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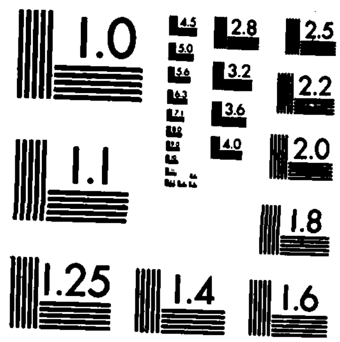

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

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THE MATHEMATICAL BASIS OF EXPOSURE CONTROL CALCULATIONS

Introduction:

The current exposure control system is based on empirical data for standard decay which is shown in graphic form in Attachment A. The graphs indicate a linear relationship between $\log(R/R_1)$ and $\log t$:

1.* $$\log(R_t/R_1) = -1.2 \log t$$

where t is the time after explosion (in hours) of the radiation reading R_t (in rads/hour), and R_1 is the radiation reading at 1 hour after the explosion. Note that this is standard decay with a decay exponent of -1.2. The current exposure control system cannot handle non-standard decay, which will be addressed later.

After rearrangement, equation 1 becomes:

2. $$R_t = R_1 t^{-1.2}$$

Integrating with respect to t between times t_a and t_b ($t_a < t_b$) gives the dose, D, received during that time interval:

3. $$D = \int R_t dt = \int_{t_a}^{t_b} R_1 t^{-1.2} dt = R_1 \int_{t_a}^{t_b} t^{-1.2} dt$$

$$= R_1(-1/0.2) \int_{t_a}^{t_b} -0.2 t^{-1.2} dt$$

$$= R_1(-5)(t^{-0.2}) \Big|_{t_a}^{t_b} = -5R_1(t_b^{-0.2} - t_a^{-0.2})$$

4. $$D = 5R_1(t_a^{-0.2} - t_b^{-0.2})$$

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*An index to the variables used in this paper (by equation number and page) is included as Attachment B.

Standard Decay, Current Stay-Time:

Stay-time is based upon the Maximum Allowable Dose, MAD, the Total Accumulated Dose, TACD, and the Allowable Dose, ALD, for a trip outside the shelter:

$$\mathrm{MAD} = \mathrm{TACD} + \mathrm{ALD}, \quad \text{or}$$

5. $$\mathrm{ALD} = \mathrm{MAD} - \mathrm{TACD}$$

Using equation 4:

$$\mathrm{ALD} = D = 5R_1(t_a^{-0.2} - t_b^{-0.2}), \quad \text{or}$$

6. $$\mathrm{ALD} = 5R_1[t^{-0.2} - (t + \mathrm{CST})^{-0.2}]$$

where CST is the Current Stay-Time. This is then solved for CST:

$$\mathrm{ALD}/5R_1 = t^{-0.2} - (t + \mathrm{CST})^{-0.2}$$

$$(t + \mathrm{CST})^{-0.2} = t^{-0.2} - \mathrm{ALD}/5R_1$$

$$t + \mathrm{CST} = (t^{-0.2} - \mathrm{ALD}/5R_1)^{-5}$$

7. $$\mathrm{CST} = (t^{-0.2} - \mathrm{ALD}/5R_1)^{-5} - t$$

Solving equation 2 for R_1 gives:

8. $$R_1 = R_t\, t^{1.2}$$

Combining equations 7 and 8 gives:

9. $$\mathrm{CST} = (t^{-0.2} - \mathrm{ALD}/5R_t\, t^{1.2})^{-5} - t$$

$$= t^{(-0.2)(-5)}(1 - \mathrm{ALD}/5R_t\, t)^{-5} - t$$

10. $$\mathrm{CST} = t[(1 - \mathrm{ALD}/5R_t\, t)^{-5} - 1]$$

Combining equations 5 and 10 gives the standard decay CST equation:

11. $$\mathrm{CST}_{std} = t[(1 - \frac{\mathrm{MAD} - \mathrm{TACD}}{5Rt})^{-5} - 1]$$

Standard Decay, Future Stay-Time:

Like CST, Future Stay-Time, FST, is also based on MAD, TACD, and ALD, but it also incorporates the dose received in shelter prior to the anticipated exit, D_{in}:

$$MAD = TACD + D_{in} + ALD, \quad \text{or}$$

12. $$ALD = MAD - TACD - D_{in}$$

Combining equations 4 and 12 gives:

13. $$ALD = MAD - TACD - 5R_1(t^{-0.2} - t_d^{-0.2})/PF$$

where t is the current time, t_d is the time of the future departure (both in hours after explosion), and PF is the shelter's protection factor. Using equation 4 again, this time in place of ALD, gives:

14. $$5R_1[t_d^{-0.2} - (t_d + FST)^{-0.2}] = MAD - TACD - \frac{5R_1}{PF}(t^{-0.2} - t_d^{-0.2})$$

Solving for FST:

$$t_d^{-0.2} - (t_d + FST)^{-0.2} = \frac{MAD - TACD}{5R_1} - (t^{-0.2} - t_d^{-0.2})/PF$$

$$(t_d + FST)^{-0.2} = t_d^{-0.2} - \frac{MAD - TACD}{5R_1} + (t^{-0.2} - t_d^{-0.2})/PF$$

$$t_d + FST = \left(t_d^{-0.2} - \frac{MAD - TACD}{5R_1} + \frac{1}{PF}(t^{-0.2} - t_d^{-0.2})\right)^{-5}$$

$$FST = \left(t_d^{-0.2} - \frac{MAD - TACD}{5R_1} + \frac{t^{-0.2} - t_d^{-0.2}}{PF}\right)^{-5} - t_d$$

15. $$FST = \left(t_d^{-0.2} + \frac{TACD - MAD}{5R_1} + \frac{t^{-0.2} - t_d^{-0.2}}{PF}\right)^{-5} - t_d$$

Combining equations 8 and 15 gives the standard decay FST equation:

16. $$FST_{std} = \left(t_d^{-0.2} + \frac{TACD - MAD}{5Rt^{1.2}} + \frac{t^{-0.2} - t_d^{-0.2}}{PF}\right)^{-5} - t_d$$

Non-standard Decay:

One major flaw in the current exposure control system is its inability to allow for variable or non-standard decay. In this type of decay, the "1.2" exponent of equation 2 (and subsequent calculations) varies from 0.9 to 2.0 as a result of weapons variations, overlapping fallout patterns (resulting in multiple peaks of radiation), weathering, etc. Realistically, this must be taken into consideration. (See *Effects of Nuclear Weapons*, 1977, para 9.151.) This section deals with the derivation of CST and FST equations for variable decay exponents.

Equation 2 becomes:

17. $$R_t = R_1 t^{-n}$$

where n is the decay exponent. Therefore:

18. $$D = \int R_t dt = \int_{t_a}^{t_b} R_1 t^{-n} dt = R_1 \int_{t_a}^{t_b} t^{-n} dt$$

$$= R_1 \left(\frac{1}{1 - n}\right) \int_{t_a}^{t_b} (1 - n)\, t^{-n}\, dt$$

$$= R_1 \left(\frac{1}{1 - n}\right) (t^{1-n}) \Big|_{t_a}^{t_b}$$

$$= \frac{R_1}{1 - n} (t_b^{1-n} - t_a^{1-n})$$

19. $$D = \frac{R_1}{n - 1} (t_a^{1-n} - t_b^{1-n})$$

Non-Standard Decay, Current Stay-Time:

For calculating variable or non-standard decay CST:

20. $$ALD = MAD - TACD$$

as before; then using equation 19:

21. $$ALD = \frac{R_1}{n - 1}\left(t^{1-n} - (t + CST)^{1-n}\right)$$

Solving for CST:

$$\frac{ALD(n - 1)}{R_1} = t^{1-n} - (t + CST)^{1-n}$$

$$(t + CST)^{1-n} = t^{1-n} - ALD(n - 1)/R_1$$

$$t + CST = \left(t^{1-n} - ALD(n - 1)/R_1\right)^{1/(1-n)}$$

22. $$CST = \left(t^{1-n} - ALD(n - 1)/R_1\right)^{\frac{1}{1-n}} - t$$

Solving equation 17 for R_1 gives:

23. $$R_1 = R_t\ t^n$$

Substituting this and equation 20 into equation 22 gives the variable (non-standard) decay CST equation:

24. $$CST_{var} = \left(t^{1-n} + \frac{(MAD - TACD)(1-n)}{R\,t^n}\right)^{\frac{1}{1-n}} - t$$

<u>Non-Standard Decay, Future Stay-Time</u>:

For calculating the variable decay FST:

25. $$ALD = MAD - TACD - D_{in}$$

as before. Combining equations 19 and 25 gives:

26. $$ALD = MAD - TACD - \frac{R_1}{n-1}(t^{1-n} - t_d{}^{1-n})/PF$$

Re-inserting equation 19 for ALD gives:

27. $$\frac{R_1}{n-1}\left[t_d{}^{1-n} - (t_d + FST)^{1-n}\right] = MAD - TACD - \frac{R_1}{PF(n-1)}(t^{1-n} - t_d{}^{1-n})$$

Solving for FST:

$$t_d{}^{1-n} - (t_d + FST)^{1-n} = \frac{(MAD - TACD)(n-1)}{R_1} - \frac{t^{1-n} - t_d{}^{1-n}}{PF}$$

$$(t_d + FST)^{1-n} = \frac{(MAD - TACD)(1-n)}{R_1} + \frac{t^{1-n} - t_d{}^{1-n}}{PF} + t_d{}^{1-n}$$

$$t_d + FST = \left[\frac{(MAD - TACD)(1-n)}{R_1} + \frac{t^{1-n} - t_d{}^{1-n}}{PF} + t_d{}^{1-n}\right]^{\frac{1}{1-n}}$$

28. $$FST = \left[\frac{(MAD - TACD)(1-n)}{R_1} + \frac{t^{1-n} - t_d{}^{1-n}}{PF} + t_d{}^{1-n}\right]^{\frac{1}{1-n}} - t_d$$

Substituting equation 23 into equation 28 gives the variable decay FST equation:

29. $$FST_{var} = \left[\frac{(MAD - TACD)(1-n)}{R\,t^n} + \frac{t^{1-n} - t_d{}^{1-n}}{PF} + t_d{}^{1-n}\right]^{\frac{1}{1-n}} - t_d$$

This is the general form for the stay-time equation; that is, the previous stay-time equations (equations 11, 16, and 24) are special cases of equation 29.

Calculation of n and t:

The use of variable decay necessitates a method of evaluating n in terms of known quantities. Different approaches are possible depending on whether the time of explosion, t_o, is known.

Known time of explosion:

If t_o is known, data from two radiation readings can be used to evaluate n. From equation 17, it can be seen that:

$$R_1 = R_a t_a^n = R_b t_b^n, \text{ so}$$

30. $$(t_b/t_a)^n = R_a/R_b$$

$$n \log(t_b/t_a) = \log(R_a/R_b)$$

31. $$n = \frac{\log(R_a/R_b)}{\log(t_b/t_a)}$$

Unknown time of explosion:

The more likely case would be that t_o is unknown or that several fallout generating blasts occurred at times different enough to preclude the selection of any single one as "the" t_o. In this case, t_o (or an apparent t_o) must be calculated.

Determination of n in this case requres a numerical approach and will be discussed later; for now we will assume that we know the value of n.

The derivation of t_o is based upon equation 30 and the following variable notation:

t_a = time elapsed between explosion and reading a
t_{ab}= time elapsed between readings a and b
$t_b = t_a + t_{ab}$ = time elapsed between explosion and reading b

Inserting the last equation into equation 30 gives:

$$\frac{t_b}{t_a} = \frac{t_a + t_{ab}}{t_a} = (R_a/R_b)^{1/n}$$

$$1 + \frac{t_{ab}}{t_a} = (R_a/R_b)^{1/n}$$

$$t_{ab}/t_a = (R_a/R_b)^{1/n} - 1$$

32. $$t_a = \frac{t_{ab}}{(R_a/R_b)^{1/n} - 1}$$

The Zulu or Local time for the time of explosion can then be determined from t_a and the time (Z or L) of reading a.

Calculation of n:

To find n in terms of measurable quantities requires a third radiation reading at time t_c. Using equation 32 and the variable t_{bc} to represent the time between readings b and c:

$$t_b = \frac{t_{bc}}{(R_b/R_c)^{1/n} - 1} = t_a + t_{ab}$$

$$\frac{t_{bc}}{(R_b/R_c)^{1/n} - 1} = \frac{t_{ab}}{(R_a/R_b)^{1/n} - 1} + t_{ab}$$

$$= t_{ab}\left[\frac{1}{(R_a/R_b)^{1/n} - 1} + 1\right]$$

$$\frac{t_{bc}}{t_{ab}}\left(\frac{1}{(R_b/R_c)^{1/n} - 1}\right) = \frac{1 + (R_a/R_b)^{1/n} - 1}{(R_a/R_b)^{1/n} - 1}$$

$$\frac{t_{bc}}{t_{ab}} = \frac{(R_a/R_b)^{1/n}\left[(R_b/R_c)^{1/n} - 1\right]}{(R_a/R_b)^{1/n} - 1}$$

$$= \frac{(R_a/R_b)^{1/n}(R_b/R_c)^{1/n} - (R_a/R_b)^{1/n}}{(R_a/R_b)^{1/n} - 1}$$

$$\frac{t_{bc}}{t_{ab}} = \frac{\left(\frac{R_a}{R_b} \cdot \frac{R_b}{R_c}\right)^{1/n} - \left(\frac{R_a}{R_b}\right)^{1/n}}{(R_a/R_b)^{1/n} - 1}$$

Simplification and multiplication by 1 gives:

$$\frac{t_{bc}}{t_{ab}} = \frac{(R_a/R_c)^{1/n} - (R_a/R_b)^{1/n}}{(R_a/R_b)^{1/n} - 1} \cdot \frac{(R_b/R_a)^{1/n}}{(R_b/R_a)^{1/n}}$$

33. $$\frac{t_{bc}}{t_{ab}} = \frac{(R_b/R_c)^{1/n} - 1}{1 - (R_b/R_a)^{1/n}}$$

Choosing the time intervals between the radiation readings so that $t_{bc} = t_{ab}$ gives an intuitively interesting result:

$$1 = \frac{t_{bc}}{t_{ab}} = \frac{(R_b/R_c)^{1/n} - 1}{1 - (R_b/R_a)^{1/n}}$$

$$1 - (R_b/R_a)^{1/n} = (R_b/R_c)^{1/n} - 1$$

$$(R_b/R_c)^{1/n} + (R_b/R_a)^{1/n} = 2$$

$$R_b^{1/n}\,(R_c^{-1/n} + R_a^{-1/n}) = 2$$

34. $$R_b^{-1/n} = \frac{R_c^{-1/n} + R_a^{-1/n}}{2}$$

Note that equation 34 resembles an averaging formula. Also note that this equation does not easily transform into an explicit equation for n; therefore, we will resort to an approximation method to find n. Continuing from equation 33:

$$t_{bc}\left(1 - (R_b/R_a)^{1/n}\right) = t_{ab}\left((R_b/R_c)^{1/n} - 1\right)$$

$$t_{bc} - t_{bc}(R_b/R_a)^{1/n} = t_{ab}(R_b/R_c)^{1/n} - t_{ab}$$

$$t_{ab} + t_{bc} = t_{ab}(R_b/R_c)^{1/n} + t_{bc}(R_b/R_a)^{1/n}$$

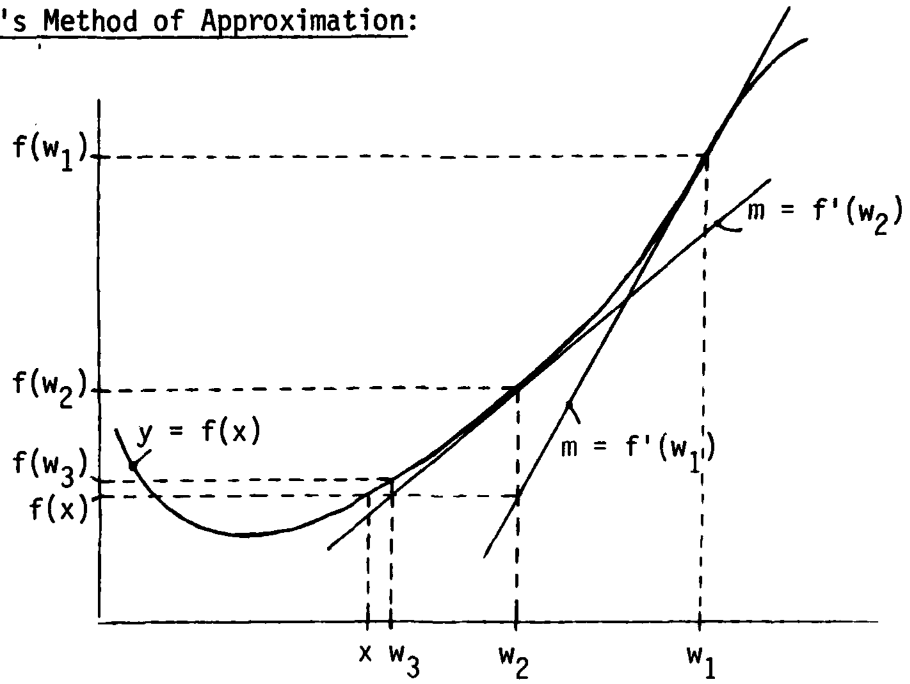
$$T = t_{ab}(R_b/R_c)^{1/n} + t_{bc}(R_b/R_a)^{1/n}$$

35. $$f(n) = 1 = \frac{t_{ab}}{T}(R_b/R_c)^{1/n} + \frac{t_{bc}}{T}(R_b/R_a)^{1/n}$$

36. $$f(n) = 1 = \frac{R_b^{1/n}}{T}(t_{ab}/R_c^{1/n} + t_{bc}/R_a^{1/n})$$

where $T = t_{ab} + t_{bc}$. Equation 36 is an implicit equation for n which can be used to develop a numerical approximation algorithm by which n can be calculated to any desired level of precision on a programmable calculator. Before developing the algorithm, the theory behind the approximation method will be discussed.

Newton's Method of Approximation:



For any function $y = f(x)$ at point $(w_1, f(w_1))$, the slope, m, of the tangent line is given by the equation

$$m = f'(w_1) = \frac{f(x) - f(w_1)}{w_2 - w_1}$$

where $f(x)$ is known and is taken as the y-value on the tangent line at w_2, and w_1 is the first "best-guess" for an approximate value for x. Solving for w_2 gives:

$$w_2 = w_1 + \frac{f(x) - f(w_1)}{f'(w_1)}$$

Note in the figure on the preceding page that w_2 more closely approximates x than does w_1, and that repeated application of the procedure ing w_2 in place of w_1 results in w_3, an even better approximation for Therefore, the general approximation equation is:

37. $$x \simeq w + \frac{f(x) - f(w)}{f'(w)}$$

where w is the current best approximation for x. The procedure can be iterated until an adequately precise approximation is reached.

<u>Approximation of n</u>:

Equation 38, then, is the basis for estimating n:

38. $$n \simeq w + \frac{1 - f(w)}{f'(w)}$$

where w is an approximation for n, $f(x) = f(n) = 1$ (from equation 36), and $f(w)$ is calculated using w for n in equation 36. Next, $f'(w)$, the first derivative at $n = w$ needs to be calculated. Starting from equation 35:

$$f(n) = \frac{t_{ab}}{T}(R_b/R_c)^{n^{-1}} + \frac{t_{bc}}{T}(R_b/R_a)^{n^{-1}}$$

$$f'(n) = \frac{t_{ab}}{T}(R_b/R_c)^{n^{-1}}(-n^{-2})\ln(R_b/R_c) + \frac{t_{bc}}{T}(R_b/R_a)^{n^{-1}}(-n^{-2})\ln(R_b/R_a)$$

39. $$f'(n) = \frac{-R_b^{1/n}}{n^2 T}\left(\frac{t_{ab}}{R_c^{1/n}}\ln(R_b/R_c) + \frac{t_{bc}}{R_a^{1/n}}\ln(R_b/R_a)\right)$$

Evaluating equations 36 and 39 at $n = w$ and substituting into equation 38 gives:

40. $$n \simeq w + \frac{1 - \frac{R_b^{1/w}}{T}\left(\frac{t_{ab}}{R_c^{1/w}} + \frac{t_{bc}}{R_a^{1/w}}\right)}{\frac{-R_b^{1/w}}{n^2 T}\left(\frac{t_{ab}}{R_c^{1/w}} \ln (R_b/R_c) + \frac{t_{bc}}{R_a^{1/w}} \ln (R_b/R_a)\right)}$$

Using the following substitutions,

41a. $$A = t_{bc}/R_a^{1/w}$$

b. $$B = T/R_b^{1/w}$$

c. $$C = t_{ab}/R_c^{1/w}$$

and multiplying by 1, gives:

$$n \simeq w + \frac{1 - \frac{1}{B}(C + A)}{\frac{-1}{w^2 B}\left(C \ln (R_b/R_c) + A \ln (R_b/R_a)\right)} \cdot \frac{B}{B}$$

$$\simeq w + \frac{B - (C + A)}{\frac{-1}{w^2}\left(C \ln (R_b/R_c) + A \ln (R_b/R_a)\right)}$$

42. $$n \simeq w + w^2 \frac{A - B + C}{C \ln (R_b/R_c) + A \ln (R_b/R_a)}$$

This is the approximation equation for n.

To find the best approximation for n, the equation is used iteratively, taking the resultant n from each step for the value of w in the next step until the desired level of precision is reached. A reasonable limit in this case is

43. $$|n - w| < 0.05$$

This method of approximation rapidly converges on n so that the limit should be reached within a few iterations. When n is found, it can then be used with equation 32 to find the time in hours since the explosion.

This can then be used with n and the current radiation reading in equations 24 and 29 to find the Current and Future Stay-Times. (An approximation procedure similar to the one described here could be used with equation 29 to calculate the time of departure from shelter, t_d, at which FST will equal the amount of time projected to be spent outside. In other words, an optimal departure time could be calculated.)

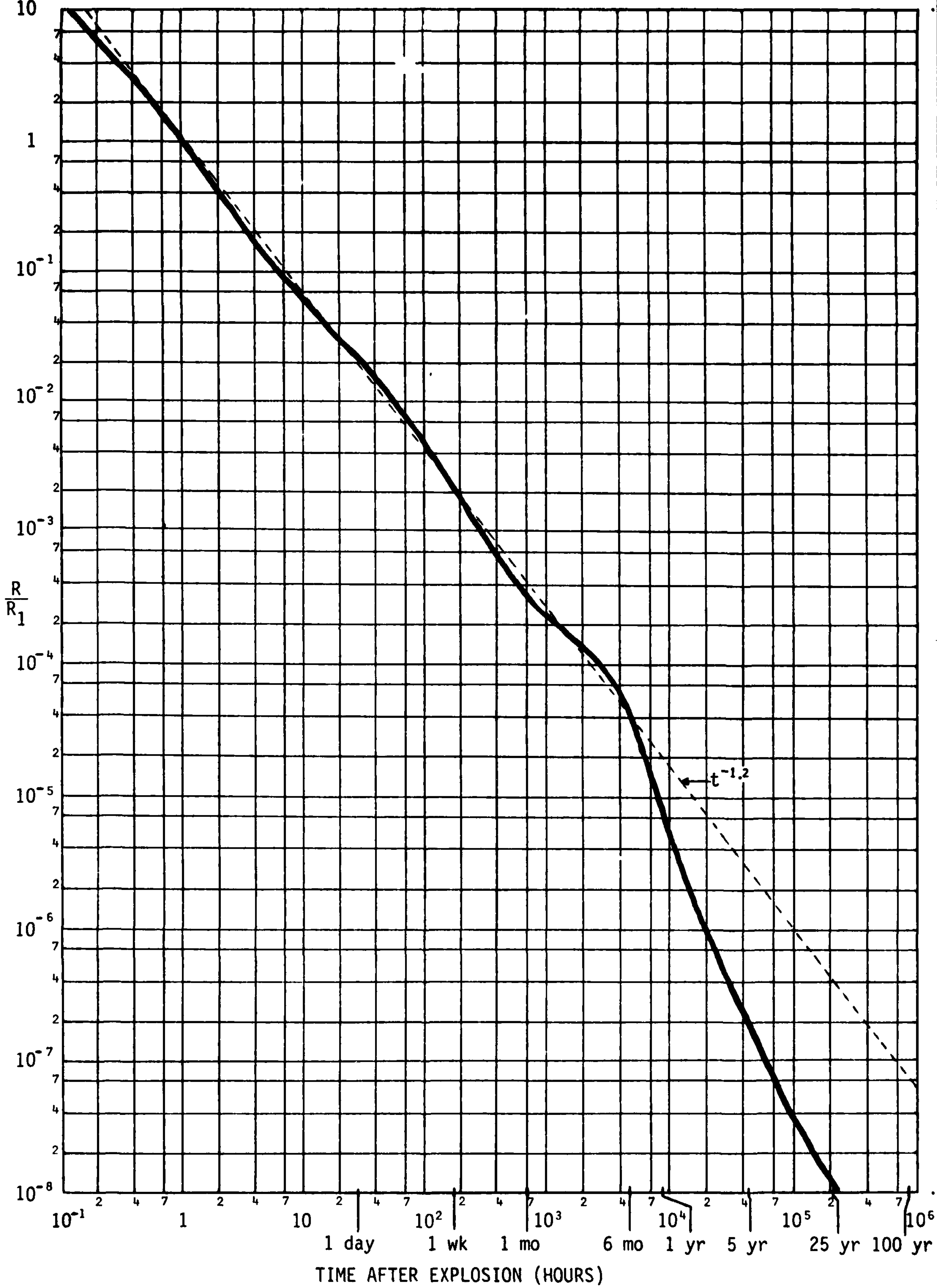
Conclusion:

It is important to note that using this analysis, exposure control calculations require knowledge of only three consecutive (and decreasing) radiation readings and the two time intervals between the readings. For this set of equations, there are no restrictions on the time intervals. Most importantly, the analysis presented here does not assume standard decay, nor does it require knowledge of the time of the nuclear detonation(s).

Reference: The Effects of Nuclear Weapons, Samuel Glasstone and Philip J. Dolan, eds., 1977. (Dept. of the Army Pamphlet 50-3).

Revised 1 March 1983
S. Mark Clardy

(From The Effects of Nuclear Weapons, Glasstone and Dolan, eds.) ATCH A



VARIABLE INDEX

Variable	Definition	First Appearance Eq'n #	Page #
A	$t_{bc}/R_a^{1/w}$ (substitution variable)	41	12
ALD	Allowable dose (rads)	5	2
B	$T/R_b^{1/w}$ (substitution variable)	41	12
C	$t_{ab}/R_c^{1/w}$ (substitution variable)	41	12
CST	Current stay-time (hrs)	6	2
D	Dose (rads)	3	1
D_{in}	Dose received in shelter before exit (rads)	12	3
FST	Future stay-time (hrs)	14	3
MAD	Maximum allowable dose (rads)	5	2
n	Decay exponent	17	4
PF	Protection factor of shelter ($= \frac{\text{outside R}}{\text{inside R}}$)	13	3
R	Radiation reading, current (rads/hr)	11	2
R_1	" " at 1 hr after explosion	1	1
R_a	" " " time t_a	30	7
R_b	" " " " t_b	30	7
R_c	" " " " t_c	33	9
R_t	" " " " t (indefinite time)	1	1
T	$t_{ab} + t_{bc}$ (substitution variable)	35	10
t	Time after explosion (hrs)	1	1
t_a	Time elapsed between explosion and R_a (hrs)	32	8
t_b	" " " " " R_b	-	8
t_c	" " " " " R_c	-	8
t_d	" " " " " departure from shelter	13	3
t_{ab}	" " " readings R_a and R_b	32	8
t_{bc}	" " " " R_b and R_c	33	9
w	Approximation for n	38	11

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