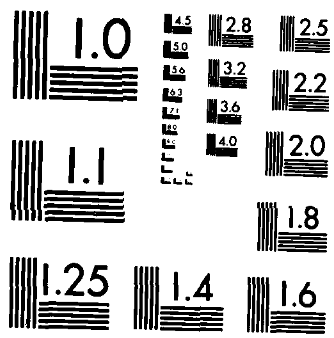
AD-A125 907 FREQUENCY DOMAIN DESIGN OF MULTIBAND FINITE IMPULSE RESPONSE DIGITAL FILT..(U) ILLINOIS UNIV AT URBANA COORDINATED SCIENCE LAB G CORTELAZZO DEC 80 R-906 1/2

UNCLASSIFIED AFOSR-79-0029 F/G 9/5 NL


1.0
4.5
5.0
5.6
6.3
2.8
2.5
3.2
2.2
3.6
4.0
2.0
1.1
1.8
1.25
1.4
1.6

AD A125907

REPORT R-906 DECEMBER, 1980 UILU-ENG 81-2237

COORDINATED SCIENCE LABORATORY

FREQUENCY DOMAIN DESIGN OF MULTIBAND FINITE IMPULSE RESPONSE DIGITAL FILTERS BASED ON THE MINIMAX CRITERION

GUIDO CORTELAZZO

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO. AD-A125	3. RECIPIENT'S CATALOG NUMBER 907
4. TITLE (and Subtitle) FREQUENCY DOMAIN DESIGN OF MULTIBAND FINITE IMPULSE RESPONSE DIGITAL FILTERS BASED ON THE MINIMAX CRITERION		5. TYPE OF REPORT & PERIOD COVERED Technical Report
		6. PERFORMING ORG. REPORT NUMBER R-906; UILU-Eng 81-2237
7. AUTHOR(s) Guido Cortelazzo		8. CONTRACT OR GRANT NUMBER(s) NSF ECS 8007075 & AFOSR-79-0029
9. PERFORMING ORGANIZATION NAME AND ADDRESS Coordinated Science Laboratory, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS National Science Foundation, Air Force Office of Scientific Research		12. REPORT DATE December 1980
		13. NUMBER OF PAGES 106
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Digital Filters - FIR multiband
Minimax design

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

It is well known that in applying minimax criteria for the frequency domain design of finite impulse response digital filters, the use of 'dont't care' regions can lead to the occurrence of undesirable resonances in the transition bands of the multiband designs. Rabiner et. al. showed how this effect can be eliminated by modifying the choice of design parameters, although their approach did not attempt to reformulate the design technique. The present work presents a modified design technique in which the desired function is specified also over the transition regions and the minimax optimization

DD FORM 1 JAN 73 1473

is carried out over the entire band $[0,\pi]$. Four example filters of Rabiner et. al. are used as examples to illustrate this modified technique. Theoretical results relating to the design technique are also presented.

FREQUENCY DOMAIN DESIGN OF MULTIBAND FINITE IMPULSE RESPONSE

DIGITAL FILTERS BASED ON THE MINIMAX CRITERION

BY

GUIDO CORTELAZZO

Laur., Universita degli Studi di Padova, 1976

THESIS

Submitted in partial fulfillment of the requirements
for the degree of Master of Science in Electrical Engineering
in the Graduate College of the
University of Illinois at Urbana-Champaign, 1980

Urbana, Illinois

ACKNOWLEDGEMENTS

I would like to acknowledge my advisors Professor Jenkins and Professor Lightner for their supporting encouragement without which this work would not have been possible.

TABLE OF CONTENTS

CHAPTER 1

INTRODUCTION

In the early 70's minimax criteria for frequency domain design of digital filters received a vast amount of attention ([1], [2]). The most popular among these formulations of the problem is the one due to Parks and McClellan [3], probably because it allows a very large class of design specifications and it is available in an efficient implementation [4].

Parks and McClellan characterized the problem of the design of linear-phase finite impulse response (FIR) digital filters as the following Chebyshev minimization problem:

$$\min_{\{h(i)\}_{i=0}^{n-1}} \max_{\omega \in \mathfrak{F}} \{\hat{W}(e^{j\omega}) | \hat{D}(e^{j\omega}) - \hat{H}(e^{j\omega})| \} \tag{1.1}$$

where

$\hat{W}(e^{j\omega})$ is a weight function (real and positive),

$\hat{D}(e^{j\omega})$ is the desired objective function,

$\hat{H}(e^{j\omega}) \triangleq \sum_{l=0}^{N-1} h(l) e^{-j\omega l}$, where N is the order of the filter,

$\hat{\mathfrak{F}} \triangleq \bigcup_{i=1}^{L} B_i$ where $B_i \triangleq \{\omega | \omega_{fi} \leq \omega \leq \omega_{si};\ \omega_{fi},\ \omega_{si} \in [0,\pi]\}$

$$\text{and } B_j \cap B_i = \begin{cases} \emptyset & j \neq i \\ B_i & j = i \end{cases} \qquad i,j = 1,2,\ldots L$$

with L denoting the number of passbands and stopbands.

Fig. 1 gives a pictorial illustration of the criterion. $\hat{W}(e^{j\omega})$ weights the error $|\hat{D}(e^{j\omega}) - \hat{H}(e^{j\omega})|$ on $\hat{\mathfrak{F}}$. Parks and McClellan mainly conceive piece-wise constant behavior for $\hat{W}(e^{j\omega})$, although they leave open the possibility of other (strictly positive) behaviors. The reason why $\hat{W}(e^{j\omega})$ cannot take on the value 0 will be explained in Chapter 2. $\hat{D}(e^{j\omega})$ is a piece-wise constant function for the case of multiple passband-stopband filters. $\hat{\mathfrak{F}}$ is taken so that it doesn't contain any discontinuity points of $\hat{D}(e^{j\omega})$. The phase linearity of the frequency response of the filter $\hat{H}(e^{j\omega})$, implies that

$$\hat{H}(e^{j\omega}) = Q(e^{j\omega})\, H(e^{j\omega})\, e^{j\beta\omega} \qquad (1.2)$$

where β is constant and both $Q(e^{j\omega})$ and $H(e^{j\omega})$ are real functions defined [4] in the following way:(*)

(*) In (1.2), for β real constant, the phase term is actually $e^{j\beta\omega}$ if we neglect to take into account phase jumps of size 2π as it is customary in the literature.

Case 1: N odd

$$Q(e^{j\omega}) = 1$$

$$H(e^{j\omega}) = \sum_{K=0}^{M} a(K) \cos(K\omega)$$

with $M = \frac{N-1}{2}$, $a(0) = h(M)$ and $a(K) = 2h(M-K)$, $K= 1,2,\ldots,M$.

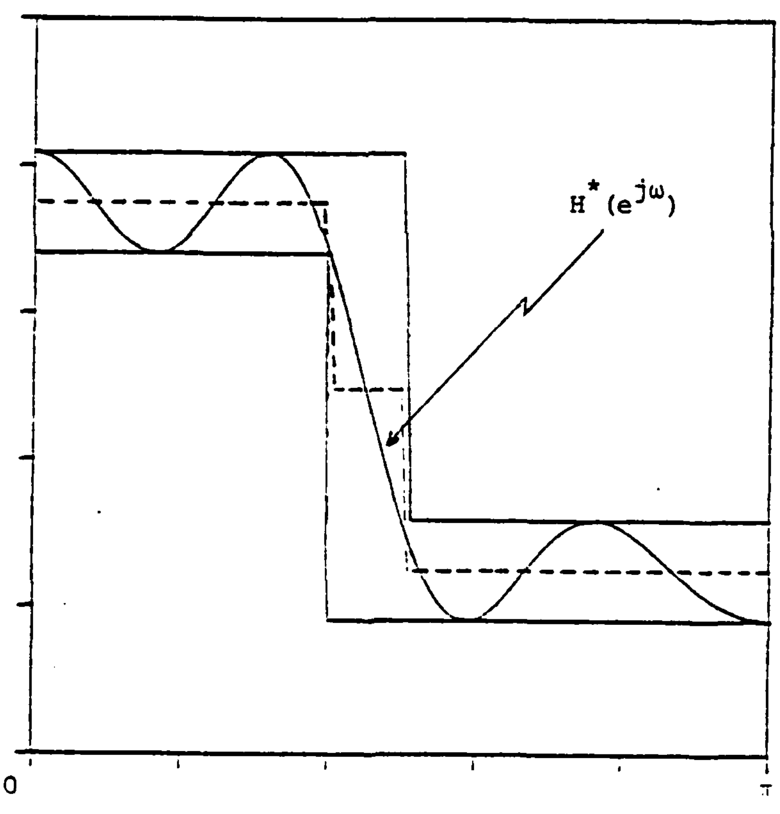


Fig. 1 Chebychev criterion in frequency domain for a low-pass filter

Case 2: N even

$$Q(e^{j\omega}) = \cos \omega$$

$$H(e^{j\omega}) = \sum_{K=0}^{M-1} a(K) \cos(K\omega)$$

where $M = \frac{N}{2}$ and a(K)'s are defined by

$$h(M-1) = \frac{1}{2} a(0) + \frac{1}{4} a(1)$$

$$h(M-K) = \frac{1}{4} a(K-1) + \frac{1}{2} a(K) \qquad K=2,3,\ldots,M-1$$

$$h(0) = \frac{1}{4} a(M-1)$$

Substituting (1.2) into (1.1) results into the expression given in (1.3)

$$\min_{\{a(K)\}_{K=0}^{M}} \{\max_{\omega\in\mathcal{J}} W(e^{j\omega})\left|D(e^{j\omega}) - H(e^{j\omega})\right|\} \qquad (1.3)$$

where

$$W(e^{j\omega}) = \hat{W}(e^{j\omega})\, Q(e^{j\omega})$$

$$D(e^{j\omega}) = \frac{\hat{D}(e^{j\omega})}{Q(e^{j\omega})}$$

$$\mathcal{J} \triangleq \{\omega | \omega\in \hat{\mathcal{J}} \text{ and } Q(e^{j\omega}) \neq 0\}$$

The sets B_i, $i = 1,2,\ldots,L$ defined above are called passbands if $D(e^{j\omega}) = 1$, $\omega\in B_i$ or stopbands if $D(e^{j\omega}) = 0$, $\omega\in B_i$. The sets $\{\omega|\omega\in[0,\pi],\ \omega_{si} < \omega < \omega_{fi}\}$, with $i = 1,2,\ldots, L-1$ constitute the so called "don't care" bands.

Equations (1.1) and (1.3) have the structure of a minimax (or Chebyshev) approximation problem for generalized (trigonometric) polynomials. The solution to this class of problems is characterized by the Alternation Theorem ([5], pp. 75` E. Ya Remez [6] provided algorithms for the computational s tion of such Chebyshev problems. J. McClellan et al. published a c uter program [4] that uses the 2nd Remez Exchange Algorithm nd the solution of a discretization of (1.3). Specifically, the problem solved in McClellan's program is:

$$\min_{\{a(K)\}_{i=0}^{M}} \max_{\omega \in \mathcal{J}_d} \{W(e^{j\omega})|D(e^{j\omega}) - H(e^{j\omega})|\} \qquad (1.4)$$

with $\mathcal{J}_d \triangleq \{\omega | \omega \in \mathcal{J}$ and $\omega = K\Delta$, $\Delta = \frac{\pi}{G \cdot M}$, K positive integer$\}$

C is a parameter (positive integers) that can be controlled by the user. It can be shown ([5], pp. 89-95) that the solution to (1.4) approaches the solution to (1.3) as Δ tends to 0. McClellan suggests that values of $L \geq 16$ produce satisfactory results. The choice of solving (1.4) instead of (1.3) is dictated for reasons of computational efficiency.

McClellan's program works very well for designing low-pass or high-pass digital filters. However, the multiband filters, i.e., the filters with a number of pass and stop-bands greater than 2, designed with McClellan's program often exhibit non monotonic (sometimes even resonant) behavior in the "don't care" bands. Figures 7, 11, 15, 19 illustrate several cases of this phenomenon.

Since McClellan's program became the tool most universally used for the design of finite impulse response filters, Rabiner, Shafer and Kaiser [7] addressed the multiband design problem in particular. The technique they propose is the following: if McClellan's program returns a multiband design with resonances, the filter specifications passed to the program are modified according to empirical strategies until a filter without resonances is obtained. Their strategies take into consideration the modification of the size of the stopbands (i.e., basically changes of $\mathcal{J}$) as well as the modification of $W(e^{j\omega})$. The number of tries, with the McClellan's program, necessary to obtain an acceptable filter varies from case to case. The implementation of multiband filters with this procedure might easily become cumbersome.

The present work reconsiders the problem of the minimax design in the frequency domain of linear phase finite impulse response (FIR) digital filters. The objective is to provide a satisfactory theoretical solution for the design of multiband filters as well as a convenient technique for the implementation of such a solution.

It was decided to use McClellan's program for the implementation of the new solution. This was natural since the new algorithm is an extension of the Parks McClellan technique and because the new algorithm would be of interest to the great number of filter designers currently using McClellan's program. Chapter 2 shows that the inadequacy of McClellan's program for the design of multiband filters lies in the formulation of the problem. Chapter 3 presents a new formulation capable of handling these difficulties. Chapter 4 introduces an implementation of the new solution and examines the results. Chapter 5 points out the relationship between the new program and the program CONRIP [8].

CHAPTER 2

LIMITS OF PARKS AND McCLELLAN'S FORMULATION

Two standard results of Approximation Theory are now introduced: the Alternation Theorem and the 2^{nd} Remez Algorithm. They respectively constitute the theoretical and the computational tools to solve Chebyshev problems.

Alternation Theorem: Let $\mathfrak{F}$ be any closed subset of $[0,\pi]$ and let $D(e^{j\omega})$ be any continuous real valued function defined on $\mathfrak{F}$. In order that $H^*(e^{j\omega})$ be the unique best approximant on $\mathfrak{F}$ to $D(e^{j\omega})$, among the class of the trigonometric polynomials of order M, it is necessary and sufficient that $E(e^{j\omega})$, defined as

$$E(e^{j\omega}) = W(e^{j\omega})\left|D(e^{j\omega}) - H^*(e^{j\omega})\right| \tag{2.1}$$

exhibits on $\mathfrak{F}$ at least $M + 2$ "alternations". Thus

$$E(e^{j\omega_i}) = -E(e^{j\omega_{i-1}}) = \pm \|E\| = \pm \max_{\omega\in\mathfrak{F}} \left|E(e^{j\omega})\right|$$

with $\omega_0 \leq \omega_1 \leq \omega_2 \leq \dots \leq \omega_{M+1}$ and $\omega_i \in \mathfrak{F}$.

Proof: The proof can be found in [5], pp. 75.

Remark: The notation of the following chapters is consistent with the one of the previous sections. $\mathfrak{F}$, H, D, W therefore are as defined in (1.3).

Notice that the Alternation Theorem is an existance theorem, it does not describe how to find the best approximant

$H^*(e^{j\omega})$. Remez algorithms are iterative procedures for generating $H^*(e^{j\omega})$. The second one applies to classes of approximating functions like the trigonometric polynomials.

2nd Remez Algorithm: Each step of the algorithm works with a set of $M + 2$ frequencies $\{\omega_K\}_{K=0}^{M+1}$. The frequencies are arbitrarily chosen at the first step and updated at successive iterations according to the particular algorithm ([5], pp. 97). The frequencies $\{\omega_K\}_{K=0}^{M+1}$ are used to determine the following system of $M + 2$ equations in the $M + 1$ $a(K)$'s and in p ($M + 2$ unknowns):

$$E(e^{j\omega_K}) = W(e^{j\omega_K}) \left| D(e^{j\omega_K}) - H(e^{j\omega_K}) \right| = -(-1)^K p \qquad (2.2)$$

for $K = 0,1,2,\ldots, M + 1$

The assumption $W(e^{j\omega}) > 0$, $\omega \in \omega_d$ allows (2.2) to be written as:

$$\begin{bmatrix} 1 & \cos\omega_0 & \cos 2\omega_0 \; \cdots\cdots & \cos M\omega_0 & \dfrac{1}{W(e^{j\omega_0})} \\ 1 & \cos\omega_1 & \cos 2\omega_1 \; \cdots\cdots & \cos M\omega_1 & \dfrac{-1}{W(e^{j\omega_1})} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \cos\omega_{M+1} & \cos 2\omega_{M+1} \cdots & \cos M\omega_{M+1} & \dfrac{(-1)^{M+1}}{W(e^{j\omega_{M+1}})} \end{bmatrix} \cdot \begin{bmatrix} a(0) \\ a(1) \\ \vdots \\ a(M) \\ p \end{bmatrix} = \begin{bmatrix} D(e^{j\omega_0}) \\ D(e^{j\omega_1}) \\ \vdots \\ D(e^{j\omega_M}) \\ D(e^{j\omega_{M+1}}) \end{bmatrix} \qquad (2.3)$$

For reasons of efficiency Parks and McClellan avoid the matrix inversion in the solution of (2.3). This is done by first calculating:

$$p = \frac{\sum_{i=0}^{M+1} b_i \ D(e^{j\omega_i})}{\sum_{i=0}^{M+1} \frac{(-1)^i \ b_i}{W(e^{j\omega_i})}}, \tag{2.4}$$

where $b_K = (-1)^K \prod_{\substack{i=0 \\ i \neq K}}^{M+1} \frac{1}{x_i - x_K}$

$x_i = \cos\omega_i$

Then the Lagrange interpolation formula in the baricentric form is used to interpolate $H(e^{j\omega})$ on the $M + 1$ points $\{\omega_K\}_{K=0}^{M}$ to obtain the values

$$c_K = D(e^{j\omega_K}) - (-1)^K \frac{p}{W(e^{j\omega_K})} \tag{2.5}$$

with $K = 0,1,\ldots,M$.

This type of interpolation gives an equiripple fit to the $M + 2$ data points. This form of interpolation is required by the 2nd Remez algorithm.

After the system is solved, the algorithm calls for the check:

$$E(e^{j\omega_d}) \leq E(e^{j\omega_K}) = p \quad , \omega_d \epsilon \mathfrak{F}_d \qquad (2.6)$$

If there is some frequency of $\mathfrak{F}_d$ not satisfying inequality (2.6), a further iteration that begins with the update of the frequencies $\{\omega_K\}_{K=0}^{M+1}$ is necessary. The algorithm continues until inequality (2.6) is satisfied over the whole $\mathfrak{F}_d$. The $H(e^{j\omega})$ obtained at this step is the best approximant $H^*(e^{j\omega})$ characterized by the Alternation Theorem.

The Parks and McClellan's FIR design technique corresponds to the application of the Alternation Theorem and of the 2nd Remez algorithm to the solution of (1.3).

It must be noticed that the "don't care" bands constitute a mathematically ambiguous feature of their formulation that can be very dangerous. In fact, one actually does "care" about the behavior of $H(e^{j\omega})$ over the whole band $[0,\pi]$ and, in particular, one wants $H(e^{j\omega})$ to be monotonic over the "don't care" bands.

Further, it is not clear that ignoring the behavior of the filter over the "don't care" bands will necessarily result in the desired monotonic response.

Before showing in detail the dangers connected with the "don't care" bands, let's review a few mathematical concepts necessary to understand them. Recall that the extrema of a function over a compact and bounded set can only occur at the boundary points of the set or at the interior points where the first derivative of the function is 0. $H(e^{j\omega})$ is a trigonometric polynomial of order M defined on $[0,\pi]$. Its derivative $\frac{dH(e^{j\omega})}{d\omega}$ can have at most M - 1 distinct 0's in $(0,\pi)$ since it is a polynomial of order M - 1. The boundary points of $M(e^{j\omega})$ are the set $\{0,\pi\}$. Therefore $H(e^{j\omega})$ can have at most M + 1 distinct extrema on $[0,\pi]$. The function:

$$E(e^{j\omega}) = H(e^{j\omega}) - D(e^{j\omega}) \qquad \omega\in\mathfrak{F} \qquad (2.7)$$

where $D(e^{j\omega})$ and $\mathfrak{F}$ are defined in formulation (1.3), is a trigonometric polynomial of order at most M on $\mathfrak{F}$. Notice that $E(e^{j\omega})$ over $[0, \pi]$ is not a polynomial of order M, because of the discontinuities of $D(e^{j\omega})$.

The cases of the low and high-pass filters will be considered first because of their special characteristics. Then the multiband filters will be discussed.

For low or high-pass filters $\mathfrak{F} \triangleq [0,\omega_p] \cup [\omega_s,\pi]$, the interior of $\mathfrak{F}$ is $\mathring{\mathfrak{F}} \triangleq (0,\omega_p)\cup(\omega_s,\pi)$ and the boundary of $\mathfrak{F}$ is $\mathfrak{F}\setminus\mathring{\mathfrak{F}} \triangleq \{0,\omega_p,\omega_s,\pi\}$. $E(e^{j\omega})$ can have at most M - 1 distinct extrema on $\mathring{\mathfrak{F}}$ (since $E(e^{j\omega})$ is a polynomial of degree at most M in $\mathfrak{F}$) and at most four extrema on the boundary $\{0,\omega_p,\omega_s,\pi\}$. Therefore $E(e^{j\omega})$ can have at most M + 3 extrema on $\mathfrak{F}$. Furthermore (2.7) implies that all the extremal points of $E(e^{j\omega})$, except possibly the extrema at ω_p and ω_s are extremal points of $H(e^{j\omega})$, namely

$$E^*(e^{j\omega}) = H^*(e^{j\omega}) - D(e^{j\omega}) \qquad \omega\in\mathfrak{F} \qquad (2.8)$$

will have at least M + 2 distinct extrema alternating in $\mathfrak{F}$ by the Alternation Theorem. Specifically, $E^*(e^{j\omega})$ can have at least M + 2 distinct extrema on $\mathfrak{F}$ either having M - 1 distinct extrema in $\mathring{\mathfrak{F}}$ and at least 3 extrema in $\{0,\omega_p,\omega_s,\pi\}$ or having M - 2 distinct extrema in $\mathring{\mathfrak{F}}$ and four extrema in $\{0,\omega_p,\omega_s,\pi\}$. More specifically $E^*(e^{j\omega})$ can have:

i) M-1 extrema in $\mathring{\mathfrak{F}}$ and extrema at $\{0,\omega_p,\omega_s,\pi\}$
ii) M-1 extrema in $\mathring{\mathfrak{F}}$ and extrema at $\{0,\omega_p,\pi\}$
iii) M-1 extrema in $\mathring{\mathfrak{F}}$ and extrema at $\{0,\omega_s,\pi\}$
iv) M-1 extrema in $\mathring{\mathfrak{F}}$ and extrema at $\{0,\omega_p,\omega_s\}$
v) M-1 extrema in $\mathring{\mathfrak{F}}$ and extrema at $\{\omega_p,\omega_s,\pi\}$
vi) M-2 extrema in $\mathring{\mathfrak{F}}$ and extrema at $\{0,\omega_p,\omega_s,\pi\}$.

Some of the cases of $E^*(e^{j\omega})$ listed above leave open the possibility of a corresponding $H^*(e^{j\omega})$ that would be unacceptable as a low or high-pass filter. Case ii), for instance could correspond to and $H^*(e^{j\omega})$ having an extremum at ω_p (see Fig. 2c). Case iv) could bring an $H^*(e^{j\omega})$ with a saddle point in (ω_p,ω_s) (see Fig. 2f) and case vi) an $H^*(e^{j\omega})$ with a local maximum in (ω_p,ω_s) (see Fig. 2e). The reasons such possibilities don't occur are explained in the following theorem.

<u>Theorem</u>: The low and high-pass digital filters designed via the Remez algorithm, according to formulation (1.3) where $B \triangleq \{\omega | \omega_p < \omega < \omega_s, \omega_p, \omega_s \in [0,\pi]\}$ is the don't care band, have the following properties:

a) ω_p and ω_s are always extremal points of $E^*(e^{j\omega})$, (i.e., the cases ii) and iii) above cannot occur)

b) ω_p and ω_s are not extremal points of $H^*(e^{j\omega})$

c) $H^*(e^{j\omega})$ is strictly monotonic on B, namely:

$$\frac{dH^*(e^{j\omega})}{d\omega} < 0 \quad , \quad \omega \in B \text{ for the low-pass filters}$$

and $$\frac{dH^*(e^{j\omega})}{d\omega} > 0 \quad , \quad \omega \in B \text{ for the high-pass filters}$$

Proof: the Proof takes into consideration each of the cases of $E^*(e^{j\omega})$ listed above and shows the necessity of properties a), b), c) for the cases whose occurrance is not a contradiction (i.e., all but case ii) and iii)).

<u>Case i) satisfies properties a), b), c)</u>

Property a) is satisfied by definition.

$E^*(e^{j\omega})$ has now $M + 3$ extrema on $\mathcal{F}$. Therefore $H^*(e^{j\omega})$ has $M + 1$ distinct extrema in $\mathcal{F}$: namely $M - 1$ in $\mathring{\mathcal{F}}$ and 2 at 0 and π, respectively. The points ω_p and ω_s therefore cannot be extrema of $H^*(e^{j\omega})$ (property b)) and further there cannot be any extremal point in B (property c)).

Case ii) and iii) contradict the Alternation Theorem

Consider first case ii), i.e. $E^*(e^{j\omega})$ has M-1 extrema on $\mathring{\mathcal{F}}$ plus 3 extreme at $\{0, \omega_p, \pi\}$. Notice that 0 and π are also extrema of $H^*(e^{j\omega})$, therefore $\frac{dH^*(e^{j\bar{\omega}})}{d\omega} = 0$ cannot occur at any $\bar{\omega} \in B$ (property c)). Also, the point ω_p cannot be an extremal point of $H^*(e^{j\omega})$ otherwise $H^*(e^{j\omega})$ also has $M + 2$ extrema on $\mathcal{F}$. Since , by assumption ω_s is not an extremal point of $E^*(e^{j\omega})$, $E^*(e^{j\omega})$ does not alternate (see fig. 2a,b) and therefore the contradiction is achieved.

A completely analogous argument proves that case iii) cannot occur.

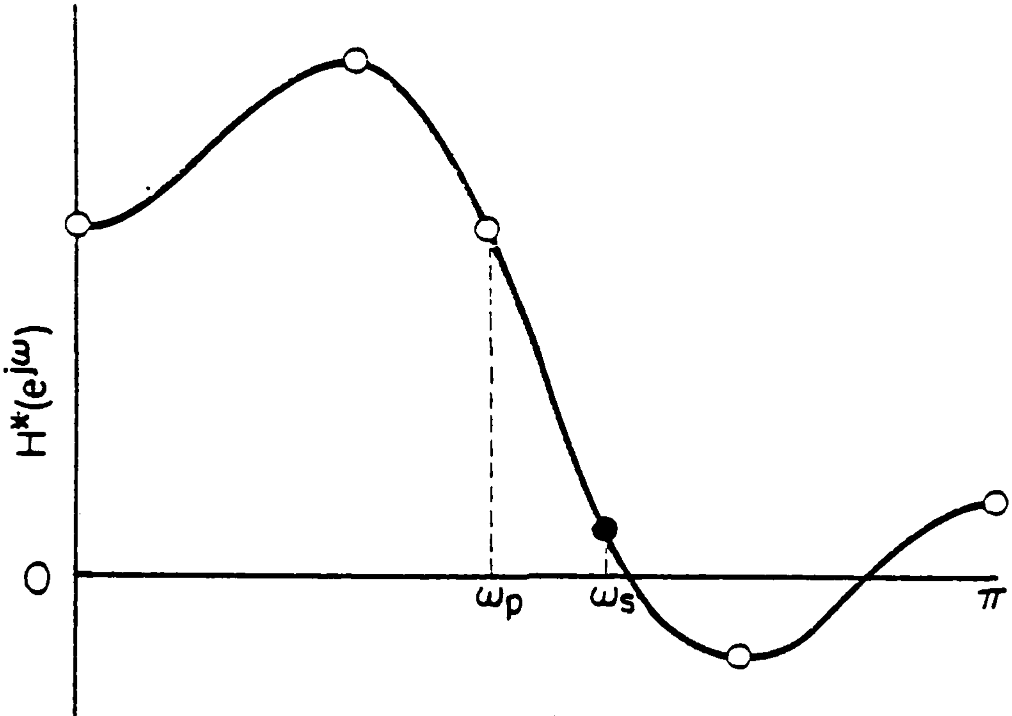
Case iv) and v) satisfy properties a), b), c)

Consider case iv) first, i.e. $E^*(e^{j\omega})$ has M-1 extrema in $\mathring{\mathcal{F}}$ plus 3 extrema at $\{0, \omega_s, \omega_p\}$. Notice that 0 is extremal point also of $H^*(e^{j\omega})$, therefore $H^*(e^{j\omega})$ has at least M extrema in $\mathcal{F}$. It is worthy to distinguish 3 subcases:

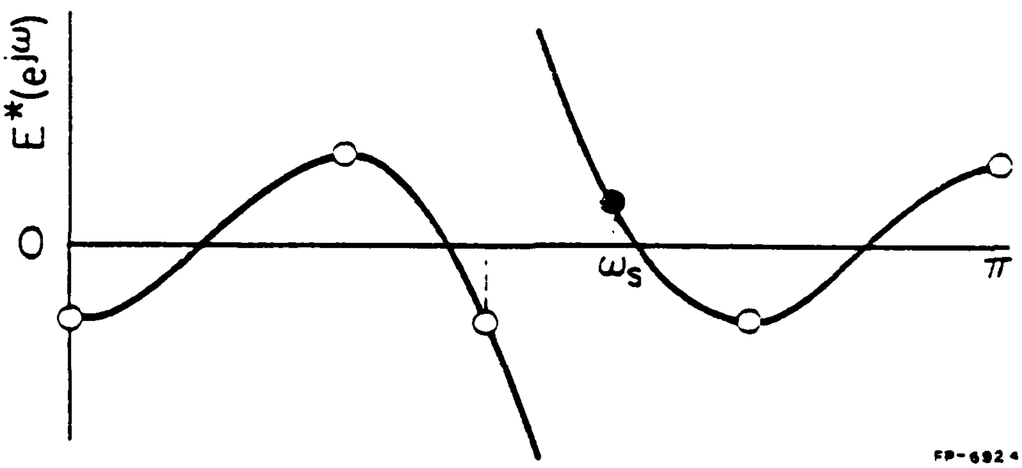
Subcase I: ω_p and ω_s are both also extrema of $H^*(e^{j\omega})$. This is a contradiction because $H^*(e^{j\omega})$ would have M+2 extrema.

Subcase II: ω_p is an extremum of $H^*(e^{j\omega})$ and ω_s is not (the case ω_s is an extremum of $H^*(e^{j\omega})$ and ω_p is not, is completely analogous and leads to the same conclusions).

By hypothesis $H^*(e^{j\omega})$ has M-1 extrema on $\mathring{\mathcal{F}}$, since $H^*(e^{j\omega})$ has at most M-1 extrema on $(0, \pi)$ it is $dH^*(e^{j\omega})/d\omega \neq 0 \ \forall \omega \in B$ (recall that the roots of derivatives correspond to the interior extremal points).



a) $H^*(e^{j\omega})$



b) $E^*(e^{j\omega})$

Fig. 2a,b. Case ii): $E^*(e^{j\omega})$ with $M-1$ extrema in $\mathring{F}$ and three extrema at $\{o, \omega_p, \pi\}$. The frequency ω_s is not extremal point of $E^*(e^{j\omega})$.

In this subcase the contradiction is achieved because $E^*(e^{j\omega})$ violates the Alternation Theorem (see Fig. 2-c,d). Therefore subcase II cannot occur.

Subcase III: Neither ω_p nor ω_s are extrema of $H^*(e^{j\omega})$. In this case property a) and b) are satisfied by assumption. $H^*(e^{j\omega})$ by assumption has M-1 extrema in $\mathring{\mathcal{F}}$, therefore $\frac{dH^*(e^{j\omega})}{d\omega} = 0$ can not occur in B otherwise $H^*(e^{j\omega})$ would have M extrema in $(0, \pi)$. This proves property c).

The same argument shows also that the properties a), b), c) are satisfied for case v) (it is enough to interchange the roles of 0 and π as extremal and non-extremal points of $E^*(e^{j\omega})$ respectively).

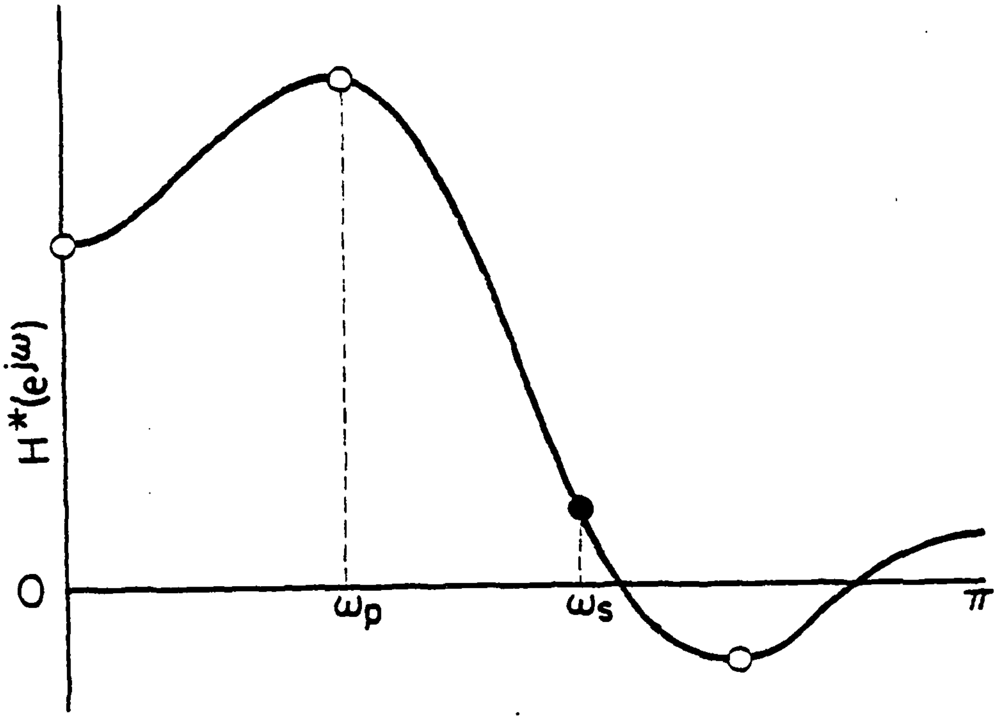
Case vi) satisfies properties a), b), c)

$E^*(e^{j\omega})$ is assumed to have four extrema at $\{0, \omega_p, \omega_s, \pi\}$ and M-2 extrema in $\mathring{\mathcal{F}}$. This implies that $H^*(e^{j\omega})$ has 2 extrema at 0 and π plus M-2 extrema in $\mathring{\mathcal{F}}$, i.e. M extrema in $\mathcal{F}$.

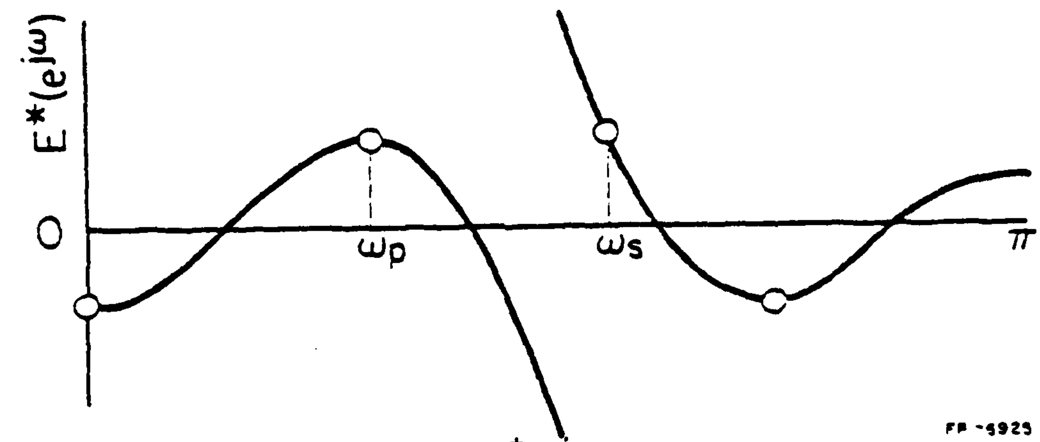
It is again convenient to distinguish three subcases.

Subcase I: ω_s and ω_p are extrema of $H^*(e^{j\omega})$. This cannot occur because $H^*(e^{j\omega})$ would have M+2 extrema in $\mathcal{F}$.

Subcase II: ω_p is an extremum of $H^*(e^{j\omega})$ and ω_s is not (equivalent to the case ω_s is an extremum of $H^*(e^{j\omega})$ and ω_p is not). $H^*(e^{j\omega})$ has by assumption M - 2 extrema in $\mathring{\mathcal{F}}$, two extrema at $\{0, \pi\}$ and one extremum at ω_p. $H^*(e^{j\omega})$ therefore has by assumption M + 1 extrema in $\mathcal{F}$. This excludes the possibility of



c) $H^*(e^{j\omega})$



d) $E^*(e^{j\omega})$

Fig. 2c,d. Case iv): $E^*(e^{j\omega})$ with $M-1$ extrema in $\overset{o}{\Omega}$ and three extrema at $\{o, \omega_p, \omega_s\}$. The frequency ω_p is assumed to be extremal point of $H^*(e^{j\omega})$.

$\frac{dH^*(e^{j\omega})}{d\omega} = 0$ at some $\omega \in B$ (property c), otherwise $H^*(e^{j\omega})$ would have $M + 2$ extrema in $\mathfrak{F}$. In this subcase contradiction is achieved because $E^*(e^{j\omega})$ violates the Alternation Theorem (the situation is analogous to the one of Fig. 2-c,d).

<u>Subcase III</u>: ω_p and ω_s are not extrema of $H^*(e^{j\omega})$. This case specializes into four situations:

(a) Maximum (or minimum) in B. I. e.:

$$\frac{dH(e^{j\bar{\omega}})}{d\omega} = 0 \,, \quad \frac{d^2H(e^{j\bar{\omega}})}{d^2\omega} \neq 0 \,, \quad \bar{\omega} \in B \tag{2.8}$$

Since $H^*(e^{j\omega_p}) \neq H^*(e^{j\omega_s})$ as long as $\omega_p \neq \omega_s$ and neither ω_p nor ω_s are extrema of $H^*(e^{j\omega})$, the presence of a maximum (or a minimum) in (ω_p, ω_s) implies also the presence of a minimum (or a maximum) in (ω_p, ω_s) (see Fig. 2e). The contradiction is achieved since this requires $H^*(e^{j\omega})$ to have $M + 2$ extrema on $[0,\pi]$.

(b) Saddle point in B (see Fig. 2f)

$$\frac{dH^*(e^{j\omega})}{d\omega} = 0 \qquad \frac{d^2H^*(e^{j\bar{\omega}})}{d^2\omega} = 0 \qquad \bar{\omega} \in B \tag{2.9}$$

Recall that a saddle point of a polynomial $H(e^{j\omega})$ corresponds to a zero of the derivative of multiplicity at least 2. By assumption $H^*(e^{j\omega})$ has $M - 2$ extrema in $\mathring{\mathfrak{F}}$, this means that $dH^*(e^{j\omega})/d\omega$ has $M - 2$ zeros in $\mathring{\mathfrak{F}}$. Condition (2.9) implies that $dH^*(e^{j\omega})/d\omega$ has a zero at least of multiplicity 2 in B. This is a contradiction because it implies that $dH^*(e^{j\omega})/d\omega$ has M zeros in $(0,\pi)$ and $dH^*(e^{j\omega})/d\omega$ is a polynomial of order $M - 1$.

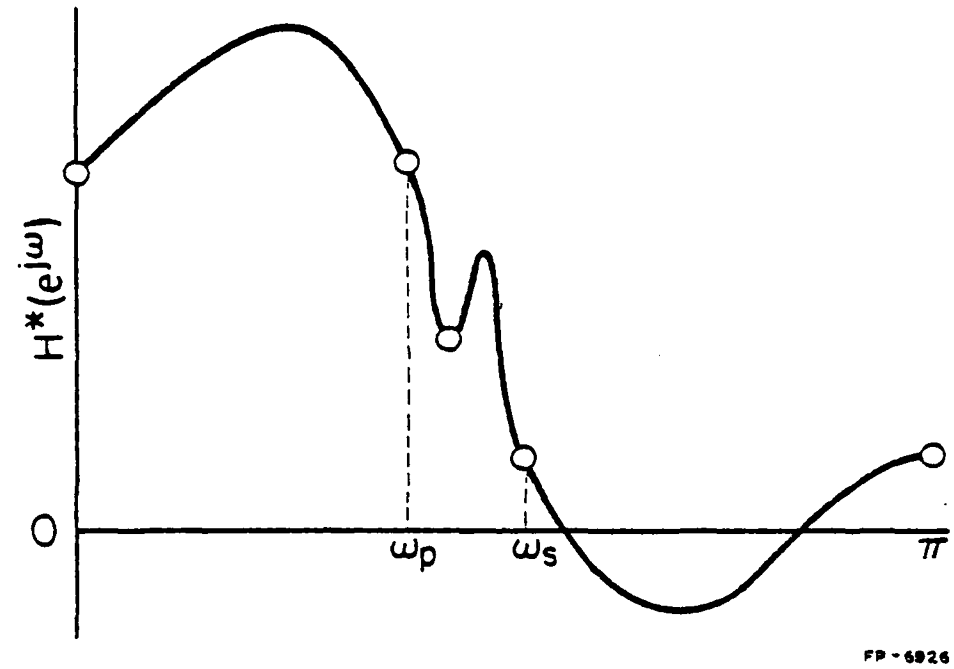


Fig. 2e. Case vi): $E^*(e^{j\omega})$ with M -2 extrema in and four extrema at $\{o,\omega_p,\omega_s,\}$ $H^*(e^{j\omega})$ is assumed to have a maximum in (ω_p,ω_s).

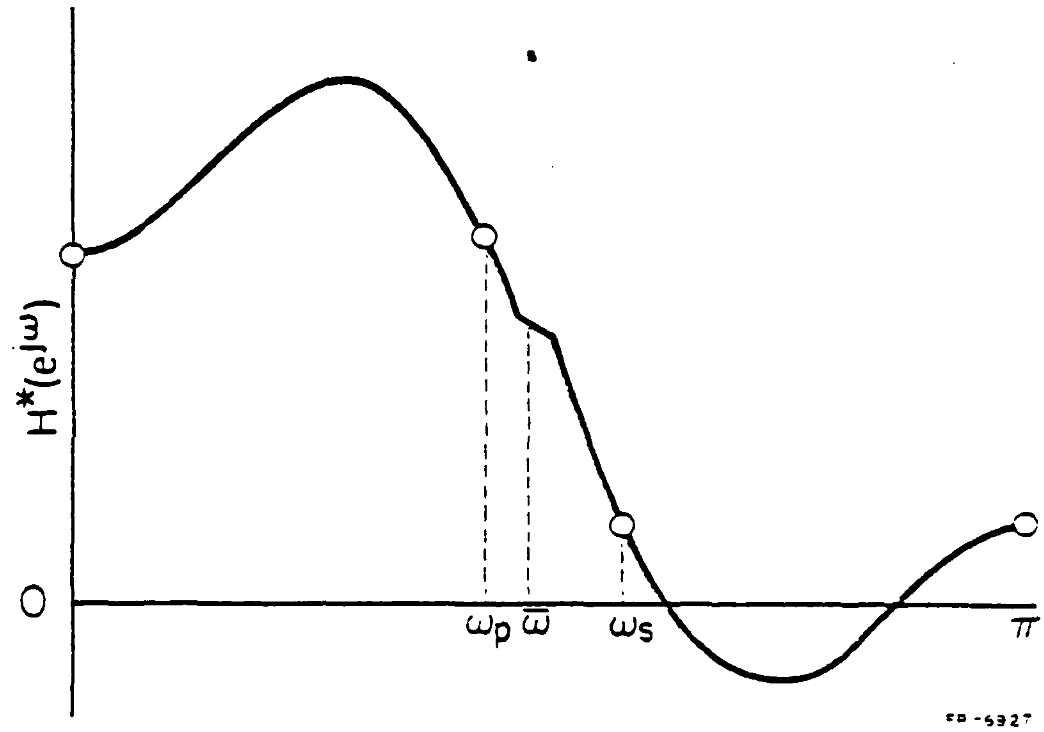


Fig. 2f. Case vi): $E^*(e^{j\omega})$ with M -2 extrema in and four extrema at $\{o,\omega_p,\omega_s,\pi\}$ $H^*(e^{j\omega})$ is assumed to have a saddle-point in (ω_p,ω_s).

(c) Further extremum of $H^*(e^{j\omega})$ in $\mathring{\mathcal{F}}$, i.e. if $\{\omega\}_{i=1}^{M-2} \subset \mathring{\mathcal{F}}$ denote the set of the extremal frequencies of $\mathring{\mathcal{F}}$ assumed by hypothesis, this circumstance is expressed as

$$\frac{dH^*(e^{j\bar{\omega}})}{d\omega} = 0 \text{ at } \bar{\omega} \in \{\omega \in \mathring{\mathcal{F}}, \omega \notin \{\omega_i\}_{i=1}^{M-2}\}.$$

This situation satisfies properties a), b), c) by assumption.

It should be noticed that, since $\bar{\omega}$ is not accounted among the extremal frequencies $\{\omega_i\}_{i=1}^{M-2}$, the Alternation Theorem imposes

$$|E^*(e^{j\bar{\omega}})| < p = \max_{\omega \in \mathcal{F}} |E^*(e^{j\omega})| = |E(e^{j\omega_i})|$$

(d) No further extremum on $(0, \pi)$. I.e., if $\{\omega_i\}_{i=1}^{M-2}$ denotes the set of the extremal frequencies of $\mathring{\mathcal{F}}$, this circumstance is expressed as

$$\frac{dH^*(e^{j\omega})}{d\omega} \neq 0 \quad \forall\, \omega \in \{\omega \in (0,\pi),\ \omega \notin \{\omega_i\}_{i=1}^{M-2},\ \omega \notin \{\omega_s, \omega_p\}\}$$

This situation satisfies properties a), b), c) by assumption.

Remark: For simplicity $E(e^{j\omega})$ has been used in the theorem according to definition (2.8) instead of definition (2.1). The theorem can be proved also for a weighted error as in (2.1) with $W(e^{j\omega}) > 0$ and real. Also in this case $E(e^{j\omega})$ turns out to have all its extremal points, except possibly ω_s and ω_p, in common with $H(e^{j\omega})$ and $E^*(e^{j\omega})$ has extrema as in the six cases previously considered.

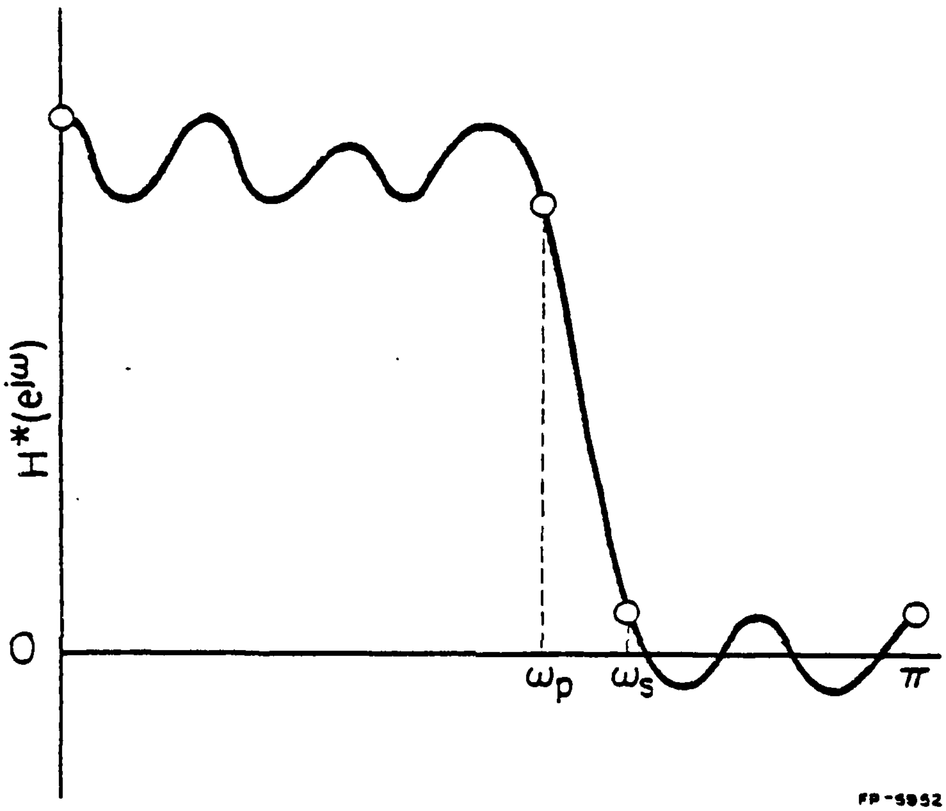


Fig. 2g. Case vi): $E^*(e^{j\omega})$ with $M-2$ extrema in $\overset{\circ}{\mathcal{J}}$ and four extrema at $\{o,\omega_p,\omega_s,\pi\}$ $H^*(e^{j\omega})$ is assumed to have a further extremum in $\tilde{\mathcal{J}}$.

The preceding theorem shows that the mathematical structure of the approximation problem (1.3) corresponding to the design of high and low-pass filters guarantees strictly monotonic behavior in the "don't care" band. It is legitimate to ask if formulation (1.3) also guarantees strict monotonicity in the "don't care" bands for multiband filters. The answer is "no". The reasons behind it can be illustrated by an example. Consider a 3-band filter like the one shown in Figure 3-a,b. In this case $\mathfrak{F} \stackrel{\Delta}{=} [0, \omega_{s1}] \cup [\omega_{f2}, \omega_{s2}] \cup [\omega_{f3}, \pi]$, the interior of $\mathfrak{F}$ is $\mathring{\mathfrak{F}} = (0, \omega_{s1}) \cup (\omega_{f_2}, \omega_{s2}) \cup (\omega_{f3}, \pi)$ and the boundary of $\mathfrak{F}$. $\mathfrak{F} \backslash \mathring{\mathfrak{F}} = \{0, \omega_{s1}, \omega_{f2}, \omega_{s2}, \omega_{f3}, \pi\}$.

$E^*(e^{j\omega})$ given by the 2nd Remez algorithm has at least $M + 2$ extrema in $\mathfrak{F}$. Therefore E^* can have the structure of any of the cases below:

(1) $M - 4$ extrema in $\mathring{\mathfrak{F}}$ and 6 extrema in $\mathfrak{F} \backslash \mathring{\mathfrak{F}}$: 1 subcase

(2) $M - 3$ extrema in $\mathring{\mathfrak{F}}$ and 5 extrema in $\mathfrak{F} \backslash \mathring{\mathfrak{F}}$: 6 subcases

(3) $M - 3$ extrema in $\mathring{\mathfrak{F}}$ and 6 extrema in $\mathfrak{F} \backslash \mathring{\mathfrak{F}}$: 1 subcase

(4) $M - 2$ extrema in $\mathring{\mathfrak{F}}$ and 4 extrema in $\mathfrak{F} \backslash \mathring{\mathfrak{F}}$: 15 subcases

(5) $M - 2$ extrema in $\mathring{\mathfrak{F}}$ and 5 extrema in $\mathfrak{F} \backslash \mathring{\mathfrak{F}}$: 6 subcases

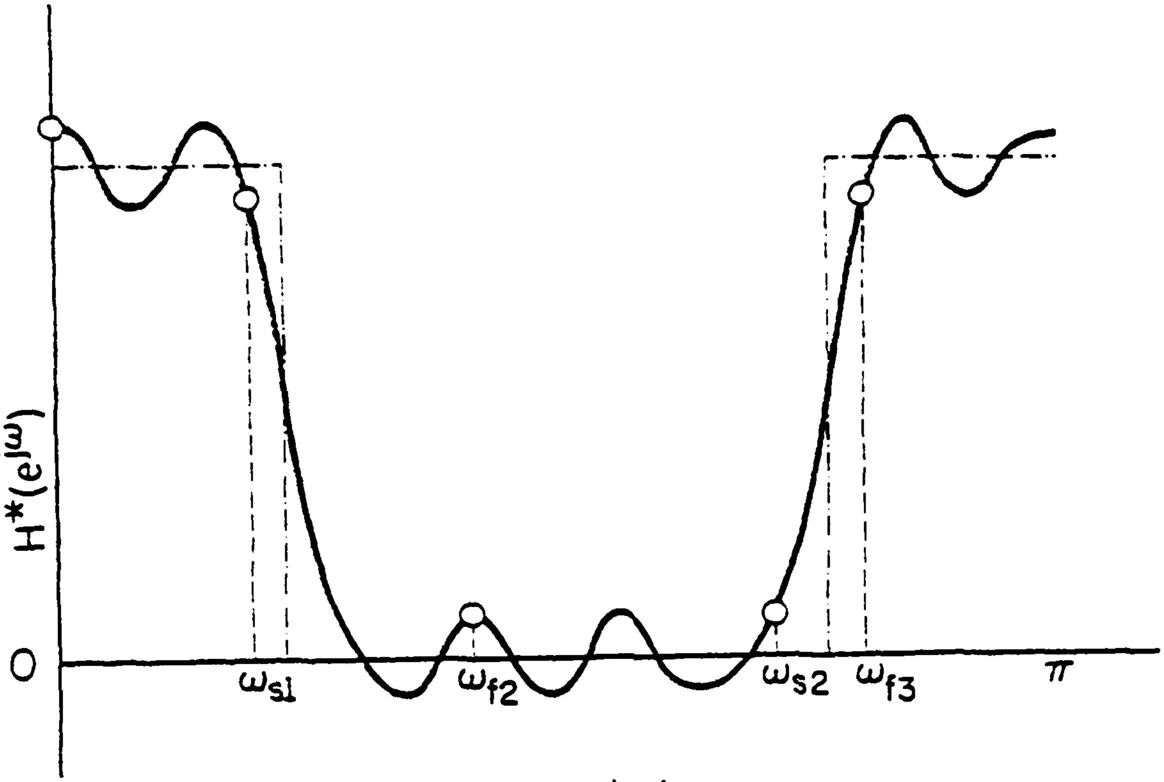
(6) $M - 2$ extrema in $\mathring{\mathfrak{F}}$ and 6 extrema in $\mathfrak{F} \backslash \mathring{\mathfrak{F}}$: 1 subcase

(7) $M - 1$ extrema in $\mathring{\mathfrak{F}}$ and 3 extrema in $\mathfrak{F} \backslash \mathring{\mathfrak{F}}$: 20 subcases

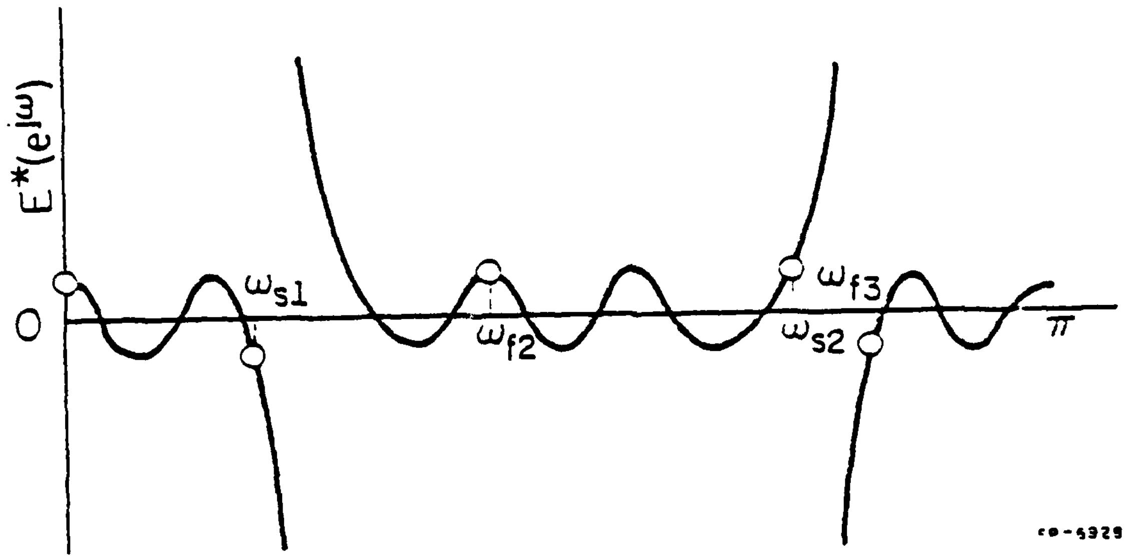
(8) $M - 1$ extrema in $\mathring{\mathfrak{F}}$ and 4 extrema in $\mathfrak{F} \backslash \mathring{\mathfrak{F}}$: 15 subcases

(9) $M - 1$ extrema in $\mathring{\mathfrak{F}}$ and 5 extrema in $\mathfrak{F} \backslash \mathring{\mathfrak{F}}$: 6 subcases

(10) $M - 1$ extrema in $\mathring{\mathfrak{F}}$ and 6 extrema in $\mathfrak{F} \backslash \mathring{\mathfrak{F}}$: 1 subcase.



a) $H^*(e^{j\omega})$



b) $E^*(e^{j\omega})$

Fig. 3a,b. Example of 3-band filter with an extremum of $H^*(e^{j\omega})$ in $(\omega_{s1},\omega_{s2})$.

Each one of these cases can specialize in many subcases depending from the points of $\mathfrak{F} \setminus \mathring{\mathfrak{F}}$ that are assumed to be extrema (similarly to what was done for the low or high-pass filters). The number of the subcases is $6!/(K!(6-K)!)$, where K is the number of extrema in $\mathfrak{F} \setminus \mathring{\mathfrak{F}}$. It is not difficult in this forest of cases to find a situation unable to guarantee monotonic behavior in the "don't care" bands without violating the Alternation Theorem or the relationships between the order of a polynomial and the number of its extrema. Select, for example, the subcase of case 9) corresponding to M-1 extrema in $\mathring{\mathfrak{F}}$ and extrema at $\{0, \omega_{s1}, \omega_{f2}, \omega_{f1}\}$. If $\omega_{s1}, \omega_{f2}, \omega_{s2}, \omega_{f3}$ are not extrema of $H^*(e^{j\omega})$, $H^*(e^{j\omega})$ has M extrema in $\mathfrak{F}$: one at 0 and M-1 in $\mathring{\mathfrak{F}}$. $H^*(e^{j\omega})$ can have a further extremum in $(\omega_{s1},\omega_{f2})$, as shown in Fig.3 a,b without causing any contradiction.

In general for an L-band filter

$$\mathfrak{F} = \bigcup_{j=1}^{L} \left[\omega_{fj}, \omega_{sj}\right]$$

and $E(e^{j\omega})$ can have up to M-1 + 2L distinct extrema in $\mathfrak{F}$. The 2nd Remez algorithm guarantees only that $E^*(e^{j\omega})$ has at least M + 2 alternating extrema in $\mathfrak{F}$. The possibility that the Remez algorithm takes into account extrema of $E^*(e^{j\omega})$ not corresponding to extremal point of $H^*(e^{j\omega})$ is very high. Therefore for an L-band filter ($L > 2$) some of the M + 1 extrema of $H(e^{j\omega})$ can occur anywhere on $[0,\pi]$. If they occur in $\mathfrak{F}$ the second Remez algorithm constrains them to give deviations from $D(e^{j\omega})$ within the final minimax error $\rho = \max_{\omega\in\mathfrak{F}} E^*(e^{j\omega})$.

If they occur in the "don't care" bands they are unconstrained and can not only spoil the monotonicity but also become resonances for the filter. Unbounded extrema of $H^*(e^{j\omega})$ in the "don't care" bands are reported [7] to occur experimentally in 9 out of 10 multiband filters designed with formulation given in (1.3).

In the next section a new formulation of the filter design problem is presented. This new formulation is immune from the drawbacks of the McClellan formulation and capable of the straightforward design of the multiband filters.

CHAPTER 3

NEW FORMULATION OF THE LINEAR-PHASE FIR DIGITAL FILTERS DESIGN PROBLEM

The elimination of the "don't care" bands in the formulation of the linear phase FIR digital filter design problem calls for the following type of formulation:

$$\min_{\{a(k)\}_{k=0}^{M}} \; \max_{\omega \in [0,\pi]} \; W(e^{j\omega}) \, |D(e^{j\omega}) - H(e^{j\omega})| \qquad (3.1)$$

where the symbols are as defined in (1.3).

The presence of $D(e^{j\omega})$, as defined for (1.3), i.e., as a piecewise constant discontinuous function, brings two major difficulties to the formulation (3.1). The first difficulty is that the discontinuities of $D(e^{j\omega})$ will seriously limit the convergence of the approximation (3.1). For instance it is easy to see that if $W(e^{j\omega})=1$ the minimax error will not become smaller than

$$\max_{\omega \in \left\{\substack{\text{discontinuity} \\ \text{points of } D(e^{j\omega})}\right\}} \frac{1}{2} |D(e^{j\omega})| . \qquad (3.2)$$

The second difficulty is that the Alternation Theorem cannot be used to characterize the solution $H^*(e^{j\omega})$ of (3.1), since it requires the continuity of the function to be approximated. It will be noticed that since the approximating functions in (3.1) are trigonometric polynomials it does not even make sense to require a discontinuous behavior from them. These reasons motivate the use of a continuous $D(e^{j\omega})$ in (3.1). Figure 4a shows an example of a piecewise

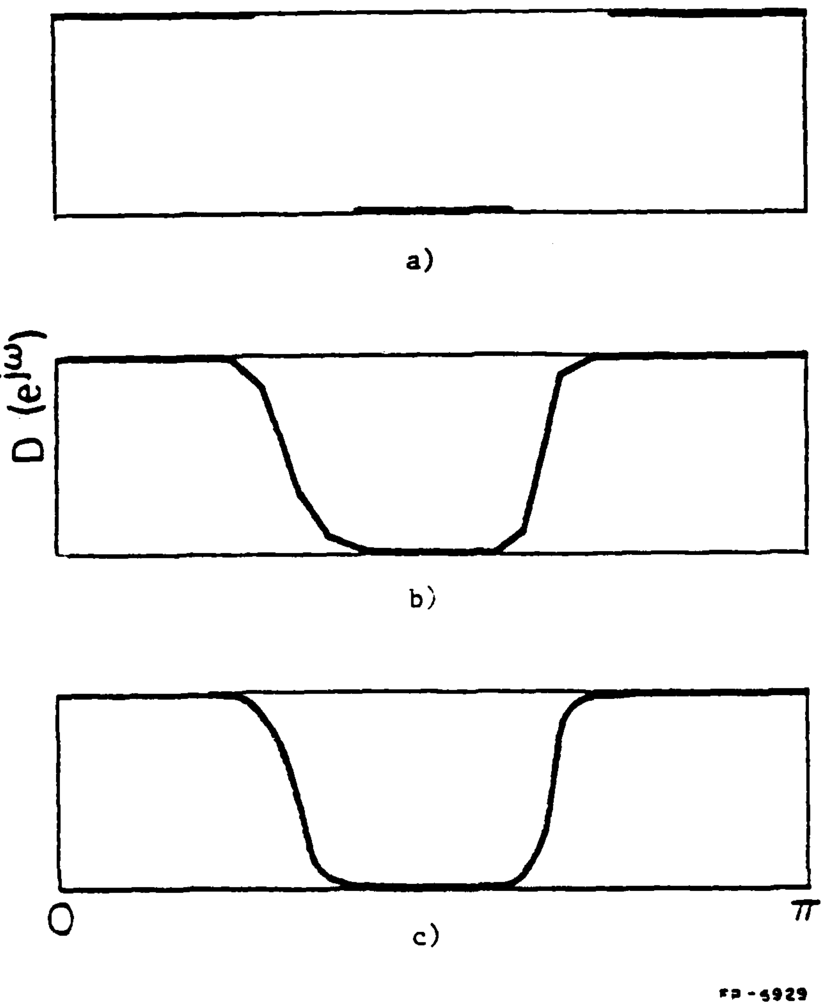


Fig. 4a - Example of $D(e^{j\omega})$ for the McClellan's formulation

b - Example of continuous $D(e^{j\omega})$ for the new formulation (technique L)

c - Example of continuous $D(e^{j\omega})$ for the new formulation (technique P) discontinuous $D(e^{j\omega})$ and Fig. 4b and c two continuous versions of $D(e^{j\omega})$ for use in the new formulation (3.1). A method for choosing such a continuous $D(e^{j\omega})$ is another major point of the new formulation of the filter design problem, whose detailed discussion will be given later in the section.

The relationship between the Parks and McClellan's formulation (1.3) and the new formulation will now be discussed. Such a comparison turns out to be rather informative and gives the motivation for the choice of $W(e^{j\omega})$ and $D(e^{j\omega})$ in the new formulation (3.1)

The Parks and McClellan formulation (1.3) can be obtained as a particular case of the new formulation (3.1). This equivalence occurs when

$$W(e^{j\omega}) = 0 \qquad \omega \in D \overset{\Delta}{=} \{\omega | \omega \varepsilon [0, \pi], \omega \notin \mathfrak{F} \} \tag{3.3}$$

is used in (3.1).

It will now be shown why the formulation of (1.3) is mathematically preferable to the formulation (3.1) with condition (3.3), although the two are absolutely equivalent in meaning.

Formulation (3.1) for $W(e^{j\omega}) > 0$ constitutes a minimization problem with respect to a weighted minimax norm defined for real functions supported by $[0, \pi]$.(*) Condition (3.3) turns (3.1) into

---

(*) A norm is a functional f over a vector space X, with the following properties:

(i) $f(x) \geq 0$ for all $x \in X$

(ii) $f(\alpha x) = |\alpha| x$ for all scalars α and $x \in X$

(iii) $f(x+y) \leq f(x) + f(y)$ for each $x, y \in X$

(iv) $f(x) = 0$ if and only if $x = 0$.

a minimization problem with respect to a weighted seminorm defined for the real functions supported by $[0, \pi]$. Such a seminorm defines a norm for the $[0, \pi]$ real functions supported by $\mathfrak{F}$ (**). To write (3.1) with condition (3.3) as formulation (1.3) corresponds to considering the seminorm for function of $[0, \pi]$ as a norm for functions of $\mathfrak{F}$. This point of view is of theoretical as well as computational utility. The theoretical utility comes from the fact that formulation (1.3) entitles us to apply to a seminorm problem the results given for the norm problems (such as the Alternation Theorem and the second Remez algorithm). In order to understand the computational utility notice that $W(e^{j\omega}) = 0$ for $\omega \in B_i$ and B_i considered as a "don't care" band will produce the same effect, namely eliminate the occurrence of extremal frequencies on B_i. In fact, $W(e^{j\omega}) = 0$ will force $E(e^{j\omega}) = 0$ on $\omega \in B_i$, therefore the second Remez Algorithm will not find any extremum of $E(e^{j\omega})$ for $\omega \in B_i$. If B_i is a "don't care" band, it is just not taken into account during the operation of the second Remez Algorithm, therefore it cannot deliver any extrema of $E(e^{j\omega})$. Therefore the total effect of the two techniques is identical, but the use of the "don't care" band is more efficient. The efficiency lies in saving the actual evaluation of $E(e^{j\omega})$ over $\omega \in B_i$ as well as in allowing the efficient non-conventional solution of system (2.3) via (2.4) and (2.5) (made possible by the fact that the weight $W(e^{j\omega})$ is never 0 in $\mathfrak{F}$).

(**) A seminorm is a functional g over a vector space X, satisfying the properties (i), (ii), (iii) above.

A computational check of the above claimed equivalence between "don't care" bands and regions supporting 0 weight was tried.

McClellan's program was used because minor modifications can convert it into a tool implementing formulation (3.1) with condition (3.3). A direct verification of the equivalence was found not to be possible since the McClellan's program does not allow 0-weights. This limitation derives from the calculation of p via (2.4). Also an indirect verification by means of small weights, ideally tending to 0, was not easy to obtain. Specifically, filters of relatively high order (above 60) can call for weights of order 10^{-7} or less in order not to exhibit any extremum over the "don't care" bands. McClellan's program starts to lose its numerical accuracy for weights of this order(as reported in [7]) and the results may not be reliable. However for filters of lower order the indirect verification is possible, as the examples of Fig 5 and 6 show.

During these experiments it was noticed that a complete removal of the extremal frequencies from the transition regions gives a better performance in terms of reducing ripple over the "care" bands than a partial removal of extremal frequencies. It was noticed, in practice, that a fewer number of extremal frequencies in the transition regions results in a smaller ripple.

The idea behind the choice of the values of $W(e^{j\omega})$ and $D(e^{j\omega})$ over the transition regions for use in (3.1) is to have as few extremal frequencies as possible over these regions (possibly none). The choice of the values of $W(e^{j\omega})$ and $D(e^{j\omega})$ over the pass-bands and the stop-bands follows the usual criteria taken for the Parks and McClellan's

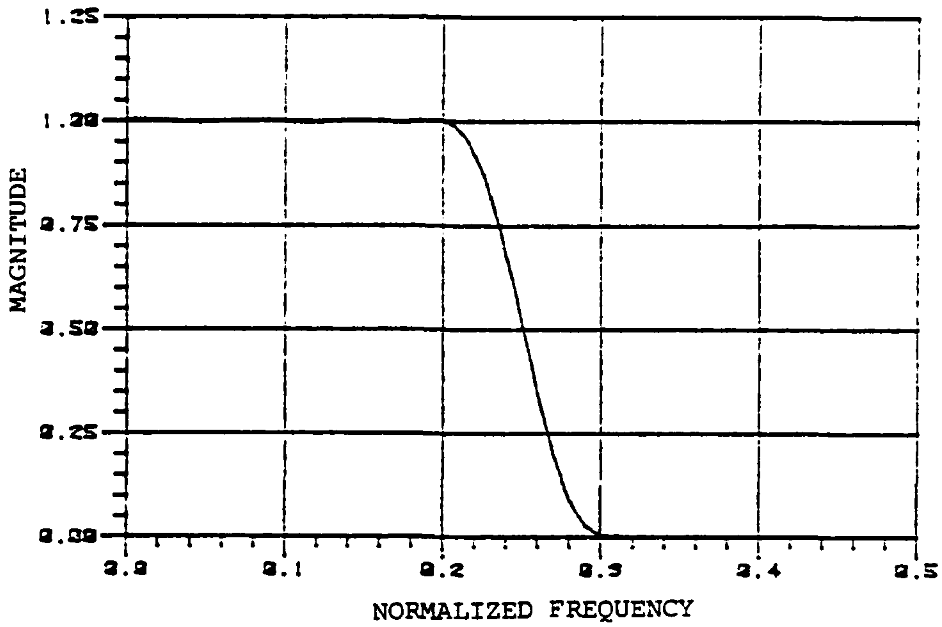


Fig. 5a McClellan's program: example of low-pass with N = 30

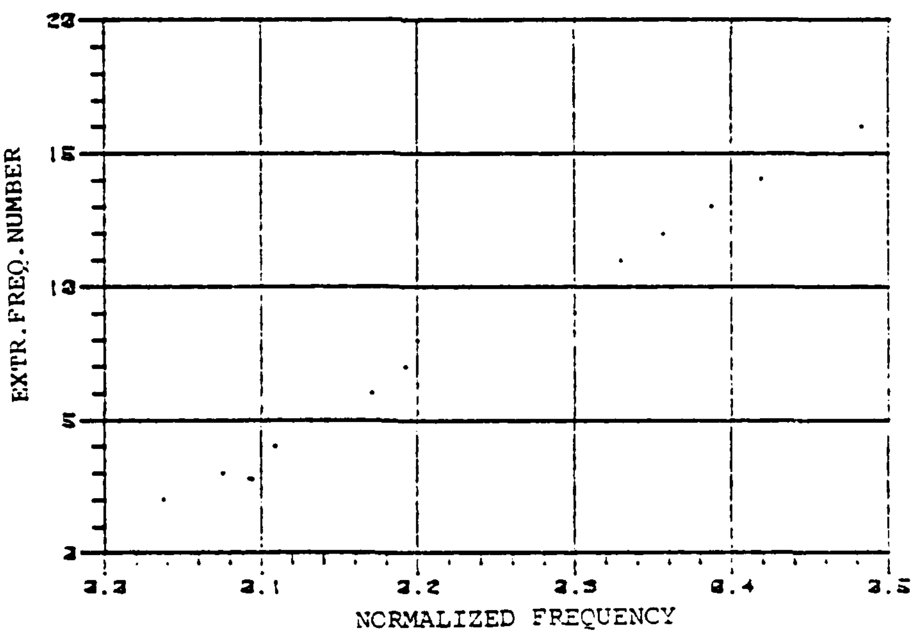


Fig. 5b Extremal frequencies of the low-pass of Figure 4a

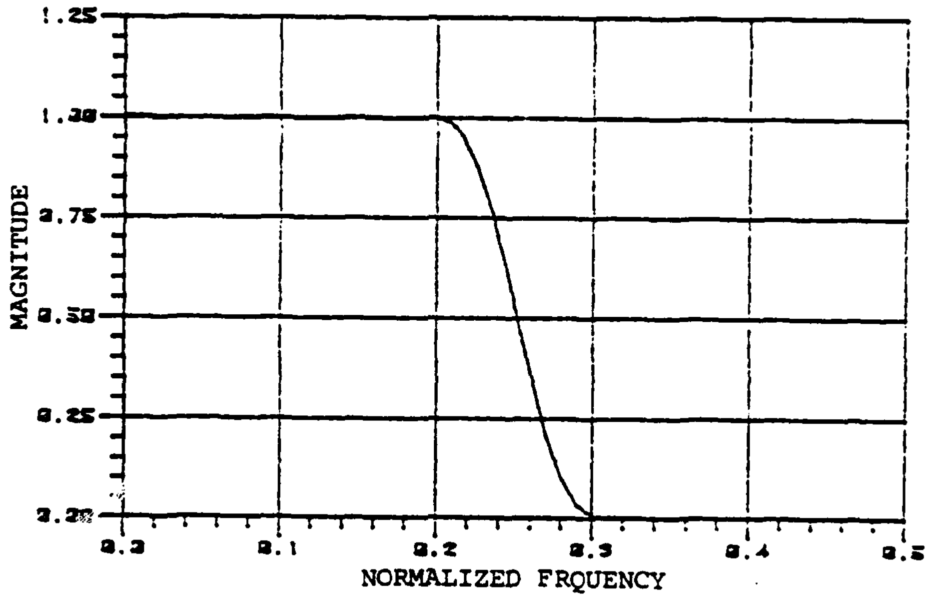


Fig. 6a The low-pass of Figure 4 designed with the new program

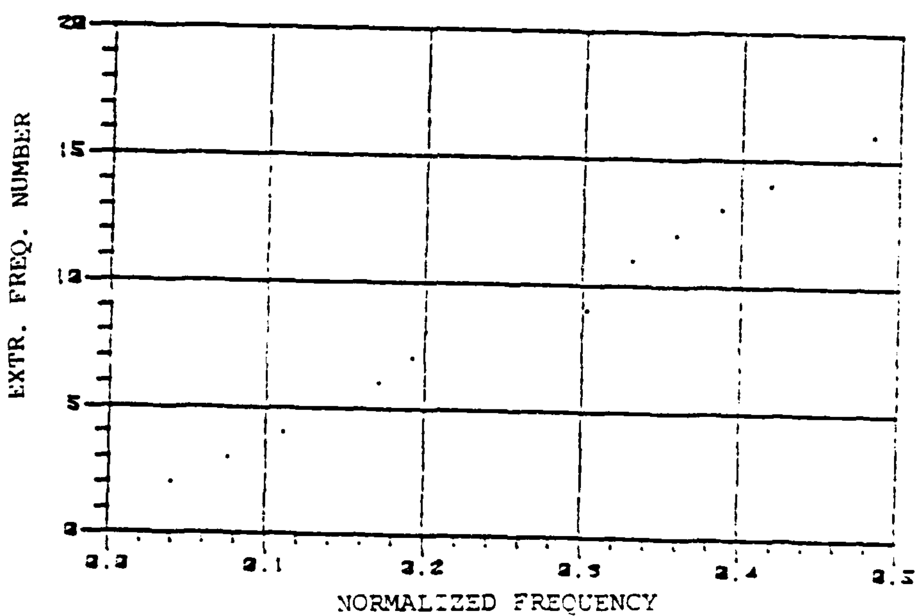


Fig. 6b Extremal frequencies of the low-pass of Figure 5a

formulation.

The $W(e^{j\omega})$ to be assigned to the transition regions comes from a trade-off between two conflicting requirements. $W(e^{j\omega})$ should be small in order to keep $E(e^{j\omega})$ small, so that no extremal frequency is detected by the Remez algorithm. But $W(e^{j\omega})$ cannot be too small, since the smaller $W(e^{j\omega})$ is, the larger the term $|D(e^{j\omega}) - H(e^{j\omega})|$ becomes for a given minimax error $E(e^{j\omega})$. As a matter of fact, the difficulty with $W(e^{j\omega}) = 0$ (or "don't care" bands) is that resonances of $H(e^{j\omega})$ (i.e. points where the term $|D(e^{j\omega}) - H(e^{j\omega})|$ is very large) can occur without being detected as extrema of $E(e^{j\omega})$. Therefore the weight over the transition regions has to be taken small, but non-zero, in order to prevent resonances.

The following two observations are useful for the choice of $D(e^{j\omega})$ over the transition regions. The first observation is that every multiband filter can be thought to be the composition of several low-pass or high-pass filters [7]. Such low and high-pass filters will be called "prototypes" in the present work. The second observation is that the prototypes, by the theorem of Chapter 2 have a monotonic transition region. Thus, they can be safely implemented with the McClellan's program. This suggests that the transition regions of the prototypes be taken as a model for the transition regions of $D(e^{j\omega})$. Two techniques for obtaining the transition regions of $D(e^{j\omega})$ have been tried: a multisegment piece-wise linear approximation of the transition regions of the prototypes, and the direct use of the transition regions of the prototypes in $D(e^{j\omega})$. For convenience these will be referred to as technique L and P, respectively. A detailed discussion of their implementation and performance will be given in the next section.

The new formulation together with technique P prevents an extremal frequency from occurring in the transition region for the design of low or high-pass filters, since the error $E(e^{j\omega})$ over the transition region is made 0 by the term $|D(e^{j\omega}) - H(e^{j\omega})|$. This result is independent of the value of $W(e^{j\omega})$ over the transition region.

In the design of multiband filters, the complete elimination of the extremal frequencies from the transition regions is not to be expected, even with technique P, since every prototype induces extremal frequencies into the transition regions of the other prototypes.

Finally, notice that the new formulation (3.1) allows a general control over the transition regions of the filter. This characteristic can be used to obtain monotonic behavior as well as any other desired behavior in the transition regions. The new formulation is therefore very suited to the design of filters for which the shape of the transition regions is important. Thus if transition band performance is important in a high or low-pass filter formulation, (3.1) is to be preferred to the McClellan's formulation. Figure 7 shows an example of a low-pass filter that could not be obtained with McClellan's formulation.

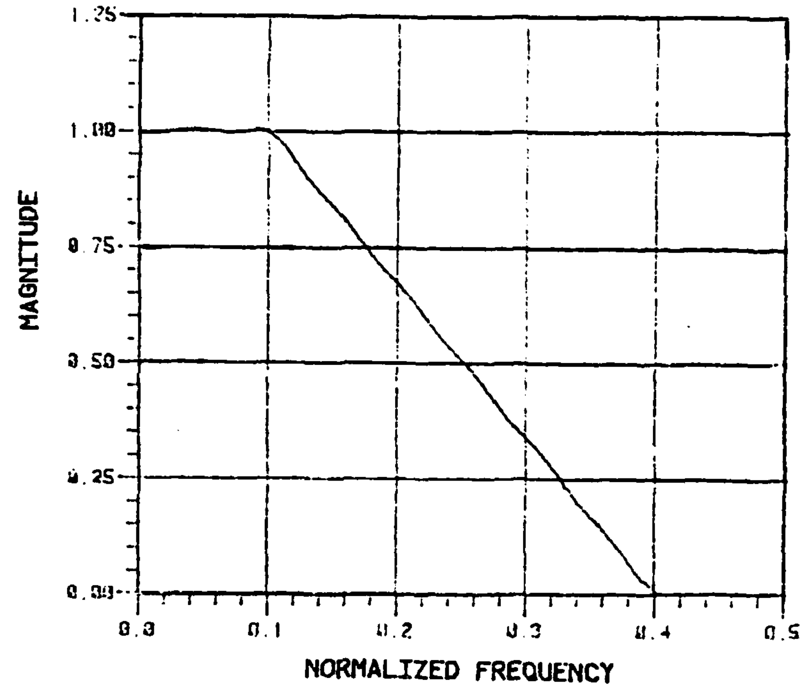
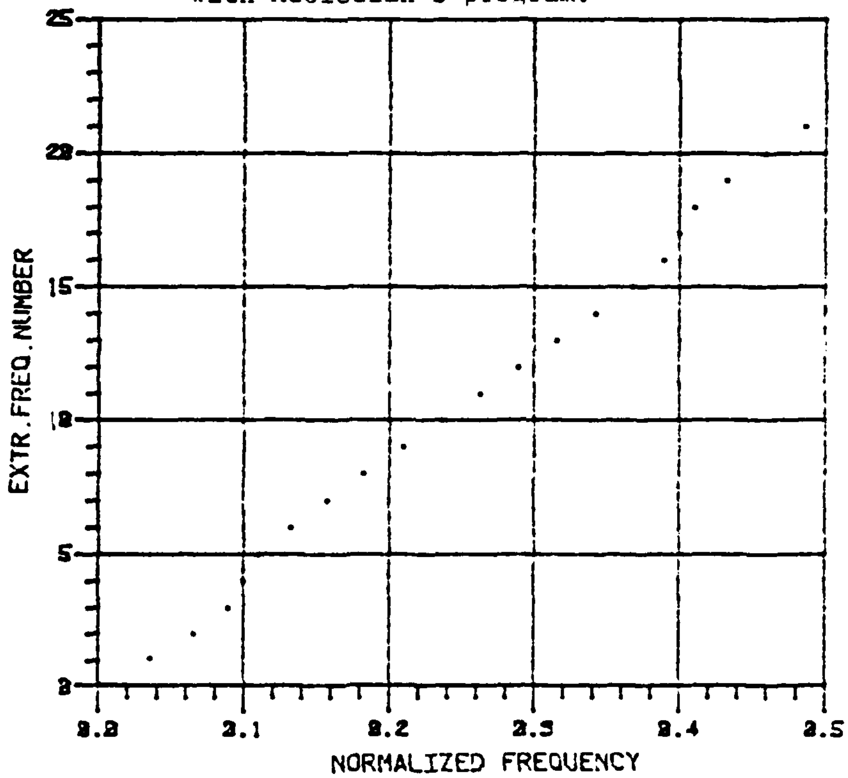


Fig. 7a Example of a filter that can be designed with the new program and can not be designed with McClellan's program.



Fib. 7b Extremal frequencies of the low-pass of of Figure 7a

CHAPTER 4

IMPLEMENTATION AND PERFORMANCE OF THE NEW FORMULATION

The new formulation of the linear phase digital filter design problem (3.1) has the structure of a minimization problem in minimax norm for trigonometric polynomials, similar to the Parks and McClellan formulation (1.3). The Alternation Theorem and the second Remez algorithm are still applicable for computing the solution to problem (3.1).

The program written by J. McClellan contains a general purpose subroutine for performing the Remez algorithm. The program is designed in three parts; an input section, a computational part, and an output section. The first part is devoted to building $W(e^{j\omega})$, $D(e^{j\omega})$ from the input data and to setting up the approximation problem. The central computational part is the implementation of the Remez algorithm. The third part is devoted to the display of the results. Such modularity facilitates possible modifications of the program.

It appeared convenient to incorporate the implementation of the new formulation (3.1) into McClellan's program. This would make the new formulation directly accessible to all who currently use McClellan's program.

The key idea of the implementation was to insert in parallel with McClellan's program an alternate pattern in 3 parts devoted to the implementation of (3.1). The first part can accept the input data peculiar to the new formulation. The second part prompts the operation of the second Remez Algorithm over the full band $[0, \pi]$. The third part is devoted to the display of the results of interest. The new program so obtained can be thought of as a modified version of the McClellan's program allowing the selection of two different "modes" of operation. Namely the "regular mode" that prompts the new program to implement formulation (1.3) (i.e. to behave exactly as the McClellan's program) and the "custom mode" that prompts the implementation of the new formulation (3.1). A user-oriented description of this program is presented in Appendix 1.

The rest of the section is devoted to the presentation of the results obtained with the new program together with some practical observations useful for the choice of $W(e^{j\omega})$ and $D(e^{j\omega})$ over the transition regions.

For comparison purposes the four filters reported in [7] as typical cases of multiband filters with non-monotonic transition regions have been designed with the McClellan's program, the new program, and CONRIP (which is another digital filter design program discussed in Chapter 5). The filters examined are labeled Design 1, 2, 3, or 4 as in [7]. The new program has been used with both the techniques L and P, described in Chapter 3.

Figures 8-23 constitute by themselves the best comments on the performance of the new program versus the other two programs. It can be seen that the new program gives strictly monotonic behavior in the transition regions also in cases where McClellan's program and CONRIP do not.

Several comments are now presented about the choice of $W(e^{j\omega})$

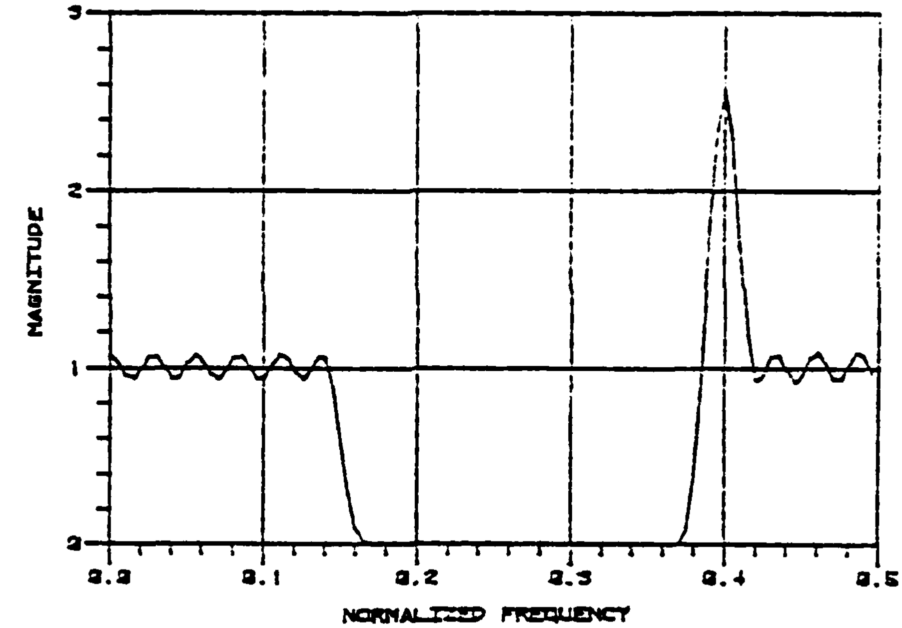


Fig. 8a - McClellan's program: $H^*(e^{j\omega})$ of Design 1

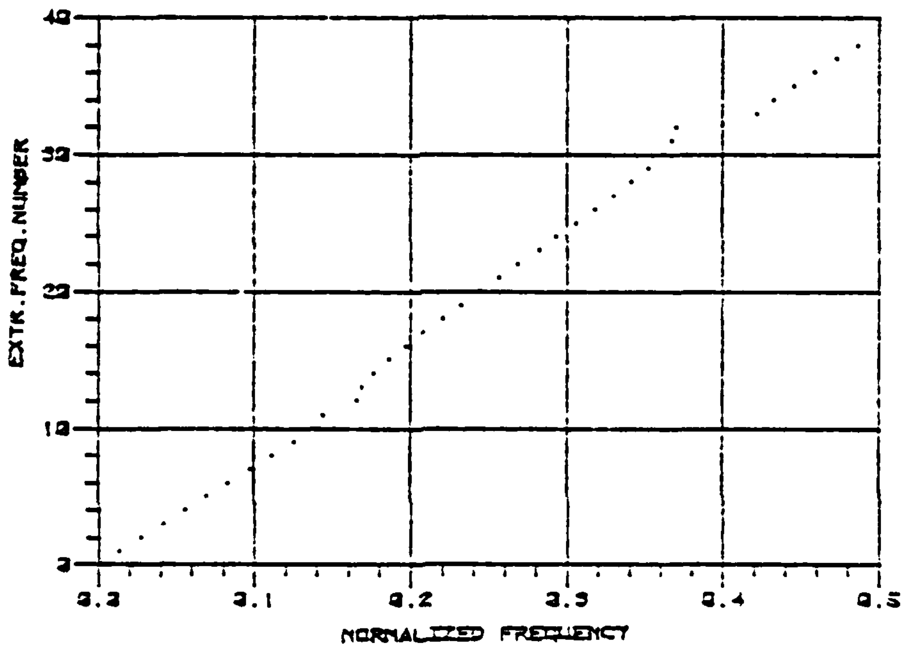


Fig. 8b McClellan's program: extremal frequencies of Design 1

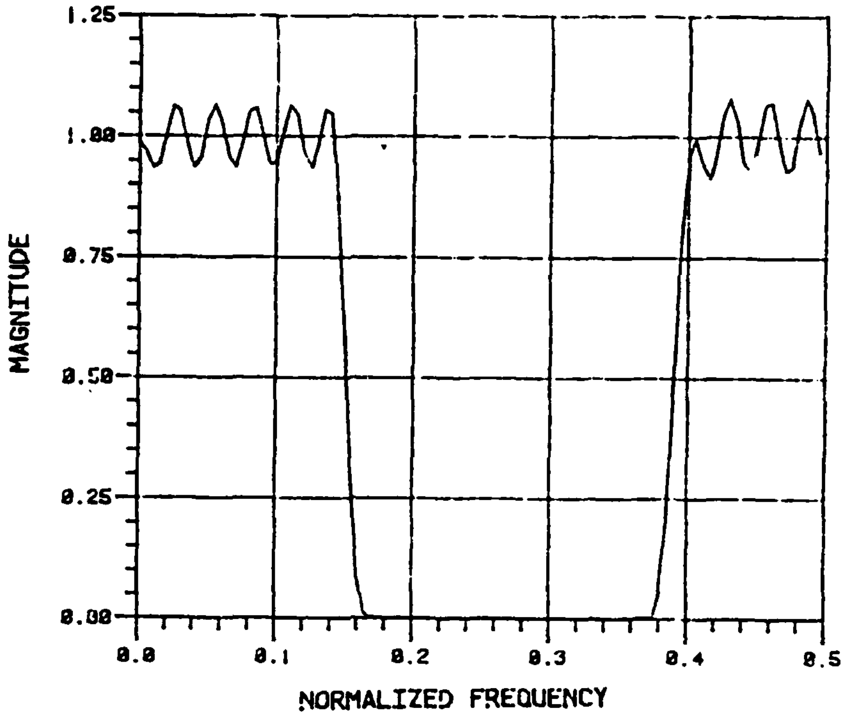


Fig. 9a New program, technique L: $H^*(e^{j\omega})$ of Design 1

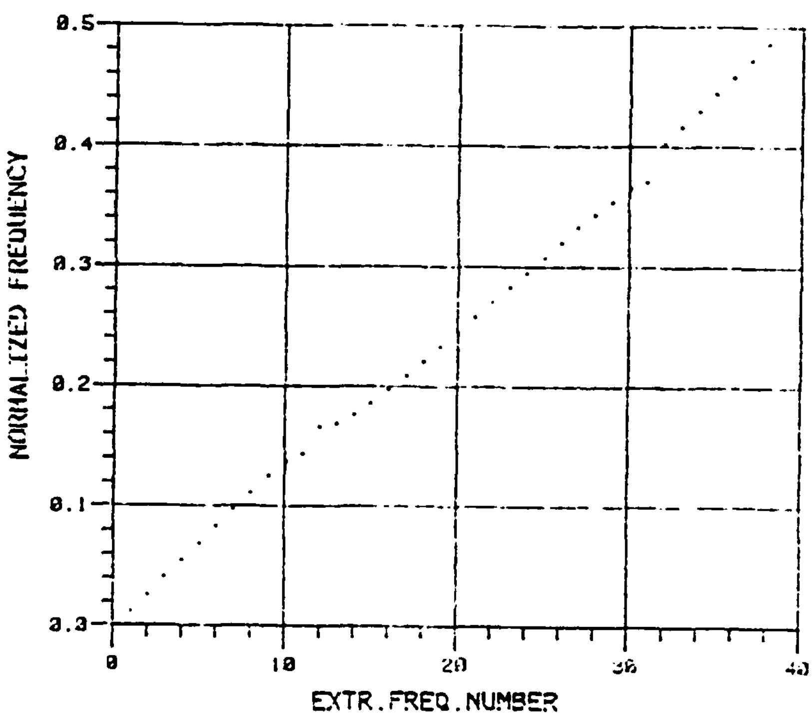


Fig 9b New program, technique L: extremal frequencies of Design 1

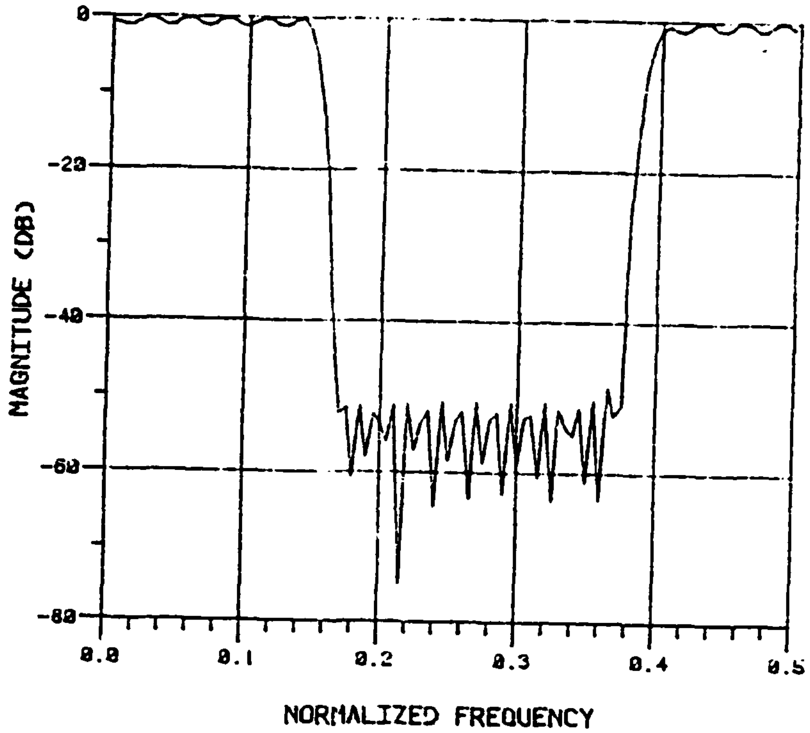


Fig. 9c New program, technique L: $H^*(e^{j\omega})$ of Design 1 in dB

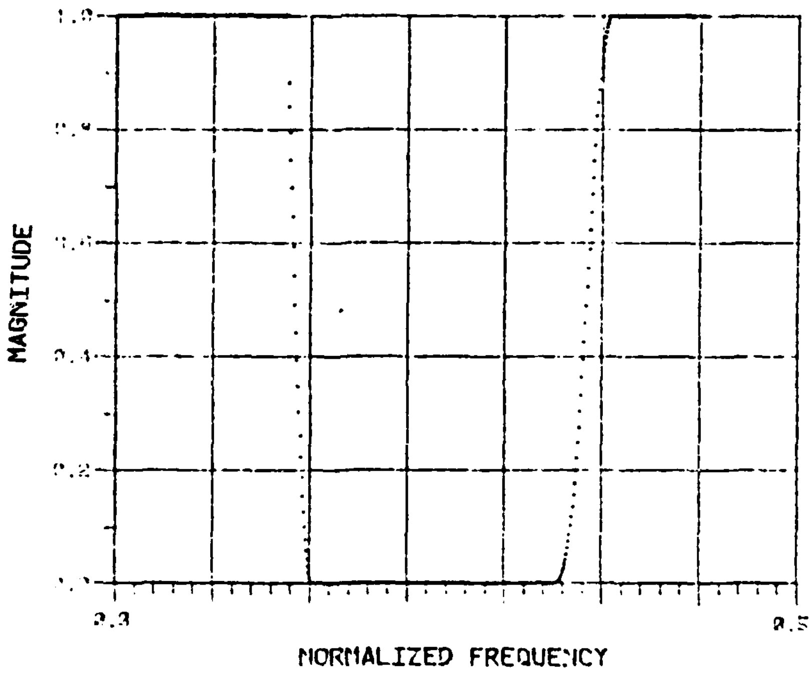


Fig. 10 New program, technique P: desired function $D(e^{j\omega})$ for Design 1

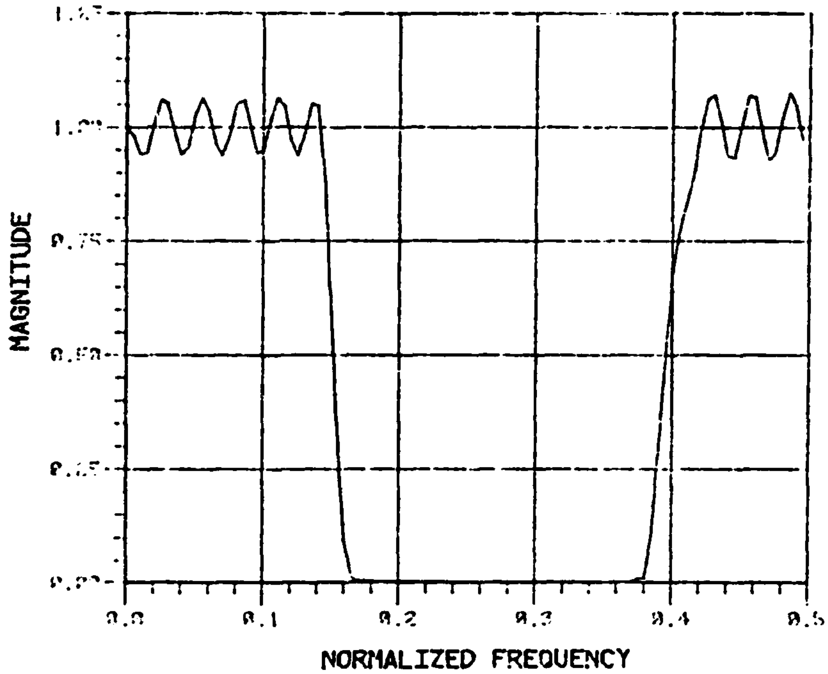


Fig. 10a New program, technique P: $H^*(e^{j\omega})$ of Design 1

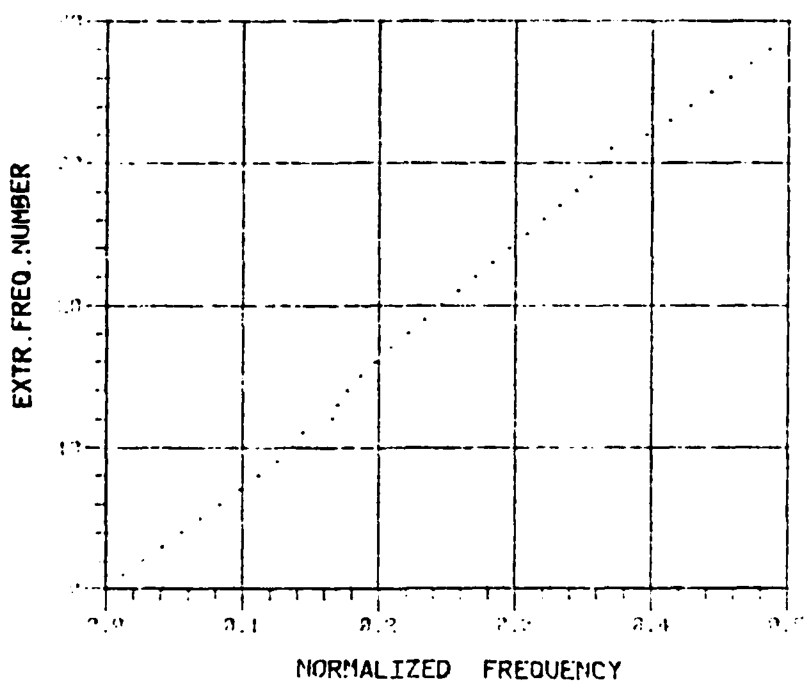


Fig. 10b New program, technique P: extremal frequencies of Design 1

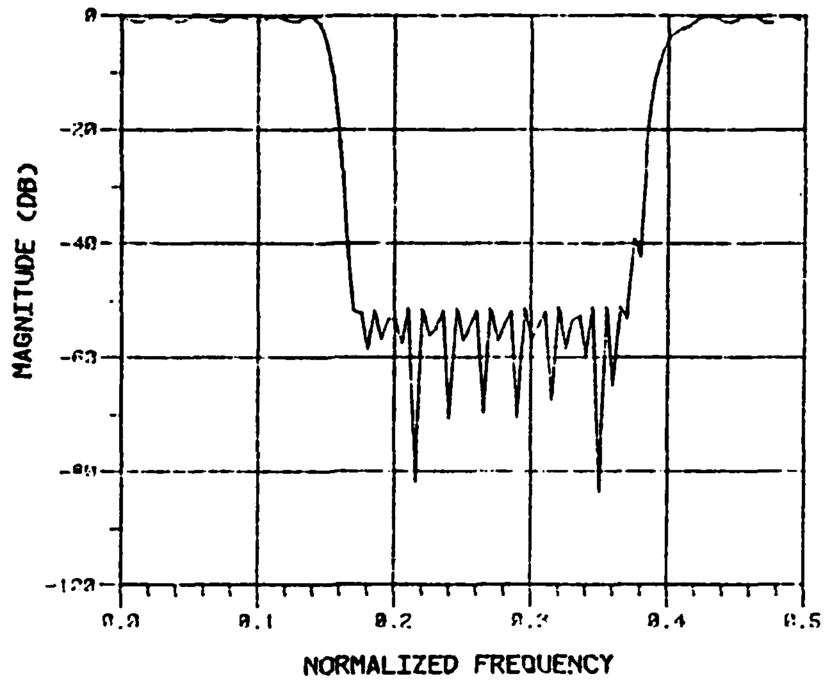


Fig. 10c New program, technique P: $H^*(e^{j\omega})$ of Design 1 in dB

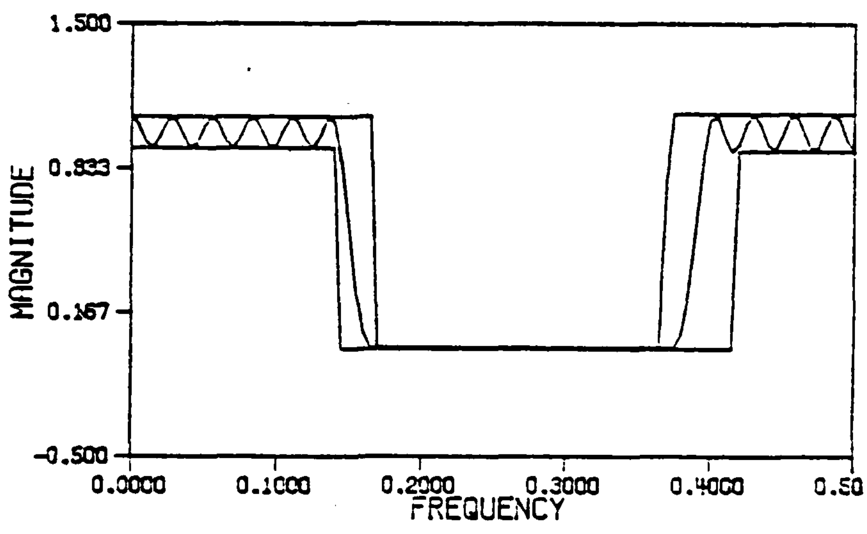


Fig. 11 CONRIP: $H^*(e^{j\omega})$ of Design 1 (from [8], pp. 66)

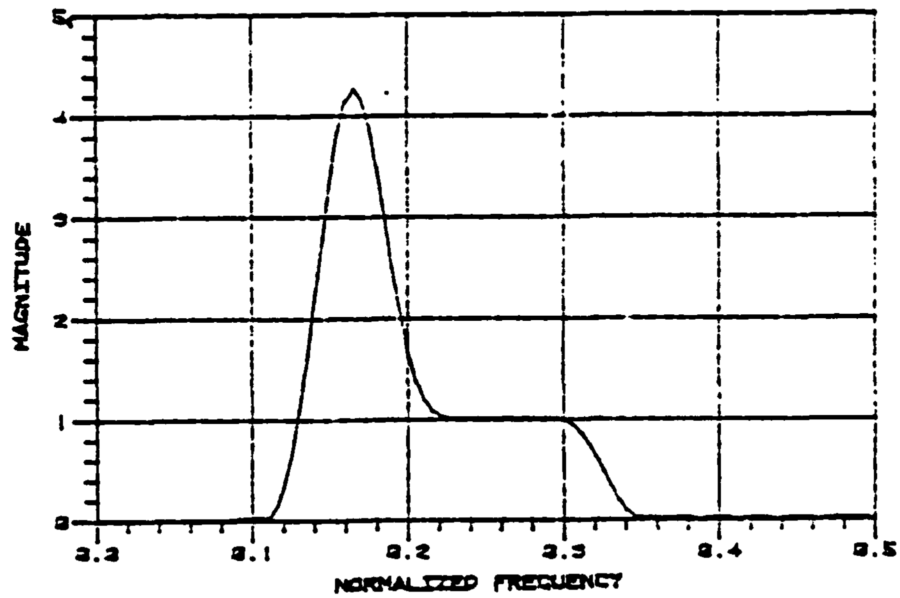


Fig. 12a McClellan's program: $H^*(e^{j\omega})$ of Design 2

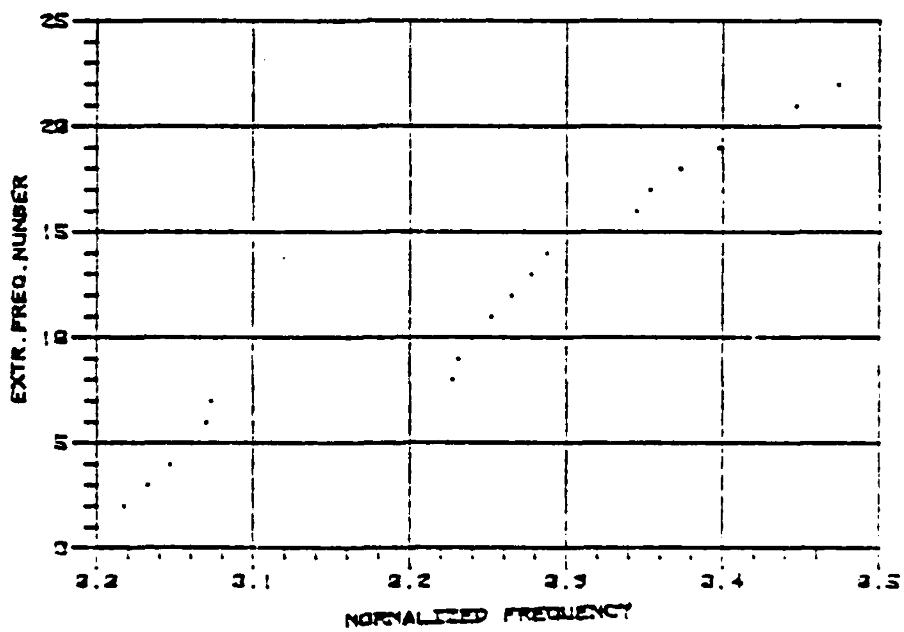


Fig. 12b McClellan's program: extremal frequencies of Design 2

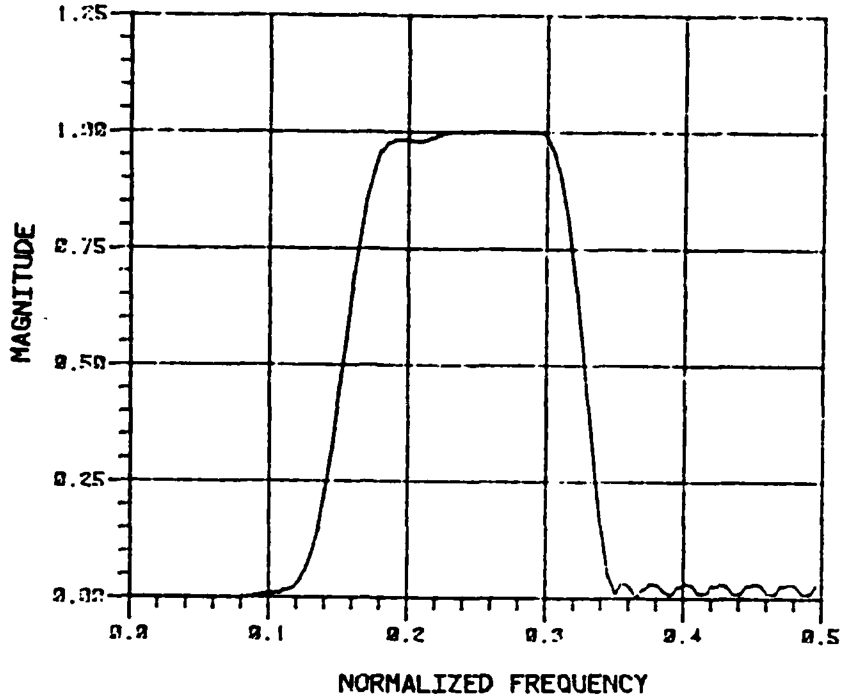


Fig. 13a New program, technique L: $H^*(e^{j\omega})$ of Design 2

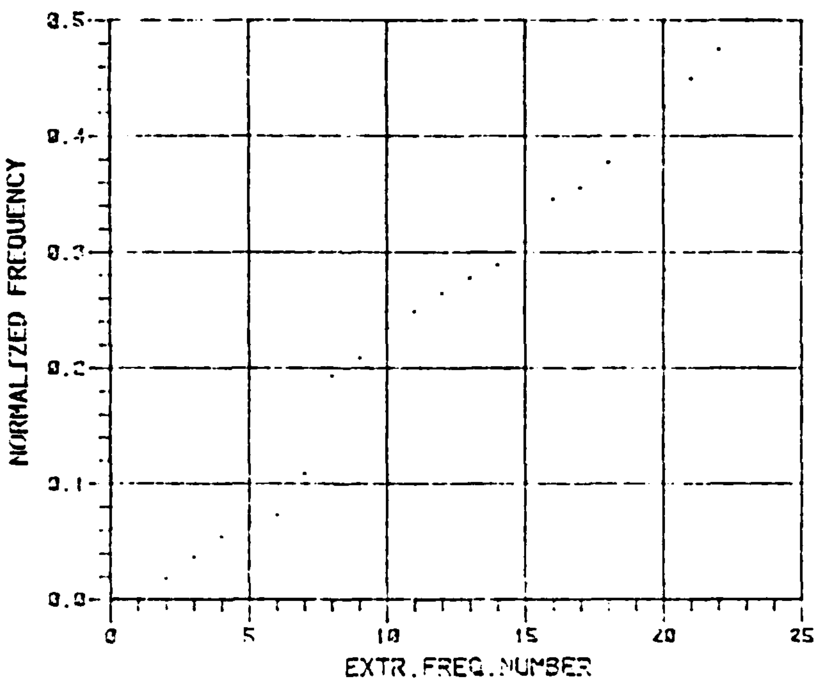


Fig. 13b New program, technique L: extremal frequencies of Design 2

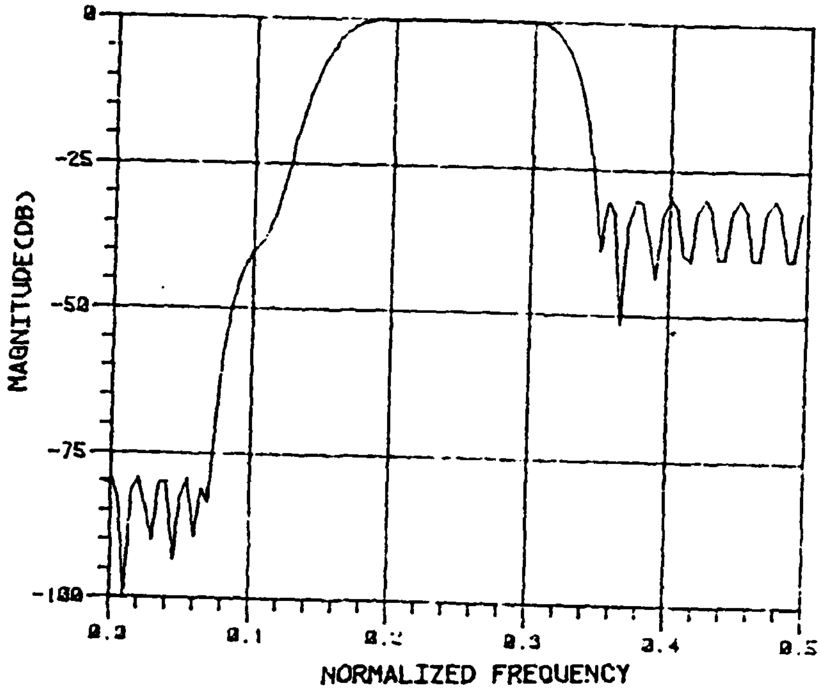


Fig. 13c New program, technique L: $H^*(e^{j\omega})$ of Design 2 in dB

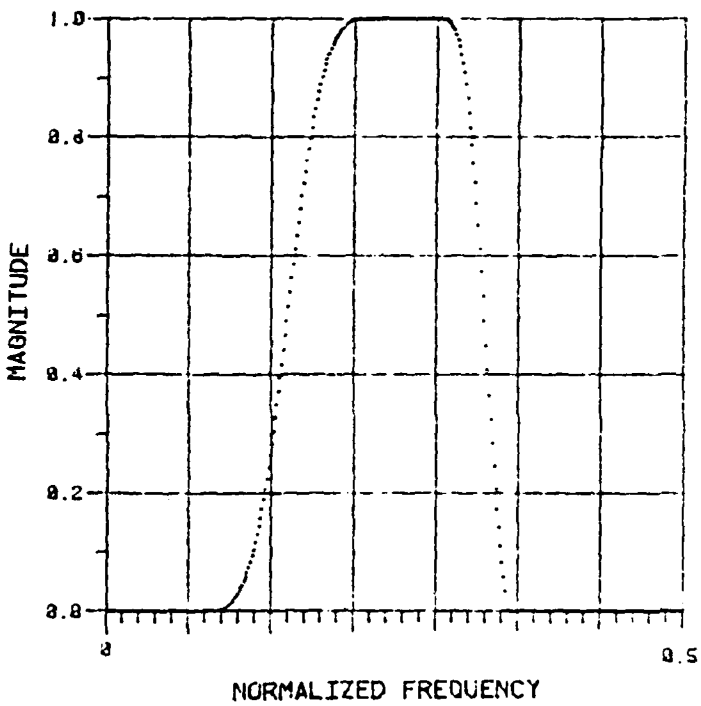


Fig. 14 New program, technique P: desired function $D(e^{j\omega})$ for Design 2

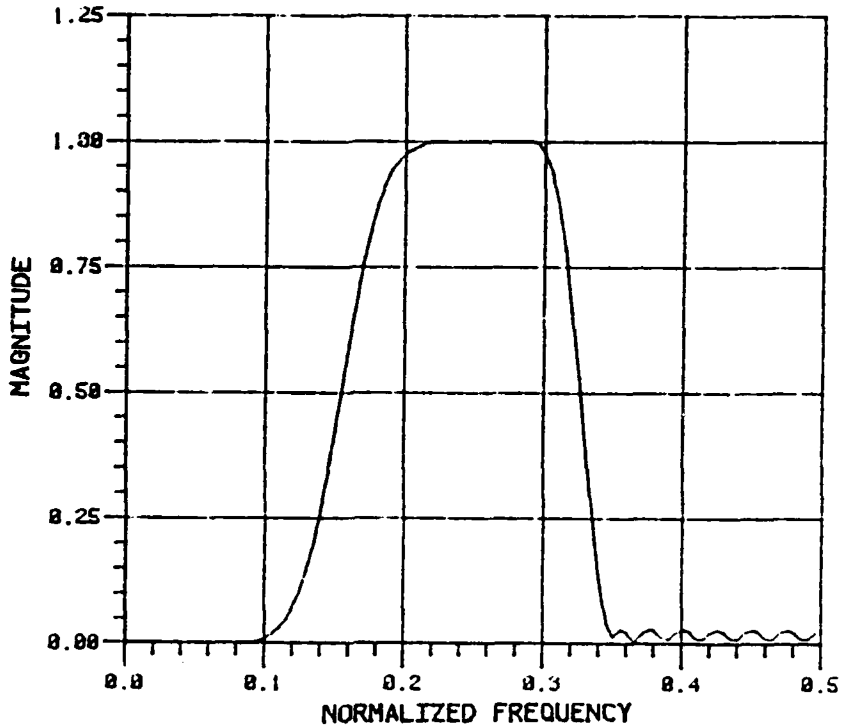


Fig. 14a New program, technique P: $H^*(e^{j\omega})$ of Design 2

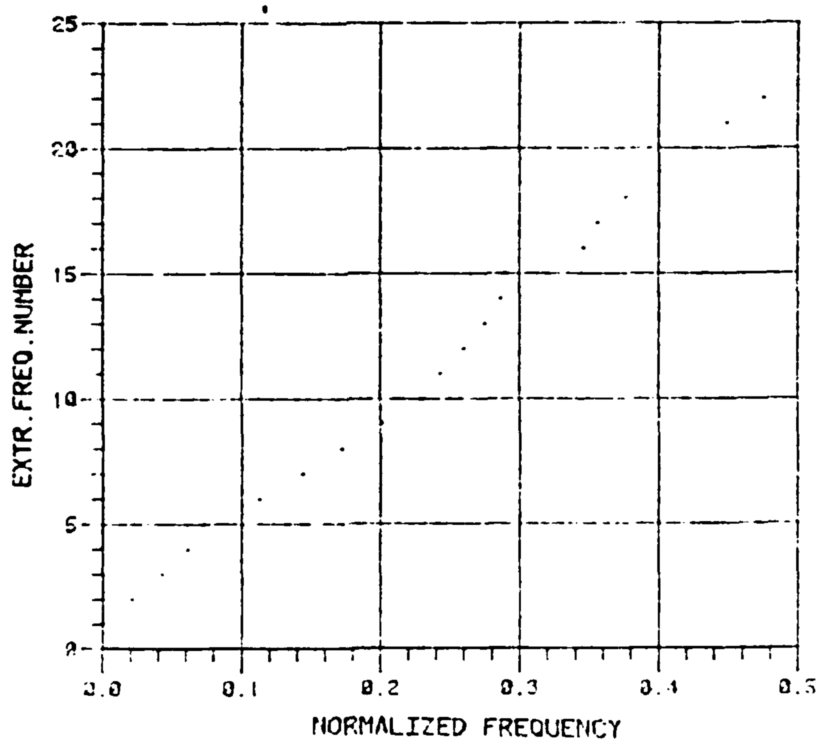


Fig. 14b New program, technique P: extremal frequencies of Design 2

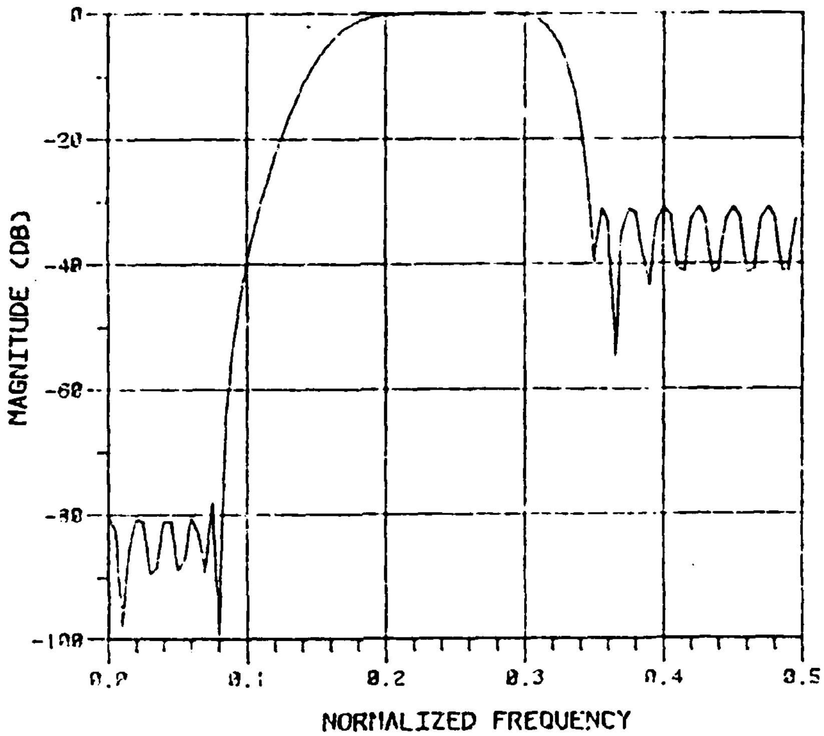


Fig. 14c New program, technique P: $H^*(e^{j\omega})$ of Design 2 in dB

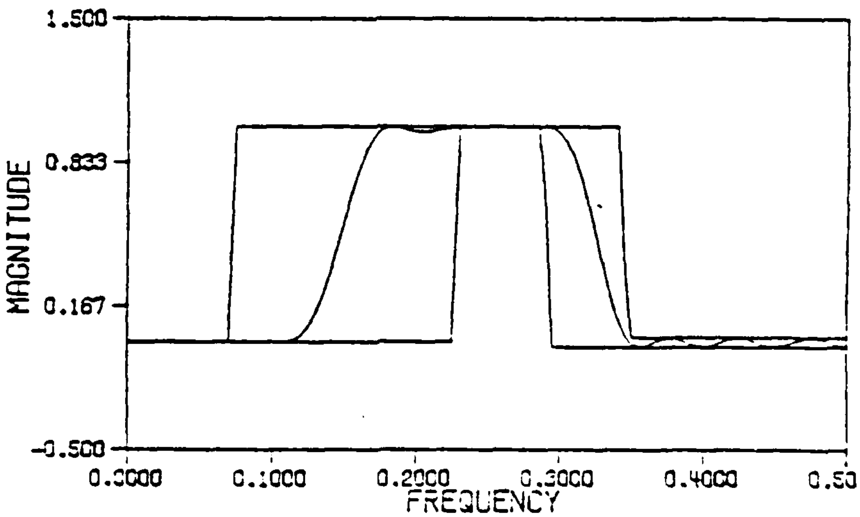


Fig. 15 CONRIP: $H^*(e^{j\omega})$ of Design 2 (from [8], pp. 69)

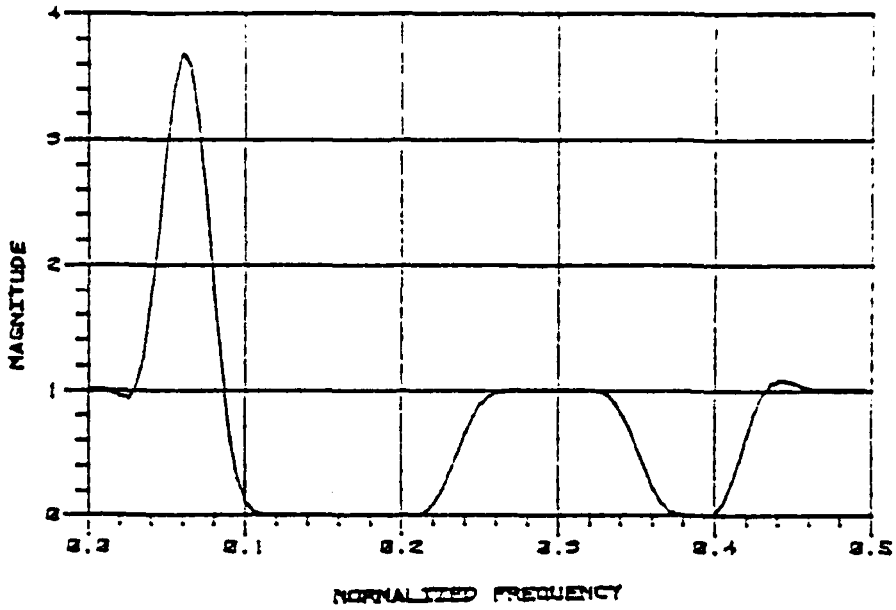


Fig. 16a McClellan's program: $H^*(e^{j\omega})$ of Design 3

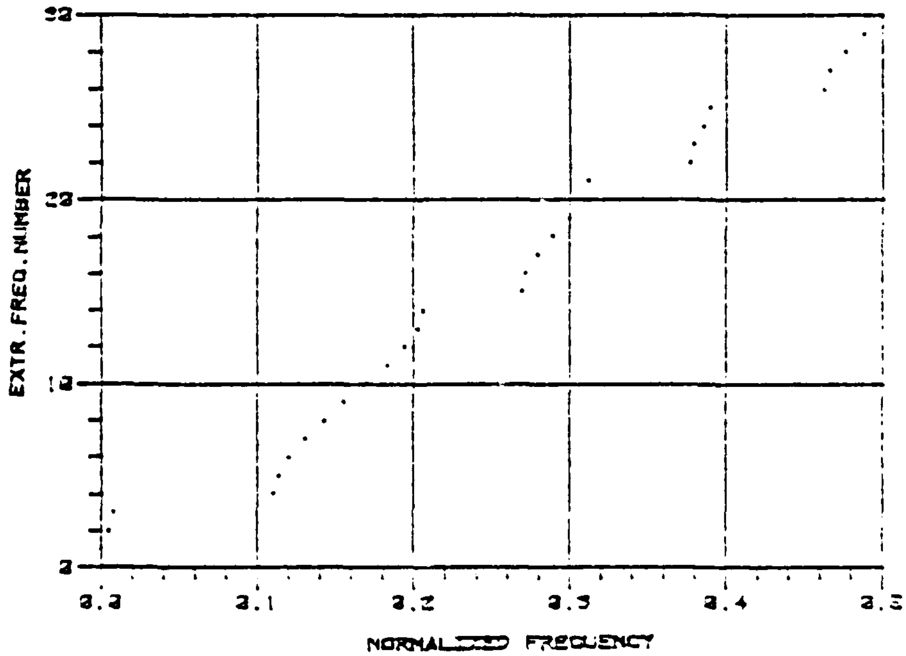


Fig. 16b McClellan's program: extremal frequencies of Design 3

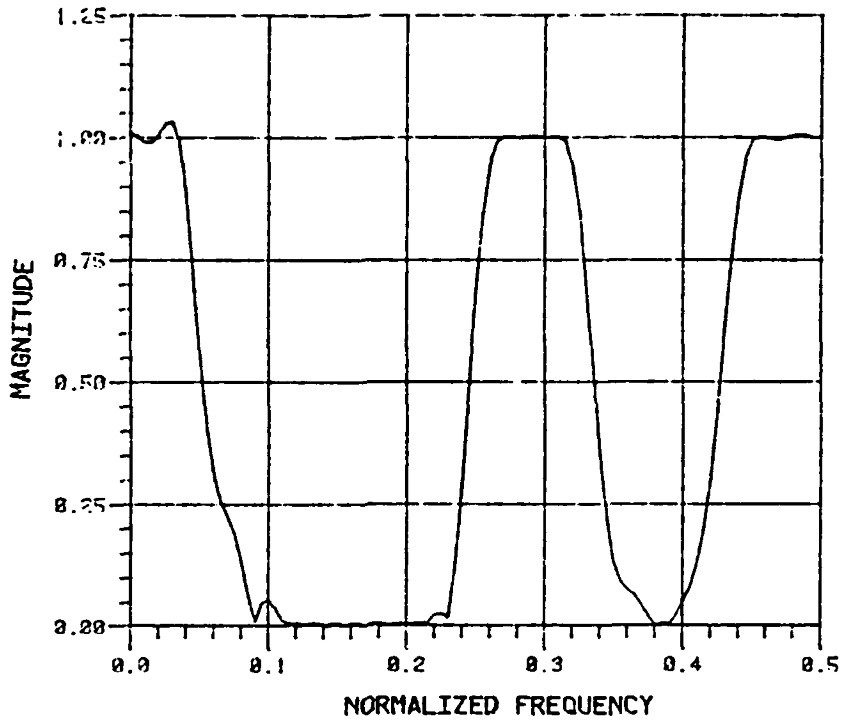


Fig. 17a New program, technique L: $H^*(e^{j\omega})$ of Design 3

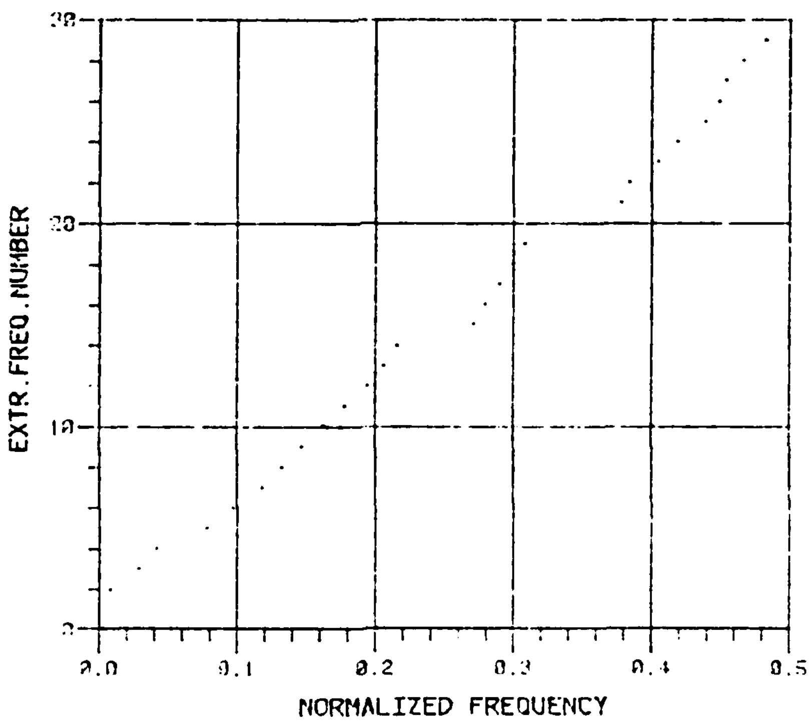


Fig. 17b New program, technique L: extremal frequencies of Design 3

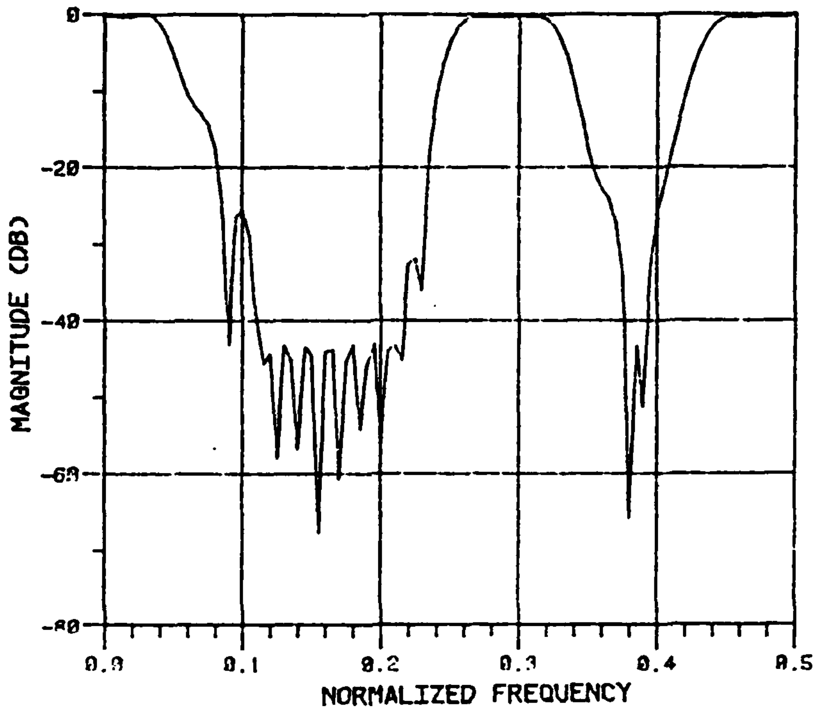


Fig. 17c New program, technique L: $H^*(e^{j\omega})$ of Design 3 in dB

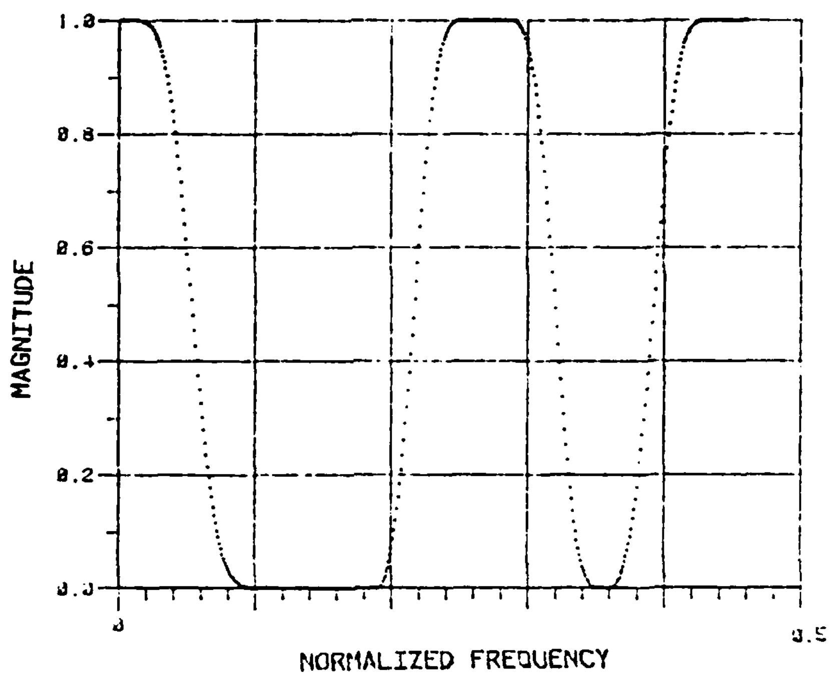


Fig. 18 New program, technique P: desired function $D(e^{j\omega})$ for Design 3

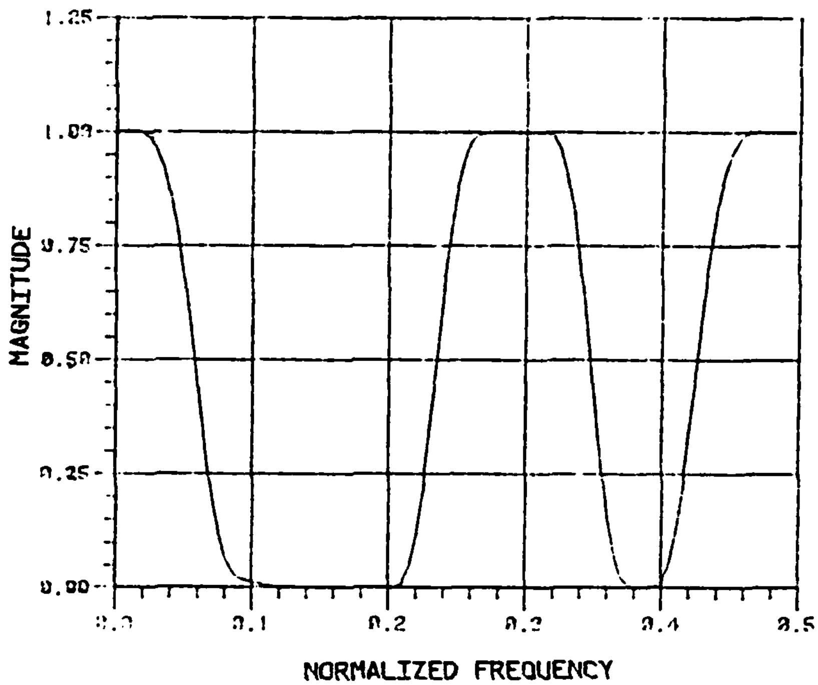


Fig. 18a New program, technique P: $H^*(e^{j\omega})$ of Design 3

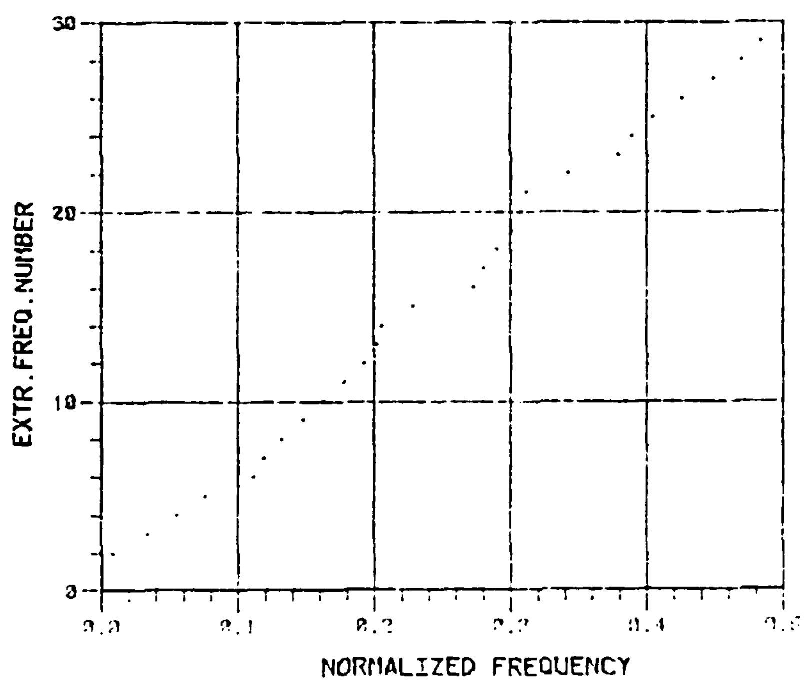


Fig. 18b New program, technique P: extremal frequencies of Design 3

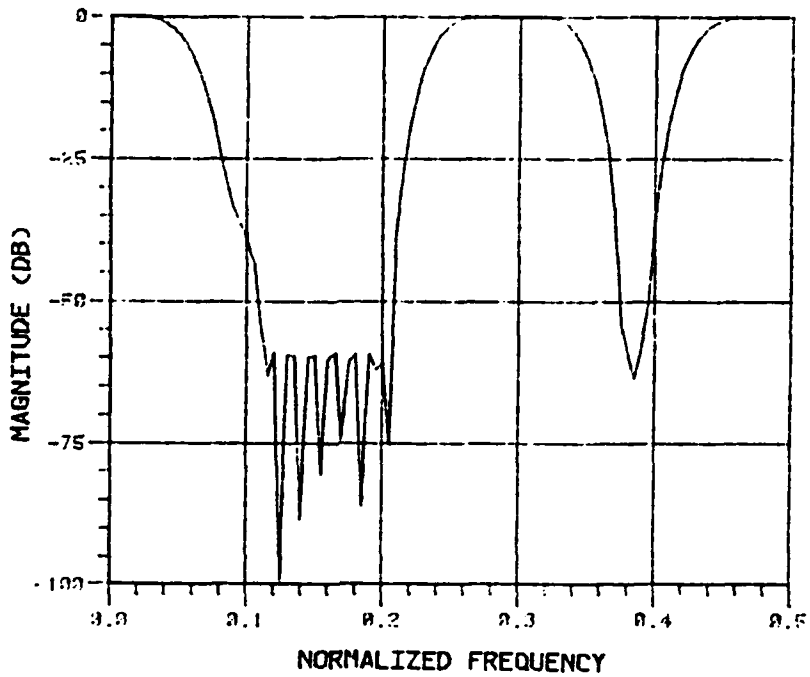


Fig. 18c New program, technique P: $H^*(e^{j\omega})$ of Design 3 in dB

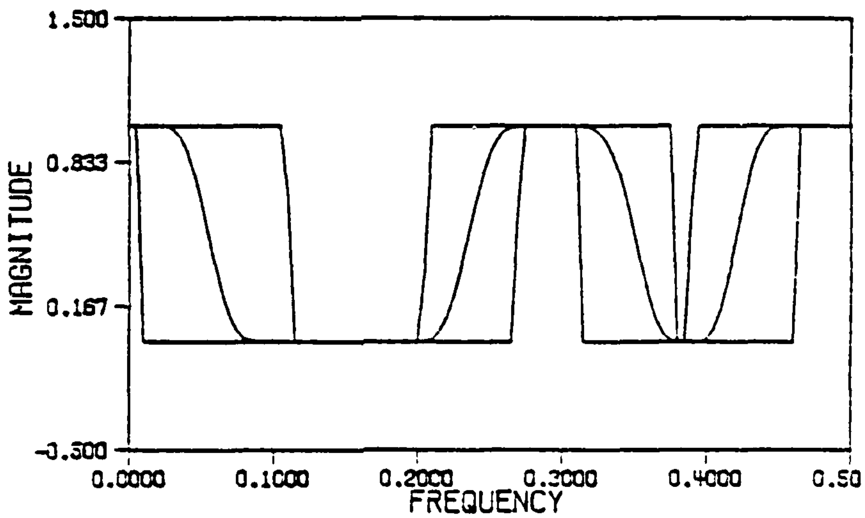


Fig. 19 CONRIP: $H^*(e^{j\omega})$ of Design 3 (from [8], pp. 72)

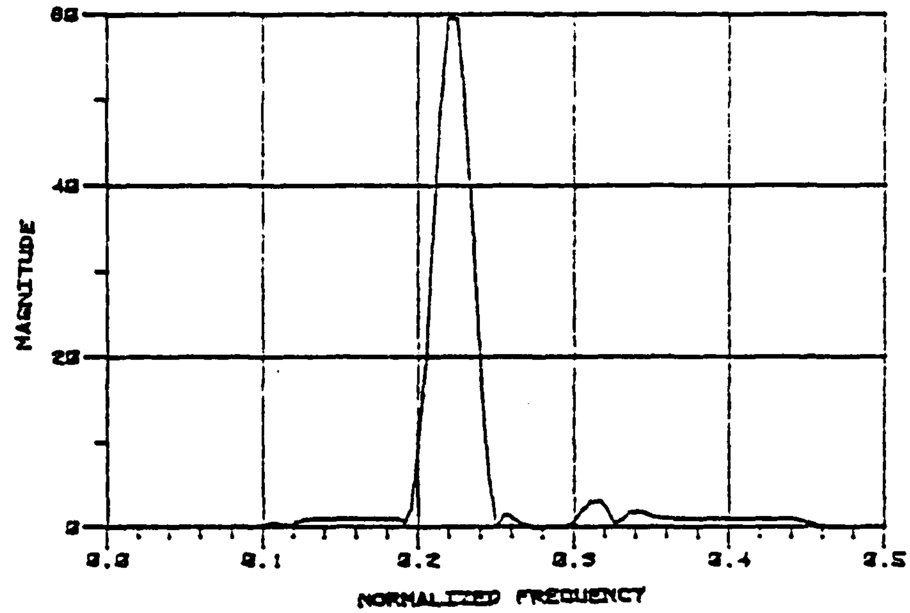


Fig. 20a McClellan's program: $H^*(e^{j\omega})$ of Design 4

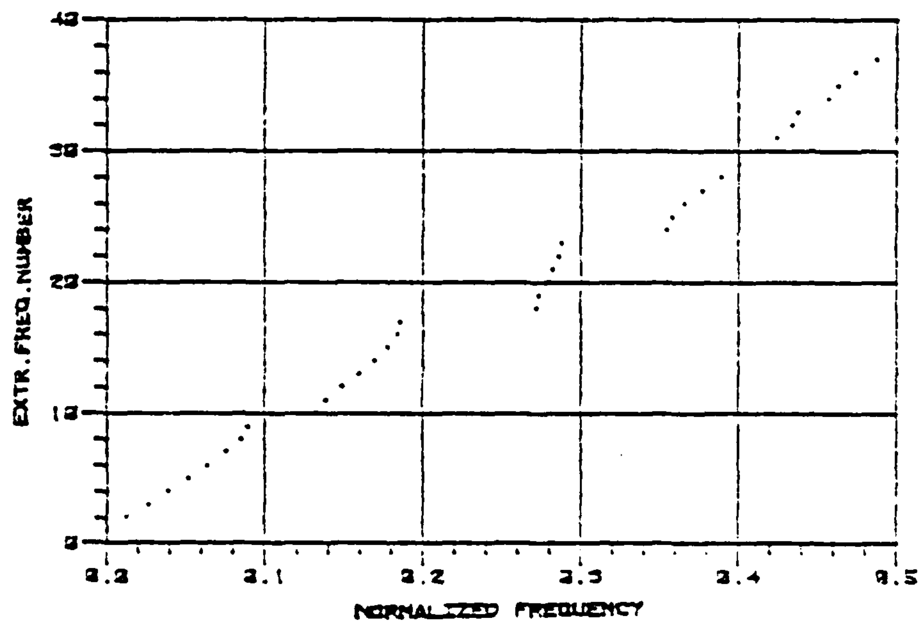


Fig. 20b McClellan's program: extremal frequencies of Design 4

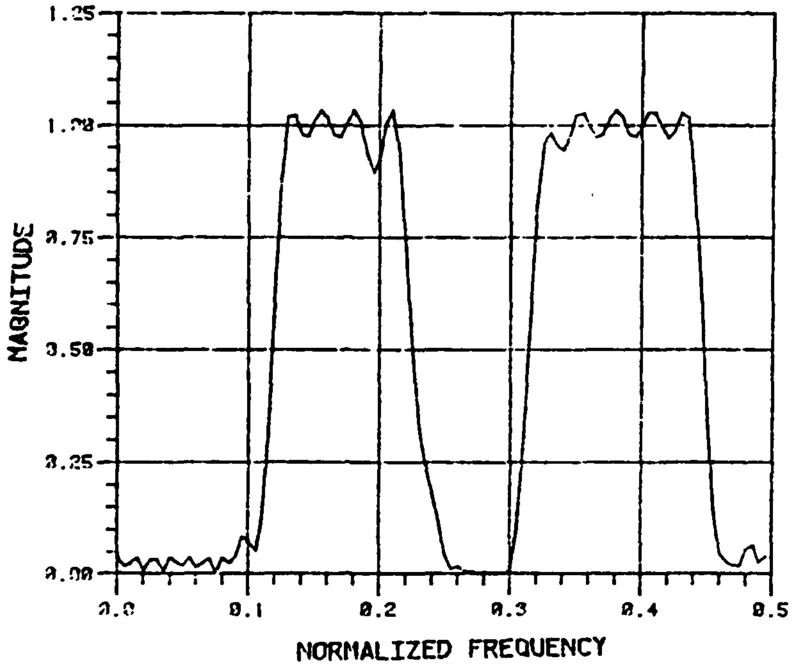


Fig. 21a New program, technique L: $H^*(e^{j\omega})$ of Design 4

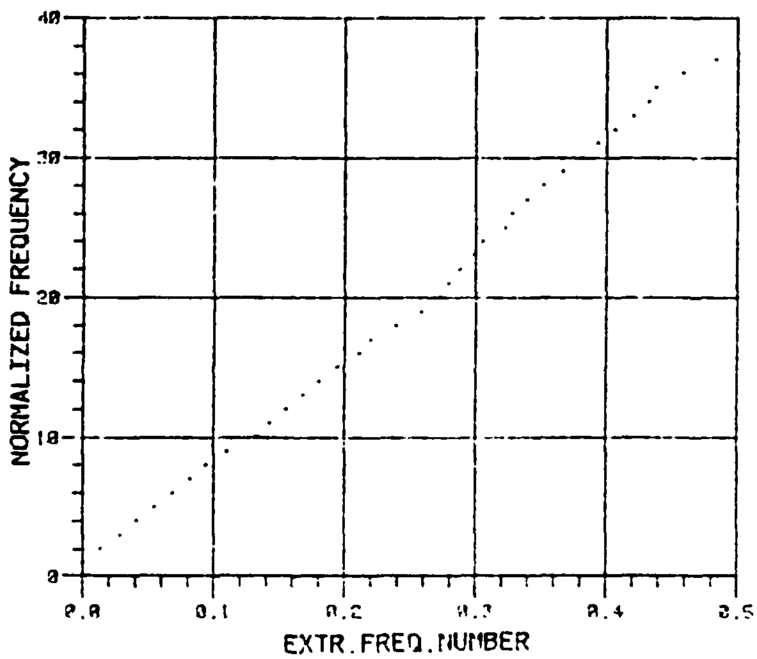


Fig. 21b New program, technique L: extremal frequencies of Design 4

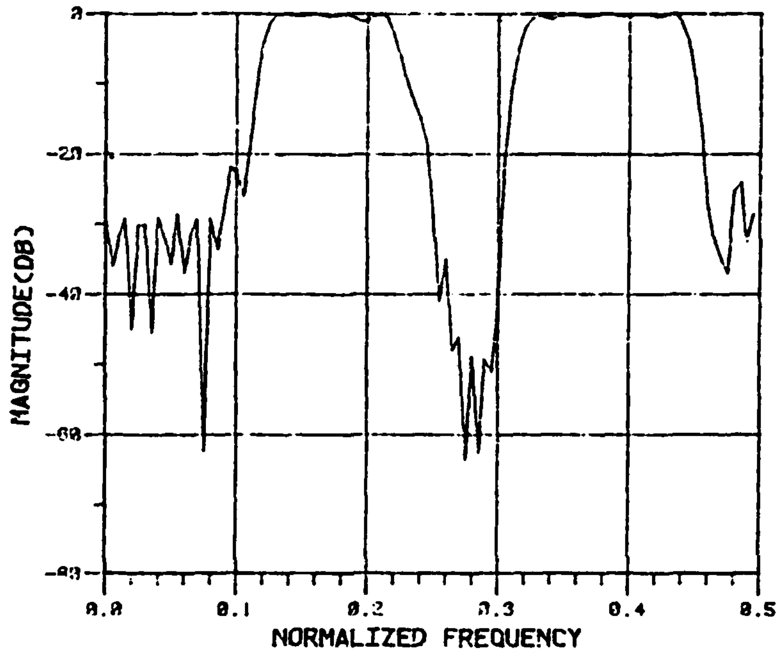


Fig. 21c New program, technique P: $H^*(e^{j\omega})$ of Design 4 in dB

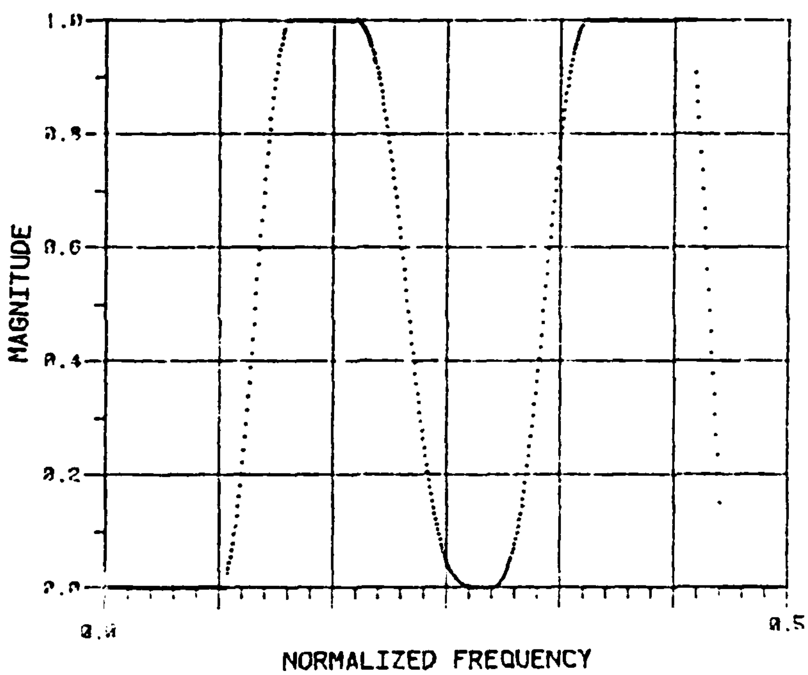


Fig. 22 New program, technique P: desired function $D(e^{j\omega})$ for Design 4

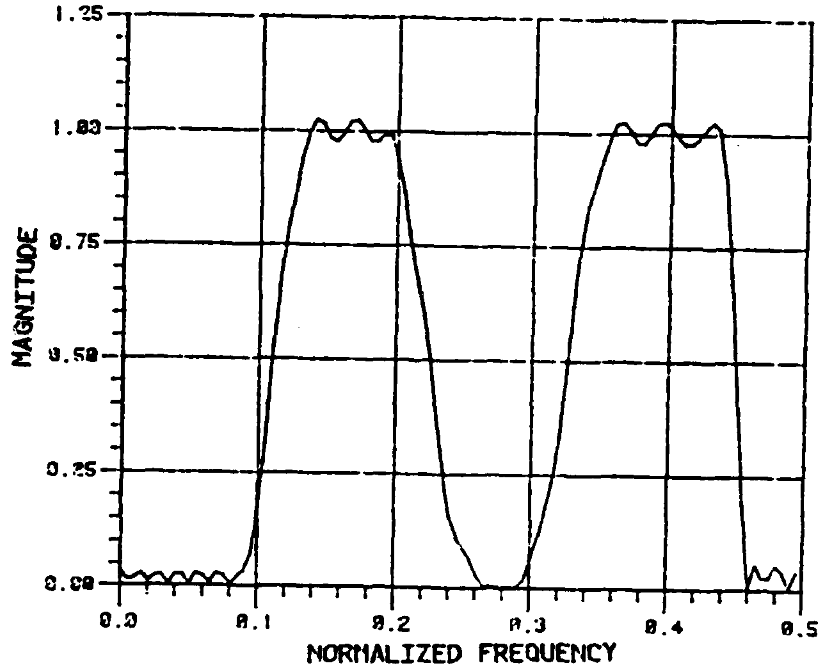


Fig. 22a New program, technique P: $H^*(e^{j\omega})$ of Design 4

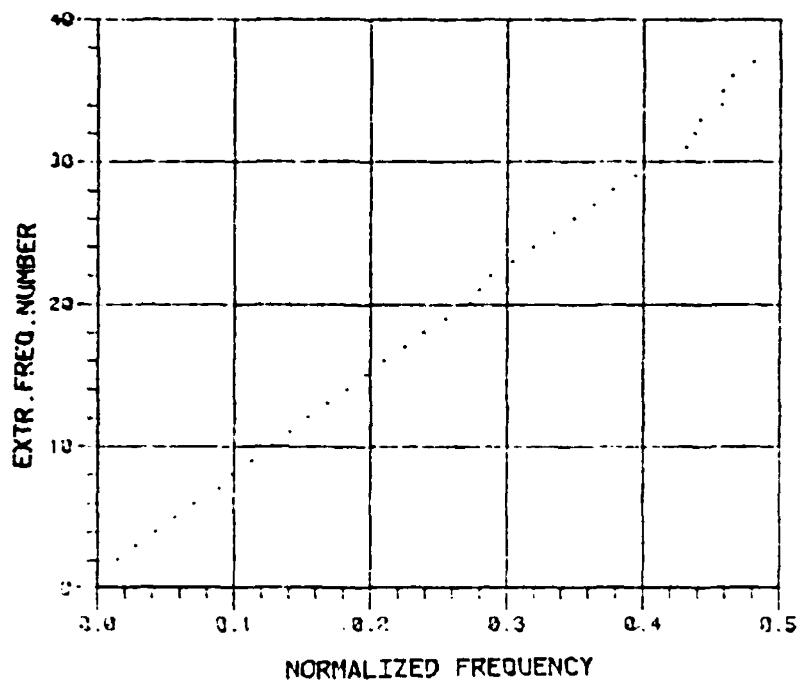


Fig. 22b New program, technique P: extremal frequencies of Design 4

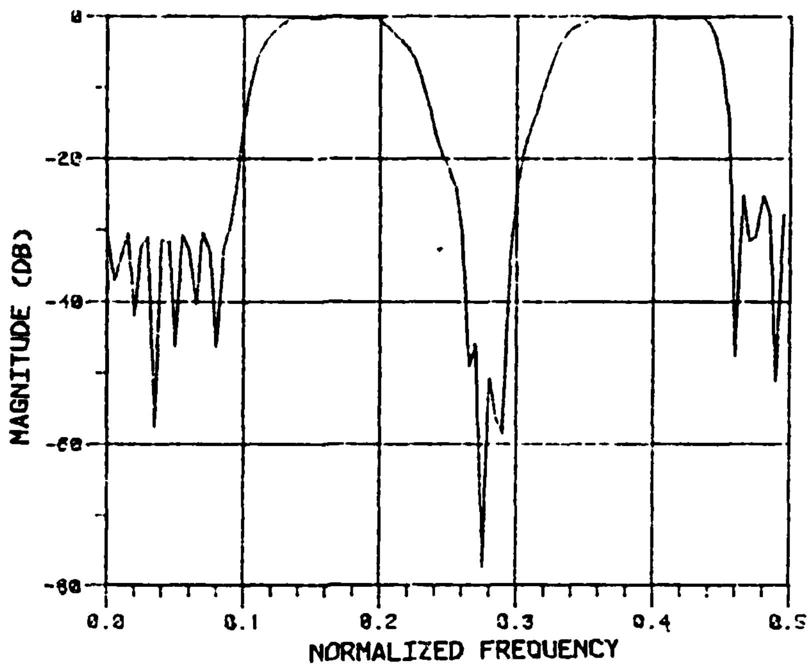


Fig. 22c New program, technique P:
$H^*(e^{j\omega})$ of Design 4 in dB

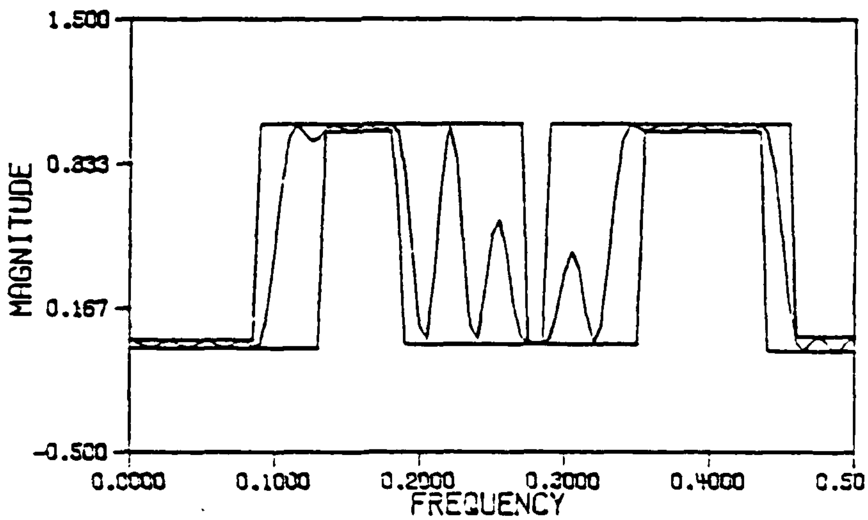


Fig. 23 CONRIP $H^*(e^{j\omega})$ of Design 4
(from [8], pp. 75)

and $D(e^{j\omega})$ for the new program when technique L, i.e., the multisegment piecewise linear simulation of the transition regions, is used. Technique L implies the subdivision of each transition region into several bands over which $D(e^{j\omega})$ has a prescribed slope. The values of $D(e^{j\omega})$ on each band are chosen to linearly approximate the filter prototypes. A criterion for the choice of the weights on each band, that seems very effective, is to assign the smallest weights to the internal sub-bands of the transition region where $D(e^{j\omega})$ is steepest. Greater weights should be assigned to the other sub-regions where the absolute value of the slope of $D(e^{j\omega})$ become smaller. For instance, if the transition region between the band B_i with weight α and the band B_{i+1} with weight β is divided into 5 sub-bands the preceding criterion can be expressed by the two following choices of the weights (it is assumed $\alpha \geq \beta$, without any loss of generality).

	Choice 1	Choice 2
Band 1	α	α
Subregion 1	$s\alpha$	0.1α
Subregion 2	$ss\alpha$	0.01α
Subregion 3	$sss\alpha$	0.001α
Subregion 4	$ss\beta$	0.01β
Subregion 5	$s\beta$	0.1β
Band 2	β	β

Choice 2 is a particular case of Choice 1 and it has been extensively used in the design examples presented in this section.

Choice 1 assumes $\alpha > s\alpha > ss\alpha > sss\alpha$ and $ss\beta < s\beta < \beta$ and it is recommended when Choice 1 gives poor results (for instance in Deisgn 4). A non-uniform (piecewise) constant choice of the weights in the transition regions seems to significantly contribute to the elimination of extremal frequencies on them. Technique P corresponds to assigning the values of the transition regions of the prototypes to the corresponding regions of $D(e^{j\omega})$. The weight assigned to a transition region with this technique should be the smallest weight still capable of giving monotonic behavior.

The order of the prototypes is a point that needs some comment. If the multiband is thought of as a cascade of prototypes, their order should be equal to the order of the multiband divided by the number of the prototypes. If instead the multiband is thought of as a parallel of prototypes their order should be equal to the order of the multiband. Actually the relationship between the multiband and the prototypes is not clear. Therefore, the question of the order of the prototypes doesn't have a definite answer. Fortunately, it appears that the order of the prototypes doesn't influence the result very much, probably because of the effect of the weight that helps keep $E(e^{j\omega})$ small over the transition regions. In the design examples shown in this section the prototypes were usually taken of the same order of the multiband. In Design 4, though, the multiband is of order 73 and the prototypes were taken of order 41. Prototypes of different orders, according to the empirical rules of Rabiner et al.[7]were tried, but no improvement over the other choices of the order was noticed.

The plots of this section show that technique P gives better monotonicity of the transition regions than technique L. However, the two techniques have comparable performance in terms of elimination of extremal frequencies from the transition regions. This is explained by the two following facts: every prototype induces extremal frequencies into the transition regions of the other prototypes, therefore, as mentioned earlier, extremal frequencies in the transition regions should be expected. Furthermore, technique L benefits from the non-uniform weights in the transition regions, while technique P, in the present implementation, doesn't have this feature. (This further modification has not been implemented because the results of technique P were already satisfactory).

To obtain a multiband filter with monotonic transition regions with the new program is very simple. Some common sense is needed to choose the weights of the transition regions, which is the only euristic part of the procedure. The rules are the following: if the initial weights give resonances, then increase their value; if the initial weights give monotonic behavior then try a smaller weight that might reduce the ripple. The criteria for the choice of the weights in the transition regions should be used in these changes of weight.

All the filter examples shown in this section were obtained on the first try, with the exception of the filter of Design 4 which required two attempts (it is not excluded that prototypes of order 73, instead of 41, would have given a satisfactory result on the first try).

CHAPTER 5

COMMENTS ABOUT CONRIP

This section presents the relationships between the new program discussed in Section 4 and the program CONRIP, written by M. T. McCallig [8]. CONRIP implements the design of FIR filters according to a "constrained ripple" formulation, due to B. J. Leon and M. T. McCallig [8]. Their formulation is the following: given continuous functions $U(e^{j\omega})$ and $L(e^{j\omega})$ on $[0, \pi]$ such that $U(e^{j\omega}) > L(e^{j\omega})$ find the polynomial

$$P(e^{j\omega}) = \sum_{K=0}^{M} a(K)\cos(K\omega) \text{ such that:}$$

(i) $L(e^{j\omega}) \le P(e^{j\omega}) \le U(e^{j\omega})$, $\omega \in [0, \pi]$

(ii) $P(e^{j\omega})$ is monotone on specified subintervals of $[0, \pi]$

(iii) $P(e^{j\omega})$ is the minimal order polynomial meeting conditions (i) and (ii).

It is worthwhile to note that the preceding formulation is not a traditional approximation problem (like (1.3) and (3.1)) and it is a full band formulation (like (3.1) and unlike (1.3)).

CONRIP allows the design of multiband filters that do not exhibit strictly monotonic behavior in the transition regions but which do not have resonances like the ones obtained with McClellan's program. The results of using CONRIP for the filters of Designs 1, 2, 3, 4 are shown in figures 11, 15, 19, and 23.

The similarity between the constrained ripple problem and the

Chebychev approximation problems (1.3) and (3.1) is greater than it might appear at first sight. In fact the computational solution of the constrained ripple problem is obtained with an algorithm (reminiscent of the second Remez algorithm) which searches for the polynomials tangent to $U(e^{j\omega})$ or $L(e^{j\omega})$ at their extrema. Two theorems of McCallig state precisely the similarity of the two methods.

Theorem 5.1 Let $H^*(e^{j\omega})$ be the solution to the Chebychev approximation problem (3.1). Let

$$p = \max_{\omega \in [0,\pi]} W(e^{j\omega}) \left| H^*(e^{j\omega}) - D(e^{j\omega}) \right| \tag{5.1}$$

The constrained ripple problem having

$$\begin{aligned} L(e^{j\omega}) &= D(e^{j\omega}) - p \\ U(e^{j\omega}) &= D(e^{j\omega}) + p \end{aligned} \tag{5.2}$$

admits unique solution $P(e^{j\omega}) = H^*(e^{j\omega})$.

Proof: [3], pp. 42. The converse of Theorem 5.1 is stated as follows.

Theorem 5.2: Let the boundary curves $U(e^{j\omega})$ and $L(e^{j\omega})$ be given such that the unique solution to the constrained ripple problem is $P(e^{j\omega})$. The Chebychev approximation problem (3.1), having

$$D(e^{j\omega}) = \frac{1}{2}(U(e^{j\omega}) + L(e^{j\omega}))$$

$$W(e^{j\omega}) = \frac{U(e^{j0}) - D(e^{j0})}{U(e^{j\omega}) - D(e^{j\omega})}$$

admits unique solution $H^*(e^{j\omega}) = P(e^{j\omega})$.

Note: The assumption $U(e^{j}) > L(e^{j})$ guarantees the denominator $U(e^{j}) - D(e^{j}) = \frac{1}{2}(U(e^{j}) - L(e^{j})) > 0$.

Proof: [8], pp. 42.

It is important to notice that Theorem 5.1 says that the approximation problem (3.1) needs to be solved in order to obtain the equivalent constrained ripple problem. (The two problems are said to be equivalent if they have the same solution). In fact the final deviation p (5.1) is not available before the solution of (3.1).

Theorem 5.2 similarly says that the constrained ripple problem needs to be solved in order to obtain the equivalent approximation problem. The order of the polynomial to use in (3.1) is not available without solving the constrained ripple problem. Some consequences of the above two theorems having practical interest are now discussed.

The following equalities

$$\begin{aligned} L(e^{j\omega}) &= D(e^{j\omega})\left[1 - \frac{1}{W(e^{j\omega})}\right] \\ U(e^{j\omega}) &= D(e^{j\omega})\left[1 + \frac{1}{W(e^{j\omega})}\right] \end{aligned} \tag{5.4}$$

can be used to solve via CONRIP the following Chebychev approximation problem:

$$\min_{M \in Z^{+}} \ \min_{\{a(K)\}_{K=0}^{M}} \ \max_{\omega \in [0,\pi]} \ W(e^{j\omega}) \left| D(e^{j\omega}) - H(e^{j\omega}) \right| \tag{5.5}$$

This last aspect of the equivalence between constrained ripple problems and approximation problems needs a comment.

Let

$$p(\beta) = \min_{\{a(K)\}_{K=0}^{M}} \max_{\omega \epsilon [0,\pi]} \beta W(e^{j\omega}) |D(e^{j\omega}) - H(e^{j\omega})| \tag{5.6}$$

with β real and positive. The best approximant $H^*(e^{j\omega})$ of (5.6) is independent of β, but the minimax error $p(\beta)$ is dependent on β. The constrained ripple formulation, as intuitively understood and as precisely stated in Theorem 5.1, is sensitive to the minimax error $p(\beta)$. Therefore the filters found via formulation (3.1) and the new program are independent of the weights. However, scaling the weights in formulation (5.5) implemented via CONRIP by (5.4), give different filters (even differences in the order of the filter must be expected).

The new formulation (3.1), on the converse, suggests different ways of using CONRIP. Perfect monotonicity of the transition regions should be obtained by picking $L(e^{j\omega})$ and $U(e^{j\omega})$ via (5.3) where the $D(e^{j\omega})$ is chosen with the help of the prototypes and the $W(e^{j\omega})$ is chosen with the criterion of Section 4. The new program presented in Section 4 can be used to implement the following problem which is related to a constrained ripple problem:

Given continuous functions $U(e^{j\omega})$ and $L(e^{j\omega})$ on $[0,\pi]$ such that $U(e^{j\omega}) > L(e^{j\omega})$, and a given order M, find the polynomial

$$P(e^{j\omega}) = \sum_{K=0}^{M} a(K) \cos (K\omega) \text{ such that:}$$

(i) $L(e^{j\omega}) - \varepsilon(M) \leq P(e^{j\omega}) \leq U(e^{j\omega}) + \varepsilon(M)$ for some

$\varepsilon(M) > 0 \quad \omega \in [0, \pi]$

(ii) $P(e^{j\omega})$ is monotone on specified subintervals of $[0,\pi]$.

It should be noted that the $\delta(M)$ of point (i) depends on M, and there exists an $\bar{M}$ such that $\forall M > \bar{M} \quad \delta(M) = 0$.

The experimental verification of the use of CONRIP to solve approximation problem (5.5) and of the use of the new program to solve problems similar to the constrained ripple problem was beyond the objective of this work. Nevertheless, such verification would be an interesting project.

CHAPTER 6

SUMMARY

The problem addressed in this work was finding a technique for the straightforward design in the frequency domain, using a minimax criterion of multiband linear phase FIR digital filters.

To the author's knowledge the only similar attempt has been the work of Rabiner, Kaiser and Schaffer [7]. Their work has the merit of having brought to general attention the problems arising in multiband FIR filter design with McClellan's program. However, their solution doesn't address the essence of the problem which lies in the inadequacy of Parks and McClellan's theoretical formulation of the filter design problem for the multiband filter case. The "strategies" proposed in [7] to design multiband filters are empirical and their application is generally not straightforward.

Chapter 2 presents an original analysis of Parks and McClellan's filter design formulation. It is clearly stated that its mathematical meaning over the band $[0,\pi]$ is that of an approximation problem not according to the Chebychev norm but according to a seminorm. The Parks and McClellan formulation is shown to be ideal from the filter design point of view for the high and low-pass filter cases. It is similarly shown that it is mathematically inadequate from the filter design point of view where there is more than one "don't care" band as in the multiband filter case.

Chapter 3 proposes a new formulation of the filter design problem. The new formulation corresponds to a minimization problem in Chebychev norm over the full band $[0, \pi]$. It requires the use of continuous desired functions $D(e^{j\omega})$. The Alternation Theorem and the second Remez algorithm still apply. The new formulation is immune from the drawbacks of the McClellan formulation and it is theoretically adequate for the multiband filter design problem.

Chapter 4 discusses an implementation of the new formulation based on McClellan's program. Considerations about the choice of $W(e^{j\omega})$ and $D(e^{j\omega})$ having practical interest are also introduced. The performance of the new program is compared to that of McClellan's program and McCallig's program CONRIP for the multiband filter case. It is emphasized that the new program seems to be the only one capable of giving strictly monotonic transition regions in a straightforward way without changing filter specifications.

Chapter 5 presents the relationships with McCallig's program CONRIP. The possibility of the use of CONRIP to implement a design criterion very close to the one presented in Chapter 3 and the possibility of the use of the new program to implement a design criterion similar to a constrained ripple problem are both introduced and discussed.

From the author's point of view McClellan's filter design formulation (1.3) is a particular case of the new formulation (3.1). This made it natural to incorporate McClellan's program as the core of a program implementing formulation (3.1). The practical result obtained in this thesis therefore is a kind of "extended McClellan's program" that is identical to the original one when appropriate, such as for the low and high-pass filters, or can be used for its new features (formulation (3.1)) when McClellan's program is not adequate, as in the design of multiband filters.

APPENDIX 1

Appendix 1 provides a user-oriented description of the new program discussed in Section 4. The program operates interactively and asks the user a sequence of questions part of which are derived from McClellan's program and part are original. The meaning of the questions in terms of McClellan's program will not be reviewed here and the word "standard" will be used in place of the actual answer for questions from McClellan's program. All the comments to the answers will be on the modifications for the new program. The questions will be numbered for convenience of reference.

1. TYPE FILTER ORDER

   Standard

2. ENTER FILTER TYPE

   Standard

3. ENTER NUMBER OF BANDS

   Enter the number of bands where $D(e^{j\omega})$ changes slope if the transition regions will be piecewise linearly simulated. Enter the total number of passband, stopband, and transition bands if the transition regions of the prototypes will be used.

4. TYPE GRID DENSITY

   Standard

5. ENTER IPRINT IPLOT

   Standard

6. ENTER LINE PRINT FLAG (0 - DO NOT PRINT, 1 - PRINT)

   Standard

7. STANDARD WEIGHT AND TARGET, TYPE 0, CUSTOM WEIGHT AND TARGET, TYPE 1

   0 selects the operation of the program as regular program of McClellan. 1 selects the "custom mode" operation, implementing formulation (3.1) (all the comments to the questions assume 1 is entered here)

8. TO CHANGE BAND EDGES TYPE 1, OTHERWISE TYPE 0

   The band edges previously entered will be kept if 0 is entered (this happens because the program can cyclically call itself). New band edges must be provided if 1 is entered.

9. ENTER THE BAND EDGES - - NO. = NBANDS

   Enter the sequence of the edges of the bands. Since the new program assumes full-band operation the edges corresponding to two contiguous bands have to be 1 sample apart. In order to facilitate this feature, the program just uses the edges 0, 0.5 and the even ones (the second edge, the fourth, and so on) to calculate the odd ones via an increment of 1 sample.

   Since the important information for this answer is the edge 0, the edge 0.5 and the even edges, the odd edges are usually assigned to dummy integers like 0 or 1(because they are convenient to type!).

10. FILTERS PROTOTYPE USED? NO = 0, YES = 1

    Enter 1 for technique P. Enter 0 if no filter prototype will be used.

11. ENTER SIMULATION FLAGS

    Question 11 appears only if 1 has been entered at question 10.

    Enter a vector associating a number with each band. If the band is a transition region the number associated must be the number of the file containing the impulse response of the prototype for that region. The number 0 has to be given for the pass and stop-bands.

12. ENTER ORDERS OF THE PROTOTYPES

This question appears only if 1 has been entered at question 10. Enter a vector associating a number with each band. If the band is a transition region the associated number must be the order of the prototypes used for it. Any number can be associated with the other bands. This feature allows the use of prototypes of different order.

13. PIECEWISE LINEAR DESIRED FUNCTION? YES = 1, NO = 0

Enter 1 for technique L. Enter 0 if technique L is not wanted. It should be pointed out that the current implementation allows also the use of technique L and technique P together, that is some transition regions can be taken from prototypes and some others can be piecewise linearly modeled.

14. ENTER DESIRED FUNCTION AT THE EDGES

This question appears only if 1 has been entered at question 13. Enter the values assumed by $D(e^{j\omega})$ at the band edges.

15. ENTER THE CONSTANT VALUES FOR EACH BAND

This question appears only if 0 has been entered at question 13. Enter 1 corresponding to the pass-bands, 0 corresponding to the stop-bands, and any number corresponding to the transition regions.

16. ENTER WEIGHT FACTORS FOR EACH BAND

Enter the weights corresponding to each band. It should be emphasized that the current implementation calls for uniform weights over each band. A piecewise-constant weight could be also obtained over a region by means of its subdivision into several bands.

Figure 24 shows an example of conversational terminal using the new program with technique P.

```
EXEC FIR.FOR,NEW:AG210.REL/SEARCH
LINK:   Loading
[LNKXCT FIR Execution]
    TYPE 1 TO CONTINUE,TYPE 0 TO STOP

 1
    TYPE FILTER ORDER

 75
    ENTER FILTER TYPE (1,2,3)

 1
    TYPE NO. OF BANDS

 5
    TYPE GRID DENSITY

 16
    ENTER IPRINT IPLOT

 1 1
    ENTER LINE PRINT FLAG (0=DO NOT PRINT,1=PRINT)
 1
    STANDARD WEIG.STARG. TYPE 0,CUSTOM W&T TYPE 1
 1
    TO CHANGE BAND EDGES TYPE 1,OTHERWISE TYPE 0

 1
    ENTER THE BAND EDGES--NO.=NBANDS

 0 0.14375973
 0 0.16532943
 0 0.37832451
 0 0.41679744
 0 0.5
   FILTERS PROTOTYPE USED?NO=0,YES=1

 1
   ENTER SIM. FLGS.

0 21 0 22 0
   ENTER ORDERS OF THE PROTOTYPES

0 75 0 75 0
   PIECEWISE LIN. DES. FUNC.?NO=0,YES=1

0
   ENTER THE CONSTANT VALUES FOR EACH BAND

1 1000 0 1000 1
   ENTER WEIGHT FACTORS FOR EACH BAND

0.04500009
0.02
.
0.015
0.02350273
   INPUT DATA IS COMPLETE
```

Fig. 24   Conversational terminal example using the new program: Design 1

APPENDIX 2

This appendix offers numerical examples of filters designed with techniques L and P. The examples correspond to the filters denoted Design 1, 2, 3, 4.

Both techniques require one to design filter prototypes. Technique L needs them as models for piecewise linearly simulating the transition regions. Technique P needs them to use the actual values of their transition regions into $D(e^{j\omega})$.

The prototypes are designed with McClellan's program. Their specifications are directly obtained from those of the multiband filter corresponding to them.

For every design example the information relative to the prototypes will be presented first, and then the information relative to the multiband filter.

(a) Technique L

Design 1

Prototype 1.1 (low-pass); Filter order = 75

Band no.	First Edge	Second Edge	$D(e^{j\omega})$	$W(e^{j\omega})$
1	0	0.16533943	1	0.04599809
2	0.1437593	0.5	0	1

Prototype 1.2 (high-pass) ; Filter order = 75

Band no.	First Edge	Second Edge	$D(e^{j\omega})$	$W(e^{j\omega})$
1	0	0.41679744	0	0.03853275
2	0.37032451	0.5	1	1

The transition region of Prototype 1.1 was modeled by means of 2 line segments. The one of Prototype 1.2 was modeled by means of 3 line segments.

Multiple passband-stopband filter 1

Band No.	1st Edge	2nd Edge	$D(e^{j\omega})$ (1st Edge)	$D(e^{j\omega})$ (2nd Edge)	$W(e^{j\omega})$
1	0	0.14375973	1	1	0.04588809
2	0	0.159	1	0.07	0.004588809
3	0	0.165	0.07	0	0.04588809
4	0	0.37032451	0	0	1
5	0	0.385	0	0.09	0.0383853275
6	0	0.4021199	0.09	0.91	0.003853275
7	0	0.41679744	0.91	1	0.03853275
8	0	0.5	1	1	0.03853275

Design 3

Prototype 3.1 (Low-pass); Filter order = 57

Band No.	1st Edge	2nd Edge	$D(e^{j\omega})$	$W(e^{j\omega})$
1	0	0.11018835	1	0.19386528
2	0.00820222	0.5	0	0.17459027

Prototype 3.2 (High-pass); Filter order = 57

Band No.	1st Edge	2nd Edge	$D(e^{j\omega})$	$W(e^{j\omega})$
1	0	0.26931373	0	0.17459027
2	0.20585967	0.5	1	1

Prototype 3.3 (Low-pass); Filter order = 57

Band No.	1st Edge	2nd Edge	$D(e^{j\omega})$	$W(e^{j\omega})$
1	0	0.37673551	1	1
2	0.31158715	0.5	0	0.18180259

Prototype 3.4 (High-pass); Filter order = 57

Band No.	1st Edge	2nd Edge	$D(e^{j\omega})$	$W(e^{j\omega})$
1	0	0.46299174	0	0.18180259
2	0.3892995	0.5	1	0.21319649

The transition regions of Prototypes 3.1, 3.2, 3.3, 3.4 were simulated by 3,3,5, and 3 line segments respectively.

Multiple passband-stopband filter 2

Band No.	1st Edge	2nd Edge	$D(e^{j\omega})$ (1st Edge)	$D(e^{j\omega})$ (2nd Edge)	$W(e^{j\omega})$
1	0	0.00820222	1	1	0.19386528
2	0	0.03439052	1	0.96	0.0193
3	0	0.04139057	0.96	0.91	0.0193
4	0	0.077	0.91	0.09	0.00174
5	0	0.084	0.09	0.04	0.0174
6	0	0.11018835	0.04	0	0.0174
7	0	0.20585967	0	0	0.17459027
8	0	0.216	0	0.065	0.017459027
9	0	0.2591734	0.065	0.935	0.001749027
10	0	0.26931373	0.935	1	0.01749027
11	0	0.31150715	1	1	1
12	0	0.31842266	1	0.953	0.01
13	0	0.3699	0.953	0.047	0.001
14	0	0.37673551	0.047	0	0.018180258
15	0	0.3892995	0	0	0.18180258

Band No.	1st Edge	2nd Edge	$D(e^{j\omega})$ (1st Edge)	$D(e^{j\omega})$ (2nd Edge)	$W(e^{j\omega})$
16	0	0.4045	0	0.006	0.018180258
17	0	0.44779124	0.006	0.994	0.0018180258
18	0	0.46299174	0.994	1	0.02139649
19	0	0.5	1	1	0.21319649

(b) Technique P

Design 2

Prototype 2.1 (High-pass) ; Filter order = 27

Band No.	1st Edge	2nd Edge	$D(e^{j\omega})$	$W(e^{j\omega})$
1	0	0.22754845	0	1
2	0.0728033	0.5	1	0.1616032

Prototype 2.2 (Low-pass) ; Filter order = 43

Band No.	1st Edge	2nd Edge	$D(e^{j\omega})$	$W(e^{j\omega})$
1	0	0.3445328	1	0.1616032
2	0.29030124	0.5	0	0.0034068

The prototypes used in this example are of different arders (their orders correspond to those suggested by Rabiner _et_ _al_. [7]).

McClellan's program stored the coefficients of Prototypes 2.1 and 2.2 in disk-files number 21 and 22 respectively.

Multiple passband-stopband filter 2

Band No.	Simulation Flags	Prototype Orders	1st Edge	2nd Edge	$D(e^{j\omega})$	$W(e^{j\omega})$
1	0	0	0	0.07280333	0	1
2	21	27	0	0.22754845	1000	0.01616032
3	0	0	0	0.29030124	1	0.1616032
4	22	43	0	0.3445328	1000	0.00340608
5	0	0	0	0.5	0	0.00340608

The dummy value 1000 in the transition regions of $D(e^{j\omega})$ has been used to indicate the use of Prototypes.

Design 4

Prototype 4.1 (High-pass) ; Filter order = 41

Band No.	1st Edge	2nd Edge	$D(e^{j\omega})$	$W(e^{j\omega})$
1	0	0.13199438	0	0.0959953
2	0.08886197	0.5	1	0.11187421

Prototype 4.2 (Low-pass) ; Filter order = 41

Band No.	1st Edge	2nd Edge	$D(e^{j\omega})$	$W(e^{j\omega})$
1	0	0.27193968	0	0.11187421
2	0.18550831	0.5	1	1

Prototype 4.3 (High-pass) ; Filter order = 41

Band No.	1st Edge	2nd Edge	$D(e^{j\omega})$	$W(e^{j\omega})$
1	0	0.35373202	0	1
2	0.28819105	0.5	1	0.11177379

Prototype 4.4 (Low-pass).; Filter order = 41

Band No.	1st Edge	2nd Edge	$D(e^{j\omega})$	$W(e^{j\omega})$
1	0	0.45732656	1	0.11177379
2	0.43737502	0.5	0	0.05401694

The prototypes in this case were taken of orders different from the order of the multiple passband-stopband filter. McClellan's program wrote the coefficients of Prototypes 4.1, 4.2, 4.3, 4.4 in disk-files number 21, 22, 23, 24 respectively.

Multiple passband-stopband filter 4

Band No.	Simulation Flags	Prototypes Order	1st Edge	2nd Edge	$D(e^{j\omega})$	$W(e^{j\omega})$
1	0	0	0	0.08886197	0	0.0959953
2	21	41	0	0.13199438	1000	0.09
3	0	0	0	0.18550831	1	0.11187421
4	22	41	0	0.27193968	1000	0.11
5	0	0	0	0.28819105	0	1
6	23	41	0	0.35373202	1000	0.11
7	0	0	0	0.43737502	1	0.11177379
8	24	41	0	0.45732656	1000	0.05
9	0	0	0	0.5	0	0.05401695

APPENDIX 3

This Appendix contains the computer printouts with all the numerical information referring to the filters shown in the plots of this work.

In order to facilitate the association of the plots with the information tables corresponding to them, the tables of this Appendix are labeled with the same number as the figures containing the plot to which they refer. Therefore, Figure 4 corresponds to Table 4, Fig. 13 corresponds to Table 13, and so on.

It should finally be noticed that the printouts of the new program used with technique L under the voice "DESIRED VALUE" report the slopes of $D(e^{j\omega})$ over each band.

TABLE 5

```
************************************************************************

                    FINITE IMPULSE RESPONSE (FIR)
                    LINEAR PHASE DIGITAL FILTER DESIGN
                    REMEZ EXCHANGE ALGORITHM

                    BANDPASS FILTER

               FILTER LENGTH =  30

               ***** IMPULSE RESPONSE *****
               H(  1)= -0.12685416E-02 = H(  30)
               H(  2)=  0.25128179E-02 = H(  29)
               H(  3)=  0.37103542E-02 = H(  28)
               H(  4)= -0.48509456E-02 = H(  27)
               H(  5)= -0.72986242E-02 = H(  26)
               H(  6)=  0.97128045E-02 = H(  25)
               H(  7)=  0.13543584E-01 = H(  24)
               H(  8)= -0.17527104E-01 = H(  23)
               H(  9)= -0.23721668E-01 = H(  22)
               H( 10)=  0.30817019E-01 = H(  21)
               H( 11)=  0.41962352E-01 = H(  20)
               H( 12)= -0.57338704E-01 = H(  19)
               H( 13)= -0.85472318E-01 = H(  18)
               H( 14)=  0.14677296E+00 = H(  17)
               H( 15)=  0.44968437E+00 = H(  16)

                    BAND  1         BAND  2          BAND
LOWER BAND EDGE    0.000000000    0.300000000
UPPER BAND EDGE    0.200000000    0.500000000
DESIRED VALUE      1.000000000    0.000000000
WEIGHTING          1.000000000    1.000000000
DEVIATION          0.002060720    0.002060720
DEVIATION IN DB  -53.719620595  -53.719620595

EXTREMAL FREQUENCIES
   0.0000000   0.0375000   0.0750000   0.1083333   0.1416667
   0.1708333   0.1916667   0.2000000   0.3000000   0.3083333
   0.3291667   0.3562500   0.3875000   0.4187500   0.4500000
   0.4833333
```

TABLE 6

```
****************************************************************

                FINITE IMPULSE RESPONSE (FIR)
                LINEAR PHASE DIGITAL FILTER DESIGN
                REMEZ EXCHANGE ALGORITHM

                BANDPASS FILTER

             FILTER LENGTH =  30

             ***** IMPULSE RESPONSE *****
             H(  1)= -0.13156184E-02 = H(  30)
             H(  2)=  0.21666221E-02 = H(  29)
             H(  3)=  0.38458906E-02 = H(  28)
             H(  4)= -0.43153422E-02 = H(  27)
             H(  5)= -0.75892045E-02 = H(  26)
             H(  6)=  0.69081429E-02 = H(  25)
             H(  7)=  0.14064823E-01 = H(  24)
             H(  8)= -0.16435316E-01 = H(  23)
             H(  9)= -0.24498261E-01 = H(  22)
             H( 10)=  0.29490709E-01 = H(  21)
             H( 11)=  0.43012072E-01 = H(  20)
             H( 12)= -0.55859960E-01 = H(  19)
             H( 13)= -0.86763254E-01 = H(  18)
             H( 14)=  0.14524734E+00 = H(  17)
             H( 15)=  0.45094332E+00 = H(  16)

                      BAND  1         BAND  2          BAND  3
LOWER BAND EDGE     0.000000000     0.202083333     0.302083333
UPPER BAND EDGE     0.200000000     0.300000000     0.500000000
DESIRED VALUE       0.000000000   -13.000000000     0.000000000
WEIGHTING           1.000000000     0.000010000     1.000000000

EXTREMAL FREQUENCIES
   0.0000000   0.0395833   0.0750000   0.1104167   0.1416667
   0.1708333   0.1916667   0.2000000   0.3020833   0.3104167
   0.3312500   0.3583333   0.3875000   0.4187500   0.4520833
   0.4833333
```

TABLE 7

```
***************************************************************************

                  FINITE IMPULSE RESPONSE (FIR)
                  LINEAR PHASE DIGITAL FILTER DESIGN
                  REMEZ EXCHANGE ALGORITHM

                  BANDPASS FILTER

             FILTER LENGTH =  40

             ***** IMPULSE RESPONSE *****
             H(  1)=  0.13185830E-02 = H(  40)
             H(  2)=  0.84947953E-03 = H(  39)
             H(  3)= -0.11423889E-02 = H(  38)
             H(  4)= -0.51609168E-03 = H(  37)
             H(  5)= -0.88455008E-03 = H(  36)
             H(  6)= -0.17963831E-02 = H(  35)
             H(  7)=  0.34925023E-03 = H(  34)
             H(  8)= -0.15960056E-02 = H(  33)
             H(  9)=  0.18752870E-02 = H(  32)
             H( 10)=  0.79289268E-03 = H(  31)
             H( 11)=  0.11679032E-02 = H(  30)
             H( 12)=  0.32317824E-02 = H(  29)
             H( 13)= -0.31413921E-02 = H(  28)
             H( 14)=  0.84815417E-03 = H(  27)
             H( 15)= -0.71878533E-02 = H(  26)
             H( 16)= -0.10580362E-01 = H(  25)
             H( 17)=  0.29695523E-02 = H(  24)
             H( 18)= -0.27065245E-01 = H(  23)
             H( 19)=  0.13483039E+00 = H(  22)
             H( 20)=  0.43368320E+00 = H(  21)

                         BAND  1         BAND  2         BAND  3
LOWER BAND EDGE      0.000000000     0.101562500     0.401562500
UPPER BAND EDGE      0.100000000     0.400000000     0.500000000
DESIRED VALUE        0.000000000    -3.333333333     0.300000000
WEIGHTING            1.000000000     1.000000000     1.000000000

EXTREMAL FREQUENCIES
   0.0000000   0.0390625   0.0671875   0.0890625   0.1000000
   0.1109375   0.1312500   0.1562500   0.1812500   0.2078125
   0.2328125   0.2593750   0.2859375   0.3125000   0.3375000
   0.3640625   0.3875000   0.4015625   0.4140625   0.4468750
   0.4829125
```

TABLE 8

```
************************************************************

                 FINITE IMPULSE RESPONSE (FIR)
                 LINEAR PHASE DIGITAL FILTER DESIGN
                 REMEZ EXCHANGE ALGORITHM

                 BANDPASS FILTER

             FILTER LENGTH =  75

             ***** IMPULSE RESPONSE *****
             H(  1) =  0.15065669E-01 = H( 75)
             H(  2) = -0.11423129E-02 = H( 74)
             H(  3) =  0.24299351E-01 = H( 73)
             H(  4) = -0.92821304E-02 = H( 72)
             H(  5) =  0.12967521E-01 = H( 71)
             H(  6) = -0.79249574E-02 = H( 70)
             H(  7) = -0.18112947E-01 = H( 69)
             H(  8) =  0.16531987E-01 = H( 68)
             H(  9) = -0.27209825E-01 = H( 67)
             H( 10) =  0.24729657E-01 = H( 66)
             H( 11) =  0.37391297E-02 = H( 65)
             H( 12) = -0.22363515E-01 = H( 64)
             H( 13) =  0.25119470E-01 = H( 63)
             H( 14) = -0.44441976E-01 = H( 62)
             H( 15) =  0.23777066E-01 = H( 61)
             H( 16) =  0.12101761E-01 = H( 60)
             H( 17) = -0.20630934E-01 = H( 59)
             H( 18) =  0.52622551E-01 = H( 58)
             H( 19) = -0.58728325E-01 = H( 57)
             H( 20) =  0.11347437E-01 = H( 56)
             H( 21) =  0.33360546E-02 = H( 55)
             H( 22) = -0.40325350E-01 = H( 54)
             H( 23) =  0.85432297E-01 = H( 53)
             H( 24) = -0.44279378E-01 = H( 52)
             H( 25) =  0.18534946E-01 = H( 51)
             H( 26) =  0.43617314E-02 = H( 50)
             H( 27) = -0.92201247E-01 = H( 49)
             H( 28) =  0.78889224E-01 = H( 48)
             H( 29) = -0.34093485E-01 = H( 47)
             H( 30) =  0.43011208E-01 = H( 46)
             H( 31) =  0.72617342E-01 = H( 45)
             H( 32) = -0.12496753E+00 = H( 44)
             H( 33) =  0.36839457E-01 = H( 43)
             H( 34) = -0.84332996E-01 = H( 42)
             H( 35) = -0.35504666E-01 = H( 41)
             H( 36) =  0.32626794E+00 = H( 40)
             H( 37) = -0.40469027E-02 = H( 39)
             H( 38) =  0.60205355E+00 = H( 38)

                        BAND  1        BAND  2        BAND  3
LOWER BAND EDGE      0.000000000    0.165339430    0.416797440
UPPER BAND EDGE      0.143759730    0.373324510    0.500000000
DESIRED VALUE        1.000000000    0.000000000    1.000000000
WEIGHTING            0.245388090    1.000000000    0.338532750
DEVIATION            0.071513966    0.003295401    0.085522085
DEVIATION IN DB    -22.975833579  -49.041834250  -21.358434340

EXTREMAL FREQUENCIES
   0.0131579   0.0271382   0.0411184   0.0550987   0.0690789
   0.0830592   0.0972395   0.1110197   0.1250000   0.1373355



   0.1437597   0.1553394   0.1696289   0.1764322   0.1958986
   0.1965394   0.2081725   0.2204381   0.2319513   0.2442868
   0.2560223   0.2649579   0.2821157   0.2936249   0.3059644
   0.3183730   0.3296131   0.3413263   0.3520171   0.3513631
   0.3676421   0.3773245   0.4217317   0.4324224   0.4455903
   0.4567332   0.4727155   0.4858764   0.5000000
```

TABLE 9

```
****************************************************************

            FINITE IMPULSE RESPONSE (FIR)
            LINEAR PHASE DIGITAL FILTER DESIGN
            REMEZ EXCHANGE ALGORITHM

            BANDPASS FILTER

        FILTER LENGTH =  75

        ***** IMPULSE RESPONSE *****
        H(  1)=  0.11877904E-01 = H(  75)
        H(  2)=  0.20746998E-02 = H(  74)
        H(  3)=  0.21593887E-01 = H(  73)
        H(  4)= -0.37731763E-02 = H(  72)
        H(  5)=  0.84473692E-02 = H(  71)
        H(  6)= -0.11012091E-01 = H(  70)
        H(  7)= -0.92722334E-02 = H(  69)
        H(  8)=  0.12157133E-02 = H(  68)
        H(  9)= -0.40313453E-02 = H(  67)
        H( 10)=  0.11510730E-01 = H(  66)
        H( 11)=  0.29104280E-02 = H(  65)
        H( 12)= -0.52381627E-02 = H(  64)
        H( 13)= -0.23280096E-02 = H(  63)
        H( 14)= -0.15925940E-01 = H(  62)
        H( 15)=  0.22222048E-02 = H(  61)
        H( 16)=  0.11596988E-01 = H(  60)
        H( 17)=  0.90277606E-03 = H(  59)
        H( 18)=  0.12372094E-01 = H(  58)
        H( 19)= -0.16995603E-01 = H(  57)
        H( 20)= -0.16801813E-01 = H(  56)
        H( 21)=  0.10191932E-02 = H(  55)
        H( 22)= -0.98078090E-02 = H(  54)
        H( 23)=  0.30937333E-01 = H(  53)
        H( 24)=  0.59254601E-02 = H(  52)
        H( 25)= -0.13458892E-01 = H(  51)
        H( 26)= -0.22436902E-02 = H(  50)
        H( 27)= -0.49243857E-01 = H(  49)
        H( 28)=  0.11637873E-01 = H(  48)
        H( 29)=  0.26744696E-01 = H(  47)
        H( 30)=  0.10465313E-01 = H(  46)
        H( 31)=  0.59024254E-01 = H(  45)
        H( 32)= -0.70872283E-01 = H(  44)
        H( 33)= -0.45562972E-01 = H(  43)
        H( 34)= -0.29541572E-01 = H(  42)
        H( 35)= -0.54702834E-01 = H(  41)
        H( 36)=  0.32418225E+00 = H(  40)
        H( 37)=  0.56971816E-01 = H(  39)
        H( 38)=  0.52162215E+00 = H(  38)
```

	BAND 1	BAND 2	BAND 3	BAND 4
LOWER BAND EDGE	0.000000030	0.144582098	0.159822368	0.165822368
UPPER BAND EDGE	0.143759730	0.159000000	0.165000000	0.370324514
DESIRED VALUE	0.000000000	-61.022540939	-11.666666667	0.000000000
WEIGHTING	0.045888090	0.004588829	2.004588829	1.000000000

	BAND 5	BAND 6	BAND 7	BAND 8
LOWER BAND EDGE	0.371146875	0.385822368	0.402942268	0.417619829
UPPER BAND EDGE	0.385200000	0.402119000	0.416797448	0.500000000
DESIRED VALUE	6.132674275	47.697476037	5.131317732	0.000000000
WEIGHTING	0.238527500	0.003853275	0.338532750	0.338532750

EXTREMAL FREQUENCIES

0.0115132	0.0263158	0.0411184	0.0559987	0.0690739
0.0838592	0.0970395	0.1118197	0.1250000	0.1373355
0.1437597	0.1658224	0.1591118	0.1765132	0.1363916
0.1973724	0.2085855	0.2209211	0.2332566	0.2455921
0.2579276	0.2702632	0.2825987	0.2949342	0.3072697
0.3196053	0.3319408	0.3434539	0.3541447	0.3648355
0.3703245	0.4029423	0.4167973	0.4291330	0.4439356
0.4579159	0.4718961	0.4858764	0.5000000	

TABLE 10

****

```
              FINITE IMPULSE RESPONSE (FIR)
              LINEAR PHASE DIGITAL FILTER DESIGN
              REMEZ EXCHANGE ALGORITHM

              BANDPASS FILTER

          FILTER LENGTH =  75

          ***** IMPULSE RESPONSE *****
          H(  1)=  0.10714460E-01 = H(  75)
          H(  2)=  0.32081376E-02 = H(  74)
          H(  3)=  0.19413180E-01 = H(  73)
          H(  4)= -0.16231102E-02 = H(  72)
          H(  5)=  0.77142632E-02 = H(  71)
          H(  6)= -0.10903163E-01 = H(  70)
          H(  7)= -0.66070236E-02 = H(  69)
          H(  8)= -0.24549452E-02 = H(  68)
          H(  9)=  0.17530442E-03 = H(  67)
          H( 10)=  0.86043161E-02 = H(  66)
          H( 11)=  0.36283944E-02 = H(  65)
          H( 12)= -0.50790241E-02 = H(  64)
          H( 13)= -0.71931121E-02 = H(  63)
          H( 14)= -0.98243626E-02 = H(  62)
          H( 15)= -0.19994325E-02 = H(  61)
          H( 16)=  0.12156566E-01 = H(  60)
          H( 17)=  0.49171109E-02 = H(  59)
          H( 18)=  0.56973981E-02 = H(  58)
          H( 19)= -0.59075154E-02 = H(  57)
          H( 20)= -0.21640208E-01 = H(  56)
          H( 21)=  0.11194784E-02 = H(  55)
          H( 22)= -0.36438931E-02 = H(  54)
          H( 23)=  0.21899086E-01 = H(  53)
          H( 24)=  0.18525902E-01 = H(  52)
          H( 25)= -0.18525993E-01 = H(  51)
          H( 26)= -0.27428000E-02 = H(  50)
          H( 27)= -0.41906998E-01 = H(  49)
          H( 28)=  0.53235060E-03 = H(  48)
          H( 29)=  0.37860741E-01 = H(  47)
          H( 30)=  0.53053126E-02 = H(  46)
          H( 31)=  0.57329729E-01 = H(  45)
          H( 32)= -0.61466346E-01 = H(  44)
          H( 33)= -0.57722297E-01 = H(  43)
          H( 34)= -0.98749716E-02 = H(  42)
          H( 35)= -0.69451725E-01 = H(  41)
          H( 36)=  0.32075917E+00 = H(  40)
          H( 37)=  0.67543831E-01 = H(  39)
          H( 38)=  0.53921176E+00 = H(  38)

                       BAND  1        BAND  2         BAND  3         BAND  4
LOWER BAND EDGE     0.000000000    0.144582098     0.166156311     0.371146478
UPPER BAND EDGE     0.143759730    0.165333943     0.370324510     0.416797440
DESIRED VALUE       1.000000000  100.000000000     0.000000000   100.000000000
WEIGHTING           0.045000090    0.040000000     1.000000000     0.030000000

                       BAND  5        BAND
LOWER BAND EDGE     0.417619800
UPPER BAND EDGE     0.500000000
DESIRED VALUE       1.000000000
WEIGHTING           0.038532750


EXTREMAL FREQUENCIES
     0.0123355   0.0271382   0.0411134   0.0550987   0.0690739
     0.0830592   0.0970395   0.1110197   0.1250000   0.1373355
     0.1437597   0.1661563   0.1694458   0.1768471   0.1867155
     0.1974063   0.2089195   0.2212552   0.2335905   0.2459260
     0.2582616   0.2705971   0.2829326   0.2960905   0.3084260
     0.3207616   0.3322737   0.3446123   0.3553010   0.3651595
     0.3703245   0.3958179   0.4130877   0.4283120   0.4431132
     0.4570935   0.4718961   0.4858754   0.5000000
```

TABLE 11

PURDUE UNIVERSITY - SCHOOL OF ELECTRICAL ENGINEERING
CIRCUITS AND SYSTEMS GROUP

[illegible] RIPPLE NONRECURSIVE DIGITAL FILTER DESIGN

****** SPECIFICATIONS ******

BANDPASS FILTER
LENGTH 75
TOUCH UPPER BOUND FIRST

****** PIECEWISE LINEAR BOUNDARY ******

BREAKPOINTS

UPPER BOUND		LOWER BOUND	
FREQUENCY	RESPONSE	FREQUENCY	RESPONSE
0	1.072000	0.	.928000
.165000	1.072000	.143000	.928000
.167000	.004000	.143700	-.004000
.362000	.004000	.417000	-.004000
.370400	1.086000	.417500	.914500
.500000	1.086000	.500000	.914500

****** BOUNDARY INTERCEPT FREQUENCIES ******

UPPER BOUNDARY

0	.02775	.05550	.08325	.11100	.13725
.17560	.19[illegible]25	.22050	.24[illegible]25	.27025	.29550
.3[illegible]	.34100	.36500	.40300	.43000	.45800
.48[illegible]					

LOWER BOUNDARY

.01[illegible]	.04150	.06925	.09725	.12475	.16800
.1[illegible]	.20350	.23300	.25775	.28300	.30775
.3[illegible]	.35500	.37275	.41750	.44400	.47200
.50000					

15 ITERATIONS TIME 16.2 SECONDS

LENGTH 75 BANDPASS FILTER
TOUCH UPPER BOUND FIRST

****** PULSE RESPONSE ******

HEXADEC				DECIMAL		HEXADEC		
0	3	5	H(1)=	.13720550E 01	=H(75)	0	3	5
0	0	9	H(2)=	.21763360E 02	=H(74)	0	0	9
0	5	E	H(3)=	.24294198E-01	=H(73)	0	5	E
8	0	C	H(4)=	-.31405080E-02	=H(72)	8	0	C
0	2	6	H(5)=	.[illegible]33E-02	=H(71)	0	2	6
8	2	5	H(6)=	-.[illegible]7384E-02	=H(70)	8	2	5
8	2	3	H(7)=	-.9215071[illegible]E-02	=H(69)	8	2	3
0	1	0	H(8)=	.1261775[illegible]E-02	=H(68)	0	1	0
8	0	F	H(9)=	-.[illegible]0603[illegible] -02	=H(67)	8	0	F
0	3	5	H(10)=	.13[illegible]886E-01	=H(66)	0	3	5
0	0	F	H(11)=	.41084008E-02	=H(65)	0	0	F
8	2	1	H(12)=	-.86640011E-02	=H(64)	8	2	1
0	0	2	H(13)=	.5661305[illegible]E-02	=H(63)	0	0	2
8	4	1	H(14)=	-.167[illegible]16E 01	=H(62)	8	4	1
0	1	3	H(15)=	.51032627E-02	=H(61)	0	1	3
0	3	3	H(16)=	.1314453[illegible]E-01	=H(60)	0	3	3
0	0	2	H(17)=	.51675774E-03	=H(59)	0	0	2
0	3	C	H(18)=	.1562541[illegible]6E-01	=H(58)	0	3	C
8	4	9	H(19)=	-.18[illegible]E-01	=H(57)	8	4	9
8	3	7	H(20)=	-.141[illegible]330E-01	=H(56)	8	3	7
0	0	9	H(21)=	.250[illegible]661E-02	=H(55)	0	0	9
8	2	4	H(22)=	-.931[illegible]E-02	=H(54)	8	2	4
0	8	9	H(23)=	.352[illegible]E-01	=H(53)	0	8	9
0	1	A	H(24)=	.670[illegible]3437E-02	=H(52)	0	1	A
8	2	B	H(25)=	-.111[illegible]931E-01	=H(51)	8	2	B
8	0	4	H(26)=	-.125[illegible]21[illegible]E-02	=H(50)	8	0	4
8	C	4	H(27)=	-.50103[illegible]E-01	=H(49)	8	C	4
0	4	0	H(28)=	.16602337E-01	=H(48)	0	4	0
0	6	1	H(29)=	.24920912E-01	=H(47)	0	6	1
0	3	3	H(30)=	.131259[illegible]E-01	=H(46)	0	3	3
0	E	A	H(31)=	.6020316[illegible]E-01	=H(45)	0	E	A
9	1	C	H(32)=	-.7316779[illegible]E-01	=H(44)	9	1	C
8	9	E	H(33)=	-.40834511E-01	=H(43)	8	9	E
8	5	7	H(34)=	-.22441468E-01	=H(42)	8	5	7
8	F	0	H(35)=	-.6174966[illegible]E-01	=H(41)	8	F	0
4	0	9	H(36)=	.30647178	=H(40)	4	0	9
0	D	4	H(37)=	.5415[illegible]265E-01	=H(39)	0	D	4
7	F	F	H(38)=	.[illegible]	=H(38)	7	F	F

TABLE 12

```
*****************************************************************************

                 FINITE IMPULSE RESPONSE (FIR)
                 LINEAR PHASE DIGITAL FILTER DESIGN
                 REMEZ EXCHANGE ALGORITHM

                 BANDPASS FILTER

            FILTER LENGTH =  43

            ***** IMPULSE RESPONSE *****
            H(  1) = -0.47725953E-02 = H(  43)
            H(  2) = -0.46714223E-02 = H(  42)
            H(  3) =  0.15323213E-01 = H(  41)
            H(  4) =  0.11555324E-01 = H(  40)
            H(  5) =  0.45492205E-02 = H(  39)
            H(  6) = -0.24466085E-01 = H(  38)
            H(  7) = -0.63199661E-01 = H(  37)
            H(  8) = -0.22153602E-01 = H(  36)
            H(  9) =  0.84450876E-01 = H(  35)
            H( 10) =  0.10757150E+00 = H(  34)
            H( 11) =  0.39417526E-01 = H(  33)
            H( 12) = -0.88695822E-01 = H(  32)
            H( 13) = -0.21962374E+00 = H(  31)
            H( 14) = -0.11773340E+00 = H(  30)
            H( 15) =  0.18300227E+00 = H(  29)
            H( 16) =  0.27713146E+00 = H(  28)
            H( 17) =  0.13421407E+00 = H(  27)
            H( 18) = -0.63452570E-01 = H(  26)
            H( 19) = -0.37983334E+00 = H(  25)
            H( 20) = -0.43255067E+00 = H(  24)
            H( 21) =  0.29900782E+00 = H(  23)
            H( 22) =  0.69941633E+00 = H(  22)
```

	BAND 1	BAND 2	BAND 3
LOWER BAND EDGE	0.000000000	0.227548450	0.344532800
UPPER BAND EDGE	0.172803330	0.299301240	0.500000000
DESIRED VALUE	0.000000000	1.000000000	0.000000000
WEIGHTING	1.000000000	0.151683270	0.003436060
DEVIATION	0.000075975	0.000475886	0.022578758
DEVIATION IN DB	-82.289902144	-66.449991169	-32.925998928

EXTREMAL FREQUENCIES

0.0000000	0.0177455	0.0320775	0.0463750	0.0596591
0.0946023	0.1728033	0.2275484	0.2333594	0.2349121
0.2510952	0.2544433	0.2772644	0.2872075	0.2993012
0.3445324	0.3530555	0.3729419	0.39798[illegible]5	0.4212373
0.4460855	0.4737942	0.5000000		

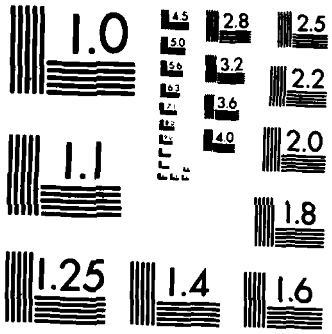
AD-A125 907 FREQUENCY DOMAIN DESIGN OF MULTIBAND FINITE IMPULSE RESPONSE DIGITAL FILT..(U) ILLINOIS UNIV AT URBANA COORDINATED SCIENCE LAB G CORTELAZZO DEC 80 R-906 2/2

UNCLASSIFIED AFOSR-79-0029 F/G 9/5 NL

END

FILMED

DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

TABLE 13

```
****************************************************************************

                    FINITE IMPULSE RESPONSE (FIR)
                    LINEAR PHASE DIGITAL FILTER DESIGN
                    REMEZ EXCHANGE ALGORITHM

                    BANDPASS FILTER

               FILTER LENGTH =  43

               ***** IMPULSE RESPONSE *****
               H(  1)=  0.12072547E-02 = H(  43)
               H(  2)= -0.76080397E-02 = H(  42)
               H(  3)=  0.97672149E-02 = H(  41)
               H(  4)= -0.23594813E-02 = H(  40)
               H(  5)=  0.13253569E-03 = H(  39)
               H(  6)=  0.82992102E-02 = H(  38)
               H(  7)= -0.13247947E-01 = H(  37)
               H(  8)= -0.12975836E-01 = H(  36)
               H(  9)=  0.16036997E-01 = H(  35)
               H( 10)=  0.27420275E-02 = H(  34)
               H( 11)=  0.25691290E-02 = H(  33)
               H( 12)=  0.26783141E-01 = H(  32)
               H( 13)= -0.26957126E-01 = H(  31)
               H( 14)= -0.48313597E-01 = H(  30)
               H( 15)=  0.25100772E-01 = H(  29)
               H( 16)=  0.90745242E-02 = H(  28)
               H( 17)=  0.10208402E-01 = H(  27)
               H( 18)=  0.11720360E+00 = H(  26)
               H( 19)= -0.39957339E-01 = H(  25)
               H( 20)= -0.27429029E+00 = H(  24)
               H( 21)=  0.22937424E-01 = H(  23)
               H( 22)=  0.34781566E+00 = H(  22)

                     BAND  1        BAND  2        BAND  3        BAND  4
LOWER BAND EDGE    0.000000000    0.074223755    0.110420455    0.122420455
UPPER BAND EDGE    0.072003300    0.109000000    0.121300000    0.179351750
DESIRED VALUE      0.000000000    0.676857283    4.166666667   14.583967062
WEIGHTING          1.000000000    0.010000000    0.001000000    0.000160320
                                                      5
                     BAND  5        BAND  6        BAND  7        BAND  8
LOWER BAND EDGE    0.180772205    0.192772205    0.228964905    0.291721695
UPPER BAND EDGE    0.191351750    0.227548450    0.290301240    0.309000000
DESIRED VALUE      4.166666667    0.676857283    0.000000000   -3.048330478
WEIGHTING          0.001603200    0.016032000    0.169320000    0.003460000

                     BAND  9        BAND 10        BAND
LOWER BAND EDGE    0.310420455    0.345953255
UPPER BAND EDGE    0.344532800    0.500000000
DESIRED VALUE    -26.538859870    0.000000000
WEIGHTING          0.000346000    0.003460000

EXTREMAL FREQUENCIES
     0.0000000   0.0184659   0.0369318   0.0539773   0.0667614
     0.0720033   0.1090000   0.1927722   0.2003972   0.2346507
     0.2400553   0.2644003   0.2786648   0.2886240   0.3090000
     0.3459533   0.3558964   0.3772033   0.3999305   0.4254987
     0.4496464   0.4752146   0.5000000
```

TABLE 14

```
*****************************************************************

          FINITE IMPULSE RESPONSE (FIR)
          LINEAR PHASE DIGITAL FILTER DESIGN
          REMEZ EXCHANGE ALGORITHM

          BANDPASS FILTER

     FILTER LENGTH =  43

     ***** IMPULSE RESPONSE *****
     H(  1)=  0.14361431E-02 = H(  43)
     H(  2)= -0.58903457E-02 = H(  42)
     H(  3)=  0.81669671E-02 = H(  41)
     H(  4)= -0.47503306E-02 = H(  40)
     H(  5)= -0.10377730E-02 = H(  39)
     H(  6)=  0.88700240E-02 = H(  38)
     H(  7)= -0.88262104E-02 = H(  37)
     H(  8)= -0.76360747E-02 = H(  36)
     H(  9)=  0.14055687E-01 = H(  35)
     H( 10)= -0.45800703E-02 = H(  34)
     H( 11)= -0.19137720E-02 = H(  33)
     H( 12)=  0.26730084E-01 = H(  32)
     H( 13)= -0.21505464E-01 = H(  31)
     H( 14)= -0.40114318E-01 = H(  30)
     H( 15)=  0.25473160E-01 = H(  29)
     H( 16)=  0.28000465E-02 = H(  28)
     H( 17)=  0.65232033E-02 = H(  27)
     H( 18)=  0.11720399E+00 = H(  26)
     H( 19)= -0.38329311E-01 = H(  25)
     H( 20)= -0.26991361E+00 = H(  24)
     H( 21)=  0.23033077E-01 = H(  23)
     H( 22)=  0.34191398E+00 = H(  22)

                      BAND  1        BAND  2       -BAND  3        BAND  4
LOWER BAND EDGE    0.000000000    0.074223785    0.228960905    0.291721695
UPPER BAND EDGE    0.072003330    0.227540450    0.290301240    0.344532000
DESIRED VALUE      0.000000000  100.000000000    1.000000000  100.000000000
WEIGHTING          1.000000000    0.016160320    0.161603200    0.003406080

                      BAND  5        BAND
LOWER BAND EDGE    0.345953255
UPPER BAND EDGE    0.500000000
DESIRED VALUE      0.000000000
WEIGHTING          0.003406080

EXTREMAL FREQUENCIES
    0.0000000   0.0213068   0.0426136   0.0610795   0.0728033
    0.1125761   0.1438261   0.1722351   0.2206442   0.2289609
    0.2431734   0.2602189   0.2744235   0.2857871   0.2903012
    0.3459933   0.3558964   0.3757828   0.3999305   0.4254907
    0.4496464   0.4752146   0.5000000
```

TABLE 15

[illegible] UNIVERSITY - SCHOOL OF ELECTRICAL ENGINEERING
CIRCUITS AND SYSTEMS GROUP

CONRIP - CONSTRAINED RIPPLE NONRECURSIVE DIGITAL FILTER DESIGN

****** SPECIFICATIONS ******

BANDPASS FILTER
LENGTH 43
TOUCH UPPER BOUND FIRST

****** PIECEWISE LINEAR BOUNDARY ******

BREAKPOINTS

UPPER BOUND		LOWER BOUND	
FREQUENCY	RESPONSE	FREQUENCY	RESPONSE
0	.000030	0	-.000030
.071000	.000030	.225000	-.000030
.074000	1.000500	.228000	.999500
.343000	1.000500	.289000	.999500
.345000	.023000	.292000	-.023000
.500000	.023000	.500000	-.023000

****** BOUNDARY INTERCEPT FREQUENCIES ******

UPPER BOUNDARY

.01200	[illegible]	.07100	.18175	.23350	.26100
[illegible]	[illegible]	[illegible]	.42100	.47450	

LOWER BOUNDARY

[illegible]	[illegible]	.10400	[illegible]	.24625	.27475
[illegible]	[illegible]	[illegible]	[illegible]	.50000	

21 ITERATIONS   TIME 8.7 SECONDS

LENGTH 43 BANDPASS FILTER
TOUCH UPPER BOUND FIRST

****** PULSE RESPONSE ******

HEXADEC			DECIMAL		HEXADEC		
8	0	2	H(1)=	-.37912282E-03=H(43)	8	0	2
8	1	9	H(2)=	-.44702616E-02=H(42)	8	1	9
0	3	6	H(3)=	.93467473E-02=H(41)	0	3	6
8	0	D	H(4)=	-.82466632E-02=H(40)	8	0	D
0	0	1	H(5)=	.21112384E-03=H(39)	0	0	1
0	1	5	H(6)=	.37305920E-02=H(38)	0	1	5
8	5	A	H(7)=	-.15646112E-01=H(37)	8	5	A
8	2	F	H(8)=	-.81132271E-02=H(36)	8	2	F
0	7	6	H(9)=	.20405457E-01=H(35)	0	7	6
0	1	C	H(10)=	.43473670E-02=H(34)	0	1	C
0	1	3	H(11)=	.33440687E-02=H(33)	0	1	3
0	6	C	H(12)=	.18715691E-01=H(32)	0	6	C
8	D	1	H(13)=	-.36136427E-01=H(31)	8	D	1
9	0	4	H(14)=	-.44326051E-01=H(30)	9	0	4
0	B	E	H(15)=	.32882547E-01=H(29)	0	B	E
0	5	5	H(16)=	.14672231E-01=H(28)	0	5	5
0	5	8	H(17)=	.15198407E-01=H(27)	0	5	8
2	8	3	H(18)=	.11108614 =H(26)	2	8	3
9	3	8	H(19)=	-.53840576E-01=H(25)	9	3	8
E	3	E	H(20)=	-.27582875 =H(24)	E	3	E
0	B	0	H(21)=	.30362177E-01=H(23)	0	B	0
7	F	F	H(22)=	.35329584 =H(22)	7	F	F

TABLE 16

FINITE IMPULSE RESPONSE (FIR)
LINEAR PHASE DIGITAL FILTER DESIGN
REMEZ EXCHANGE ALGORITHM

BANDPASS FILTER

FILTER LENGTH = 57

***** IMPULSE RESPONSE *****

```
H( 1)= -0.49951821E-03 = H( 57)
H( 2)= -0.42533974E-02 = H( 56)
H( 3)= -0.64724341E-02 = H( 55)
H( 4)= -0.15264820E-01 = H( 54)
H( 5)= -0.10875563E-01 = H( 53)
H( 6)= -0.17524962E-01 = H( 52)
H( 7)= -0.33127998E-01 = H( 51)
H( 8)= -0.13437283E-02 = H( 50)
H( 9)=  0.63575269E-03 = H( 49)
H( 10)=  0.17900015E-01 = H( 48)
H( 11)=  0.47764815E-01 = H( 47)
H( 12)=  0.59946959E-01 = H( 46)
H( 13)=  0.12323666E-03 = H( 45)
H( 14)=  0.47764988E-01 = H( 44)
H( 15)=  0.73560347E-01 = H( 43)
H( 16)=  0.26005579E-01 = H( 42)
H( 17)=  0.31012701E-01 = H( 41)
H( 18)= -0.51543957E-01 = H( 40)
H( 19)= -0.16228474E+00 = H( 39)
H( 20)= -0.12227988E+00 = H( 38)
H( 21)= -0.22269285E+00 = H( 37)
H( 22)= -0.99302620E-01 = H( 36)
H( 23)= -0.11549371E+00 = H( 35)
H( 24)= -0.17908649E+00 = H( 34)
H( 25)=  0.20444665E+00 = H( 33)
H( 26)=  0.19682734E+00 = H( 32)
H( 27)=  0.22394858E+00 = H( 31)
H( 28)=  0.10508076E+00 = H( 30)
H( 29)=  0.74889173E+00 = H( 29)
```

	BAND 1	BAND 2	BAND 3	BAND 4
LOWER BAND EDGE	0.000000000	0.110188358	0.269313730	0.376735510
UPPER BAND EDGE	0.090222222	0.205859673	0.311587150	0.389299580
DESIRED VALUE	1.000000000	0.000000000	1.000000000	0.000000000
WEIGHTING	0.193865260	0.174590273	1.000000000	0.181802582
DEVIATION	0.000559764	0.000619211	0.000108138	0.000594646
DEVIATION IN DB	-65.072832114	-64.163232111	-79.322831379	-64.514832221

	BAND 5	BAND
LOWER BAND EDGE	0.462991740	
UPPER BAND EDGE	0.500000000	
DESIRED VALUE	1.000000000	
WEIGHTING	0.213196490	
DEVIATION	0.000597982	
DEVIATION IN DB	-65.599432386	

EXTREMAL FREQUENCIES

0.0000000	0.0353879	0.0682022	0.1101884	0.1134211
0.1195800	0.1360025	0.1425159	0.1554470	0.1694556
0.1823800	0.1942401	0.2020608	0.2058597	0.2693137
0.2714669	0.2790120	0.2887103	0.2994861	0.3081868
0.3115872	0.3767355	0.3784987	0.3853562	0.3892996
0.4629917	0.4662245	0.4759228	0.4877762	0.5000000

TABLE 17

```
*****************************************************************

                    FINITE IMPULSE RESPONSE (FIR)
                    LINEAR PHASE DIGITAL FILTER DESIGN
                    REMEZ EXCHANGE ALGORITHM

                    BANDPASS FILTER

               FILTER LENGTH =  57

               ***** IMPULSE RESPONSE *****
               H(  1)=  0.79203505E-02 = H(  57)
               H(  2)=  0.94909472E-02 = H(  56)
               H(  3)= -0.27987331E-02 = H(  55)
               H(  4)=  0.82379902E-02 = H(  54)
               H(  5)=  0.69408833E-02 = H(  53)
               H(  6)=  0.32174408E-02 = H(  52)
               H(  7)= -0.21296233E-02 = H(  51)
               H(  8)= -0.10001187E-01 = H(  50)
               H(  9)= -0.33430563E-02 = H(  49)
               H( 10)=  0.37174686E-02 = H(  48)
               H( 11)= -0.58588446E-02 = H(  47)
               H( 12)= -0.32349384E-01 = H(  46)
               H( 13)=  0.15631400E-01 = H(  45)
               H( 14)=  0.24460521E-02 = H(  44)
               H( 15)= -0.25627013E-01 = H(  43)
               H( 16)= -0.11624765E-01 = H(  42)
               H( 17)= -0.12087293E-01 = H(  41)
               H( 18)=  0.13470426E-01 = H(  40)
               H( 19)= -0.18906471E-01 = H(  39)
               H( 20)= -0.30134279E-02 = H(  38)
               H( 21)= -0.29018004E-01 = H(  37)
               H( 22)=  0.10086105E+00 = H(  36)
               H( 23)=  0.54207845E-01 = H(  35)
               H( 24)= -0.99689795E-01 = H(  34)
               H( 25)=  0.21601600E+00 = H(  33)
               H( 26)=  0.99937877E-01 = H(  32)
               H( 27)=  0.79622979E-01 = H(  31)
               H( 28)= -0.81521534E-01 = H(  30)
               H( 29)=  0.44312917E+00 = H(  29)

                      BAND  1        BAND  2        BAND  3        BAND  4
LOWER BAND EDGE    0.000000000    0.009279806    0.035460106    0.062460156
UPPER BAND EDGE    0.008292229    0.034390520    0.061390570    0.077000000
DESIRED VALUE      0.000000000   -1.527399640   -7.142806123  -23.027608136
WEIGHTING          0.193865280    0.019380000    0.019380000    0.001740020

                      BAND  5        BAND  6        BAND  7        BAND  8
LOWER BAND EDGE    0.078077586    0.085077586    0.111265936    0.206937256
UPPER BAND EDGE    0.084000000    0.110158350    0.205859670    0.216000000
DESIRED VALUE     -7.142857143   -1.527396724    0.000000000    6.410047799
WEIGHTING          0.017400030    0.017400000    0.174590270    0.017400000

                      BAND  9        BAND 10        BAND 11        BAND 12
LOWER BAND EDGE    0.217077586    0.260250986    0.270391316    0.312604736
UPPER BAND EDGE    0.259173460    0.269313730    0.311587150    0.318422660
DESIRED VALUE     20.151296463    6.410047799    0.000000000   -6.875858564
WEIGHTING          0.001740000    0.017400000    1.000000000    0.213000000

                      BAND 13        BAND 14        BAND 15        BAND 16
LOWER BAND EDGE    0.319500246    0.370977586    0.377813096    0.390377086
UPPER BAND EDGE    0.369990000    0.376735510    0.389299500    0.404500000
DESIRED VALUE    -17.599977200   -6.875858568    0.000000000    0.394723858
WEIGHTING          0.001200000    0.218180250    0.181802580    0.218180250

                      BAND 17        BAND 18        BAND 19        BAND
LOWER BAND EDGE    0.405577586    0.448868826    0.464069326
UPPER BAND EDGE    0.447791240    0.462991740    0.500000000
DESIRED VALUE     22.822169134    0.394723858    0.000000000
WEIGHTING          0.021319649    0.213196490    0.213196490

EXTREMAL FREQUENCIES
    0.0402000    0.0082022    0.0286754    0.0413906    0.0750776
    0.0969310    0.1177315    0.1317401    0.1460263    0.1629901
    0.1780763    0.1942401    0.2058597    0.2160000    0.2713659
    0.2790120    0.2897879    0.2994461    0.3201764    0.3115872
    0.3778131    0.3842730    0.4245000    0.4135286    0.4389828
    0.4480688    0.4542568    0.4662245    0.4823883    0.5000000
```

TABLE 18

****

FINITE IMPULSE RESPONSE (FIR)
LINEAR PHASE DIGITAL FILTER DESIGN
REMEZ EXCHANGE ALGORITHM

BANDPASS FILTER

FILTER LENGTH = 57

```
***** IMPULSE RESPONSE *****
H(  1)= -0.36535244E-03 = H( 57)
H(  2)= -0.17480977E-02 = H( 56)
H(  3)= -0.33479100E-03 = H( 55)
H(  4)= -0.38065546E-03 = H( 54)
H(  5)=  0.16206552E-02 = H( 53)
H(  6)=  0.18767609E-02 = H( 52)
H(  7)= -0.51857960E-02 = H( 51)
H(  8)=  0.82525085E-02 = H( 50)
H(  9)=  0.67055328E-02 = H( 49)
H( 10)= -0.40431054E-02 = H( 48)
H( 11)=  0.48685175E-02 = H( 47)
H( 12)= -0.10316302E-01 = H( 46)
H( 13)=  0.98179195E-02 = H( 45)
H( 14)= -0.19374000E-03 = H( 44)
H( 15)= -0.27103711E-01 = H( 43)
H( 16)= -0.14500032E-01 = H( 42)
H( 17)=  0.82516235E-02 = H( 41)
H( 18)= -0.26077136E-02 = H( 40)
H( 19)= -0.44869722E-01 = H( 39)
H( 20)=  0.21610013E-01 = H( 38)
H( 21)= -0.19073316E-01 = H( 37)
H( 22)=  0.73623419E-01 = H( 36)
H( 23)=  0.51494776E-01 = H( 35)
H( 24)= -0.99130330E-01 = H( 34)
H( 25)=  0.22192184E+00 = H( 33)
H( 26)=  0.11386566E+00 = H( 32)
H( 27)=  0.51512294E-01 = H( 31)
H( 28)= -0.86017394E-01 = H( 30)
H( 29)=  0.46533464E+00 = H( 29)
```

	BAND 1	BAND 2	BAND 3	BAND 4
LOWER BAND EDGE	0.000000000	0.109279806	0.111265936	0.206937256
UPPER BAND EDGE	0.100222220	0.110190350	0.205859670	0.269313730
DESIRED VALUE	1.000000000	100.000000000	0.000000000	100.000000000
WEIGHTING	0.193865280	0.019000000	0.174590270	0.017000000

	BAND 5	BAND 6	BAND 7	BAND 8
LOWER BAND EDGE	0.270391316	0.312664736	0.377813096	0.390377166
UPPER BAND EDGE	0.311587150	0.376735510	0.389299580	0.462991740
DESIRED VALUE	1.000000000	100.000000000	0.000000000	100.000000000
WEIGHTING	1.000000000	0.018000000	0.131802580	0.018000000

	BAND 9	BAND
LOWER BAND EDGE	0.464069326	
UPPER BAND EDGE	0.500000000	
DESIRED VALUE	1.000000000	
WEIGHTING	0.213196490	

EXTREMAL FREQUENCIES

0.0000000	0.0202222	0.0340643	0.0556160	0.0760902
0.1112659	0.1188090	0.1317401	0.1479039	0.1629901
0.1780763	0.1922549	0.2017632	0.2058597	0.2204490
0.2725465	0.2804896	0.2897879	0.3005637	0.3081066
0.3115872	0.3417594	0.3748907	0.3892996	0.4043858
0.4259375	0.4485668	0.4694573	0.4834659	0.5000000

TABLE 19

PURDUE UNIVERSITY - SCHOOL OF ELECTRICAL ENGINEERING
CIRCUITS AND SYSTEMS GROUP

CONRIP - CONSTRAINED RIPPLE NONRECURSIVE DIGITAL FILTER DESIGN . .

****** SPECIFICATIONS ******

BANDPASS FILTER
LENGTH 57
TOUCH UPPER BOUND FIRST

****** PIECEWISE LINEAR BOUNDARY ******

BREAKPOINTS

UPPER BOUND		LOWER BOUND	
FREQUENCY	RESPONSE	FREQUENCY	RESPONSE
0	1.000600	0	.999400
.100000	1.000600	.007000	.999400
.112000	.000600	.010000	-.000600
.204000	.000600	.268000	-.000600
.292000	1.000110	.271000	.999900
.375000	1.000110	.310000	.999900
.378000	.000600	.313000	-.000600
.386000	.000600	.461000	-.000600
.951000	1.000510	.461000	.999500
.500000	1.000510	.500000	.999500

****** BOUNDARY INTERCEPT FREQUENCIES ******

UPPER BOUNDARY

0	.02150	.11200	.13325	.16375	.19100
.20400	.27250	.28975	.30700	.37800	.38675
.45300	.47250	.50000			

LOWER BOUNDARY

.00650	.08825	.11925	.14850	.17825	.20050
.27100	.28025	.29950	.31000	.38075	.39275
.46400	.48625				

23 ITERATIONS TIME 15.2 SECONDS

LENGTH 57 BANDPASS FILTER
TOUCH UPPER BOUND FIRST

****** PULSE RESPONSE ******

HEXADEC				DECIMAL		HEXADEC		
0	0	5	H(1)=	.12115744E-02	=H(57)	0	0	5
8	0	8	H(2)=	-.21438715E-02	=H(56)	8	0	8
0	0	7	H(3)=	.17813266E-02	=H(55)	0	0	7
0	0	3	H(4)=	.85598504E-03	=H(54)	0	0	3
0	0	6	H(5)=	.15465814E-02	=H(53)	0	0	6
0	1	7	H(6)=	.57070921E-02	=H(52)	0	1	7
8	1	C	H(7)=	-.70009784E-02	=H(51)	8	1	C
0	2	C	H(8)=	.10782256E-01	=H(50)	0	2	C
0	1	1	H(9)=	.43471703E-02	=H(49)	0	1	1
8	1	1	H(10)=	-.43244030E-02	=H(48)	8	1	1
0	0	C	H(11)=	.29198684E-02	=H(47)	0	0	C
8	3	6	H(12)=	-.13105034E-01	=H(46)	8	3	6
0	1	F	H(13)=	.77009791E-02	=H(45)	0	1	F
8	2	4	H(14)=	-.87677349E-02	=H(44)	8	2	4
8	5	E	H(15)=	-.22798981E-01	=H(43)	8	5	E
8	6	D	H(16)=	-.26518143E-01	=H(42)	8	6	D
0	2	A	H(17)=	.10277325E-01	=H(41)	0	2	A
8	0	4	H(18)=	-.99819231E-03	=H(40)	8	0	4
8	C	E	H(19)=	-.49890790E-01	=H(39)	8	C	E
0	8	8	H(20)=	.33036085E-01	=H(38)	0	8	8
8	5	9	H(21)=	-.21593783E-01	=H(37)	8	5	9
1	5	7	H(22)=	.83077641E-01	=H(36)	1	5	7
0	D	C	H(23)=	.53311426E-01	=H(35)	0	D	C
9	7	9	H(24)=	-.91072820E-01	=H(34)	9	7	9
3	8	4	H(25)=	.21743439	=H(33)	3	8	4
1	D	E	H(26)=	.11552929	=H(32)	1	D	E
0	D	E	H(27)=	.53710914E-01	=H(31)	0	D	E
9	A	6	H(28)=	-.10205765	=H(30)	9	A	6
7	F	F	H(29)=	.49464096	=H(29)	7	F	F

TABLE 20

```
********************************************************************

                    FINITE IMPULSE RESPONSE (FIR)
                    LINEAR PHASE DIGITAL FILTER DESIGN
                    REMEZ EXCHANGE ALGORITHM

                    BANDPASS FILTER

                FILTER LENGTH =  73

                ***** IMPULSE RESPONSE *****
                H(  1)= -0.30673043E-01 = H( 73)
                H(  2)= -0.56223003E-01 = H( 72)
                H(  3)=  0.93176918E-01 = H( 71)
                H(  4)=  0.11737416E+00 = H( 70)
                H(  5)= -0.12394306E+00 = H( 69)
                H(  6)= -0.19059447E+00 = H( 68)
                H(  7)=  0.14796549E+00 = H( 67)
                H(  8)=  0.27505695E+00 = H( 66)
                H(  9)= -0.71452794E-01 = H( 65)
                H( 10)= -0.42615065E-02 = H( 64)
                H( 11)= -0.18622781E-03 = H( 63)
                H( 12)=  0.59877447E+00 = H( 62)
                H( 13)=  0.53153044E-02 = H( 61)
                H( 14)= -0.67377122E-02 = H( 60)
                H( 15)= -0.90375252E+00 = H( 59)
                H( 16)=  0.67345323E-02 = H( 58)
                H( 17)=  0.12205115E-01 = H( 57)
                H( 18)= -0.40909657E+00 = H( 56)
                H( 19)= -0.15409092E-01 = H( 55)
                H( 20)= -0.25206136E-02 = H( 54)
                H( 21)=  0.17228691E-01 = H( 53)
                H( 22)=  0.14032531E-01 = H( 52)
                H( 23)= -0.10170011E-01 = H( 51)
                H( 24)= -0.19284083E-01 = H( 50)
                H( 25)=  0.13492231E-01 = H( 49)
                H( 26)=  0.25416628E-01 = H( 48)
                H( 27)= -0.73764497E+00 = H( 47)
                H( 28)= -0.29413977E-01 = H( 46)
                H( 29)= -0.32838460E-02 = H( 45)
                H( 30)=  0.29244843E-01 = H( 44)
                H( 31)=  0.15346945E-01 = H( 43)
                H( 32)= -0.22497625E-01 = H( 42)
                H( 33)= -0.27417566E-01 = H( 41)
                H( 34)=  0.15926355E-01 = H( 40)
                H( 35)=  0.32259987E-01 = H( 39)
                H( 36)= -0.60997982E+00 = H( 38)
                H( 37)= -0.31309804E-01 = H( 37)

                      BAND  1        BAND  2        BAND  3        BAND  4
LOWER BAND EDGE     0.000000000    0.131994350    0.271939060    0.353732020
UPPER BAND EDGE     0.069801970    0.185508313    0.290191850    0.437375020
DESIRED VALUE       0.000000000    1.000000000    0.000000000    1.000000000
WEIGHTING           0.095995300    0.111874210    1.000000000    0.111773790
DEVIATION           0.019181520    0.016458983    0.001841336    0.016473777
DEVIATION IN DB   -34.342334818  -35.671940846  -54.697343414  -35.664139957

                      BAND  5        BAND
LOWER BAND EDGE     0.457324560
UPPER BAND EDGE     0.500000000
DESIRED VALUE       0.000000000
WEIGHTING           0.254216900
DEVIATION           0.034088116
DEVIATION IN DB   -29.347939979

EXTREMAL FREQUENCIES
    0.0000000    0.0126689    0.0241824    0.0348514    0.0515203
    0.0633400    0.0751089    0.0844595    0.0889620    0.1319944
    0.1387511    0.1488863    0.1590214    0.1691565    0.1776825
    0.1835147    0.1855043    0.2719397    0.2736289    0.2778518
    0.2920748    0.2962979    0.2981911    0.3537320    0.3571104
    0.3655563    0.3765361    0.3883604    0.4001847    0.4120090
    0.4238334    0.4331239    0.4373758    0.4573266    0.4623941
    0.4733739    0.4860874    0.5000000
```

TABLE 21

FILTER LENGTH = 73

***** IMPULSE RESPONSE *****

H(1)=	-0.13841604E-01	=	H(73)
H(2)=	-0.17299667E-01	=	H(72)
H(3)=	0.78933243E-02	=	H(71)
H(4)=	-0.33125931E-02	=	H(70)
H(5)=	0.13319767E-01	=	H(69)
H(6)=	-0.61100603E-04	=	H(68)
H(7)=	-0.22162603E-03	=	H(67)
H(8)=	-0.36641035E-02	=	H(66)
H(9)=	0.70681245E-02	=	H(65)
H(10)=	-0.90133430E-02	=	H(64)
H(11)=	-0.19589351E-01	=	H(63)
H(12)=	0.19652863E-01	=	H(62)
H(13)=	0.56658900E-02	=	H(61)
H(14)=	0.13015645E-01	=	H(60)
H(15)=	0.38018350E-02	=	H(59)
H(16)=	0.22555529E-02	=	H(58)
H(17)=	-0.21850033E-01	=	H(57)
H(18)=	-0.16277675E-02	=	H(56)
H(19)=	0.10614013E-01	=	H(55)
H(20)=	-0.31173401E-01	=	H(54)
H(21)=	0.13735736E-01	=	H(53)
H(22)=	0.20551809E-01	=	H(52)
H(23)=	0.35601586E-01	=	H(51)
H(24)=	-0.20205279E-01	=	H(50)
H(25)=	0.96075556E-02	=	H(49)
H(26)=	-0.44162980E-01	=	H(48)
H(27)=	-0.19313069E-01	=	H(47)
H(28)=	0.32487387E-01	=	H(46)
H(29)=	-0.31525190E-01	=	H(45)
H(30)=	0.10247624E-01	=	H(44)
H(31)=	0.69317832E-01	=	H(43)
H(32)=	0.16447476E+00	=	H(42)
H(33)=	-0.21135559E+00	=	H(41)
H(34)=	-0.53053197E-01	=	H(40)
H(35)=	-0.94746001E-01	=	H(39)
H(36)=	-0.88390454E-01	=	H(38)
H(37)=	0.44631891E+00	=	H(37)

	BAND 1	BAND 2	BAND 3	BAND 4
LOWER BAND EDGE	0.000000000	0.009726565	0.095844595	0.126700945
UPPER BAND EDGE	0.008861970	0.095000000	0.125856350	0.131994300
DESIRED VALUE	0.000000000	7.963017352	29.232232587	7.963017352
WEIGHTING	0.095995300	0.095995300	0.009599520	0.211187421

	BAND 5	BAND 6	BAND 7	BAND 8
LOWER BAND EDGE	0.132838979	0.186352905	0.206392585	0.220292585
UPPER BAND EDGE	0.185500310	0.205547990	0.219447990	0.238000000
DESIRED VALUE	0.000000000	-1.247524911	-0.992005755	-37.731760602
WEIGHTING	0.111874210	0.037200000	0.037200000	0.012000000

	BAND 9	BAND 10	BAND 11	BAND 12
LOWER BAND EDGE	0.238844595	0.252744595	0.272784275	0.289035645
UPPER BAND EDGE	0.251900000	0.271939680	0.288191050	0.305000000
DESIRED VALUE	-0.992005755	-1.247524911	0.000000000	3.569525448
WEIGHTING	0.072000000	0.100000000	1.000000000	0.120000000

	BAND 13	BAND 14	BAND 15	BAND 16
LOWER BAND EDGE	0.305844595	0.322653545	0.354576615	0.438219615
UPPER BAND EDGE	0.321808950	0.353732020	0.437375020	0.442201380
DESIRED VALUE	52.353061910	1.379516460	0.000000000	-24.862427899
WEIGHTING	0.011177379	0.111773790	0.111773790	0.011770000

	BAND 17	BAND 18	BAND 19	BAND
LOWER BAND EDGE	0.443246175	0.453344595	0.458171155	
UPPER BAND EDGE	0.452500000	0.457326560	0.500000000	
DESIRED VALUE	-73.797728195	-24.462427899	0.000000000	
WEIGHTING	0.001177000	0.011770000	0.054016940	

EXTREMAL FREQUENCIES

0.0000000	0.0135135	0.0270716	0.0413851	0.0548956
0.0684122	0.0810811	0.0930849	0.1102027	0.1325390
0.1429731	0.1556430	0.1683119	0.1801363	0.1947959
0.2130156	0.2194460	0.2380446	0.2578122	0.2727543
0.2795417	0.2861911	0.2903262	0.3050000	0.3226535
0.3277211	0.3395454	0.3522144	0.3664009	0.3799145
0.3934280	0.4069415	0.4204952	0.4322793	0.4373752
0.4581712	0.4635040	0.5000000		

TABLE 22

****

FINITE IMPULSE RESPONSE (FIR)
LINEAR PHASE DIGITAL FILTER DESIGN
REMEZ EXCHANGE ALGORITHM

BANDPASS FILTER

FILTER LENGTH = 73

***** IMPULSE RESPONSE *****

H(1)=	0.10487799E-01	= H(73)
H(2)=	0.43064915E-02	= H(72)
H(3)=	0.11046296E-02	= H(71)
H(4)=	-0.27349799E-03	= H(70)
H(5)=	-0.44698226E-03	= H(69)
H(6)=	0.73632801E-04	= H(68)
H(7)=	0.16039041E-02	= H(67)
H(8)=	-0.33225994E-03	= H(66)
H(9)=	-0.46925598E-02	= H(65)
H(10)=	0.88660452E-02	= H(64)
H(11)=	-0.74092546E-02	= H(63)
H(12)=	0.10559631E-01	= H(62)
H(13)=	-0.95729131E-02	= H(61)
H(14)=	0.12428996E-01	= H(60)
H(15)=	-0.13374346E-01	= H(59)
H(16)=	0.28143582E-02	= H(58)
H(17)=	-0.53676082E-02	= H(57)
H(18)=	-0.63036692E-02	= H(56)
H(19)=	0.10048472E-01	= H(55)
H(20)=	-0.62006673E-02	= H(54)
H(21)=	0.17646047E-01	= H(53)
H(22)=	0.40990057E-03	= H(52)
H(23)=	0.31161783E-01	= H(51)
H(24)=	-0.42637122E-01	= H(50)
H(25)=	-0.35655913E-02	= H(49)
H(26)=	-0.18297664E-01	= H(48)
H(27)=	-0.15981285E-01	= H(47)
H(28)=	0.21913598E-01	= H(46)
H(29)=	-0.56487949E-02	= H(45)
H(30)=	0.23814112E-01	= H(44)
H(31)=	0.42134109E-01	= H(43)
H(32)=	0.16042809E+00	= H(42)
H(33)=	-0.21753963E+00	= H(41)
H(34)=	-0.77274711E-01	= H(40)
H(35)=	-0.73921977E-01	= H(39)
H(36)=	-0.72160657E-01	= H(38)
H(37)=	0.47424255E+00	= H(37)

	BAND 1	BAND 2	BAND 3	BAND 4
LOWER BAND EDGE	0.000000000	0.089706565	0.132938975	0.186352985
UPPER BAND EDGE	0.088861970	0.131994380	0.185598310	0.271939660
DESIRED VALUE	0.000000000	100.000000000	1.000000000	100.000000000
WEIGHTING	0.095995300	0.090000000	0.111374210	0.110000000

	BAND 5	BAND 6	BAND 7	BAND 8
LOWER BAND EDGE	0.272754275	0.289035645	0.354576615	0.438219615
UPPER BAND EDGE	0.288191150	0.353732020	0.437375020	0.457326560
DESIRED VALUE	0.000000000	100.000000000	1.000000000	100.000000000
WEIGHTING	1.000000000	0.110000000	0.111773790	0.050000000

	BAND 9	BAND
LOWER BAND EDGE	0.458171155	
UPPER BAND EDGE	0.500000000	
DESIRED VALUE	0.000000000	
WEIGHTING	0.054016940	

EXTREMAL FREQUENCIES

0.0000000	0.0143581	0.0278716	0.0422297	0.0565878
0.0709459	0.0888620	0.0981525	0.1125106	0.1277133
0.1404403	0.1539539	0.1683119	0.1826701	0.1964888
0.2108016	0.2243597	0.2387178	0.2547651	0.2727843
0.2795410	0.2871424	0.3042383	0.3194411	0.3346438
0.3498019	0.3630224	0.3773807	0.3934260	0.4120090
0.4305901	0.4373752	0.4415982	0.4573246	0.4581712
0.4649279	0.4801334	0.5000000		

TABLE 23

[illegible] UNIVERSITY - [illegible] OF ELECTRICAL ENGINEERING
CIRCUITS AND SYSTEMS GROUP

CONRIP - CONSTRAINED RIPPLE NONRECURSIVE DIGITAL FILTER DESIGN

****** SPECIFICATIONS ******

BANDPASS FILTER
LENGTH 73
[illegible] UPPER BOUND FIRST

****** PIECEWISE LINEAR BOUNDARY ******

BREAKPOINTS

UPPER BOUND FREQUENCY	UPPER BOUND RESPONSE	LOWER BOUND FREQUENCY	LOWER BOUND RESPONSE
0	.020000	0	-.020000
.087000	.020000	.130000	-.020000
[illegible]	1.017000	.133000	.983000
[illegible]	1.017000	.184000	.983000
.273000	.002000	.187000	-.002000
[illegible]	.002000	.352000	-.002000
.290000	1.017000	.355000	.983000
.456000	1.017000	.436000	.983000
.455000	.035000	.439000	-.035000
.500000	.035000	.500000	-.035000

****** BOUNDARY INTERCEPT FREQUENCIES ******

UPPER BOUNDARY

0	[illegible]	[illegible]	.07675	.11450	.13950
.16175	.18125	[illegible]	.27300	.28000	.28700
.21550	[illegible]	.39025	.41475	.43550	.47475
[illegible]					

LOWER BOUNDARY

[illegible]	[illegible]	[illegible]	[illegible]	.13300	.15050
[illegible]	[illegible]	[illegible]	[illegible]	.28475	.32100
[illegible]	[illegible]	[illegible]	.42600	.46100	.48725

[illegible] ITERATIONS TIME 13.[illegible] SECONDS

LENGTH 73 BANDPASS FILTER
[illegible] UPPER BOUND FIRST

****** PULSE RESPONSE ******

HEXADEC			DECIMAL	HEXADEC		
0	8	4	H(1)= .31618769E-01=H(73)	0	8	4
8	1	0	H(2)= -.387[illegible]57E-02=H(72)	8	1	0
8	6	D	H(3)= -.26250320E-01=H(71)	8	6	D
8	2	0	H(4)= -.78057427E-02=H(70)	8	2	0
0	7	2	H(5)= .27243440E-01=H(69)	0	7	2
8	1	1	H(6)= -.40652775E-02=H(68)	8	1	1
8	5	5	H(7)= -.20449265E-01=H(67)	8	5	5
8	3	2	H(8)= -.12073114E-01=H(66)	8	3	2
0	1	C	H(9)= .69220074E-02=H(65)	0	1	C
0	D	C	H(10)= .52612636E-01=H(64)	0	D	C
0	1	2	H(11)= .44603264E-02=H(63)	0	1	2
8	B	0	H(12)= -.42005935E-01=H(62)	8	B	0
8	5	A	H(13)= -.21685945E-01=H(61)	8	5	A
0	C	3	H(14)= .46739315E-01=H(60)	0	C	3
8	4	0	H(15)= -.15410504E-01=H(59)	8	4	0
8	7	5	H(16)= -.28100530E-01=H(58)	8	7	5
8	0	5	H(17)= -.13494905E-02=H(57)	8	0	5
8	1	1	H(18)= -.41109234E-02=H(56)	8	1	1
0	7	5	H(19)= .23121917E-01=H(55)	0	7	5
0	7	9	H(20)= .29125972E-01=H(54)	0	7	9
0	4	3	H(21)= .16107832E-01=H(53)	0	4	3
8	A	D	H(22)= -.41422129E-01=H(52)	8	A	D
0	3	7	H(23)= .13293200E-01=H(51)	0	3	7
8	9	0	H(24)= -.34162329E-01=H(50)	8	9	0
0	2	1	H(25)= .78931394E-02=H(49)	0	2	1
8	3	9	H(26)= -.13655455E-01=H(48)	8	3	9
8	9	C	H(27)= -.37350844E-01=H(47)	8	9	C
0	6	4	H(28)= .23923465E-01=H(46)	0	6	4
0	A	C	H(29)= .41236724E-01=H(45)	0	A	C
0	D	F	H(30)= .53510783E-01=H(44)	0	D	F
0	0	A	H(31)= .24134024E-02=H(43)	0	0	A
1	E	9	H(32)= .11705221 =H(42)	1	E	9
B	9	4	H(33)= -.21891493 =H(41)	B	9	4
9	1	6	H(34)= -.66639096E-01=H(40)	9	1	6
9	2	0	H(35)= -.68902279E-01=H(39)	9	2	0
9	1	E	H(36)= -.68410327E-01=H(38)	9	1	E
7	F	F	H(37)= [illegible]34564 =H(37)	7	F	F

BIBLIOGRAPHY

1. O. Herrmann, "On the Design of Nonrecursive Digital Filters with Linear Phase," Elec. Lett., Vol. 6, No. 11, 1970, pp. 328-329.

2. E. Hofstetter, A. V. Oppenheim and J. Siegel, "A New Technique for the Design of Nonrecursive Digital Filters," Proc. Fifth Annual Princeton Conf. Inform. Sci. Systems, 1971, pp. 64-72.

3. T. W. Parks and J. H. McClellan, "Chebychev Approximation for Nonrecursive Digital Filters with Linear Phase," IEEE Trans. on Circuit Theory, Vol. CT-19, No. 2, pp. 189-194, March 1972.

4. J. M. McClellan, T. M. Parks and L. R. Rabiner, "A Computer Program for Designing Optimum FIR Linear Phase Digital Filters," IEEE Trans. Audio Electroacoust., Vol. AU-21, pp. 506-526, Dec. 1973.

5. E. W. Cheney, Introduction to Approximation Theory, New York: McGraw-Hill, 1966.

6. E. Y. Remez, General Computational Methods of Chebyshev Approximation. The Problems with Linear Real Parameters, United States Atomic Energy Commission, AEC-t2-4491, Book 1, pp. 76-101.

7. L. R. Rabiner, J. F. Kaiser and R. W. Shaffer, "Some Considerations in the Design of Multiband Finite-Impulse-Response Digital Filters," IEEE Trans. Acoustics, Speech and Signal Processing, Vol. ASSP-22, No. 6, pp. 462-472, Dec. 1974.

8. M. T. McCallig and B. J. Leon, Design of Nonrecursive Digital Filters to meet Maximum and Minimum Frequency Response Constraints, School of Electrical Engineering, Purdue University, Report TR-EE-75-43, Dec. 1975.

4-8

DTI