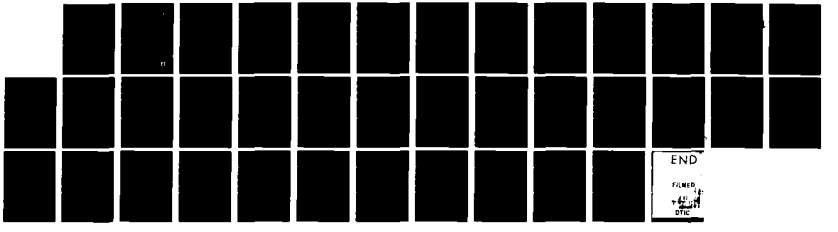
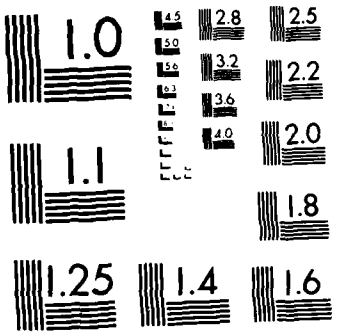
AD-A126 321 ON THE DIFFERENCE BETWEEN WAVES AND TURBULENCE IN A STRATIFIED FLUID(U) AIR FORCE GEOPHYSICS LAB HANSCOM AFB MA E M DEWAN 02 NOV 82 AFGL-TR-82-0352 1/1

UNCLASSIFIED F/G 20/4 NL





MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

AFGL-TR-82-0352
PHYSICAL SCIENCES RESEARCH PAPERS, NO. 661

On the Difference Between Waves and Turbulence in a Stratified Fluid

EDMOND M. DEWAN

2 NOVEMBER 1982

AERONOMY DIVISION PROJECT 6687
AIR FORCE GEOPHYSICS LABORATORY
HANSCOM AFB, MASSACHUSETTS 01731

AIR FORCE SYSTEMS COMMAND, USAF

This report has been reviewed by the ESD Public Affairs Office (PA) and is releasable to the National Technical Information Service (NTIS).

This technical report has been reviewed and is approved for publication.

Alva T Stair, Jr.

DR. ALVA T. STAIR, Jr
Chief Scientist

Qualified requestors may obtain additional copies from the Defense Technical Information Center. All others should apply to the National Technical Information Service.

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFGL-TR-82-0352	2. GOVT ACCESSION NO. AD-A126	3. RECIPIENT'S CATALOG NUMBER 321
4. TITLE (and Subtitle) ON THE DIFFERENCE BETWEEN WAVES AND TURBULENCE IN A STRATIFIED FLUID		5. TYPE OF REPORT & PERIOD COVERED Scientific. Final. 6. PERFORMING ORG. REPORT NUMBER PSRP No. 661
7. AUTHOR(s) Edmond M. Dewan		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Air Force Geophysics Laboratory (LKD) Hanscom AFB Massachusetts 01731		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 66870507 62101F
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Geophysics Laboratory (LKD) Hanscom AFB Massachusetts 01731		12. REPORT DATE 2 November 1982 13. NUMBER OF PAGES 39
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified 15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Waves
Turbulence
Atmospheric physics
Optical turbulence
Polution
C_N^2

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

What are the theoretical and experimental physical differences between waves and turbulence? The motivation behind this question is related to the practical problems associated with laser beam propagation and pollution transport in the atmosphere. Because turbulence causes mixing and waves do not, one must not regard turbulence as a field of random waves. The power density spectrum of velocity fluctuations, when taken alone, cannot be used to distinguish between waves and turbulence. Its physical interpretation

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

Unclassified

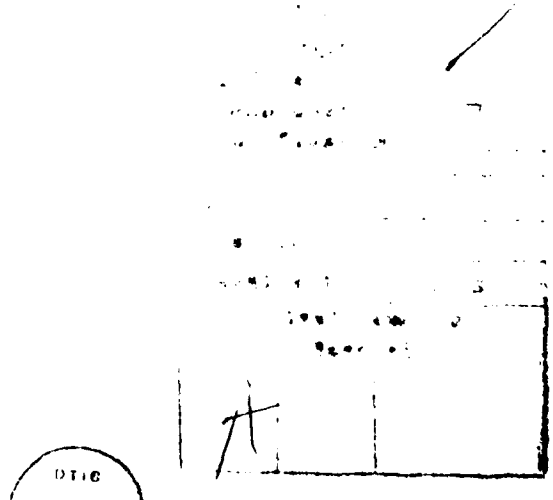
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

20. Abstract (Contd)

can differ radically depending upon which type of motion is involved. Two approaches are used here to differentiate theoretically and experimentally between waves and turbulence. The first involves the degree of interaction between modes. Theoretically, this leads to a new interpretation of the buoyancy length, k_B^{-1}, and it raises questions of the nature of the buoyancy subrange of turbulence. Experimentally, it leads to new suggestions for distinguishing between wave and turbulent motion, for example, by means of bi-spectral coherence. The second approach depends on the mixing property of turbulence. Theoretically, the mixing is related to strong mode-interaction, which is physically due to a vortex-stretching cascade process. Experimentally one can use mixing to distinguish between waves and turbulence by means of a procedure suggested by Busch. The latter procedure uses the cross-spectrum between vertical velocity and potential temperature fluctuations. It is described here in some detail and is generalized for possible application to the case of two-dimensional turbulence. In addition, a wave cascade model is proposed in order to explain power spectra observed in project HICAT, and a test is suggested for this theory.

Preface

I wish to thank Ralph Shapiro and Thomas VanZandt for many stimulating conversations on this topic. I also appreciate the comments made by Earl Good, Chris Stergis, and Jerome Weinstock.



Contents

1.	INTRODUCTION	7
2.	"RANDOM WAVES" VS "TURBULENCE"	10
3.	THE BUOYANCY LENGTH	11
4.	EXPERIMENTAL CRITERIA TO DISTINGUISH BETWEEN WAVES AND TURBULENCE	12
5.	REMARKS ON THE RELATION BETWEEN STRONG MODE-INTERACTION AND "MIXING", AND SPECTRAL CONSIDERATIONS	14
6.	TWO-DIMENSIONAL TURBULENCE	18
7.	THE WAVE CASCADE EXPLANATION FOR A $k^{-5/3}$ SPECTRUM	19
8.	ON THE NATURE OF THE BUOYANCY SUBRANGE OF TURBULENCE AND A SCALE-LENGTH CRITERION	24
9.	CONCLUSIONS	28
	REFERENCES	29
	BIBLIOGRAPHY	33
	APPENDIX A: NEW DEVELOPMENTS IN TURBULENCE THEORY	35

Illustrations

1. Power Spectra From Project HICAT (Crooks et al[1]) 8
2. Vortex-Stretching Mechanism in a Turbulent Cascade (After Tennekes and Lumley[24]) 16
3. Schematic Representation of a Power Spectrum of Waves and Turbulence in a Steady-State Cascade 25

On the Difference Between Waves and Turbulence in a Stratified Fluid

1. INTRODUCTION

Velocity fluctuations in the stratosphere that were presumed to be turbulence were measured (Crooks et al[1]) by means of an instrumented U-2 aircraft in project HICAT. Figure 1 shows some power spectra of such fluctuations plotted on a log-log scale. Typical of all such data in their report, these spectral display a -5/3 slope. This happens to be the same slope as that predicted by Kolmogorov[2] for the case of "inertial range" turbulence. Some authors such as Lilly et al[3] have indeed interpreted this spectrum as being in the inertial range of turbulence.

Other authors have noted that the buoyancy length is too small to permit an inertial range interpretation (Zimmerman and Loving[4]) and that a more likely explanation would be that it is some sort of buoyancy subrange turbulence. In other words, the inertial range assumption appears to be contradicted by the fact that the 5/3 spectrum is almost entirely at scales larger than the buoyancy length (in Figure 1). Following this suggestion, both Weinstock[5] and Dewan[6], using different arguments, attempted to show that it is theoretically possible to have a buoyancy subrange with a -5/3 dependence. This is in contrast to the original theories of the buoyancy subrange proposed by Bolgiano,[7] Shur,[8] and Lumley.[9,10]

(Received for publication 29 October 1982)

Because of the large number of references cited above, they will not be listed here. See References, page 27.

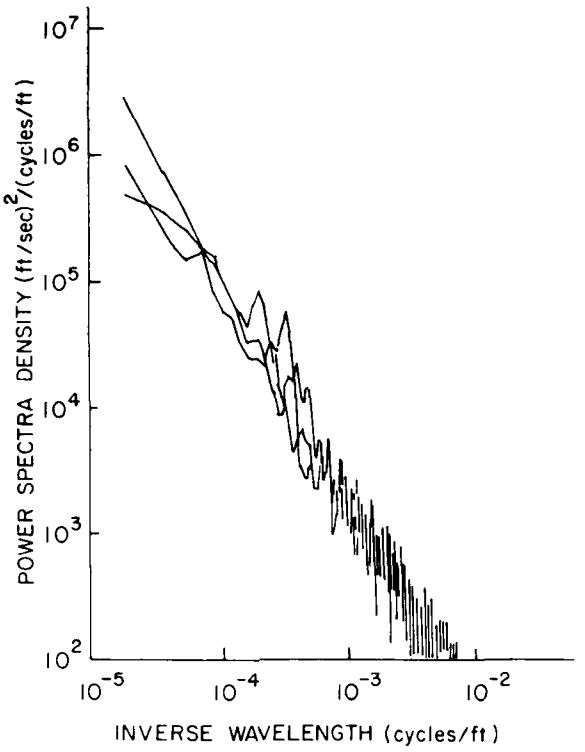


Figure 1. Power Spectra From Project HICAT (Crooks et al[1])

These original theories and others are summarized best by Lin et al,[11] who also generalized the theory of buoyancy subrange turbulence. The original theories predicted a much steeper slope for this subrange, that is, -3 for the Lumley-Shur theory and -11/5 for that of Bolgiano.

While an inertial range approach toward the understanding of Figure 1 appears reasonable at first sight, it seems untenable if one accepts the experimental evidence (see Barat and Aimedieu,[12] Crane,[13] Anderson,[14] and

---

11. Lin, J. T., Panchev, S., and Cermak, J. (1969) A modified hypothesis on turbulence spectra in the buoyancy subrange of stably stratified shear flow, Radio Sci. 4:1333-1337.

12. Barat, J., and Aimedieu, P. (1981) The external scale of clear air turbulence derived from the vertical ozone profile: application to vertical transport measurement, J. Appl. Met. 20:275-280.

13. Crane, R.K. (1980) Radar measurements of wind at Kwajalein, Radio Sci. 15:383-394.

14. Anderson, A.D. (1957) Free-air turbulence, J. Meteorol. 14:477-494.

Rosenberg and Dewan[15]) concerning the average thickness of turbulent layers in the stratosphere. These cited references agree that the thickness, which presumably represents an "outer length," is of the order of 100 m, which is several orders of magnitude smaller than the largest scales of Figure 1. While one piece of experimental evidence seems to indicate that 1.0-km thick layers are not uncommon (Crooks et al[16]), it has been argued elsewhere (Dewan[17]) that the latter evidence was based on a dubious assumption, namely that turbulence was responsible for the fluctuations in question. Instead, these fluctuations seem more likely to be due to waves. This is supported by both outer-length considerations and buoyancy-length considerations. In any case, one purpose of the present report is to explain Figure 1, with the aid of the hypothesis that the data of Crooks et al[1,16] represent primarily waves and not inertial range turbulence.

This leads to the main issue of this paper, which is to explain the theoretical and experimental difference between waves and turbulence. Some authors have referred to randomly distributed gravity waves as "turbulence". As will be shown later, however, such an approach appears to be even less useful than the position taken in the present paper [illegible] turbulence are mutually exclusive in some of their properties. [illegible] mainly upon the publications of Phillips[18,19] [illegible], and Stewart[20] the relevant properties of waves and turbulence are [illegible] and [illegible] briefly.

The important [illegible] between wave and turbulence is that ideally waves create no mixing, transport (as examples in [illegible] problems), or generation of [illegible] (for example, in the case of laser beam propagation [illegible]

That is, waves without decay, dissipation, or mode interactions. Such waves do not exist in the real world; but, at times they serve as good approximations.

15. Rosenberg, N.W., and Dewan, E.M. (1967) Stratospheric Turbulence and Vertical Effective Diffusion Coefficients, AFCRL-TR-67-0511, AD 662 197.

16. Crooks, W., Hobart, F., Propnet, D., et al. (1967) Project HICAT, An Investigation of High Altitude Clear Air Turbulence, AD 834 080.

17. Dewan, E.M. (1968) Stratospheric wave spectra resembling turbulence, Science, 204, 832-835.

18. Phillips, O.M. (1966) The Dynamics of the Upper Ocean, Cambridge University Press (1st ed.), 261 pp.

19. Phillips, O.M. (1977) The Dynamics of the Upper Ocean, Cambridge University Press (2nd ed.), 336 pp.

20. Poisson, N.I., et al. (1968) Waves and turbulence, Radio Sci., 4, 1357-1370.

21. Stewart, R.W. (1969) Turbulence and waves in a stratified atmosphere, Radio Sci., 4, 1269-1278.

2. "RANDOM WAVES" VS "TURBULENCE"

Phillips[18,19] considered the case of a field of random internal gravity waves. In particular, he pointed out that in this situation the transition to turbulence would not follow the usual sequence of "instability" (for example, Kelvin-Helmholtz or Rayleigh-Taylor) followed by a sudden "turbulent breakdown". Instead, the "degeneration to turbulence" would be due to the development of stronger interactions between wave modes. As the waves increase in amplitude and slope, the interactions between modes would increase. When such interactions become sufficiently "promiscuous" (to use his apt term) one would then have a turbulent cascade. More precisely, Phillips considered wave interactions that involve a "resonant triad". The latter involves a three-wave interaction that satisfies certain special conditions. In this case, he showed that the interaction time, T_i, is of the order

$$T_i \sim (k_1 \alpha_1 k_2 \alpha_2)^{-1/2} \tag{1}$$

where the k's and α's refer to wave numbers and fluid particle velocities, respectively. The subindices refer to two of the three particular waves in the resonant triad (energy goes into the third wave from waves numbered 1 and 2). As the wave amplitudes increase, so also do the particle velocities; hence, from Eq. (1) T_i decreases. This decrease of T_i is a measure of the increase in the strength of the interaction. The shorter T_i becomes, the less wave-like do the waves in question become in the sense that they do not obey a dispersion relation. In other words, if a mode interaction is of sufficient strength to cause T_i to become comparable to a wave period (inverse Brunt-Vaisala frequency), the wave does not propagate but, instead, is a short-lived local entity better known as a "turbulent eddy". In the case of turbulent eddies there is no longer the requirement for a resonant triad in order to have interaction. The following are two of the crucial distinctions between waves and turbulent eddies: (a) waves obey dispersion relations; eddies or turbulent modes do not, and (b) waves last a long time and propagate whereas the opposite holds for eddies.

Ideally, a wave has infinite duration ($T_i = \infty$) and will "linearly superpose" with other waves (that is, no non-linear interactions). In the real world such requirements can only be approximated. In what follows we will not use so idealized a definition. Instead we shall use the term "waves" in a sense that allows finite duration (such that the decay time is much greater than a buoyancy period). We will also use "waves" in a sense that allows for weak non-linear interactions such that T_i is much greater than a buoyancy period.

Ideally, an eddy would lose its identity within one wave oscillation period and within one wavelength due to its strong interactions with other modes of oscillation. This is presumed to be a good description of turbulence in the real world.

3. THE BUOYANCY LENGTH

The previous considerations can shed new light on the physical meaning of the so-called "buoyancy length" that arises in Lumley's theory[9, 10] of the buoyancy subrange of turbulence. (See also the review of Phillips[22].) This buoyancy length is defined by

$$L_B = \frac{1}{2\pi} \left(\frac{\varepsilon}{N_B^3} \right)^{1/2} \tag{2}$$

where ε is the dissipation rate, N_B the buoyancy frequency, and C is a constant of proportionality. According to Lumley, this length separates the scales of the buoyancy subrange of turbulence from the inertial subrange, the latter of course containing the smaller scales. We now derive Eq. (2) from an entirely different (and more physical) approach than given by Lumley.

For convenience, suppose we consider the case of Eq. (1) where $k_1 = k_2 \equiv k$ and $\omega_1 = \omega_2 = N_B$. Also, we take the case where $T_i = N_B^{-1}$, which is to say that we consider the borderline case where the interaction time is of the order of the minimum wave period. This is the case where, according to the previous section, the oscillations or waves are turbulent-like in their promiscuity of interaction. From Eq. (1), we have

$$k = N_B \tag{3}$$

In other words, the wave mode in question gives up all its kinetic energy to a mode interaction cascade in one period of its oscillation. Now, the cascade must eventually lose energy at the *molecular* level at the small-scale end at the rate ε. Taking v^2 as the kinetic energy per unit of mass, and assuming the cascade to be conservative, we arrive at

22. Phillips, O.M. (1967) On the Bolgiano and Lumley-Shur Theories of the buoyancy subrange, in Atmospheric Turbulence and Radio Wave Propagation, A.M. Yaglom, and V.I. Tatarsky, Eds., NAUKA, Moscow, 374 pp.

$$\varepsilon = \frac{v^2}{N_B^{-1}} \tag{4}$$

Eliminating v in Eq. (3) by means of Eq. (4) and solving for k, we arrive at

$$k = \left(\frac{N_B^3}{\varepsilon}\right)^{1/2} , \tag{5}$$

which is exactly the same, in effect, as Eq. (2) for k_B.

This argument reveals a surprising result. The buoyancy length does not merely divide the isotropic from the anisotropic turbulence. In addition, it also gives the minimum wavelength for a horizontally propagating gravity wave. Many new questions are raised by the above considerations about the nature of buoyancy turbulence. For this reason, these issues will be discussed at length in Section 8.

4. EXPERIMENTAL CRITERIA TO DISTINGUISH BETWEEN WAVES AND TURBULENCE

Busch[20] and Stewart[21] have suggested criteria that would enable one to determine experimentally whether or not a given field of fluctuations is due to turbulence or to waves. The most promising of these criteria is based on the fact that turbulence causes mixing whereas waves do not. In the discussion given previously, the strength of the mode interactions was the criterion, and this raises the question of what the physical connection between these two different criteria might be. As will be explained, they are indeed closely related physically. In terms of practical applications concerning pollution transport and chemistry, as well as optical turbulence, it is the mixing property of turbulence that is the one of greatest interest.

Stewart[21] suggested that one simultaneously measure the vertical velocity fluctuations and potential temperature fluctuations. (In place of temperature one could measure any other scalar quantity such as the concentration or mixing ratio of a neutrally buoyant substance provided it had a significant vertical gradient). From these measurements one would then calculate the coherence between these

two fluctuations and this would give the degree of vertical mixing taking place. It would, in fact, give the vertical transport of the scalar quantity due to the turbulence. In the case of waves, this flux would be zero.

Busch[20] suggested a similar idea, but he introduced a useful scale dependence as follows. Instead of coherence he suggested that one calculate a cross-spectrum of the vertical velocity and temperature fluctuations. This spectrum, the reader will recall, has a real part (the co-spectrum) and an imaginary part (the quadrature spectrum). For an ideal wave, the vertical velocity and temperature fluctuations would be $(\pm\pi/2)$ out-of-phase or in quadrature. The cross-spectrum would be purely imaginary and there would be no net vertical transport. In the case of turbulence, there would be transport or mixing, and the phase of the cross-spectrum would be $(\pm\pi)$, that is, it would be real and with no imaginary component. The phase angle, which we shall call β, will in general depend on k, and it is defined as

$$\beta(k) \equiv \arctan\left(\frac{\text{Im } \Phi_{w'\theta'}(k)}{\text{Re } \Phi_{w'\theta'}(k)}\right) , \tag{6}$$

where $\Phi_{w'\theta'}(k)$ is the cross-spectrum between fluctuations of vertical velocity w', and potential temperature θ'. In general, therefore, for those values of k where $\beta = 90^{o}$ or 270^{o}, the motion would be wave-like; and, where $\beta = 0^{o}$, or 180^{o}, it would be turbulent-like. In between these two extreme cases the motion would be neither pure turbulence nor pure waves.

Axford[23] tested this approach experimentally in the stratosphere. He found that, for the cases of well-defined trains of waves, $\beta = 90^{o} \pm 10^{o}$. Furthermore, be found that the coherence spectrum [or $\{[\text{Im } \Phi_{w'\theta'}(k)]^2 + [\text{Re } \Phi_{w'\theta'}(k)]^2\}^{1/2}$ normalized by the individual power spectra of w' and θ'] had values greater than 0.8. This second finding is consistent with the idea that a wave-like disturbance has an extended periodic pattern in space. It gives a second test for waves to use in conjunction with the above "β test". In the cases of turbulent motion at the scale of k, Axford found that $-45^{o} < \beta(k) < +45^{o}$, or $135^{o} < \beta(k) < 225^{o}$ were valuable as criteria.

In view of these theoretical and experimental findings we arrive at one of the conclusions of this paper; namely it would be desirable to apply such tests to the kind of data described by Crooks et al,[1,16] that is, those upon which Figure 1 is based.

---

23. Axford, D. N. (1971) Spectral analysis of an aircraft observation of gravity waves, Quart. J. Met. Soc. 97:313-321.

Other suggestions will be found in Busch.[20] We only mention that the individual power spectra of θ' and of w' when taken in combination would also be helpful in distinguishing between turbulence and waves. In the case of turbulence, they would be comparable in shape and magnitude. In the case of waves they would differ greatly in magnitude with the θ' spectrum being much smaller than the w' spectrum, especially at high values of k.

5. REMARKS ON THE RELATION BETWEEN STRONG MODE-INTERACTION AND "MIXING", AND SPECTRAL CONSIDERATIONS

The purpose of this section is to (a) explain the physical relationship between strong interactions between modes of oscillation and the mixing property of turbulence. In the process we shall examine the physical nature of turbulence, going well beyond the previous discussions; and (b) examine the ambiguous nature of the interpretation of the power spectrum in this context. The latter seems to have been a frequent source of confusion in the literature. We shall conclude with some remarks about the use of higher-order spectra for the purpose of directly measuring strong interactions.

We begin by asking, "What is the connection between promiscuous mode interaction and mixing (that is to say, mixing in physical space)?" Consider first the case of idealized, small-amplitude waves that exhibit perfect superposition or non-interaction between each other. Take, for example, the case of a field of random internal waves. Can these cause mixing? A single wave would not cause mixing because the motion would be periodic, and a particle of fluid would periodically return to a fixed location in space. Two waves of commensurate frequency would obviously have the same effect, but the time between returns would be longer [being $2\pi((q/\omega_1) + (p/\omega_2))$ where the ω_i are the wave frequencies and p and q are integers]. In other words, the motion would still be periodic. What happens in the case where there are a very large number of waves with incommensurate frequencies? Since we assume complete superposition, any fluid particle will remain permanently within a certain neighborhood even though it will never return to a certain exact location, as would have been the case for periodic motion. More importantly, nearest neighbors will remain nearest neighbors, so there is no effective diffusion or mixing or net transport of any kind.

The question we next consider is, "How do strong mode-interactions give rise to mixing (or "diffusion")?" If a wave mode decays in a very short time, particles will not be left in their initial positions. A random superposition of such [illegible], non-propagating modes (that is, eddies) would therefore be expected to cause some degree of transport. While this picture opens up the possibility for mixing, the random vorticity property of turbulence makes the case for mixing far more convincing. This description will be found in Tennekes and Lumley,[24] and [illegible]. One should imagine that the eddy motion is associated with not only random vorticity but a "strain-rate field" (or random deformations) as well. [illegible], the random deformations that cause the particles, which start [illegible], to eventually become widely separated or diffused. The question [illegible] "How do strong deformations arise from strong interactions [illegible]?"

[illegible] Lumley[24] represent the mode interactions as a cascade in k or [illegible]. This cascade from small to large k is simply [illegible] represen- [illegible] famous poem, "Big whirls have little whirls that feed on [illegible] whirls [illegible] lesser whirls, and so on to viscosity." [illegible] are due to vortex stretching. That is to say, in the [illegible] the deformation (vortex-stretch mechanism) is the mode inter- [illegible] based on the diagram of Tennekes and Lumley,[24] shows how a [illegible] large-scale strain rate can cause the stretching of a vortex [illegible]. The vortex that becomes stretched has its vorticity amplified [illegible] conservation of angular momentum. As shown in the figure, the [illegible] direction of negative strain rate has its vorticity decreased. [illegible] is an increase in vorticity at the smaller scale, and [illegible] energy is transferred from larger to smaller scales. Tennekes [illegible] of this process in Chapter [illegible], which [illegible] the physics of [illegible]

[illegible]

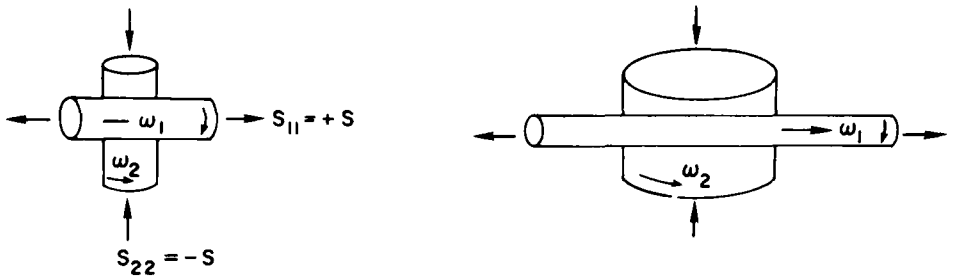


Figure 2. Vortex-Stretching Mechanism in a Turbulent Cascade (after Tennekes and Lumley[24]). The stress field is given by the arrows. Larger-scale deformation gives energy to the small-scale vortices by stretching them. This causes the small-scale vortices to increase their spin rotation due to conservation of angular momentum

the cascade. The discussion is continued in an article by Tennekes[25] where he gives a convincing argument that shows that a large eddy breaks up into two "daughters,* each of which is about half as big as the mother". He then adds, "The evolution of and interaction between eddies involve non-linear mixing, both in coordinate space and in wave number space." As was already mentioned, the former is due to the random rates of strain caused by the cascade process. It should now be clear why strong mode-interaction implies mixing.** The deformation, since it separates nearest neighbors of fluid particles, is the cause of the mixing. The deformation, in turn, is due to mode interactions.

The above sort of cascade picture led Kolmogorov[2] to his famous -5/3 spectrum by means of a dimensional argument as follows. Since the power density spectrum of the velocity fluctuations, $\Phi(k)$, has the dimensions of v^2/k or $[L^3]\,[T^{-2}]$; and since we assume, by virtue of the conservative cascade, that $\Phi(k)$ depends only on ε and k with dimensions $[L^2]\,[T^{-3}]$ and L^{-1} respectively, it follows that

$$\Phi(k) = \alpha \varepsilon^{2/3} k^{-5/3} \tag{7}$$

---

*That is, one large vortex feeds two smaller ones until the large one is exhausted. Onsager[26] envisioned the same factor of 2.

**Recently, two new mathematical approaches have appeared that shed light on some of the classic problems of this subject. The interested reader may consult the Appendix for a brief sketch of these.

26. Onsager, L. (1949) Statistical hydrodynamics, Nuovo Cimento 6(Ser. 9, Supp. 2):279-287.

where α is a constant of order unity. Note that in this derivation it was not necessary to assume locality of interaction in k-space. Rather, all that is used is the assumption that ε does not depend on k.

We now turn to what is perhaps the key to the confusion that leads to statements like "turbulence is a field of random waves"; and "if the spectrum has -5/3 slope, this means that it represents cascade or inertial range turbulence". The key issue, in my opinion, lies in the interpretation of the power spectrum. Tennekes[25] brings this out very clearly. There are, in fact, two possible interpretations. In the case of waves one can imagine the Fourier components as representing individual waves propagating through the entire sample of fluid under consideration. This is a very natural way to consider Fourier components because this is the way the subject is taught in courses. On the other hand, when one considers turbulent motion, one is not looking at extended waves but, instead, at short-lived, highly-localized modes of disturbance or eddies. One is considering a large collection or assemblage of these entities (approximating an ensemble) and the Fourier component does not refer to a single entity such as a wave. Instead, as Tennekes puts it, "We conclude that a Fourier coefficient should not be confused with an eddy. Fourier coefficients are associated with an ensemble of eddies, and when one analyzes the dynamics of turbulence in wave number space one should use the plural only: (the collective of) eddies at or near a certain wave number are involved." The reader should consult his work for his use of wave packet and quantum mechanical analogies, which are very illuminating.

Of course, in view of the preceding discussion, there is a complete range of dynamics from waves to, shall we say, "turb-undulence", and finally to turbulence; and a power spectrum alone says nothing about this aspect of the situation. As was already explained, a cross-spectrum can bring out the difference between waves and turbulence. This method depends directly upon the mixing property in coordinate space.

Can the mixing property in k-space be used to design a spectral criterion between waves and turbulence? The existence of "higher order spectra" suggests that, in principle, this could be done. Bi-spectra, and their properties were investigated, for example, by Hasselmann et al.[27] An extended discussion of higher-order spectra in general will be found in Brillinger,[28] and a special degenerate form of the bi-spectrum designed to detect coupling between sum and difference

27. Hasselmann, K., Munk, W., and MacDonald, G. (1963) Bispectra of ocean waves, in Time Series Analysis, M. Rosenblatt Ed., J. Wiley and Sons, N.Y., [illegible]

28. Brillinger, D.R. (1965) An introduction to polyspectra, Ann. Math. Statist. 36, 1351-1374.

frequencies will be found in Dewan.[29] Such spectra measure non-linear interactions between "wave-modes". In the absence of such non-linear interactions the bi-spectrum, for example, would be zero.

Unfortunately, the idea of using higher-order spectra to measure the transfer rate of energy between modes of oceanic internal waves has problems associated with it that render it impractical at this time (as explained by McComas[30]). On the other hand, the application of higher-order spectra to atmospheric turbulence has yet to be explored. In the case of turbulence, there is every reason to be optimistic about the usefulness of higher-order spectral measurements. For the present, then, only mixing in coordinate space is available for measuring "degree of turbulent interactions" in a fluctuation field. However, higher-order spectra may very likely play a role in the future.

6. TWO-DIMENSIONAL TURBULENCE

The term "two-dimensional turbulence" is well known, and was first discussed by Onsager[26] in terms of a "reverse cascade". More recently, Kraichnan[31] discussed this two-dimensional, reversed cascade (large k to small k) of energy, and he showed that it would have a $k^{-5/3}$ dependence. Leith and Kraichnan[32] subsequently showed that in two-dimensional turbulence there is also the possibility of a forward cascade of enstrophy that would have a k^{-3} dependence for the spectrum (see also Charney[23]). Both Stewart[21] and Gage[34] have proposed that some of the observations of $k^{-5/3}$ one-dimensional spectra in the atmosphere may be the result of a reverse two-dimensional cascade. This leads directly to the question of whether or not Figure 1 could be explained along such lines.

An experimental approach to the answer to this question would be to invent a modification of the Busch and Stewart approach that would render it appropriate to two-dimensional turbulence. We now attempt to do this. In the previous case, we had three-dimensional turbulence in which was imbedded a scalar quantity, θ (the potential temperature), which had a vertical gradient. Vertical transport due

---

29. Dewan, E.M. (1969) Nonlinear Cross-spectral Analysis and Pattern Recognition, AFCRL-69-0026, AD 6870851.

30. McComas, C.H. (1978) Private communication.

31. Kraichnan, R.H. (1967) Inertial ranges in two dimensional turbulence, Phys. of Fluids 10:1417-1423.

32. Leith, C.E., and Kraichnan, R.H. (1972) Predictability of turbulent flows, J. Atmos. Sci. 29:1041-1058.

33. Charney, J.G. (1971) Geostrophic turbulence, J. Atmos. Sci. 28:1087-1095.

34. Gage, K.S. (1979) Evidence for a $k^{-5/3}$ law inertial range in mesoscale two-dimensional turbulence, J. Atmos. Sci. 36:1950-1954.

to mixing provided the criterion for the presence of turbulence. In the case of (horizontal) two-dimensional turbulence, the analogous situation would have to prevail, that is, there would have to be a scalar quantity θ present that had a horizontal gradient. Two-dimensional turbulence would then be signaled by transport via the horizontal velocity fluctuations in the direction of $\vec{\nabla}\theta$, and one would merely substitute that velocity component for what we called w previously. The test would be formally identical to what was given previously. Whether such a test would be practical remains to be seen; for, in the previous situation vertical mean motion was negligible. In contrast, there would be horizontal mean motions that would have to be eliminated from the analysis.

Theoretically, there are two problems with a two-dimensional turbulence explanation of Figure 1. The first is the question of how energy could be supplied to the small-scale end of the spectrum. The second is the question of how to explain the observation that the vertical velocity fluctuations have the same magnitude and power spectrum slope as the horizontal velocity components in (Figure 1).

There is one complication in this experimental criterion for turbulence that was not pointed out by Busch, but that we should mention for completeness. The cross-spectra involved are based on one-dimensional trajectories, that is, we deal with one-dimensional spectra. It is well known that such spectra are subject to spatial aliasing effects. This, in turn, could introduce an ambiguity into the meaning of $\beta(k)$, or phase, if more than one wave gave the same k component. In principle this could be remedied by making use of higher dimensional spectra; however, the notable success of Axford[23] with this technique suggests that my concern is premature.

7. THE WAVE CASCADE EXPLANATION FOR A $k^{-5/3}$ SPECTRUM

McComas[35] and McComas and Bretherton[36] have investigated internal wave interactions and the transfer of action* in k-space. Their work is primarily

---

35. McComas, C.H. (1977) Equilibrium mechanisms within the oceanic internal wave field, J. Phys. Oceanog. 7:836-845.

36. McComas, C.H., and Bretherton, F.P. (1977) Resonant interaction of oceanic waves, J. Geophys. Res. 82:1397-1412.

*Wave action is defined as wave energy divided by frequency (see Phillips[19]).

built upon the former researches of Phillips[18] and Hasselman[37,38] among others. Briefly, their treatment assumes that the waves are essentially linear and that the interaction between modes is weak. Phillips[18] showed that there is a way by which internal waves can interact in a resonant manner provided that the temporal frequencies and wave numbers of a triad of waves obey a certain condition. This condition is

$$k_1 \pm k_2 = k_3$$

$$\omega_1 \pm \omega_2 = \omega_3 \quad . \tag{8}$$

(In the case of surface waves there must be a four-wave interaction.)

The physical mechanism (by which energy can be transferred from two interacting modes into a third mode) resembles forced simple harmonic oscillation. Two modes interact by means of the convective, non-linear term in the equations of motion and, as a result, they generate sum and difference frequencies [see Eq. (8)]. The third wave (number 3), is assumed to have both a spatial and temporal resonance with this "beat frequency phenomenon" caused by the interaction between waves numbered 1 and 2. Wave three is thus "driven" in a manner analogous to a simple harmonic oscillation with a forcing term on the right-hand side of the equation.

McComas[35] shows that among all the possibilities for Eq. (8) to hold, there are three distinct limiting configurations compatible with the dispersion relations; and these he labels "induced diffusion", "elastic scattering", and "subharmonic instability". These allow a simplified and physical way to further understand the transfer between wave modes. He, for the purpose of this paper, provides us with a proof that a wave–wave energy cascade can exist. Beyond this one fact our present work needs no further details.

Parenthetically, it should be mentioned at this point that the thrust of the work by McComas and Bretherton[36] is primarily to provide a theory for the "Garrett-Munk" Spectrum,[39,40] which represents a universal wave spectrum to be found associated with internal waves in the ocean. A desirable achievement

---

37. Hasselmann, K. (1962) On the non-linear energy transfer in a gravity wave spectrum, Part 1, J. Fluid Mech. 12:481-500.

38. Hasselmann, K. (1963) On the non-linear energy transfer in a gravity wave spectrum, Part 2 and Part 3, J. Fluid Mech. 15:273-281, 385-398.

39. Garrett, C., and Munk, W. (1972) Space-time scales of internal waves, Geophys. Fluid Dyn. 2:225-264.

40. Garrett, C., and Munk, W. (1975) Space-time scales of internal waves: a progress report, J. Geophys. Res. 80:291-297.

would be to explain the atmospheric data such as that exemplified by Figure 1. Far from being "universal", such data are found only on rare occasions in which the wave intensities are so great that they were originally identified as turbulence. Such disturbances are found on the order of 5 percent of the time. Furthermore, the disturbance is found to be confined to altitude thicknesses of 1 to 2 km (as shown by Crooks et al[1,16]). At present it is not clear if our conclusions have any application to the Garrett-Munk (GM) spectra; but this cannot be ruled out at this time. On the other hand, they do appear to have relevance to Figure 1.

In Dewan,[17] an alternative existence proof of the possibility of an energy wave cascade is given. There, the interaction between wave scales is explained not by resonance, but by the effect of deformations caused by large waves, which in turn can "feed" small waves. The next step, if one were to work in parallel with the bizarre history of turbulence theory (that is, Richardson's famous poem followed by Kolmogorov's dimensional argument) would be to create a new poem, "Big waves have little waves that feed on deformation and little waves have lesser waves, to turbulent dissipation". Needless to say, the poem is not a prerequisite to making a dimensional argument nor does it reflect the resonant mode interaction approach of this paper. Instead, it reflects an alternative approach given in Dewan.[17]

In one sense there is a very close parallel between the wave cascade theory given in Dewan[17] and that given in McComas.[35] The latter involves interactions with time scales given by Eq. (1) or its simplification given by $T_i \sim (kv)^{-1}$, which was used in Section 3. In contrast, the treatment given in Dewan[17] involves production terms of the form $\overline{uw}\,\partial\bar{U}/\partial z$ where $\bar{U}$ represents average larger-scale velocity fluctuations, and u and w the smaller scale ones. These production terms represent transfer from larger to smaller scales. In this case, however, $T_i = (\partial\bar{U}/\partial z)^{-1}$. However, in a wave cascade, U is oscillatory and its shear is given by $\bar{U}k$ thus (T_i^{-1}) is physically the same quantity as that given in resonance theory. Instead of $\varepsilon_w = v^2 (vk)$ for resonant mode interactions (where ε_w is the transfer rate) one would have $\varepsilon_w = v^2\,\partial v/\partial z$ which, in effect, is the same quantity. The cascade in question must involve transfer from the large to the small scales. This is well known because turbulence itself (at the small scale) is dissipative and ultimately converts motion into heat. It cannot represent the original source. The latter must be at large scales. True, turbulence can generate gravity waves; however, it itself, must be generated by shears that arise from larger scale fluctuations. For more discussion of this the reader should consult Dewan.[17]

In any case we now make the assertion that the wave power spectrum depends only on k and "ε_w" (which is the dissipation rate for the waves due to their interactions). In order to calculate the actual kinetic energy dissipation rate per unit volume one must multiply ε_w by ρ, the mass density.

We are now in a position to write down the expression for the wave cascade one-dimension spectrum. One need only repeat the dimensional argument of Kolmogorov[2] given previously to arrive at

$$\Phi_W(k) = \alpha_W \varepsilon_W^{2/3} k^{-5/3} \quad . \tag{9}$$

Here the subscript W signifies waves.

One could object that, unlike turbulence, waves must obey a dispersion relation that, in turn, must depend on the buoyancy frequency. In its simplest form, this dispersion relation is

$$\omega = N_B \cos \theta \, ^* \tag{10}$$

where θ is the angle ($\theta \leq 1/2\pi$) between the horizontal direction and k (Turner[41]). Thus, how can one assume that $\Phi(k)$ does not depend upon N_B? My answer is that we presently are assuming a cascade in steady-state and in which negligible mixing is taking place. (We, like McComas, assume that the interactions are weak in comparison to turbulence.) That is to say, ε_W output would equal the input so long as energy is not given up in the form of the potential energy due to mixing in a stratified environment. This implies that buoyancy does not affect ε_W. It is therefore reasonable to suppose that a change in N_B would affect only the amplitude of the waves in such a way that the kinetic energy for a scale, k, would remain unaltered. In other words, our dimensional argument is physically permitted.

The other facts that make the hypothesis of a wave cascade a useful one are (a) the experimental values of the spectra in fact give us a $k^{-5/3}$, dependence, (b) as will be seen, ε_W can be related to observation, and (c) one can experimentally test the "wave field" hypothesis via the previously described "Busch Test" to verify that it is indeed a wave field. In other words, this approach is testable.

Finally, we now consider the theoretical shape of a spectrum that extends through ranges of k including both the waves and turbulence. This requires that we obtain the answers to the following three questions: (a) "At what value of k is the transition between $\Phi_W(k)$ and $\Phi_T(k)$ (the turbulence spectrum)?, (b) "In

---

* More generally, $\omega^2 = N_B^2 \cos^2 \theta + \omega_i^2 \sin \theta$ where ω_i is the inertial frequency (see Garrett and Munk[39, 40]).

41. Turner, J.S. (1973) Buoyancy Effects in Fluids, Cambridge University Press, 367 pp.

what way is ε_W related to ε_T (turbulent dissipation)"? and, (c) "What are the numerical values for α_W and α_T?"

The first question is easily answered on the basis of the previous discussion. The "buoyancy length" derived above gives us the boundary between horizontally propagating waves and isotropic turbulence. While this "boundary" is a transition, and is hence smooth, it is given by k_B where

$$k_B = \left(\frac{N_B^3}{\varepsilon_T}\right)^{1/2} \cdot c \tag{11}$$

[see Eq. (5)] where c is a constant of order unity.

The answer to the second question (regarding the relation between the wave and turbulence dissipation rates ε_W, and ε_T respectively) is obtained from the assumption that the cascade is conservative. Energy conservation gives

$$\rho\ \varepsilon_W V_W = \rho\ \varepsilon_T V_T \tag{12}$$

where V_W represents the volume occupied by the waves and V_T that of the turbulence. The reason V_T and V_W are not equal in a stratified medium is that turbulence in the latter is known to occur in thin layers occupying a small fraction of the total volume. Thus we arrive at

$$\frac{\varepsilon_W}{\varepsilon_T} = \frac{V_T}{V_W} \ . \tag{13}$$

Since $V_W > V_T$, it follows that $\varepsilon_W < \varepsilon_T$. Knowing ε_W, one could calculate ε_T if the volume ratio were known. It should be noted that if one were to ascertain $\varepsilon_W/\varepsilon_T$ by means of the spectrum (assuming at that point that α_T and α_W were known) then one would have a measure of V_T/V_W that would in turn have some pragmatic utility in problems concerning pollution transport and optical transmission through turbulence. Specifically, V_T/V_W would be a measure of the volume occupied by turbulence relative to the overall volume of the disturbed region.

The third question (concerning the universal constants of order unity, α_T, and α_W) can only be answered by experiment. One already knows that α_T is around 1.5 (Tennekes and Lumley[24]) but it depends somewhat on the velocity component

in question. (One does not yet know the value of α_W but, according to Bond,[42] there is a 90 percent a priori chance that it lies between 0.1 and 10.)

Figure 3 shows a schematic representation of this spectrum assuming that the measurement is made from a probe within a turbulent layer.* In this figure no account has been taken of the spatial aliasing effects to be expected in the case of one-dimensional spectra. Gifford[43] showed an example where there was a -5/3 slope that continued to wavelengths three to five times larger than would appear in a spectrum without aliasing.

To illustrate this effect, let us consider a specific numerical example. Suppose that k_B^{-1} = 50 m (a typical value for HICAT spectra such as in Figure 1). In this case, λ_B = 315 m. If aliasing were to extend the -5/3 slope to scales five times larger than λ_B, that would take it out to 1,575 m. Figure 1, however, has the slope of -5/3 extending to about one order of magnitude beyond this length.

It would be very useful if velocity spectra and cross spectra [Eq. (6)] were measured in stable fluids in order to test the idea that, at wavelengths much larger than λ_B, waves exist, and much less than λ_B, turbulence exists, and that both can have a -5/3 slope but with $\varepsilon_W \neq \varepsilon_T$ [see Eq. (15)]. Equation (6), which is what I have called the Ozmidov test, could presumably discern between waves and turbulence. Up to this point, we have left out buoyancy turbulence.

8. ON THE NATURE OF THE BUOYANCY SUBRANGE OF TURBULENCE AND A SCALE LENGTH CRITERION

In the previous [illegible] the case of waves with maximum N_{max} [illegible] departs from the horizontal ([illegible] horizontally), the [illegible] We now [illegible] between [illegible]

[illegible]

[illegible]

[illegible]

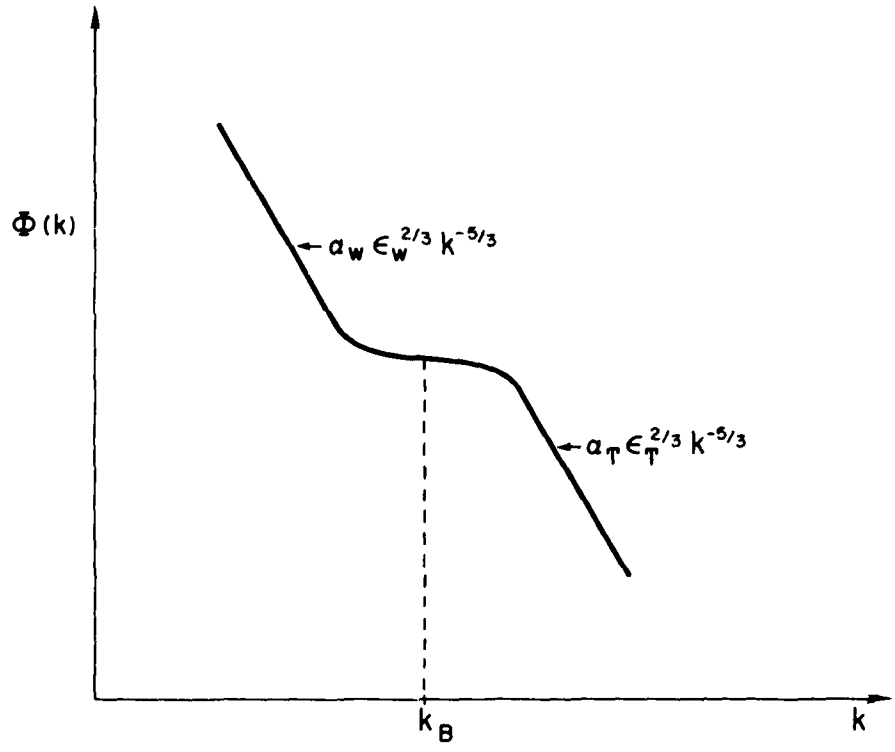


Figure 3. Schematic Representation of a Power Spectrum of Waves and Turbulence in a Steady-State Cascade. Note that the part of the curve that connects the "wave" and "turbulent" parts has been given an arbitrary shape

Using Eq. (10) we obtain

$$T_i = (N_B \cos\theta)^{-1} \tag{14}$$

in place of $T_i = N_B^{-1}$ used previously. Similarly, in place of Eq. (4) we use

$$\varepsilon = v^2 N_B \cos\theta \quad . \tag{15}$$

As a result, in place of (5) for k_B, we now get

$$K_B = \left(\frac{N_B^3 \cos^3\theta}{\varepsilon} \right)^{1/2} \quad . \tag{16}$$

$$k_B^{-2} = \frac{v^3}{N_B^3} \quad \frac{1}{\ell_o} \quad . \tag{18}$$

Using the fact that (v/ℓ_o) is of the same order as the mean vertical shear (Reference 24), we can write the Richardson number, R_i in the form

$$R_i = \frac{N_B^2}{(v/\ell_o)^2} \quad . \tag{19}$$

A typical value for R_i in cases where shear induced turbulence exists is (1/4). Using this it is easy to obtain

$$k_B^{-1} = \ell_o(2.83) \quad . \tag{20}$$

Thus, within a constant of order unity, the outer length and buoyancy length are equal.

This raises some interesting questions. Usually ℓ_o defines the scale at which the turbulence is generated. But k_B^{-1} is the minimum scale for buoyancy subrange turbulence. How then can there be buoyancy turbulence if the energy cascades to smaller scales? Could it be the case that buoyancy turbulence must be generated by a larger scale horizontal shear? This is not an unreasonable possibility since, as we have seen, this kind of turbulence involves horizontal eddies (especially at large values of λ). Another possibility that comes to mind is that a reverse cascade may be involved in buoyancy turbulence. It is well known that two-dimensional turbulence can involve a reverse cascade, and for large λ we have seen that buoyancy turbulence becomes two dimensional.

Leaving these questions for future research, we now turn to the use of Eq. (16) as a way to distinguish turbulent from wave fluctuations. It has the unfortunate drawback that it requires knowledge of the value of θ for the $\vec{k}$ under consideration. In principle such information would be available if the complete three-dimensional wave-number spectrum were measured. While such measurements might be possible in a laboratory, they are usually impossible to obtain in atmospheric [illegible]. On the other hand, our discussion suggests that there may be special [illegible] where the direction of the fluctuations can be used as a wave turbulence [illegible]. For example, suppose we consider a case where, for sufficiently [illegible], the only waves in a boiling situation that are trapped have $\theta = 0$. All other waves would escape from the duct leaving behind only the anisotropic

turbulence of large λ and the horizontally propagating waves. As we have seen, such turbulence would involve mainly horizontal motion. Thus vertical motion would be due to waves and horizontal motion would be due to eddies, making the distinction merely a matter of measuring the direction of the velocity fluctuations.

9. CONCLUSIONS

Waves and turbulence are idealizations for two extremely different types of fluctuations in a stable fluid. Waves represent the extreme of linear (superposable) dissipationless motion. Turbulence represents very strong mode-interactions, mixing, and dissipation. Waves have non-local, long-lived propagating patterns, and turbulence has local, short-lived, non-propagating patterns.

I have used "waves" in this paper in a manner that departs somewhat from the ideal type in that some weak non-linear interactions are permitted. In any case, the cross-spectral test of Busch was discussed and it should prove useful in distinguishing between waves and turbulence. There is reason to hope that bi-spectra could also play a future role in making this experimental distinction. More specifically, the "bi-coherence" defined in McComas[30] could be used to estimate the degree of turbulent non-linear interaction.

Figure 1 presents us with an interesting scientific mystery. On one hand, it is clear that the inertial turbulence interpretation is ruled out. A wave cascade possibility has been proposed. Buoyancy turbulence or two-dimensional turbulence (they seem to be related) may be involved. The answers to the questions raised will probably necessitate the use of some of the wave turbulence criteria proposed in this report.

Hopefully this work will prove useful in the practical problems of pollution transport and optical turbulence. If it turns out that the cascade hypothesis is related to universal wave spectra (Garrett and Munk[39,40]), this approach could lead to better ways to predict shear structure giving rise to turbulence. If it could lead to a better understanding of the intense trapped gravity wave structure of the type presumably observed by Crooks et al,[1,16] perhaps this could lead to ways to predict high C_N^2 conditions.*

---

* One mechanism that might account for these high-intensity trapped waves could involve over-reflection, for example.

References

1. Crooks, W., Hoblit, F., Prophet, D. et al (1968) Project HICAT: An Investigation of High Altitude Clear Air Turbulence, AD 847497.

2. Kolmogorov, A. (1941) The local structure of turbulence in incompressible viscous fluid for very large Reynolds' numbers, in Turbulence, S. Friedlander, and L. Topper, Eds., Interscience Publishers, N.Y., 187 pp.

3. Lilly, D.K., Waco, D.E., and Adelfang, S.I. (1974) Stratospheric mixing estimated from high-altitude turbulence measurements, J. Appl. Met. 13:488-493.

4. Zimmerman, S.P., and Loving, N. (1975) Turbulent dissipation and diffusivities in the stratosphere using Richardson's technique, The National Stratosphere of 1974, CIAP Monograph 1, Final Report, DOT-CIAP, DOT-TST-75-51.

5. Weinstock, J. (1978) On the theory of turbulence in the buoyancy subrange of stably stratified flows, J. Atmos. Sci. 35:634-649.

6. Dewan, E.M. (1976) Theoretical Explanation of Spectral Slopes in Stratospheric Turbulence Data and Implications for Vertical Transport, AFGL-TR-76-0247, AD A036307.

7. Bolgiano, R. (1959) Turbulent spectra in a stably stratified atmosphere, J. Geophys. Res. 64:2226-2229.

8. Shur, G.N. (1962) Experimental studies of the energy spectrum of atmospheric turbulence. Proceedings of the Central Aerological Observatory, 43; and, (1962) Tsentral'naia Aerologicheskai Observatoriia, Moscow, Trudy 13:79-90.

9. Lumley, J.L. (1964) The spectrum of nearly inertial turbulence in a stably stratified fluid, J. Atmos. Sci. 21:99-102.

10. Lumley, J.L. (1967) Theoretical aspects of research on turbulence in stratified flows, in Atmospheric Turbulence and Radio Wave Propagation, A.M. Yaglom, and V.I. Tatarsky, NAUKA, Moscow, 374 pp.

11. Lin, J. T., Panchev, S., and Cermak, J. (1969) A modified hypothesis on turbulence spectra in the buoyancy subrange of stably stratified shear flow, Radio Sci. 4:1333-1337.

12. Barat, J., and Aimedieu, P. (1981) The external scale of clear air turbulence derived from the vertical ozone profile: application to vertical transport measurement, J. Appl. Met. 20:275-280.

13. Crane, R.K. (1980) Radar measurements of wind at Kwajalein, Radio Sci. 15:383-394.

14. Anderson, A.D. (1957) Free-air turbulence, J. Meteorol. 14:477-494.

15. Rosenberg, N.W., and Dewan, E.M. (1975) Stratospheric Turbulence and Vertical Effective Diffusion Coefficients, AFCRL-TR-75-0519, AD A019701.

16. Crooks, W., Hoblit, F., Prophet, D. et al (1967) Project HICAT: An Investigation of High Altitude Clear Air Turbulence, AD 846086.

17. Dewan, E.M. (1979) Stratospheric wave spectra resembling turbulence, Science 204:832-835.

18. Phillips, O.M. (1966) The Dynamics of the Upper Ocean. Cambridge University Press, (1st ed.), 336 pp.

19. Phillips, O.M. (1977) The Dynamics of the Upper Ocean. Cambridge University Press, (2nd ed.), 336 pp.

20. Busch, N.E. (1969) Waves and turbulence, Radio Sci. 4:1377-1379.

21. Stewart, R.W. (1969) Turbulence and waves in a stratified atmosphere, Radio Sci. 4:1269-1278.

22. Phillips, O.M. (1967) On the Bolgiano and Lumley-Shur Theories of the buoyancy subrange, in Atmospheric Turbulence and Radio Wave Propagation, A.M. Yaglom, and V.I. Tatarsky, Eds., NAUKA, Moscow, 374 pp.

23. Axford, D.N. (1971) Spectral analysis of an aircraft observation of gravity waves, Quart. J. Met. Soc. 97:313-321.

24. Tennekes, H., and Lumley, J.L. (1972) A First Course in Turbulence, MIT Press, Cambridge, Mass., 300 pp.

25. Tennekes, H. (1977) Turbulence: diffusion, statistics, spectral dynamics, in Handbook of Turbulence, W. Frost, and T.H. Moulden, Eds., Plenum Press, 498 pp.

26. Onsager, L. (1949) Statistical hydrodynamics, Nuovo Cimento 6(Ser. 9, Supp. 2): 279-287.

27. Hasselmann, K., Munk, W., and MacDonald, G. (1963) Bispectra of ocean waves, in Time Series Analysis, M. Rosenblatt, Ed., J. Wiley and Sons, N.Y., 497 pp.

28. Brillinger, D.R. (1965) An introduction to polyspectra, Ann. Math. Statist. 36:1361-1374.

29. Dewan, E.M. (1969) Nonlinear Cross-spectral Analysis and Pattern Recognition, AFCRL-69-0026, AD 6870851.

30. McComas, C.H. (1978) Private communication.

31. Kraichnan, R.H. (1967) Inertial ranges in two dimensional turbulence, Phys. of Fluids 10:1417-1423.

32. Leith, C.E., and Kraichnan, R.H. (1972) Predictability of turbulent flows, J. Atmos. Sci. 29:1041-1058.

33. Charney, J.G. (1971) Geostrophic turbulence, J. Atmos. Sci. 28:1087-1095.

34. Gage, K.S. (1979) Evidence for a $k^{-5/3}$ law inertial range in mesoscale two-dimensional turbulence, J. Atmos. Sci. 36:1950-1954.

35. McComas, C.H. (1977) Equilibrium mechanisms within the oceanic internal wave field, J. Phys. Oceanog. 7:836-845.

36. McComas, C.H., and Bretherton, F.P. (1977) Resonant interaction of oceanic waves, J. Geophys. Res. 82:1397-1412.

37. Hasselmann, K. (1962) On the non-linear energy transfer in a gravity wave spectrum, Part 1, J. Fluid Mech. 12:481-500.

38. Hasselmann, K. (1963) On the non-linear energy transfer in a gravity wave spectrum, Part 2 and Part 3, J. Fluid Mech. 15:273-281, 385-398.

39. Garrett, C., and Munk, W. (1972) Space-time scales of internal waves, Geophys. Fluid Dyn. 2:225-264.

40. Garrett, C., and Munk, W. (1975) Space-time scales of internal waves: a progress report, J. Geophys. Res. 80:291-297.

41. Turner, J.S. (1973) Buoyancy Effects in Fluids, Cambridge University Press, 367 pp.

42. Bond, W.N. (1929) The magnetude of non-dimensional constants, Phil. Mag. 7:722.

43. Gifford, F. (1959) The interpretation of meteorological spectra and correlations, J. Meteorol. 16:344-346.

Bibliography

Andronov, A.A., Vitt, A.A., and Khaiken, S.E. (1966) Theory of Oscillators, Pergamon Press, 815 pp.

Dold, A., and Eckmann, B. (1976, 1977) Turbulence Seminar, Springer Verlag, N.Y., 155 pp.

Dold, A., and Eckmann, B. (1976) Turbulence and Navier Stokes Equation, Springer Verlag, N.Y., 194 pp.

Feigenbaum, M.J. (1978) Quantitative universality for a class of nonlinear transformations, J. Stat. Phys. 19:25-52.

Lee, Y.W. (1960) Statistical Theory of Communication, MIT Press, Cambridge, Mass., 509 pp.

Lorenz, E.N. (1963) Deterministic nonperiodic flow, J. Atm. Sci., 20:130-141.

Mandelbrot, B.B. (1977) Fractals, Forms, Chance, and Dimension, Freeman and Co., 365 pp.

Minorsky, N. (1947) Introduction to Non-Linear Mechanisms, Edwards, 464 pp.

Ruelle, D., and Takens, F. (1971) On the nature of turbulence, Commun. Math. Phys. 20:167-192.

Treve, Y.M. (1978) Theory of chaotic motion with application to controlled fusion research, in Topics in Nonlinear Dynamics, S. Jorna, Ed., American Institute of Physics, pp. 147-220.

Appendix A

New Developments in Turbulence Theory

New work in the mathematical approach to turbulence enhances our understanding of the chaotic nature of turbulent mixing. This appendix is intended to bring this new information to the reader's attention. The major unresolved question in turbulence theory is, "How do the random or chaotic motions, which supposedly are the solutions of the purely deterministic equations of fluid motion, come into existence?" The new findings demonstrate a physical mechanism that makes such a phenomenon possible.

It is very well known that, as the Reynolds number is increased, a critical value can be reached where the laminar flow makes a transition to turbulent flow. The transition, in some cases, is spread over a range of Reynolds numbers, that is, there is a series of transitions or bifurcations in the complexity of the motion. As Reynolds number increases, complexity increases until a point is reached where, suddenly, the motion becomes totally chaotic.

The "breakthrough" concept that explains the chaotic behavior is known as a "strange attractor" and it was introduced by Lorenz,[A1] in the context of the Benard instability. By Fourier decomposing certain variables of the partial differential equations for the problem he obtained an infinite set of ordinary differential equations. These he truncated to three equations, and hence his work represents a severe mutilation of the actual equations. In any case, it led to the first discovery

A1. Lorenz, E.N. (1963) Deterministic nonperiodic flow, J. Atm. Sci. 20:130-141.

of strange attractors. Subsequently, Ruelle and Takens[A2] arrived at strange attractors for turbulence theory from an entirely different direction, namely from the qualitative theory of differential equations. They did much to explain both chaotic motion and also the Reynolds number bifurcation sequence described previously. Most recently, the work of Feigenbaum[A3] provided an especially clear way to understand both the transition to increasing degrees of complexity of motion as well as the subsequent transition to total chaos.

In order to understand qualitatively the meaning of the "strange attractor" concept, one can start with the simpler entity known as an attractor. The simplest examples of an attractor occur in works on non-linear mechanics (for example, Andronov et al[A4] or Minorsky[A5]). Consider, for example, the motion of an electronic oscillator as modeled by the van der Pol equation. Such motion is conveniently examined in the x-v (that is, displacement and velocity) or "phase plane". For example, if the oscillator were to start with initial conditions near $x = 0$ and $v = 0$, the trajectory of the solution would form an outward-going spiral that asymptotically approaches an ellipse that is known as "limit cycle". The latter trajectory represents an equilibrium between input and output of energy over the cycle. If the cycle were to start with large values of x and v, the trajectory would spiral inward towards the limit cycle. The limit cycle thus appears to attract all trajectories no matter what their initial conditions might be, and hence the term "attractor" is used.

The adjective "strange", when employed in the present context, designates a key aspect of the explanation of chaotic motion. It refers to the fact that such an attractor is of fractional dimension or is a "fractal" as is described at great length in the book by Mandelbrot.[A6] As he explains, a fractal curve (or at least one major type of such curve) is one of which each piece is a reduced scale version of the whole. Such curves have the property that, as their length is measured at ever higher resolutions, it increases indefinitely. The dimension, D, is given by

$$D = \log N / \log (1/r) \qquad (A1)$$

A2. Ruelle, D., and Takens, F. (1971) On the nature of turbulence, Commun. Math. Phys. 20:167-192.

A3. Feigenbaum, M.J. (1980) Universal behavior in nonlinear systems, Los Alamos Science Summer:4.

A4. Andronov, A.A., Vitt, A.A., and Khaiken, S.E. (1966) Theory of Oscillators, Pergamon Press, New York, 815 pp.

A5. Minorsky, N. (1947) Introduction to Non-Linear Mechanics, Edwards, 464 pp.

A6. Mandelbrot, B.B. (1977) Fractals, Form, Chance, and Dimension, Freeman and Co., 365 pp.

where N is the number of parts in a whole piece of the curve and r is the so-called similarity ratio. For example, consider the case of a square which is subdivided into N squares; the "similarity ratio" would be $r = 1/N^{1/2}$. That is to say, each side of the small squares would have this ratio to the length of the original square. Here, one can obtain D = 2 from Eq. (A1). But D = 2 dimensions is the ordinary situation for plane figures. Thus Eq. (A1) gives the usual results for non-fractal objects. The simplest example of a fractal is the so-called "Koch curve"; however the derivation of D = log 4/log 3 = 1.2618 for that case (as well as the discussion of many other examples of fractals) will not be given here. (See Mandelbrot.[A6]) Thus, fractals are of fractional dimension.

It has been shown by Lorenz,[A1] Ruelle et al,[A2] and Feigenbaum,[A3] that if a non-linear system has an attractor in its phase space, which is also a fractal, then the system can exhibit chaotic motion. It follows that total mixing will take place in coordinate space because total mixing occurs (in the fractal sense) in the phase space.

The following citations will be useful for further reference. In particular, the works by Dold et al,[A7,A8] Swinney et al,[A9] and Treve[A10] are good review papers on the subject.

A7. Dold, A., and Eckmann, B. (1976/1977) Turbulence Seminar, Springer Verlag, N.Y., 155 pp.

A8. Dold, A., and Eckmann, B. (1976) Turbulence and Navier Stokes Equation, Springer Verlag, N.Y., 194 pp.

A9. Swinney, H.L., and Gollub, J.P. (1981) Hydrodynamic Instabilities and the Transition to Turbulence, Springer Verlag, N.Y., 292 pp.

A10. Treve, Y.M. (1978) Theory of chaotic motion with application to controlled fusion research, Topics in Nonlinear Dynamics, S. Jorna, Ed., American Institute of Physics, pp. 147-220.

References

A1. Lorenz, E. N. (1963) Deterministic nonperiodic flow, J. Atm. Sci. 20: 130-141.

A2. Ruelle, D., and Takens, F. (1971) On the nature of turbulence, Commun. Math. Phys. 20:167-192.

A3. Feigenbaum, M. J. (1980) Universal behavior in nonlinear systems, Los Alamos Science Summer:4.

A4. Andronov, A. A., Vitt, A. A., and Khaiken, S. E. (1966) Theory of Oscillators, Pergamon Press, 815 pp.

A5. Minorsky, N. (1947) Introduction to Non-Linear Mechanics, Edwards, 464 pp.

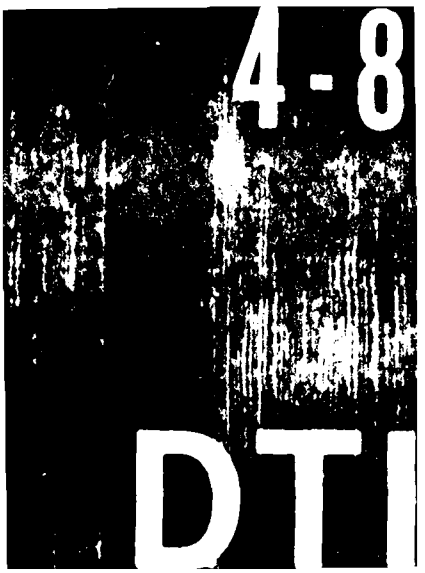
A6. Mandelbrot, B. B. (1977) Fractals, Form, Chance, and Dimension, Freeman and Co., 365 pp.

A7. Dold, A., and Eckmann, B. (1976/1977) Turbulence Seminar, Springer Verlag, N. Y., 155 pp.

A8. Dold, A., and Eckmann, B. (1976) Turbulence and Navier Stokes Equation, Springer Verlag, N. Y., 194 pp.

A9. Swinney, H. L., and Gollub, J. P. (1981) Hydrodynamic Instabilities and the Transition to Turbulence, Springer Verlag, 292 pp.

A10. Treve, Y. M. (1978) Theory of chaotic motion with application to controlled fusion research, Topics in Nonlinear Dynamics, S. Jorna, Ed., American Institute of Physics, pp. 147-220.


4-8
DTI