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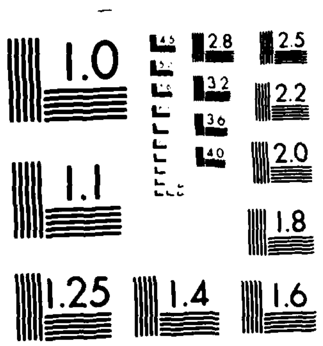

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TERRAIN MESOROUGHNESS DESCRIPTION AND ITS APPLICATION TO MOBILITY AND COVER

Richard A. Weiss

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TERRAIN MESOROUGHNESS DESCRIPTION AND
ITS APPLICATION TO MOBILITY AND COVER

Richard A. Weiss
Mobility Systems Division
U. S. Army Engineer Waterways Experiment Station
Vicksburg, Mississippi 39180

ABSTRACT. The mobility of vehicles traveling off the road has always been of interest to military planners. In fact, the survivability of military vehicles on a battlefield depends to a large extent on the degree of mobility and cover afforded by the terrain. In order to develop accurate measures of the mobility and cover characteristics of terrain, a study was made of the statistical description of terrain elevation variations on a scale smaller than shown on a topographic map. This study shows that a useful set of terrain descriptors are the standard deviations of the detrended elevation and its derivatives. The probabilities of a vehicle encountering specified values of slope or finding cover in a specified interval are calculated in terms of the terrain roughness descriptors. These probabilities can be used to quantify terrain areas of a battlefield and develop map overlays showing patches which indicate the degree of slopes expected to be encountered and the amount of cover that should be available. Numerical values of the probabilities are calculated for several terrain areas.

1. INTRODUCTION. A significant part of the determination of the survivability of military vehicles on a battlefield is an estimation of their mobility on the battlefield terrain and an estimation of the degree of cover afforded by the terrain. The detection of a target and the subsequent firepower accuracy, including hit and kill probabilities, depend on the percent of a target that is exposed to fire and on the speed at which a vehicle can move across the battlefield terrain. This paper presents a method of estimating the mobility characteristics and available cover of battlefield terrain and specifies the terrain parameters required to accomplish this.

This is the second of a two-part study of the methods used for a quantitative description of terrain roughness. The first part considered the description of terrain microroughness with applications to the prediction of the dynamic response of military vehicles operating on rough terrain.[1,2] An analytical description of microroughness is necessary for the design of track and wheel suspension systems and also for the design of optical observation and sighting devices, gun stabilization systems, and many other complex weapons systems that are part of modern military vehicles. Many of the analytical procedures introduced for the description of microroughness can be used to describe large-scale variations of terrain elevation.

The second part of the terrain roughness study deals with large-scale elevation variations on a scale appropriate for the description of the mobility, cover, and concealment properties of a terrain area.[3] For the purposes of this paper the large-scale terrain elevations can be separated into two classes: mesoroughness which describes the terrain elevation variations on a scale between microroughness and the elevations described by a topographic map, and macroroughness which is essentially the terrain elevations given by contours on a topographic map. The mesoroughness is not obtainable from a standard topographic map, and the terrain elevation variations of the mesoroughness may afford cover for vehicles on a battlefield.

Aside from vegetation, the cover, concealment, and mobility characteristics of a mesoroughness elevation profile, as seen from a point on a contour of a topographic map, are determined by the magnitude and character of its elevation variations. Therefore, an analytical description of terrain mesoroughness is required for the prediction of cover and mobility, and the U. S. Army Engineer Waterways Experiment Station (WES) was requested to develop improved analytical representations of terrain mesoroughness and to incorporate this development into cover and mobility indices for military mapping purposes.

Measurement of Terrain Elevation Profiles. A study of terrain roughness begins with the measurement of an elevation profile. A simple and very accurate method of obtaining a profile uses the theodolite and surveyor's staff, but this method is very time-consuming. The measuring wheel method is quicker but has the disadvantages of disturbing the terrain and not being useful in very rough terrain. Terrain elevation measurements done by WES utilize the theodolite and surveyor's staff.

Photogrammetric methods of measuring terrain elevation profiles are simpler and faster because extended areas can be measured by areal surveys.[4,5] They have the further advantages of not affecting terrain conditions, and measuring all terrain features such as vegetation, roads, rivers, ditches, etc., as well as elevations. Photogrammetry has several disadvantages including the facts that vegetation often impairs terrain elevation measurements, evaluating photographs is time-consuming and expensive, and the resolution of elevations is limited especially for high altitude photographs.[6]

Terrain profiles determined by any method are presented as measurements of elevation at discrete points along some predetermined line. The effect of this discrete scanning is to remove from the actual elevation profile frequencies higher than $(\Delta L)^{-1}$ where ΔL = scanning length , so that scanning can be represented as a low pass filter. The spatial frequency $(\Delta L)^{-1}$ is essentially the Nyquist frequency associated with the measurement of the elevation profile and enters the calculation of power spectra through the spectral window functions for the slope and curvature. Photogrammetric methods tend to underestimate the values of the standard deviations of elevation, slope, and curvature and their associated power spectra especially in the high frequency regions.

Macroterrain Roughness. A battlefield area can be divided into a grid of square areas at whose vertices the terrain elevations are specified. These elevations are recorded on a topographic map at uniform spacings of generally about 10 or 100 m. These terrain elevations form a three-dimensional surface called the macroterrain roughness. The macroroughness describes the large-scale trend of elevation variations, and is suitable for the descriptions of cover and vehicle mobility over large areas.[7] Terrain elevation variations on a scale less than the 100-m grid length are not obtainable from a topographic map.

Mesoterrain Roughness. Superimposed on the macroterrain (the trend given by the elevations specified at the 100-m intervals of a topographic map) are the elevation variations on a scale less than the 100-m interval--the terrain mesoroughness. Troops and equipment may possibly be concealed and covered with the terrain elevation variations of the 100-m interval connecting two elevation contours of a topographic map, whereas a straight line or other extrapolation of elevation between these two points would not suggest this.

The slopes occurring within the mesoroughness may differ considerably from that obtained from a topographic map, and therefore the actual vehicle mobility in a battlefield area may be considerably different from the predicted by the slopes obtained from a topographic map. Knowledge of the mesoroughness terrain elevations will be critical for estimating transit time of vehicles across a battlefield. Obstacles of various sizes and shapes are also expected to occur within the mesoroughness and these will affect the degree of mobility in a given area. The mesoroughness description is necessary for the prediction of cover, concealment, and mobility over small distance scales and small time scales.

The mesoroughness elevation varies throughout a battlefield area, and, accordingly, the amount of cover for a target and the mobility of a vehicle can vary rapidly over small distances (~10 m). Therefore, it will be of advantage for military purposes to be able to estimate the mesoroughness for the areas of a battlefield. A number of mesoroughness descriptors need to be developed in order to do this. The battlefield is divided into areas within which the mesoroughness descriptors have essentially the same values. The mesoroughness descriptors can be estimated for a given area by sampling a number of mesoterrain elevation profiles in the area.

A reduction of the size of the grid spacing of the macroterrain roughness descriptions would adequately incorporate the mesoroughness, but this would require large computer storage capacity. A more reasonable procedure is to describe the mesoroughness by a set of stochastic variables.

The determination of the descriptors of the random mesoroughness component requires the removal of the trend of the terrain elevation data. A detrending procedure is described in Part II of this paper. With a properly selected filter constant the detrended elevation profile gives the mesoroughness elevations between two points on a topographic map. The detrending is accomplished by a computer program RFNWUD.[3] It is assumed that the mesoroughness irregularities can be described by a zero mean stationary Gaussian (normal) random process.[8,9] The mesoroughness displacement and its derivatives will be represented by zero-mean Gaussian processes which are described by the standard deviations of displacement, slope, curvature, etc. The standard deviations of the mesoroughness displacemenet and its derivatives to any order are calculated from a detrended elevation program by the computer program RFNWUD.

Cover and Mobility Characteristics. Cover refers to protection from direct weapons fire, while the term concealment refers to features that would interrupt the line of sight. For instance, vegetation may afford concealment but not necessarily cover. Vegetation is not considered in this report, and only cover (concealment) due to terrain elevation variations is treated. The degree of available cover afforded a target is measured by the fraction of target height that is covered, i.e. the depth of the hole in which the vehicle sits compared to the vehicle's height.

The mobility of vehicles depends on a large number of terrain features including: vegetaion, soil strength, terrain slopes, etc. However, the primary terrain roughness variable that enters mobility calculations is the slope expected to be encountered along some path.[10] Each vehicle is associated with a critical slope which it cannot negotiate.

Because the mesoterrain roughness is represented by stochastic variables, the average mesoroughness displacement and the fraction of target height that is covered can only be described by a probability theory. This is also true of the slopes expected to be encountered on a battlefield area. With the assumption of zero mean Gaussian distributions to describe the mesoroughness, the calculation of average values and probabilities of encountering specified terrain characteristics is relatively simple and is done in Part III. These average values and probabilities are expressed in terms of the mesoroughness descriptors (standard deviations of terrain displacement and its derivatives) and can serve as indices to delineate patches on a map that have similar cover and mobility characteristics.

The basic objective of this paper is the development of new mesoroughness terrain descriptors. The objectives and scope of this paper are shown in Figures 1a and 1b.

2. STATISTICAL DESCRIPTION OF MESOTERRAIN ROUGHNESS. This section develops the parameters necessary for the description of terrain mesoroughness. It is assumed that the mesoroughness can be described by a zero-mean stationary Gaussian random process which is obtained from a measured elevation profile by a suitable detrending process. The frequency content of a stationary random process is described by the power spectrum or alternatively by the autocorrelation or autocovariance functions.[11] It is shown that that autocovariance functions are completely determined by the standard deviations of the mesoterrain elevation and all its derivatives. Therefore, the basic mesoroughness descriptors are the standard deviations of the mesoterrain elevation and its derivatives.

The standard deviations of mesoroughness displacement and its derivatives are calculated from detrended terrain elevation data. The accuracy of the values of the numerical derivatives obtained from the elevation data decreases for the higher derivatives. But many terms (higher derivatives) are required for a useful power series expansion of the autocorrelation function for large argument. Therefore, the power series method of calculating the autocorrelation function and the power spectrum is more formal than practical, and for numerical calculations it is more useful to represent the power spectrum as a polynomial whose coefficients are determined by using only the standard deviations of displacement, slope, and curvature.[1]

Although the power series representations for the autocorrelations functions are only of formal value for determining the power spectra, they can be used to calculate the probabilities of finding specified mesoterrain displacements and slopes in a small spatial interval of the terrain. These probabilities are important for the description of cover and mobility characteristics.

Detrending. Except for relatively flat areas, a macroterrain elevation profile cannot be represented as a stationary random process because it has a trend, i.e. a variation of the statistical parameters along the length of the profile. In general the nonstationary character of the measured elevation profile has to be removed by a detrending procedure in order to obtain the standard deviations of the resulting mesoroughness. In the cases where no trend exists the measured elevation data can be processed directly.[1] The nonstationary property of an elevation profile can be removed by a detrending procedure which removes the long wavelength (the trend) components as shown in Figure 2a.

The mesoroughness will be treated as a stationary random process. The mesoroughness is extracted from the measured elevation profile by using the following exponentially weighted moving average.[12]

$$\zeta(x) = h(x) - \frac{1}{2\lambda}\int_0^\infty \Big[h(x - a) + h(x + a)\Big] e^{-a/\lambda}\, da \tag{1}$$

where

ζ = mesoroughness displacement
h = terrain elevation
λ = detrending constant (filter constant)
x = horizontal distance
a = integration variable

The detrending procedure given in Equation 1 removes all wavelength components of the macroterrain elevations that are greater than λ . A computer program RFNWUD was developed to accomplish the detrending.[3]

Plausibility arguments can be given to select a value for λ . If the measurement interval of the macroroughness is L the filter constant can be taken to be

$$\lambda \sim L \tag{1a}$$

in order to remove the trend of the macroterrain as shown in Figure 2b. Another possible choice for λ would account for the elevation difference between two points on the macroterrain of a topographic map as follows

$$\lambda \sim \sqrt{L^2 + H^2} \tag{1b}$$

where H = elevation difference . Equations 1a and 1b represent intuitive possibilities; in fact the values of the parameter λ should be chosen to produce agreement between the probability calculations of cover and mobility given in Part III and the measured cover and mobility characteristics of an area.

Mesoterrain Roughness Description. Terrain elevation is specified at finite intervals of horizontal distance and is generally a continuous function whose derivatives are discontinuous at the points of measurement. For the purpose of calculating standard deviations the derivatives at each point can be assumed to be the slope of the straight line segment to its right or left as shown in Figure 2c. To simplify notation the n'th derivative of the mesoterrain displacement $\xi(x_i)$ at the point x_i will be written as

$$\xi_n(x_i) = \frac{d^n\xi}{dx_i^n} \tag{2}$$

where $n = 0, 1, 2, \ldots$ As usual the zeroth derivative is just the displacement itself

$$\xi_o(x_i) = \xi(x_i) \tag{3}$$

The root mean square (rms) values of these derivatives are calculated as follows

$$\Sigma_o^2 = \frac{1}{N}\sum_{i=1}^{N} \xi^2(x_i) = \frac{1}{N}\sum_{i=1}^{N} \left[\xi_o(x_i)\right]^2 \tag{4}$$

$$\Sigma_1^2 = \frac{1}{N}\sum_{i=1}^{N} \left(\frac{d\xi}{dx_i}\right)^2 = \frac{1}{N}\sum_{i=1}^{N} \left[\xi_1(x_i)\right]^2 \tag{5}$$

$$\Sigma_2^2 = \frac{1}{N}\sum_{i=1}^{N} \left(\frac{d^2\xi}{dx_i^2}\right)^2 = \frac{1}{N}\sum_{i=1}^{N} \left[\xi_2(x_i)\right]^2 \tag{6}$$

$$\Sigma_3^2 = \frac{1}{N}\sum_{i=1}^{N} \left(\frac{d^3\xi}{dx_i^3}\right)^2 = \frac{1}{N}\sum_{i=1}^{N} \left[\xi_3(x_i)\right]^2 \tag{7}$$

or in general

$$\Sigma_n^2 = \frac{1}{N}\sum_{i=1}^{N} \left(\frac{d^n\xi}{dx_i^n}\right)^2 = \frac{1}{N}\sum_{i=1}^{N} \left[\xi_n(x_i)\right]^2 \tag{8}$$

where

Σ_o = rms value of displacement

Σ_1 = rms value of slope

Σ_2 = rms value of second derivative (curvature)

Σ_3 = rms value of third derivative of displacement

Σ_n = rms value of n'th derivative of displacement

N = number of points where elevations are measured

The number N is generally large being of the order of hundreds or thousands.

The mean values of the macroterrain displacement and its derivatives are calculated as follows

$$\xi_n^M = \frac{1}{N}\sum_{i=1}^{N} \frac{d^n\xi}{dx_i^n} = \frac{1}{N}\sum_{i=1}^{N} \xi_n(x_i) \tag{9}$$

The standard deviations of macroroughness displacement and derivatives are given by[11]

$$\sigma_n^2 = \Sigma_n^2 - \left(\xi_n^M\right)^2 \tag{10}$$

The standard deviations can also be written as

$$\sigma_n^2 = \frac{1}{N}\sum_{i=1}^{N} \psi_n^2(x_i) \tag{11}$$

where

$$\psi_n(x_i) = \xi_n(x_i) - \xi_n^M \tag{12}$$

are the values of mesoterrain displacement and its derivatives measured from their mean values, i.e. their values for a zero mean process since $\psi_n^M = 0$. The computer program RFNWUD calculates the standard deviations of mesoterrain displacement, slope, curvature, etc. from measured elevation data using the formulas 1 through 10.

Autocorrelation and Autocovariance Functions. The autocorrelation and autocovariance functions of the mesoterrain elevation and its derivatives are defined by[11]

$$R_n(\alpha) = \frac{1}{N} \sum_{i=1}^{N} \xi_n(x_i)\ \xi_n(x_i + \alpha) \tag{13a}$$

$$C_n(\alpha) = R_n(\alpha) - \left(\xi_n^M\right)^2 \tag{13b}$$

where

$R_n(\alpha)$ = autocorrelation function of n'th derivative of elevation

$C_n(\alpha)$ = autocovariance function of n'th derivative of elevation

n = 0, 1, 2, 3, ...

The autocovariance functions can also be written as[11]

$$C_n(\alpha) = \frac{1}{N} \sum_{i=1}^{N} \psi_n(x_i)\ \psi_n(x_i + \alpha) \tag{14}$$

where $\psi_n(x_i)$ is given by Equation 12. Because of the stationarity assumption, the parameters Σ_n, σ_n, and ξ_n^M are independent of the interval α. From Equations 8, 11, 13a, and 14 it follows that

$$R_n(0) = \Sigma_n^2 \tag{15a}$$

$$C_n(0) = \sigma_n^2 \tag{15b}$$

Useful quantities that describe the random variables ψ_n are the correlation coefficients[11]

$$r_n(\alpha) = \frac{C_n(\alpha)}{C_n(0)} \tag{16}$$

where n = 0, 1, 2, 3, ... The correlation coefficients describe the statistical properties of the mesoterrain displacement and all its derivatives. The correlation coefficients will be used in Part III to calculate the probability of encountering specified displacements and slopes in a given spatial interval.

The autocorrelation and autocovariance are even functions and therefore have the following even power Taylor series expansions[13]

$$R_n(\alpha) = R_n(0) + \frac{R_n^{(2)}(0)}{2!}\alpha^2 + \frac{R_n^{(4)}(0)}{4!}\alpha^4 + \ldots \tag{17a}$$

$$C_n(\alpha) = C_n(0) + \frac{C_n^{(2)}(0)}{2!}\alpha^2 + \frac{C_n^{(4)}(0)}{4!}\alpha^4 + \ldots \tag{17b}$$

where

$R_n^{(j)}(0)$ = j'th derivative of $R_n(\alpha)$ evaluated at $\alpha = 0$

$C_n^{(j)}(0)$ = j'th derivative of $C_n(\alpha)$ evaluated at $\alpha = 0$

The correlation coefficients are given by Equations 16 and 17b to be

$$r_n(\alpha) = 1 + \frac{1}{2!}\frac{C_n^{(2)}(0)}{C_n(0)}\alpha^2 + \frac{1}{4!}\frac{C_n^{(4)}(0)}{C_n(0)}\alpha^4 + \ldots \tag{18}$$

It can be shown that the autocovariance function and its derivatives are related to the standard deviations of the displacement and its derivatives in the following way[3]

$$C_n(0) = \sigma_n^2 \tag{19}$$

$$C_n^{(2)}(0) = -\sigma_{n+1}^2 \tag{20}$$

$$C_n^{(4)}(0) = \sigma_{n+2}^2 \tag{21}$$

$$C_n^{(6)}(0) = -\sigma_{n+3}^2 \tag{22}$$

$$C_n^{(2s)}(0) = (-1)^s\,\sigma_{n+s}^2 \tag{23}$$

Similar expressions hold for the $R_n^{(j)}(0)$ coefficients with the σ's replaced by the Σ's .

The Taylor series of the autocovariance functions (Equation 17b) and the correlation coefficients (Equation 18) can therefore be written as

$$C_n(\alpha) = \sigma_n^2 - \frac{\sigma_{n+1}^2\alpha^2}{2!} + \frac{\sigma_{n+2}^2\alpha^4}{4!} - \ldots \tag{24}$$

$$r_n(\alpha) = 1 - \frac{1}{2!}\left(\frac{\sigma_{n+1}}{\sigma_n}\right)^2\alpha^2 + \frac{1}{4!}\left(\frac{\sigma_{n+2}}{\sigma_n}\right)^2\alpha^4 - \ldots \tag{25}$$

In this way the correlation coefficients can be evaluated in terms of the standard deviations of mesoterrain displacement and its derivatives. The general expressions for these functions are

$$C_n(\alpha) = \sigma_n^2 \sum_{j=0}^{\infty} (-1)^j \frac{S_{nj}}{(2j)!} \alpha^{2j} \tag{26}$$

$$r_n(\alpha) = 1 + \sum_{j=1}^{\infty} (-1)^j \frac{S_{nj}}{(2j)!} \alpha^{2j} \tag{27}$$

where

$$S_{no} = 1 \tag{28}$$

$$S_{n1} = \left(\frac{\sigma_{n+1}}{\sigma_n}\right)^2 \tag{29}$$

$$S_{n2} = \left(\frac{\sigma_{n+2}}{\sigma_n}\right)^2 \tag{30}$$

$$S_{nj} = \left(\frac{\sigma_{n+j}}{\sigma_n}\right)^2 \tag{31}$$

The coefficients appearing in the power series expansion of the correlation coefficients have a simple physical interpretation if it is noticed that the characteristic wavelengths for the mesoterrain displacement and its derivatives are given as follows

$$\lambda_n = 2\pi \frac{\sigma_n}{\sigma_{n+1}} \tag{32}$$

where

λ_n = wavelength of n'th derivative of the mesoroughness displacement. Specifically,

$\lambda_o = 2\pi\sigma_o/\sigma_1$ = characteristic wavelength of displacement

$\lambda_1 = 2\pi\sigma_1/\sigma_2$ = characteristic wavelength of slope

$\lambda_2 = 2\pi\sigma_2/\sigma_3$ = characteristic wavelength curvature

$\lambda_3 = 2\pi\sigma_3/\sigma_4$ = characteristic wavelength of the third derivative

and so on. Therefore from Equations 31 and 32 it follows that

$$S_{nj} = \left(\frac{2\pi}{\lambda_n} \frac{2\pi}{\lambda_{n+1}} \frac{2\pi}{\lambda_{n+2}} \cdot \cdot \cdot \frac{2\pi}{\lambda_{n+j-1}} \right)^2 \tag{33}$$

for $j = 1, 2, 3, \ldots \infty$.

In terms of these wavelengths the correlation coefficients are written as

$$r_n(\alpha) = 1 - \frac{1}{2!} \left(\frac{2\pi\alpha}{\lambda_n} \right)^2 + \frac{1}{4!} \left(\frac{2\pi\alpha}{\lambda_n} \frac{2\pi\alpha}{\lambda_{n+1}} \right)^2 - \frac{1}{6!} \left(\frac{2\pi\alpha}{\lambda_n} \frac{2\pi\alpha}{\lambda_{n+1}} \frac{2\pi\alpha}{\lambda_{n+2}} \right)^2 + \ldots \tag{34}$$

If the following set of dimensionless numbers are introduced

$$T_n = \frac{2\pi\alpha}{\lambda_n} \tag{35}$$

the correlation coefficients can be written as

$$r_n(\alpha) = 1 - \frac{1}{2!} T_n^2 + \frac{1}{4!} (T_n T_{n+1})^2 - \frac{1}{6!} (T_n T_{n+1} T_{n+2})^2 + \ldots \tag{36}$$

$$= 1 + \sum_{j=1}^{\infty} (-1)^j \frac{1}{(2j)!} (T_n T_{n+1} T_{n+2} \ldots T_{n+j-1})^2 \tag{37}$$

Written out in full the expressions for the correlation coefficients of mesoterrain displacement, slope, and curvature are respectively

$$r_o = 1 - \frac{1}{2} T_o^2 + \frac{1}{24} (T_0 T_1)^2 - \frac{1}{720} (T_o T_1 T_2)^2 + \frac{1}{40320} (T_o T_1 T_2 T_3)^2 - \ldots \tag{38}$$

$$r_1 = 1 - \frac{1}{2} T_1^2 + \frac{1}{24} (T_1 T_2)^2 - \frac{1}{720} (T_1 T_2 T_3)^2 + \frac{1}{40320} (T_1 T_2 T_3 T_4)^2 - \ldots \tag{39}$$

$$r_2 = 1 - \frac{1}{2} T_2^2 + \frac{1}{24} (T_2 T_3)^2 - \frac{1}{720} (T_2 T_3 T_4)^2 + \frac{1}{40320} (T_2 T_3 T_4 T_5)^2 - \ldots \tag{40}$$

with the restriction that $T_j << 1$. Figure 3 shows typical correlation coefficients calculated using Equations 38 through 40 and the computer program MESO. These curves show how the calculation breaks down for large spatial distance (large T_j) because only a finite number of standard deviations are accurately obtained from a measured elevation profile. In other words the use of a truncated series restricts these results to small values of spatial distance α .

Power Spectra. The power spectrum measures the frequency content of a random process. It is defined as the Fourier transform of the autocorrelation function as follows[11,13]

$$P_n^*(k) = \frac{1}{\pi} \int_{-\infty}^{\infty} R_n(\alpha) e^{-ik\alpha} d\alpha \tag{41}$$

$$R_n(\alpha) = \frac{1}{2} \int_{-\infty}^{\infty} P_n^*(k) e^{ik\alpha} dk \tag{42}$$

where

$k = 2\pi\Omega$
Ω = spatial frequency
$n = 0, 1, 2, 3, \ldots$

The definition of the power spectrum is physically valid only if the average values of the random processes have physical significance, because according to Equation 13b, the average values ξ_n^M appear in the definition of the autocorrelation functions $R_n(\alpha)$.

However, for the mesoroughness description the autocovariance functions $C_n(\alpha)$ are of more physical interest because elevation profiles are measured from an arbitrary baseline and this leads to arbitrary average values ξ_n^M for the mesoroughness profile. The autocovariance functions, however, describe the mesoterrain roughness relative to the average values, i.e. it takes the average values to be the baseline and therefore describes zero mean random processes. For terrain roughness descriptions the power spectra of physical interest are defined by the following Fourier transform pairs

$$P_n(k) = \frac{1}{\pi} \int_{-\infty}^{\infty} C_n(\alpha) e^{-ik\alpha} d\alpha \tag{41a}$$

$$C_n(\alpha) = \frac{1}{2} \int_{-\infty}^{\infty} P_n(k) e^{ik\alpha} dk \tag{42a}$$

where

$$k = 2\pi\Omega$$

Equation 41a is the formal definition of the power spectra for mesoterrain displacement, slope, curvature, etc. This equation can be used to determine $P_n(k)$ provided $C_n(\alpha)$ is completely specified for all α, and this is never the case for actual terrain elevation data. In particular the integral (Equation 41a) converges only if $C_n(\alpha) \to 0$ for large α, i.e., $C_n(\alpha)$ must be known for large α and must approach zero in this limit. This precludes using the series expansion in Equation 26, which is valid only for small α, because in this case

$$\begin{aligned} P_n(k) &= \frac{\sigma_n^2}{\pi} \sum_{j=0}^{\infty} (-1)^j S_{nj} \int_{-\infty}^{\infty} \alpha^{2j} e^{-ik\alpha} d\alpha \\ &= \frac{2\sigma_n^2}{\pi} \sum_{j=0}^{\infty} (-1)^j S_{nj} F_j(k) \end{aligned} \tag{43}$$

where

$$F_j(k) = \int_0^{\infty} \alpha^{2j} \cos(k\alpha) d\alpha \tag{44}$$

The integrals $F_j(k)$ do not converge, so that the power spectrum cannot be obtained from a power series expansion of the autocovariance function.

The use of a finite set of data points precludes the complete determination of $C_n(\alpha)$ using Equation 14 and therefore $P_n(k)$ cannot be determined for the full range of frequencies $0 < \Omega < \infty$ by using Equation 41a. A finite set of data points results from the finite length L of the elevation profile and from the fact that the elevation profile is measured at intervals $\Delta L = L/N$, so that the limits on the domain of definition of $C_n(\alpha)$ are $\Delta L < \alpha < L$. Then the spatial frequencies have the following bounds $L^{-1} < \Omega < (\Delta L)^{-1}$, so that the lack of information about $C_n(\alpha)$ for large α leads to a lack of information about $P_n(\Omega)$ for small Ω, while the lack of information about $C_n(\alpha)$ for small α leads to an uncertainty in the values of $P_n(\Omega)$ for high frequencies. The upper frequency limit $(2\Delta L)^{-1}$ often occurs in information theory and is called the Nyquist frequency.[11]

In order to determine the behavior of $P_n(\Omega)$ for $\Omega < 1/L$ and for $\Omega > 1/\Delta L$ a model approach for the mesoterrain roughness power spectrum is adopted. The procedure uses the following mathematical model for the power spectra of mesoterrain displacement, slope, curvature, etc.[1,2]

$$P_o(\Omega) = C\Omega^{-2} + D\Omega^{-3} + E\Omega^{-4} \tag{45}$$

$$P_1(\Omega) = (2\pi)^2\Omega^2\left[\frac{\sin(\pi\Omega\Delta L)}{\pi\Omega\Delta L}\right]^2 P_o(\Omega) \tag{46}$$

$$P_2(\Omega) = (2\pi)^4\Omega^4\left[\frac{\sin(\pi\Omega\Delta L)}{\pi\Omega\Delta L}\right]^4 P_o(\Omega) \tag{47}$$

$$P_n(\Omega) = (2\pi)^{2n}\Omega^{2n}\left[\frac{\sin(\pi\Omega\Delta L)}{\pi\Omega\Delta L}\right]^{2n} P_o(\Omega) \tag{48}$$

The coefficients C, D, and E of the three-parameter power spectrum model are evaluated from the values of σ_o, σ_1, and σ_2 that are obtained from detrended (and in some cases undertrended) elevation profile data measured at intervals of length ΔL.[1,2] This model predicts the five basic types of power spectra that are shown in Figure 4. Spectral types 3 and 4 exhibit no trend, so that for these cases it is possible to use undertrended elevation data to determine the power spectrum.[1,2] The five spectral types can be used to classify mesoterrain areas.

3. MESOTERRAIN COVER AND MOBILITY PROBABILITIES. The main objective of this paper is the development of a quantitative method for estimating the effects of a specified mesoterrain roughness on the survivability of a vehicle on a battlefield. Survivability depends in part on the mobility and cover characteristics of the mesoroughness. The mobility characteristics are described by the degree of slopes expected to be encountered, while the cover afforded a vehicle is described by the expected amplitudes and widths (wavelengths) of the hills and holes of the mesoroughness.

This part of the paper uses the terrain descriptors defined in Part II to calculate the probabilities for encountering specified values, and ranges of values, of mesoterrain elevation and slope. For the description of available cover the specified value of the mesoterrain displacement is the depth of a hole, generally equal to the vehicle height or larger, which will afford cover to a vehicle. For mobility considerations a critical slope may be specified which would limit a vehicle's performance.

As described in Part II the mesoroughness elevation variation and its derivatives are described by stochastic variables whose parameters are the standard deviations σ_o, σ_1, σ_2 ... of the displacement and its derivatives. For the calculation of cover and slope probabilities, a Gaussian distribution is assumed for the random variables of terrain displacement and its derivatives.

Two kinds of probability index are calculated in this paper: (a) the probability of finding a given mesoroughness elevation and slope at a fixed point in the battlefield area, and (b) the probability of encountering a specified mesoroughness elevation and slope in a specified small interval of the battlefield terrain. The first probability is useful in the case where the vehicle motion (or travel time) is not important; while the second type of probability is used for moving vehicles looking for a covered firing position. The two situations are quite distinct and an area having large probability of cover at a fixed point in the area may exhibit a relatively small probability of finding cover in a specified small distance interval.

For a fixed point in the battlefield the probability of cover can be evaluated if the standard deviation of mesoroughness elevation σ_0 is known for the area which contains the point. The probability of a moving vehicle finding cover in a specified interval will also depend on the spatial frequency content of the mesoterrain roughness. Thus a relatively smooth area (of long wavelengths) will afford less cover in a specified interval than an undulating area (of shorter wavelengths) with the same standard deviation of mesoroughness elevation. The calculation of the probability of finding a specified elevation (cover) in a given interval will be shown to depend on σ_1 as well as σ_0, so that for the case of a moving vehicle looking for a covered firing position the standard deviation of slope is a critical terrain roughness descriptor.

The probability of encountering a given mesoterrain elevation or range of elevations (cover) in a specified interval of distance corresponds to finding cover in a corresponding specified interval of time given by $\alpha = \mu t$, where α = specified distance, μ = vehicle speed, and t = specified time. If the time interval is taken to be the time interval between successive rounds fired by an enemy gun t_f, then the vehicle had better be able to find cover in the distance $\alpha_f = \mu t_f$ else its chances for survival will be small. Therefore it is of practical value to calculate the probability of finding cover in a specified distance in terms of the roughness descriptors and to use these probabilities as an index to delineate areas of a battlefield having different degrees of cover.

In a similar way it will be shown that the probability of encountering a specified slope in a given interval will depend on the standard deviation of the second derivative σ_2 as well as on the standard deviation of the slope σ_1. Therefore for mobility problems over short distances in a battlefield the curvature roughness parameter is as important as the slope roughness parameter. This probability is important to assess the possible values of slopes that may be encountered in a dash for cover in the interval $\alpha_f = \mu t_f$.

<u>Probability Density Functions for a Point in the Battlefield</u>. The calculation of probabilities of finding specified values of the mesoterrain elevation and its derivatives at a point in the battlefield requires the calculation of probability density functions and probability distribution functions.[11,13] These functions are commonly used in probability theory. Of particular interest to military problems will be the probability of encountering specified values of elevation and slope.

This paper assumes that the mesoterrain elevation, slope, curvature, and all higher derivatives are independent stationary random processes whose distribution about their mean values are given by Gaussian probability density functions defined by

$$p_G^M(\xi_n) = \frac{1}{\sqrt{2\pi}\sigma_n} e^{-\left(\xi_n - \xi_n^M\right)^2 \Big/ \left(2\sigma_n^2\right)} \tag{49}$$

If the stochastic variables are measured from their mean values by $\psi_n = \xi_n - \xi_n^M$, the zero mean Gaussian distribution is given by

$$p_G(\psi_n) = \frac{1}{\sqrt{2\pi}\sigma_n} e^{-\psi_n^2 \Big/ \left(2\sigma_n^2\right)} \tag{50}$$

where as defined in Part II the variables ξ_n and ψ_n correspond to the detrended elevation profile assumed to describe the mesoroughness as follows

ξ_n = n'th derivative of mesoterrain roughness elevation measured from arbitrary level

ψ_n = n'th derivative of the mesoterrain elevation measued from its mean value

Only the standard deviations σ_n are required to describe the Gaussian distributions. The probability density functions determine the probability of encountering a specified value of ξ_n.

The probability density functions have the following properties for $n = 0, 1, 2, 3, \ldots$

$$\int_{-\infty}^{\infty} p_G^M(\xi_n)d\xi_n = 1 \tag{51}$$

$$\left\langle \xi_n \right\rangle = \int_{-\infty}^{\infty} \xi_n p_G^M(\xi_n)d\xi_n = \xi_n^M \tag{52}$$

$$\left\langle \xi_n^2 \right\rangle = \int_{-\infty}^{\infty} \xi_n^2 p_G^M(\xi_n)d\xi_n = \sigma_n^2 \tag{53}$$

and for the zero mean probability density functions

$$\int_{-\infty}^{\infty} p_G(\psi_n)d\psi_n = 1 \tag{54}$$

$$\left\langle \psi_n \right\rangle = \int_{-\infty}^{\infty} \psi_n p_G(\psi_n) d\psi_n = 0 \tag{55}$$

$$\left\langle \psi_n^2 \right\rangle = \int_{-\infty}^{\infty} \psi_n^2 p_G(\psi_n) d\psi_n = \sigma_n^2 \tag{56}$$

Typical shapes of the $p_G^M(\xi_n)$ and $p_G(\psi_n)$ functions are given in Figure 5a.

A futher descriptor of the zero mean Gaussian distribution is the average of positive values only of the stochastic variables

$$\left\langle \psi_n \right\rangle^+ = \int_{0}^{\infty} \psi_n p_G(\psi_n) d\psi_n = \frac{\sigma_n}{\sqrt{2\pi}} \tag{55a}$$

Therefore $\sigma_n/\sqrt{2\pi}$ is a measure of the average amplitude of the variation of the stochastic variables about its mean value of zero.

If joint probabilities are required it should be remembered that the derivatives ξ_n are not completely arbitrary in the sense that σ_0 , σ_1 , σ_2 , σ_3 , etc. are not completely independent.[1,2] Therefore the joint distribution is not simply the product of two or more proability density functions, but requires the introduction of correlation coefficients between the various derivatives.[11,13] This has not been studied in this paper.

Probability Distribution Function for a Point in the Battlefield. The probability distribution function defined for the zero mean Gaussian distribution is

$$\begin{aligned} P_G(\beta_n) &= \frac{1}{\sqrt{2\pi}\sigma_n} \int_{-\infty}^{\beta_n} e^{-\psi_n^2/\left(2\sigma_n^2\right)} d\psi_n \\ &= \int_{-\infty}^{\beta_n} p_G(\psi_n) d\psi_n \end{aligned} \tag{57}$$

where β_n = a specified value of ψ_n for n = 0, 1, 2, 3, ... This function determines the probability that $\psi_n \leq \beta_n$, i.e.,[11,13]

$$\text{Prob } (\psi_n \leq \beta_n) = P_G(\beta_n) \tag{58}$$

The probability that $\psi_n > \beta_n$ is then given by

$$\text{Prob}\ (\psi_n > \beta_n) = 1 - P_G(\beta_n) \tag{59}$$

The probability distribution functions for Gaussian distributions are often written as

$$P_G(\beta_n) = \text{erf}\left(\frac{\beta_n}{\sigma_n}\right) + \frac{1}{2} \tag{60}$$

where the error function erf(x) is defined as[13]

$$\text{erf}(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-z^2/2} dz \tag{61}$$

A graph of the function erf(x) is shown in Figure 5b.

Probability of Cover for a Vehicle at a Point in the Battlefied. The zero mean Gaussian probability distribution representing the mesoterrain elevation variations can be used to estimate the probability of finding cover for a vehicle at some point in a battlefield. The condition for complete cover is assumed to be that the vehicle sits in a depression whose depth is equal or greater than the height of the vehicle $\psi_o \leq - H_v$, where H_v = vehicle height (see Figure 5c). The probability for finding cover is

$$\begin{aligned}\text{Prob}\ (\psi_o \leq - H_v) &= P_G(-H_v)\\ &= \frac{1}{2} + \text{erf}(-H_v/\sigma_o)\\ &= \frac{1}{2} - \text{erf}(H_v/\sigma_o)\end{aligned} \tag{62}$$

A typical graph of the function $P_G(-H_v)$ is given in Figure 5d. The relevant mesoroughness parameter is the standard deviation of the mesoterrain elevation.

Slope Probability at a Point in the Battlefield. For slope probabilities the situation is somewhat different because both positive and negative values of these parameters must be considered. The probability distribution function for slope is expressed in terms of a critical slope ψ_1^C such that mobility is possible for $|\psi_1| < \psi_1^C$. The probability for slope ψ_1 being outside this range is

$$\mathrm{Prob}\left(\psi_1 < -\psi_1^c \text{ or } \psi_1 > \psi_1^c\right) = \frac{1}{\sqrt{2\pi}\sigma_1}\int_{-\infty}^{-\psi_1^c} e^{-\psi_1^2/\left(2\sigma_1^2\right)} d\psi_1$$

$$+ \frac{1}{\sqrt{2\pi}\sigma_1}\int_{\psi_1^c}^{\infty} e^{-\psi_1^2/\left(2\sigma_1^2\right)} d\psi_1$$

$$= \frac{2}{\sqrt{2\pi}\sigma_1}\int_{-\infty}^{-\psi_1^c} e^{-\psi_1^2/\left(2\sigma_1^2\right)} d\psi_1 \tag{63}$$

$$= 2P_G\left(-\psi_1^c\right) = 1 - 2\,\mathrm{erf}\left(\psi_1^c/\sigma_1\right)$$

Then the probability for $|\psi_1| < \psi_1^c$ is

$$\mathrm{Prob}\left(|\psi_1| < \psi_1^c\right) = 1 - 2\,P_G\left(-\psi_1^c\right) = 2\,\mathrm{erf}\left(\psi_1^c/\sigma_1\right) \tag{64}$$

and this can be used as a mobility index. This function appears in Figure 5e. A similar analysis can be done for the curvature and higher derivatives. Wheeled vehicles have $\psi_1^c = 0.3$ and tracked vehicles have $\psi_1^c = 0.45$.

The situation of a moving vehicle seeking cover in a specified distance and of encountering a specified range of slopes in this interval can also be quantified by the theory of probability. The probabilities of encountering specified values of mesoterrain elevation and slope in a given interval can be expressed in terms of the autocovariance functions defined in Part II and in terms of the standard deviations of the elevations and its derivatives.

Probability Density Functions in an Interval of Travel. The probability density functions for encountering specified values of the zero mean Gaussian random processes ψ_n , given by $\psi_n = \beta_n$, in a small interval α are given by[13]

$$p_n(\alpha) = \frac{1}{\pi}\sqrt{\frac{2\left[C_n(0) - C_n(\alpha)\right]}{C_n(0)}}\;\frac{e^{-1/2(\beta_n/\sigma_n)^2}}{\sqrt{2\pi}\sigma_n} \tag{65}$$

where $n = 0, 1, 2, 3, \ldots$ refer to the mesoterrain elevation and its derivatives. These probability density functions can be rewritten in terms of the correlation coefficients defined in Equation 16 as follows[13]

$$p_n(\alpha) = \frac{1}{\pi}\sqrt{2\left[1 - r_n(\alpha)\right]}\ \frac{e^{-1/2(\beta_n/\sigma_n)^2}}{\sqrt{2\pi}\sigma_n}$$

$$= \frac{1}{\pi}\sqrt{2\left[1 - r_n(\alpha)\right]}\ p_G(\beta_n,\sigma_n) \tag{66}$$

The probability density functions can be rewritten in terms of the standard deviations using the power series expansions in Equations 25 and 27 as follows

$$p_n(\alpha) = \frac{\alpha}{\pi} M_n \frac{e^{-1/2(\beta_n/\sigma_n)^2}}{\sqrt{2\pi}\sigma_n} = \frac{\alpha}{\pi} M_n p_G(\beta_n,\sigma_n) \tag{67}$$

where

$$M_n = \sqrt{\frac{2}{2!}\left(\frac{\sigma_{n+1}}{\sigma_n}\right)^2 - \frac{2}{4!}\left(\frac{\sigma_{n+2}}{\sigma_n}\right)^2 \alpha^2 + \frac{2}{6!}\left(\frac{\sigma_{n+3}}{\sigma_n}\right)^2 \alpha^4 - \ldots}$$

$$= \sqrt{2\sum_{j=1}^{\infty} (-1)^{j+1} \frac{S_{nj}}{(2j)!}\alpha^{2(j-1)}} \tag{68}$$

where S_{nj} is defined in Equation 31.

The probability density functions can also be written in terms of the dimensionless wavelength parameters T_n given in Equation 35 as follows

$$p_n(\alpha) = \frac{T_n\Phi_n}{\pi} \frac{e^{-1/2(\beta_n/\sigma_n)^2}}{\sqrt{2\pi}\sigma_n} \tag{69}$$

where

$$\Phi_n = \frac{\alpha}{T_n} M_n = \frac{\lambda_n M_n}{2\pi} = \frac{\sqrt{2(1 - r_n)}}{T_n}$$

$$= \sqrt{1 - \frac{2}{4!} T_{n+1}^2 + \frac{2}{6!} (T_{n+1}T_{n+2})^2 - \frac{2}{8!} (T_{n+1}T_{n+2}T_{n+3})^2 + \cdots}$$

$$= \sqrt{1 + \sum_{j=2}^{\infty} (-1)^{j+1} \frac{2}{(2j)!} (T_{n+1}T_{n+2}T_{n+3} \cdots T_{n+j-1})^2} \tag{70}$$

$$= \sqrt{1 + \sum_{j=2}^{\infty} (-1)^{j+1} \frac{2}{(2j)!} \left(\frac{\lambda_n}{2\pi}\right)^2 S_{nj}\alpha^{2(j-1)}}$$

For reference, the first few Φ_n are

$$\Phi_0 = \sqrt{1 - \frac{2}{4!} T_1^2 + \frac{2}{6!} (T_1T_2)^2 - \frac{2}{8!} (T_1T_2T_3)^2 + \cdots} \tag{71}$$

$$\Phi_1 = \sqrt{1 - \frac{2}{4!} T_2^2 + \frac{2}{6!} (T_2T_3)^2 - \frac{2}{8!} (T_2T_3T_4)^2 + \cdots} \tag{72}$$

$$\Phi_2 = \sqrt{1 - \frac{2}{4!} T_3^2 + \frac{2}{6!} (T_3T_4)^2 - \frac{2}{8!} (T_3T_4T_5)^2 + \cdots} \tag{73}$$

For small values of α the values of Φ_n are nearly unity.

The probability density functions (Equation 69) for an interval in the mesoterrain can be written in terms of the Gaussian probability density function as follows

$$p_n(\alpha) = \frac{\alpha\sigma_{n+1}}{\pi\sigma_n} \Phi_n p_G(\beta_n, \sigma_n) = \frac{2\alpha}{\lambda_n} \Phi_n p_G(\beta_n, \sigma_n) \tag{74}$$

Consider now the special cases of mesoterrain elevation, slope, and curvature

$$p_0(\alpha) = \frac{\alpha\sigma_1}{\pi\sigma_0} \Phi_0 p_G(\beta_0, \sigma_0) = \frac{2\alpha}{\lambda_0} \Phi_0 p_G(\beta_0, \sigma_0) \tag{75}$$

$$p_1(\alpha) = \frac{\alpha\sigma_2}{\pi\sigma_1} \Phi_1 p_G(\beta_1, \sigma_1) = \frac{2\alpha}{\lambda_1} \Phi_1 p_G(\beta_1, \sigma_1) \tag{76}$$

$$p_2(\alpha) = \frac{\alpha\sigma_3}{\pi\sigma_2} \Phi_2 p_G(\beta_2,\sigma_2) = \frac{2\alpha}{\lambda_2} \Phi_2 p_G(\beta_2,\sigma_2) \tag{77}$$

A result similar to Equation 75, but involving the power spectrum, has already appeared in the literature.[14] From Equation 75, it is clear that in addition to the elevation Gaussian probability density function $p_G(\beta_o,\sigma_o)$, a spatial frequency term σ_1/σ_o appears in the expression for the mesoterrain elevation probability density function. Therefore, to first order the standard deviations σ_o and σ_1 , determine the elevation probability density function. The standard deviations of the second and higher order derivatives enter to a lesser degree through the function Φ_o . Likewise Equation 76 shows that the probability density function for encountering a specified slope in a given interval is proportional to σ_2/σ_1 in addition to the slope Gaussian probability density function.

Probability Distribution Function in an Interval of Travel. The probability distribution functions associated with the probability density functions given in Equation 74 are

$$\begin{aligned} P(\beta_n) &= \int_{-\infty}^{\beta_n} p_n d\psi_n \\ &= \frac{\alpha\sigma_{n+1}}{\pi\sigma_n} \Phi_n \int_{-\infty}^{\beta_n} p_G(\psi_n,\sigma_n)d\psi_n \\ &= \frac{\alpha\sigma_{n+1}\Phi_n}{\pi\sigma_n} P_G(\beta_n) = \frac{2\alpha}{\lambda_n} \Phi_n P_G(\beta_n) \end{aligned} \tag{78}$$

where $P_G(\beta_n)$ is related to the error function as in Equation 60.

Cover Available in a Specified Interval of Travel. The probability of finding cover for a vehicle of high H_v in a specified interval of the mesoroughness is calculated in terms of the probability distribution function (Equation 78) as follows

$$\begin{aligned} \text{Prob}(\psi_o < -H_v) &= P(-H_v) \\ &= \frac{\alpha\sigma_1\Phi_o}{\pi\sigma_o} P_G(-H_v) = \frac{2\alpha}{\lambda_o} \Phi_o P_G(-H_v) \\ &= \frac{\alpha\sigma_1\Phi_o}{\pi\sigma_o} \left[\frac{1}{2} - \text{erf}(H_v/\sigma_o)\right] \end{aligned} \tag{79}$$

The probability for finding cover in a specified mesoterrain interval depends on σ_1 as well as σ_o through the ratio $\sigma_1/\sigma_o \sim 1/\lambda_o$. In other words Equation 79 shows that for the same values of σ_o the mesoterrain containing longer elevation wavelengths affords less cover in a specified distance. The standard deviations of the second and higher elevation derivatives also influence the degree of cover through the functions Φ_o defined in Equation 71.

Degree of Mobility Possible in a Specified Interval of Travel. The problems of determining a mobility probability index for mapping purposes amounts to calculating the probability that $|\psi_1| < \psi_1^c$ in a specified interval, where ψ_1^c = critical slope beyond which a vehicle cannot go. This is calculated as follows

$$\begin{aligned} \mathrm{Prob}\left(\psi_1 < -\psi_1^c \ \text{ or } \ \psi_1 > \psi_1^c\right) &= \int_{-\infty}^{-\psi_1^c} p_1 d\psi_1 + \int_{\psi_1^c}^{\infty} p_1 d\psi_1 \\ &= \frac{2\alpha\sigma_2\Phi_1}{\pi\sigma_1} P_G\left(-\psi_1^c\right) \\ &= \frac{4\alpha}{\lambda_1} \Phi_1 P_G\left(-\psi_1^c\right) \\ &= \frac{2\alpha\sigma_2\Phi_1}{\pi\sigma_1}\left[\frac{1}{2} - \mathrm{erf}\left(\psi_1^c/\sigma_1\right)\right] \end{aligned} \tag{80}$$

Then the probability of finding slopes less than the critical slope is given by

$$\begin{aligned} \mathrm{Prob}\left(|\psi_1| < \psi_1^c\right) &= \frac{2\alpha\sigma_2\Phi_1}{\pi\sigma_1} \mathrm{erf}\left(\psi_1^c/\sigma_1\right) = \frac{4\alpha}{\lambda_1} \Phi_1 \ \mathrm{erf}\left(\psi_1^c/\sigma_1\right) \\ &= \frac{2\alpha\Phi_1}{\lambda_1}\left[1 - 2P_G\left(-\psi_1^c\right)\right] \end{aligned} \tag{81}$$

The probability of encountering a critical slope in a mesoterrain interval depends on σ_2 as well as on σ_1 through the characteristic slope wavelength $\sigma_2/\sigma_1 \sim 1/\lambda_1$. The standard deviations of the third and higher derivatives also affect the calculation, but to a smaller degree, through the function Φ_1 defined in Equation 72.

Average Distance Between Covered Positions and Between Points of Critical Slope. The average distance that a vehicle would have to travel between two covered positions and between two adjacent points where the critical slope value occurs can be estimated from the values of the characteristic mesoterrain elevation wavelength λ_o and the characteristic mesoterrain slope wavelength λ_1 that are given in Equation 32. The simplest assumptions give the following results

$$\alpha_o \sim \lambda_o \frac{H_v}{\sigma_o} = 2\pi \frac{H_v}{\sigma_1} \tag{82}$$

$$\alpha_1 \sim \lambda_1 \frac{\psi_1^c}{\sigma_1} = 2\pi \frac{\psi_1^c}{\sigma_2} \tag{83}$$

where

α_o = average distance between two adjacent covered positions for a vehicle of height H_v

α_1 = average distance between two slope mobility failures for a vehicle whose critical slope is ψ_1^c .

Numerical Analysis of Terrain Roughness Probabilities. Macroterrain elevation data were available for only two terrain sites - Freiensteinau and Wetzlar both located in West Germany. A 1-mile section of macroterrain elevation data was selected from each site. Smaller sections of 100-m length within the one mile sections, the mesoterrain elevation profiles, were analyzed and the results compared the descriptors of the full 1-mile section.

The standard deviation of the detrended elevation can be used as a measure of the relative roughness of the Freiensteinau and Wetzlar sites. These standard deviations appear in Figure 6a in terms of the reciprocal of the detrending parameter. In terms of this descriptor the Freiensteinau site is about four times more rough than the Wetzlar site. Nevertheless, as seen in Figures 6b through 6d, the characteristic wavelengths λ_o and λ_1 are roughly the same for the two sites.

The power spectra of the terrain displacement were calculated using Equation 45 and the techniques developed in References 1 and 2. The results appear in Figures 7a and 7b from which it is clear that several different types of power spectra can occur for the 100-m sections within a 1-mile terrain elevation profile. Therefore the power spectrum type will vary along a terrain section, and the mesoterrain roughness power spectra is expected to be different from the macroterrain roughness power spectra.

Figure 8a shows the Gaussian cumulative probability for terrain elevations as calculated from Equations 60 and 62. Figure 8b shows the results of using Equation 62 to calculate the Gaussian cumulative probability for finding cover for a vehicle of height H_v = 8 ft in terms of the detrending parameter λ . The detrending parameter enters the probability calculation through the function $\sigma_o(\lambda)$ given in Figure 6a. Equation 79 is used to calculate the cumulative probability for finding cover in a 1-ft unit interval, and the results are shown in Figure 8c. A peak occurs in the Freiensteinau probability curve in Figure 8c because the probability varies inversely with λ_o as in Equation 79, and λ_o is a rapidly decreasing function of λ^{-1} as given in Figure 6b.

Figures 8d through 8f give the probability distribution functions for encountering critical slopes using Equations 63, 80, and 81. The critical slopes were taken to be 0.3 for wheeled vehicles and 0.45 for track-laying vehicles. The standard deviation of the slopes at the Wetzlar and Freinsteinau terrain sites are about an order of magnitude smaller than the critical slopes so that the probability of encountering these critical slope values is vanishingly small. No terrain data is available for a site which would exhibit high probabilities for encountering the critical slopes. In order to obtain some numerical results the elevations at the Freiensteinau and Wetzlar sites were arbitrarily magnified by a factor of 10. Figures 8d through 8f give the critical slope probability distribution functions for this artificial situation. However since relative probabilities are of interest, it is clear that Freiensteinau has the higher probability for encountering a critical slope value.

Mobility and Cover Map Overlays. The cover and mobility probabilities can be used to construct military map overlays on which areas having distinct mobility or cover characteristics are isolated to form a patchwork. Each patch of a map overlay for a specified vehicle is associated with a probability index of cover or mobility as calculated from the standard deviations of the elevation and its derivatives. Several elevation profiles are measured for each area to determine a descriptive set of values for σ_o , σ_1 , σ_2 , and so on. In this way map overlays can be produced in a logical fashion from some elevation profiles measured in each area.

4. CONCLUSIONS. This paper develops a formalism for developing cover and mobility map overlays. The formalism is based on a rigorous application of probability theory to the description of random terrain elevation data. Terrain descriptors obtained from elevation profiles measured in a battlefield area are used to develop cover and mobility indices for the purpose of determining patches on a map overlay having distinct cover and mobility characteristics.

The studies of mesoterrain roughness gave the following conclusions:

a. The mesoterrain roughness can be extracted from a measured elevation profile by a detrending procedure with a proper choice of filter constant (Part II).

b. The basic set of terrain descriptors required for the complete specification of mesoroughness are the standard deviations of the mesoterrain elevation and its derivatives (Part II).

c. The probabilities for encountering specified ranges of mesoterrain elevation or slope in a given distance can be determined from the standard deviations of mesoterrain elevation and its derivatives, and these probabilities can be used as indices for determining patches on a map that have distinct cover and mobility characteristics (Part III).

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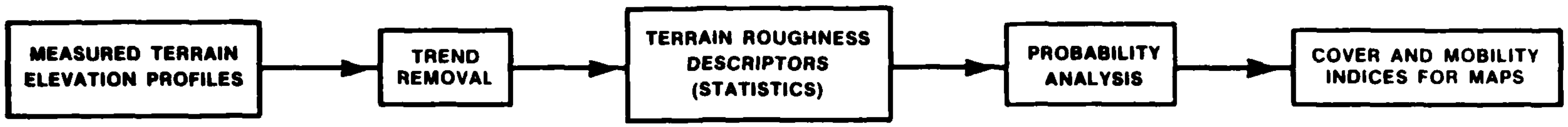
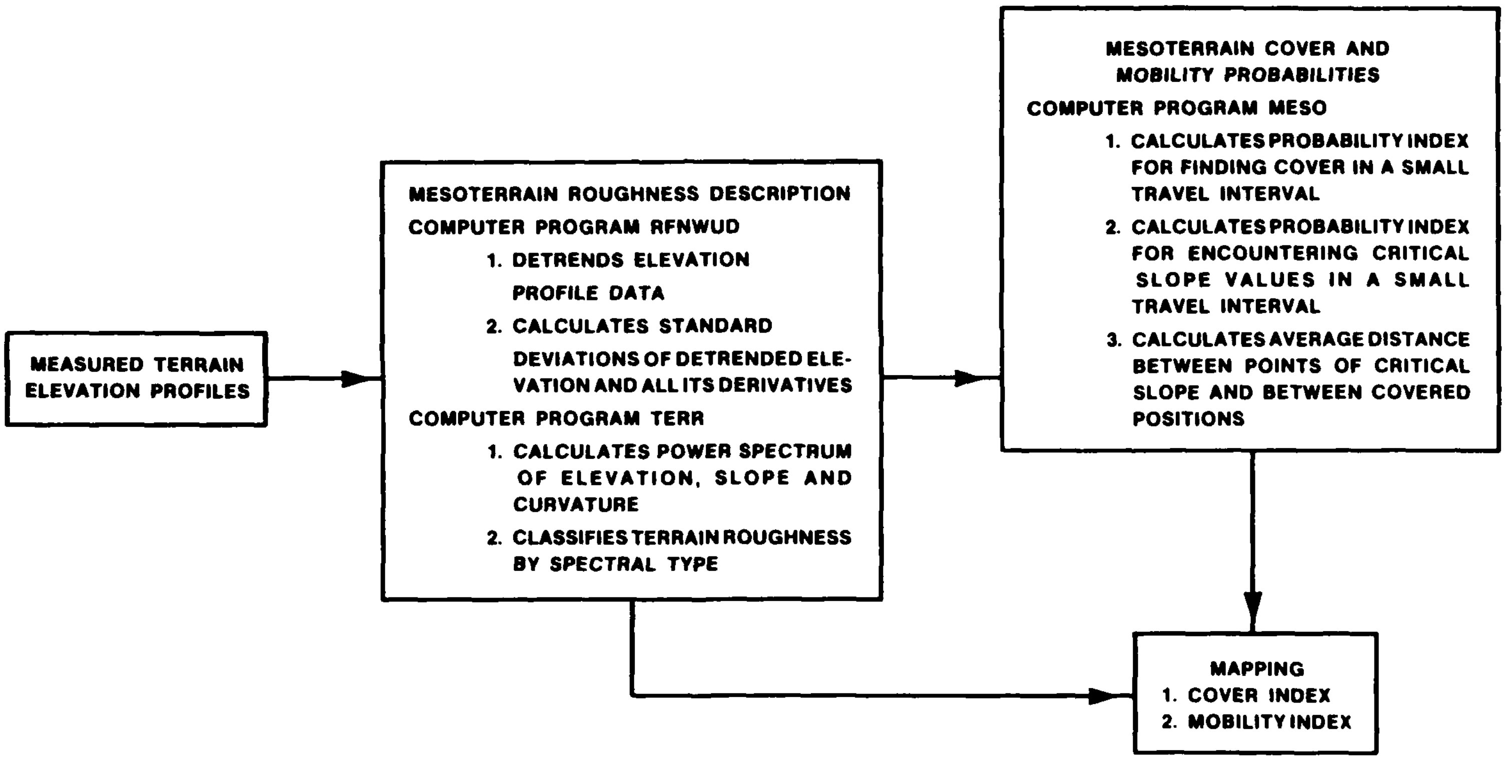

MEASURED TERRAIN ELEVATION PROFILES
TREND REMOVAL
TERRAIN ROUGHNESS DESCRIPTORS (STATISTICS)
PROBABILITY ANALYSIS
COVER AND MOBILITY INDICES FOR MAPS

a.


MEASURED TERRAIN ELEVATION PROFILES
MESOTERRAIN ROUGHNESS DESCRIPTION
COMPUTER PROGRAM RFNWUD
1. DETRENDS ELEVATION PROFILE DATA
2. CALCULATES STANDARD DEVIATIONS OF DETRENDED ELEVATION AND ALL ITS DERIVATIVES
COMPUTER PROGRAM TERR
1. CALCULATES POWER SPECTRUM OF ELEVATION, SLOPE AND CURVATURE
2. CLASSIFIES TERRAIN ROUGHNESS BY SPECTRAL TYPE
MESOTERRAIN COVER AND MOBILITY PROBABILITIES
COMPUTER PROGRAM MESO
1. CALCULATES PROBABILITY INDEX FOR FINDING COVER IN A SMALL TRAVEL INTERVAL
2. CALCULATES PROBABILITY INDEX FOR ENCOUNTERING CRITICAL SLOPE VALUES IN A SMALL TRAVEL INTERVAL
3. CALCULATES AVERAGE DISTANCE BETWEEN POINTS OF CRITICAL SLOPE AND BETWEEN COVERED POSITIONS
MAPPING
1. COVER INDEX
2. MOBILITY INDEX

b.

Figure 1

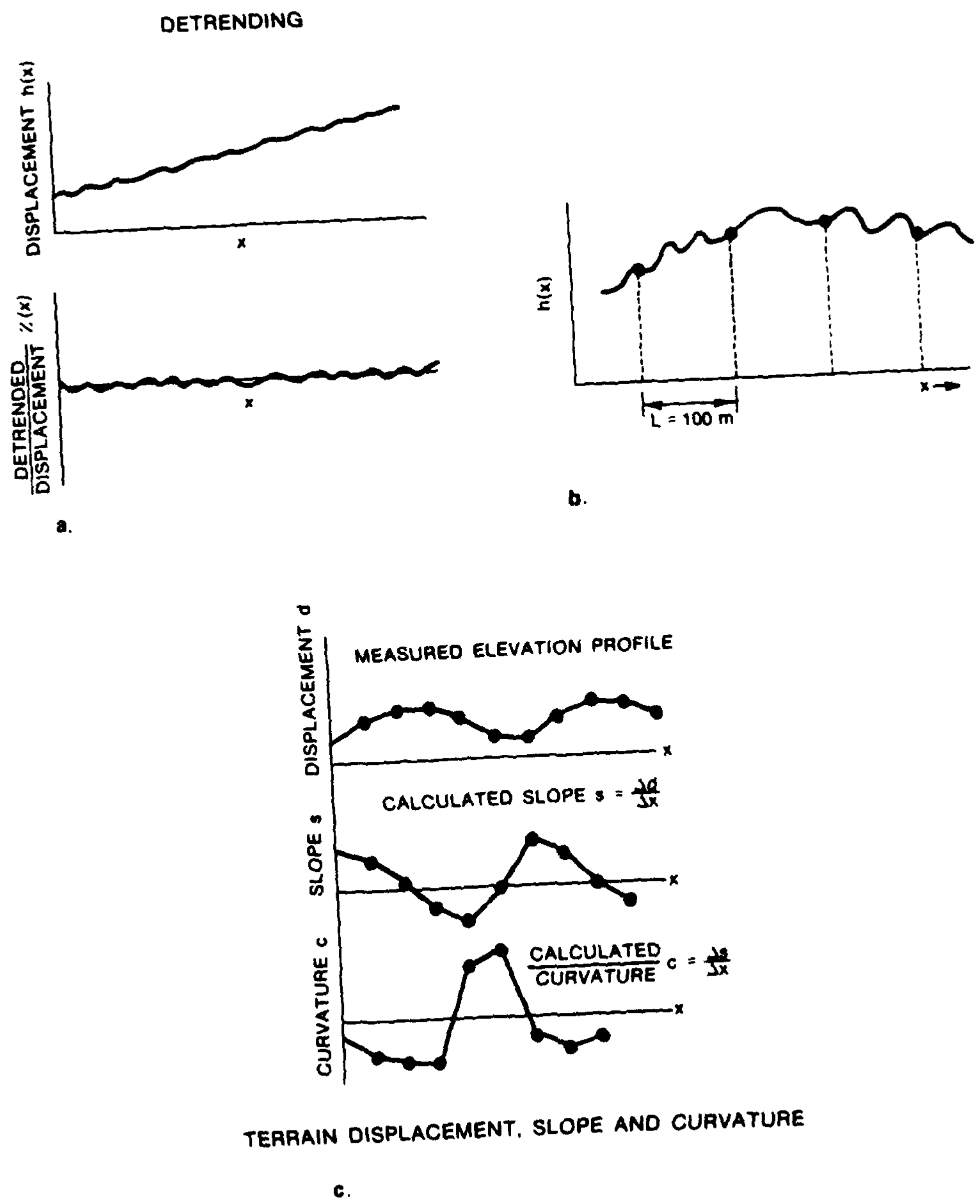

DETRENDING
DISPLACEMENT h(x)
x
DETRENDED DISPLACEMENT
x
a.
h(x)
L = 100 m
x
b.
DISPLACEMENT d
MEASURED ELEVATION PROFILE
x
CALCULATED SLOPE s = Δd/Δx
SLOPE s
x
CALCULATED CURVATURE c = Δs/Δx
CURVATURE c
x
TERRAIN DISPLACEMENT, SLOPE AND CURVATURE
c.

Figure 2

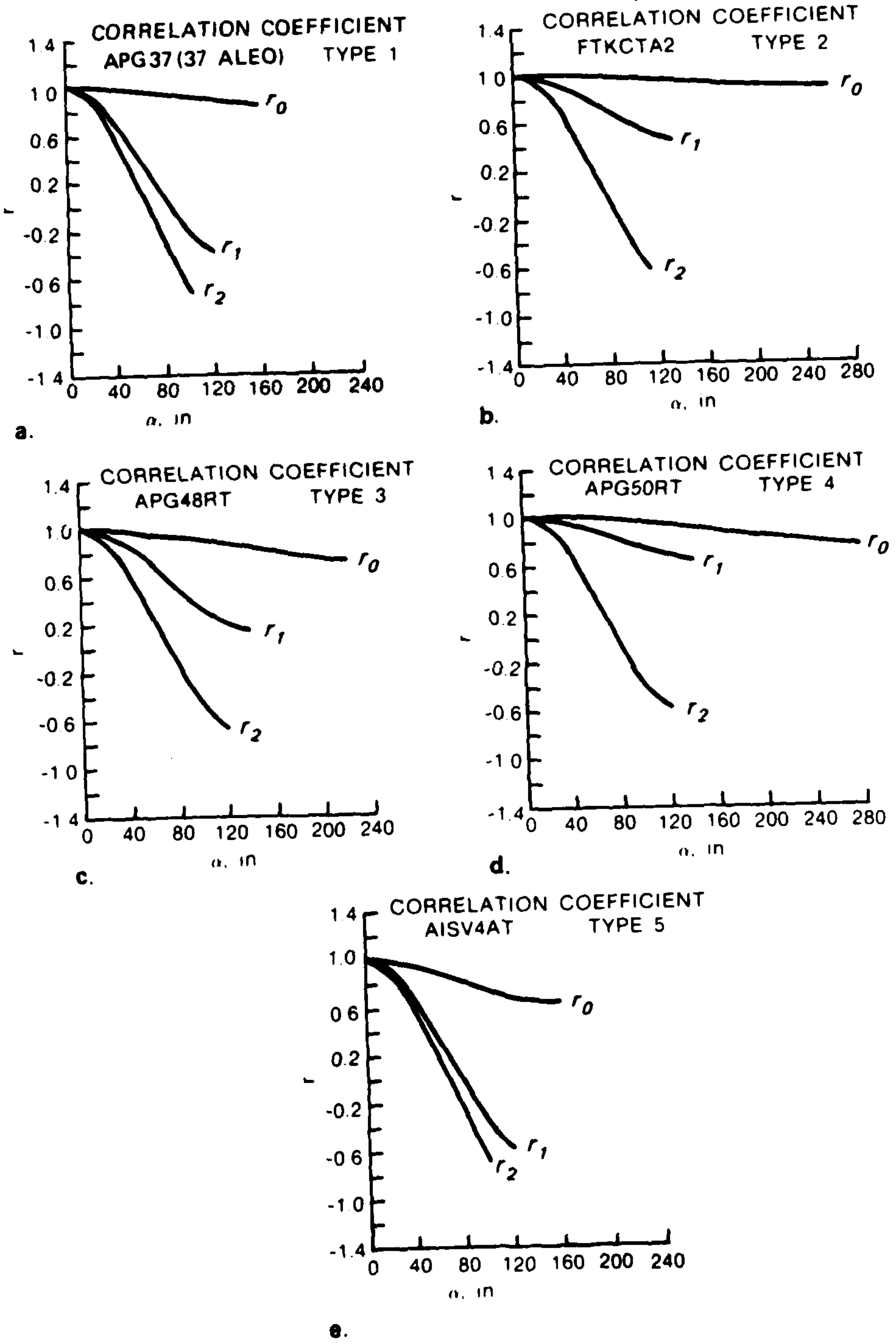

CORRELATION COEFFICIENT
APG37 (37 ALEO) TYPE 1
r
α, in
a.
CORRELATION COEFFICIENT
FTKCTA2 TYPE 2
b.
CORRELATION COEFFICIENT
APG48RT TYPE 3
c.
CORRELATION COEFFICIENT
APG50RT TYPE 4
d.
CORRELATION COEFFICIENT
AISV4AT TYPE 5
e.

Figure 3

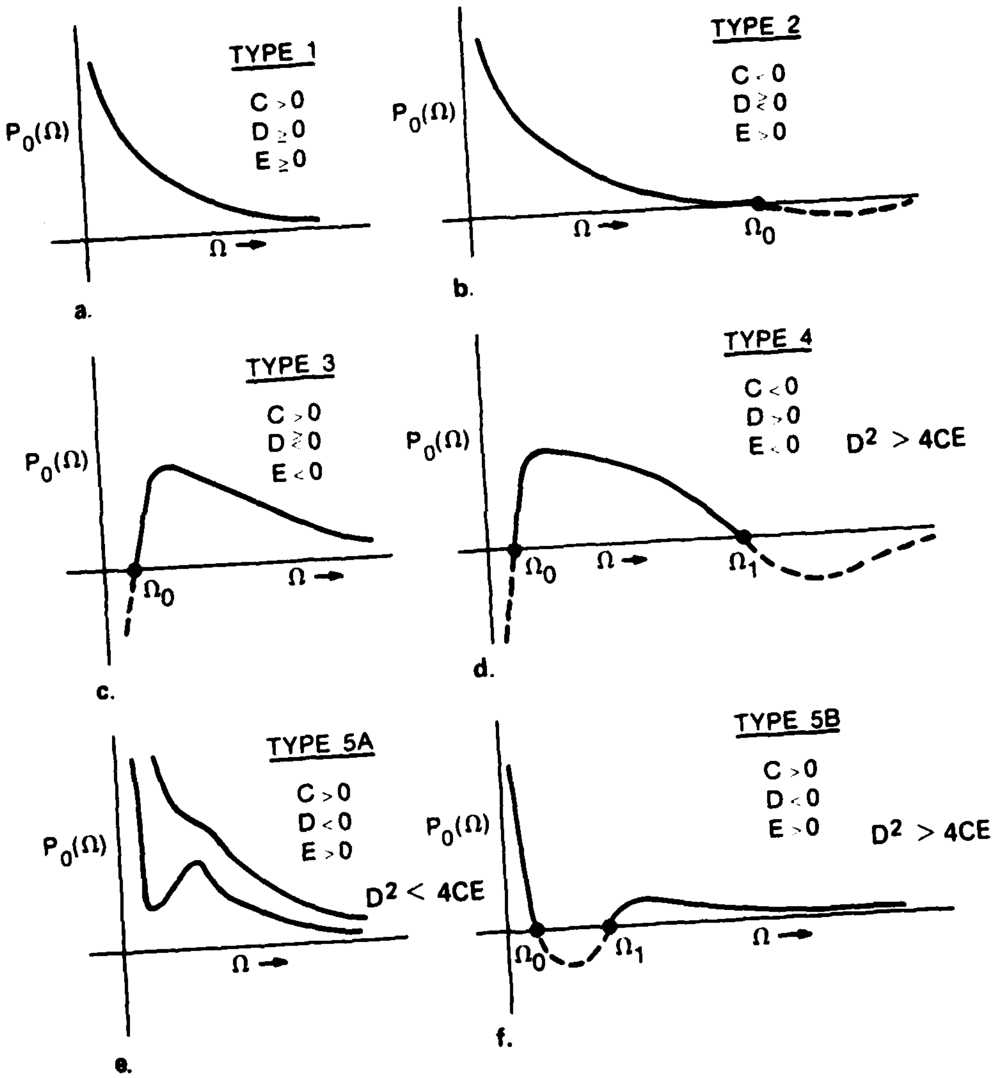

TYPE 1
C > 0
D ≥ 0
E ≥ 0
$P_0(\Omega)$
Ω →
a.
TYPE 2
C < 0
D ≷ 0
E > 0
$P_0(\Omega)$
Ω →
Ω_0
b.
TYPE 3
C > 0
D ≷ 0
E < 0
$P_0(\Omega)$
Ω_0
Ω →
c.
TYPE 4
C < 0
D > 0
E < 0
$D^2 > 4CE$
$P_0(\Omega)$
Ω_0
Ω →
Ω_1
d.
TYPE 5A
C > 0
D < 0
E > 0
$D^2 < 4CE$
$P_0(\Omega)$
Ω →
e.
TYPE 5B
C > 0
D < 0
E > 0
$D^2 > 4CE$
$P_0(\Omega)$
Ω_0
Ω_1
Ω →
f.

Figure 4

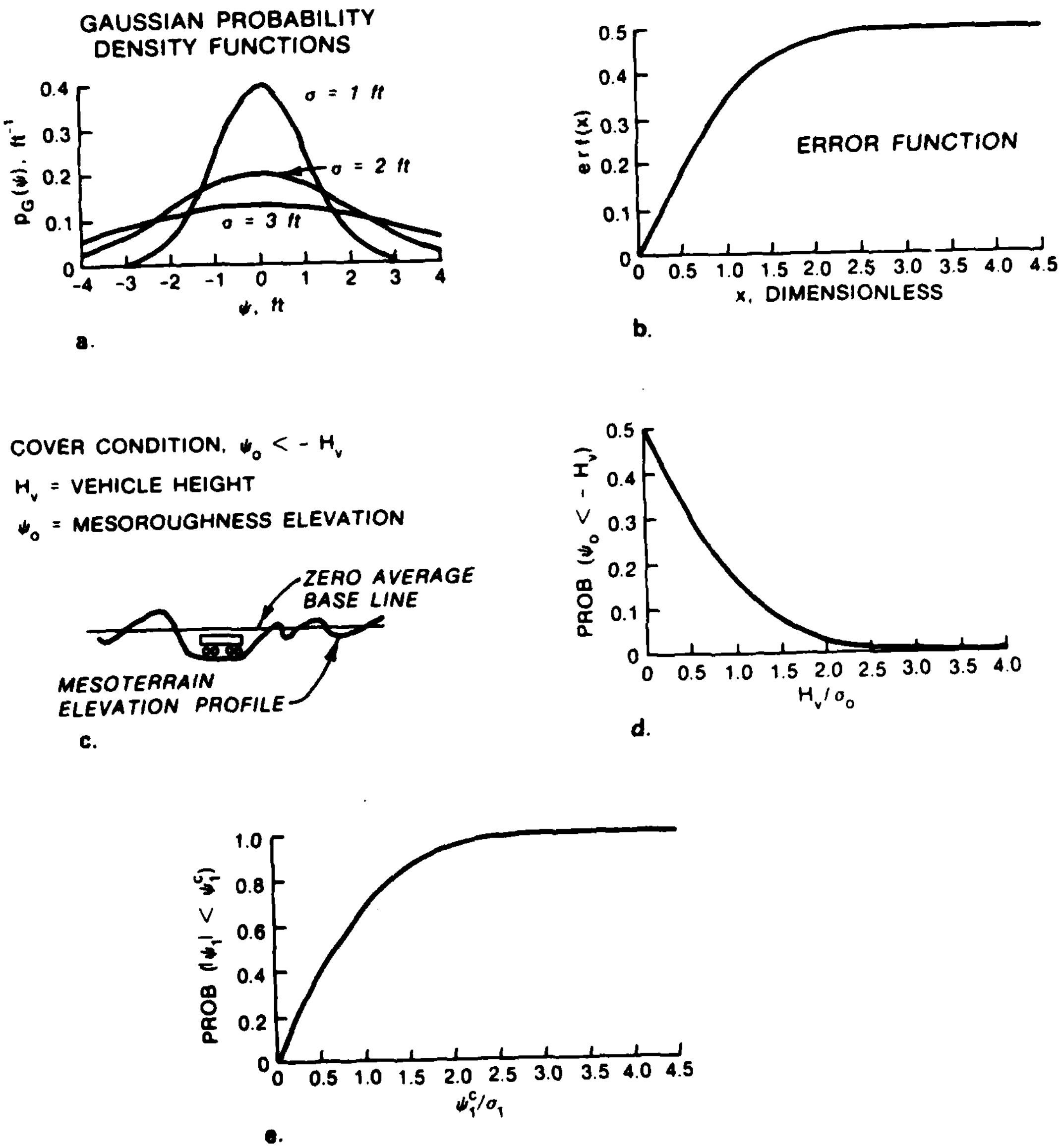
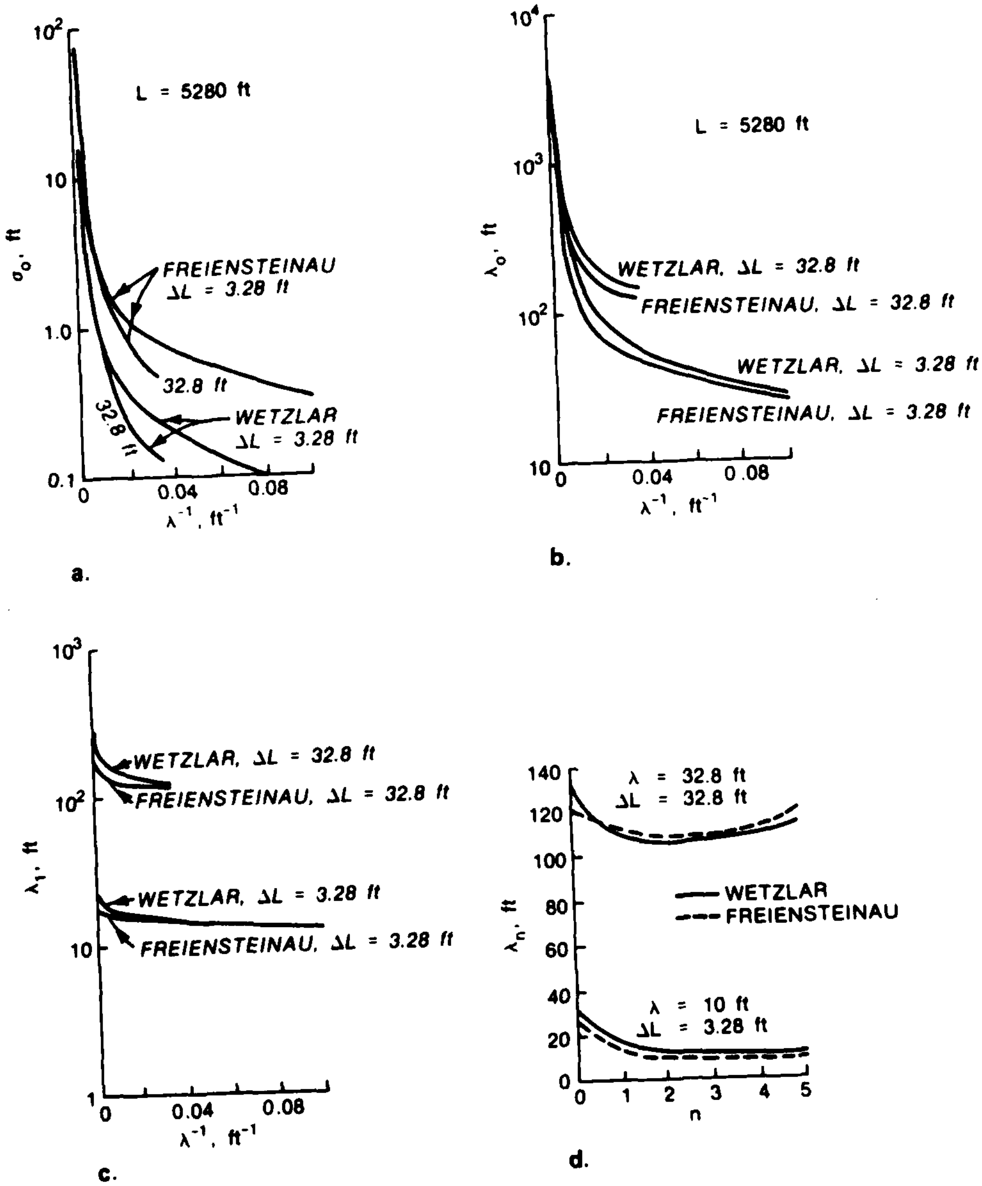

GAUSSIAN PROBABILITY DENSITY FUNCTIONS
σ = 1 ft
σ = 2 ft
σ = 3 ft
p_G(ψ), ft^-1
ψ, ft
a.
ERROR FUNCTION
erf(x)
x, DIMENSIONLESS
b.
COVER CONDITION, ψ_o < - H_v
H_v = VEHICLE HEIGHT
ψ_o = MESOROUGHNESS ELEVATION
ZERO AVERAGE BASE LINE
MESOTERRAIN ELEVATION PROFILE
c.
PROB (ψ_o < - H_v)
H_v / σ_o
d.
PROB (|ψ_1| < ψ_1^c)
ψ_1^c/σ_1
e.

Figure 5



L = 5280 ft
FREIENSTEINAU
ΔL = 3.28 ft
32.8 ft
32.8 ft
WETZLAR
ΔL = 3.28 ft
σ_o, ft
λ^{-1}, ft^{-1}
a.
L = 5280 ft
WETZLAR, ΔL = 32.8 ft
FREIENSTEINAU, ΔL = 32.8 ft
WETZLAR, ΔL = 3.28 ft
FREIENSTEINAU, ΔL = 3.28 ft
λ_o, ft
λ^{-1}, ft^{-1}
b.
WETZLAR, ΔL = 32.8 ft
FREIENSTEINAU, ΔL = 32.8 ft
WETZLAR, ΔL = 3.28 ft
FREIENSTEINAU, ΔL = 3.28 ft
λ_1, ft
λ^{-1}, ft^{-1}
c.
λ = 32.8 ft
ΔL = 32.8 ft
WETZLAR
FREIENSTEINAU
λ = 10 ft
ΔL = 3.28 ft
λ_n, ft
n
d.

Figure 6

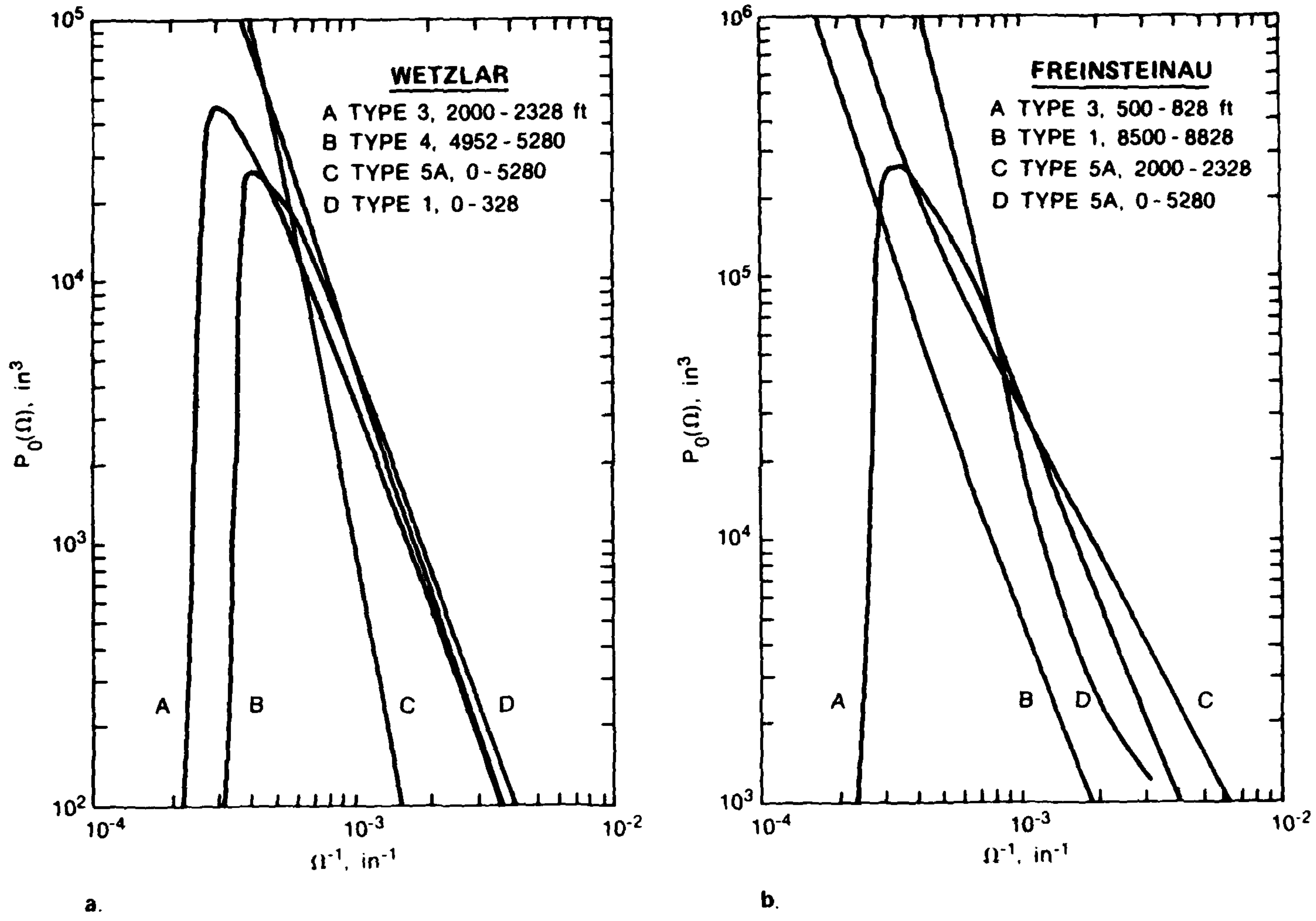
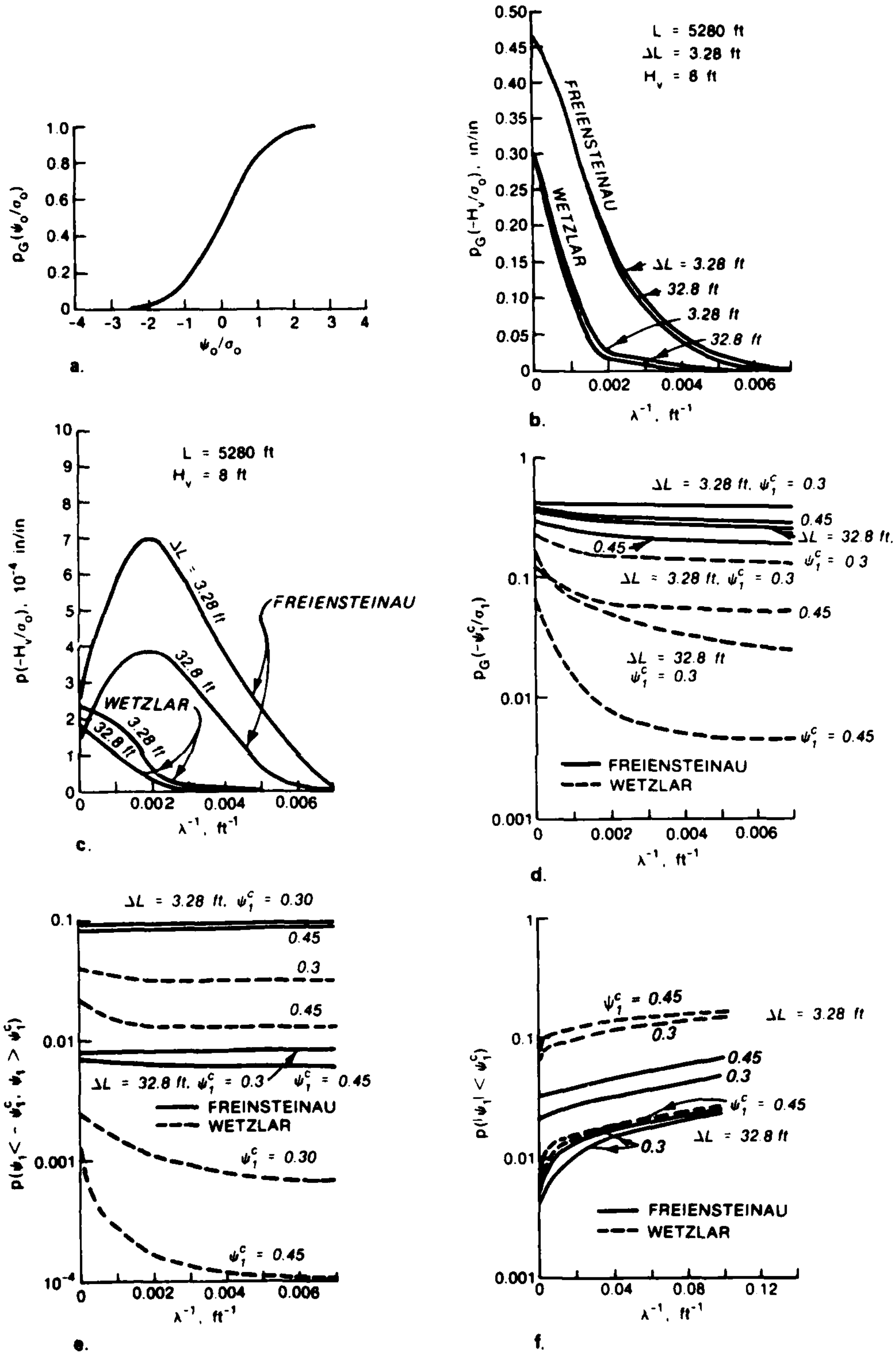

WETZLAR
A TYPE 3, 2000 - 2328 ft
B TYPE 4, 4952 - 5280
C TYPE 5A, 0 - 5280
D TYPE 1, 0 - 328
$P_0(\Omega)$, in^3
Ω^{-1}, in^{-1}
a.
FREINSTEINAU
A TYPE 3, 500 - 828 ft
B TYPE 1, 8500 - 8828
C TYPE 5A, 2000 - 2328
D TYPE 5A, 0 - 5280
$P_0(\Omega)$, in^3
Ω^{-1}, in^{-1}
b.

Figure 7



a.
b.
L = 5280 ft
ΔL = 3.28 ft
H_v = 8 ft
FREIENSTEINAU
WETZLAR
c.
L = 5280 ft
H_v = 8 ft
d.
FREIENSTEINAU
WETZLAR
e.
FREINSTEINAU
WETZLAR
f.
FREIENSTEINAU
WETZLAR

Figure 8

MED

8