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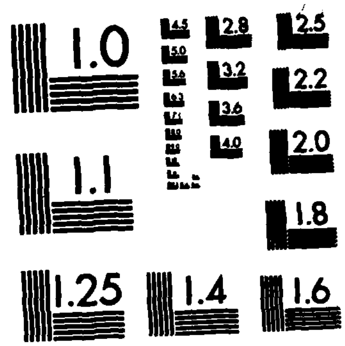
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A NOTE ON BEAMFORMING ALGORITHMS

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LEXINGTON MASSACHUSETTS

Abstract

Two procedures for specifying element weightings for antenna arrays are described. The first procedure is used for gain optimization and the second for shaping the array gain pattern. Applications of these procedures including beamwidth and sidelobe reduction are included.

CONTENTS

1. INTRODUCTION

This note presents two beamforming algorithms applicable to arrays of antenna elements. Both algorithms are derived from the standpoint of determining a set of system parameters to optimize a specified system performance index and both utilize a constrained optimization approach.

The first algorithm addresses the problem of gain optimization for arbitrary arrays. The formalism and the solution of this problem is an illustration of the minimum energy method [1] which is closely related to the maximum likelihood method [2, 3] used in adaptive arrays. It is presented primarily as a means of introducing a formalism which can be applied to a second algorithm that addresses the question of shaping the beam to achieve a desired antenna pattern. This latter algorithm can be used to control both the amplitude and phase of the antenna pattern at specified angles and thereby provides a solution to a variety of problems faced by the array designer. Examples of the solutions included in this paper are maximization of gain, beamwidth reduction and control of sidelobe levels. Other problems which are amenable to this approach include not only amplitude shaping of array patterns but also phase front shaping.

The two beamforming algorithms are described in the second section of this note followed by examples in the third section and a summary in the final section.

2. ALGORITHM FORMALISM

Consider a linear array of N elements, not necessarily equally spaced, as shown in Fig. 1. Each element has a power pattern $\nu(\theta, \phi)$ a complex amplitude of excitation a_i, $i=1,\ldots,N$ and is located at d_i, $i=1,\ldots,N$. The array factor which is the electric field radiated by an array of isotropic elements can be written as

$$E(\theta) = A^T \epsilon_\theta^* \tag{1}$$

where the vectors A and ϵ_θ are the complex column vectors defined below, T denotes the transpose and * the conjugate operation.

$$A = \begin{bmatrix} a_1 \\ \cdot \\ \cdot \\ \cdot \\ a_N \end{bmatrix} \qquad \epsilon_\theta = \begin{bmatrix} e^{ikd_1\cos\theta} \\ \cdot \\ \cdot \\ \cdot \\ e^{ikd_N\cos\theta} \end{bmatrix} \tag{2}$$

Here, k is the wavenumber defined as $2\pi/\lambda$, where λ is the wavelength of the radiated field. The total power density radiated by this array is therefore given by:

$$S(\theta, \phi) = \nu(\theta, \phi)\ |E(\theta)|^2$$

where

$$|E(\theta)|^2 = A^\dagger \epsilon_\theta\ \epsilon_\theta^\dagger\ A = A^\dagger \textstyle\sum_\theta A. \tag{3}$$

Here the † symbol signifies Hermitian conjugate and $\sum_\theta$ is the rank one matrix defined below:

$$\textstyle\sum_\theta = \epsilon_\theta\ \epsilon_\theta^\dagger. \tag{4}$$

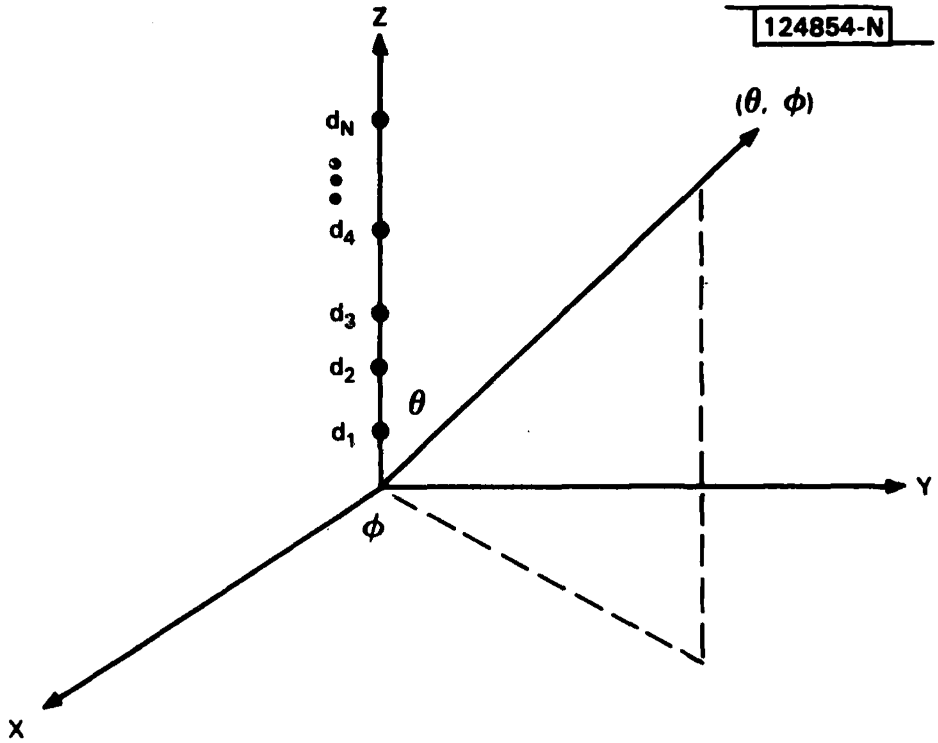

Fig. 1. Array Geometry

The total power radiated over 4π steradians is given by:

$$P = \frac{1}{4\pi} \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta S(\theta, \phi) d\theta$$

which can be written as

$$P = A^{\dagger} \Phi A \qquad (5)$$

where the matrix Φ has elements ϕ_{mn}

$$\phi_{mn} = \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} \nu(\theta, \phi)\, e^{-ik(d_n - d_m)\cos\theta} \sin\theta \, d\theta. \qquad (6)$$

In order to maximize the gain of this array in a specific direction (θ_o, ϕ_o) we minimize the total power radiated over 4π steradians, P, subject to the constraint that the power radiated in the direction (θ_o, ϕ_o) is equal to a constant, for example, unity. We therefore minimize the total power radiated over all angles except in the desired direction (θ_o, ϕ_o). This optimization is equivalent to maximizing the gain $S(\theta_o, \phi_o)/P$ in that direction. This is accomplished by using the technique of Lagrange multipliers and minimizing the cost function

$$J = P + \lambda \, [1 - S(\theta_o, \phi_o)]. \qquad (7)$$

For an isotropic element power pattern, (i.e. $\nu(\theta, \phi) = 1$), eq. (7) reduces to

$$J = A^{\dagger}\Phi A + \lambda \, [1 - A^{\dagger} \textstyle\sum_{\theta_o} A] \qquad (8)$$

where

$$\phi_{mn} = \sin\, [k(d_n - d_m)]/[k(d_n - d_m)].$$

Taking the derivative with respect to A and equating to zero we have

$$\frac{\partial J}{\partial A} = 2\Phi A - 2\lambda \Sigma_{\theta_o} A = 0 \tag{9}$$

resulting in the eigenvalue equation

$$\Phi^{-1} \Sigma_{\theta_o} A = \lambda^{-1} A = \mu A \tag{10}$$

The eigenvalues and eigenvectors of $\Phi^{-1}\Sigma_{\theta_o}$ can be calculated by a simulataneous diagonalization of Σ_{θ_o} and Φ but the solution of (10) is more easily obtained by recalling that Σ_{θ_o} is a matrix of rank one. The solution for the optimum weights A, is the eigenvector of (10) corresponding to the single non-zero eigenvalue. It is straightforward to show that this solution is

$$A = \Phi^{-1} \epsilon_{\theta_o} \tag{11}$$

and the non-zero eigenvalue is

$$\mu = \epsilon^{\dagger}_{\theta_o} \Phi^{-1} \epsilon_{\theta_o} \tag{12}$$

It can also be easily shown that the gain of the array at θ_o, i.e. the maximum gain, is equal to the eigenvalue, μ. Equations (11) and (12) represent the complete solution of the gain optimization problem.

We next turn our attention to an algorithm for specifying the complex field pattern of the array at a number of specified angles while minimizing the total radiated power. Solutions of this problem can be used for designing arrays with specified beamwidths, reducing sidelobes in particular directions, altering phase fronts, etc. The formalism for this problem is similar to the one described above, but in this case the constraints are levied on the radiated fields in specified directions rather than the radiated power in one particular direction. The constraints take the form

$$E(\theta_i) = A^T \epsilon^*_{\theta_i} = c_i \qquad i=1,\ldots,m \tag{13}$$

where c_i, the amplitude of the field in the direction θ_i is a complex number and the number of constraints, m, is smaller or equal to the number of elements in the array.

The criterion function to be minimized is given by

$$J = A^T \Phi A + \sum_{i=1}^{m} \mu_i (c_i - A^T \epsilon^*_{\theta_i}) \tag{14}$$

where μ_i are the Lagrange multipliers.

The solution for the antenna element weights is given by

$$A = 1/2 \sum_{i=1}^{m} \mu_i \Phi^{-1} \epsilon^*_{\theta_i} \tag{15}$$

where the values of the Lagrange multipliers μ_i may be determined from the m constraint equations shown in (13) i.e.

$$\sum_{j=1}^{m} \mu_j \, \epsilon^{\dagger}_{\theta_j} \, \Phi^{-1} \epsilon_{\theta_i} = 2c_i \tag{16}$$

Equations (15) and (16) completely specify the desired solution for the antenna element weights for the beamshaping problem. They may be combined into a single equation by defining the matrix

$$H = \left[\epsilon_{\theta_1} \middle| \epsilon_{\theta_2} \middle| \cdots \middle| \epsilon_{\theta_m} \right]$$

and the vectors μ and C

$$\mu = \begin{bmatrix} \mu_1 \\ \cdot \\ \cdot \\ \cdot \\ \mu_m \end{bmatrix} \qquad C = \begin{bmatrix} C_1 \\ \cdot \\ \cdot \\ \cdot \\ C_m \end{bmatrix}$$

so that (16) may be written as

$$H^{\dagger}\Phi^{-1}H\mu = 2C$$

and therefore

$$\mu = 2(H^{\dagger}\Phi^{-1}H)^{-1}C$$

resulting in the following equation for the antenna element weights

$$A = \Phi^{-1}H(H^{\dagger}\Phi^{-1}H)^{-1}C \ . \tag{17}$$

Equations (11) and (17) represent the major result of this work. The former prescribes the weights of the antenna elements for maximizing the array gain in a particular direction and the latter prescribes the element weights for shaping the beam to meet specific constraints.

3. EXAMPLES

In this section we apply the equations of the previous section to the problem of reducing the beamwidth achievable by the conventional endfire and broadside arrays. We will show that by application of eq. (17) the beamwidth can be reduced at the expense of a reduction in gain and a resulting increase in sidelobe level. For the case of the endfire array we include an example of sidelobe reduction in addition to beamwidth reduction to further illustrate the application of eq. (17).

We consider, for both examples, an array of five isotropic elements, equally spaced at 5 inch intervals operating at 2.0 GHz, which results in a 0.85λ spacing between elements. We first examine the case of an endfire array, that is the element weights and phases are chosen to provide maximum gain in the direction of the line of the array.

Figure 2a shows the gain pattern of the array obtained from a solution of eq. (11) for maximizing the gain in the direction $\theta_0=0°$. A similar result would be obtained using eq. (17) for the case of a single constraint in the direction of $\theta_1=0$ deg. This conventional end-fire array solution results in an endfire gain of 5.8 and a beamwidth (measured between the half-power points) of approximately 50 deg. Using eq. (17) to reduce the beamwidth to 30° we obtain the gain pattern shown in fig. 2b. As a result of reducing the beamwidth by 40%, the endfire gain has been reduced by 45% to a value of 3.15 and the total power radiated increased by 80%. In addition, the gain in the broadside direction has become appreciably greater than in the endfire direction. In many applications, this particular sidelobe level increment may not be particularly worrisome but there may be some applications where a large gain at 90 deg. may bo undesirable. Figure 2c shows the result of reducing the gain at 90 deg. while maintaining the endfire beamwidth at 30 deg. The endfire gain has now been reduced to a value of 2.4, which is less than half of the original gain shown in fig. 2a and the total radiated power is 2.4 times greater than the value shown in that case. We have, however, been able to achieve a 30 deg. beamwidth and in addition, significantly reduce the extremely high broadside sidelobe level incurred in the intermediate result in which beamwidth control was the only constraint. The cost of this specific sidelobe reduction has been an increase in overall sidelobe level over the entire angular range.

Turning to the case of a broadside array, we see the result of maximizing the gain in fig. 3a. The beamwidth of the broadside lobe is 15 deg. for this array. Reducing the beamwidth to 9° results in the gain pattern indicated in fig. 3b. The broadside gain has been reduced by 45% and the radiated power increased by 80% as a result of this 40% reduction in beamwidth .

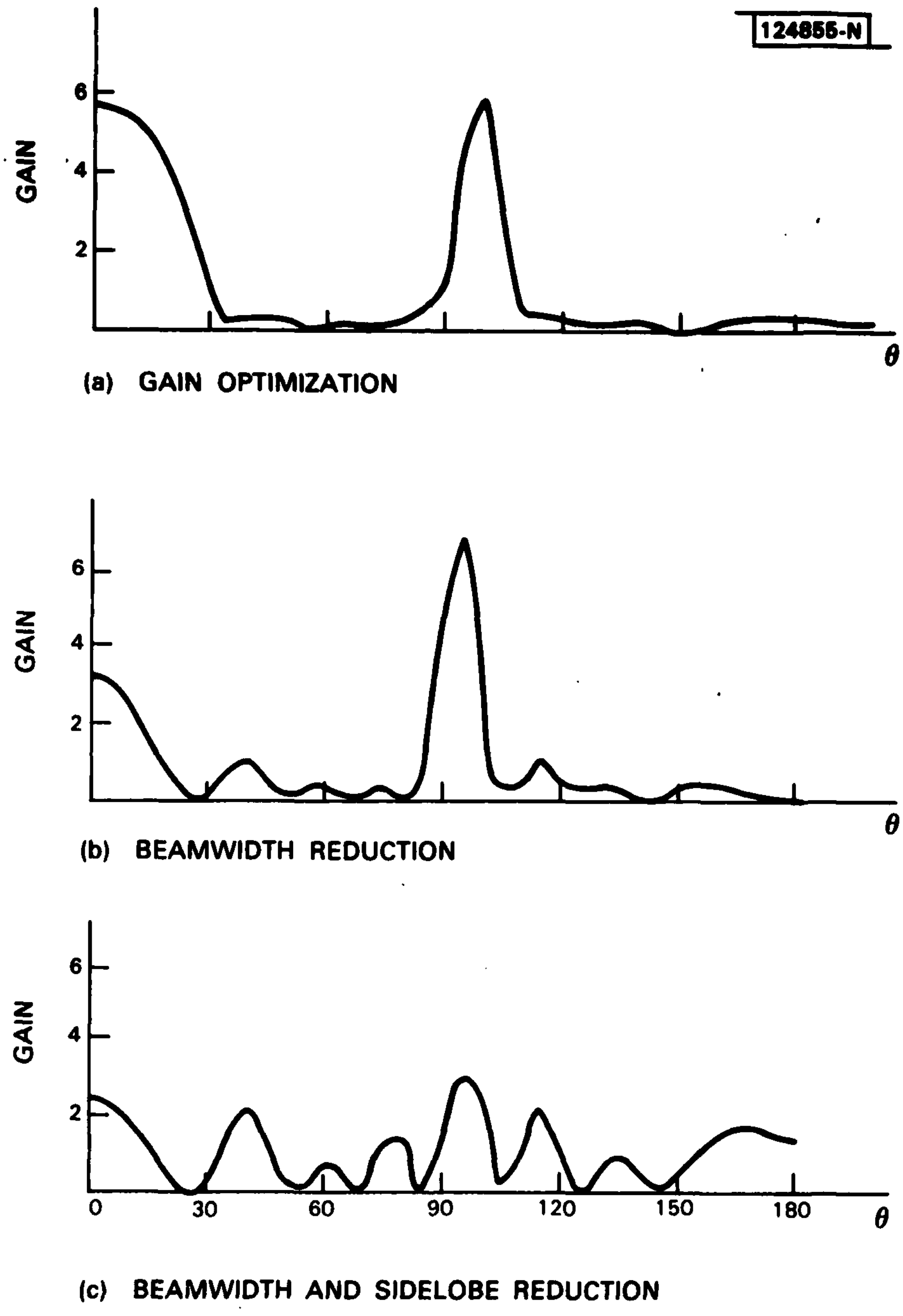

Fig. 2. Endfire Array Gain Patterns

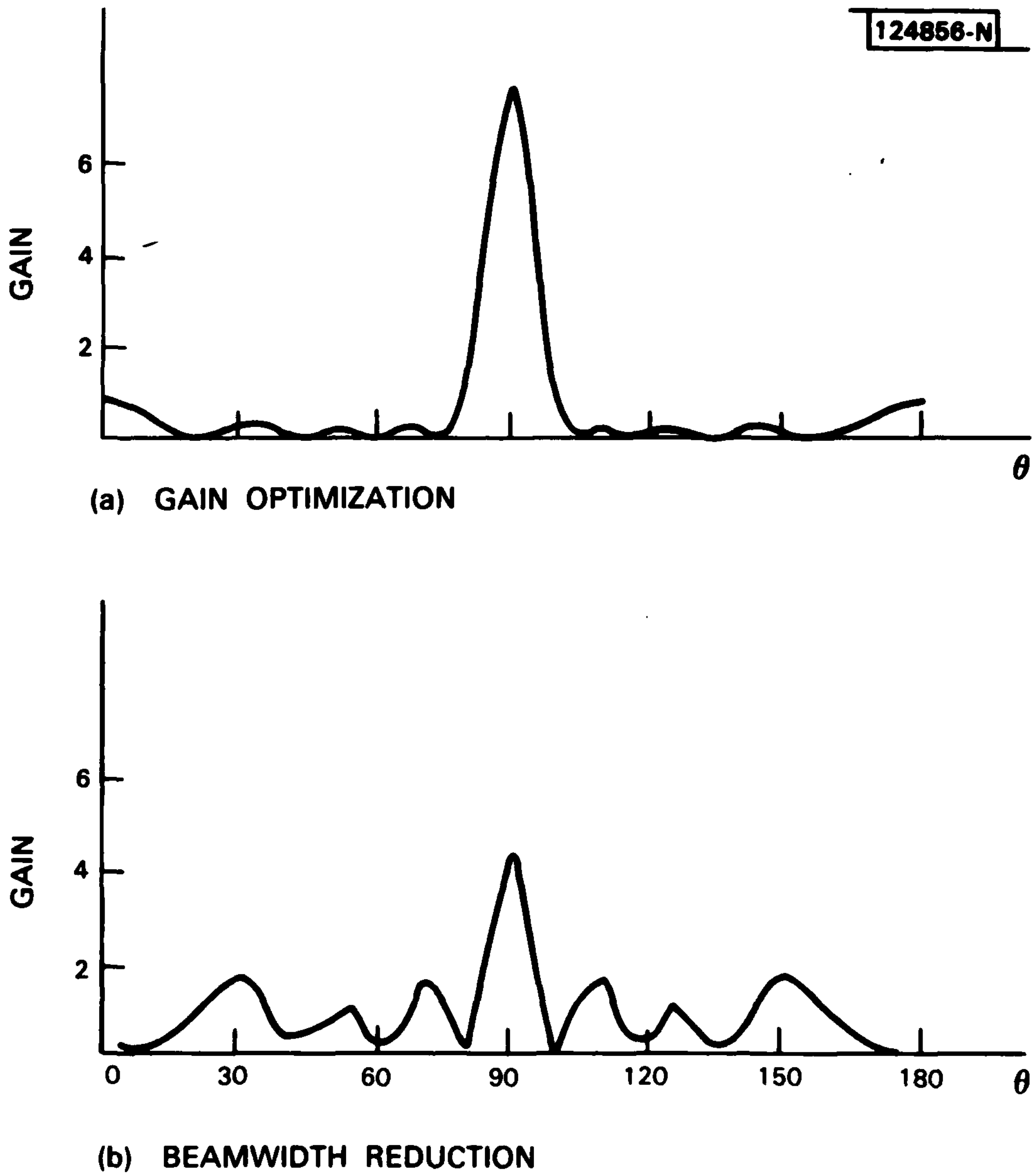

Fig. 3. Broadside Array Gain Patterns

This result is similar to that achieved for the case of the endfire array in that the reduction in mainlobe gain and increase in radiated power level are the same for an equivalent amount of beamwidth reduction. The increase in sidelobe level is evident in the increased gains at 70 and 110 deg. as well as at other regions of the gain pattern. However, in this case unlike the endfire case there is no large increase in a particular sidelobe so we will not illustrate sidelobe reduction for this array.

4. SUMMARY

A procedure for determining the weights of array elements to shape both the amplitude and phase of the radiated field has been presented along with an algorithm for determining the weights that maximize the gain in a specific direction.

The beamshaping procedure is a constrained optimization method which minimizes the total radiated power while constraining the complex value of the radiated field at specifc angles. While this provides both amplitude and phase control over certain portions of the radiated field, the value of the gain cannot be controlled over these regions. This results in trade-offs between array parameters (e.g. gain, beamwidth and sidelobe level) that the array designer must judge for each application. Examples of typical results and trade-offs were presented for the case of the conventional end-fire and broadside arrays.

REFERENCES

1. D. H. Johnson and S. R. DeGraaf, "Improving the Resolution of Bearing in Passive Sonar Arrays by Eigenvalue Analysis," IEEE Trans. Acoust., Signal Processing, Vol. ASSP-30, pp. 638-647, 1982.
2. J. Capon, "Maximum-Likelihood Spectral Estimation," Nonlinear Methods of Spectral Analysis, Topics in Applied Physics, Volume 34, ed. by S. Haykin (Springer-Verlag, New York, 1979).
3. R. N. McDonough, "Application of the Maximum-Likelihood Method and the Maximum-Entropy Method to Array Processing," Nonlinear Methods of Spectral Analysis, Topics in Applied Physics, Volume 34, ed. by S. Haykin (Springer-Verlag, New York, 1979).

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