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LOCATION AND THE SPECTRAL DENSITY ESTIMATION OF MULTIPLE SOURCES[*]

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ABSTRACT

A new non parametric method for estimating the locations and power spectral densities of multiple wide-band sources from measurements provided by an array of sensors, is described. The proposed method treats with the same case multiple sources and multipath propagation. Direction-of-arrival estimation for wide-band sources is obtained as a limiting case of the proposed method.

I. INTRODUCTION

In radar, sonar and seismology one is frequently interested in determining the locations and the spectral densities ("signatures") of radiating sources from measurements provided by an array of sensors. The signals received by the sensors consist, in the simplest case, of sources to the sensors, may exist.

The conventional method for estimating the location of the source is based on a two-step procedure. First, the time-difference-of-arrival (TDOA) of the the propagating signal to the different sensors are estimated using a generalized correlator (see e.g., Knapp and Carter (1976)). Then, these TDOAs estimates are used to derive the corresponding lines-of-position whose "intersection" yields the location of the source. This method has serious shortcomings, the major one being the inability of the generalized correlator estimator to cope effectively with multiple sources and with multipath propagation (see e.g., Carter (1981)).

An interesting attempt to overcome this problem was outlined by Morf et al. (1979). Their basic idea was to use parametric models for the sources and the additive noises and then to use system identification techniques to estimate these parameters from the received signals. The TDOA estimates are then extracted from these parameters. This idea was further elaborated and developed by Porat and Friedlander (1981) and Nehorai and Morf (1982). The shortcoming of this method, as with any parametric method, is its sensitivity to the assumed model.

The maximum likelihood processor for estimating the location of multiple wideband sources has been recently presented by Wax and Kailath (1982a). This processor requires the knowledge of the spectral density matrix of the sources and hence its applicability is limited to the cases in which such knowledge is available.

In the special case that the sources are narrowband and are located in the far-field of the array, the problem degenerates to the estimation of the direction-of-arrival and the center-frequency of the radiating sources, namely to the 2-D harmonic retrieval problem. This problem has been addressed by Wax et al. (1982b), where a suboptimal method, based on the eigenstructure of the covariance matrix of the received signals, has been presented. This method is an extension of the method presented by Schmidt (1979) (see also Schmidt (1981)) for the special case that all the sources are co-frequency. A similar method for this special case, based on the eigenstructure of the spectral density matrix, has been presented by Bienvenu (1979) (see also Bienvenu and Kopp (1980), (1981)).

In this paper a new approach is presented for the problem of estimating the locations and the spectral density matrix of wideband sources. The approach is nonparametric and hence robust and can cope with multiple sources and multipath propagation. It is a one-step procedure based on the estimation of the eigenstructure of the spectral density matrix of the received signals. It extends the approach of Schmidt (1979) and Bienvenu (1979) for the case of wideband sources located in the near-field of the array. It is shown that the eigenvector subspace corresponding to the repeated smallest eigenvalue of the spectral density matrix contains all the information on the locations of the impinging sources. Algorithms for extracting this information that enable trade-offs between resolution and accuracy, are presented. Simulation results that demonstrate the performance of the proposed algorithms are also presented.

II. PROBLEM FORMULATION

Assume that we have m sensors and d $(d < m)$ sources distributed in the plane. Let (x_i, y_i) denote the coordinates of the i-th source. Each source is assumed to emit a zero-mean wide-sense-stationary signal that propagates radially with speed c. Denoting the received signal at the i-th sensor by $r_i(t)$, we can write

$$r_i(t) = \sum_{k=1}^{d} a_{ik} s_k(t - \tau_{ik}) + n_i(t) \quad (1)$$

$$-\frac{T}{2} \le t \le \frac{T}{2}, \quad 1 \le i \le m$$

where

$s_k(t)$ = the signal radiating by the k-th source

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τ_{ik} = the propagation time from the k-th source to the i-th sensor

a_{ik} = the attenuation from the k-th source to the i-th sensor

$n_i(t)$ = the additive noise at the i-th sensor

We assume that the noises $\{n_i(t)\}_{i=1}^{m}$ are wide-sense-stationary with zero-mean and *identical* spectral densities, and that they are uncorrelated with each other and with each of the signals $\{s_k(t)\ ,\ k = 1, \ldots, d\}$.

Since the "observation" interval $T_o = [-\frac{T}{2}, \frac{T}{2}]$ is finite, we can represent the received signals (1) by either a Fourier series

$$r_i(t) = \frac{1}{T^{1/2}} \sum_{n=-\infty}^{\infty} R_i(\omega_n) e^{-j\omega_n t} \tag{2}$$

(which implies the periodicity of $r(\cdot)$ outside the interval T_o) or by the inverse Fourier-transform of a Whittaker (sampling) series:

$$r_i(t) = F^{-1}\left\{ T^{1/2} \sum_{n=-\infty}^{\infty} R_i(\omega_n) \frac{\sin(\omega_n \frac{t}{2} - n\pi)}{(\omega_n \frac{t}{2} - n\pi)} \right\} \tag{3}$$

(which implies that our processes vanish almost everywhere outside the observation interval). In each case, the Fourier coefficients $\{R_i(\omega_n)\ ,\ \omega_n \in B$, where B is the bandwidth of the processes $\}$, are given by

$$R_i(\omega_n) = \frac{1}{T^{1/2}} \int_{-\frac{T}{2}}^{\frac{T}{2}} r_i(t)\, e^{-j\omega_n t}\, dt \tag{4a}$$

where

$$\omega_n = \frac{2\pi n}{T}, \quad n = 0, 1, 2, \cdots \tag{4b}$$

Note that since $r_i(\cdot)$ is real,

$$R_i(\omega_{-n}) = R_i^*(\omega_n) \quad \text{for all } n \tag{4c}$$

where $*$ denotes the complex conjugate. Thus we need only to consider positive "frequencies", i.e., ω_n with $n > 0$.

Taking the Fourier coefficients of both sides of (1), assuming $\tau_{ki} \ll T$, we obtain

$$R_i(\omega_n) = \sum_{k=1}^{d} a_{ik} S_k(\omega_n) e^{-j\omega_n \tau_{ik}} + N_i(\omega_n) \quad i=1,\ldots,m \tag{5}$$

or rewritten in matrix notation

$$\mathbf{R}(\omega_n) = \mathbf{A}(\omega_n)\mathbf{S}(\omega_n) + \mathbf{N}(\omega_n) \tag{6a}$$

where $\mathbf{R}(\omega_n)$, $\mathbf{S}(\omega_n)$ and $\mathbf{N}(\omega_n)$ are the $m \times 1$ vectors

$$\mathbf{R}(\omega_n) = \begin{bmatrix} R_1(\omega_n) \\ \vdots \\ R_m(\omega_n) \end{bmatrix} \quad \mathbf{S}(\omega_n) = \begin{bmatrix} S_1(\omega_n) \\ \vdots \\ S_m(\omega_n) \end{bmatrix}$$

$$\mathbf{N}(\omega_n) = \begin{bmatrix} N_1(\omega_n) \\ \vdots \\ N_m(\omega_n) \end{bmatrix} \tag{6b}$$

and $\mathbf{A}(\omega_n)$ is the $m \times d$ matrix

$$\mathbf{A}(\omega_n) = \begin{bmatrix} a_{11} e^{-j\omega_n \tau_{11}} & \cdots & a_{1d} e^{-j\omega_n \tau_{1d}} \\ \vdots & & \vdots \\ a_{m1} e^{-j\omega_n \tau_{m1}} & & a_{md} e^{-j\omega_n \tau_{md}} \end{bmatrix}. \tag{6c}$$

Note that the i-th column of $\mathbf{A}(\omega_n)$ corresponds to the attenuations and delays of the i-th source to the different sensors. *Motivated by this we define the "location vector"* of the i-th source as,

$$\mathbf{A}_{a_i,\tau_i}(\omega_n) = \begin{bmatrix} a_{1i} e^{-j\omega_n \tau_{1i}} \\ \vdots \\ a_{mi} e^{-j\omega_n \tau_{mi}} \end{bmatrix} \tag{6d}$$

Observe that since $\mathbf{S}(\omega_n)$ and $\mathbf{N}(\omega_n)$ are zero-mean, so is $\mathbf{R}(\omega_n)$. Thus it follows from (6) that the covariance matrix of $\mathbf{R}(\omega_n)$ is given by

$$E\mathbf{R}(\omega_n)\mathbf{R}^+(\omega_n) = \mathbf{A}(\omega_n)E\mathbf{S}(\omega_n)\mathbf{S}^+(\omega_n)\mathbf{A}^+(\omega_n) + E\mathbf{N}(\omega_n)\mathbf{N}^+(\omega_n) \tag{7}$$

where $+$ denotes the complex-conjugate transpose. Now, it is well known (see, e.g., Whalen (1971), p. 81) that if the observation interval is large compared to the correlation time of the processes $\{r_i(t)\}_{i=1}^{m}$, $\{s_k(t)\}_{k=1}^{d}$ and $\{n_i(t)\}_{i=1}^{m}$, then

$$E\mathbf{R}(\omega_n)\mathbf{R}^+(\omega_n) = \mathbf{K}(\omega_n) \tag{8.a}$$

$$E\mathbf{S}(\omega_n)\mathbf{S}^+(\omega_n) = \mathbf{P}(\omega_n) \tag{8.b}$$

$$E\mathbf{N}(\omega_n)\mathbf{N}^+(\omega_n) = \mathbf{Q}(\omega_n) \tag{8.c}$$

where $\mathbf{K}(\omega_n)$, $\mathbf{P}(\omega_n)$ and $\mathbf{Q}(\omega_n)$ are the power spectral density matrices of the processes $\{r_i(\cdot)\}$, $\{s_k(\cdot)\}$ and $\{n_i(\cdot)\}$, respectively, at the frequency ω_n. Now, since we have assumed that the noises $\{n_i(\cdot)\}$ are uncorrelated processes with the same spectral densities it follows that the noise power spectral matrix is given by

$$\mathbf{Q}(\omega_n) = \sigma^2(\omega_n)\mathbf{I} \tag{9}$$

where $\sigma^2(\omega_n)$ is the (scalar) power spectral density of each of the noises $\{n_i(t)\}$ at the frequency ω_n and $\mathbf{I}$ is the identity matrix.

Equation (7) can then be rewritten as

$$\mathbf{K}(\omega_n) = \mathbf{A}(\omega_n)\mathbf{P}(\omega_n)\mathbf{A}^+(\omega_n) + \sigma^2(\omega_n)\mathbf{I}\ . \tag{10}$$

which is the basic relation in the forthcoming analysis.

III. EIGENDECOMPOSITION OF THE SPECTRAL DENSITY MATRIX

Observing the structure of the spectral density matrix, as given by (10), assuming that the rank of $\mathbf{A}(\omega_n)$ is d (i.e., that the "location vectors" of the d sources are linearly independent) and that $\mathbf{P}(\omega_n)$ is positive definite, it can easily be shown (see e.g., Schmidt (1979) or Bienvenu (1979)) that the minimal eigenvalue of $\mathbf{K}(\omega_n)$ is equal to $\sigma^2(\omega_n)$ with multiplicity $m-d$, and that the corresponding eigenvector subspace is orthogonal to the columns of the matrix $\mathbf{A}(\omega_n)$, namely, to the "*location vectors*" of the sources.

Thus, denoting by $\{\lambda_i(\omega_n)\}$ and $\{\mathbf{V}_i(\omega_n)\}$ the eigenvalues and eigenvectors, respectively, of $\mathbf{K}(\omega_n)$, where

$$\lambda_1(\omega_n) \le \lambda_2(\omega_n) \le \ldots \le \lambda_m(\omega_n)$$

it follows that

$$\lambda_1(\omega_n) = \ldots = \lambda_{m-d}(\omega_n) = \sigma^2(\omega_n) \qquad (11.a)$$

and

$$\{V_i(\omega_n), i = 1,\ldots,m-d\} \perp \{A_{x_i,y_i}(\omega_n), i = 1,\ldots,d\}, \qquad (11.b)$$

where $\perp$ denotes orthogonality.

In practice $K(\omega_n)$ is not known, so that an *estimate* $\hat{K}(\omega_n)$ must be formed from the data. The eigenvalues and eigenvectors of $\hat{K}(\omega_n)$ obey only asymptotically the relations (11). The multiplicity of the smallest $m-d$ eigenvalues of $K(\omega_n)$ (11a) is reflected in the estimate $\hat{K}(\omega_n)$ as a "cluster" of the smallest $m-d$ eigenvalues, and the orthogonality condition (11.b) is reflected as

$$\sum_{i=1}^{m-d} |A_{x_i,y_i}(\omega_n)\hat{V}_i^*(\omega_n)|^2 \approx 0 \qquad (12)$$

Since (12) holds for every $\omega_n \in B$, two reasonable "measures" of the "closeness to orthogonality" over the whole bandwidth B are either the sum or the product of the individual "measures" (12) at each frequency bin, in the sense that

$$\sum_{\omega_n \in B} \sum_{i=1}^{m-d} |A_{x_i,y_i}(\omega_n)\hat{V}_i^*(\omega_n)|^2 \approx 0 \qquad (13a)$$

and also

$$\prod_{\omega_n \in B} \sum_{i=1}^{m-d} |A_{x_i,y_i}(\omega_n)\hat{V}_i^*(\omega_n)|^2 \approx 0 \qquad (13b)$$

IV. DETECTION OF THE NUMBER OF SOURCES

Determining the $m-d$ eigenvectors that correspond to the repeated smallest eigenvalue requires the knowledge of the number of sources d. In practice d is usually unknown and hence it must also be *estimated* from the data.

The problem of estimating d is equivalent to the problem of determining the multiplicity of the smallest eigenvalue of the spectral density matrix $K(\omega_n)$. This problem can be formalized as a hypothesis test

$$H_d(\omega_n) : \lambda_1(\omega_n) = \quad = \lambda_{m-d}(\omega_n)$$
$$A : \lambda_1(\omega_n) \neq \quad = \lambda_{m-d}(\omega_n)$$

It can be shown (see e.g Priestley et al. (1973)) that the likelihood-ratio statistic for this problem is given by the ratio of the geometric mean and the arithmetic mean of $m-d$ smallest eigenvalues of $K(\omega_n)$.

$$L_d(\omega_n) = \frac{\prod_{i=1}^{m-d} \hat{\lambda}_i(\omega_n)^{1/m-d}}{\frac{1}{m-d}\sum_{i=1}^{m-d} \hat{\lambda}_i(\omega_n)} \qquad (14a)$$

Under the null hypothesis the statistic $-2\ln L_d(\omega_n)$ is asymptotically distributed as χ^2 with $(m-d)^2-1$ degrees of freedom (see Priestley et al. (1973)).

To minimize the probability of error, this test should be performed for every $\omega_n \in B$. Now, since the Fourier-coefficient corresponding to different frequencies are independent it follows that likelihood ratio statistic for the different frequencies are independent. Thus, the composite likelihood ratio statistics for determining the number of sources is given by

$$L_d = \prod_{i=1}^{M} L_d(\omega_i) \qquad (14b)$$

and is distributed as χ^2 with $M[(m-d)^2-1]$ degrees of freedom. The way to implement this test is to apply it sequentially to $d = 0,1,\ldots,m-1$ and to choose $\hat{d}$ as the value of d for which $-2\ln L_d$ crosses the χ^2 significance level threshold.

V. SOURCE LOCATION ESTIMATION

Let $A_{x,y}(\omega_n)$ denote the "location vector" corresponding to a source at the point (x,y) and let $\{A_{x,y}(\omega_n)\}_{\omega_n \in B}$ denote the collection of the "location vectors" of all possible locations $\{(x,y)\}$ in the plane. These vectors can either be measured in the field or computed analytically, using the appropriate propagation model, if measuring is not feasible. The estimate of the sources locations is obtained by computing and plotting

$$\frac{1}{\sum_{\omega_n \in B} \sum_{i=1}^{m-d} |A_{x,y}(\omega_n)\hat{V}_i^*(\omega_n)|^2} \qquad (15a)$$

or

$$\frac{1}{\prod_{\omega_n \in B} \sum_{i=1}^{m-d} |A_{x,y}(\omega_n)\hat{V}_i^*(\omega_n)|^2} \qquad (15b)$$

as a 2-D function of (x,y). The d peaks of these 2-D function constitute a good estimate of the locations $\{(x_i,y_i)\}_{i=1}^{d}$, since for this values the denominators of (15) should be "close" to zero according to (13). The two estimators give in (15) differ in their resolution and stability properties. For the first estimator (15a), it is clear that there will be a peak in the point (x,y) if and only if the "location vector" $A_{x,y}(\omega_n)$ corresponding to this point is "closely orthogonal", at *all* frequencies $\omega_n \in B$, to the eigenvectors $\{V_i(\omega_n), i = 1,\ldots,m-d\}$. The second estimator (15b), will have a peak at a point (x,y) even if the "location vector" $A_{x,y}(\omega_n)$ corresponding to this point is "closely orthogonal", at only a *subset* of the frequencies $\omega_n \in B$, to the eigenvectors $\{V_i(\omega_n), i = 1,\ldots,m-d\}$. Thus the first estimator will show lower resolution but higher stability then the second estimator. Other estimators that enable a more delicate trade-off between resolution and stability are discussed by Wax et al.(1992b).

VI. SOURCE SPECTRAL DENSITY ESTIMATION

Having at hand the estimates $\{(\hat{x}_i,\hat{y}_i)\}_{i=1}^{d}$ of the sources' location we can construct an estimate $\hat{A}(\omega_n)$ of the matrix $A(\omega_n)$ simply by stacking together the d "location vectors" corresponding to the estimated positions of the d sources. Thus

$$\hat{A}(\omega_n) = [A_{\hat{x}_1,\hat{y}_1}(\omega_n) \cdots A_{\hat{x}_d,\hat{y}_d}(\omega_n)] \qquad (16)$$

Now, since the eigenvalues and eigenvectors of $A(\omega_n)P(\omega_n)A^*(\omega_n)$ are $\{\lambda_i(\omega_n) - \sigma^2(\omega_n), V_i(\omega_n), i = m-d+1,\ldots,m\}$ then from the well known spectral theorem of matrix theory (see e.g., Strang (1981)) an estimate of $A(\omega_n)P(\omega_n)A^*(\omega_n)$ can be constructed by

$$\hat{A}(\omega_n)\hat{P}(\omega_n)\hat{A}^*(\omega_n) = \sum_{i=m-d+1}^{m} [\hat{\lambda}_i(\omega_n) - \hat{\sigma}^2(\omega_n)]\hat{V}_i(\omega_n)\hat{V}_i^*(\omega_n) \qquad (17a)$$

where $\hat{\sigma}^2(\omega_n)$ is the estimate of $\sigma^2(\omega_n)$, given by

$$\hat{\sigma}^2(\omega_n) = \frac{1}{m-d}\sum_{i=1}^{m-d} \hat{\lambda}_i(\omega_n) \qquad (17b)$$

Rewriting this in matrix notation we obtain

$$\hat{A}(\omega_n)\hat{P}(\omega_n)A^*(\omega_n) = \hat{V}(\omega_n)[\hat{\Lambda}(\omega_n) - \hat{\sigma}^2(\omega_n)I]\hat{V}^*(\omega_n) \qquad (18a)$$

where

$$\hat{V}(\omega_n) = [\hat{V}_{d-m+1}(\omega_n) \ \cdots \ \hat{V}_m(\omega_n)] \qquad (18b)$$

and

$$\hat{\Lambda}(\omega_n) = \begin{bmatrix} \hat{\lambda}_{m-d+1}(\omega_n) & & \\ & \ddots & \\ & & \hat{\lambda}_m(\omega_n) \end{bmatrix} \qquad (18c)$$

Thus, solving (18) for the desired spectral density matrix $P(\omega_n)$, we obtain

$$\hat{P}(\omega_n) = [\hat{A}^*(\omega_n)A(\omega_n)]^{-1}A^*(\omega_n)\hat{V}(\omega_n)[\hat{\Lambda}(\omega_n) - \hat{\sigma}^2(\omega_n)I] \cdot \hat{V}^*(\omega_n)\hat{A}(\omega_n)[A^*(\omega_n)A(\omega_n)]^{-1} \qquad (19)$$

VII. DIRECTION-OF-ARRIVAL ESTIMATION OF WIDE-BAND SOURCES

Direction-of-arrival estimation is a limiting case of source location estimation corresponding to sources at "infinity". However, instead of deriving the direction-of-arrival estimation algorithm by limiting arguments let us go back to basics and see how to modify equation (1) for this case. Clearly $\{\tau_{ik}\}$ and $\{a_{ik}\}$ are meaningless for sources at infinity. A way around this problem is to define the reference not at sources' locations but at some arbitrary point in the plane, say, the origin of the coordinate axes. Equation (1) can then be rewritten as

$$r_i(t) = \sum_{k=1}^{d} a'_i(\vartheta_k)s_k(t - \tau_i(\vartheta_k)) + n_i(t) \qquad (20)$$

$$-\frac{T}{2} \le t \le \frac{T}{2} \qquad 1 \le i \le m$$

where

$\tau'_i(\vartheta_k)$ = the propagating delay between the i-th sensor and the reference point for a wavefront impinging from direction ϑ_k

$a'_i(\vartheta_k)$ = amplitude response of the i-th sensor to a wavefront impinging from direction ϑ_k

The analysis parallels that of the general case discussed in the previous sections, with the only difference that the *location vector* degenerates to a *direction vector* given by

$$A_{\vartheta_i}(\omega_n) = \begin{bmatrix} a'_{1i}e^{-j\omega_n\tau_1(\vartheta_i)} \\ \vdots \\ a'_{mi}e^{-j\omega_n\tau_m(\vartheta_i)} \end{bmatrix}$$

The direction-of-arrival estimation algorithm proceeds in the same manner as in the general source location problem, except that (15) are now 1-D functions plotted over all possible directions. The d peaks of this 1-D functions constitute a good estimate of the d unknown directions $\{\vartheta_i\}_{i=1}^{d}$.

VIII. SIMULATION RESULTS

Because of space limitations we will present only the result of a single simulation. The array consisted of three sensors in positions (1,0), (0,0) and (0,1) and of two sensors in positions (−1,5) and (10,20). The signals were independent ARMA processes with identical spectral density centered at 0.25 and of bandwidth 0.05. The additive noises were white processes and the signal-to-noise ratio was 10 dB. The estimation of the spectral density matrix was done by the periodogram method using an FFT of 64 points with a total of 300x64 samples. The results, using the estimator given by (15a) are shown in Fig.1. The two peaks corresponding to the two sources are clearly seen.

IX. CONCLUDING REMARKS

A new non-parametric approach for passive localization and identification of wideband sources, has been presented. The approach is capable of handling multiple correlated sources so that multipath propagation is included as a special case. Direction-of-arrival of multiple wideband sources is also included as a special case.

The method presented is based on the eigenstructure of the spectral density matrix of the received signals. It is an extension of the method of Schmidt (1979) and Bienvenu (1979) to the case of wideband sources. It was shown that the eigenvector sub-space corresponding to the repeated smallest eigenvalue of the spectral density matrix contains all the information on the sources locations. Algorithms for extracting this information from an estimate of the spectral density matrix of the received signals, that enable trade-offs between resolution and accuracy, have been presented. Simulation results that demonstrate the performance of the proposed algorithms have also been presented.

We should note that though the proposed method is non-parametric, in the sense that it was derived for non-parametrized signals, the estimation of the spectral density matrix of the received signals can be done by *any* method, non-parametric or parametric, whichever is believed to give a better estimate.

Computation-wise, the method is quite expensive since it involves eigenvalue-eigenvector decomposition of the spectral density matrix at each *frequency* bin of the received signals. However, since all these eigndecompositions can be carried out in parallel, using special-purpose hardware, the computation time can be reduced up to nearly real-time.

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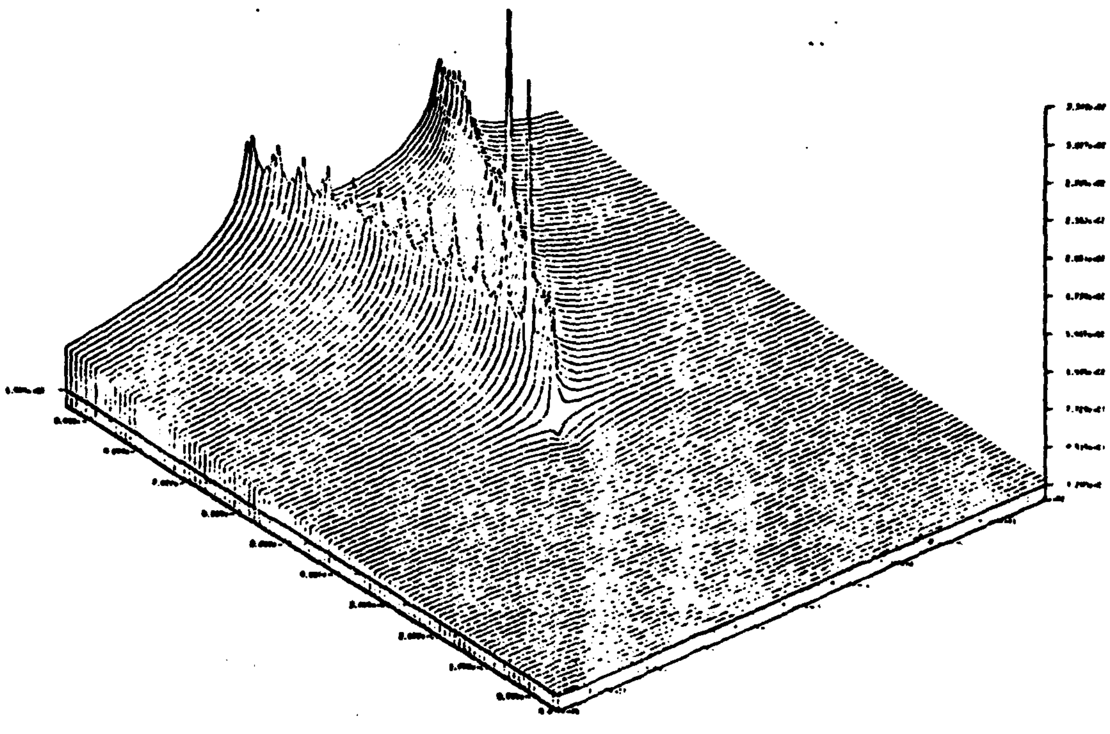

FIGURE 1