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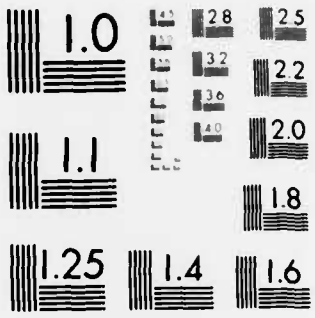

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The problem of estimating the form of a surface, given a set of points lying on that surface, is a fundamental concern in automated cartography. A good deal of effort has been spent investigating the problem by using particular terrain modeling schemes, but no general approach has previously been developed. In this paper, we examine the problem of terrain estimation from an information-theoretic viewpoint. Geometric analysis shows that there is an essentially unique criterion for the optimal choice of an estimated terrain surface given limited knowledge of that surface. We briefly review this theory, and then turn to the practical question of developing versatile and efficient software for manipulating terrain

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INFORMATION-THEORETIC SURFACE MODELING

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BIOGRAPHICAL SKETCHES

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ABSTRACT

The problem of estimating the form of a surface, given a set of points lying on that surface, is a fundamental concern in automated cartography. A good deal of effort has been spent investigating the problem by using particular terrain modeling schemes, but no general approach has previously been developed. In this paper, we examine the problem of terrain estimation from an information-theoretic viewpoint. Geometric analysis shows that there is an essentially unique criterion for the optimal choice of an estimated terrain surface given limited knowledge of that surface. We briefly review this theory, and then turn to the practical question of developing versatile and efficient software for manipulating terrain elevation data based on the information-theoretic analysis. The contour to grid estimation system CONTOGES was implemented at the U.S. Army Engineer Topographic Laboratories (USAETL) as a preliminary check of the theoretical work. We discuss the particular algorithm and the results of a test of the software system.

INTRODUCTION

The problem of estimating the form of the earth's surface from a limited knowledge of that surface is not new. With the advent of widespread computer technology over the past two decades, and the corresponding demand for high-quality digital terrain models (DTM's), the problem has become particularly acute. To date the elevation estimation

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problem has been addressed in an ad-hoc manner. For example, the decision to use certain functions with "desirable" continuity properties for interpolation is an arbitrary one, even though it seems to have an intuitive basis. Previously proposed terrain estimation schemes cannot be extended to other domains such as surface estimation in higher dimensional spaces. It is the purpose of this paper to suggest a general way of approaching the problem of surface estimation from limited data.

Before discussing the details of our work, it is worth while emphasizing the motivation for the approach which we have taken. Many techniques which work well for some terrain features fail when presented with others. Thus, a linear interpolation scheme can have nice behaviour on hill sides. but do poorly when presented with hilltops. Expansions in terms of more complicated functions, such as splines, have smooth behaviour in a mathematical sense, but at the cost of introducing spurious features. Yet we know that the problem is not insoluble in principle, since cartographers have been producing acceptable maps from limited data for centuries. The cartographic license which goes into the manual production of maps is not linear, but the underlying principle may still be mathematically well defined. The work described in this paper suggests that there is a well-defined principle, that it is essentially unique, and that, with suitable engineering assumptions, it can be used as the basis for automated generation of DTM's from sparse data of arbitrary form.

The starting point in our approach to the problem of terrain estimation from limited data is the assumption that there is some sense in which we may speak of the information content of a terrain surface. When estimating a surface from sparse data, it is appropriate to select that particular surface which is consistent with our knowledge and at the same time has the minimum information content. Any other choice assumes information which we do not have. Next we develope the functional form of the measure of information of a surface. We assume that a value on the surface may be estimated from knowledge of its neighborhood. Using only that assumption, we can use the tensor calculus to prove that the information intrinsic to a surface must be expressible in terms of the Gaussian curvature. Since there has been some discussion of surface estimation using the mean curvature*, we discuss the cartographic basis for the two approaches. In particular, we show that, since the mean curvature is not intrinsic to the surface, it cannot be calculated from angles and distances measured on the surface. To show that this philosophy is of practical value, we discuss its application to a particular problem: the generation of a gridded DTM from digitized contour data. By introducing suitable engineering assumptions, the

* I. C. Briggs, Geophysics, V. 41 (1976), p. 1377.

abstractions of information theory and tensor calculus are reduced to a set of versatile and efficient algorithms which have been implemented on the PDP 11/45 computer at USAETL. Finally, we comment on the implications of our work, both for the specific problem of contour-to-grid estimation and for the general problem of digital terrain modeling.

INFORMATION THEORY AND INTUITION

When a cartographer manually estimates a terrain surface, as in logical contouring, his estimate is based on the information (e.g., contours) in the vicinity of the area he is estimating. His estimate is smooth; no depressions or rises are introduced unless they are indicated by the available data. These elements of cartographic intuition can be mathematically formulated by assuming that there exists a measure of the information content of the surface which is local, and that valid surface estimates minimize the information content of the interpolated surface. The information function is assumed to be positive semi-definite: there are no surfaces of negative information content. Finally, since a cartographer's intuition is independent of the choice of any particular coordinate system, the information of a surface should be intrinsic to the surface. These three assumptions, locality, positivity, and coordinate independence of the measure of information are the basic assumptions of information theory.* Since the assumptions seem reasonable in a cartographic context, it is appropriate to begin the study of terrain estimation with them. We thus have the first fundamental equation,

$$H = \int dxdy\ I[x,y] \qquad (1)$$

where H is the information content of the total surface, I[x,y] is the local measure of information content, x and y are suitable coordinates defined over the two dimensional surface of interest, and the integral is taken over the entire surface.

Without additional information, this is all that we can say. Some will argue that, since the nature of the processes underlying the formation of terrain are not well understood in a mathematical sense, there is no basis to our approach. This is not the case, however, since we do have one additional fact at our disposal: in generating DTM's, we are modeling a <u>surface</u> on which we can measure distances. This is true of the earth's surface but not of an arbitrary function of two variables. (think about the distance between two points on the earth's surface (x1,y1,z1) and (x2,y2,z2), and compare that with the notion of a "distance"

* J.Shore and R. Johnson, <u>IEEE Transactions on Information Theory</u>, V.26 (1980), p. 26.

between the temperatures at the two points (x1,y1,t1) and (x2,y2,t2). Both z and t are functions of the two variables x and y, yet only the former makes sense). It is this last piece of information which allows us to make useful statements about the information content of a terrain model.

TENSOR CALCULUS AND THE GAUSSIAN CURVATURE

In the last section, we saw that the information content of a two dimensional surface could be written as the integral of a local function of the two coordinates used to map the surface. The function is intrinsic to the surface and so must be invariant under arbitrary coordinate transformations. Finally we have seen that the earth's surface is special in that a distance function is defined on it. Mathematically speaking, the earth's surface forms a two-dimensional metric space. The task of cataloging the invariant functions intrinsic to metric spaces of arbitrary dimensionality is well known. The formalism needed to discuss such issues in detail, the tensor calculus, is complex. For our purposes, we need only a few fundamental results, and therefore refer the interested reader to standard modern textbooks.*

The tensor calculus, when applied to two dimensional surfaces, yields a powerful result. All invariant functions intrinsic to two dimensional surfaces can be expressed in terms of a single invariant function: the Gaussian curvature. This result allows us to write the information at a given position as an arbitrary functional of the Gaussian curvature

$$I[x,y] = I(K[x,y]) \qquad (2)$$

Before proceeding to the application of this result to engineering problems, it is important to discuss the meaning of the Gaussian curvature in a cartographic context. Consider a particular point on the earth's surface. If we take a vertical plane passing through the point, then the curvature in the direction of the plane is defined to be the inverse of the radius of the osculating circle at that point. If we denote the maximum of such curvatures by k1, and the minimum by k2, then the mean curvature M is defined by

$$M = (k1+k2)/2 \qquad (3)$$

while the Gaussian curvature K is defined by the product

$$K = k1 \cdot k2 \qquad (4)$$

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* See, for example, S.Weinberg, Gravitation and Cosmology, (Wiley, New York, 1978).

The Gaussian curvature is named after Karl Friedrich Gauss, a mathematician who was also a surveyor. He proved that the integral of the Gaussian curvature over the surface of a triangle is equal to the sum of the interior angles minus 180 degrees. The limit of this integral as the triangle is shrunk to a point is the Gaussian curvature at that point. The Gaussian curvature is just what a surveyor would measure on the earth's surface (see Figure 1). As a corollary, it follows that the mean curvature cannot be measured by such techniques. For example, consider the triangle in Figure 2(a). It lies on a flat plane, and the sum of the interior angles is 180 degrees. Both Gaussian and mean curvatures vanish. By changing our coordinates, however, so as to "roll" the plane into the cylinder of Figure 2(b), the sum of the interior angles of the triangle remain unchanged, but now the mean curvature is non-zero. The mean curvature is therefore not intrinsic to the surface.

ENGINEERING CONSIDERATIONS

In the last section, we saw that the information intrinsic to the earth's surface can be expressed as an arbitrary functional of the Gaussian curvature. This is not useful unless the functional can be approximated by a simple function. To do this, we introduce two constitutive assumptions: that the curvature of the earth's surface is not rapidly varying and that it is small. Both assumptions are reasonable as long as we are not interested in very small features not well represented by the data. In problems of terrain estimation from sparse data, such features will not be present in the input data except as noise. The first of these assumptions allows us to neglect derivatives of the Gaussian curvature. The second allows us to expand the information I[K] in a Taylor series:

$$I[K] = \sum_{n=1}^{\infty} A_n \cdot K^n / n! \qquad (5)$$

where the A are some unknown coefficients. If we now recall that the information must be positive, then the lowest order non-trivial term which contributes is the square curvature. Our approximations thus allow us to write

$$I[K] = K^2 \qquad (6)$$

where we have absorbed the constant A_2 in the definition of our unit of information.

With equation (6), we are close to the position of applying our approach to practical problems of terrain estimation. The one remaining problem is defining K in terms of the structure of a DTM, rather than in terms of a continuous

function. The key is to further exploit Gauss' theorem on integral curvature. As noted by T. Regge, in the context of numerical calculations in General Relativity,* the appropriate way of treating curved spaces on digital computers is to approximate them by triangulated networks. The invariant integral of a function of the curvature over a curved surface is replaced by the sum of that function evaluated at the vertices of the triangulation. The Gaussian curvature at a vertex V is given by the deficit angle

$$K(V) = 2\pi - \sum_i \alpha_i \qquad (7)$$

where the α_i are the interior angles of the triangulation with vertex V (see Figure 3). For example, the Gaussian curvature at each of the four vertices of a regular tetrahedron is 180 degrees, while the Gaussian curvature at each of the vertices of a cube is 90 degrees.

Regge's formalism can be directly applied to terrain estimation from a model based on a triangulated irregular network**, but it must be extended if we are to deal with terrain estimation from more familiar DTM's based on digitized contours or rectangular grids. It is clear that some triangulation of the surface in the neighborhood of the point of interest is needed. Optimal results can be expected from careful consideration of possible triangulations.*** Because of severe constraints on software development time, we chose the triangulation of Figure 4 for the initial implementation of our approach. This triangulation was used as the basis for the algorithms in the contour-to-grid-estimation-system CONTOGES implemented at USAETL.****

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* T. Regge, Il Nuovo Cimento, V. 19 (1961), p. 558.

** T. Peucker, et. al., Proceedings of the Digital Terrain Models Symposium, American Society of Photogrammetry, American Congress of Surveying and Mapping (1978), p. 516.

*** See, for example, H.Fuchs, et.al, Communications of the ACM, V. 20 (1977), p. 693.

**** S. Norton and C. Taylor, An Information-Theoretic Contour-to-Grid-Estimation System: CONTOGES, USAETL Technical Report, in preparation.

RESULTS

The test of any terrain modeling theory is the performance of computer programs embodying the theory. The problems generally associated with contour-to-grid interpolation include flattening of hilltops, shifting of terrain features, and introduction of spurious features. In addition, there are frequently problems such as a "staircase" effect between contours which are not associated with the interpolation scheme but are due to the terrain sampling mechanism.

To focus on problems associated with the interpolation scheme, and to allow limited comparison with "ground truth," the CONTOGES software was tested on data derived from an existing elevation data base. To demonstrate the flexibility of inputs, we generated a 601x201 input grid with elevations falling on every tenth diagonal. Contour data can also be considered as a sparse elevation matrix, and therefore can be input directly into our routines. Although CONTOGES requires a gridded elevation input, it is straight forward to extend the implementation to accept any form of elevation data. The 601x201 sparse grid with elevations at approximately 20% of the nodes was processed by CONTOGES. The output was a 601x201 dense grid with elevations at all of the nodes. In order to check for slope continuity and systematic errors orthonormal shaded relief was chosen as the technique for viewing the data. Figure 5 shows the original data base when illuminated from the upper left. Figure 6 shows the output of the test just described. Note that the hilltops are round, the ridgelines are sharp (without the use of ridgeline data), and that no spurious features are evident. The noise in Figure 6 is largely due to the use of shaded relief since we are looking at the derivatives of the surface rather than at elevations. Inspection of the output shows that the noise corresponds to fluctuations which are typically of the quantization size of the input data and does not seem to be a problem of the estimation scheme.

The other element of concern in the evaluation of the performance of the CONTOGES software is execution time. Running on a PDP 11/45 minicomputer, the generation of the full 601x201 data set requires approximately 20 minutes. Despite the non-linearity of the approach, CONTOGES is competitive in execution time with existing programs for grid estimation from contour data.

DISCUSSION

Preliminary testing of the contour-to-grid-estimation system CONTOGES suggests that the techniques described above may form the basis for production software for the generation of gridded DTM's from digitized contour data. Further testing

on typical production data sets, as well as careful consideration of where these ideas should be applied in a production system will be necessary to determine the changes, if any, which need to be made in software used for large scale generation of gridded data bases. It is apparent that the information-theoretic approach to terrain estimation is free of the problems found in other approaches without severe degredation of processing time.

Perhaps of greater importance in the long run is the application of the ideas outlined in this paper to other problems of terrain modeling. If the preliminary results presented here are substantiated in further testing, the information theoretic approach will prove to be of general utility in generating any kind of DTM from any other. In particular, an interesting application might be the generation of triangulated irregular networks from available terrain models such as gridded or digitized contour DTM's. Another possibility is the treatment of noisy data by extending this formalism to the consideration of fractal surfaces.*

ACKNOWLEDGEMENTS

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* B. Mandelbrot, Fractals, (W.H. Freeman, San Francisco, 1977).

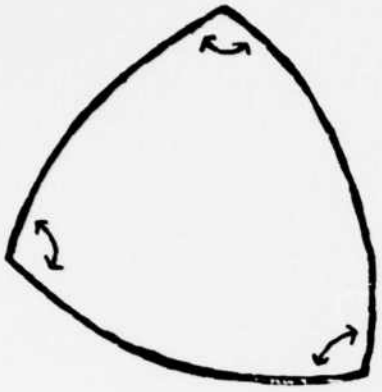

Figure 1. Gauss' Integral Theorem

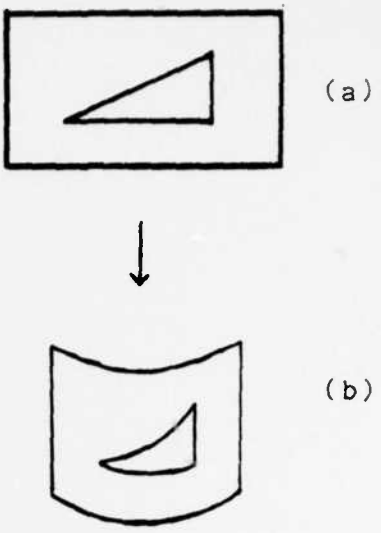

Figure 2. Gaussian Curvature is intrinsic to the surface; mean curvature is not.

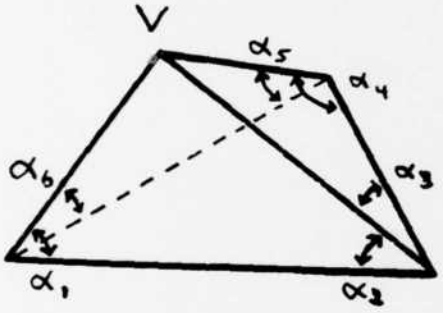

Figure 3.

$$K(V) = \pi - \sum_{i=1}^{6} \alpha_i$$

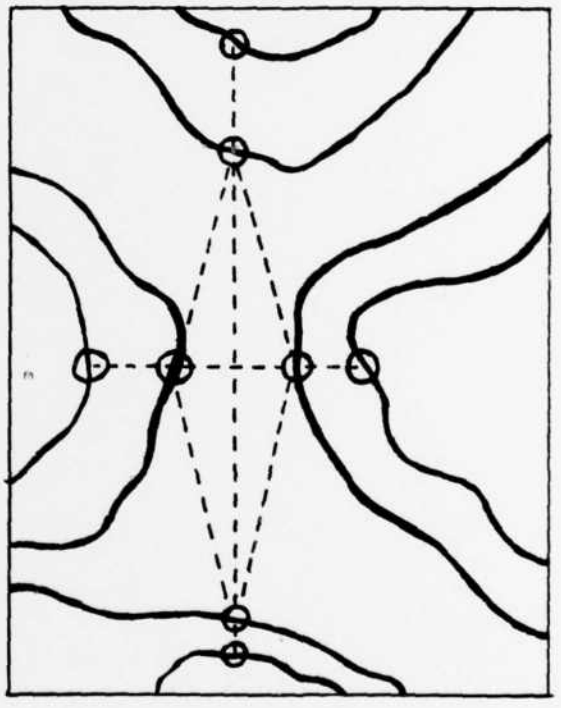

Figure 4.
A Triangulation Scheme

Figure 5. 601x201 Test Data Set.

Figure 6. Output Generated by CONTOGES from Sparse Data.

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