. All other J-statistics were well beyond the 0.950 significance level. The hypothesis of a random element or a lack of concordance among the 12 interpreters was evident when the consistently bad compression C3 and the consistently good compression C1 were removed from consideration.

The next step was to remove C1, C2, and C3 to determine whether the hypothesis of random ranking was tenable when the three compression schemes C4, C5, and C6 were evaluated. The averaged ranking over the 12 interpreters and 45 images turned out to be the following:

C4 : 1.91
C5 : 2.08
C6 : 2.00

The largest J-statistic was 56, which from table B2 is at the 0.967 significance level. All other J-statistics are less than or equal to 54, which is at the 0.950 significance level. The hypothesis of a lack of concordance is definitely in evidence here. Finally, C4, C5, and C6 were removed from consideration and the averaged rankings over the 12 interpreters and 45 images turned out to be the following:

C1 : 1.09
C2 : 1.94
C3 : 2.97

The smallest J-statistic was 206, which is beyond all of the entries for N = 12 in table B2. The hypothesis of a random ranking must be rejected in this case.

The second test utilized existing theory to test whether the six experts were in agreement with the six non-experts. Hotelling's T^2 statistic was used to test the equality of averaged rankings between the two groups.[4] The abandonment of the mathematical purity proclaimed in the Background Section is in this case only a minor accommodation to expediency. Except for the assumptions of multivariate normal distributions and equal covariance matrices, the conditions associated with the experiment are identical to the requirements of the two sample T^2 statistics. The tests on the equality of the average rankings between experts and non-experts were performed over the same experimental conditions described above in the ranking tests. When the null hypothesis of equal means is true, then the quantity F given below has the F-distribution with stated degrees of freedom.

[4]D. Morrison, Multivariate Statistical Methods, McGraw Hill Book Company, New York, 1967.

$$F = \frac{11-M}{10} T^2$$

with degrees of freedom M and 11-M

$$T^2 = 3D^T S^{-1} D$$

D = Mean Vector Difference

S - Pooled Covariance Matrix

When the mean differences associated with all six compression schemes were tested, none of the 45 test statistics exceeded the $F_{6,5} = 3.11$, 90 percent significance level. In the case where C1 was removed from consideration, none of the 45 test statistics exceeded the $F_{5,6} = 3.40$, 90 percent significance level. When C2 was removed, the test statistic associated with image #42 was significant, and when C3 was removed, the test statistic associated with images #36 and #42 was significant. In the case where C1 and C2 were removed from consideration, the test statistic associated with image #27 exceeded the $F_{4,7} = 3.98$, 90 percent significance level. When C1 and C2 were removed, the test statistic associated with image #36 was excessive, and when C2 and C3 were removed, the test statistic associated with image #42 exceeded the 95 percent level of significance ($F_{4,7} = 6.09$). When C1, C2, and C3 were removed from consideration, none of the 45 test statistics exceeded the $F_{3,8} = 5.25$, 90 percent significance level.

DISCUSSION

The mathematician will recognize that much of the work here is a fishing expedition rather than executing a previously designed statistical experiment. It must be recalled, however, that the main purpose of the work was to demonstrate that the analyst need not, in this day of relatively inexpensive computer CPU time, resort to a procrustean method when analyzing experimental results. In fact, exact cumulative probability tables were developed for the experiment at hand. That a great deal of searching through the data and, in a few cases, resorting to procrustean methods to obtain certain results is readily admitted.

The various tests for random ranking among the 12 interpreters showed that a strong consistency among the interpreters for several of the compressions masked a randomness or lack of concordance among the remaining compressions. When all six compression schemes were considered, the null hypothesis of random ranking among the 12 interpreters was decidely rejected. Whereas, when C1, C2, and C3 were removed from consideration, only 1 of the 45 J-statistics exceeded the 95 percent significance level. This result cannot be used to reject the hypothesis. Note that the probability of getting at least 1 value out of 45 beyond the 95 percent significance point is 0.900 when the null hypothesis is true. This result is derived from the cumulative binomial distribution since under the null hypothesis the J-statistic values are independent and there is a five percent chance for each to fall in the critical region. The tests for random ranking among the 12 interpreters also showed that a definite lack of concordance among the interpreters for several of the compressions tended to mask a strong consistency among the remaining compressions. This was shown when C4, C5, and C6, were removed from consideration.

The several tests to determine if there were any differences between expert interpreters and non-expert interpreters did not reject the hypothesis of no difference. When C2 was removed from consideration, two sample F statistics exceeded the 90 percent significance level. Note that in this case the probability of getting at least 2 values out of 45 beyond the 90 percent significance level is 0.948 when the null hypothesis of equal means is true. When C1, C2, and C3 were removed from consideration, none of the 45 values were beyond the 90 percent critical point. Under the null hypothesis of equal means, the likelihood of this result is only 0.009. Either an unlikely occurrence has taken place or the assumption of a multivariate normal distribution is exaggerated.

The inconsistency noted in the last paragraph is another reason for using exact probability distributions wherever possible. In order for us to get an idea about the random (or non-random) nature of the C4, C5, and C6 ranking results, the 12 * 45 = 540 3-component ranks were extracted from the comparison history and summarized in tables 2 and 3.

TABLE 2. EXPERT RESULTS

RANKINGS

Interpreter	(222)	(123)	(132)	(213)	(231)	(312)	(321)
1	10	7	11	4	4	5	4
2	15	2	5	5	12	2	4
3	13	6	6	7	4	4	5
4	9	8	8	5	7	6	2
5	7	10	3	7	6	6	6
6	10	6	5	8	6	3	7
Totals	64	39	38	36	39	26	28
Prob. Est.	0.237	0.144	0.141	0.133	0.144	0.096	0.104

TABLE 3. NON EXPERT-RESULTS

RANKINGS

Interpreter	(222)	(123)	(132)	(213)	(231)	(312)	(321)
1	9	6	6	6	7	8	3
2	12	4	5	10	8	2	4
3	11	4	10	4	10	4	2
4	12	7	5	7	6	5	3
5	9	4	8	3	8	6	7
6	11	2	15	4	4	3	6
Totals	64	27	49	34	43	28	25
Prob. Est.	0.237	0.100	0.181	0.126	0.159	0.104	0.095

If the rankings were entirely random, then from appendix A the inconsistent rankings (2,2,2) should occur with a probability of 0.250; whereas, the six consistent rankings should each occur with a probability of 0.125. If random ranking is assumed, as described in appendix A, then the observed counts of the several rankings when compared to the expected count can be tested by using the chi-square goodness of fit test. The expert's X^2_5 estimate was 5.3, which is not significant; whereas, the non-expert's value is 15.1, which is significant at the 99 percent level. The expert's results demonstrate more consistency than the non-expert's results. Both sets of results appear to show a reluctance in allowing compression C4 to be ranked third.

It can be concluded that C4, C5, and C6 are, for all practical purposes, equivalent under the experimental conditions and that C1 > (C4, C5, and C6) > C2 > C3. The strong relationship (C1 > C2 > C3) tended to deny a lack of concordance when the original null hypothesis of the six schemes being random was tested. The sub-hypothesis of C4, C5, and C6 producing a lack of concordance was supported by the data when C1, C2, and C3 were excluded from the analysis. Other, unrelated, multivariate analysis work at ETL has also produced uncertain or ambiguous results.[5] In the referenced work, it was determined that a simple 2-component signature did just as well, and in some cases better, than a large component descriptor in segmenting an aerial image. There is a suspicion here that the linear models used in multivariate analysis do not adequately represent the unknown structure relating variables of interest.

CONCLUSIONS

1. Experimental tests should use an exact statistical design and develop relevant probability data when needed.

2. An extension of Kendall's probability of concordance was developed for a limited number of parameters and shown to be useful.

3. There is a growing concern over the validity of many multivariate analyses using large numbers of components, where unknown internal relationships tend to contaminate results.

[5] M. Crombie, N. Friend, and R. Rand, Feature Component Reduction Through Divergence Analysis, U.S. Army Engineer Topographic Laboratories, Fort Belvoir, VA, ETL-0305, October 1982.

APPENDIX A. RANDOM RANKING OF M ITEMS BY N JUDGES

The numerical ranking of M items is determined by making choices over the $C_{M,2}$ possible binary comparisons of the M items. The rank of a specific item is determined by adding 1 to the number of times the item was chosen. Since a specific item is compared (M-1) times, the rank values will range from 1 to M. Note that this type of ranking procedure can produce ties. The purpose here is to describe the development of statistical tables for testing the null hypothesis of random rankings among N judges.

The density function and associated distribution function were developed using Monte Carlo methods. The two functins, p(J) and P(J) respectively, are characterized by two parameters, namely N, the number of judges and M, the number of items to be ranked. If the experimental results are organized into N rows and M columns, and if R_j is the expected sum of the J^{th} column, then the random variable J is defined as follows:

$$J = \sum_{j=1}^{M} (R'_j - R_j)^2$$

R'_j : observed sum of the J^{th} column

Assuming that the rankings are random, then each of the N judges' rankings will sum to M(M+1)/2 for a total of NM (M+1)/2. Since no one of the items is favored, the expected value for each of the M columns is R_j = N(M+1)/2.

For example, let M = 3 and consider the following matrix that represents results from the I^{th} judge:

$$V_I = \begin{matrix} 1 & v_{12} & v_{13} \\ v_{21} & 1 & v_{23} \\ v_{31} & v_{32} & 1 \end{matrix}$$

If v_{LK} = 1, then item L was judged to be superior to item K. Note that if v_{LK} = 1, then v_{KL} = 0. The ones along the diagonal pertain to the ones added to each score. The J^{th} rank is determined by summing the J^{th} row of V. There are 2^3 ways that ones can be distributed over V. In general, there are 2^{QM} ways where QM = M(M-1)/2. There are 3! ways that consistent rankings may occur, and in general, there are M! ways that consistent rankings may occur. The eight possible V_I are listed as follows:

$$\begin{array}{ccc}1&1&1\\0&1&1\\0&0&1\end{array} \;=>\; \begin{array}{c}3\\2\\1\end{array} \qquad \begin{array}{ccc}1&0&0\\1&1&0\\1&1&1\end{array} \;=>\; \begin{array}{c}1\\2\\3\end{array} \qquad \begin{array}{ccc}1&1&1\\0&1&0\\0&1&1\end{array} \;=>\; \begin{array}{c}3\\1\\2\end{array}$$

$$\begin{array}{ccc}1&0&0\\1&1&1\\1&0&1\end{array} \;=>\; \begin{array}{c}1\\3\\2\end{array} \qquad \begin{array}{ccc}1&1&0\\0&1&0\\1&1&1\end{array} \;=>\; \begin{array}{c}2\\1\\3\end{array} \qquad \begin{array}{ccc}1&0&1\\1&1&1\\0&0&1\end{array} \;=>\; \begin{array}{c}2\\3\\1\end{array}$$

$$\begin{array}{ccc}1&1&0\\0&1&1\\1&0&1\end{array} \;=>\; \begin{array}{c}2\\2\\2\end{array} \qquad \begin{array}{ccc}1&0&1\\1&1&0\\0&1&1\end{array} \;=>\; \begin{array}{c}2\\2\\2\end{array}$$

Note that the latter two arrays are transposes of one another and in fact are logically inconsistent. The first array implies that the first item was judged superior to the second item, which in turn was judged superior to the third item. The array also implies that the third item was judged superior to the first item.

The distribution of J under the null hypothesis was developed by generating pseudo random numbers from one to eight and then selecting one of the eight possible rankings. This exercise was performed N times to produce one value of J. A large number of J-values were generated in this manner to estimate the distribution function.

When there are M = 4 items, there are $2^6 = 64$ possible rankings of which 4! = 24 are consistent. The 24 consistent rankings and 40 possible ties were organized into a (64 X 4) array and sampled by generating a random number between 1 and 64. As before, the exercise was performed N times to produce one value of J.

Two sets of distribution values were generated for M = 3 and for M = 4. In the first case, ties were not allowed, and in the second case, they were allowed. When there are M = 5 items, there are $2^{10} = 1024$ possible rankings, of which 5! = 120 are consistent. The simple enumeration of all possible cases was discarded in favor of the sampling procedure described next. If M = 5, then M(M-1)/2 = 10 random values, either one or zero, were developed and inserted into V_I according to the rules defined above for M = 3. This procedure was repeated N times to produce one value of J. This procedure was used for M = 5 and for M = 6. In these cases, distribution values, where ties were allowed, were calculated, but not values where ties were not allowed.

It should be noted that rankings are equivalent to scores in this exercise. Thus, a higher score implies a higher rank. If an ordinal ranking is desired, then the scores should be modified by the relation (M-S+1). For example, if M = 5 and a particular scoring was (1,3,5,4,2), then the equivalent ordinal ranking is (5,3,1,2,4).

APPENDIX B. PROBABILITY TABLES

Monte Carlo methods were used to generate the cumulative probability values given in tables B1 through B8. Tables B1 and B2 pertain to N = 1 to 12 judges and M = 3 items. The eight possible rankings were organized and stored in the same order as described in appendix A. The results in table B1, where ties were not allowed, were developed by random sampling among the first six scores; whereas, all eight scores were sampled for table B2. The same general procedure was used to generate tables B3 through B6. The 64 possible rankings were organized so that the first 24 scores were valid scores, and the last 40 scores were the possible ties.

When M = 5, there are 120 consistent scores and 904 possible ties, and when M = 6, there are 720 consistent scores and 32,048 possible ties. The method described above was discarded in favor of the second method described in appendix A for these cases. In both cases, ties were allowed; however, only even values of N (up to N = 12) were calculated for M = 6.

In all cases, 400,000 J-values, as described in appendix A, were computed to develop the sample density functions and finally the sample cumulative functions. The results are presented to three decimal digits. Several of the probability tables were generated using 100,000 J-values, and the vast majority of these values differed by no more than one in the third decimal place from those generated from 400,000 J-values. The largest discrepancy was three digits in the third decimal place.

Table B 1 $M = 3$: TIES NOT ALLOWED

					N					
J	3	4	5	6	7	8	9	10	11	12
0	.056	.069	.046	.044	.036	.032	.029	.026	.024	.022
2	.472	.347	.309	.261	.231	.206	.187	.170	.156	.145
6	.639	.570	.479	.430	.381	.347	.314	.290	.268	.249
8	.807	.727	.633	.570	.514	.470	.431	.399	.371	.346
14	.972	.875	.818	.748	.696	.646	.603	.564	.531	.499
18	1.000	.930	.876	.816	.764	.715	.672	.632	.597	.565
24		.958	.907	.858	.810	.765	.722	.684	.649	.617
26		.995	.960	.928	.889	.851	.813	.777	.744	.713
32		1.000	.976	.948	.915	.881	.846	.813	.780	.750
38			.992	.971	.949	.921	.893	.865	.836	.809
42			.999	.988	.973	.953	.931	.907	.883	.859
50			1.000	.992	.979	.962	.943	.922	.899	.877
54				.994	.984	.970	.952	.933	.913	.892
56				.998	.992	.982	.969	.954	.938	.920
62				1.000	.996	.990	.981	.969	.956	.942
72					.997	.992	.984	.974	.962	.949
74					.999	.995	.990	.982	.973	.962
78					1.000	.998	.994	.988	.981	.973
86						.999	.996	.993	.987	.980
96						.999	.997	.994	.989	.983
98						1.000	.999	.997	.993	.989
104							.999	.998	.996	.993
114							1.000	.999	.997	.995
122								.999	.998	.996
126								1.000	.999	.997
128									.999	.998
134									.999	.999
146									1.000	.999
150										.999
152										.999
158										1.000

Table B 2 M = 3 : TIES ALLOWED

	N									
J	3	4	5	6	7	8	9	10	11	12
0	.109	.085	.068	.058	.050	.044	.039	.036	.032	.030
2	.566	.465	.395	.344	.302	.270	.244	.224	.206	.191
6	.777	.675	.595	.534	.480	.437	.400	.370	.345	.321
8	.918	.830	.753	.687	.628	.579	.536	.500	.467	.438
14	.988	.948	.900	.850	.801	.755	.713	.674	.640	.607
18	1.000	.978	.943	.903	.861	.819	.779	.743	.709	.676
24		.987	.962	.929	.895	.858	.822	.789	.756	.725
26		.998	.989	.972	.950	.924	.897	.869	842	.814
32		1.000	.995	.983	.965	.944	.921	.897	.872	.846
38			.998	.992	.982	.968	.951	.933	.914	.891
42			1.000	.998	.993	.984	.973	.960	.945	.929
50				.999	.995	.988	.979	.968	.955	.940
54				.999	.996	.991	.984	.974	.963	.950
56				1.000	.999	.995	.991	.985	.976	.967
62					1.000	.998	.995	.991	.985	.978
72						.999	.996	.993	.988	.982
74						.999	.998	.995	.992	.988
78						1.000	.999	.997	.995	.992
86							1.000	.999	.997	.995
96								.999	.998	.996
98								1.000	.999	.998
104									.999	.999
114									1.000	.999
122										.999
126										1.000

Table B 3 M = 4; N ODD. TIES NOT ALLOWED

J	N = 3	5	7	9	11
1	.042	.026	.017	.012	.009
3	.090	.056	.037	.027	.020
5	.271	.143	.093	.068	.052
9	.391	.229	.155	.114	.090
11	.475	.292	.200	.149	.117
13	.554	.348	.242	.181	.143
17	.659	.439	.315	.239	.190
19	.700	.480	.349	.267	.213
21	.794	.556	.411	.318	.256
25	.825	.593	.443	.346	.280
27	.853	.629	.477	.375	.305
29	.925	.702	.544	.435	.358
33	.946	.740	.583	.470	.389
35	.967	.774	.618	.503	.419
37	.982	.790	.635	.519	.433
41	.998	.838	.691	.574	.484
43	.999	.848	.704	.587	.496
45	1.000	.877	.739	.624	.531
49		.893	.761	.647	.554
51		.906	.780	.668	.574
53		.924	.806	.695	.602
57		.933	.820	.712	.618
59		.945	.840	.735	.642
61		.955	.858	.756	.665
65		.966	.878	.782	.693
67		.968	.883	.787	.699
69		.977	.900	.809	.724
73		.930	.907	.819	.735
75		.983	.915	.830	.747
77		.988	.927	.847	.766
81		.991	.937	.863	.785
83		.993	.944	.873	.798
85		.994	.948	.880	.806
89		.997	.959	.897	.828
91		.998	.962	.902	.835
93		.998	.965	.907	.841
97		.998	.967	.912	.847
99		.999	.970	.918	.855
101		.999	.977	.931	.873
105		1.000	.980	.938	.882
107			.982	.942	.889
109			.984	.946	.895
113			.987	.951	.902
115			.988	.954	.906

Table B 3 (Continued) M = 4, N ODD: TIES NOT ALLOWED

	N				
J	3	5	7	9	11
117			.990	.959	.914
121			.991	.962	.919
123			.992	.964	.922
125			.993	.968	.930
129			.995	.973	.937
131			.996	.976	.943
133			.996	.977	.945
137			.997	.979	.949
139			.997	.981	.951
141			.998	.982	.955
145			.998	.984	.958
147			.998	.985	.959
149			.999	.987	.964
153			.999	.989	.967
155			.999	.990	.970
157			.999	.990	.971
161			.999	.992	.975
163			.999	.992	.975
165			1.000	.993	.977
169				.993	.978
171				.994	.980
173				.995	.982
177				.995	.983
179				.996	.984
181				.996	.985
185				.997	.987
187				.997	.987
189				.998	.989
193				.998	.989
195				.998	.990
197				.998	.991
201				.998	.992
203				.999	.992
205				.999	.993
209				.999	.994
211				.999	.994
213				[illegible]	.995
217				.999	.995
219				.999	.995
221				.999	.996
225				1.000	.996
227					.997
229					.997
233					.997

Table B 3 (Continued)   M = 4, N ODD: TIES NOT ALLOWED

			N		
J	3	5	7	9	11
235					.997
237					.998
241					.998
243					.998
245					.998
249					.998
251					.998
253					.999
257					.999
259					.999
261					.999
265					.999
267					.999
269					.999
273					.999
275					.999
277					.999
281					1.000

Table B 4 M = 4, N EVEN: TIES NOT ALLOWED

J	N = 4	6	8	10	12
0	.008	.004	.002	.002	.002
2	.071	.042	.029	.022	.017
4	.099	.060	.042	.031	.024
6	.199	.125	.088	.066	.052
8	.245	.156	.109	.083	.065
10	.323	.211	.150	.115	.091
12	.351	.228	.163	.125	.099
14	.476	.321	.235	.181	.145
16	.492	.332	.243	.188	.150
18	.568	.390	.289	.226	.182
20	.610	.426	.319	.249	.202
22	.645	.458	.346	.272	.222
24	.676	.487	.372	.294	.240
26	.759	.569	.443	.355	.293
30	.800	.614	.485	.391	.325
32	.810	.625	.494	.399	.333
34	.842	.662	.530	.431	.362
36	.859	.663	.551	.450	.379
38	.895	.730	.597	.494	.418
40	.906	.745	.612	.508	.431
42	.923	.770	.639	.534	.454
44	.932	.782	.652	.546	.466
46	.946	.804	.675	.569	.488
48	.948	.807	.679	.573	.492
50	.964	.837	.714	.608	.527
52	.967	.845	.723	.618	.536
54	.981	.873	.758	.655	.572
56	.986	.886	.775	.672	.58[illegible]
58	.988	.892	.782	.681	.597
62	.993	.911	.808	.709	.627
64	.994	.912	.810	.711	.628
66	.997	.927	.932	.737	.655
68	.998	.934	.843	.750	.668
70	.999	.940	.852	.761	.680
72	.999	.944	.859	.769	.689
74	1.000	.957	.880	.795	.717
76		.957	.884	.800	.722
78		.963	.891	.809	.732
80		.965	.894	.814	.736
82		.968	.900	.822	.746
84		.971	.906	.829	.754
86		.977	.919	.847	.775
88		.978	.921	.851	.779
90		.983	.933	.866	.798
94		.986	.940	.877	.811
96		.987	.942	.880	.815

Table B 4 (Continued) M = 4, N EVEN: TIES NOT ALLOWED

J	N = 4	6	8	10	12
98		.990	.949	.891	.828
100		.991	.951	.894	.832
102		.992	.954	.899	.837
104		.993	.958	.905	.845
106		.994	.961	.911	.853
108		.995	.963	.911	.856
110		.996	.969	.924	.869
114		.997	.972	.930	.878
116		.997	.975	.934	.883
118		.998	.977	.938	.889
120		.998	.978	.941	.893
122		.999	.981	.947	.901
126		.999	.985	.953	.910
128		.999	.985	.954	.911
130		.999	.986	.955	.914
132		.999	.986	.957	.917
134		1.000	.989	.963	.925
136			.990	.964	.927
138			.991	.967	.932
140			.991	.968	.934
142			.992	.970	.936
144			.992	.970	.937
146			.994	.975	.944
148			.994	.975	.945
150			.995	.978	.949
152			.995	.979	.951
154			.996	.981	.954
158			.996	.982	.957
160			.997	.982	.958
162			.997	.984	.960
164			.997	.985	.963
166			.998	.986	.965
168			.998	.987	.966
170			.998	.988	.969
172			.998	.989	.969
174			.999	.990	.972
176			.999	.990	.973
178			.999	.991	.974
180			.999	.991	.975
182			.999	.993	.977
184			.999	.993	.978
186			.999	.994	.980
190			.999	.994	.980
192			.999	.994	.980
194			1.000	.995	.983
196				.995	.983

Table B 4 (Continued) M = 4, N EVEN: TIES NOT ALLOWED

J	4	6	8	10	12
198				.995	.985
200				.996	.985
202				.996	.986
204				.996	.986
206				.997	.988
208				.997	.988
210				.997	.989
212				.997	.990
214				.998	.990
216				.998	.991
218				.998	.991
222				.998	.992
224				.998	.992
226				.998	.993
228				.998	.993
230				.999	.994
232				.999	.994
234				.999	.995
236				.999	.995
238				.999	.995
242				.999	.995
244				.999	.996
246				.999	.996
248				.999	.996
250				.999	.997
254				1.000	.997
256					.997
258					.997
260					.997
262					.997
264					.998
266					.998
268					.998
270					.998
272					.998
274					.998
276					.998
278					.999
280					.999
282					.999
286					.999
288					.999
290					.999
292					.999

Table B 4 (Continued)   M = 4, N EVEN: TIES NOT ALLOWED

			N		
J	4	6	8	10	12
294					.999
296					.999
298					.999
300					.999
302					.999
304					.999
306					.999
308					.999
310					.999
312					.999
314					1.000

Table B 5 M = 4, N ODD: TIES ALLOWED

J	N = 3	N = 5	N = 7	N = 9	N = 11
1	.112	.058	.036	.026	.019
3	.224	.122	.079	.057	.043
5	.475	.282	.190	.141	.109
9	.647	.420	.296	.224	.177
11	.747	.512	.372	.285	.227
13	.821	.587	.436	.339	.273
17	.897	.691	.536	.426	.350
19	.924	.733	.579	.466	.384
21	.962	.803	.655	.536	.449
25	.973	.833	.690	.571	.483
27	.981	.858	.723	.605	.515
29	.993	.906	.788	.674	.582
33	.996	.927	.820	.711	.620
35	.998	.944	.848	.744	.655
37	.999	.951	.860	.759	.670
41	1.000	.969	.896	.807	.723
43		.973	.904	.817	.735
45		.981	.924	.840	.768
49		.986	.936	.863	.789
51		.989	.945	.877	.806
53		.992	.956	.896	.829
57		.994	.962	.906	.842
59		.996	.969	.919	.859
61		.997	.975	.930	.875
65		.998	.981	.943	.893
67		.998	.982	.945	.897
69		.999	.987	.956	.912
73		.999	.989	.959	.918
75		.999	.990	.964	.925
77		1.000	.992	.970	.935
81			.994	.975	.944
83			.995	.979	.950
85			.995	.980	.953
89			.99[illegible]	.985	.962
91			.998	.987	.965
93			.998	.988	.967
97			.998	.989	.969
99			.998	.990	.972
101			.999	.993	.978
105			.999	.994	.981
107			.999	.995	.982
109			1.000	.995	.984
113				.996	.986
115				.99[illegible]	.987
117				.997	.989

Table B 5 (Continued) M = 4, N ODD: TIES ALLOWED

J	N = 3	5	7	9	11
121				.997	.990
123				.998	.991
125				.998	.992
129				.999	.993
131				.999	.994
133				.999	.995
137				.999	.995
139				.999	.996
141				.999	.996
145				.999	.997
147				1.000	.997
149					.998
153					.998
155					.998
157					.998
161					.999
163					.999
165					.999
169					.999
171					.999
173					.999
177					.999
179					.999
181					1.000

Table B 6 M = 4, N EVEN: TIES ALLOWED

	N				
J	4	6	8	10	12
0	.015	.008	.005	.004	.003
2	.153	.093	.062	.046	.036
4	.209	.129	.088	.065	.051
6	.387	.250	.178	.135	.107
8	.457	.302	.218	.167	.133
10	.568	.392	.289	.225	.181
12	.597	.417	.310	.242	.196
14	.736	.548	.422	.337	.277
16	.750	.561	.435	.348	.286
18	.815	.633	.500	.406	.338
20	.849	.673	.540	.442	.370
22	.874	.707	.574	.474	.400
24	.894	.736	.604	.503	.427
26	.941	.810	.686	.583	.502
30	.960	.846	.728	.627	.544
32	.963	.854	.737	.637	.554
34	.973	.879	.770	.672	.590
36	.978	.893	.789	.693	.610
38	.987	.920	.828	.737	.655
40	.990	.928	.839	.750	.669
42	.993	.941	.859	.775	.695
44	.994	.946	.868	.785	.707
46	.996	.955	.884	.805	.728
48	.996	.957	.886	.808	.732
50	.998	.968	.907	.837	.764
52	.998	.970	.913	.844	.773
54	.999	.980	.932	.870	.804
56	1.000	.983	.940	.883	.818
58		.985	.944	.888	.825
62		.989	.955	.906	.848
64		.989	.956	.907	.849
66		.992	.964	.921	.868
68		.994	.968	.928	.877
70		.994	.972	.934	.885
72		.995	.974	.938	.891
74		.997	.980	.950	.908
76		.997	.982	.952	.911
78		.998	.984	.956	.917
80		.998	.984	.957	.919
82		.998	.986	.960	.924
84		.998	.987	.964	.929
86		.999	.990	.970	.939
88		.999	.991	.971	.941
90		.999	.993	.976	.950

Table B 6 (Continued) M = 4, N EVEN: TIES ALLOWED

			N		
J	4	6	8	10	12
94		1.000	.994	.980	.956
96			.995	.981	.957
98			.996	.984	.962
100			.996	.985	.964
102			.996	.986	.966
104			.997	.987	.969
106			.997	.989	.971
108			.998	.989	.973
110			.998	.991	.977
114			.999	.993	.979
116			.999	.993	.981
118			.999	.994	.983
120			.999	.995	.984
122			.999	.996	.986
126			1.000	.996	.988
128				.996	.989
130				.997	.989
132				.997	.990
134				.998	.992
136				.998	.992
138				.998	.993
140				.998	.993
142				.998	.994
144				.998	.994
146				.999	.995
148				.999	.995
150				.999	.996
152				.999	.996
154				.999	.996
158				.999	.997
160				.999	.997
162				.999	.997
164				1.000	.998
166					.998
168					.998
170					.998
172					.998
174					.998
176					.998
178					.999
180					.999
182					.999
184					.999
186					.999
190					.999
192					.999

Table B 6 (Continued) M = 4, N EVEN: TIES ALLOWED

J	N: 4	6	8	10	12
194					.999
196					.999
198					.999
200					1.000

Table B 7 M = 5: TIES ALLOWED

N

J	3	4	5	6	7	8	9	10	11	12
0	.004	.002	.001	.001	.001	.001	.000	.000	.000	.000
2	.059	.036	.025	.018	.013	.010	.008	.007	.006	.005
4	.124	.079	.055	.040	.031	.024	.019	.016	.013	.011
6	.228	.151	.106	.078	.061	.048	.039	.032	.027	.023
8	.310	.210	.150	.113	.088	.071	.057	.047	.040	.034
10	.438	.308	.227	.173	.137	.110	.091	.076	.065	.056
12	.471	.336	.249	.191	.151	.122	.101	.084	.072	.063
14	.587	.436	.333	.261	.210	.172	.144	.121	.104	.091
16	.664	.505	.394	.312	.254	.210	.176	.149	.129	.113
18	.718	.559	.442	.354	.291	.242	.204	.173	.151	.132
20	.757	.600	.481	.388	.321	.269	.228	.194	.170	.149
22	.812	.663	.542	.444	.372	.314	.268	.230	.201	.177
24	.844	.701	.581	.481	.406	.345	.296	.255	.224	.197
26	.892	.765	.648	.547	.466	.400	.346	.300	.265	.234
28	.904	.782	.667	.566	.484	.416	.362	.314	.278	.246
30	.925	.814	.702	.602	.519	.450	.393	.342	.304	.271
32	.938	.836	.728	.629	.545	.475	.416	.365	.325	.289
34	.959	.874	.776	.681	.597	.526	.464	.409	.367	.328
36	.965	.887	.793	.701	.617	.546	.483	.427	.384	.344
38	.975	.908	.822	.733	.651	.580	.516	.459	.414	.372
40	.982	.924	.844	.759	.678	.608	.544	.486	.439	.396
42	.985	.932	.857	.774	.695	.625	.561	.502	.455	.411
44	.988	.942	.873	.795	.717	.648	.584	.525	.477	.432
46	.993	.958	.898	.827	.754	.686	.623	.563	.514	.468
48	.994	.961	.905	.836	.763	.696	.633	.574	.525	.478
50	.996	.970	.922	.858	.791	.725	.664	.605	.555	.508
52	.997	.973	.927	.866	.800	.736	.675	.616	.566	.518
54	.998	.979	.939	.883	.821	.759	.699	.642	.592	.544
56	.999	.983	.948	.897	.837	.778	.721	.663	.613	.565
58	.999	.987	.957	.910	.854	.797	.741	.685	.635	.588
60	.999	.988	.960	.915	.860	.804	.749	.693	.644	.596
62	1.000	.991	.966	.925	.874	.821	.767	.713	.664	.616
64		.993	.971	.934	.887	.836	.784	.731	.683	.635
66		.994	.976	.943	.899	.851	.802	.750	.702	.655
68		.995	.978	.947	.905	.857	.809	.757	.711	.664
70		.996	.982	.954	.916	.871	.825	.775	.729	.683
72		.997	.984	.957	.921	.878	.832	.784	.738	.693
74		.998	.987	.965	.932	.893	.851	.805	.761	.716
76		.998	.989	.969	.938	.901	.861	.816	.773	.728
78		.999	.990	.971	.942	.907	.868	.824	.781	.737
80		.999	.992	.974	.948	.913	.875	.833	.792	.749
82		.999	.993	.978	.953	.921	.886	.845	.805	.762
84		.999	.994	.979	.956	.925	.890	.849	.810	.768
86		1.000	.995	.983	.962	.934	.902	.864	.827	.787
88			.996	.985	.966	.940	.908	.872	.836	.796

Table B 7 (Continued) L = 5: TIES ALLOWED

J	N: 3	4	5	6	7	8	9	10	11	12
90			.997	.987	.969	.945	.916	.881	.846	.807
92			.997	.988	.971	.947	.919	.885	.850	.812
94			.998	.990	.976	.954	.928	.897	.864	.828
96			.998	.991	.978	.957	.932	.902	.870	.835
98			.998	.992	.980	.961	.937	.908	.878	.844
100			.998	.993	.982	.964	.941	.912	.883	.849
102			.999	.994	.983	.966	.944	.916	.888	.855
104			.999	994	.985	.969	.949	.922	.895	.863
106			.999	.996	.987	.974	.955	.931	.905	.875
108			.999	.996	.988	.975	.956	.933	.907	.877
110			.999	.997	.990	.978	.961	.939	.915	.887
112			1.000	.997	.990	.979	.963	.942	.918	.891
114				.997	.991	.981	.966	.946	.924	.898
116				.998	.992	.983	.969	.949	.928	.902
118				.998	.993	.984	.971	.953	.933	.908
120				.998	.994	.985	.973	.956	.936	.912
122				.998	.994	.987	.975	.959	.940	.917
124				.999	.995	.988	.977	.962	.943	.921
126				.999	.996	.989	.979	.964	.947	.926
128				.999	.996	.990	.980	.966	.950	.929
130				.999	.996	.991	.982	.970	.954	.934
132				.999	.997	.991	.983	.970	.955	.936
134				.999	.997	.993	.985	.974	.959	.942
136				.999	.998	.994	.987	.976	.962	.945
138				1.000	.988	.994	.987	.977	.964	.948
140					.998	.994	.988	.978	.965	.949
142					.998	.995	.989	.980	.968	.953
144					.998	.995	.990	.981	.969	.955
146					.999	.996	.991	.983	.973	.959
148					.999	.996	.992	.984	.974	.960
150					.999	.997	.992	.985	.975	.963
152					.999	.997	.993	.986	.977	.964
154					.999	.997	.994	.987	.979	.968
156					.999	.998	.994	.988	.980	.969
158					.999	.998	.995	.989	.981	.971
160					.999	.998	.995	.989	.982	.972
162					1.000	.998	.995	.990	.983	.973
164						.998	.996	.991	.984	.975
166						.999	.996	.992	.986	.977
168						.999	.996	.992	.986	.978
170						.999	.997	.993	.987	.980
172						.999	.997	.993	.988	.980
174						.999	.997	.994	.989	.981
176						.999	.998	.994	.990	.983
178						.999	.998	.995	.990	.984

Table B 7 (Continued) M = 5: TIES ALLOWED

	N									
J	3	4	5	6	7	8	9	10	11	12
180						.999	.998	.995	.991	.984
182						.999	.998	.996	.991	.985
184						.999	.998	.996	.992	.986
186						1.000	.998	.996	.993	.987
188							.999	.996	.993	.998
190							.999	.997	.994	.989
192							.999	.997	.994	.989
194							.999	.997	.994	.990
196							.999	.998	.995	.991
198							.999	.998	.995	.991
200							.999	.998	.995	.992
202							.999	.998	.996	.992
204							.999	.998	.996	.992
206							.999	.998	.996	.993
208							.999	.998	.997	.994
210							.999	999	.997	.994
212							.999	999	.997	.994
214							1.000	.999	.997	.995
216								.999	.997	.995
218								.999	.998	.995
220								.999	.998	.996
222								.999	.998	.996
224								.999	.998	.996
226								.999	.998	.996
228								.999	.998	.997
230								.999	.998	.997
232								.999	.999	.997
234								1.000	.999	.997
236									.999	.997
238									.999	.998
240									.999	.998
242									.999	.998
244									.999	.998
246									.999	.998
248									.999	.998
250									.999	.998
252									.999	.998
254									.999	.999
256									.999	.999
258									.999	.999
260									1.000	.999
262										.999
264										.999
266										.999
268										.999

Table B 7 (Continued) M = 5: TIES ALLOWED

	N									
J	3	4	5	6	7	8	9	10	11	12
270										
272										.999
274										.999
276										.999
278										.999
280										.999
282										.999
284										.999
										1.000

Table B 8 M = 6, N EVEN: TIES ALLOWED

N

J	4	6	8	10	12
0	.000	.000	.000	.000	.000
2	.007	.003	.001	.001	.001
4	.024	.009	.005	.003	.002
6	.046	.019	.010	.006	.004
8	.083	.036	.019	.012	.008
10	.125	.056	.031	.019	.012
12	.158	.072	.040	.025	.017
14	.214	.102	.058	.037	.024
16	.273	.135	.078	.050	.034
18	.308	.156	.091	.058	.040
20	.362	.190	.112	.073	.050
22	.419	.227	.137	.090	.062
24	.461	.257	.157	.104	.072
26	.512	.294	.183	.123	.086
28	.557	.329	.208	.140	.099
30	.592	.357	.228	.155	.110
32	.636	.395	.257	.177	.126
34	.676	.431	.285	.198	.142
36	.700	.455	.304	.212	.154
38	.737	.493	.333	.236	.172
40	.771	.529	.364	.260	.191
42	.788	.548	.380	.273	.201
44	.813	.578	.407	.295	.219
46	.839	.610	.436	.319	.239
48	.854	.631	.457	.336	.253
50	.872	.657	.481	.357	.270
52	.888	.681	.505	.378	.288
54	.901	.701	.526	.396	.304
56	.916	.727	.552	.420	.325
58	.926	.746	.573	.439	.341
60	.933	.758	.587	.452	.353
62	.943	.778	.610	.474	.372
64	.952	.798	.632	.496	.392
66	.957	.810	.647	.511	.405
68	.963	.825	.666	.528	.422
70	.969	.841	.687	.550	.442
72	.972	.851	.700	.563	.454
74	.976	.863	.717	.580	.471
76	.980	.875	.734	.598	.489
78	.982	.883	.744	.609	.500
80	.985	.894	.761	.627	.518
82	.987	.903	.775	.643	.533
84	.989	.909	.785	.654	.545
86	.991	.918	.799	.672	.562
88	.992	.926	.812	.687	.578

Table B 3 (Continued)   M = 6, N EVEN: TIES ALLOWED

N

J	4	6	8	10	12
90	.993	.931	.820	.697	.588
92	.994	.936	.830	.709	.601
94	.995	.943	.842	.724	.617
96	.996	.947	.850	.735	.628
98	.997	.952	.859	.746	.641
100	.997	.956	.867	.758	.653
102	.998	.959	.874	.766	.663
104	.998	.964	.883	.779	.677
106	.998	.967	.891	.790	.690
108	.999	.969	.896	.797	.697
110	.999	.973	.903	.808	.710
112	.999	.975	.910	.818	.722
114	.999	.977	.914	.823	.729
116	.999	.979	.920	.832	.739
118	1.000	.982	.926	.841	.751
120		.983	.930	.848	.758
122		.985	.934	.855	.768
124		.986	.939	.863	.777
126		.987	.942	.868	.784
128		.989	.946	.875	.793
130		.990	.950	.881	.801
132		.990	.952	.885	.806
134		.992	.956	.892	.815
136		.993	.960	.898	.823
138		.993	.962	.902	.829
140		.994	.965	.907	.836
142		.994	.967	.912	.843
144		.995	.969	.916	.848
146		.995	.971	.920	.854
148		.996	.973	.924	.860
150		.996	.975	.928	.865
152		.997	.977	.932	.871
154		.997	.979	.936	.876
156		.997	.980	.938	.880
158		.998	.981	.942	.885
160		.998	.983	.945	.891
162		.998	.984	.948	.894
164		.998	.985	.950	.899
166		.998	.986	.954	.904
168		.999	.987	.956	.907
170		.999	.988	.958	.911
172		.999	.989	.960	.914
174		.999	.990	.962	.918
176		.999	.991	.964	.922
178		.999	.991	.966	.925
180		.999	.992	.968	.927
182		.999	.992	.970	.931

Table B 8 (Continued)   M = 6, N EVEN: TIES ALLOWED

J	N: 4	6	8	10	12
184		.999	.993	.972	.934
186		1.000	.993	.973	.936
188			.994	.975	.939
190			.994	.976	.942
192			.995	.977	.944
194			.995	.979	.947
196			.995	.980	.949
198			.996	.981	.951
200			.996	.982	.953
202			.997	.983	.955
204			.997	.984	.957
206			.997	.985	.959
208			.997	.986	.961
210			.997	.987	.963
212			.998	.987	.964
214			.998	.988	.966
216			.998	.989	.968
218			.998	.989	.969
220			.998	.990	.970
222			.998	.990	.971
224			.999	.991	.973
226			.999	.992	.974
228			.999	.992	.975
230			.999	.992	.976
232			.999	.993	.978
234			.999	.993	.978
236			.999	.994	.980
238			.999	.994	.981
240			.999	.994	.981
242			.999	.995	.982
244			.999	.995	.983
246			.999	.995	.984
248			.999	.996	.985
250			1.000	.996	.985
252				.996	.986
254				.996	.987
256				.997	.987
258				.997	.988
260				.997	.988
262				.997	.989
264				.997	.989
266				.997	.990
268				.998	.990
270				.998	.991
272				.998	.991
274				.998	.992

Table B 8 (Continued) M = 6, N EVEN: TIES ALLOWED

J	N = 4	6	8	10	12
276				.998	.992
278				.998	.992
280				.998	.993
282				.998	.993
284				.999	.993
286				.999	.994
288				.999	.994
290				.999	.994
292				.999	.995
294				.999	.995
296				.999	.995
298				.999	.995
300				.999	.996
302				.999	.996
304				.999	.996
306				.999	.996
308				.999	.996
310				.999	.997
312				.999	.997
314				.999	.997
316				1.000	.997
318					.997
320					.997
322					.997
324					.998
326					.998
328					.998
330					.998
332					.998
334					.998
336					.998
338					.998
340					.998
342					.998
344					.998
346					.999
348					.999
350					.999
352					.999
354					.999
356					.999
358					.999
360					.999
362					.999
364					.999
366					.999
368					.999

Table B 8 (Continued) M = 6, N EVEN: TIES ALLOWED

	N				
J	4	6	8	10	12
370					.999
372					.999
374					.999
376					.999
378					.999
380					.999
382					1.000