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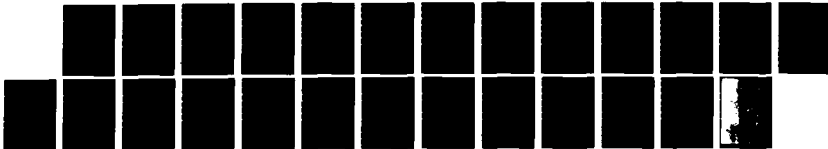
INPUT DEFLECTION REQUIREMENTS FOR QUANTIZERS FOLLOWED
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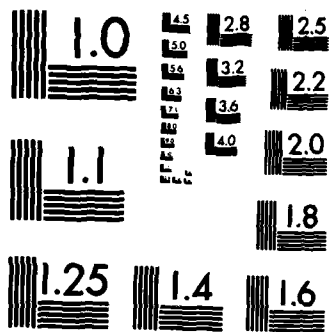
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NAVAL UNDERWATER SYSTEMS CENTER
NEW LONDON LABORATORY
NEW LONDON, CONNECTICUT 06320

Technical Memorandum

INPUT DEFLECTION REQUIREMENTS FOR QUANTIZERS
FOLLOWED BY GREATEST-OF DEVICE AND INTEGRATOR

Date: 24 August 1978

Prepared by:

Albert H. Nuttall
ALBERT H. NUTTALL
Special Projects Dept.
Adv. Sys. Tech. Div.

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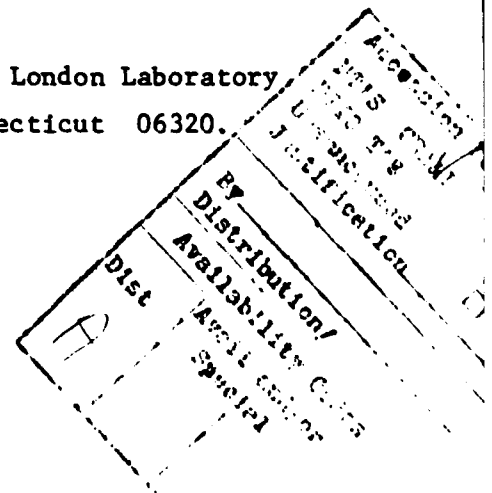
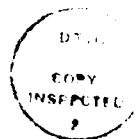
ABSTRACT

Derivations and programs are presented for evaluation of the required input deflection, for specified output deflection, of a system composed of quantizers followed by a Greatest-Of device and an integrator. The important parameters are: L, the number of levels in each quantizer (along with their breakpoints and step heights); M, the number of statistically independent samples summed in the integrator; N, the number of statistically independent input channels; and d , the system output deflection. Several numerical examples are presented. Extension of some earlier results for a system without quantizers (Ref. 1) is also made, and additional results given.

ADMINISTRATIVE INFORMATION

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The author of this memorandum is located at the New London Laboratory of the Naval Underwater Systems Center, New London, Connecticut 06320.



INTRODUCTION

Practical realizations of desired systems often incorporate approximations to ideal devices, for the sake of expense and equipment complexity. In particular, quantization is frequently employed, since it facilitates data processing. In an earlier study (Ref. 1), the required input signal-to-noise ratio for a Greatest-Of device followed by an integrator was determined; here, we wish to extend the analysis to a system which also has quantization prior to the Greatest-Of device. Surprisingly, it will turn out that the analysis of this more-complicated data processor is much simpler and quicker (via computer aid) than that of the earlier system without quantization. It is advantageous for the reader to be familiar with the assumptions, notation, and results of Ref. 1 before reading this memorandum.

SYSTEM DEFINITION AND ASSUMPTIONS

The system of interest is depicted in figure 1. The input channels x_1, \dots, x_N are assumed statistically independent and contain either (a) no signal in any channel, or (b) signal in one channel only. (A method for handling statistically-dependent inputs, by defining an effective number of statistically-independent channels, is described in appendix A. Thus the present results are applicable to a wider class of inputs than presumed here, under proper interpretation of the value of N.) The probability density functions of a single channel random variable x , under the two hypothesis (a) and (b), are denoted by $p_0(x)$ and $p_1(x)$, respectively, for the noise-only channels and the signal-bearing channel. The noise-only channels are identically-distributed; however, the analysis could be extended to the case of different distributions on each channel.

The Greatest-Of device is described mathematically by

$$y = \max\{f(x_1), \dots, f(x_N)\}. \quad (1)$$

Now we shall limit consideration to non-linear no-memory devices f that are monotonically non-decreasing; that is

$$f(a) \leq f(b) \quad \text{if } a < b. \quad (2)$$

Then (1) can be written in the equivalent form

$$y = f(\max\{x_1, \dots, x_N\}). \quad (3)$$

This implies that the system in figure 2 is equivalent to that of figure 1, when (2) is satisfied. In practice, figure 1 might be preferred for a quantizer f , because the Greatest-Of device need only handle a discrete set of inputs, rather than the continuous inputs in figure 2. The reason for pointing out (3) is that the processor in figure 2 is more convenient to analyze than figure 1; however, the system of figure 1 is capable of analysis for a general non-monotonic quantizing nonlinearity f , as will become apparent by the methods to follow.

The output of the integrator in figures 1 and 2 is assumed to consist of a sum of M statistically-independent samples. (For a discrete sum of dependent samples, an effective number of independent samples can be defined in a manner identical to that in Appendix A; see Ref. 2, Appendix B. Alternatively, a continuous integration can also be modelled as an equivalent discrete summation of an effective number of independent samples, by a slight modification of Appendix A; see for example, Ref. 3, Appendix A. Thus, the present results are applicable to a wider class of integrators than presumed here, under proper interpretation of the value of M .)

DEFINITION OF OUTPUT DEFLECTION

Since the integrator output is

$$z = \sum_{m=1}^M y_m, \quad (4)$$

we have for its mean and variance, respectively,

$$\mu_z = M \mu_y, \quad \sigma_z^2 = M \sigma_y^2, \quad (5)$$

using the independence of samples $\{y_m\}$. (Actually, we need only require $\{y_m\}$ be uncorrelated.)

We define a system output deflection (prior to the threshold comparison) as the physically-meaningful quantity

$$d_0 = \frac{\mu_{z1} - \mu_{z0}}{\sigma_{z0}}, \quad (6)$$

where subscripts 0 and 1 denote, respectively, the signal-absent and signal-present cases. In an earlier study (Ref. 1), we presumed that output z was Gaussian (as it would reasonably be, for large M). The way this assumption appeared in the results of the earlier analysis was in the setting of the value of d_0 in (6), for prescribed false alarm and detection probabilities; see Ref. 1, eqs. (6)-(16). Here, we will not assume z is Gaussian; rather, we will simply require deflection d_0 to take values in the neighborhood*, 3-5, knowing that relatively good performance, in terms of false alarm and detection probabilities, is then attainable through appropriate choice of threshold T in figure 1. The actual numerical values selected for d_0 for plotting purposes will, however, correspond exactly to those used earlier*, for comparison purposes. The purpose of this discussion is to point out that the results actually have applicability to a wider class of outputs, z , than presumed in the original work (Ref. 1), under proper interpretation of d_0 as a physically-useful (although statistically incomplete) measure of performance.

Substituting (5) in (6), we have

$$d_0 = \sqrt{M} \frac{\mu_{y1} - \mu_{y0}}{\sigma_{y0}}. \quad (7)$$

SPECIALIZATION TO QUANTIZERS

In order to evaluate (7), we need to evaluate the moments (see figure 2)

$$\overline{y^p} = \overline{f^p(w)} = \int dw q(w) f^p(w), \quad (8)$$

*For example, $-\Phi^{-1}(P_F) = 3.090$ and 4.753 for $P_F = 10^{-3}$ and 10^{-6} , respectively; see Ref. 1, eqs. (16) and (10).

where $q(w)$ is the probability density function of w . (We will impose subscripts 0 and 1 when necessary.) For general monotonic nonlinearities f , this is a complicated expression. However, when f is a quantizer, (8) takes a particularly simple form. Consider the general monotonic quantizer characterization in figure 3. Without loss of generality, $L \geq 1$; otherwise, the quantizer output is independent of its input. Also $h_\ell < h_{\ell+1}$ for $0 \leq \ell \leq L-1$, and $b_\ell < b_{\ell+1}$ for $1 \leq \ell \leq L-1$ ($L \geq 2$).

Then (8) becomes simply*

$$\begin{aligned} \bar{y}^\nu &= h_0^\nu \int_{-\infty}^{b_1} dx q(x) + \sum_{\ell=1}^{L-1} h_\ell^\nu \int_{b_\ell}^{b_{\ell+1}} dx q(x) + h_L^\nu \int_{b_L}^{\infty} dx q(x) \\ &= h_0^\nu Q(b) + \sum_{\ell=1}^{L-1} h_\ell^\nu [Q(b_{\ell+1}) - Q(b_\ell)] + h_L^\nu [1 - Q(b_L)] \\ &= h_L^\nu - \sum_{\ell=1}^L Q(b_\ell) [h_\ell^\nu - h_{\ell-1}^\nu], \end{aligned} \quad (9)$$

where $Q(w)$ is the cumulative distribution function of w . In particular, the quantities needed in (7) are (re-establishing subscripts)

$$\begin{aligned} \mu_{y_0} &= h_L - \sum_{\ell=1}^L Q_0(b_\ell) [h_\ell - h_{\ell-1}], \\ \mu_{y_1} &= h_L - \sum_{\ell=1}^L Q_1(b_\ell) [h_\ell - h_{\ell-1}], \\ \bar{y}_0^2 &= h_L^2 - \sum_{\ell=1}^L Q_0(b_\ell) [h_\ell^2 - h_{\ell-1}^2], \\ \sigma_{y_0} &= (\bar{y}_0^2 - \mu_{y_0}^2)^{1/2}. \end{aligned} \quad (10)$$

The cumulative distribution functions of w under the signal-absent and signal-present cases, respectively, are, using the statistical independence of $\{x_1, \dots, x_N\}$ (see figure 2),

$$\begin{aligned} Q_0(w) &= P_0^N(w), \\ Q_1(w) &= P_0^{N-1}(w)P_1(w), \end{aligned} \quad (11)$$

*By contrast, the results for no non-linear device, i.e. $f(x) = x$, are much more complicated; see Ref. 1, eqs. (17), (18), and (30) (upper line).

where $P_0(x)$ and $P_1(x)$ are the cumulative distribution functions of x under the signal-absent and signal-present alternatives, respectively; see Definitions subsection above.

SPECIALIZATION TO GAUSSIAN INPUTS

This specialization to Gaussian inputs is made solely for the purpose of evaluating (10) and (11) for a particular example. It is not fundamental, and can be replaced by another more-appropriate example if desired. For Gaussian inputs, we have

$$\begin{aligned} P_0(x) &= \Phi\left(\frac{x-m_0}{\sigma}\right), \\ P_1(x) &= \Phi\left(\frac{x-m_1}{\sigma}\right), \end{aligned} \quad (12)$$

where m_0, m_1 are the means, and σ is the standard deviation (assumed identical), for the two signal hypotheses. It is convenient to define an input deflection (analogous to output deflection (6)) as

$$d_i = \frac{m_1 - m_0}{\sigma}. \quad (13)$$

(This quantity was denoted by r in Ref. 1). Employing (11) and (12) in (10), we can now evaluate

$$\begin{aligned} Q_0(b_l) &= \Phi^N\left(\frac{b_l - m_0}{\sigma}\right), \\ Q_1(b_l) &= \Phi^{N-1}\left(\frac{b_l - m_0}{\sigma}\right) \Phi\left(\frac{b_l - m_1}{\sigma}\right). \end{aligned} \quad (14)$$

The quantizer break-points $\{b_l\}$ will be specified in terms of input parameters m_0 and σ (see figure 1); i.e., let

$$b_l = m_0 + \sigma v_l, \text{ or } \frac{b_l - m_0}{\sigma} = v_l, \quad 1 \leq l \leq L, \quad (15)$$

where v_l is a "normalized" breakpoint. Then (14) becomes

$$\begin{aligned}
 Q_0(b_2) &= \Phi^N(v_2), \\
 Q_1(b_2) &= \Phi^{N-1}(v_1) \Phi(v_2 - d_i),
 \end{aligned}
 \tag{16}$$

where we also utilized (13). The $\{v_2\}$ in (15) can be negative or positive.

SUMMARY OF ANALYTICAL RESULTS FOR QUANTIZERS PRESENT

Output deflection d_0 is given by (7). The quantities needed in (7) are given by (10). And the latter quantities are given by (16). The necessary input parameters are L , $\{b_2\}_0^L$, $\{v_2\}_1^L$, M , N , d_1 . A program for the evaluation of d_0 is given in appendix B. We will set d_0 equal to 3.090 or 4.753, which as noted earlier, correspond to $P_F = 10^{-3}$, $P_D = .5$ or $P_F = 10^{-6}$, $P_D = .5$ respectively, and solve for the required value of input deflection d_1 , for specified quantizer, number of input channels, and number of independent samples in the integrator.

The quantizer example to be investigated is an eight-level (3 bit) device as shown in figure 4. That is, all the steps are of equal spacing and height, and extend from the noise-only mean m_0 to $m_0 + 2\sigma$. Thus from figure 3 and (15),

$$v_2 = \frac{\lambda - 1}{3}, \quad 1 \leq \lambda \leq L = 7.
 \tag{17}$$

We will characterize this spacing by $\Delta v = 1/3$.

RESULTS

The input deflection, d_1 , required for $M = 16$ is presented in figure 5, as a function of N , the number of independent input channels. Similar results for $M = 32$ and 64 are presented in figures 6 and 7.

Since the quantizer in figure 4 only spans the range $(m_0, m_0 + 2\sigma)$, it is anticipated that better performance might be attained if the quantizer covered a wider range. In figure 8, a comparison of quantizers (for $M = 32$) which cover the range $(m_0, m_0 + 6\Delta v)$ is made, for $\Delta v = 1/3, .5$, and $.7$. It is

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observed that best performance (lowest d_i) is realized when $\Delta v = .5$; that is, the quantizers then cover the range $(m_0, m_0 + 3\sigma)$. Results for other cases are easily available by means of the program in appendix B. In particular, some of the $\{k_i\}$ could be negative if desired.

EXTENSION OF EARLIER RESULTS

In Ref. 1, analytical and graphical results were presented for the system without quantizers. In particular, in Ref. 1, eqs. (21) and (31), the quantities

$$\begin{aligned}
 a_N &= N \int dx x \phi(x) \Phi^{N-1}(x), \\
 b_N &= N \int dx x^2 \phi(x) \Phi^{N-1}(x),
 \end{aligned}
 \tag{18}$$

were found necessary. By use of some results in Ref. 4, closed form expressions for some of these quantities have been found, which augment eqs. (22) and (32) in Ref. 1. They are listed in Table 1 below.

N	a_N	b_N
1	0	1
2	$1/\sqrt{\pi}$	1
3	$1.5/\sqrt{\pi}$	$1 + \frac{\sqrt{3}}{2\pi}$
4	$\frac{3}{\sqrt{\pi}} \left(1 - \frac{2}{\pi} \arctan \frac{1}{2}\right)$	$1 + \frac{\sqrt{3}}{\pi}$
5	$\frac{5}{\sqrt{\pi}} \left(1 - \frac{3}{\pi} \arctan \frac{1}{2}\right)$?

Table 1. Values of a_N and b_N

The graphical results in Ref. 1 were for very large values of M. Here we present some additional results* for M = 16, 32, 64, 128, in figures 9-10, respectively. The labelling on these figures corresponds to the deflection criteria cited earlier in this memorandum, rather than that in Ref. 1. These results also allow for comparison with the quantizer results given in figures 5-8 above.

*A program for the system without quantizers is given in Appendix C.

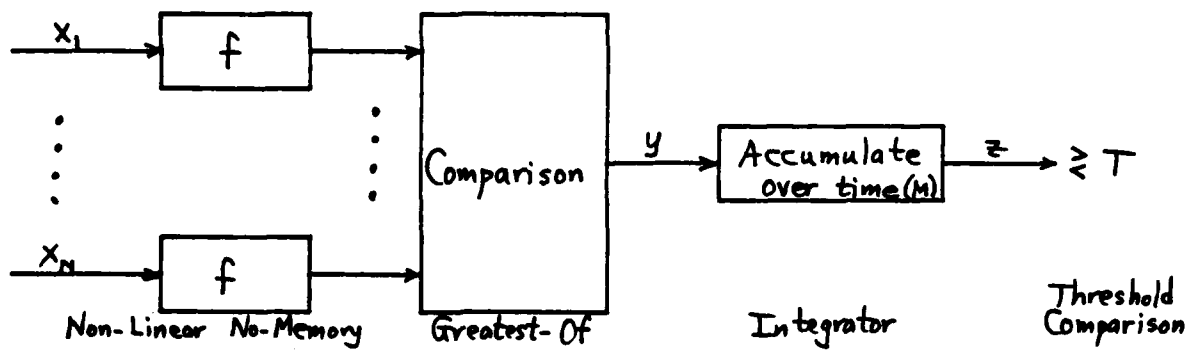


Figure 1. Generic Block Diagram

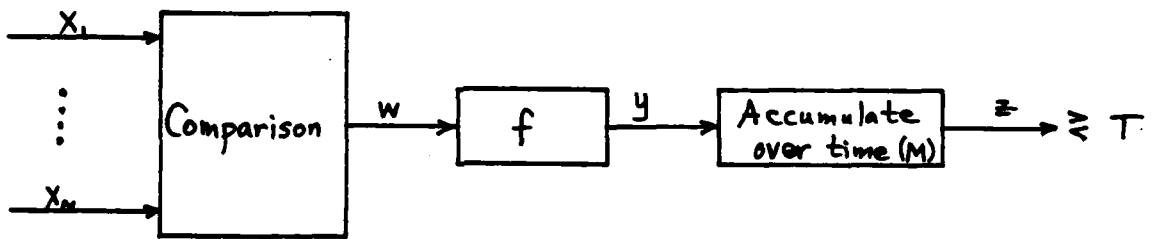


Figure 2. Equivalent Processor For Monotonic Non-Linearity

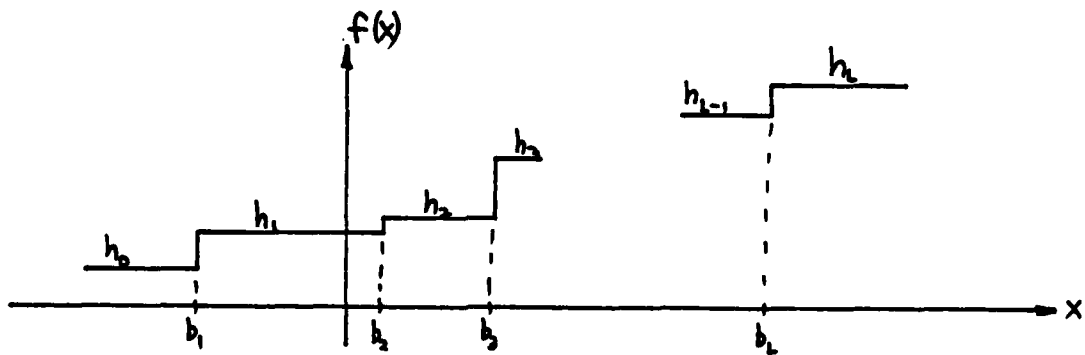


Figure 3. Quantizer Characterization

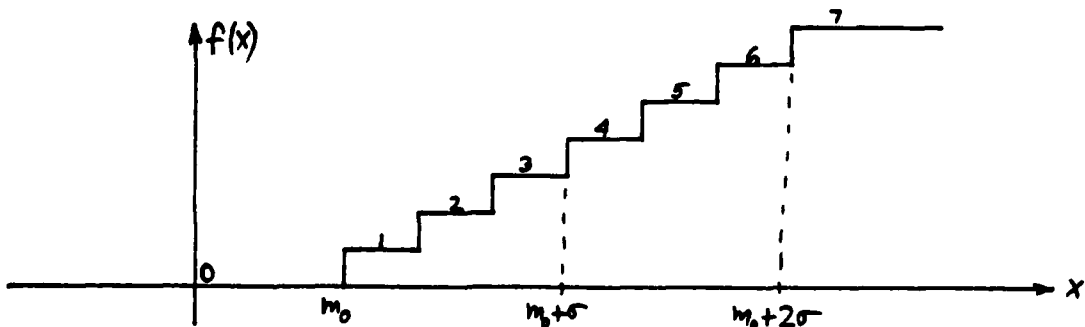


Figure 4. Quantizer Example

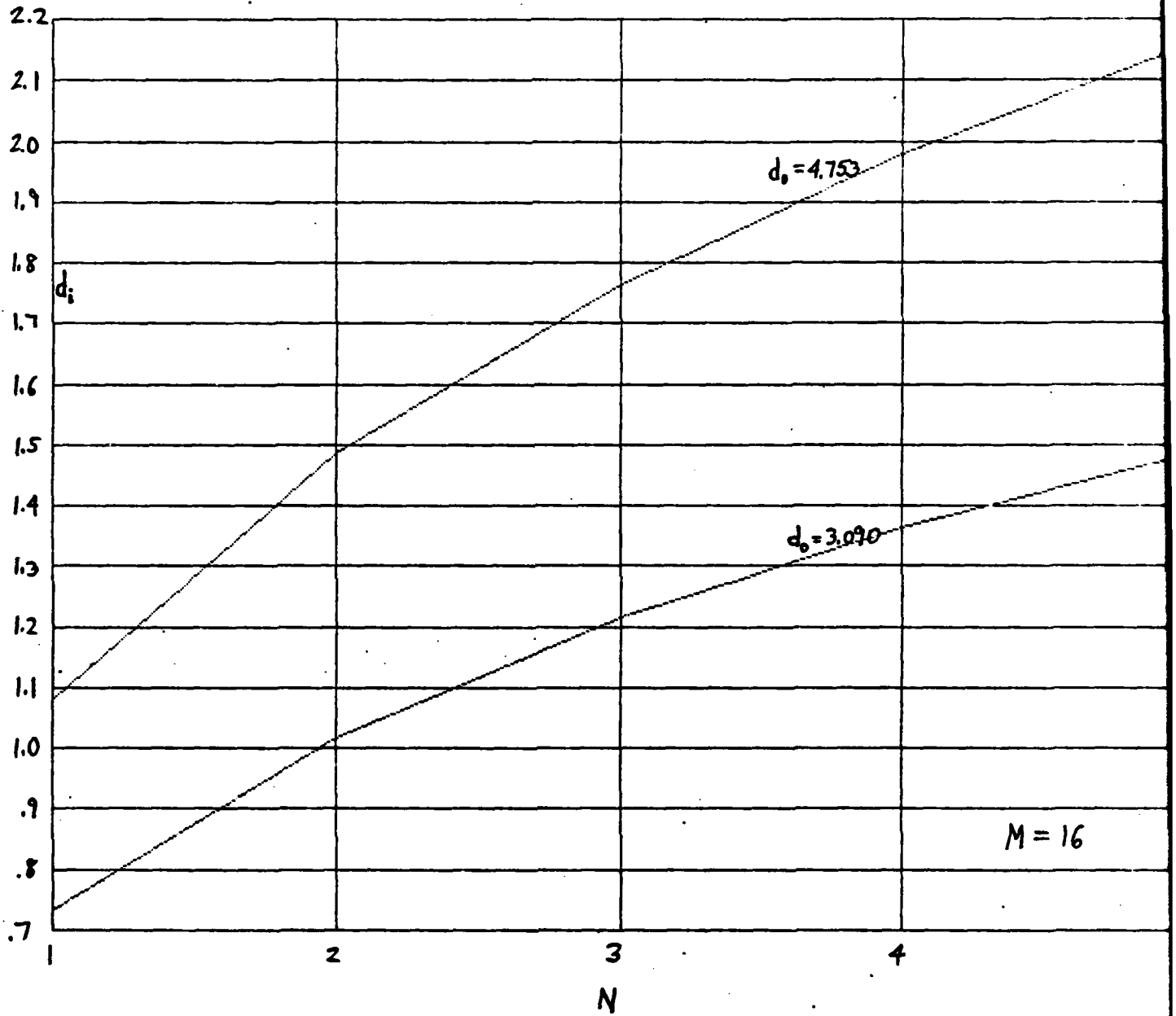


Figure 5. Input Deflection Requirements for $M=16$

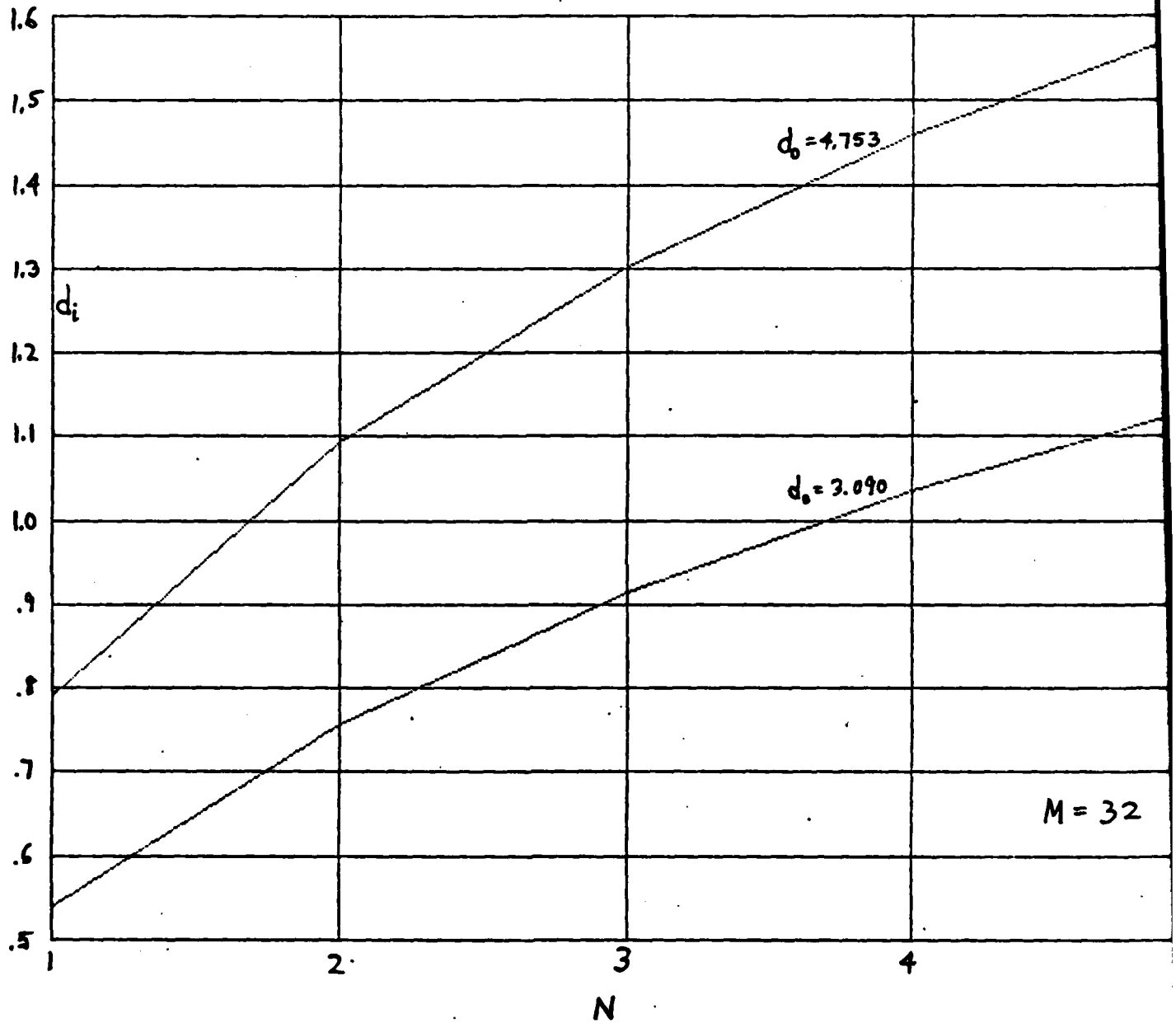


Figure 6. Input Deflection Requirements for $M = 32$

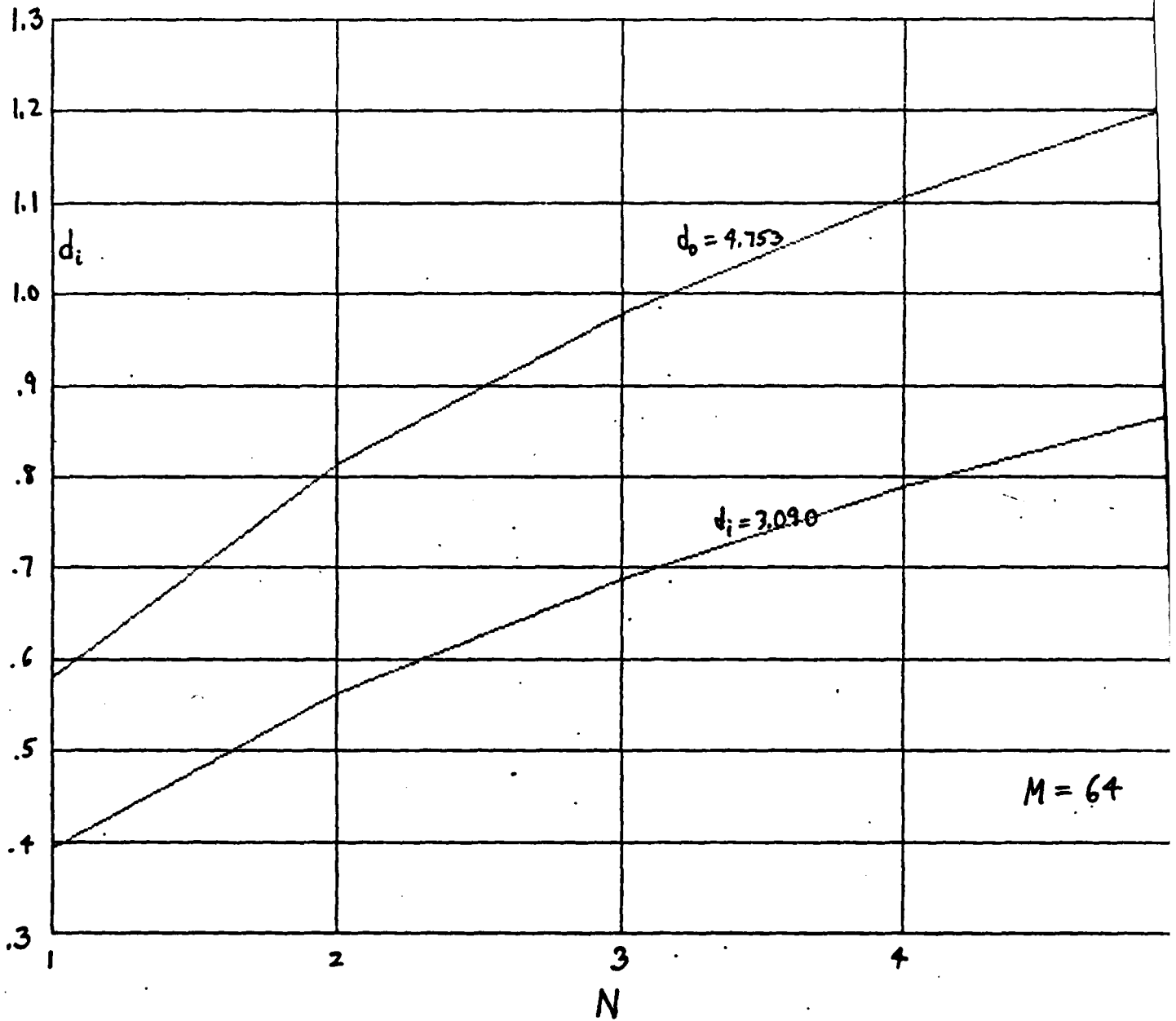


Figure 7. Input Deflection Requirements for $M = 64$

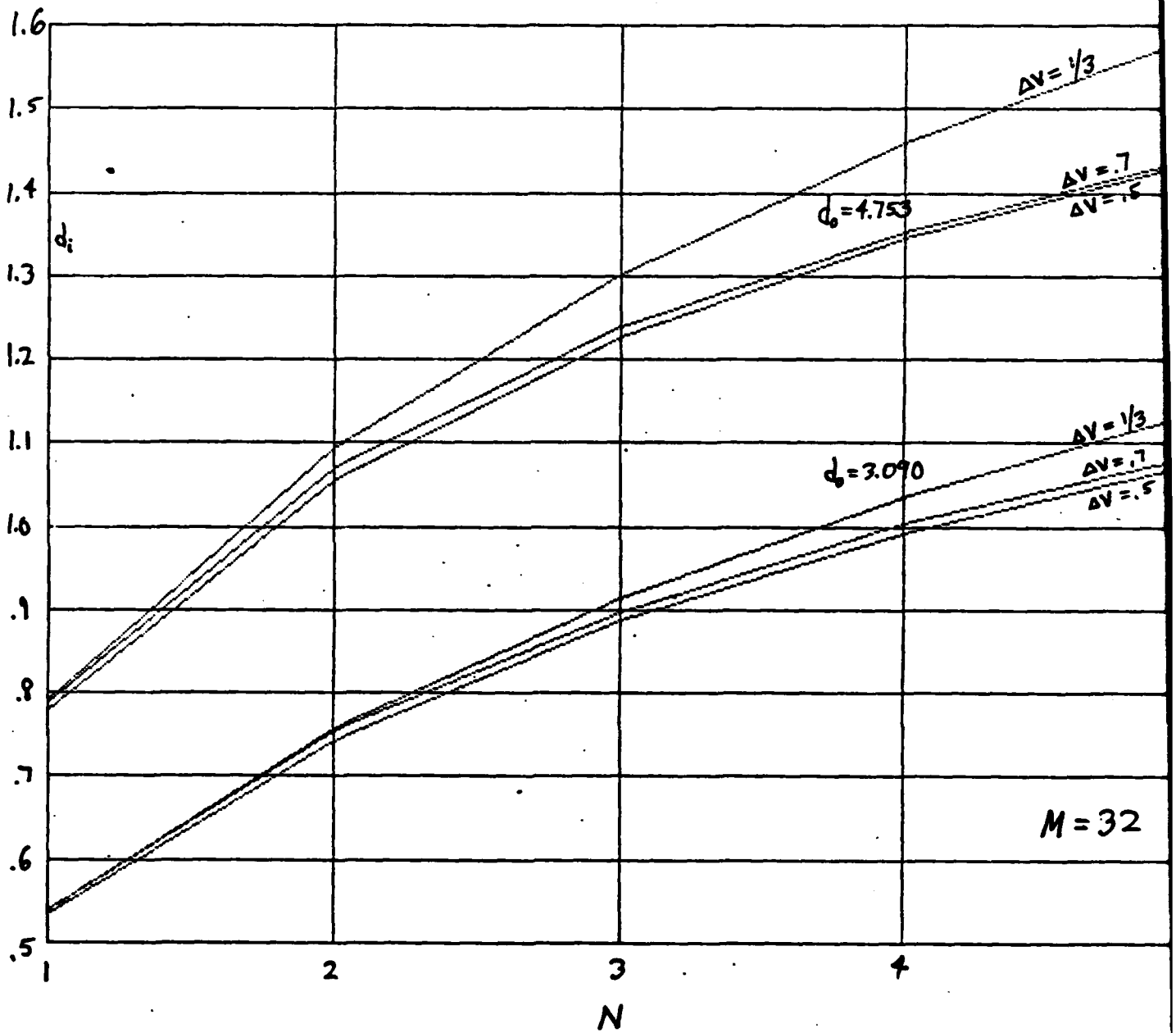


Figure 8. Comparison of Quantizers, for $M = 32$

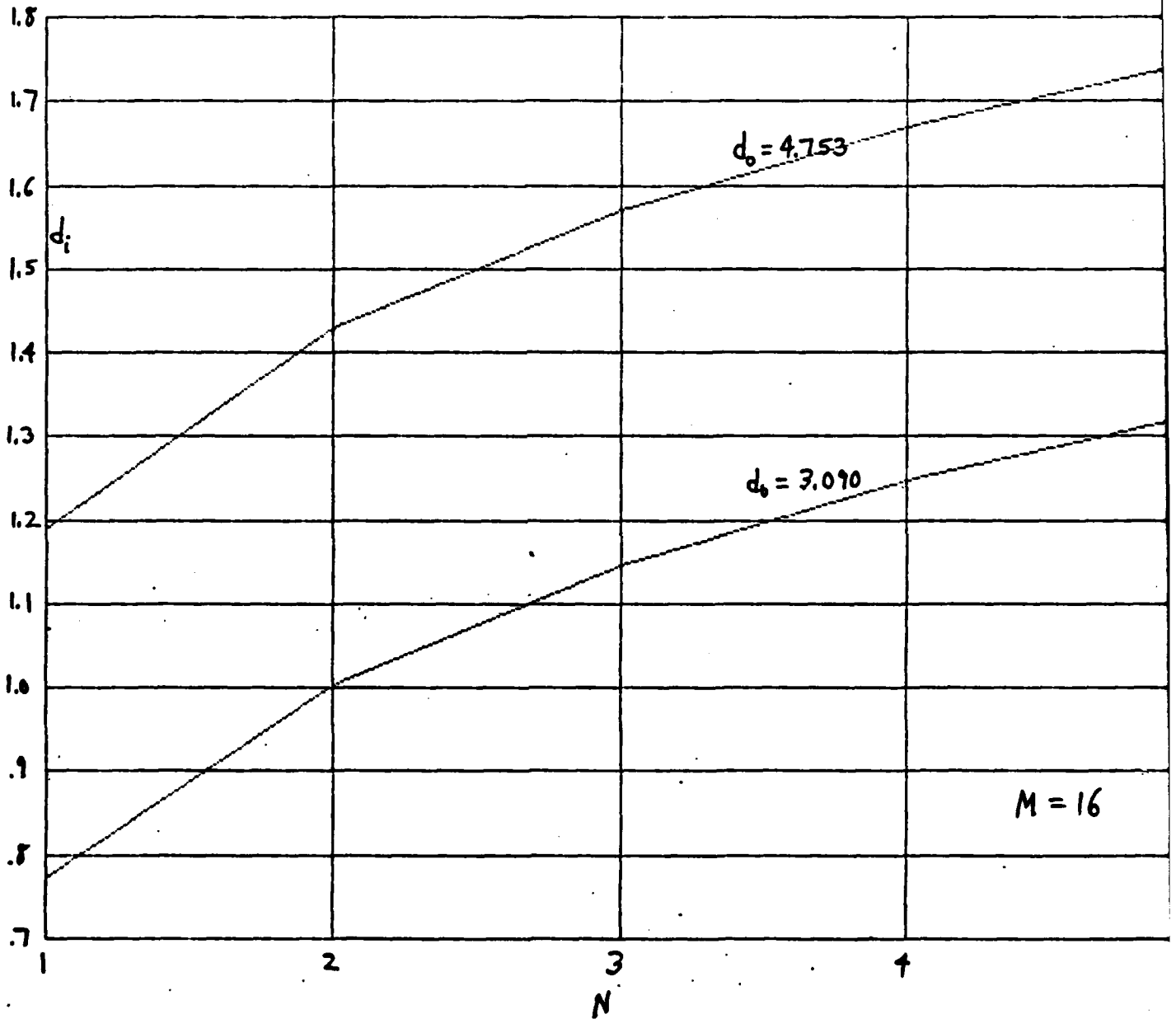


Figure 9. Input Deflection Requirements for $M=16$, No Quantizers

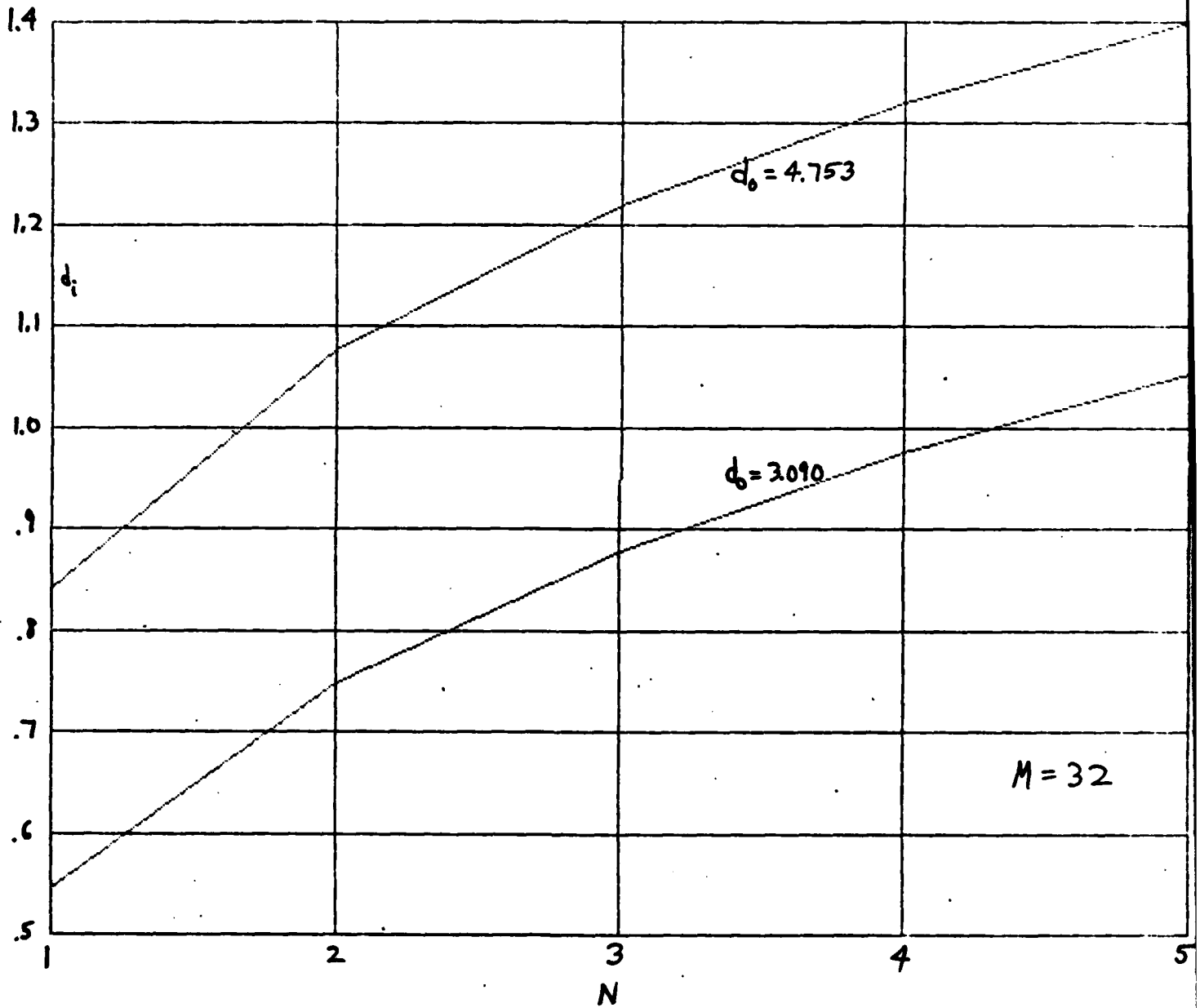


Figure 10. Input Deflection Requirements for $M=32$, No Quantizers

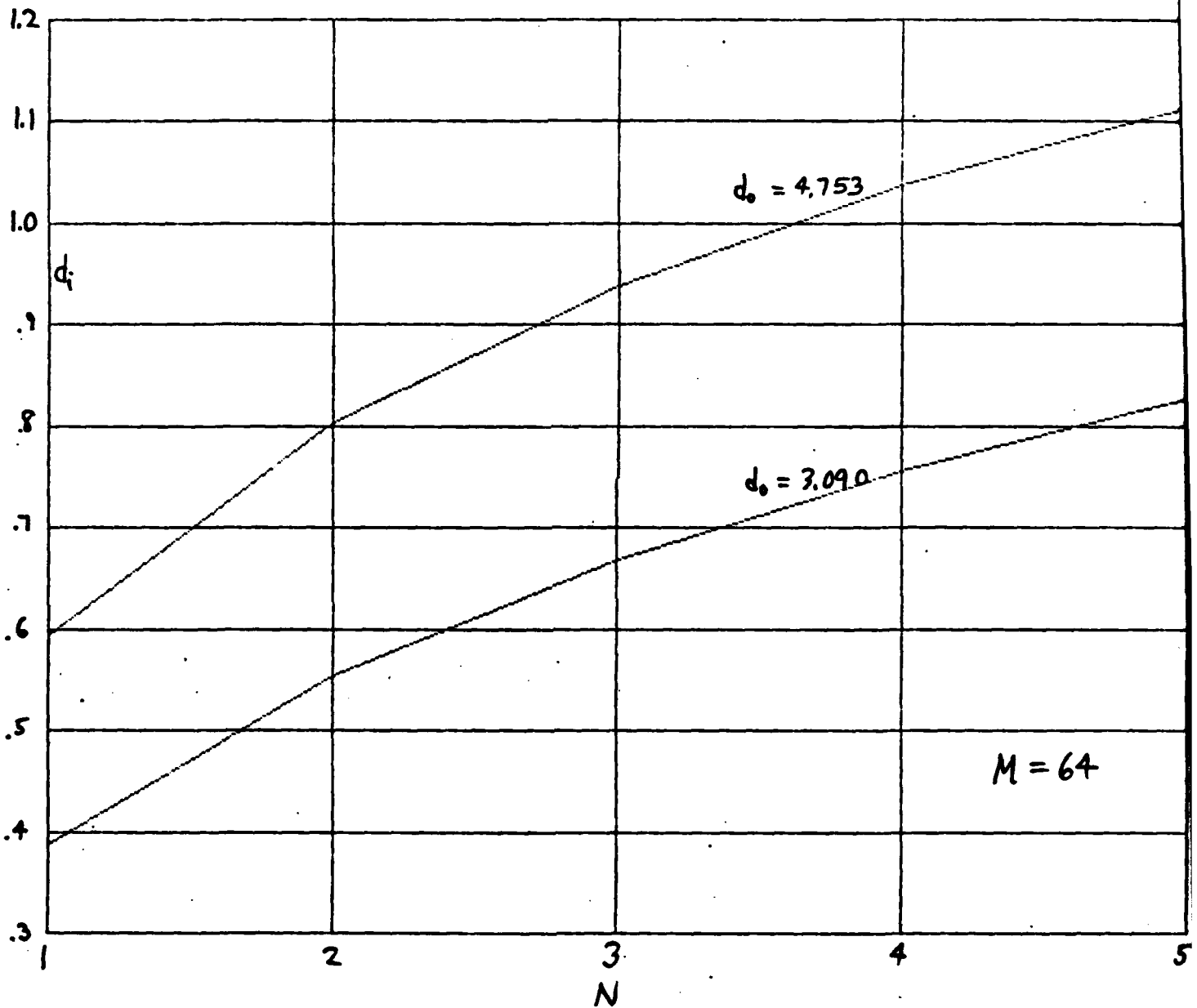


Figure 11. Input Deflection Requirements for $M=64$, No Quantizers

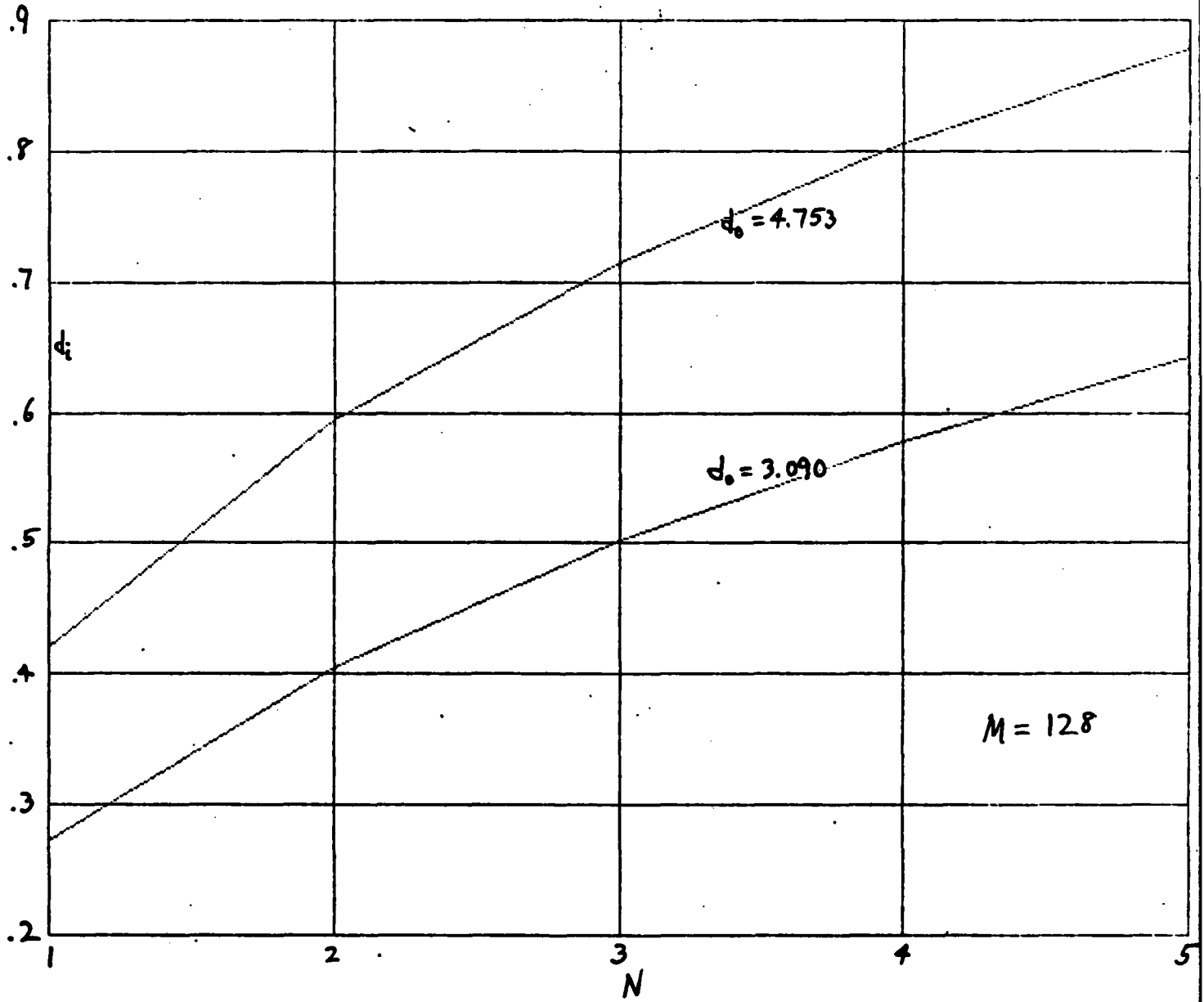


Figure 12. Input Deflection Requirements for $M=128$, No Quantizers

APPENDIX A. Definition of Effective Number of Independent Inputs

Consider random variables x_1, \dots, x_N which are identically distributed (noise-only case), but not necessarily independent. Define

$$z = \sum_{n=1}^N x_n. \quad (\text{A-1})$$

Then

$$\mu_z = \bar{z} = \sum_{n=1}^N \bar{x}_n = N\mu_x. \quad (\text{A-2})$$

Let

$$a_n = x_n - \mu_x; \quad \bar{a}_n = 0, \quad \overline{a_n^2} = \overline{(x_n - \mu_x)^2} = \sigma_x^2, \quad (\text{A-3})$$

and

$$\overline{a_m a_n} = \sigma_x^2 \rho_{mn} \quad (\rho_{nn} = 1, \quad 1 \leq n \leq N). \quad (\text{A-4})$$

Then

$$z - \bar{z} = \sum_{n=1}^N (x_n - \bar{x}_n) = \sum_{n=1}^N a_n, \quad (\text{A-5})$$

and

$$\sigma_z^2 = \overline{(z - \bar{z})^2} = \sum_{m,n=1}^N \overline{a_m a_n} = \sigma_x^2 \sum_{m,n=1}^N \rho_{mn}. \quad (\text{A-6})$$

Define

$$N_{\text{eff}} = \frac{\mu_z^2 / \sigma_z^2}{\mu_x^2 / \sigma_x^2} = \frac{N^2}{\sum_{m,n=1}^N \rho_{mn}}. \quad (\text{A-7})$$

$$\left. \begin{array}{l} \text{If } \rho_{mn} = \delta_{mn}, \quad N_{\text{eff}} = N. \\ \text{If } \rho_{mn} = 1, \quad N_{\text{eff}} = 1. \end{array} \right\} \quad (\text{A-8})$$

These check intuition.

So the results in this memorandum could be used for dependent inputs x_1, \dots, x_N , if N is replaced by

$$N_{\text{eff}} = \frac{N^2}{\sum_{m, n=1}^N p_{mn}} \quad (\text{A-9})$$

This applies even if N_{eff} is not an integer; this yields no mathematical difficulty.

APPENDIX B. Program for System with Quantizers

The following Basic program is written for the Hewlett Packard 9845A. The quantizer parameters are entered in lines 40, 50, and 60. d_1 , N, and M are input in lines 10, 20, and 30 respectively.

```

10 INPUT D1          ! D1 >= 0    INPUT DEFLECTION
20 N=5              ! N >= 1    NUMBER OF INPUT CHANNELS
30 M=32            ! M > 0     NUMBER OF TERMS IN ACCUMULATOR
40 L=7             ! L >= 1    NUMBER OF LEVELS -1 IN QUANTIZER
50 DATA 0,1,2,3,4,5,6,7
51 REM L EQUI-SIZED VERTICAL JUMPS FROM 0 UP TO L
60 DATA 0,.33333,.66667,1,1.33333,1.66667,2
61 REM L-1 EQUI-SPACED HORIZONTAL JUMPS FROM MEAN TO 2*SIGMA
70 DIM H(0:7),V(1:7)
80 MAT READ H,V
90 MUY0=MUY1=H(L)
100 Y02AV=H(L)^2
110 T1=N-1
120 FOR L1=1 TO L
130 T2=FN(P(V(L1)))
140 T3=T2^T1
150 Q0=T3*T2
160 Q1=T3*FN(P(V(L1)-D1))
170 T4=H(L1)-H(L1-1)
180 MUY0=MUY0-Q0*T4
190 MUY1=MUY1-Q1*T4
200 Y02AV=Y02AV-Q0*(H(L1)^2-H(L1-1)^2)
210 NEXT L1
220 SIGY0=SQR(Y02AV-MUY0^2)
230 DO=SQR(M)*(MUY1-MUY0)/SIGY0 ! OUTPUT DEFLECTION
240 PRINT "L =";L,"M =";M,"N =";N
250 PRINT "D1 =";D1,"Do =";Do
260 STOP
270 DEF FN(P) ! PHI
280 P=ABS(P)
290 IF P<7 THEN 320
300 P=0
310 GOTO 350
320 P=1/(1+.2316419*P)
330 P=P*(.31938153-P*(.356563782-P*(1.781477937-P*(1.821255978-P*1.33027443)))
340 P=P*EXP(-.5*P^2)*.398942280401
350 IF P<0 THEN 370
360 P=1-P
370 RETURN P
380 FNEND
390 END

```


APPENDIX C. Program for System without Quantizers

The following Basic program is written for the Hewlett Packard 9845A; it is essentially the same as Ref. 1, appendix B. The inputs are r and N in lines 30 and 40. Observation of the comments in lines 170-190 is

```

10 REM      TM No. TC-13-75, APPENDIX B
20 REM      NOTICE LINES A, B, C, AND D BELOW
30 DEF FNF(R)
40 A:      N=8                ! INPUT N >= 2
50 C1=1/SQR(2*PI)
60 S1=-8
70 S2=8
80 S3=(FNS(S1,R,N,C1)+FNS(S2,R,N,C1))*0.5
90 S4=0
100 S5=2
110 S6=(S2-S1)*0.5
120 S7=2/3
130 Loop:  FOR S8=1 TO S5-1 STEP 2
140 S4=S4+FNS(S1+S6*S8,R,N,C1)
150 NEXT S8
160 Fold=F
170 B:      F=(S3+2*S4)*S6*S7-.01541      ! RHS OF (35) SUBTRACTED
180 C:      IF S5<128 THEN 200             ! NEED MORE THEN 128 ?
190 D:      IF ABS(F-Fold)<1E-5 THEN RETURN F ! ERROR TOLERANCE OK ?
200 S3=S3+S4
210 S4=0
220 S5=S5*2
230 S6=S6*.5
240 GOTO Loop
250 FNEND
260 DEF FNS(X,R,N,C1)
270 T1=FNF(X,C1)
280 T2=EXP(-.5*X^2)
290 T3=X-R
300 IF N=2 THEN 330
310 T4=T1^(N-2)
320 GOTO 340
330 T4=1
340 S=C1*X*T4*((N-1)*T2*FNF(T3,C1)+T1*EXP(-.5*T3^2)-N*T2*T1)
350 RETURN S
360 FNEND
370 DEF FNP(P0,C1) ! PHI
380 P=ABS(P0)
390 IF P<7 THEN 420
400 P=0
410 GOTO 450
420 P=1/(1+.2316419*P)
430 P=P*(.31938153-P*(.356563782-P*(1.781477937-P*(1.821255978-P*1.33027448)))
440 P=C1+P*EXP(-.5*P0^2)
450 IF P0<0 THEN 470
460 P=1-P
470 RETURN P
480 FNEND

```

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