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TOWARD A CONTINUUM THEORY OF FRACTURE(U)  
CARNEGIE-MELLON UNIV PITTSBURGH PA DEPT OF MATHEMATICS  
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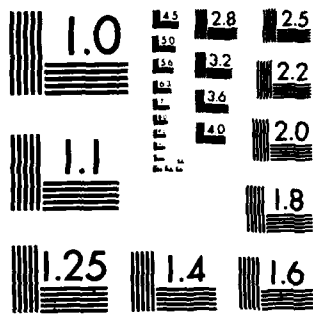
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This report reviews the work done on Grant AFOSR-76-3013 during the period 6/1/76-2/1/82. The research discussed here lies in the following areas: mechanics and thermodynamics of fracture, porous flow, optimal cooling of viscoelastic materials, theories of internal damage, variational principles.

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REPORT 21

FINAL REPORT (AFOSR-76-3013)



TOWARD A CONTINUUM THEORY OF FRACTURE

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### Research Summary

We have established a path-independent integral for linear elastodynamics (Report 1). For a plane crack this integral is related to the dynamic stress intensity factor, and, unlike previous results of this type (cf. Nilsson, Int. J. Solids Structures (1973), 1107), is valid for a running crack. We have extended the J-integral to bi-materials; in particular, to crack propagation along the interface between two materials (Report 4). This result should be applicable to fracture studies involving composite materials. We have also developed path-independent integrals for thermoelasticity (Report 13) and for elastostatics and elastodynamics (Report 5). The latter work involves pairs of fields and might be useful in problems involving mixed fracture modes.

Variational principles are important tools for the approximate solution of boundary-value problems. There are many types of variational principles, and each has its advantages and disadvantages. We have developed a method of utilizing a combination of variational principles, each for a given sub-region of the underlying region of space, so as to best utilize the chief benefits of the individual principles (Report 3). Patched principles of this type are particularly useful in solving problems involving cracks and other stress concentrations. Indeed, one usually wants the stress computed accurately near the crack, but specific results for the far field are not so important; it is therefore natural to patch together two principles,

one appropriate to the near-crack field and one for the far field. Patched principles are also useful in solving transonic flow problems. Here we use different principles in the elliptic and hyperbolic regions. The results of some numerical experiments for the Tricomi problem seem to indicate that our patched principle, when used in conjunction with the finite element method, leads to accuracy which is second-order in the mesh spacing, as compared to the standard numerical methods of solving this problem, which are only first-order.<sup>1</sup>

With a view toward solving actual crack and stress concentration problems, we have developed patched principles for linear elasticity (Report 6).

A linear theory was developed for the diffusion of a fluid through an elastic solid (Report 2). The basic equations have analogs in linear thermoelasticity theory, with either concentration or pore pressure playing the role of temperature. The usefulness of these analogies in the solution of actual problems is discussed.

We have also developed a theory of diffusion in which moisture is allowed to exist in two phases (Report 11). We believe that a model of this type might be applicable to polymeric materials reinforced with fibers: one phase might account for moisture flowing through the matrix, the second for moisture trapped around the fibers. Two phases might also be appropriate in situations for which moisture exists in free

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<sup>1</sup>The methods developed here are now being used at NASA-Langley, where a computer code based on our variational principle has been written.

and bound states. To study the behavior of our model we solved (in closed form) the following simple problem: a one-dimensional sample is dry at time  $t = 0$  and is suddenly subjected to a moist environment. Interestingly, the graph of total moisture versus  $\sqrt{t}$  shows a double plateau<sup>1</sup> for sufficiently small sample lengths, an effect not present in classical Fickian diffusion. Our model might also be appropriate to the study of hydrogen embrittlement in solids, in particular, to hydrogen trapping.

We have also developed basic thermodynamic laws for brittle fracture (Reports 7, 8, 10). Using these, we were able to show that the Griffith criterion is necessary for crack initiation, even in the presence of heat conduction; interestingly, the underlying stored energy turns out to be the free energy corresponding to the (not necessarily constant) initial temperature field. Report 10 presents our initial attempts to establish a thermodynamics of the cohesive zone preceding a running crack. We have developed appropriate forms of the first two laws for the body and for the cohesive zone, and have studied implications of these laws for elastic and viscoelastic cohesive zones.

We have used this theory to obtain explicit solutions (under certain approximative assumptions) for the temperature rise near the tip of a fast running crack (Report 17). For a (crack) surface temperature which is much higher than the

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<sup>1</sup>Such a double plateau is evident in the experimental results of McKague, Reynolds, and Halkias (J. Eng. Materials Tech. 98 (1976) 92) for 4-ply laminates of 5208 epoxy resin impregnated with graphite fibers.

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temperature of the surrounding environment, our solution agrees with experimental results of Fuller, Fox, and Field for Polymethyl Methacrylate (Proc. Roy. Soc. London ~~341A~~ (1975) 537).

We have given a clear derivation of several important results related to the energy release rate for a sharp crack in a homogeneous elastic body (Reports 9, 14). Our main result (Report 14) is a decomposition of the dynamic energy release rate into the usual quasi-static energy release rate plus a nonpositive dynamic contribution; thus for a dynamic solution the energy release rate computed using the classical quasi-static formula is larger than the actual dynamic energy release rate.

We have shown (Report 15) that the well-known formulae of Irwin and Achenbach for the quasi-static energy release rate are generally not valid for nonlinear elastic materials.

In Report 16 we proved that for a crack starting from the apex of a wedge the initial energy release rate vanishes. Thus for such problems the Griffith criterion is not applicable.

As is well known, Griffith's value for the fracture load corresponding to an infinite cracked plate under uniform tension at infinity is in error. Report 18 gives a theoretical explanation of Griffith's error.

When a fiber-reinforced viscoelastic material is cooled from cure temperature to room temperature residual stresses result because the coefficient of thermal expansion of the matrix differs from that of the fibers. To study this phenomenon in detail, we considered a thin, thermorheologically simple,



viscoelastic plate reinforced by fibers lying in the plane of the plate, and solved the following problem (Report 12):

Of all temperature paths  $\theta(t)$ ,  $0 \leq t \leq T$ , which have prescribed values at  $t = 0$  and  $t = T$ , find a path which renders the residual stress at  $t = T$  a minimum.<sup>1</sup> The Euler equation associated with this problem is a complicated nonlinear integral equation, which we used to show that optimal paths generally exhibit jump discontinuities at  $t = 0$  and  $t = T$ . For a Maxwell material with an exponential shift function we were able to solve the Euler equation in closed form. We used this result to compute the optimal temperature path for Polymethyl Methacrylate with initial and final temperatures  $90^\circ\text{C}$  and  $80^\circ\text{C}$ , respectively, and with  $T = 8$  hours. The optimal path produces a residual stress of 11 psi as compared to 246 psi for a linear path. The importance of this work lies not only in its industrial applications, but also in the fact that it represents a truly nonstandard optimization problem with a strong intrinsic mathematical value.

Certain highly filled polymeric materials, such as those used in solid rocket-propellants, exhibit a stress softening due to internal damage which renders the stress lower than expected whenever the material is unloaded from a stress

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<sup>1</sup>This problem was brought to our attention by Dr. Y. Weitsman (Dept. of Civil Engineering, Texas A.M.) during his stay at MRL, Wright-Patterson AFB, and we consulted with him during his initial investigations.

maximum. Previous attempts<sup>1</sup> to study internal damage resulted in models which are quite complicated in nature, a factor which renders intuitive insight difficult and which leads to a complicated program for the characterization of real materials. Indeed, all previous characterizations using these models were based on "least-squares" procedures in conjunction with a large experimental base, with the entire base being used in one fell swoop to determine all of the underlying constants. As is well known, procedures of this type can lead to large errors when the model is extrapolated outside of the experimental range. Report 19 gives our initial attempts to develop a simple theory of stress softening.<sup>2</sup> The theory presented is based on two fundamental ingredients. The first is the virgin stress, which represents the stress the material would experience in the absence of softening. This stress is assumed governed by a constitutive equation of the type encountered in linear viscoelasticity. The second ingredient is a damage function, which gives the true stress when the virgin stress and its past maximum are known. We show that our theory predicts effects observed in

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<sup>1</sup>Farris, R. J., The stress-strain behavior of mechanically degradable polymers. In: Polymer Networks: Structural and Mechanical Properties. A. J. Chompff and S. Newman, eds., New York: Plenum (1971); Farris, R. J. and J. E. Fitzgerald, Deficiencies of viscoelastic theory as applied to solid propellants. Bull. MANNAF Mech. Beh. Wkg. Group, 8th Mtg., CPIA Pub. 193, March 1970; Quinlan, M. J., Materials with variable bonding. Arch. Rational Mech. Anal. 68, 165-181 (1978).

<sup>2</sup>An extension of the theory presented here is now being used at Los Alamos National Laboratory to characterize high explosives.

practice during stress relaxation and during repeated ramp loadings. We show further that, in contrast to previous theories, the characterization of real materials using our model is quite simple: (i) ramp loadings with constant strain-rate can be used to determine the virgin relaxation function; that is, the function that appears in the linear constitutive equation for the virgin stress; (ii) certain types of simple sawtooth loadings can be used to find the damage function. Other experiments, such as stress-relaxation, can then be utilized as consistency checks.

Report 20 gives a simple rate-independent model for materials undergoing internal damage. The measure of damage is the past strain maximum. Experimental data justifying this theory are given.

We found that the theory presented in Report 19 is better cast in terms of strain (rather than stress) maxima, and a start was made in this direction. We also studied various methods of characterizing real materials - in particular, explosives and solid propellants - and began a systematic development of the necessary computational software.

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15. Yatomi, C., On Irwin's and Achenbach's expressions for the energy release rate. *Int. J. Fracture* 18 (1982) 233.

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18. Yatomi, C., On Griffith's theory. Int. J. Fracture 16 (1980) R239.
19. Gurtin, M. E., On a simple nonlinear model for viscoelastic materials that exhibit stress softening. Report only.
20. Gurtin, M. E. and E. C. Francis, On a simple rate-independent model for damage. J. Spacecraft Rockets 18 (1981) 285.

### Consultations to Other Laboratories

Dr. M. J. Quinlan, during his stay at AFRPL (Edwards AFB), developed a model for nonlinear viscoelastic behavior. Quinlan's model includes the effects of damage, and therefore should be applicable in the stress analysis of rocket grains. The principal investigator spent much time consulting with Dr. Quinlan in his investigations.

Report 12 (on optimal temperature paths for thermorheologically simple viscoelastic materials) was the result of consultations with Dr. Y. Weitsman of MRL (Wright-Patterson AFB).

### Students Receiving Ph.D.'s on This Grant

Lea F. Murphy (Thesis: Minimization of work and stress in linear viscoelasticity).

Chikayoshi Yatomi (Thesis: On the dynamic energy release rate in elastic crack propagation).

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