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# SELECTION OF REPETITION FREQUENCIES FOR LASER RANGEFINDERS

by

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## ABSTRACT

This paper presents some methods for determining repetition frequencies for laser rangefinders. Using these methods, repetition frequencies for air-to-air, air-to-ground, and ground-to-air laser rangefinders can be correctly selected.

### I. FOREWORD

Laser rangefinders are noted for high rangefinding accuracy, low bulk, light weight, good antijamming ability, and excellent lowaltitude performance, and at present they have obtained wide use in various types of fire control systems. Since the high speed of the target, the interceptor aircraft, or both gives rise to a rapid relative motion between the weapon system and the target, in laser rangefinders used for small antiaircraft guns and aircraft fire control systems, lowering the repetition frequency will not ensure the hit accuracy of the weapon system; raising it is also not necessary and, likewise, it does not help to increase the bulk, the weight, or the power consumption. Therefore, how to rationally select rangefinder repetition frequencies (hereafter called simply repetition frequencies) becomes a crucial problem which must be solved when designing ground-to-air, air-to-ground, and air-to-air type laser rangefinders.

The fire control system requires real-time input into it of the

target distance in order to calculate the moment to fire or the lead angle of the weapon. For a moving target, the process of measuring target distance is essentially the process of carrying out sampling of continuously changing distance values. Based on the sampling theorem  $\mathbb{H}$ , when the sampling frequency is equal to or more than two times the maximum frequency  $f_m$  contained in the signal (assuming the frequency has a finite bandwidth, i.e.,  $-f_m < f < f_m$ ), it will be able to pass through an ideal low-pass filter and the original signal will be extracted intact. Consequently, the selection of rangefinder repetition frequencies is summed up as the problem of conducting spectral analysis on target distance information in order to determine its maximum frequency component. Below we discuss two types of laser rangefinders.

## II. AIR-TO-AIR LASER RANGEFINDERS

We used a typical aircraft fire control system as an example to conduct analysis on repetition frequencies of the air-to-air rangefinder. Aircraft fire control systems often take the plane which is defined by the velocity vectors of both the interceptor aircraft and the target aircraft as the basis of analysis. It is assumed that the target is in uniform linear motion (i.e., Vm = const) and that the interceptor aircraft is in constant speed flight (i.e.,  $V_1 = const$ ). Since the duration of an air combat attack is normally relatively brief, the range of the projectile relatively short, and the gravitational fall negligible, when the maneuverability of the target is relatively poor (such as a bomber) the above assumed and simplified spatial model is an approximation which holds true [2]. Under these simplified assumptions the following attack courses can be developed [2]:

1. The Pure-Pursuit Attack: The velocity vector of the interceptor aircraft is aimed at the target at all times. Infrared missiles are commonly aimed and launched using this attack mode.

2. The Lead-Pursuit Attack: The velocity vectors of the interceptor aircraft are at all times kept pointed at an impact point where the projectile and the target will collide. At present, a large number of aircraft cannon and rockets are aimed and fired using this attack mode.

3. The Lead-Collision Attack: The interceptor aircraft flies

at a constant heading aimed at a projectile-target collision point which is at a preselected launch range. This attack mode normally is used for launching high-powered missiles or rocket salvos and requires much less maneuverability of the interceptor aircraft.

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The characteristics of these three attack courses can be shown by fire control triangles as in Figure 1. In the target polar coordinate system, approach angle q and relative distance D are taken as the main parameters and the following fire control equations can be obtained

for pure-pursuit attack, see Fig. 1(a);

$$\dot{\mathbf{p}} = -V_{m}\cos q - V_{1} = -V_{m}(\cos q + K) \tag{1}$$

$$\dot{q}D = V_{m} \sin q \tag{2}$$

for lead-pursuit attack, see Fig. 1(b);

$$\dot{\mathbf{p}} = -V_{m}\cos q - V_{1}\cos \varphi \approx -V_{m}(\cos q + K) \tag{3}$$

$$\dot{q}D = aV_{m}\sin q \tag{4}$$

and lead-collision attack, see Fig. 1(c).

$$\dot{D} = -V_m(\cos q + K\cos \varphi) \tag{5}$$

$$D = V_m T \cos q + (V_1 T + D_{x_0}) \cos \varphi \tag{6}$$

In the above expressions,

$$K = \frac{V_1}{V_m} \tag{7}$$

$$a = 1 - \frac{V_{\perp}}{V_{dp}} \tag{8}$$

where  $V_1$  is the velocity of the interceptor aircraft,  $V_m$  is the velocity of the target,  $V_{dp}$  is the mean velocity of the projectile,  $\varphi$  is the lead angle,  $D_{x_0}$  is the preset relative range. From expressions (1)  $\sim$  (6) it is not difficult to derive the relative trajectory equations

for pure-pursuit, 
$$D_{tg}^{\kappa}(q/2)\sin q = D_{otg}^{\kappa}(q_{o}/2)\sin q_{o}$$
 (9)

for lead-pursuit,  $D^* tg^{\kappa}(q/2) \sin q = D^* stg^{\kappa}(q_0/2) \sin q_0$ 

and for lead-collision

$$D = \frac{D_{x_0} \sin \lambda}{\sin q \left(1 + (V_1/V_m) \cos \lambda\right) - (V_1/V_m) \sin \lambda \cos q}$$
(11)

where A is the course intersection angle;  $D_0$  and  $q_0$  are the relative distance and approach angle when t = 0, also called initial distance and occupied approach angle.



The problem now is to derive equations of distance with change in time, beginning with the above equations, then carry out Fourier transformation and find the maximum frequency component. However, for pure-pursuit and lead-pursuit there are no analytical solutions for distance with change in time. For the former we conducted numerical solutions based on the Parseval theorem [3] with the aid of an electronic computer; for the latter we used Guillemin's impulse approximation method [2, 4] to conduct frequency spectrum analysis. We introduce these below.

From expressions (2), (4), (9), and (10), taking q as a parameter to carry out numerical integration we can obtain a numerical solution for distance information D(t) for both the pure-pursuit and leadpursuit attack courses. Thus, based on the Parseval theorem, the total energy value of the signal is

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$$W = \int_{-\infty}^{+\infty} D(t) D^{\bullet}(t) dt \qquad (12)$$

 $W = \int_{-\infty}^{+\infty} S(f) S^*(f) df$ (13)

and

(10)

where

$$S(f) = \{ (14) \}$$

is the Fourier transform for D(t);  $S(f)S^{\bullet}(f) = |S(f)|^{\circ}$  is called the energy spectrum density function of D(t). Thus we can find the energy contained within any given bandwidth (-f, +f) as

$$W_{f} = \int_{-f}^{+f} |S(f)|^{2} df = \int_{-f}^{+f} S(f) S^{\bullet}(f) df$$
(15)

Comparing expression (15) to expression (12), we can then use the frequency components obtained when  $W_{\rm f}/W$  arbitrarily approaches 1 (such as 0.99) as the maximum frequency component  $f_{\rm m}$ . Based on the above, we used a DJS-14 computer to find the energy spectrum densities for D(t) under conditions of pure-pursuit and lead-pursuit as shown in Figures 2 and 3, respectively. Table 1 shows the frequency component  $f_{\rm m}$  which corresponds to  $W_{\rm f}/W$ =0.99 as well as the required repetition frequency  $f_{\rm c}$ .

Table 1. The Minimum Repetition Frequency f required for rangefinding during air-to-air attacks

这当方式	$\mathcal{D}_{5}$	40	Im	$f = 2f_{\rm cm}$
	(5; 00)	(c) (x)	(d) (G)	(e (it, it)
<sub>建道章</sub> (十)	5,000	. 90°	1.53	3.06
	5,000	150*	0.38	0.76
	10,000	90*	0.56	1.12
	10,000	150°	0.19	0.38
87.2 <i>4 (9</i> )	2,000	90°	4.00	8.00
	2,000	: 3., *	1.11	2.22

$$(V_m = 500 \text{ m/s}, \text{ K} = 1.2, a = 0.5)$$

KEY: (a) attack mode; (b) meters; (c) degrees; (d) Hertz; (e) times/second; (f) pure-pursuit; (g) lead-pursuit

It can be seen from Table 1 that when target velocity  $V_m$ , velocity ratio K, and coefficient 4 are fixed, the required repetition frequency f increases as the initial distance  $D_0$  and occupied approach angle  $q_0$  decrease. These results coincide with the characteristics of the energy spectrum curves in Figures 2 and 3. From Figures 2 and 3 we can see that when the numerical values of  $D_0$  and  $q_0$  are relatively large, the energy spectrum curves are rather pointed, the amplitude of the peak values is high, and there is an ample

low-frequency component; as  $D_0$  and  $q_0$  decrease, the energy spectrum curves gradually level out, the amplitude of the peak value drops, and the high-frequency component noticeably increases. This is because, for a given course of attack, as  $D_0$  and  $q_0$  decrease, the relative distances change more rapidly with change in time.



Fig. 2(a). Energy spectrum density for D(t) in pure-pursuit attack mode  $D_0 = 5000m$ ,  $V_m = 500m/s$ , K = 1.2;  $|S(0)|^2 = 1.98 \times 10^9$ , when  $q_0 = 90^\circ$ ,  $|S(0)|^2 = 1.21 \times 10^{10}$  when  $q_0 = 150^\circ$ 



Fig. 2(b). Energy spectrum density for D(t) in pure-pursuit attack mode  $D_0 = 10,000m$ ,  $V_m = 500m/s$ , K = 1.2;  $S(0) = 3.1 \times 10^{10}$ , when  $q_0 = 90^{\circ}$ ,  $|S(0)|^2 + 1.99 \times 10^{11}$  when  $q_0 = 150^{\circ}$ 





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For the lead-pursuit attack mode the relative motion trajectory is described by expression (11). Expression (11) can be rewritten as

$$D = P_{1}\cos(q - a) \tag{16}$$

where

-

$$P = D_{\tau_0} \sin \lambda / \left\{ \left( \frac{V_1}{V_m} \sin \lambda \right)^2 + \left( 1 + \frac{V_1}{V_m} \cos \lambda \right)^2 \right\}^{1/2}$$
(17)

$$\alpha = tg^{-1} \left\{ - \left( 1 + \frac{V_{\perp}}{V_{m}} \cos \lambda \right) / \left( \frac{V_{\perp}}{V_{m}} \sin \lambda \right) \right\}$$
(18)

Expression (16) is the standard form for "equations of a line" in the polar coordinate system, while P is the distance between the line in question and the origin of the coordinates, which we call pass-course slant range, and **4** is its polar coordinate angle. From fire control triangle Figure 1(c) it is not difficult to find that the velocity of relative motion is

$$\dot{R} \equiv V = V_m \left\{ \left( \frac{V_1}{V_m} \sin \lambda \right)^2 + \left( 1 + \frac{V_1}{V_m} \cos \lambda \right)^2 \right\}^{1/2}$$
(19)

Since the course intersection angle  $\lambda$  is constant in the leadcollision attack, the value of V is constant. This shows that the relative motion is uniform velocity, straight-line motion. Assuming pass-course time of t=0, we obtain an equation for relative distance with change in time

$$D(t) = P(1 + (t/\tau)^{*})^{t/*}$$

where

$$\tau = P/V \tag{21}$$

(20)

Such a time function cannot analytically directly determine the Fourier transform for expression (20). Even if the above method of treatment could be used here to carry out a numerical solution, since we must perform three numerical integrations to determine each point on the  $W_1^*$  curve and the machine time used will be quite long, we would rather use the Guillemin impulse method to approximate its Laplace transform, then letting  $s = j\omega$  we can determine the frequency spectrum characteristics. The dominant idea of the Guillemin impulse method is converting the integrand in Laplace integration into a set of impulses in order to use the analytical method to carry out

integration. Now letting the Laplace transform approximation expression for D(t) in expression (20) be  $D^{\frac{1}{2}}(5)$ , then from [2,4] we obtain

$$D^{\bullet}(S)/P = \frac{1}{S^{s}} - \frac{1}{\tau^{2}} \left(e^{st} - e^{st}\right)$$
(22)

and letting S-wje, we obtain a frequency characteristic of

$$D^{\bullet}(j\omega)/P = \frac{1}{(j\omega)^2} - \frac{1}{z^2} (e^{j\omega z} - e^{-j\omega z})$$
(23)

Hence, the frequency spectrum function becomes

$$|D^{\bullet}(j\omega)/P| = 2\sin\omega\tau/\omega^{\circ}\tau^{\circ}|$$
(24)

We take the first zero point of the frequency spectrum function as the maximum frequency component. Assuming that

$$\omega_{n}\tau = \pi \tag{25}$$

and assuming that

hence

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$$m = 1/2\tau$$
 (26)

The problem now is how to find  $\boldsymbol{\tau}$ . Therefore, we must first find course intersection angle  $\boldsymbol{\lambda}$ . Using Figure 1(c), from the theorem of sines we know that  $\boldsymbol{\lambda}$  must satisfy the following expression

 $2\pi f_m \tau = \pi$ 

$$D_{\rm s}/\sin(\pi - \lambda) = (k_{\rm s}T + D_{\rm rs}), \, \sin g_{\rm o} \tag{27}$$

and from the theorem of cosines we obtain

$$(V_{1}T + D_{z_{0}})^{2} = D_{0}^{2} + (V_{m}T)^{2} - 2D_{0}V_{m}T\cos q_{0}$$
(28)

From expression (27) and (28) we can obtain fly-over time T. When  $V_1$  and flight altitude H are constant, missile velocity  $V_G$  is constant. Hence,  $V_{cl_{f_0}} = D_{x_0} = \text{const}(t_{f_0} \text{ is preset projectile flight})$ time), i.e., the preset relative launch range  $D_{x_0}$  is a constant. When  $D_0$ ,  $q_0$ ,  $V_1$ , and  $V_m$  are known, we then can obtain  $\tau$  from expressions (17), (19), (21), and (28) and thus we obtain  $f_m$ . With  $V_1 =$ 600m/s and  $V_m = 500m/s$ ,  $\tau$  and  $f_m$  obtained for different (initial condition)  $D_0$  and  $q_0$  at several different preset relative launch ranges are listed in Table 2. It is obvious from Table 2 that the maximum frequency component  $f_m$  increases as the occupied approach angle  $q_0$  and preset relative launch range  $D_{x_0}$  decrease. Using the above computer method for finding a numerical solution for the case where  $D_0 = 7000m$ ,  $q_0 = 90^\circ$ , and D = 300m, we obtained a frequency component of 0.59Hz which contained 99% of the energy and using the Guillemin impulse approximation method we obtained fm = 0.7Hz. Thus the energy contained within the given bandwidth was over 99%. It is obvious that the approximation method is sufficiently accurate. Its advantage is that the solution of the problem of repetition frequency can simply be summed up as the solution of time constant  $\tau$ , and thus the problem is greatly simplified. It is also suitable for spectrum analysis of various "pass-course" problems.

D	Do	90	τ	$f_m = 1/2 \tau$	$f = 2f_m$
(*)(a)	(*) (a)	(度) (6)	(砂) (С)	(赫) (d)	(次/步)(e)
7,200	10,000	90 *	7.59	0.065	0.13
7.200	10,000	150*	32.60	0.015	0.03
3,600	7.000	30°	4.74	0.11	0.22
3,600	7,000	150*	16.36	0.03	0.06
300	7,000	90*	0.71	0.7	1.4
300	7,000	130°	1.14	0.44	0.38

Table 2. The Minimum Repetition Frequency f Required for Rangefinding in the Lead-Collision Attack  $(V_1 = 600 \text{m/s}, V_2 = 500 \text{m/s})$ 

KEY: (a) meters; (b) degrees; (c) seconds; (d) Hz; (e) times/second

III. AIR-TO-GROUND AND GOUND-TO-AIR LASER RANGEFINDERS

First we use level bombing as an example to analyze the minimum repetition frequency required when the laser rangefinder is set up as a bomb sight. Based on level bombing principles, in order for the bomb to hit the target the bomb run must be as shown in Figure 4 [2]. Let H = flight altitude, D = slant range to target M, P = pass-course slant range, V = flight speed of the aircraft, and d = horizontal offset. When t = 0, D = P. Hence, distance D with change in time t must satisfy

$$D/P = \{1 + (1/\tau)^2\}^{1/2}$$
(29)

where

$$\mathbf{r} = P/V \tag{30}$$



KEY: (a) wind direction

Fig. 4. Distances in "Pass-Course" Problems with Change in Time

It is obvious that expressions (29) and (30) are identical to expressions (20). and (21). Therefore, expression (26) can be used to find the maximum frequency component fm. Table 3 shows the repetition frequencies required after entering the bomb run at a typical flight speed of 300m/s when bombing at low altitude and at medium and high altitude. We can see from Table 3 that the repetition frequency required for low-altitude bombing is much higher than for highaltitude bombing. Therefore, when designing we must take the repetition frequency which satisfies the requirements of low-altitude bombing as the standard.

	(4) <sup>(4)</sup> 2	a de	5 ··· -	ie di
Deres (C)	120	600	1.200	12.000
11月 (1)	3	0.0	33	00
$\tau = P(V(W)(e)$	0.4	2	4	40
$f_{m} = 1/2 \tau (33) (f)$	1.25	0.25	0.125	0.0125
$f = f_{\mathcal{M}}(\mathcal{K} / \mathcal{P})(g)$	2.5	0. <b>5</b>	0-23	ñ. 923

Table 3. Typical Values for Laser Rangefinder Repetition Frequencies in Bomber Fire Control Systems

REY: (a) low-altitude bombing; (b) medium and high-altitude bombing; (c) meters; (d) meters/second; (e) seconds; (f) Hz; (g) times/second

The repetition frequencies used for ground-to-air laser rangefinders in small anti-aircraft fire control systems are analyzed below. It is common knowledge that small anti-aircraft guns are at the present time effective weapons for low-altitude defense. Since laser rangefinders have good anti-jamming characteristics, superior low-altitude performance, and high rangefinding accuracy, they have obtained increasingly wide use in small anti-aircraft fire control systems. A typical small anti-aircraft fire control system is designed under the assumption of the target moving at a constant velocity in a straight line and this approximately holds true during the the brief time of low altitude firing. With the target in level flight, its course is the same as in Figure 4, only point M reprecents the location of the guns or the rangefinder and d now represents the shortcut [TRANSLATOR'S NOTE: i.e., shortest distance] to the track. With a target velocity of 350m/s, the typical values of required repetition frequencies are shown in Table 4.

Table 4. Typical Values for Laser Rangefinder Repetition Frequencies in Small Anti-aircraft Fire Control Systems (V = 350m/s)

H (*) <b>(</b> 4)	<b>4)</b> 100		500		1,000	
d(*)(a)	0	100	0	100	0	500
$P = \sqrt{H^2 + d^2}$ (*)(a)	100	141	500	510	1,000	1,118
$\tau = P/V(b)$ (b)	0.29	0.4	1.43	1.46	2.86	3.19
$f_m = 1/2 - (\frac{1}{2})$ (G)	1.75	1.25	0.35	0.34	0.175	0.157
$f = 2f_m(\mathcal{K}/\mathfrak{H})$ (d)	3.5	2.5	0.7	0.68	0.35	0.31

#### KEY: (a) meters; (b) seconds; (c) Hz; (d) times/second

It can be seen from Figure 4 that repetition frequency f increases as flight altitude H and shortcut to track d decrease. Obviously, f also increases as target velocity increases.

## IV. BRIEF SUMMARY

This paper has presented a number of methods for determining laser rangefinder repetition frequencies. These methods can be used for correctly selecting repetition frequencies for air-to-air, airto-ground, and ground-to-air laser rangefinders. Several typical illustrative examples which have been discussed can bring to light some general trends.

1. Repetition frequency  $\mathbf{f}$  required for air-to-air rangefinding primarily depends upon the mode of attack. Under conditions where the values of  $V_m$ ,  $V_1$ , and  $\boldsymbol{a}$  are fixed,  $\mathbf{f}$  increases as  $q_0$ ,  $D_0$ , or  $D_{\mathbf{x}_c}$ decrease. The repetition frequency required in lead-pursuit attack is the highest, and can be more than 8 times/second. 2. The repetition frequency f required for air-to-ground rangefinding primarily depends upon the altitude of the bomber. The f required for a low-altitude bomber is much greater than that for a high-altitude bomber, with a typical value of more than 2.5 times/ second.

3. The repetition frequency f required for ground-to-air rangefinding primarily depends upon target velocity, altitude and track shortcut. For low-altitude, high-speed targets, the f required for a track shortcut of zero is high, having a typical value of more than 3.5 times/second.

It should be pointed out that in the above discussion it is also assumed that the return rate is 100%. In actuality, the return rate is often less than 100% and the repetition frequency must be increased proportionately. If  $\eta$  is the return rate and F is the actually required repetition frequency, then under conditions where the system has a reasonable return rate,  $F = f/\eta$ , where f is the repetition frequency required when  $\eta = 100$ %. As for the matter of return rates when a laser rangefinder is fitted to various types of tracking systems and fire control systems, this is beyond the scope of the discussion in this paper and will be discussed in another paper.

\* \* \* \*

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