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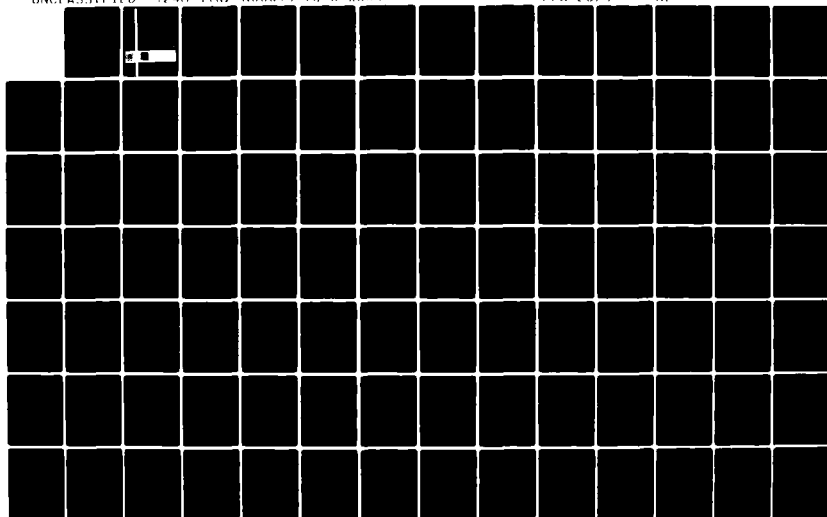
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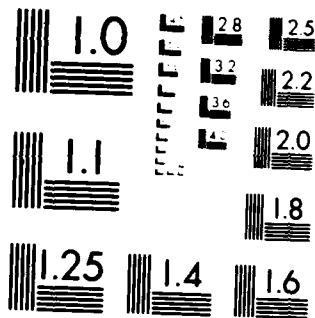
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Technical Report No. 2  
Contract N00014-75-C-0694; NR 097-395

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FLOWS WITH TEMPERATURE-DEPENDENT PROPERTIES

Engineering Experiment Station  
University of Arizona  
Tucson, Arizona 85721

30 June 1982

Interim Technical Report

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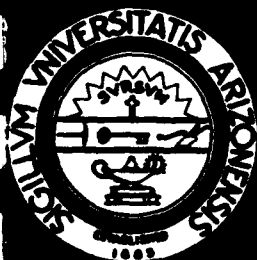
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**TUCSON, ARIZONA 85721**

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CONVECTIVE HEAT TRANSFER IN INTERNAL GAS FLOWS \*  
WITH TEMPERATURE-DEPENDENT PROPERTIES

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30 June 1982

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CONVECTIVE HEAT TRANSFER IN INTERNAL  
GAS FLOWS WITH TEMPERATURE-DEPENDENT  
PROPERTIES

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Abstract

Literature on internal convective heat transfer to gases and their binary mixtures at heating rates which cause significant variation of the temperature-dependent transport properties, is reviewed to 1982. First, laminar flows, which are easiest to predict and most difficult to measure, are discussed. Then the problems of treating turbulent flows are considered, followed by the perplexing topic of laminarization - transition from a turbulent flow to a state which may be predicted as if there were no turbulent transport present. The review concludes with a few generalizations that seem pertinent.

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# NOMENCLATURE

a, b, d	exponents
$A_{cs}$	cross sectional area
$A^+$	constant or function in van Driest turbulence model
$c_p$	specific heat at constant pressure
D	diameter; $D_h$ , hydraulic diameter, $4A_{cs}/P$
$f$	friction factor, $2g_c \tau_w / (G^2 \nu)$ ; $f_s$ , $\tau_w$ based on velocity gradient at wall; $f_{app}$ , based on $\tau_{w,app}$ , eqn. I-14
g	gravitational acceleration
$g_c$	dimensional constant
G	average mass flux, $\dot{m}/A_{cs}$
Gr	Grashof number, e.g., $gD^3(\rho_b - \rho_w)/\nu^2$
Gr*	modified Grashof number, $gD^3/\nu^2$
Gz	Graetz number, $RePr/(x/D)$
h	heat transfer coefficient, $q_w''/(t_w - t_b)$
H	specific enthalpy
k	thermal conductivity; also turbulence kinetic energy
$K_{-1}, K_{k1}, \dots$	property coefficients, eqn. II-6
$\lambda$	mixing length
L	length; also dissipation length scale
$L^+$	non-dimensional length for velocity boundary layer, $4L/(D_h Re)$
$L^*$	non-dimensional length for thermal boundary layer $4L/(D_h RePr)$

NOMENCLATURE--continued

$m, n$	exponents
$\dot{m}$	mass flow rate
$M$	molal mass ("molecular weight")
$Nu$	Nusselt number, $hD_h/k$
$p$	pressure
$Pr$	Prandtl number, $\mu c_p/k$
$Pr_t$	turbulent Prandtl number, $\varepsilon_m/\varepsilon_h$
$q_w''$	wall heat flux
$q^+$	non-dimensional wall heat flux, $q_w''/(Gc_p T)$
$Q^+$	non-dimensional wall heat flux for laminar flow, $q_w''D_h/(kT)$
$r$	radial coordinate
$R$	gas constant for a particular gas
$Re$	Reynolds number, $GD_h/\mu$
$St$	Stanton number, $h/Gc_p$
$t$	temperature (relative), e.g., °C; also time
$T$	absolute temperature
$u$	streamwise velocity component
$u^+$	non-dimensional velocity, $u/\sqrt{g_c \tau_w/\rho}$
$v$	transverse velocity component
$\bar{v}_b$	bulk velocity, $G/c_b$
$x$	axial or streamwise coordinate
$y$	transverse coordinate measured from wall, e.g., $r_w - r$
$y^+$	non-dimensional transverse coordinate, $(y/\sqrt{g_c \tau_w/\rho})$
$Z$	compressibility

NOMENCLATURE—continuedSubscripts

a	overall average lengthwise
app	apparent
b	properties evaluated at bulk temperature
cp	based on constant property idealizations
e	entry, inlet
eff	effective
f	based on film temperature, $(T_w + T_b)/2$
fd	fully developed or asymptotic value
i	inlet, initial, index
m	modified
ref	reference
trans	transition
turb	turbulent
w	evaluated at the wall
x	properties evaluated at local position
$\infty$	freestream or asymptotic value

NOMENCLATURE--continuedGreek symbols

$\delta$	boundary layer thickness
$\epsilon$	perturbation parameter eqn. II-6; also dissipation of turbulence kinetic energy
$\epsilon_h$	eddy diffusivity for heat
$\epsilon_m$	eddy diffusivity for momentum
$\theta$	non-dimensional temperature difference, eqn. II-6
$\mu$	absolute viscosity
$\nu$	kinematic viscosity, $\mu/\rho$
$f$	friction factor (Russian)
$\rho$	density
$\tau_w$	wall shear stress



## I. Introduction

### I.1. Initial comments

In the case of gases, convective heat transfer always causes property variation. Density, thermal conductivity and viscosity all vary at approximately the same rate as the absolute temperature. Therefore, the question is not whether the fluid transport properties vary but, instead, when does it matter and how may one account for the property variation when it does.

The study of heat transfer to gases with property variation is important for a number of reasons. First, such studies are necessary in order to determine the limits to which classical analyses, with their idealizations of constant fluid properties, can be taken and still have reliable predictions. Secondly, such studies provide guidance for designers when property variation is significant. Typically, the results are provided in the form of correlations which are valid for the case of particular thermal boundary conditions. With the advent and availability of high speed digital computers, the information provided to the designers can now be at a more fundamental level than empirical correlations since numerical programs have become available for general use in companies of at least moderate size. Correlations, however, are still useful for system optimization studies involving a wide range of parameters. Thirdly, the study of turbulence phenomena in cases other than simple shear layers can provide important information for the development of advanced turbulence models [Bradshaw, 1976]. In particular, the study of situations with strong heating emphasizes a need for accurate determination of such quantities as the effective viscosity and the turbulent Prandtl number in the flow immediately adjacent to the heated wall. These quantities provide means of developing wall functions for boundary conditions used in higher order turbulence models [Launder and Spalding, 1972].

Interest in gas-cooled nuclear reactors, nuclear propulsion systems and possible applications to cooling of the blanket in proposed fusion reactors has necessitated close inspection of problems involving heat transfer to gases with large temperature gradients and, thus, strongly varying transport properties. Often equipment is designed to operate in the turbulent flow regime but "laminar flow may be employed at low circulation rates or at low power operation or obtained unintentionally during coast down and during steady state natural circulation following pump failure" [Bonilla, 1958]. Under such non-ideal operating conditions, the design of heat exchangers becomes much more difficult and accurate design criteria become essential for efficient and reliable operation. In designs where pumping power must be kept low in proportion to heat transfer, as is frequently the case in the design of compact heat exchangers, laminar flow conditions may provide the optimum operation.

An interesting application of results involving significant variation of fluid properties due to their temperature dependencies is provided by the regeneratively cooled rocket nozzle [Krueger and Curren, 1966]. Combustion gases at temperatures of the order of 3000 K flow through the nozzle, typically with a circular cross section. Acceleration is induced by the converging geometry but is inhibited partly by cooling of the combustion gases. The walls are limited to temperatures of the order of 1200 K so the bulk-to-wall temperature ratio is of the order of 5/2. On the other side in the cooling channels, a dense fluid - the fuel serving as a coolant - flows at entering temperatures slightly above cryogenic conditions so wall-to-bulk temperature ratios of the order of ten or so can occur in that gas.

Studies with approximately constant wall heat flux as the thermal boundary condition can represent the situation in tubular heat exchangers of approximately equal capacity rates [Kays and London, 1964] and in electrical-resistance heating applications. Gas cooled (typically air or helium) solar collectors for process heating can have significant property variation

in the streamwise direction, but usually the transverse temperature gradients are small so that the property variation in the cross-stream direction is not too significant.

The closed Brayton cycle, or closed gas turbine cycle, has been suggested as an efficient, compact, versatile system for application to the generation of electric power, for propulsion of ships, aircraft, buses and rail units, and for power systems in satellites, space stations and undersea. Thermodynamic cycle studies have shown that considerable savings in the total cost and size of a gas turbine engine can be achieved by mixing helium with a heavier gas; the increase in density reduces the size of the turbo-machines while the reduction in thermal conductivity increases the size of the heat exchanger, so that an optimum occurs at an intermediate molecular weight.

Non-circular ducts are used for gas cooled nuclear power reactors, for compact heat exchangers in the process industries and for regeneratively cooled rocket nozzles. Applications of the annular configuration occur in other nuclear reactor cooling channels, double pipe heat exchangers, sleeve valves, turbomachinery wearing rings and balancing drums and some types of catalytic devices. In gas turbine cycles, for low pressure operation, the regenerative heat exchanger is typically constructed of parallel plates, with short fins attached forming additional parallel surfaces. For high cycle pressure ratios one expects to use circular tubes for the basic heat exchanger surface due to structural constraints.

An example of the danger of using classical analyses directly and ignoring the potential effect of property variation is given by Figure 1 taken from some earlier work of the author [McEligot, Coon and Perkins, 1970]. Predictions made with the assumption of a specified wall heat flux as the

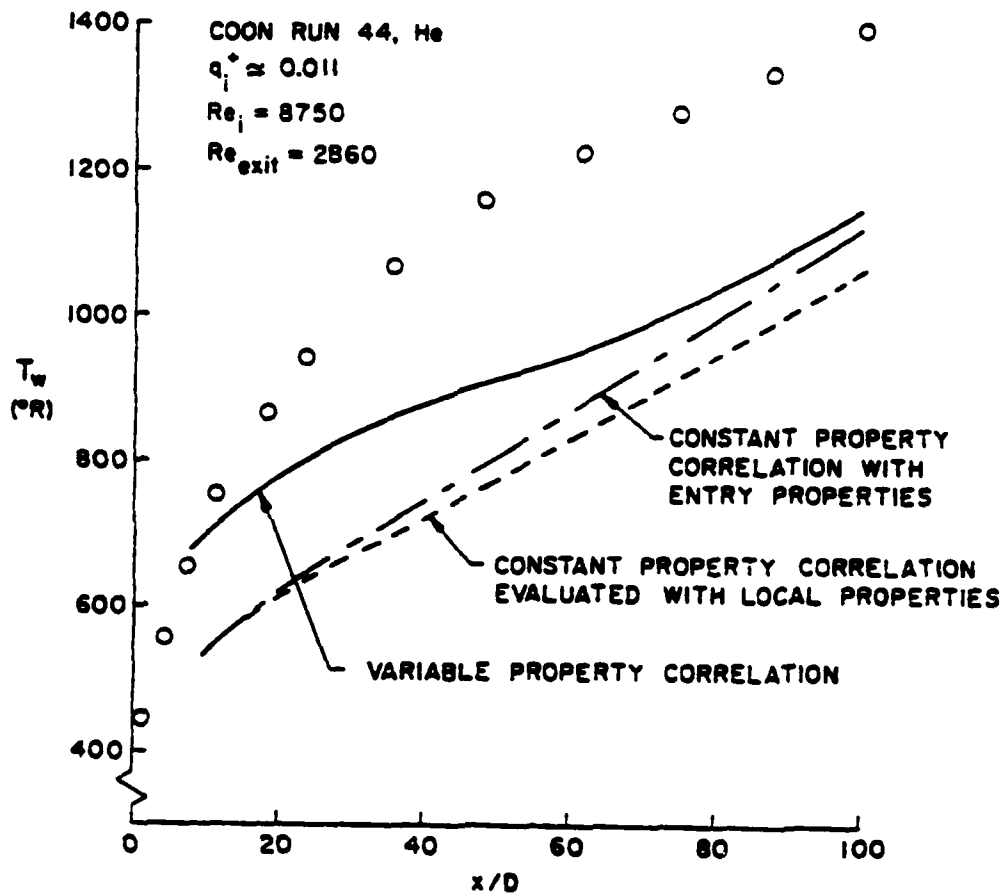


Fig. 1. Possible difficulties in predicting internal gas flow with strong property variation. [McEligot, Coon and Perkins, Int. J. Heat Mass Transfer, 1970].

thermal boundary condition are compared to data taken in an experiment with flow through a resistively heated tube, which approximates that boundary condition. One sees immediately that the analysis based on the assumption of constant properties significantly under-predicts the wall temperatures which are actually measured. Thus, if one were designing equipment for use within specified limits for allowable maximum tube temperatures, ignoring the effects of property variation could lead to a catastrophic failure. A particular application of the same sort would be a high temperature gas cooled nuclear reactor where the fission rate determines the local value of the wall heat flux distribution. In order to design gas cooled nuclear reactors for high power densities, the effects of property variation on the convective heat transfer behavior must be considered.

## I.2. Properties of gases

The pertinent properties of some typical gases and their mixtures are presented in Figure 2; they have been derived using a Lennard-Jones (6-12) potential employed in the Chapman-Enskog kinetic theory [Hirschfelder, Curtiss and Bird, 1964]. The numbers adjacent to the curves plotted represent the molal mass ("molecular weight") of the gas or mixture shown. For these noble gases, one observes that there is no large variation of viscosity as the concentration or molal mass is varied. On the other hand, the thermal conductivity varies over more than an order of magnitude from the high value of helium to the relatively low value of xenon. By plotting the curves on logarithmic coordinates, it may be seen that the variation with absolute temperature is approximately the same for both viscosity and thermal conductivity for all of these gases and their mixtures. That is, if one represents the variation by a power law dependence, the exponent would fall in the range from 0.7 to 0.8. The specific heat varies inversely as the molal mass and,

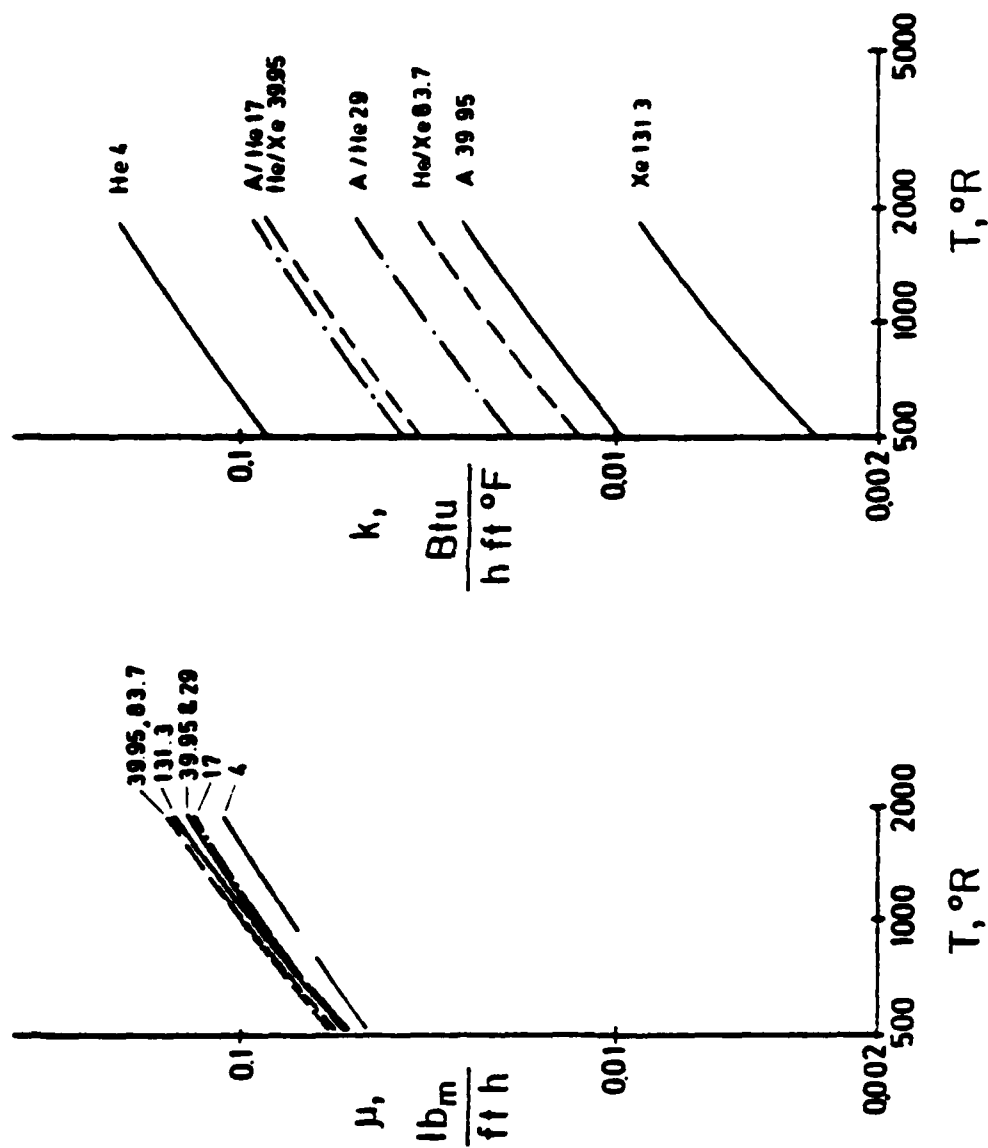


Figure 2. Property variation of gas mixtures. IMcElligot, Taylor and Durst, Int. J. Heat Mass Transfer, 1977).

as a fair approximation, most mixtures can be represented by the perfect gas idealization for the density.

A surprise to many investigators is the resulting variation in the Prandtl number. Since it is  $2/3$  for the pure noble gases and approximately 0.7 for other common gases, most expect that it would be approximately the same value for their mixtures. Figure 3 shows that not to be the case. Typically the curves for binary mixtures show a decrease in the Prandtl number as the molal mass is increased from that of the lighter of the two components until a minimum Prandtl number is reached and then there is a gradual increase back to the value for the other pure component. Our studies so far have shown that the minimum decreases as the ratio between the molal masses of the heavy and light components increases. Although only the argon/helium and xenon/helium systems are shown in the figure, other mixtures we have investigated yield curves of the same shape. A mixture of hydrogen and carbon dioxide yields a minimum Prandtl number of about  $1/3$  [Serksnis, Taylor and McEligot, 1978] while experiments recently conducted in the author's laboratory were at a Prandtl number of about 0.16 with a mixture of hydrogen and xenon. One difficulty with conducting experiments with these mixtures, and therefore designing systems employing these mixtures, is that often basic property measurements for the mixtures desired are not readily available. In particular for the hydrogen/xenon system it is necessary to use predicted properties since only a few measurements have been taken of that particular binary system and those were only near room temperature [Taylor, Bauer and McEligot, 1982]. Prediction of transport properties is addressed by Kestin [1982] in his recent review.

While the Prandtl number varies substantially with the concentration of the mixtures, there is no great change as the temperature is changed from near room temperature to the order of 1000 K. This observation is a con-

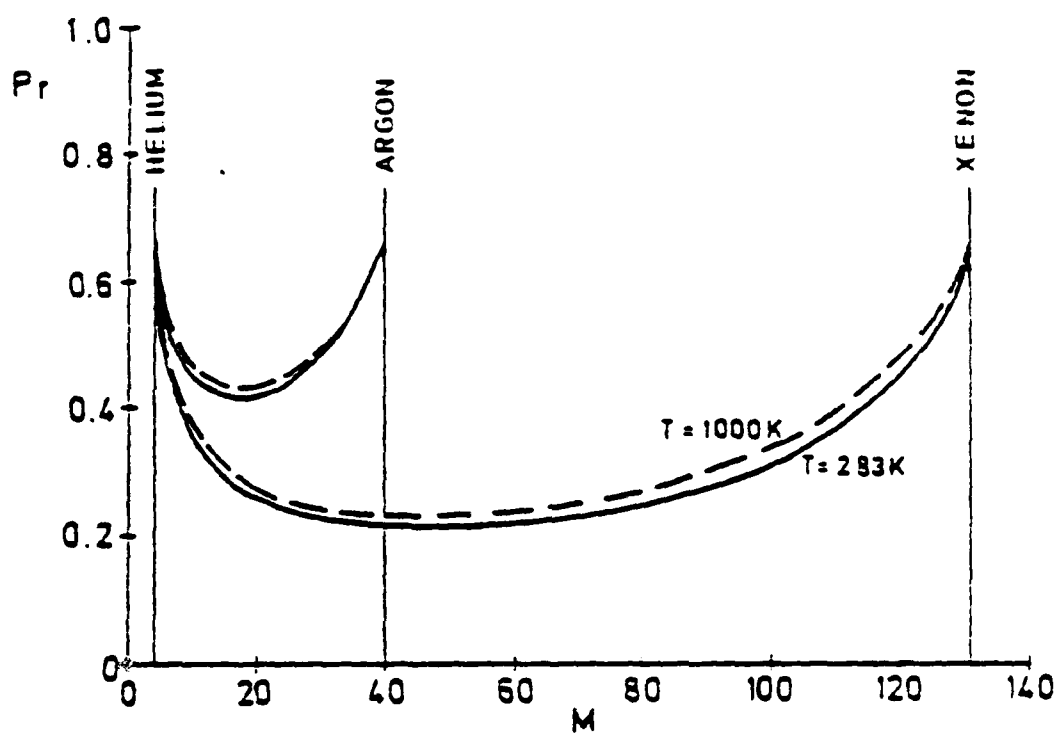


Figure 3. Prandtl number variation of gas mixtures. [McEligot, Taylor and Durst, Int. J. Heat Mass Transfer, 1977].



sequence of the earlier comment that exponents in a power law representation of thermal conductivity and viscosity are approximately the same.

### 1.3. Phenomena involved

By considering a situation with a strong heating rate, and therefore large temperature differences, one can note a number of phenomena which can be important in studies of the current sort. For convenience, the typical laboratory experiment involving an approximately constant wall heat flux is taken as an example. The heating increases the bulk temperature approximately linearly with axial distance,  $x$ , as shown in Figure 1. The wall temperature is higher in order to heat the gas; it increases from the inlet value of the bulk temperature, may go through a maximum [Perkins and Worsoe-Schmidt, 1965], and then at large distances downstream gradually approaches the same slope as the variation of bulk temperature with distance. Since the ratio between the wall temperature and the bulk temperature is often used for design correlations [Kays and Crawford, 1980], it is appropriate to consider its variation with position; at the start of heating the two are equal so the ratio is unity, then it increases to a maximum (typically at 10-20 diameters) and decreases downstream. Eventually this ratio approaches unity again as the bulk temperature becomes very high. Consequently, we have the paradox that, for a case of strong heating, the results far downstream could be expected to agree with the predictions of a constant properties analysis - if the tube does not melt first.

If one defines the Reynolds number based on bulk fluid properties as  $Re = GD/\mu_b$  where  $\mu_b$  is the viscosity evaluated at the bulk temperature, then its value will continuously decrease as the axial distance increases with heating. A perfect gas approximation would show the bulk density,  $\rho_b$ , like-

wise to decrease as  $x$  increases. By applying an integral continuity relationship, one can see that the velocity must continuously increase in the stream-wise direction with strong heating.

Consideration of the above trends shows that heating a gas at high power densities can cause large variations in the temperature dependent properties both along and across the duct. Since the thermal conductivity changes continuously, no fully established condition is exactly possible but, as noted above, eventually the constant properties idealization may be approached. The flow accelerates. This phenomenon usually provides a stabilizing influence [Schlichting, 1968]. In a turbulent flow, acceleration has been shown to reduce the apparent turbulent bursting rate [Chambers, Murphy and McEligot, 1982] near the wall. Such a reduction can be expected to lead to a thickening of the viscous layer of a turbulent flow and a consequent reduction in the convective heat transfer parameters. Since the bulk density and the density of the fluid at the wall can differ considerably, buoyancy effects can become important. These various effects can also be expected for situations with strong cooling, leading to the reverse of most of the trends described above, so one can expect variations of the same phenomena in that case as well.

The term viscous sublayer is defined variously by different investigators. In the present review the terminology suggested by Bradshaw [1971] is adopted. Thus, we define the linear sublayer as the region where molecular effects dominate the mean velocity profile to give  $u^+ \approx y^+$ ; for high Reynolds number, adiabatic flow the linear sublayer typically extends to  $y^+ \approx 5$ . The viscous sublayer is the region where both molecular effects and turbulent effects are significant; e.g., the range  $5 \lesssim y^+ \lesssim 30$  for high Reynolds number, adiabatic flow. The viscous sublayer then usually includes the region of maximum production of turbulence kinetic energy and its behavior is expected to

be important due to that observation alone. To relate positions in wall coordinates,  $y^+$ , to physical coordinates, it is worth recalling that  $(y/r_w) = y^+/y_c^+$ , where  $y_c^+$  is the numerical value of  $y^+$  at the centerline. With high heating rates the bounds of the linear and viscous sublayers can change from the familiar values expected from isothermal measurements.

An irritating result of the variation of properties is that such familiar non-dimensional governing parameters as the Reynolds number become ambiguous. One must be careful in applying correlations offered by various authors to be sure that the definition of the Reynolds number is the same as that used by the authors when they developed the correlation. One sees wall, film and bulk versions of Reynolds numbers plus modifications thereof. The definitions are generally interrelated so that one can be calculated from another provided the investigator has available the values of the wall temperature and the bulk temperature at the point of concern [McEligot, 1967]:

$$\begin{aligned} Re_b &= \frac{\rho_b V_b D}{\mu_b} = \frac{GD}{\mu_b} ; & Re_f &= \frac{GD}{\mu_f} = Re_b \frac{\mu_b}{\mu_f} \\ Re_w &= \frac{GD}{\mu_w} ; & Re_{wm} &= \frac{V_b D}{\nu_w} = \frac{\rho_w V_b D}{\mu_w} = Re_b \frac{\mu_b \rho_w}{\mu_w \rho_b} \end{aligned}$$

and so forth. As pointed out by Ward Smith [1962], use of bulk properties is usually most convenient for the designer in a situation where the wall heat flux distribution is specified because it can be determined locally by an energy balance from the start of the heating. Thus, the bulk Reynolds number, defined above, is usually easiest to use but does not necessarily collapse the data into the most convenient correlation.

In order to provide some depth to this review, the topics treated will primarily emphasize single phase flow (gas) at a sufficiently low Mach num-

ber that the effects of compressibility and viscous dissipation are small. Predominantly forced convection will be emphasized and the idealized problem will usually be two-dimensional, either a circular tube with symmetric thermal boundary conditions or the hypothetical case of a parallel plate duct of infinite width with symmetric heating. In general we will not be concerned with any significant secondary flows induced by buoyancy so that the orientation of the duct would not be important. However, most of the experiments reported have been conducted with vertical tubes with the flow upwards. For the most part the experiments and analyses conducted to date have studied smooth surfaces and this review will do likewise.

The reader is referred to a number of earlier reviews which provide introductions to the current topic; in some cases much more information is given on some of the additional topics which are pertinent to duct flows but are not being considered in this review. For duct flows in general there are references such as the survey by Miller [1978] which is recommended to the designer and the compendium by Ward Smith [1980]. The most extensive compilation for duct flow with heat transfer in general is probably the contribution of Kays and Perkins [1982] to the Handbook of Heat Transfer which is now under revision. For the problem of convective heat transfer with property variation, Chapter 14 of the text by Kays and Crawford [1980] is recommended to the unfamiliar reader as an introduction to the present review. Earlier reviews of the effects of variable properties have been provided by Petukhov [1970], specifically for turbulent flow in circular tubes, and by Leontiev [1966] for the turbulent boundary layer. Heat transfer to fluids near their critical points has been studied by Hall's group at Manchester; these studies showed that one of the most important factors was buoyancy and several surveys of these two topics have been presented by his group [Hall, 1971; Hall and Jackson, 1978]. In order to be able to ignore buoyant effects entirely for heated flow in a vertical tube Adebiyi and Hall suggest that the quotient

$Gr_b Re_b^{2.7}$  be less than about  $5 \times 10^{-6}$  where the Grashof number is defined as  $(\rho_b - \rho_w)gD^3/(\rho_b \nu_b^2)$ .

The topic of laminarization, or the so-called reverse transition from an initially turbulent flow to one appearing to be laminar, has been summarized by Narasimha and Sreenivasan [1979]. Rodi [1980] recently reviewed advances in the modeling of turbulent flows and demonstrated the state-of-the-art for a wide range of applications. The Proceedings of the International Heat Transfer Conference, held in München in September 1982, include several survey papers related to the present article: Gersten on advanced boundary layer theory, Patankar on numerical methods, Zukauskas on forced convection, Petukhov, Polyakov and Martynenko on mixed convection in ducts, Hirata, Tanaka, Kawamura and Kasagi on turbulent flows and Kestin on transport properties.

#### I. 4. Governing equations and parameters

The flow of a gas through a tube or annulus, being heated or cooled strongly, is usually described by the following internal boundary layer equations:

Continuity,

$$\frac{\partial(\rho u)}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} (\rho v r) = 0 \quad (I-1)$$

Momentum in axial direction,

$$\rho^u \frac{\partial u}{\partial x} - \rho^v \frac{\partial u}{\partial r} = -g_c \frac{dp}{dx} + \frac{1}{r} \frac{\partial}{\partial r} (\mu_{eff} r \frac{\partial u}{\partial r}) \quad (I-2)$$

Thermal energy,

$$\rho^u \frac{\partial H}{\partial x} + \rho^v \frac{\partial H}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{k_{eff}}{c_p} r \frac{\partial H}{\partial r} \right) \quad (I-3)$$

In these equations the dependent variables are the time mean values of the velocity components  $u$ , in the axial direction, and  $v$ , perpendicular to the wall, plus the enthalpy or temperature. Comparable equations written in rectangular coordinates are employed for the case of a wide duct constructed of parallel plates.

In an internal flow the pressure distribution cannot be determined in advance. Since under the boundary layer approximations pressure only varies with streamwise distance, the additional governing equation required must be one-dimensional; it is an integral continuity equation,

$$\dot{m} = \int_{A_{cs}} \rho u dA_{cs} \quad (I-4)$$

The idealizations implied in these governing equations are axial symmetry, negligible viscous dissipation, low Mach number, insignificant body forces such as gravity, and that the internal boundary layer approximations apply. When one compares these equations to their incompressible, or constant property counterparts, one finds that it has been necessary to retain the density variation in the continuity equation due to its variation with temperature. Further, even for laminar flow, the transport coefficients in the diffusion terms must be retained within the derivatives since they vary with temperature which, in turn, varies with position. With the thermal conductivity as a function of temperature, the thermal energy equation becomes non-linear and with the viscosity as a function of temperature the x-momen-

tum equation becomes coupled to the thermal energy equation. Thus, one has a parabolic system of three coupled, partial differential equations and one integral equation to solve for  $u(x, r)$ ,  $v(x, r)$ ,  $H(x, r)$  and  $p(x)$ . For turbulent flows the effective viscosity,

$$\mu_{\text{eff}} = \mu + \mu_{\text{turb}} \quad (\text{I-5})$$

and the effective thermal conductivity,

$$k_{\text{eff}} = k + k_{\text{turb}} \quad (\text{I-6})$$

must be specified by alternate means. These may also involve the solution of partial differential equations for turbulence quantities when advanced turbulence modeling is employed [Launder and Spalding, 1972].

Property relationships for the gas may be represented by a perfect gas approximation,

$$\frac{p}{\rho} \approx RT \quad (\text{I-7})$$

or with a compressibility function,

$$\frac{p}{\rho} = ZRT \quad (\text{I-8})$$

The variations in the transport properties can be represented by power law relationships

$$\frac{\mu}{\mu_i} = \left( \frac{T}{T_i} \right)^a, \quad \frac{k}{k_i} = \left( \frac{T}{T_i} \right)^b, \quad \frac{c_p}{c_{p,i}} = \left( \frac{T}{T_i} \right)^d \quad (\text{I-9})$$

or by other functions or, in the case of numerical solutions, by tabulated functions. Temperature is deduced from the enthalpy via the definition of the specific heat at constant pressure,  $c_p = (\partial H / \partial T)_p$ , in conjunction with the low Mach number assumption, i.e.,  $\Delta p / p \ll 1$ .

Boundary conditions usually employed are the no-slip condition at an impermeable wall and either a specified wall heat flux or a specified wall temperature. Most idealized analyses, and the experiments to test them, have assumed symmetry across the center plane of a rectangular duct or axial symmetry in tubes and annuli. Initial conditions would include specified temperature and velocity profiles existing at the start of the heating, or cooling, and an initial pressure.

Dimensional analysis of the above equations reveals the parameters governing the solution to be  $Re_i$ ,  $Pr$ ,  $a$ ,  $b$ ,  $d$ ,  $R$  or  $M$  (which disappears from the problem if the pressure is approximately constant along the duct) and a heating rate parameter evolving from the thermal boundary condition.

Usually these parameters are evaluated at the initial condition but, as noted above, the Prandtl number does not vary significantly with temperature. Thus, the Reynolds number would be defined as

$$Re_i = \frac{GD}{\mu_i} = \frac{4 \dot{m}}{\pi D \mu_i} \quad (I-10)$$

and the viscosity would be evaluated at the entering bulk temperature,  $T_{bi}$ . For turbulent flow the non-dimensional heating rate is often taken as

$$q^+ = \frac{q_w}{G c_{pi} T_i} \quad (I-11)$$



and, as a consequence of non-dimensionalizing as in the Graetz problem, the parameter

$$Q^+ = \frac{q_w'' r_w}{k_i T_i} \quad (I-12)$$

is appropriate for laminar flow [Worsoe-Schmidt and Leppert, 1965]. For the property variation parameters, a and b are typically of the order of 0.7 to 0.8, while d is of the order of 0 to 0.1 and is often taken to be zero.

With the resulting distribution, the analyst can calculate the local convective heat transfer coefficient from "Newton's Law of Cooling",

$$q_w''(x) = h(x) [t_w(x) - t_b(x)] \quad (I-13)$$

and a local friction factor, f, with any definition he wishes. In reducing data from an experiment, designed to test such analytical predictions, one can use the adiabatic wall temperature,  $t_{aw}$ , in place of the bulk temperature to account for slight Mach number effects. However, since most experiments to date have been conducted in ducts of very small dimensions, the experimenter does not have the luxury of choosing his representation of the friction factor indiscriminately.

Without measured profiles the experimental data are one-dimensional, varying with streamwise position x. The flow accelerates or decelerates so momentum terms must also be included in the balance used to evaluate a local value of the wall shear stress and, in turn, a local friction factor [McEligot, Smith and Bankston, 1970]. In the one-dimensional momentum balance the term for change of momentum becomes  $d(G^2 A_{cs}/\rho_b)/dx$ , which is not the same as the value obtained by integrating the velocity and density

profiles; so one only deduces an apparent wall shear stress,

$$\tau_{w,app} = - \frac{D_h}{4} \frac{d}{dx} \left[ p + \frac{G^2}{\rho g_c} \right] \quad (I-14)$$

The result is that the experiment usually provides either a local or an averaged value of an apparent friction factor  $f_{app} = 2g_c \tau_{w,app} / (G^2/c)$ , for comparison to the predictions of the analyses. On the other hand, most analyses present  $f_s$ , with  $\tau_w$  evaluated from the slope of the mean velocity profile at the wall. The two definitions can yield significantly different values. It would be desirable if the analyst would provide predictions phrased in terms of the same definitions used by the experimenter in order to avoid some unnecessary differences. An alternate form of result would be a non-dimensional pressure defect

$$\bar{p} = \rho_i g_c (p_i - p) / G^2 \quad (I-15)$$

which is easily deduced by both the analyst and the experimenter, and is probably the information desired by the designer.

Kays and Crawford [1980] emphasize the presentation of results as correlations in terms of either (1) a reference temperature (such as the familiar film temperature,  $T_f$ ) or (2) a property ratio parameter, such as  $T_w/T_b$ . In the reference temperature method one attempts to define a means of determining a temperature at which the properties can be evaluated so that the results of constant property analyses can be applied directly. In the property ratio method for gases, one modifies the constant property correlations by multiplying them by the wall-to-bulk temperature ratio raised to some power giving predictions of the form

$$\frac{Nu}{Nu_{cp}} = \left( \frac{T_w}{T_b} \right)^n \quad \text{and} \quad \frac{f}{f_{cp}} = \left( \frac{T_w}{T_b} \right)^m \quad (I-16)$$

The friction factor usually employed is the so-called "Fanning" friction factor based on the wall shear stress rather than the pressure drop.

As shown by Kays and Crawford, a number of investigators have put considerable effort into the problem of determining slight differences in the exponents  $n$  and  $m$ . Such differences in the values of exponents are probably not warranted when one considers the experimental uncertainties of the experimental data which are being correlated [Kline and McClintock, 1953]. For example at  $T_w/T_b = 2.5$  the difference between an exponent of 0.4 and 0.5 is less than 10 %. Unfortunately, too few measurements are presented with realistic estimates of the experimental uncertainties included. These techniques are generally appropriate only for downstream results and, even then, one cannot generalize from one geometry or thermal boundary condition to another or conclude that a property ratio method is better or worse than the reference temperature method. One must also be cautious when employing these techniques to make sure that the constant property portion of the correlation is evaluated in terms of parameters defined as the original author intended; it is easy to confuse wall Reynolds numbers and modified wall Reynolds numbers, etc., and the subscripting is not always sufficient to identify which is intended.

#### 1.5. Conspectus

The remainder of the review will be organized as follows. First, laminar flows, which are easiest to predict and most difficult to measure, will be discussed. Then the problems of treating turbulent flows are considered followed by the perplexing topic of laminarization - transition from a turbulent flow to a state which may be predicted as if there were no turbulent transport present. The review concludes with a few generalizations that seem pertinent.

Studies in the program at the University of Arizona, and earlier at Stanford University, are emphasized to illustrate the historical development of the topic and to demonstrate the interplay of experiment and analysis (numerical) usually necessary. Reference to other work supplements the discussion as appropriate and provides a further indication of the state of the art, but the reference list is not intended to be all inclusive; readers interested in a specific subsidiary topic - such as a particular geometry - should treat this review as a starting point in their own search.

## II. Laminar flow

### II.1. Heated tubes

Provided all pertinent phenomena are included, convective heat transfer to internal laminar flow can be predicted unambiguously because the effective viscosity and effective thermal conductivity are reduced to their molecular values. Thus, equations (I-1) - (I-4) can be solved numerically from the initial condition to a position sufficiently far downstream that the results begin to be described by constant property results (because the temperature ratio approaches unity).

The early predictions taking into account the effect of physical property variation considered a hypothetical fully developed situation, typically with variation of only one property - the viscosity. Thus, the emphasis was more on the heating and cooling of liquids than gases [Lee et al., 1938; Yamagata, 1940; Pigford, 1955]. Deissler [1951], and later Bradley and Entwistle [1965], added variation of the thermal conductivity. It is from these early works that the figure familiar in many texts, demonstrating the distortion of the velocity profile due to variable viscosity, has propa-

gated. Presumably it is reasonable for the cooling and heating of liquids, as presented by Pigford, but should not be taken seriously for gases because it does not account for the acceleration induced by the change in density. In a gas this acceleration induces distortions in the opposite direction from those due to the viscosity variation and they tend to dominate. While Pigford briefly mentioned effects on gases, he concentrated on liquids and introduced density variation in a buoyancy term in order to account for effects of mixed convection; comparison to data for water and oils showed reasonable agreement.

Experiments with strongly heated or cooled flow through circular tubes demonstrated the need for more complete solutions in the case of flow of gases [Kays and Nicoll, 1963; Davenport and Leppert, 1965]. The experiments of Davenport and Leppert were conducted in an electrically heated tube, giving an approximately constant wall heat flux except for heat losses; an unheated entry length provided for development of a parabolic velocity profile before heating, as in the case of the idealized thermal entry analyses available for constant properties. These investigators found that when the local heat transfer parameters were evaluated at the local bulk temperature, there was no large difference between the predictions of a constant property analysis, such as by Sparrow, Hallman and Siegel [1957], and the data. On the other hand, the local apparent friction factor was found to increase substantially as the wall-to-bulk temperature ratio was increased,

$$f \cdot Re_b \approx 16 (T_w/T_b)^{1.4}. \quad (\text{II-1})$$

Based on the collation of a large number of experiments, Taylor [1967] showed that the constant properties prediction worked reasonably well if the modified wall Reynolds number was employed and the density in the friction factor was evaluated at the bulk temperature,

$$f \cdot \text{Re}_{\text{wm}} \approx 16$$

(II-2)

for heating of helium, hydrogen, nitrogen and air at temperature ratios to about four and  $\text{Re}_{\text{wm}} \lesssim 3000$  (which includes some data with  $\text{Re}_b$  up to 22,000). For these gases his correlation can be transformed to  $f \cdot \text{Re}_b \approx 16 (T_w/T_b)^{1.7}$ , which is about twenty percent higher than Davenport and Leppert at  $T_w/T_b \approx 2$ .

The first reasonably complete solution for laminar heat transfer to a gas, flowing in a tube with property variation dependent upon the gas temperature, was developed by Worsoe-Schmidt and Leppert [1965]. They employed a numerical solution of the coupled partial differential equations governing the thermal entry problem (e.g., equations (I-1) to (I-4) with the turbulent transport suppressed). The fluid considered was air and the authors concentrated on heating with a constant wall heat flux after the flow had become fully developed in an unheated starting region. Air properties were represented in terms of power law relationships in terms of the absolute temperature. Effects of buoyancy forces were also considered in order to treat superposed free and forced convection and one additional solution for a constant wall temperature was obtained but was not emphasized. Results presented include the velocity and temperature distributions and, of more interest to design engineers, the axial variation of the local Nusselt number. When both the Nusselt number and the local Graetz number were based on the local bulk properties, the differences from constant property predictions were slight, with the greatest difference occurring in the immediate thermal entry region ( $Gz \gtrsim 0.06$ ). Noticing a regularity in the entry region behavior for pure forced convection, McEligot and Swearingen [1966] later suggested a correlation of these numerical results. The effect on the local value of the friction factor, defined in terms of the wall shear stress based on velocity gradient at the wall, was found to be approximately proportional to the local

wall-to-bulk temperature ratio,

$$f \cdot \text{Re}_{\text{bx}} = 16 (T_w / T_b) \quad (\text{II-3})$$

In their numerical solution Worsoe-Schmidt and Leppert provided a more realistic description of the flow field than their predecessors. Their approach led the way to the typical current technique for prediction of laminar heat transfer in ducts with variable fluid properties.

At first it appeared that the numerical predictions of friction factors by Worsoe-Schmidt and Leppert disagreed with the measurements of Davenport and Leppert. Kays and Crawford [1980] attribute the difference to difficulty in the experiment. Typically velocities are low in laminar experiments so that pressure differences along a tube are small. Likewise, low velocities lead to low convective heat transfer coefficients so the heat losses from the tube may make it difficult to deduce the heat flux to the gas and, consequently, the local bulk temperature unless very careful precautions are taken. However, there is a further important explanation: the difference in definitions of  $f$  employed by Worsoe-Schmidt and Leppert and by Davenport and Leppert as introduced in Section I.4. Using an analysis in some ways comparable to that of Worsoe-Schmidt and Leppert, Bankston and McEligot [1970] extended the range of heating rates involved in order to exaggerate the differences in the definitions. The pertinent results are presented as Figure 4. The friction factor defined in terms of the velocity gradient at the wall ( $f_s$ ) follows the trend of the temperature ratio approximately as suggested by Worsoe-Schmidt and Leppert. It is seen, however, that the predicted value of  $f_{\text{app}}$  - the apparent friction factor (defined as used by the experimenters via equation I-14) - is higher than  $f_s$ , corresponding to a higher value of the exponent in a property ratio representation. Thus, the agreement between the experiment and numerical predictions is better than originally believed.

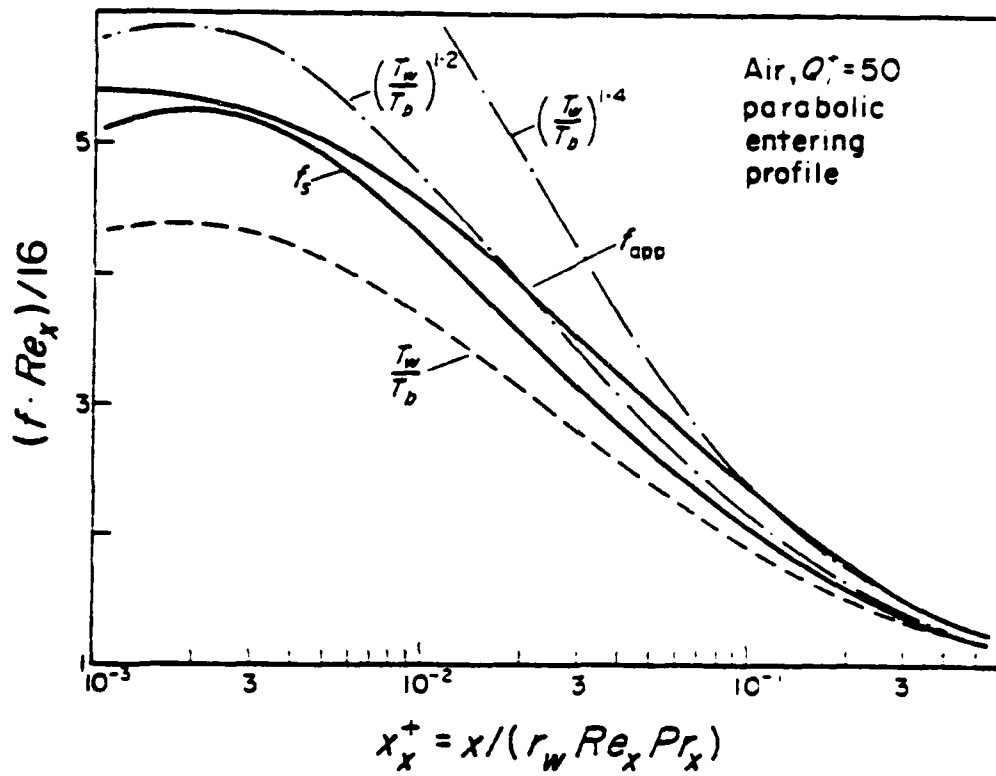


Figure 4. Friction in laminar flow of gases in tubes with property variation.  
[Bankston and McEligot, Int. J. Heat Mass Transfer, 1970].



Worsoe-Schmidt [1966] extended his numerical program to include the properties of helium and carbon dioxide in order to examine the effects of physical properties differing from those of air. Helium was chosen to represent the behavior of monatomic gases and carbon dioxide because the temperature dependencies of its transport properties differs markedly from those of air. He employed power laws for the thermal conductivity and viscosity of helium with exponents  $a = b = 0.65$ ; the specific heat and, therefore, the Prandtl number were independent of temperature. For carbon dioxide the viscosity was represented by Sutherland's law and the thermal conductivity was expressed by an empirical expression of the values suggested by Hilsenrath et al. [1955] and Vines [1960]. Again a constant wall heat flux was chosen as the thermal boundary condition after the establishment of a parabolic velocity profile in an unheated entry. Solutions were computed with inlet Mach numbers sufficiently small that expansion work and viscous dissipation were negligible throughout the tube. The computations were carried out for conditions corresponding to pure forced convection and to significant buoyancy forces in the thermal entry region. For carbon dioxide only temperatures below the onset of dissociation were considered. Results for local Nusselt numbers and local friction factors are tabulated in the publication. It was found that only slight differences existed between air and helium whereas carbon dioxide calculations gave up to 18 % higher values in the entrance region. Also, reduced heat transfer resistance for carbon dioxide apparently led to a faster approach to the asymptotic value. Worsoe-Schmidt correlated the heat transfer predictions for air and helium as follows:

$$Nu_{bx} = Nu_{cp}(Gz_b) + 0.025 \sqrt{Q^+} (Gz_b - 3) (Gz_b - 20) / Gz_b^{3/2} \quad (II-4)$$

for  $3 < Gz_b < 1000$  and  $0 \leq Q^+ \leq 20$ ,

with  $Nu_b = Nu_{cp}(Gz_b)$  for  $Gz_b \geq 3$ .

For moderate values of the temperature ratios, the friction factors for both helium and carbon dioxide agreed well with the earlier expression for air. However, at higher heating rates with carbon dioxide, the friction factor showed a somewhat stronger dependence on the heating rate than with air.

Incited by the work of Worsoe-Schmidt and by a visit from Spalding [Patankar and Spalding, 1967], in the late 1960's Bankston and McEligot developed a numerical program for the prediction of heat transfer to the turbulent and laminarizing flow of gases in tubes with strong property variation [McEligot and Bankston, 1969]. Their numerical technique was subsequently simplified and extended to a number of applications involving laminar flow. For a circular tube, Bankston and McEligot [1970] considered (1) constant fluid properties with variation of the hydrodynamic entrance length, (2) variable air properties with a high heating rate and parabolic initial velocity profiles, in order to examine the difference between the apparent friction factor and the friction factor deduced from the velocity gradient at the wall as discussed above, and (3) variable air properties with a uniform entering velocity profile. The thermal boundary condition was a constant wall heat flux in each case.

Presler [1971a] found that, when heating air, velocity profiles in the downstream section are more blunt than Poiseuille profiles because of continued flow acceleration in the region near the heated wall. He emphasizes that this result is exactly opposite from the predicted effect of heat transfer on gases as cited in most texts. He points out that the pioneer work of Deissler [1951] assumed a fully developed flow; this idealization eliminated the transverse velocity and axial velocity gradients from the momentum equation to be solved. Thus, in Deissler's work the heat transfer affected the flow only through the thermal variation of the fluid viscosity and thermal conductivity. Presler's numerical predictions for cooling [1971b] likewise refute

the earlier conclusions based on fully developed assumptions.

Shumway and McEligot [1971] extended the technique of Bankston and McEligot to flow in annuli and obtained solutions for a radius ratio of 0.25 for both fully developed and uniform velocity profiles at the entry. There are many combinations of boundary conditions possible in the annular geometry. Either wall may be heated with a specified heating rate distribution or a specified temperature distribution. The entering profile may be either uniform or fully developed for the purposes of analysis. In all, there are sixteen logical, "basic" combinations for any one radius ratio in any one fluid. For each of the sixteen cases, one set of predictions was obtained at a strong heating rate with air properties.

In contrast to flow in a circular tube, a meaningful fully established condition can exist in annuli while a significant transverse property variation still exists. (The same comment could be made for flow between parallel plates with the same boundary conditions.) With one wall held at a constant wall temperature  $T_w$  and the other at a constant wall heating rate, eventually the temperature and velocity profiles become invariant with distance. The convection terms drop out of the governing energy equation. Then it becomes uncoupled from the momentum equation. Thus, the problem becomes one of simple radial conduction with temperature-dependent thermal conductivity. The non-dimensional solution is

$$\frac{T}{T_w} = [(c_1 \ln \frac{r}{r_o} - c_2)(b - 1)]^{\frac{1}{b-1}} \quad (\text{II-5})$$

with the thermal conductivity represented as  $k/k_{\text{ref}} = (T/T_{\text{ref}})^b$ . The constants  $c_1$  and  $c_2$  depend on the magnitude of the wall boundary conditions. Once  $T(r)$  is determined, a non-dimensional bulk temperature, Nusselt num-

ber and friction factor can be calculated. However, in contrast to the constant properties idealization these results are not unique because the temperature profiles, and therefore the velocity profiles, depend on the gas properties and the heating rates.

Tabulations of the thermal entry results are provided in Shumway's dissertation [1969]. It was concluded that for flow in the thermal entry with a fully developed velocity profile at the start of heating the effects of property variation are comparable to those found in previous numerical analyses for circular tubes and parallel plate ducts. That is, the effects on Nusselt number are slight when phrased in terms of local bulk properties, but the friction factor varies significantly with the wall-to-bulk temperature ratio. However, for the "fundamental" annuli solutions the effect on friction factor is not as strong as for circular tubes since only the one wall is heated. Numerical experiments with fictitious gases showed that the dominant factors affecting heat transfer are the variation in density and thermal conductivity, but the effects are in opposite directions so that the change in  $Nu(x)$  is slight. For problems where the velocity profile is uniform when heating starts, the effects of property variation show the same trends, but the flow development dominates the problem at the entry so the effects of property variation are less striking than for heating a flow after the velocity profile has already become fully developed.

Laminar heat transfer data are difficult to obtain. Particularly difficult are measurements for comparison with predictions in the immediate thermal entry region. In order to verify that his numerical results did not overlook any significant phenomena for his application, Shumway [1969] modified his numerical program to the limit where the annulus became a circular tube; he then obtained data at high heating rates in a tubular apparatus designed to minimize experimental uncertainties. The computer program was

designed to accept varying wall heat flux or wall temperature distributions as input.

In the typical experiment, an electrode is attached to the tube so that the tube itself serves as the electrical resistance heater. Unfortunately, the electrode often acts as a thermal "sink" inducing axial conduction along the test section and thereby reducing the heat flux from the constant value intended. Variation of electrical resistivity due to its temperature dependence also leads to a varying volumetric energy generation rate while heat losses increase with tube wall temperature, which increases in the flow direction. The axial conduction in the tube wall also extends below the electrode, which increases in temperature itself by the same process, and thereby the tube wall is heated before the gas reaches the electrode. Thus, there is a thermal boundary layer already existing before the location where the idealized "start of heating" occurs. Consequently, there is usually considerable variation of the wall heat flux in the vicinity of the electrode where the heating would ideally start.

Numerical predictions for the idealized situation show the local Nusselt number to vary approximately in the same manner as the Leveque solution for a constant wall heat flux, i.e., proportional to  $x^{-1/3}$  [Worsoe-Schmidt, 1967]. Data commonly show a more gradual increase as  $x$  approaches zero (see Fig. 5). In order to overcome the discrepancy Shumway (1) employed the observed wall temperature variation as the thermal boundary condition in his program for the region before reaching the electrode and (2) then, beyond the idealized start of heating, changed to the wall heat flux variation deduced in the experiment. As shown in Fig. 5 this description yields better agreement with the data than the idealized situation. For this particular experiment, it is seen that by  $x^+ \approx 0.01$  both approaches are in reasonably close agreement. Measurements of the apparent friction factor agree well

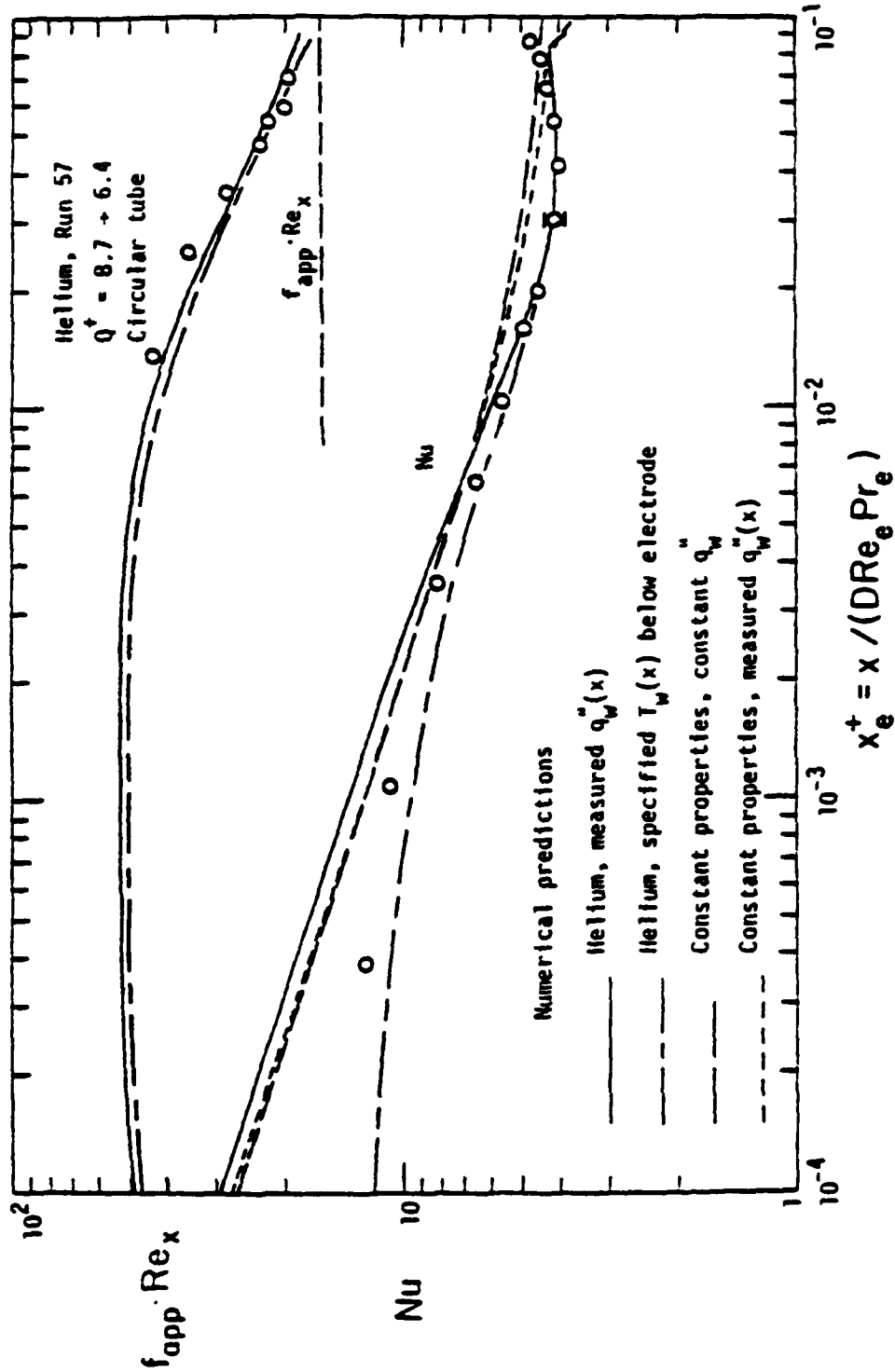


Figure 5. Comparison of measurements to predictions for thermal entry region in laminar gas flow with temperature-dependent properties. [Shumway and McEligott, Nuc. Sci. Engr., 1971].

with the predictions too, as also shown on Fig. 5.

Superposition is not strictly valid when the fluid properties vary significantly with temperatures since the governing energy equation then becomes non-linear. However, for laminar flow of gases and constant wall heating rates in circular tubes, solutions accounting for property variation do not differ from the eigenvalue solution based on properties provided the axial coordinate,  $x^+$ , is evaluated with the local bulk properties as indicated above. Thus, it seems likely that, even with properties varying, superposition might provide reasonable predictions if evaluated in this same manner. Bankston and McEligot [1969] examined this thought. They applied their numerical solution to predict the heat transfer parameters for sinusoidal heat flux distributions of successively higher maximum heating rates and compared the results to the predictions by superposing the constant property solution [Kays, 1966]. Agreement was surprisingly good. Discrepancies were within seven percent and the accuracy was adequate even for small axial distances. A possible explanation for this result is that the heat flux is small at short distances and, further, variation of fluid properties tends to shorten the thermal entry region when heating the fluid. Consequently, it appears that superposition by their equation 4 can be recommended, using the basic constant properties solution, for laminar flow of a gas for some axial heat flux distributions even with properties varying due to heating. However, the range of conditions for which the approach is accurate has not yet been examined extensively to the knowledge of this author.

## II.2. Cooling

Although current texts [Kays and Crawford, 1980] still discriminate between heating and cooling in correlations for property variation, the basic

difference is the thermal boundary condition. While most results in the previous section were for heating with a constant wall heat flux, cooling experiments typically involve equipment which imposes an approximately constant wall temperature, e.g., a water jacket. As Kays and Crawford show, heat transfer predictions for gases in laminar flow are sensitive to the choice of the thermal boundary condition.

The cooling of very high temperature gases by a liquid can usually be approximated by a constant wall temperature due to the relative thermal resistances and relative capacity rates [Kays and London, 1964], particularly if the liquid boils. Typical examples include the use of plasma generators for chemical synthesis, magnetogasdynamic generators, the nozzles of proposed gas core nuclear rockets, and waste heat recovery from gas turbine exhausts. Some of the most extreme temperature ratios between wall and fluid have been observed with gas flow through arc heaters, leading to property variation of more than an order of magnitude across the flow.

Discontinuities are not observed in the results when one conducts an analysis investigating both heating and cooling, e.g., see Fig. 3 by Schade and McEligot [1971]; there is a continuous variation. The differences in correlations for heating and for cooling seen in the texts are likely to be due to (1) differences in thermal boundary conditions and/or (2) differences in the ranges in which experiments have been conducted and correlated. Petukhov [1970] addressed the latter difficulty for turbulent flow.

In order to develop a rational prediction method which does not require distinguishing between "heating" and "cooling", Gersten [1981] applied a perturbation technique. To date he has attacked the flat plate boundary layer with a constant wall temperature and fully developed channel flow. He expands the functions representing the property dependence as series in a



perturbation parameter  $\epsilon$  and the non-dimensional temperature difference,  $\Theta$ , which is the dependent variable; for example,

$$\begin{aligned}\frac{\mu}{\mu_{\infty}} &= 1 + K_{\mu 1} \epsilon - \frac{1}{2} K_{\mu 2} \epsilon^2 + \dots \\ \frac{k}{k_{\infty}} &= 1 - K_{k 1} \epsilon - \frac{1}{2} K_{k 2} \epsilon^2 + \dots\end{aligned}\quad (\text{II-6})$$

where  $\epsilon = (T_w - T_{\infty})/T_w$  and  $\Theta = (T - T_{\infty})/(T_w - T_{\infty})$ . Truncating the series at the first two terms gives a linear representation of the property variation.

Gersten's predictions take the form

$$\begin{aligned}\frac{f}{f_{cp, \infty}} &= 1 + A_1 \epsilon + A_2 \epsilon^2 + \dots \\ \text{and} \quad \frac{Nu}{Nu_{cp, \infty}} &= 1 + B_1 \epsilon + B_2 \epsilon^2 + \dots\end{aligned}\quad (\text{II-7})$$

The perturbation solution gives the explicit form of the coefficients

$$\begin{aligned}A_1 &= (K_{\rho 1} + K_{\mu 1}) f_1(\text{Pr}_{\infty}) \\ B_1 &= K_{\rho 1} g_1(\text{Pr}_{\infty}) + K_{\mu 1} g_2(\text{Pr}_{\infty}) + K_{k 1} g_3(\text{Pr}_{\infty}) + K_{c 1} g_4(\text{Pr}_{\infty}) \\ A_2 &= f(\text{Pr}_{\infty}, K_{\rho 1}, K_{\mu 1}, K_{k 1}, K_{\rho 2}, K_{\mu 2})\end{aligned}\quad (\text{II-8})$$

Since these coefficients involve the property coefficients,  $K_{\mu 1}$ ,  $K_{k 1}$  etc., they can be evaluated directly from knowledge of the properties of the fluid.

The results can also be converted to the property ratio method, by

expanding in a binomial series,

$$\frac{f}{f_{cp}} = \left( \frac{T_w}{T_\infty} \right)^m \approx 1 + m\epsilon + \frac{m}{2} \epsilon^2 + \dots \quad (\text{II-9})$$

and comparing, to give  $m = A_1 + a_1 \epsilon_g$  where  $a_1$  is an explicit function of the coefficients,  $A_1$ . Thus, Gersten's approach determines the exponent for a representation by the property ratio method explicitly. Appropriate definitions of reference temperatures for the reference temperature method can also be determined explicitly by comparison between the results and the desired form of the correlation. The present results of Gersten, truncated at the first term, are probably useful primarily for including the effect of property variation for lower and moderate heat transfer rates. The predictions of the boundary layer analysis can be used for the entrance region of tubes or ducts with uniform entering profiles, but would become invalid at high heat transfer rates since the deceleration or acceleration which is induced would invalidate his assumption of a constant free-stream velocity. Application of this technique to fully developed channel flow is now in process.

The cooling of gases with extreme temperature differences has been measured by Wethern and Brodkey [1963], Massier, Back and Roschke [1969] and Schmidt and Leppert [1970]. In the latter experiment the gas was partially ionized. Back, Cuffel and Massier [1969] also provided some data from a laminar boundary layer at the entrance of a cooled tube with a turbulent core flow.

The numerical prediction program of Worsoe-Schmidt and Leppert [1963] was extended by Incropera and Leppert [1967] to treat laminar plasma flow in a circular tube and, in turn, was further modified by Schmidt and Leppert [1970]. The latter authors found that the analyses accurately predicted the measured heat transfer from the gas after five to ten diameters,

but in the first few diameters the predictions were highly sensitive to the shapes of the assumed initial enthalpy and velocity profiles near the wall.

Bender [1968] developed a numerical technique for both constant wall temperature and constant wall heat flux with developed and uniform entering profiles; results for  $0.1 \leq Pr \leq 30$  were presented for constant properties, but variable properties results emphasized water and oil. Shumway and McEligot [1971] developed solutions for constant wall temperatures and other boundary conditions in annuli, but only performed the calculated predictions for heating situations. As noted in the next section, Schade and McEligot [1971] provided a few tabulations of local Nusselt numbers and friction factors for cooling in a duct. Such numerical solutions are still being developed [Collins, 1980] for simple circular tubes.

In conjunction with the experiments of Massier, Back and Roschke [1969], Back [1972] developed a numerical technique for the treatment of a very high temperature gas (argon at  $T \approx 8000$  K) flowing in the entrance region of an externally cooled tube. His paper details some of the numerical difficulties encountered in solving the governing equations with severe cooling.

Back [1973] then applied his numerical technique for a range of cooling situations where the inlet velocity and total enthalpy profiles were considered to be uniform. A power law dependence was chosen for the relationship between the viscosity and the temperature; the Prandtl number was considered constant so that the thermal conductivity was taken to vary approximately as the 0.7 power of the static temperature. The thermal boundary condition was taken to be a constant wall temperature with wall-to-inlet enthalpy ratios of 0.04, 0.5 and 0.99, the last one corresponding to a nearly isothermal flow. Thus, a range from very strong cooling to very slight cooling was covered.

For the flow with the highest cooling rate, the core flow decelerates and causes a slight rise in pressure in the hot inlet region. When defined in terms of properties evaluated at the inlet temperature, for short distances ( $x^+ \lesssim 0.01$ ) the friction factor decreases relative to isothermal flow and agrees reasonably well with predictions from a laminar boundary layer analysis, also by Back [1968]. Near the wall the velocity profile was steeper for a highly cooled flow than for a nearly isothermal flow, but it was less steep for a moderately cool flow - presumably due to the smaller magnitude of the velocity in the core region compared to a nearly isothermal flow. Thus, the flow field was found to be strongly influenced by heat transfer. On the other hand, the average heat transfer coefficient did not vary much with wall cooling; Back hypothesized that the heat flux normalized by the driving potential was relatively insensitive to the amount of cooling and to acceleration or deceleration in the inlet region. These heat transfer observations are consistent with those of Kays and Nicoll [1963] who obtained comparable measurements with an approximately fully established velocity profile as the inlet condition.

### II.3. Non-circular ducts

The most extensive compilation of results for heat transfer and wall friction for laminar forced convection in ducts of a wide variety of shapes was presented by Shah and London [1978]. Only constant fluid properties were considered but this reference should be consulted by the reader for laminar flow in any duct of complex shape when considering the limiting condition of low temperature differences; it also provides reference conditions when attempting to predict results for situations where gas property variation is significant.

Recently, Dalle Donne [1982] has found that the friction correlations of Dalle Donne and Bowditch [1963] and Taylor [1967] for downstream flow in circular tubes can be extended to cylinders of other cross sections. Therefore,

the friction results of Shah and London could be expected to give useful predictions for long ducts with property variation if the modified wall Reynolds number is used with the circumferentially-averaged wall temperatures.

Kettleborough [1967] developed an iterative numerical program for the solution of laminar flows through a parallel plate channel of very small spacing relative to the length. The problem was related to air flow through gas bearings. Although including the convective terms in the governing equations, the grid spacing in the axial direction was sufficiently large that Kettleborough concentrated on a fully developed downstream condition. He examined the effect of various terms in the equations including compressibility, the convective terms and temperature dependent properties as well as considering several thermal boundary conditions. Since the results were not generalized they are primarily useful to demonstrate various effects which may be important in that application.

Schade and McEligot [1971] converted the computer program of Bankston and McEligot [1970] to flow between infinitely wide parallel plates and considered heating and cooling of air with a constant wall temperature and uniform inlet conditions as well as the heating of air at a constant wall heat flux and uniform entering velocity profile. Tabulations of the heat transfer and frictional results are presented in the publication. A comparable study is reported by Christian and Hitchcock [1971]. For flow between parallel plates with a fully developed velocity profile at the entrance and a constant wall heat flux, Swearingen and McEligot [1971] obtained solutions by an alternate numerical technique. In addition to air, calculations were conducted at one heating rate for both helium and a fictitious, compressible, perfect gas which had constant transport properties but varying density.

In order to improve the performance of closed gas turbine cycles, it has been suggested that inert gases be mixed to optimal concentrations for thermal efficiency or power-to-weight ratio. Mixtures such as helium and argon or helium and xenon have been considered. As noted earlier, the Prandtl number of such mixtures can be considerably less than the value of

0.7 associated with most common gases. For application to the regenerative heat exchanger (which could be a plate-fin configuration), McEligot, Taylor and Durst [1977] extended the numerical program of Schade and McEligot [1971] to lower Prandtl numbers and concentrated on flows with simultaneously developing boundary layers. Since the flow rates and specific heats are approximately the same on both sides of a regenerative heat exchanger, the appropriate thermal boundary condition is a constant wall heat flux which may be positive or negative depending on which side of the surface one is considering. The non-dimensional heat flux,  $Q^+ = q_w'' D_h / k_i T_i$ , was varied from -2 to 100. Results were presented as average coefficients for the heat exchanger application but tabulated values of the local parameters were also included. The technique of Schlünder [1974] was used to correlate the results.

McEligot, Taylor and Durst also used the results to examine the possibility of improving the exponents used in a property ratio method for describing the heat transfer and wall friction parameters for the parallel plate geometry. Plotted results are presented in terms of bulk properties and in terms of film properties in Fig. 6a for the heat transfer predictions and Fig. 6b for wall friction. The constant property results for the heat transfer show a slight dependence on Prandtl number in the thermal entrance due to the difference in the growth rates of the thermal and momentum boundary layers but, when using bulk properties, the predictions show only a very slight effect of property variation at a given Prandtl number. On the other hand, the Nusselt number predictions in terms of film properties show a significant decrease in the predicted Nusselt number as the heating rate increases.

The wall friction results are another story, particularly at low and moderate heating rates. No benefit was found in attempting to define the non-dimensional length in terms of average film or average bulk properties. When

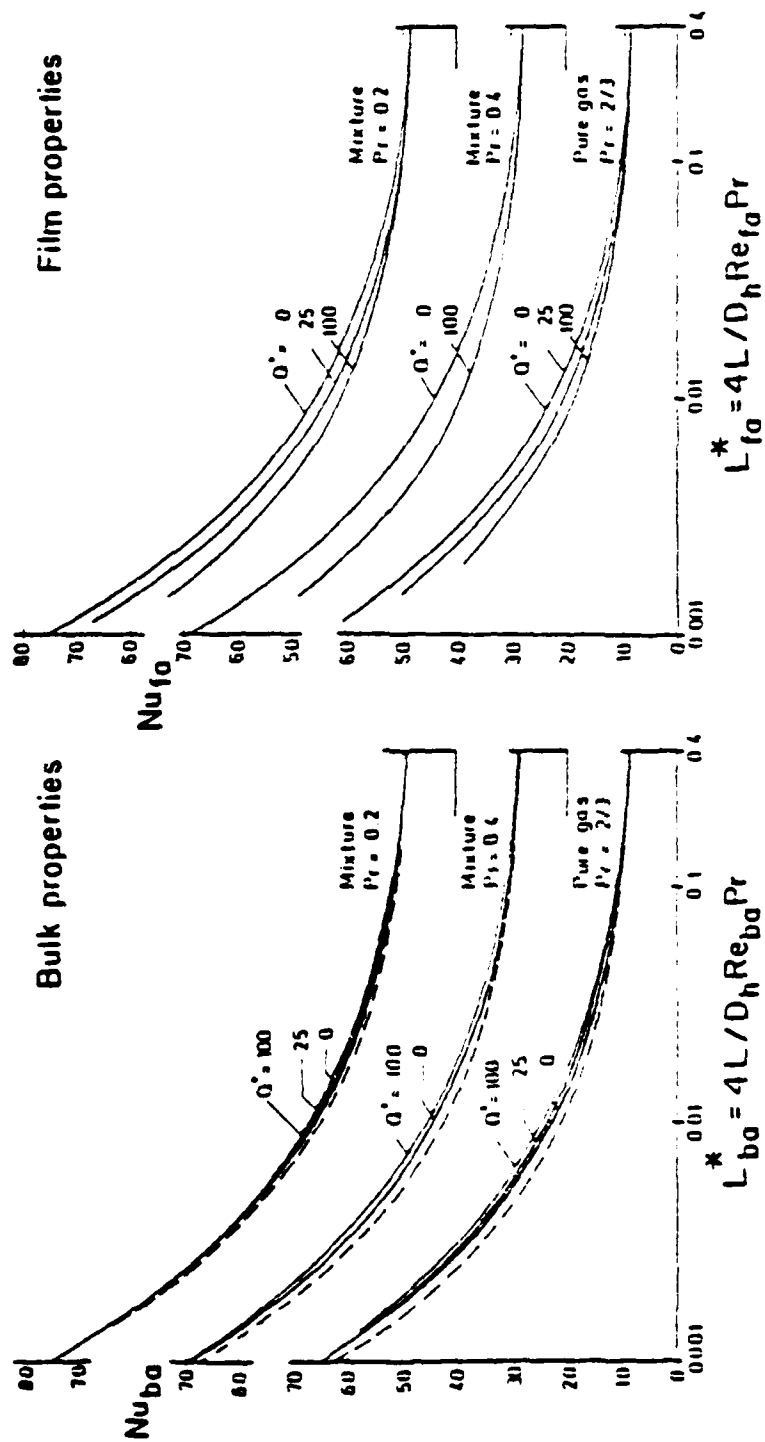


Figure 6. Laminar flow of mixtures between parallel plates with effect of property variation. [McHigot, Taylor and Durst, Int. J. Heat Mass Transfer, 1977].

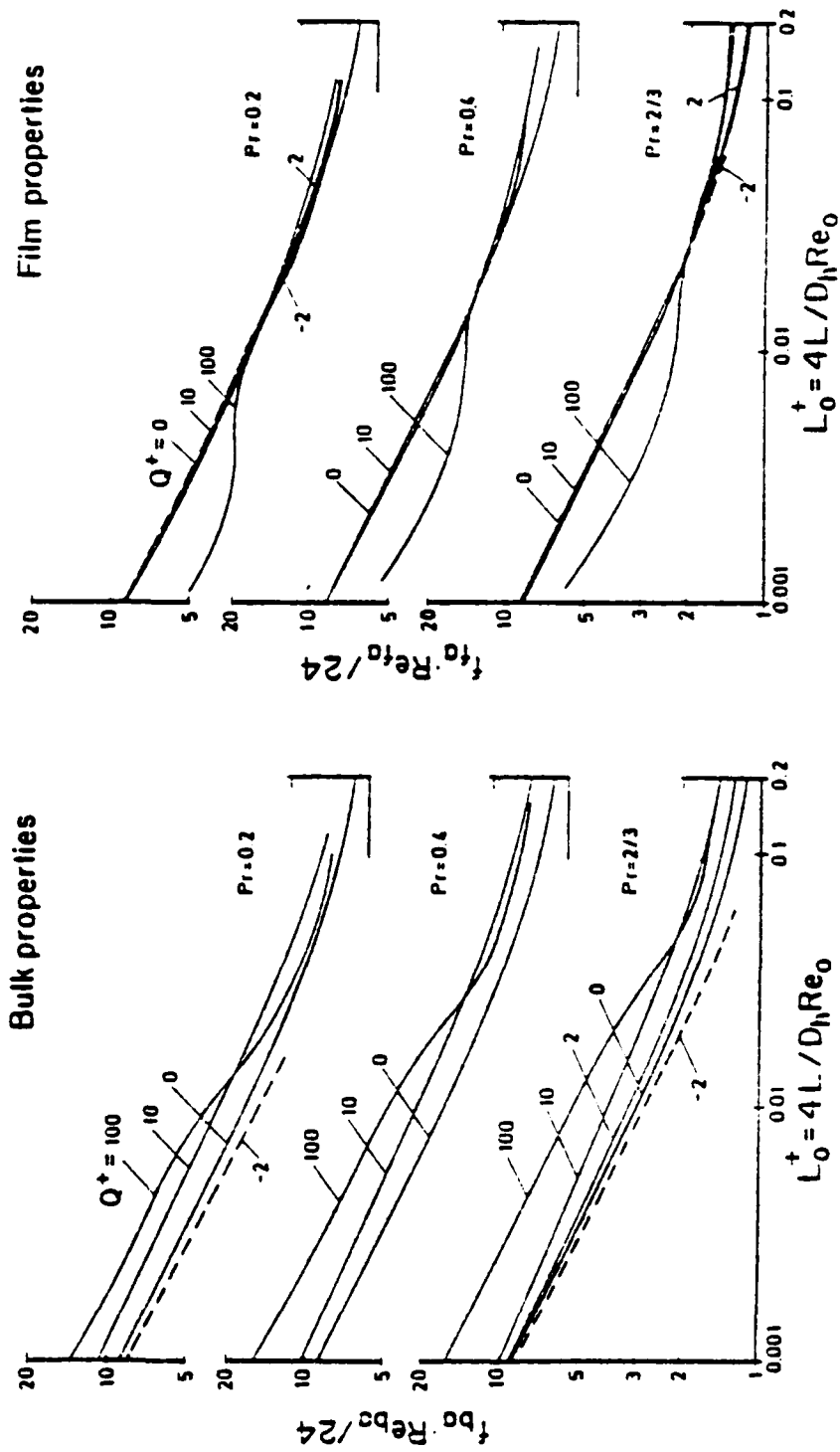


Figure 6---continued.



using the entry Reynolds number in defining the length, it was found that the average apparent friction factor predictions collapsed quite well on the constant property results if a film temperature reference was used for the properties whereas a large variation was observed if bulk properties were used. In terms of a property ratio representation for friction, the exponent would be of the order of one to two depending on the heating rate and the non-dimensional length. It would be of interest to examine the earlier results in other geometries and for other thermal boundary conditions to determine whether a comparable simplification for the apparent friction factor would apply in terms of film properties; to the author's knowledge this question has not been examined.

The correlations recommended by McEligot, Taylor and Durst for the average parameters are

$$Nu_{ba} = [8.235^2 - 1.931^2 / (Pr^{0.254} L_{ba}^*)]^{1/2}$$

and

(II-10)

$$f_{fa} \cdot Re_{fa}^{1/24} = [1 - 0.0788/L_i]^{1/2}$$

where the subscript a indicates an overall average and b or f represents the temperature at which the properties are evaluated (e.g.,  $T_{ba}$ ).

For heat transfer an exponent of 0.005 could be applied with a temperature ratio method but the effect is too small to be significant.

While conceptually one may predict numerically flow with property variation in non-circular ducts, such as those with rectangular cross sections, it becomes a three-dimensional problem requiring considerably more computer storage and time than for the flow in a tube or between infinitely wide parallel plates. Accordingly, to consider the flow in very small square ducts formed in commercial practice, K.R. Perkins, Schade and McEligot [1973] performed experiments instead. For such geometries the commercially

prepared tubing does not have infinitely small corner radii, so a numerical solution would require some significant modification of the simple rectangular grid in order to describe the heat transfer in the corner regions. The thickness of the resistively heated tube was such that the wall conduction parameter showed the experimental boundary conditions to approximate the analytical idealization of a specified axial heating rate with locally constant temperature around the periphery of the duct. Again the velocity profile was allowed to develop before the start of heating in order to approximate conditions for which constant property solutions would provide limiting values. Perkins, Schade and McEligot adapted the Leveque solution for a constant wall heat flux [Worsoe-Schmidt, 1967] to the square tube by estimating an average peripheral Nusselt number, with the wall velocity gradient taken as a value corresponding to the average peripheral friction factor, giving

$$Nu \approx 0.652 (f \cdot D_h)^{1/3} [2x/(D_h Re_{D_h} Pr)]^{-1/3} \quad (II-11)$$

Further downstream a numerical solution by Montgomery and Wibulswas [1966] was available for comparison. Their analysis appeared to be confirmed for  $x^+ > 0.01$ , the lowest value they presented. Over the range of heating rates for which experiments were conducted,  $1.1 \leq Q^+ \leq 2.5$ , it was concluded that, in terms of local bulk properties, the effects of helium property variation were a slight increase of the Nusselt number in the thermal entry and a negligible effect downstream, as with the numerical results for symmetric circular tubes. The local data showed the apparent wall friction factor to agree reasonably well with the correlation of Davenport and Leppert [1965], but a slightly lower exponent could be justified.

### III. Turbulent flow

#### III.1. Constant property conditions

Petukhov [1970] reviewed the status of heat transfer and wall friction in fully developed turbulent pipe flow. Both constant properties and variable physical properties were considered; his extensive review is recommended to the readers as an introduction to the present section of the current review.

##### III.1.a. Correlations

The limiting situation for heat transfer to gases with property variation is that of very low temperature differences where the results can be predicted via the constant property idealization. Webb [1971] has examined a range of data for fully developed turbulent flow in smooth tubes; he concluded that a relation due to Petukhov and Popov, given below, provides the best agreement with the measurements. Webb also provided a derivation of his own prediction which is comparable. Although validity was claimed for Prandtl numbers of 0.5 and above, the lowest Prandtl number for which a comparison is demonstrated in Webb's paper is for  $Pr \approx 1.2$ , water. The main difficulty in applying the results of Webb or of Petukhov and Popov for conditions with varying properties is that the recommended heat transfer correlation requires a knowledge of the friction factors. However, the effect of severe property variation on the friction factor is not necessarily known well in advance and only a few measurements of the friction factor have been obtained for strongly heated or cooled gas flows to provide guidance.

For fully established, gas flows in circular tubes, McAdams [1954] recommends that the coefficient of the empirical Dittus-Boelter relation be

modified to 0.021 giving

$$Nu = 0.021 Re^{0.8} Pr^{0.4} \quad (III-1)$$

This relationship was based on data for common gases so it was appropriate only for Prandtl numbers of the order of 0.7 or so. Based on a semi-empirical analysis, Kays [1966] recommended the relation

$$Nu = 0.022 Re^{0.8} Pr^{0.6} \quad (III-2)$$

for the range  $0.5 < Pr < 1.0$  for the thermal boundary condition of a constant wall heat flux. Sleicher and Rouse [1975] correlated analytical and experimental results, but over a larger range ( $0.1 < Pr < 10^5$ ) obtaining

$$Nu_b = 5 + 0.015 Re_f^a Pr_w^b \quad (III-3)$$

with

$$a = 0.88 - 0.24/(4 + Pr_w)$$

$$b = 1/3 + 0.5 \exp \{-0.6 Pr_w\}$$

The subscripts b, f and w refer to evaluation of properties at bulk, film and wall temperatures, respectively. While this correlation is awkward for the design engineer with a simple hand calculator to use when property variation is significant since film, bulk and wall properties must all be evaluated, for constant properties there is no ambiguity.

Petukhov [1970] indicated that the analytical results of Petukhov and Popov could be represented as

$$Nu = \frac{(\xi/8) Re Pr}{K_1(\xi) + K_2(Pr) \sqrt{\xi/8} (Pr^{2/3} - 1)} \quad (III-4)$$

where

$$\xi = (1.82 \log_{10} Re - 1.64)^{-2}$$

$$K_1(\xi) = 1 + 3.4\xi \quad K_2(Pr) = 11.7 + 1.8 Pr^{-1/3}$$

within one percent for the range  $0.5 < Pr < 200$ . He also noted that Petukhov and Kirillov suggested taking the constants  $K_1$  and  $K_2$  as equal to 1.07 and 12.7, respectively, in order to simplify the equation. The consequent accuracy in representing the results of the analysis is of the order of 5-6 % for the same range of Prandtl number.

Gnielinski [1975] collected a large amount of data from the literature and recommends the relationship

$$Nu = 0.0214 (Re^{0.8} - 100) Pr^{0.4} \left( \frac{T_b}{T_w} \right)^{0.45} \left[ 1 + \left( \frac{D}{L} \right)^{2/3} \right] \quad (\text{III-5})$$

for heating for the range  $0.6 < Pr < 1.5$  for gases. When the property variation and the entry region terms are suppressed, one can see that his recommendation is approximately the same as McAdams' version of the Dittus-Boelter correlation (with property variation, bulk temperature is used as the reference). For gases, his collection of data was apparently constrained to air, helium and carbon dioxide so the actual gas measurements would have been for  $Pr \approx 0.7$  only.

Until recently, apparently no data were available to test the Prandtl number dependencies represented in each of the above correlations. Indeed, many well-known texts do not recognize that it is possible to have  $Pr < 0.6$  for gases. As may be seen from Fig. 7, which compares them, most of the correlations agree reasonably well for common gases; however, as the Prandtl number is lowered there are significant differences in the predictions.

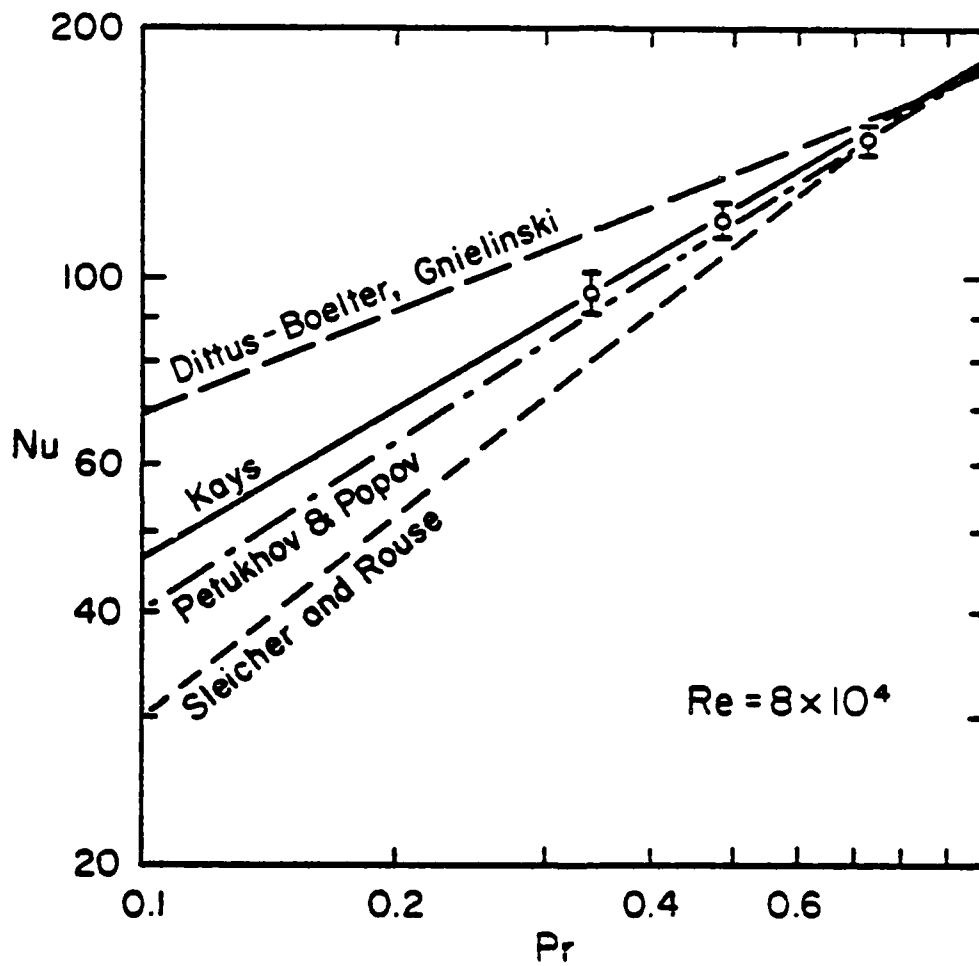


Figure 7. Heat transfer to gases with fully established conditions, constant properties. Comparison of correlations to measurements from gas mixtures.

By employing binary mixtures of light and heavy gases, the author and his coauthors have obtained basic data which can be used to test these correlations as the Prandtl number is varied. Measurements were taken at various heating rates and were extrapolated to the constant property condition by methods comparable to the approach of Malina and Sparrow [1964]. Thus, a direct comparison to analyses and correlations for constant properties can be made. Pickett, Taylor and McEligot [1979] employed mixtures of helium and argon in order to obtain  $Pr \approx 0.42$  and  $0.49$ , while Serksnis, Taylor and McEligot [1978] conducted measurements at  $Pr \approx 1/3$  by using a mixture of hydrogen and carbon dioxide. In more recent work, not yet published, Taylor, Bauer and McEligot have obtained additional data in the range  $0.16 \lesssim Pr \lesssim 0.3$  by mixing hydrogen with xenon and helium with xenon. The data for  $Re \approx 80,000$  have been superimposed on Fig. 7; one can see that there is good agreement with the simple correlation (III-2) proposed by Kays and with Petukhov's suggestion (III-4).

The danger of extrapolating heat transfer correlations based on data at  $Pr \approx 0.7$  is emphasized by Pierce [1981] in an application to the closed gas turbine cycle. At  $Pr = 0.2$ , the range of correlations is 90 percent. He shows that a consequence of using a Colburn correlation (i.e., with  $Pr^{1/3}$ ) would be to underestimate the surface area and volume of a closed cycle recuperator by about 60 percent.

#### III.1.b. Axial variation of thermal boundary conditions

For the prediction of the heat transfer parameters in a thermal entry of a circular tube with the flow already developed, the analysis of Sparrow, Hallman and Siegel [1957] can be used for the thermal condition of a constant wall heat flux. For uniform wall temperature the earlier paper

by Sleicher and Tribus [1956] provides a solution with the related eigenvalues. As described by Kays [1966], these solutions may in turn be applied by the method of superposition in order to provide predictions for varying wall temperature distributions and varying wall heat flux distributions. However, for a varying wall heat flux, Hasegawa and Fujita [1968] have presented an improved relation for superposition so that convergence occurs more rapidly as additional eigenvalues in the series are used; this is an important consideration because often only the first few eigenvalues are known for a turbulent flow.

The approach of Hasegawa and Fujita has been adapted by Bankston and McEligot [1969] to employ correlations of the thermal entry behavior in turbulent flow for situations where the eigenvalues are not available. Main advantages of using the latter approach with their correlation are (1) the Nusselt number approaches infinity as  $x$  goes to zero, whereas with a finite number of eigenvalues the Nusselt number approaches a finite maximum, (2) a close fit to the turbulent Leveque solution at shorter axial distances than would be possible with the five eigenvalues available, and (3) continuous dependence on Reynolds number instead of individual tabulations at specific Reynolds numbers. Bankston and McEligot demonstrate by comparison to an "exact" numerical solution that, for a simple sinusoidal heat flux distribution, the first method can give errors of the order of a factor of 2 or greater. The approach of Hasegawa and Fujita would agree within 20 % after one diameter, and the use of the correlation provides predictions which agree within about 2 % for  $x/D = 0.1$  and greater.

There do not appear to be any eigenvalue solutions available for the thermal entry problem for the lower Prandtl number gas mixtures. They could be derived by the techniques of Sparrow, Hallman and Siegel [1957] and others, but is probably just as convenient to apply a numerical program such as those of Patankar and Spalding [1967], Bankston and McEligot [1970], etc.



### III.1.c. Turbulent Prandtl number

Simple numerical models for the prediction of heat transfer to turbulent flows employ the turbulent Prandtl number,  $Pr_t = \epsilon_m / \epsilon_h$ , to estimate the effective thermal conductivity for use in solution of the thermal energy equation (I-3) [A. J. Reynolds, 1975]. As shown in the reviews by Reynolds and by others there still exists considerable uncertainty concerning the variation of  $Pr_t$  as the molecular Prandtl number varies. At  $Pr \approx 0.7$ , some idealized analyses suggest that  $Pr_t$  is greater than one while others indicate that  $Pr_t$  is less than one. Most experiments to measure the turbulent Prandtl number have utilized probe measurements of the radial distribution of mean velocity, mean temperature, heat flux and shear stress. Such measurements become inherently more uncertain as the wall is approached. Unfortunately, predictions of heat transfer parameters are most sensitive to the assumed value of the turbulent Prandtl number in the region where it is most uncertain. By numerical experiments with their predictions of heat transfer in thermal entry regions, McEligot, Pickett and Taylor [1976] demonstrated that it was possible to obtain good estimates of an effective turbulent Prandtl number for the viscous region ( $y^+ \lesssim 30$ ) by measurement of the local Nusselt number in heating experiments in small tubes. This is the region which dominates the thermal resistance, an observation which explains the sensitivity to the choice of the turbulent Prandtl number near the wall.

The thermal entry data from the helium-argon measurements of Pickett, Taylor and McEligot [1979] and the hydrogen-carbon dioxide measurements of Seksnis, Taylor and McEligot [1978] were compared to predictions from their numerical program for the conditions of the experiment in order to determine the variation of the turbulent Prandtl number near the wall as the molecular Prandtl number was reduced from about 0.7 to 1/3. Over the range of Reynolds numbers and Prandtl numbers studied, they

found the turbulent Prandtl number ranged from 0.9 to 1.1 with an uncertainty of about 10 %. Thus, for this region, the results do not differ substantially from Reynolds analogy. Further from the wall any  $Pr_t$  from 1/2 to 2 will probably do for design calculations.

A plausible physical explanation of the observations of the author and his colleagues follows. Scriven [1977] has suggested in a personal conversation that Reynolds' analogy corresponds to an impulsive transport model. This view is consistent with an heuristic explanation provided in Kays' text [1966] for Reynolds analogy and for mixing length models of turbulence. The technique comparing  $Nu(x)$  emphasizes measurement of  $Pr_t$  in the viscous layer,  $y^+ \gtrsim 30$ . In this region, flow visualization observations by Corino and Brodkey [1969] and others found the majority of the turbulent momentum transport to occur via "bursts" and "sweeps" which are more abrupt than the other phases of the motion. One would expect the same to be true of energy transport as well and, therefore, that Reynolds analogy would be a reasonable approximation for turbulent transport in the viscous layer. Earlier models for the turbulent Prandtl number [A. J. Reynolds, 1975] instead considered the diffusive transport of momentum and energy from turbulent eddies as a gradual process.

Application of the technique of McEligot, Pickett and Taylor requires that the numerical program have accurate velocity and eddy diffusivity profiles for fully developed flow. In the work of Pickett, Taylor and McEligot and Serksnis, Taylor and McEligot they were generated from the van Driest mixing length model [1956] modified by a Reichardt "middle law" [1951]. Although the van Driest model currently seems quite successful and has been used widely for calculation of the wall region in turbulent flows [Launder and Spalding, 1972] its detailed behavior in the viscous layer has not been definitively determined. Confidence in its validity is provided by close agreement between predicted and measured friction factors for adiabatic flow, but it is

probably best to consider the above results for  $Pr_t$  to be dependent on the assumption of a van Driest model for describing the momentum transport in the viscous layer.

As noted by Antonia [1980], the behavior of  $Pr_t$  near the wall can be inferred from Taylors' series expansions of the time mean velocity and temperature profiles  $\bar{u}(r)$ ,  $\bar{t}(r)$  and the turbulent shear stress  $-\rho\overline{uv}(r)$  and turbulent heat flux  $\rho c_p \overline{vt}(r)$  profiles provided accurate data are available. The range of validity of this approach was not given by Antonia, but since terms of order  $(y^+)^5$  and  $(y^+)^6$  are neglected it is likely that the results should be limited to very small values of  $y^+$ , presumably in the linear layer where turbulent transport can usually be neglected relative to molecular diffusion in gas flows.

The techniques of advanced turbulence modeling, discussed later in section III.3, can also be applied to generate predictions of  $Pr_t$  as demonstrated by Jischa and Rieke [1979]. For the region beyond the viscous layer in a fully developed flow, their solutions of the modeled transport equations for turbulence kinetic energy, turbulence heat flux and turbulence mass flux predict  $Pr_t = C + B/Pr$  where  $C$  and  $B$  are empirical constants. Further work is necessary to extend predictions to the dominant viscous layer.

### III.1.d. Friction factor

The adiabatic ("Fanning") friction factor for fully developed flow in circular tubes is conveniently represented by the correlation of Drew, Koo and McAdams [1932],

$$f = \frac{g_c \tau_w}{\rho V^2 / 2} = 0.0014 + 0.125 Re^{-0.32} \quad (\text{III-6})$$

for  $3000 \lesssim Re \lesssim 300,000$ . For turbulent flow in other geometries the Reynolds number is evaluated with the hydraulic diameter,  $D_h = 4A_{cs}/P$ .

For rectangular ducts and for annuli, Jones and Leung [1981] claim that the hydraulic diameter alone is insufficient to correlate geometric effects accurately. Considering constant property flows, they have developed a modified Reynolds number based on a laminar equivalent diameter and have derived a correlation procedure which, they feel, applies universally for smooth circular tubes, smooth rectangular channels and smooth concentric annuli for both laminar and turbulent flows. Results are presented in the form of a Colebrook equation for turbulent flow with the Reynolds number based on the laminar equivalent diameter. The laminar equivalent diameter for circular geometry is given as the product of the hydraulic diameter and a function  $\Phi^*(a)$ ,

$$\Phi^* = \frac{1}{(1-a)^2} \left[ 1 + a^2 - \frac{1-a^2}{\ln(1/a)} \right] \quad (\text{III-7})$$

where  $a$  is the radius ratio; for rectangular ducts  $\Phi^*$  is a function of aspect ratio [Jones, 1976]. In their development the friction factor is defined in terms of the streamwise pressure gradient, so the results are likely to be useful only as a limiting prediction in cases where only one surface is heated since it is the wall shear stress at the surface with substantial heat transfer that will be modified most. However, this relation probably could be used to represent the constant properties prediction in a property-ratio method for variable properties, if desired. Jones and Leung estimate the uncertainty to be about 5 percent.

### III.2. Experiments

Perukhov [1970] reviewed the experiments on strongly heated, quasi-fully developed flow which were available to about the mid 1960's. At the time, most data had been obtained in small circular tubes with electrical heating providing an approximately constant wall heat flux. A few experiments had been conducted with water cooling the outside of tubes, yielding an approximately constant wall temperature. Thus, most heating experiments were with a constant heating rate while most cooling experiments were with the alternate thermal boundary condition, a constant wall temperature. Recommended correlations for turbulent flow in tubes in terms of property ratio methods suggested different exponents for heating and cooling [Kays, 1966].

Since most experiments had been conducted in small tubes, the results could be presented only in terms of wall parameters. However, this restriction to small tubes is also partly a necessity when desiring to measure dominant forced convection at relatively low Reynolds numbers, since the effects of buoyancy forces can be a problem with large wall-to-bulk temperature differences. In general, the tubes were too small to insert probes in order to obtain meaningful measurements of temperature and velocity profiles. Further - although some of the early measurements examining the effects of strong property variation had pressure taps located along the test section and, therefore, local friction factors could have been deduced - most presented only overall pressure drops or average friction factors.

During the ensuing decade and a half, the emphasis in experiments has shifted away from the question of heat transfer in quasi-fully developed flow in circular tubes to other effects. Measurements have been taken at lower Reynolds numbers to investigate the phenomenon of laminarization and have considered other geometries, such as annuli and non-circular tubes. Additional

data have become available for the situation of severe cooling and some velocity and temperature distributions have been obtained in that situation.

Zukauskas and Slanciauskas [1978] briefly reviewed a number of aspects of forced convection in channel flows. Petukhov, Polyakov and Shekhter [1978] surveyed theoretical and experimental papers which considered the effects of buoyancy forces on turbulent flow in both vertical and horizontal pipes; most of the references presented were in Russian. A comparable review was presented by Jackson and Hall [1978].

### III.2.a. Wall parameters and correlations

In most of the published experiments the goal has been to obtain empirical correlations of the heat transfer and/or wall friction parameters. Only a few have been interrelated with analytical (usually numerical) approaches. However, when sufficient information is included the data can provide useful tests of turbulence models.

Reports available until the early 1960's on gaseous heat transfer experiments for heating with significant property variation were summarized by McEligot, Magee and Leppert [1965]. Most presented overall average coefficients which are not useful in predicting maximum wall temperatures with severe heating. Only a few provided sequences of incremental average coefficients or local coefficients which could be used to test analyses of the downstream region (i.e., beyond entry effects). Local heat transfer data spanned the range  $7500 < Re < 645,000$  with 2.5 as the maximum temperature ratio. Almost no local friction measurements were available. Delpont [1964] extended the range of heat transfer data for air and carbon dioxide to  $Re \approx 10^6$  but only at moderate temperature ratios ( $1.1 < (T_w/T_b) < 1.4$ ).

The first data on local friction factor distributions were presented by LeI'chuk and Dyadyakin [1959] (which became available in English in 1962) for simultaneously developing thermal and momentum boundary layers. Dalle Donne and Bowditch [1963] and McEligot [1963] then provided local friction data for a thermal boundary layer developing in a tube after a fully established velocity profile existed. Comparison to adiabatic predictions at the same local bulk Reynolds number showed only a slight reduction in friction factor as the temperature ratio was increased in the downstream region. Near the start of heating the apparent friction factor increased slightly. Based on these data, McEligot, Magee and Leppert [1965] suggested that, for locations greater than 30 diameters from the start of heating, the local apparent friction factor be represented as

$$f/f_{cp}(Re_b) = (T_w/T_b)^{-0.1} \quad (III-8)$$

At the time it appeared that the semi-theoretical predictions for hypothetical fully established conditions, such as the approach of Deissler [Deissler and Eian, 1952; Botje, 1956] substantially overestimated the effect of heating on the friction factor.

Magee and McEligot [1968] correlated data from their own measurements and others for the thermal entry to the form

$$\frac{f}{f_{cp}(Re_b)} = \left( \frac{T_w}{T_b} \right)^{\frac{1}{2} - \frac{1}{6} \ln(x/D)} \quad (III-9)$$

for distances from  $5 \leq x/D \leq 55$  for air. With helium the correlation over-corrected slightly. The results of LeI'chuk and Dyadyakin are somewhat higher, presumably due to the abrupt entrance used. No sensible correlation

could be discerned when the parameters were calculated with properties based on the film temperature.

The most extreme ranges of property variation obtained for common gases (other than the studies of super-critical gases) have been by Taylor [1963, 1965] and Perkins and Worsoe-Schmidt [1965]. Taylor [1963] reached  $T_w/T_b \approx 5.6$  by employing a tungsten tube so he could operate at surface temperatures up to 3100 K. Perkins and Worsoe-Schmidt [1965] precooled the inlet flow in a liquid nitrogen bath to obtain temperature ratios of the order of 7. By also precooling with liquid nitrogen Taylor [1965] then extended his range of data through temperature ratios of 8. At their conditions, one effect of the high heating rates was to lead to a maximum in the wall temperature near the start of heating, rather than at the exit as is normally expected from constant property predictions. Although the existence of the early peak in wall temperature could be predicted by some existing correlations which included the effect of property variation, it is a situation where an unexpected failure could occur (and perhaps did in one of the earlier experiments of Magee and McEligot) if the designer did not use local results to predict the wall temperature distribution along the entire tube! The tabulated data of Perkins and Worsoe-Schmidt [1965] provide a severe test for turbulence modeling.

The effects of temperature induced property variation on heat transfer to mixtures of gases with lower Prandtl numbers were studied by Pickett, Taylor and McEligot [1979] and Serksnis, Taylor and McEligot [1978] as part of the program described in section III-1. They found poor agreement with the correlation of Sleicher and Rouse [1975], presented above as equation (III-3), but an alternate recommendation of those authors,



$$Nu_b = 5 + 0.012 Re_b^{0.83} (Pr + 0.29)(T_w/T_b)^q \quad (III-10)$$

with

$$q = -\log_{10}(T_w/T_b)^{1/4} - 0.3 \quad \text{for } x/D > 40$$

generally agreed with the data within 10 % even though its validity was not claimed to include the lower Prandtl numbers of these experiments. The heat transfer data were also correlated by a modification of the Dittus-Boelter relationship involving a stronger dependence on the Prandtl number, as mentioned in section III.1.a, and a property ratio method to account for property variation,

$$Nu_b = 0.021 Re_b^{0.8} Pr^{0.55} [(T_w/T_b)^{-0.4} + 0.85 D/x] \quad (III-11)$$

Ninety percent of the data are predicted to within ten percent for the range  $2.1 < x/D < 81.6$  with the greatest discrepancy (up to 15 % underprediction) occurring for the highest heating rates. Overall average friction factors agreed with a correlation proposed by Taylor [1967].

Empirical correlations for heating with constant wall heat flux by Sleicher and Rouse [1975] and Gnielinski [1975] were presented in section III-1. Many are available for various ranges of data. Marien and Richards [1966] list sixteen for the purpose of predicting heat transfer to helium alone at high heating rates! For the designer caught without her hand calculator or slide rule, Gregorig [1973] has provided a nomogram. Simoneau and Hendricks [1964, 1965] correlated a wide range of early data. Examining both average and local downstream heat transfer coefficients, they found that downstream the heat transfer coefficient could be correlated as

$$h = K G^{0.8} D^{-0.2} \sqrt{T_w/T_b} \quad (III-12)$$

where  $K \cong 0.021 k^{0.6} c_p^{0.4} / \mu^{0.4}$  is approximately independent of temperature for common gases at ordinary temperatures. The dimensional constant is unique for each gas, e.g.,  $K \approx 0.0038 \text{ Btu}/(\text{R lbm}^{0.8} \text{ hr}^{0.2} \text{ ft}^{0.2})$  for air. This relation is simple in form and should be particularly attractive for parametric studies.

For gases for which the property dependence can be represented by power laws, Petukhov, Kurganov and Gladuntsov [1973] developed the relation

$$\frac{\text{Nu}_b}{\text{Nu}_{cp}} = \left( \frac{k_w}{k_b} \right)^{1/3} \left( \frac{c_{p,w}}{c_{p,b}} \right)^{1/4} \left( \frac{T_w}{T_b} \right)^{-\left[ 0.53 + \phi\left(\frac{x}{D}\right) \log \frac{\mu_w}{\mu_b} \right]} \quad (\text{III-13})$$

with  $\phi(x/D)$  being a tabulated function. Rms accuracy of the order of five percent is claimed for heating with constant wall heat flux and cooling over the ranges  $0.5 \lesssim T_w/T_b \lesssim 4$  and  $\text{Re}_b \gtrsim 7000$ .

Taylor has collected a large number of measurements from his own experiments and those of others. He notes that the reference temperature method worked well for the average friction factors of the early experiments at wall-to-bulk temperature ratios less than 2.5, for long tubes and high Reynolds numbers, but demonstrates that in general the concept is not adequate [Taylor, 1967]. Being measured, these friction factors would be apparent friction factors. He concluded that to discriminate between the likelihood of laminar flow and turbulent flow the modified wall Reynolds number,  $\text{Re}_{wm}$ , should be used for predicting transition. For both heating and cooling he was able to correlate approximately 1100 data points for the ranges  $0.3 < T_w/T_b < 7.4$  at axial distances greater than 5 diameters by the relation

$$\frac{f}{f_{cp}(Re_{wm})} = \left( \frac{T_w}{T_b} \right)^{-0.57-1.59 D/x} \quad (III-14)$$

with 88 % of the points agreeing within 10 % [Taylor, 1970]. From an examination of almost 5000 data points for heating of hydrogen, helium, nitrogen and carbon dioxide with temperature ratios up to 23 and locations greater than 2 diameters from the start of heating, he found the correlation

$$Nu_b = 0.023 Re_b^{0.8} Pr_b^{0.4} \left( \frac{T_w}{T_b} \right)^{-0.57+1.59 D/x} \quad (III-15)$$

to collapse 90 % of the data within  $\pm 25$  %.

The possible effects of compressibility were examined by Lel'chuk and Elfimov [1964] by measuring local heat transfer coefficients to argon at Mach numbers to 0.96,  $1.1 < T_w/T_b < 2.7$  and  $4 \times 10^3 \lesssim Re_i \lesssim 4 \times 10^5$ . The downstream data showed within a scatter of 6 % that their correlation in terms of wall properties (unfortunately, with  $Re_w$  not defined) was independent of Mach number. The heat transfer coefficient was apparently based on the adiabatic wall temperature with the recovery factor taken as  $\sqrt[3]{Pr}$ .

Interaction with a radiating gas was measured by Larsen, Lord and Farman [1970] by heating water vapor at various pressures. Superposition of convective and radiative heat transfer was found valid in the thin gas limit. For intermediate optical thicknesses they found that the radiative contribution approximately counteracted the usual reduction in convective heat transfer due to property variation.

Data for cooling with approximately constant wall temperatures have

been obtained with mixtures of combustion products [Brim and Eustis, 1970], with air in an air/water countercurrent heat exchanger [Zucchetto and Thorsen, 1973] and with air, nitrogen and argon from electric arc heaters [Rozhdestvenskii, 1970; Magdasiev, 1972; Ambrazyavichus, Zukauskas and Valatkya-vichyus, 1973; Valatkevicius, Zukauskas and Ambrazevicius, 1973]. These downstream data show no significant effect of the temperature ratio when local properties are evaluated at the bulk temperature and generally can be correlated as

$$Nu_b \cong 0.022 Re_b^{0.8} Pr_b^{0.4} \quad (III-16)$$

for  $T_w/T_b > 0.12$  [Rozhestvenskii, 1970]. Brim suggested a slightly different coefficient and Prandtl number dependence ( $Pr^{0.6}$ ) and, using the log mean temperature difference for average coefficients, Zucchetto and Thorsen found a slight dependence on temperature ratio. Valatkevicius, Zukauskas and Ambrazevicius found the friction factor unaffected as well when evaluated at the bulk Reynolds number.

For cooling of air and carbon dioxide at wall-to-bulk temperature ratios as low as 0.25, Taylor [1970] found no discernable effect of the heating rate and recommended equation (III-16) with 0.023 as the coefficient. Eighty percent of the almost 1200 data points examined agreed within about 15 %.

Trends apparently differ in the thermal entry region depending on the inlet configuration. If an unheated tubular entry has not been provided, the flow is comparable to an external boundary layer for the first several diameters so parameters can be defined in terms of axial distance with the free-stream temperature corresponding to the entering temperature. With air flow tripped at the entrance of a short tube following a smooth contraction, Back, Cuffel and Massier [1969] found an increase in the heat transfer parameters

as  $T_w/T_i$  was reduced if inlet temperature served as the reference; film temperature correlated the data better. They noted that the data of Zellnick and Churchill [1958] and Chi and Spalding [1966] showed the same trends. On the other hand Ambrazevicius, Valatkevicius and Kezelis [1977] with entry from an arc heater at various angles found no significant effect of cooling rate when applying flat plate boundary layer correlations with inlet properties. With fully developed flow before heating, the data of Wolf [1959] showed no significant trend when local bulk properties were employed. A difficulty with the measurements of cooling in entry regions is that often the scatter in the data is greater than the effect claimed so the designer should be cautious; further, the apparent origin of the turbulent boundary layer can not necessarily be predicted well.

Data for cooling of air flowing turbulently between parallel plates with strong property variation has been reported by Mueller [1968]. Test section surfaces were copper plates cooled by a counter flow of water. With insulated sides the aspect ratio was 8 to 1. The cooling section started immediately after the entering contraction and was about 100 hydraulic diameters in length. Plate spacing of 5 mm prohibited obtaining temperature and velocity profiles, so Mueller presented only overall average Nusselt numbers and average apparent friction factors. Maximum inlet temperature was about 1100 °K while the walls were at about 300 °K. Average wall-to-bulk temperature ratios covered the range from 0.4 to 0.7 approximately. These experiments have been used by Schade and McEligot [1971] to calibrate a numerical prediction for turbulent flow between parallel plates with strong cooling. In general, the data indicated a property ratio dependence for the average apparent bulk friction factor with an exponent of -0.3 based on bulk properties. The heat transfer could be predicted approximately by a reference temperature method using the film temperature as reference.

Experiments measuring heat transfer coefficients and friction factors to air in non-circular ducts have been conducted by Campbell and H.C. Perkins [1968] for a triangular duct and by Battista and H.C. Perkins [1970] and K.R. Perkins, Schade and McEligot [1973] in square ducts with resistive heating. For the triangular tube, Campbell and Perkins found a smaller exponent on the temperature ratio term in the friction factor correlation than for circular ducts while the heat transfer results were slightly more dependent on heating rate. The recommended relations were

$$\frac{f_b}{f_{cp}(Re_{wm})} = \left( \frac{T_w}{T_b} \right)^{-0.40 + (D_h/x)^{0.67}} \quad (III-17)$$

and

$$Nu_b = 0.021 Re_b^{0.8} Pr^{0.4} \left( \frac{T_w}{T_b} \right)^{-0.7} \left[ 1 + \left( \frac{x}{D_h} \right)^{-0.7} \left( \frac{T_w}{T_b} \right)^{0.7} \right]$$

with the hydraulic diameter taken as the significant dimension in the definition of the non-dimensional parameters. Battista and Perkins found that the same heat transfer correlation could be used for the square tube for the range  $21,000 < Re < 49,000$  and axial distances greater than twenty-two diameters; Perkins, Schade and McEligot confirmed their results and extended the validity to an entering Reynolds number of about 4000, provided the acceleration parameter,  $K = (\nu/V_b^2) dV_b/dx$ , was less than about  $1.7 \times 10^{-6}$  and the axial distances greater than about 12 diameters. (At shorter axial distances the correlation underpredicts the measured Nusselt numbers.)

Taylor [1968, 1970] has examined the prediction of heat transfer and friction in the coolant tubes of regenerative rocket nozzles as on the Nerva and PHOEBUS-2 nuclear rockets; the cross sections of these tubes may be approximated by low aspect ratio rectangles with one or two ends

rounded with a radius equal to the half spacing. In addition to measurements in vertical ducts with circular cross sections, Petersen and Kaiser [1974] also measured heat transfer to helium in a cross section of a trefoil shape at Reynolds numbers from 2,000 to 70,000 and temperature ratios from 1.1 to 1.6; the latter configuration corresponds to a cooling channel in a nuclear reactor of the DRAGON type.

In measurements of the friction and heat transfer characteristics of swirling air flow induced by twisted strip promoters, Thorsen and Landis [1968] found different exponents for the temperature ratio in a heat transfer correlation for heating than for cooling, with it being substantially less for cooling (-0.1 in place of -0.32). The effect on pressure drop was also slight (-0.1). These results are consistent with the observations for flow through circular tubes without inserts.

Applications to coolant channels in gas cooled reactors have made studies of annuli popular. Data and correlations are presented by Larkin [1965], Kutukcuoglu [1970], Michiyoshi, Mitani and Takahashi [1970], Dalle Donne and Meerwald [1973] Vilemas and Nemira [1977, 1978] and Nemira, Vilemas and Simonis [1979a, 1979b, 1980].

### III.2.c Structure

While measurements of wall parameters - such as Nusselt numbers and apparent friction factors - can be the ultimate test as far as the designer is concerned, data on the behavior of the dependent variables -  $\bar{u}(x, r)$ ,  $\bar{t}(x, r)$ , etc. - are of interest to the analyst attempting to model a strongly heated gas flow. Probes must be small to reduce disturbances to the flow measured and to obtain adequate spatial resolution in terms of  $\bar{t}^+$  for the sensor. Since

the sensor has a significant finite size, even in the case of the laser Doppler anemometer [Durst, Melling and Whitelaw, 1976], the test section dimensions must be relatively large. Large diameters lead to large Grashof numbers. The experimentalist wishing to measure the flow structure in forced convection to gases with transport property variation faces a choice: (1) reduce the Grashof number to avoid buoyancy forces by reducing the temperature differences, and therefore reduce the variation of properties, or (2) maintain a substantial heating rate and chance investigating mixed convection with modification of turbulence structure by buoyancy forces as well. Consequently, there are few measurements available of mean or turbulence structure related to the current topic.

For heating, with dominant forced convection and an approximately constant wall heat flux, reliable measurements of the mean velocity distributions and mean temperature distributions, not to mention the more difficult turbulence quantities, seem to be missing. Those developing advanced turbulence models desire this information in order to tune their techniques or to test their results. Some of the reasons that the necessary profile distributions are not available are (1) that the viscous layer of a turbulent flow is very small although, since it dominates the thermal resistance, it is probably the most important region to measure, (2) in order to examine dominant forced convection and avoid significant buoyancy forces, duct diameters themselves must be small as mentioned above, (3) significant measurement errors can occur due to the large temperature differences between the duct wall and the fluid. Hot wire probe methods can suffer from thermal radiation to the supports and, with low velocities in the viscous layer, significant thermal conduction between supports and the sensor. Further, differential thermal expansion from one side of the probe support to the other can introduce a positioning error which cannot be calculated as a practical matter and is difficult to calibrate.



One of the few studies of strongly heated flow where velocity measurements have been conducted is the study by Tanaka and Noto [1973] in which buoyancy forces were examined in both upflow and downflow in a circular tube. The tube diameter was 10 mm, with nitrogen at 40-70 atmospheres and heating was by direct current. The Grashof number was varied through a range from predominantly forced convection through predominantly natural convection. To measure the velocities, a double sensor probe was employed to determine the time of flight of thermal disturbances in the flow by comparison of the two signals. Velocity profiles were obtained at a "downstream" location with Grashof numbers, based on film properties, ranging from about  $2 \times 10^6$  to  $4 \times 10^6$  while the Reynolds number was varied from 5300 to 17,000. The velocity profiles show reasonable agreement with the predictions of an approximate analysis, apparently the same general technique as used by Tanaka et al. [1973]. For the lowest Reynolds number (again based on film properties) the predicted mean shear stress profile became negative in the central core for upflow and in the vicinity of  $y/r_w \approx 0.2$  a peak in the mean velocity occurred, both in measurements and predictions. The authors note that the heat transfer parameters are predicted reasonably well by the approximate theory provided that the data fall in the forced convection range; when the natural convective effects become of the same order as the forced convection the heat transfer coefficient in upflow was approximately half the predicted value.

Tanimoto and Hanratty [1963] obtained hot wire anemometer measurements with "small" heat fluxes at the wall. Bremhorst and Bullock [1970, 1973] studied velocity and temperature spectra in a horizontal tube of 127 mm (5 in.) i. d. with a temperature difference of 9°K ( $T_w/T_b \approx 1.03$ ); at  $Re \approx 54,500$  they believed the data showed evidence of buoyancy effects.

With an apparently horizontal tube of diameter 45.7 mm and an

entering temperature difference of about 80 °K, Hishida and Nagano [1979] measured mean velocity, temperature and fluctuation profiles at  $Re \approx 4 \times 10^4$  in a quasi-developed flow. With heating by a steam jacket, the thermal boundary condition was an approximately constant wall temperature. The entering temperature ratio of about 1.3 could be expected to cause only slight to moderate property variation downstream. The probe consisted of a 5  $\mu$ m resistance thermometer of length  $l^+ \approx 70$  with a single 5  $\mu$ m hot wire of length  $l^+ \approx 35$  located  $\Delta x^+ \approx 25$  downstream, in order to measure  $t$  and  $u$  simultaneously. Since Blackwelder and Haritonidis [1981] have shown a dependence on length for hot wires longer than  $l^+ \approx 20$ , the results should be considered qualitative. Oscilloscope traces showed the temperature and velocity signals to be anti-correlated, i.e. the temperature increased as the velocity decreased, as would be expected. The data with heating showed  $u^+$  to be raised in the viscous layer and reduced in the core in comparison to isothermal data. From the profiles,  $Pr_t$  was estimated as 0.87 for  $y^+ \gtrsim 80$ . The product -ut which might be expected to correspond to the instantaneous turbulent heat flux,  $\rho c_p vt$ , showed sharp peaks relative to its average values at  $y^+ \approx 8$  and 12; these records are reminiscent of the Reynolds stress measurements of Eckelmann [1974] and co-workers in the viscous layer and would imply impulsive transport as dominating the turbulent heat transfer in that region. From auto-correlations the average bursting periods were estimated and were found to be approximately the same on non-dimensional bases as those measured by Kim, Kline and Reynolds [1971] and Ueda and Hinze [1975] in unheated boundary layers.

Velocity and temperature profiles in the inlet of a tube cooling at temperature ratios up to about 1 to 10 have been measured by Kezhyalis, Valatkyavichyus and Ambrazyavichyus [1979], but their calorimetric probe was of a size of approximately twenty percent of the tube diameter. Consequently, the viscous layer was missed entirely and blockage problems must

have been significant. The trends of the data might be compared to numerical predictions but sufficient details are not presented for a meaningful direct comparison.

For cooling at inlet temperature ratios of 0.3 to 1, Mizushima, Matsumoto and Yoneda [1976] determined mean velocity and temperature profiles in the quasi-developed region of a horizontal tube after sixty diameters. Reynolds numbers varied from 6000 to 20,000 so three-dimensional mixed convection was likely for some experimental runs. The probe consisted of a pitot tube and a thermocouple mounted side-by-side; turbulence measurements were not possible. The closest approach to the wall was  $y^+ \approx 30$ . Deduced mixing length profiles for the turbulent core agreed with the prediction of Reichardt [1951] within experimental uncertainty and velocity profiles, calculated with a modified van Driest model [1956] near the wall (with  $y^+$  evaluated via  $\gamma_w$  but  $\nu(y)$  for the viscosity), showed fairly good agreement with the data. Nusselt numbers and friction factors were also presented.

### III.2.d. Design considerations

Petukhov [1970] has demonstrated that apparent differences between correlations with differing exponents on the temperature ratio term are not always significant. Frequently the coefficient in the equation differs also and when correlations are plotted on the same basis (e.g.,  $Nu(T_w/T_b)$ ) they are found to yield comparable values in regions where the data are mutually valid. His analysis predicts that the appropriate exponent increases as the temperature ratio increases for heating. His Fig. 12 shows reasonably good agreement amongst the data of various experiments provided individual empirical correlations are not extrapolated beyond the limits of the data from which they were deduced.

Since the differences between correlations are often only slight when plotted in absolute terms for the range of validity of the data, it can be useful for design purposes or feasibility studies to choose a correlation from which the wall temperature can be deduced explicitly when the wall heat flux is known. Usually the process is iterative, involving estimating the wall temperature in order to estimate the heat transfer coefficient so the wall temperature can be calculated in turn. For convenience, a correlation in terms of bulk properties, with the temperature ratio having an exponent of  $-1/2$ , allows a direct solution for the wall temperature via the binomial theorem once the bulk temperature is known from an energy balance. Likewise, McEligot, Smith and Bankston [1970] have pointed out that for a quasi-developed region the correlation of Kutateladze and Leont'ev [1964] can be inverted to

$$\frac{T_w}{T_b} \approx \left[ 1 - \frac{Q^+}{2 \text{Nu}_{cp}(\text{Re}_b)} \right]^{-10/3} \quad (\text{III-18})$$

where  $Q^+$  is defined as  $q_w'' r_w / k_b T_b$ .

The design engineer is interested in the friction factor to predict pressure drop and, hence, pumping power or inlet pressure. If one considers designs, such as gas cooled nuclear power reactors or nuclear rockets, where the pressure in the coolant passages is sufficiently high that one can assume  $(\Delta p/p)$  is much less than unity, then one-dimensional force and energy balances lead to

$$-\frac{d[p/(cV^2/2g_c)]}{d[x/D]} \approx 8q_x^+ + 4f_x \quad (\text{III-19})$$

The first term on the right side represents acceleration of the flow due to heating, while the second term arises from the familiar wall friction. As

the heating is increased, its effect on wall friction increases slightly but the term representing acceleration increases much more rapidly. The fraction of the pressure drop caused by friction decreases as the heating rate is increased. Consequently, the importance of obtaining accurate predictions of the friction factor declines as the heating rate increases as far as pressure drop predictions are concerned. However, some correlations of heat transfer parameters are dependent on the wall shear stress or friction factor (based on the velocity gradient at the wall); presumably for such purposes, the predictions of wall temperature would depend significantly upon knowledge of the friction factor.

Another consequence of the above relation is that as the heating rate increases so does the difficulty in obtaining accurate measurements of the friction factors. Usually the pressure drop is measured rather than the wall shear stress. At a specified Reynolds number and density, the frictional pressure drop decreases with heating while the overall pressure drop increases. The percent uncertainty in measured friction factor increases both for this reason and because the uncertainty in the bulk temperature affects the deduction of the acceleration pressure drop. For example, for the conditions of the highest heating rate in the data of Perkins and Worsøe-Schmidt [1965], a 2 % uncertainty in the determination of the pressure gradient near the start of heating would lead to approximately a 12 % uncertainty in the apparent friction factor deduced.

### III.3. Analyses

It might appear to the casual reader that, due to the success of the above experimental investigations in correlating measured data for strongly heated or cooled flows, analytical predictions are not needed by the designer.

Such reasoning would be dangerous. It has been noted that most experiments are conducted with either a constant wall heating rate or, for cooling, an approximately constant wall temperature as the thermal boundary condition. While there are direct applications - a solar collector for the former and cooling of the exhaust of a gas turbine for the latter - practice in general does not allow the luxury of restricting oneself to these idealized boundary conditions. The gas cooled nuclear reactor is a case in point. The typical heating rate distribution might be represented as sinusoidal in an active core and an exponential decay in the region where the control rods are inserted.

Bankston and McEligot [1969] examined whether a superposition technique could be applied to a sinusoidal distribution at a moderate heating rate, a case where superposition had been successful for laminar flow, and found significant differences in turbulent flow. The eigenvalue solutions for constant heat flux do not provide reasonable predictions as the heating rate is increased as they can for laminar flow. (However, in work in progress during this writing, Dalle Donne and Tartaglia [1982] are finding that the superposition technique of Kurganov and Petukhov [1974] is effective for moderately varying  $q_w''(x)$  as in experiments where the electrical resistance and heat loss vary significantly with temperature.)

Since results for varying properties differ significantly from constant property predictions in the thermal entry and an axially varying thermal boundary condition is essentially a continuous thermal entry, means must be provided to bridge the gap from the constant boundary conditions of experiments to the varying ones in actual practice. Analyses provide the means.

As for laminar flow the coupled governing equations (I-1 to 4) describing the thermal entry problem can be solved numerically. The numerical technique itself is now relatively straight-forward and routine. The difficulties arise in providing appropriate estimates of the diffusivities for turbulent flow,  $\mu_{eff}$  and  $k_{eff}$ . The question becomes how to account for effects of the fluid property variation on these quantities or their equivalents (i.e.,  $\epsilon_m$ ,  $\epsilon_h$ ,  $Pr_t$ , etc.).

In comparing predictions of turbulence models to experimental measurements, it is important to make the comparison in terms of the appropriate results. Likewise, some experimental results are more important than others for the testing of turbulence models. As far as the design engineer is concerned, he is normally interested in being able to predict wall temperatures or heating rates and pressure drop. These are quantities which are sensitive to an accurate prediction of the behavior of the viscous region near the wall. So in addition to velocity and temperature distributions in the fluids, the analyst should carefully check his predictions of local Nusselt or Stanton numbers and apparent friction factors.

Agreement of mean velocity profiles can be misleading. For example, Fig. 8 demonstrates the situation at a moderately high Reynolds number as the behavior in the viscous region for a simple turbulence model is varied by adjusting the thickness of the viscous layer (by varying  $A^+$  in a van Driest model). Though the results were calculated under the idealization of constant fluid properties, comparable results could be expected if property variation had been allowed. One can see that though the mean velocity profiles appear to differ only slightly when presented in linear coordinates (Fig. 8a) substantial differences would be seen if values were available in the viscous layer and were plotted in terms of wall definitions on semi-logarithmic coordinates (Fig. 8b). As  $A^+$  is varied from 13 to 26 to 52, the predicted Nusselt number changes from 196 to 155 to 112 and the friction factor varies from 0.00609 to 0.00468 to 0.00329, respectively. Thus, both the heat transfer and the friction behavior varies by almost a factor of two although this result would not be obvious from a comparison of the velocity profiles as plotted in Figure 8a.

Petukhov [1970] reviewed the literature to the early 1960's, concentrating on quasi-developed conditions well beyond the thermal entry. At that time high-speed digital computers were only beginning to be applied to solve

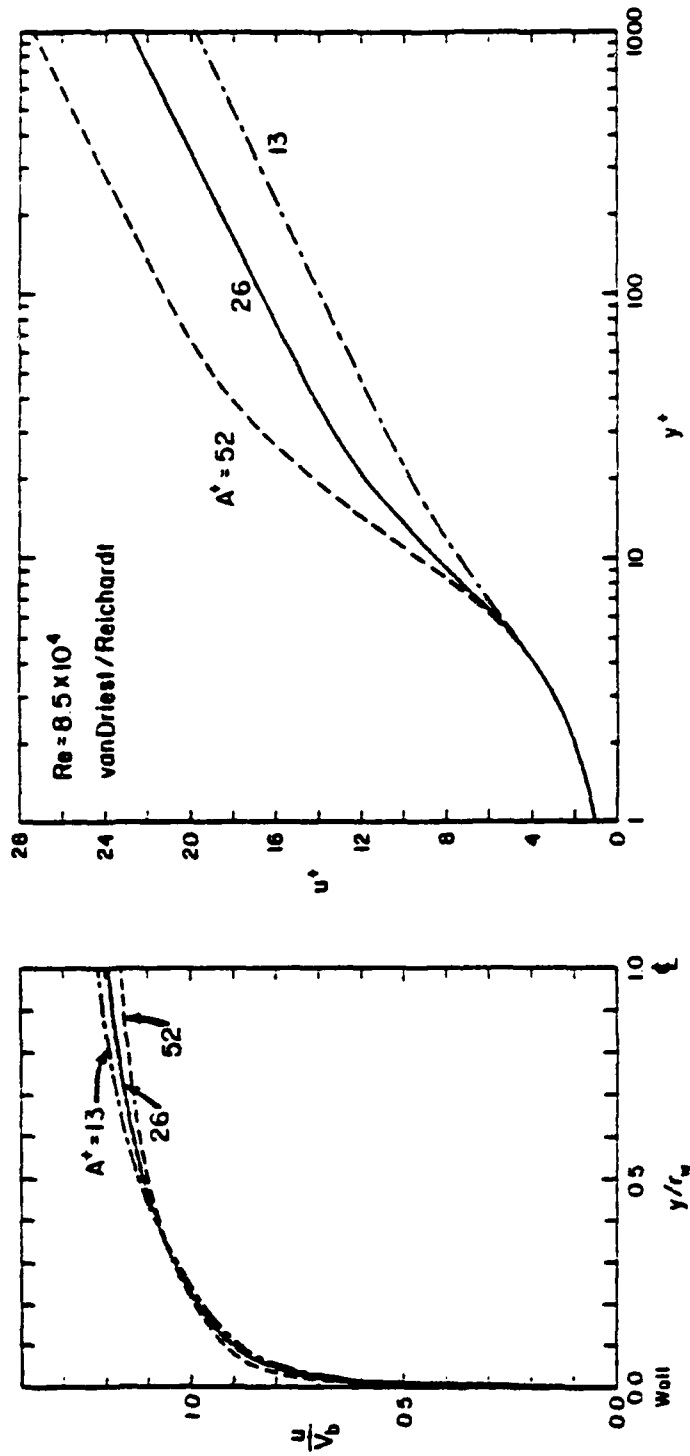


Figure 8. Effect of viscous layer on predictions of velocity profiles for fully developed, turbulent flow. Constant fluid properties.



the set of partial differential equations governing convective heat transfer, so he presented integral analyses based on the approaches of Deissler [1955], Goldmann [1954] and his own work. The analysis of Petukhov and Popov, which he presents, typifies these approaches. Liquids, gases and supercritical fluids were treated. Petukhov [1970] correlated their analytical results for the hypothetical fully developed flow with varying gas properties as

$$\frac{Nu_b}{Nu_{cp}(Re_b)} = \left( \frac{T_w}{T_b} \right)^{-[0.3 \log_{10}(T_w/T_b) + 0.36]} \quad (\text{III-20})$$

for heat transfer, and

$$\frac{f_b}{f_{cp}(Re_b)} = \left( \frac{T_w}{T_b} \right)^{-0.6 + 5.6 (Re_{wm})^{-0.38}} \quad (\text{III-21})$$

for wall friction. The predictions for friction were compared to the measurements of Lel'chuk and Dyadyakin [1959], McEligot, Magee and Leppert [1965] and Perkins and Worsoe-Schmidt [1965]. He suggested that although the experimental data showed significant differences themselves, the general trend was a weaker relationship between the friction factor and heating rate than predicted analytically. Again the measured friction factors would have been apparent friction factors,  $f_{app}$ , while the analysis predicted a friction factor based on the velocity gradient at the wall,  $f_s$ .

One of the simplest, approximate analyses for a hypothetical fully developed flow [McEligot, 1967] predicts

$$\frac{Nu_b}{Nu_{cp}(Re_b)} \approx \left( \frac{T_w}{T_b} \right)^{d - \frac{1}{2} - \frac{an}{2} + am} \quad (\text{III-22})$$

where  $d$  is the exponent for the variation of specific heat and  $m$  and  $n$  are exponents for constant property flow, i.e.,  $f = c/Re^n$  and  $St = C/Re^m$ . For moderate heating, these exponents simplify to approximately -0.3 for air and most common gases, in agreement with the predictions of Petukhov and Popov at low temperature ratios.

Petukhov concluded "Disagreement ... may be attributed to imperfect methods of estimating the effect of physical properties on turbulent diffusivity. Therefore, further refinement ... demands enhanced study of turbulent diffusivity with respect to variable physical properties."

By applying approximate analyses considering separate layers across a tube, Tanaka et al. [1973] have derived estimates of the criteria under which buoyancy and acceleration become important in strongly heated flows. The approach is somewhat reminiscent of that of Hall and Jackson [1969]. They have also modified the approach of Goldmann [1954] to predict temperature and velocity profiles, and therefore heat transfer coefficients, in the downstream region of tubes with property variation and near the critical point as well. The Reichardt representation for the eddy diffusivity near a wall was employed with  $K$  taken as 0.4 and 7.15 as the value for  $y_1^+$ . Predictions for forced convection with variable properties were not compared to gas data, but likelihood of a transition from turbulent to laminar flow due to strong heating was discussed.

Initial attempts to model the thermal entry applied boundary layer integral techniques [Deissler, 1953; Wolf, 1958]. However, in the limiting case of constant fluid properties, these analyses do not agree with the exact solution of Sparrow, Halimann and Siegel [1957] or the careful data of Hall and Price [1961] and with moderate variation of fluid properties they disagreed with Wolf's [1959] data by up to 30 %.

In order to determine whether the discrepancies were caused by the method of solution or by other phenomena, Magee [1964] developed a numerical solution of the governing thermal energy equation (I-3) in its partial differential form instead. The velocity profile was considered fully developed at the start of heating and a uniform wall heat flux was assumed thereafter. Velocity profiles were evaluated at the local Reynolds number at each axial position in order to evaluate the axial convective term and, via Reynolds analogy, to estimate the distribution of  $\epsilon_h$  across the tube. Magee used the same velocity profile as Wolf had employed in his integral solution of the thermal energy equation. The radial velocity component was neglected and gas properties were expressed as power law functions of the temperature. Magee's solution for the lowest heating rate,  $q^+ = 0.00024$ , agreed closely with the analysis of Sparrow, Hallmann and Siegel and the data of Hall and Price. At high heating rates, however, the predictions essentially agreed with those of Wolf after the first few diameters. As  $q^+$  increased, the difference from the data in the thermal entry region increased, at  $q^+ \approx 0.005$  reaching about 30 %. This comparison demonstrated that one could not use an assumed velocity profile as basic information for predictions of heat transfer in the thermal entry region with strongly varying fluid properties. It leads to the suggestion that the coupling with the momentum equation should be included in the solution technique.

The comparison between predictions and data showed that the observed reduction in the Nusselt number was more severe than was predicted by the available analyses. And it appeared that  $f$  was reduced less. Some possible explanations would be (a) experimental uncertainty or error, (b) difference in definitions, (c) effects on eddy diffusivity ratio or turbulent Prandtl number, (d) effects missing from turbulence models and/or (e) incomplete governing equations [McEligot, Smith and Bankston, 1970]. In order to investigate these possibilities, Bankston and McEligot developed a

flexible, reliable numerical method for solving the coupled partial differential equations (I-1 to I-4) involved in the thermal entry problem [McEligot and Bankston, 1969; Bankston and McEligot, 1970; McEligot, Smith and Bankston, 1970].

The numerical technique of Bankston and McEligot evolved from the approaches suggested by Worsoe-Schmidt and Leppert [1965] and Patankar and Spalding [1967]; details of the method are provided in the 1970 paper by Bankston and McEligot. Essentially, the approximating finite difference equations were derived from basic principles by setting up finite control volumes, applying appropriate balances and using simple approximations for the rate equations. Dependent variables were  $H$ ,  $u$ ,  $\rho v$  and  $p$ . The numerical representations of the rate equations were chosen so that the energy equation and the momentum equation were linear and implicit when phrased for algebraic solutions. Only the transverse diffusion terms and the axial convective terms were used in setting up the coefficients of the unknown quantities; all other terms were included in the "source" term (except pressure).

The  $x$  momentum equation and integral continuity equation were solved simultaneously by a version of the technique of Worsoe-Schmidt and Leppert which involves a solution of a tri-diagonal matrix with an additional column. The energy equation was represented as a tri-diagonal matrix which was solved by applying the recurrence relations presented by Richtmyer [1957]. The governing equations were solved successively and were then iterated to handle the non-linearities caused by the temperature-dependent property variation as well as by the axial convective term in the momentum equation.

Investigators' tabulations of local wall heat flux were approximated by spline functions to specify the boundary condition. Fluid properties were entered as power-law functions or were tabulated. Trial turbulence models

were entered as sub-routines with effective viscosity and thermal conductivity formed as sums of molecular and turbulent contributions for use in the general program. To date only simple eddy diffusivity models and mixing length models have been employed in the subroutine for the turbulence model.

Mesh spacing was varied in both the radial and axial directions. In order to investigate the effect of the turbulence model in the viscous layer for low Reynolds number flows, the first point of calculation in the radial direction was at  $y^+ \approx 0.5$  or less. To conserve computer time, mesh parameters were chosen to give heat transfer and wall friction results within about  $\pm 1\%$  of converged values.

The numerical program was first used to select a turbulence model for application to situations of gas flow with strongly varying properties. Initial choice was made by comparison of predictions to the most extreme heating rate in the experiments of Perkins and Worsoe-Schmidt [1965], their run 140. With property variation, one does not know in advance how to extend an existing turbulence model which has been developed from measurements of adiabatic flows. Typically, quantities such as the viscosity, density and wall shear stress appear in the algebraic descriptions and one question is: at what temperatures or locations should these quantities be evaluated, e.g., pointwise local, wall integral, etc.? Eleven versions of existing models, such as those of Reichardt [1951]; Sparrow, Hallmann and Siegel [1957] (essentially a version of Deissler's model); van Driest [1956], etc., were evaluated with wall and pointwise local properties and were tested against run 140. Wall temperature predictions, local Nusselt numbers and local apparent friction factors were compared to the measurements. Best agreement was found with a van Driest model with the exponential term evaluated at the wall properties,

$$1 = 0.4y \left[ 1 - \exp \left( \frac{-y \sqrt{g_c \tau_w / \rho_w}}{26\nu_w} \right) \right] \quad (\text{III-23})$$

Even so, wall temperatures were over-predicted somewhat in the immediate thermal entry and under-predicted downstream. Using local pointwise properties, e.g.,  $\nu(T(r))$  in equation (III-25), yielded predictions of the wall temperature which were about 30% lower than the data. From these comparisons, one could conclude that the eddy diffusivity models tested were inadequate for predictions for strong heating at a constant wall heat flux. Mixing length models fared better.

Once the turbulence model was selected, comparisons of numerical predictions were also made to a number of other experiments by Perkins and Worsoe-Schmidt [1965] and McEligot, Magee and Leppert [1965], at various heating rates, with good success. Both wall temperatures and axial pressure drop distributions were compared. The predictions were also performed for comparison to three interesting experiments by Petukhov, Kirillov and Maidanik [1966]. The data of Perkins and Worsoe-Schmidt and of McEligot, Magee and Leppert were obtained with approximately constant wall heat flux and an unheated entrance region. Petukhov, Kirillov and Maidanik varied the entering and the thermal boundary conditions, also choosing an abrupt entrance and a wall heat flux distribution rapidly increasing in the streamwise direction. Again the numerical results predicted the trends well with the absolute agreement being comparable to that obtained in the earlier comparisons. And again, wall temperatures were over-predicted for the thermal entry and under-predicted further downstream. Overall, agreement with heat transfer data for extreme heating was favorable. McEligot, Smith and Bankston [1970] observed that solving the complete boundary layer equations with axial effects plus selection of an alternate turbulence model resolved several of the earlier discrepancies between experiments and analyses.

Examination of the predicted velocity profiles for moderate and strong heating provides some insight into the effects caused by moderate and severe heating and, therefore, property variation. In Fig. 9 profiles at several axial stations are compared with the profile in the unheated entrance

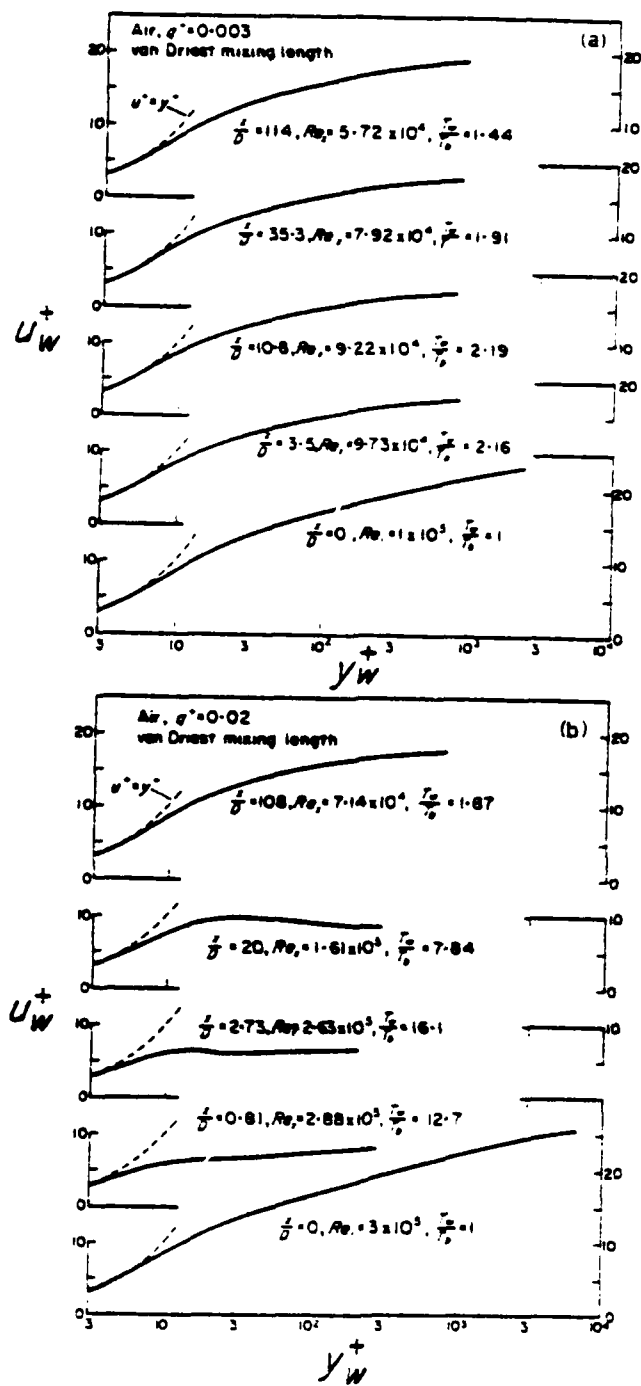


Figure 9. Predicted variation of velocity profiles in streamwise direction for turbulent flow of gases with heating allowing for temperature-dependent properties. [Bankston and McEligot, Inc. J. Heat Mass Transfer, 1970].

region; semi-logarithmic  $u_w^+ (y_w^+)$  coordinates are used. The most obvious effect of heating is a reduction in magnitudes which is caused by the increased wall viscosity used in the definition of these variables. For the moderate heating rate,  $q^+ = 0.003$ , the shapes remain comparable to the entering profile. It appears that the profiles depart from the adiabatic behavior in the viscous layer at slightly higher values of  $y_w^+$ . And since  $y_w^+$  is already foreshortened by the increased wall viscosity, it becomes apparent that heating causes a thickening of the viscous layer in terms of physical distance,  $y_w$ . The thicker viscous layer implies less turbulent transport of energy, so the local thermal resistance is increased and the Nusselt number is decreased. For the more severe heating rate, leading to temperature ratios up to 16, the shapes of the velocity profiles are not comparable to the entering profile until well downstream, where the temperature ratio is again modest. After a few diameters a peak in the velocity distribution appears near the wall rather than at the centerline (thereby causing difficulty for mixing length models such as the one used). One can see that the use of integral boundary layer analyses, with  $u^+ (y^+)$  similarity assumed for the profiles, would not be justified for such strongly heated thermal entry regions. It appears that the velocity profile diverges from the linear layer prediction of adiabatic flow at a smaller value of  $y_w^+$  in this region; however, the location where the viscous transport is approximately equal to the turbulent transport ( $\epsilon_m = \nu$ ) is over twenty times thicker than predicted by the adiabatic profile for the same local Reynolds number. Therefore, specification of a constant value of  $y_w^+$  or  $u_w^+$  as a criterion for viscous sublayer thickness appears meaningless for these highly heated conditions.

The results of the numerical predictions allow comparison of  $f_{app}$ , the apparent friction factor, to  $f_s$ , the friction factor based on wall velocity gradient as in many analyses (see section 1.4 or Nomenclature). In the thermal entry region,  $f_{app}$  increases as the heating rate is increased and as  $x/D$  is reduced. On the other hand,  $f_s$  decreases with heating rate and



remains relatively constant through the thermal entry region. Beyond several diameters the apparent friction factor also decreases below the value predicted by a constant property analysis for the same value of the local bulk Reynolds number. At low and moderate heating rates,  $f_{app}$  is always greater than  $f_s$ . This observation partially explains the discrepancy between experiments and predictions - a weaker relation between friction factor and heating rate in experiments than analyses - as noted by Petukhov [1970]. Different definitions of the friction factor have been used by the experimentalists than by the analysts and the results should not be the same. With numerical predictions of both velocity profiles and the axial pressure gradient, it is no difficulty for the analysts to provide predictions in the form in which the experimentalist is forced to reduce his data. When the velocity profile becomes severely distorted, as it does at  $q^+ \gtrsim 0.01$ , the axial variation of  $f_{app}$  also becomes distorted and  $f_{app}$  can even become negative in the extreme.

The downstream trends of the integral parameters are presented in Fig. 10 for a number of heating rates and entering Reynolds numbers. The primary results are the normalized parameters plotted against the property variation parameter  $T_w/T_b$ . This is the form that the experimentalist often uses for correlation purposes. The symbols represent the beginning of the downstream variation for each run, at about 40 diameters after the start of heating. Results are plotted as solid lines which progress from right to left as  $T_w/T_b$  decreases downstream. Two regions are apparent in each sub-figure. As each run proceeds, it approaches an asymptotic behavior downstream at low  $T_w/T_b$ . Approaching this range are curves representing the transition from the higher values in the immediate thermal entry. The asymptotic heat transfer behavior is in approximate agreement with the correlation proposed by Kutateladze and Leont'ev [1964]. Since Figure 10a does not

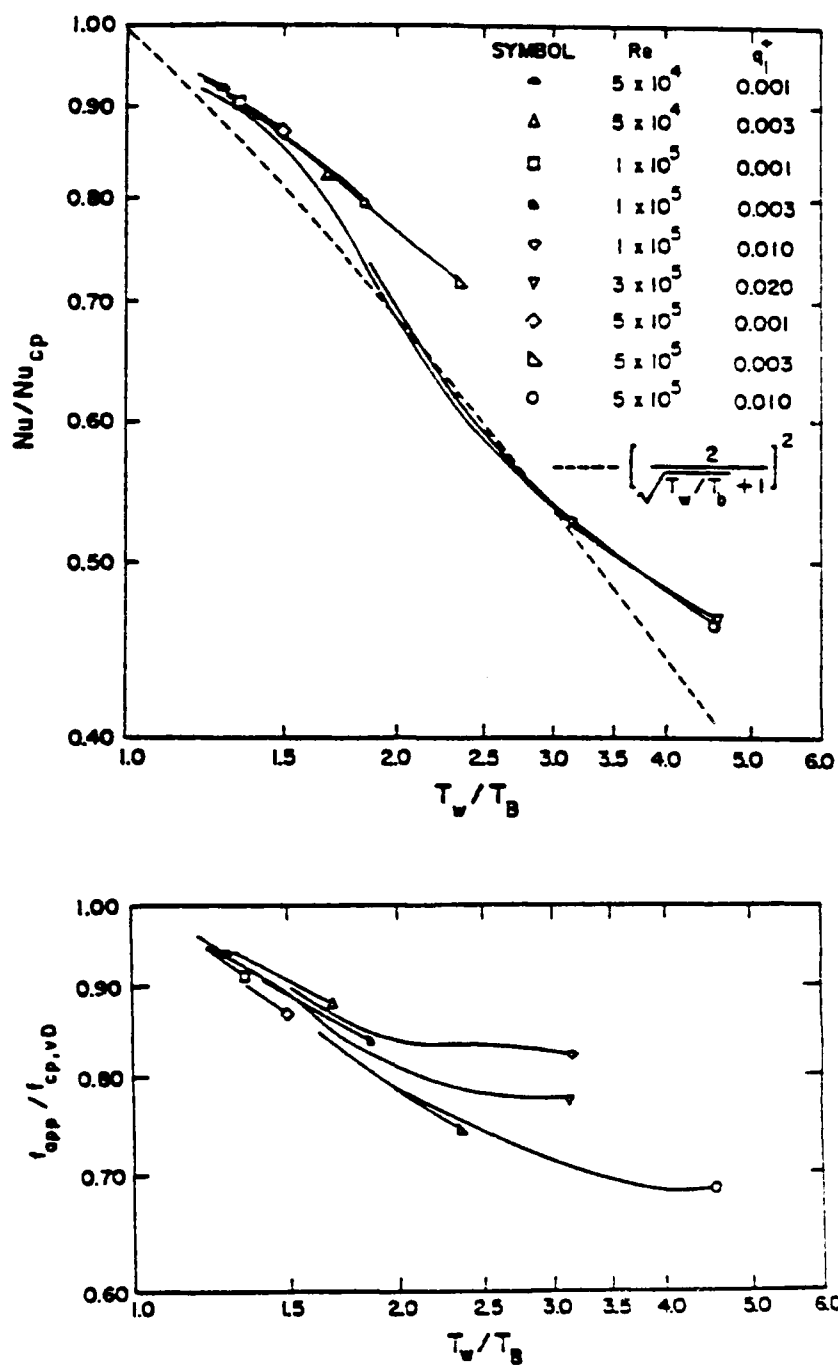


Figure 10. Quasi-developed turbulent flow of gases in tubes. Constant wall heat flux,  $x/D \gtrsim 40$ . [McEligot, Smith and Bankston, ASME J. Heat Transfer, 1970]

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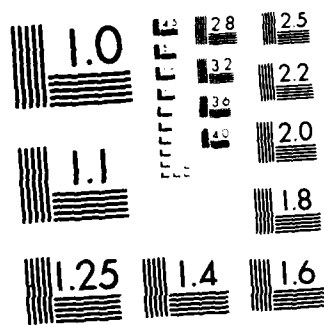
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show a simple power law dependence, experimental correlations presented as such will represent only the region in which the majority of their data are collected. Thus, at low heating rates the film temperature reference is adequate [McEligot, 1967]. The downstream data of Perkins and Worsoe-Schmidt ranged from  $1.5 \lesssim T_w/T_b \lesssim 3$  and their correlation consequently shows a steeper slope and higher intercept. The maximum  $q_i^+$  of McEligot, Magee and Leppert [1965], about 0.004, was between the values of these studies and so was their correlation. The correlations are consistent with the trends of Fig. 10. From Fig. 10b, one may estimate an exponent of about -0.25 for a power law correlation of  $f_{app}$  in terms of local bulk Reynolds numbers downstream.

The predictions of Bankston and McEligot [1970], just discussed, were based on the assumption of Reynolds' analogy ( $Pr_t = 1$  or  $\epsilon_m = \epsilon_h$ ) in determining the effective thermal conductivity in conjunction with the turbulence model selected for determining the effective viscosity. It was pointed out in section III.1. that thermal entry measurements in the range  $0.3 \lesssim Pr \lesssim 0.7$  justify Reynolds analogy to within about 10 % for conditions of constant fluid properties. As a consequence of their experimental technique, Pickett, Taylor and McEligot [1979] and Serksnis, Taylor and McEligot [1978] also obtained information on the effect of property variation on the apparent turbulent Prandtl number in the wall region for gases with lower Prandtl numbers than air and helium. Though their comparisons at strong heating rates were not exhaustive, they were able to conclude that use of  $Pr_{tw} \approx 1$ , deduced in low heating rate experiments for  $0.3 \lesssim Pr \lesssim 0.7$ , provided adequate predictions when property variation became significant at  $Re_i \gtrsim 3 \times 10^4$ .

Comparable numerical predictions with mixing length models have been presented by Malik and Pletcher [1978] for heating in annuli, Schade and McEligot [1971] for cooling between parallel plates, Wassel and Mills

[1979] for cooling in rough pipes and Kays [Kays and Crawford, 1980] for comparison to a tube flow correlation. Comparison to experiments has been spotty in the above studies, presumably because accurate, detailed data are limited. Differing choices of  $Pr_{tw}$  and in evaluation of properties in the expression for  $l(y)$  have been used and success has been claimed in each case, so there is no single, definitive model. Since most comparisons involved heat transfer results, apparent agreement could be obtained by adjusting either  $Pr_{tw}$  or the momentum transport model or both. Further evaluation of the numerical models and the data - with careful estimates of experimental uncertainties - is necessary for these cases. Whether a mixing length model can be adequate for predicting heat transfer and wall friction parameters with severe property variation has not yet been completely proven - nor refuted. The evidence to date implies that most data for heat transfer parameters can be approximately matched by adjusting a few quantities in a mixing length model for the range of the particular set of experiments, but there is not universal agreement on the appropriate treatment of those quantities (e.g.,  $y^+(T, y)$ ,  $Pr_t(y)$ , etc.).

Other analytical techniques which might be applied include advanced turbulence modeling, surface renewal approaches and large eddy simulation.

Advanced turbulence models have been developed to predict the variation of the turbulent shear stress by solution of an appropriate governing equation instead of applying an empirical algebraic relation for  $\mu_{eff}$  directly. The background and some early models have been reviewed by Launder and Spalding [1972] and recently Rodi [1980] has demonstrated a wide range of applications.

A typical, popular approach, the  $k-\epsilon$  model, has been recommended by Launder and Spalding [1974]. The turbulent viscosity is modeled as

$$\mu_t = c_\mu \rho k^2 / \epsilon \quad (\text{III-24})$$

where  $k$ , the turbulence kinetic energy, and  $\epsilon$ , its dissipation, are determined from the solution of a coupled set of governing partial differential equations. For turbulent regions beyond the viscous layer, one form of these equations is:

turbulence kinetic energy,  $k$

$$\frac{Dk}{Dt} = \frac{1}{\rho} \frac{\partial}{\partial x_k} \left[ \frac{\mu_t}{Pr_k} \frac{\partial k}{\partial x_k} \right] + \frac{\mu_t}{\rho} \left( \frac{\partial U_i}{\partial x_k} + \frac{\partial U_k}{\partial x_i} \right) \frac{\partial U_i}{\partial x_k} - \epsilon \quad (\text{III-25})$$

dissipation rate of turbulence kinetic energy,  $\epsilon$

$$\frac{D\epsilon}{Dt} = \frac{1}{\rho} \frac{\partial}{\partial x_k} \left[ \frac{\mu_t}{Pr_\epsilon} \frac{\partial \epsilon}{\partial x_k} \right] + \frac{C_1 \mu_t}{\rho} \frac{\epsilon}{k} \left( \frac{\partial U_i}{\partial x_k} + \frac{\partial U_k}{\partial x_i} \right) \frac{\partial U_i}{\partial x_k} - C_2 \frac{\epsilon^2}{k}$$

where  $Pr_k$ ,  $Pr_\epsilon$ ,  $c_\mu$ ,  $C_1$  and  $C_2$  are empirical functions, hypothesized to be constants. The modeling of the transport of  $k$  and  $\epsilon$  has been chosen so the resulting equations have the same mathematical character as the  $x$ -momentum equation (I-2), i.e., parabolic, so the same subroutine can be used for the numerical solution of each equation. For simple flows where wall similarity applies, wall functions (relating conditions at  $y^+ = 0$  and  $y^+ \approx 50$ ) are used as boundary conditions [Launder and Spalding, 1974]. For gas flows with strong property variation, a key problem is modification of the wall function to account for the modification of the viscous layer which apparently occurs.

To allow for effects on  $k$  and  $\epsilon$  in the viscous layer in low-Reynolds-number flows, Jones and Launder [1972] added to equations (III-25) terms modeling their viscous transport. Launder and Amman, in turn, modified

this version to handle variation of temperature-dependent transport properties but results are not yet available [Launder, personal communication]. Kawamura [1979a] tested the  $k-\epsilon$  model and two others against some data from Perkins and Worsoe-Schmidt [1965] and laminarizing data; as described later in section IV.5., he found a  $k-k_L$  model to provide the best results ( $L$  is a turbulence length scale). Abdelmaguid and Spalding [1979] applied the  $k-\epsilon$  model to flows in pipes with buoyancy effects with a wall function treatment for the boundary. Hauptmann and Malhotra [1980] used the  $k-\epsilon$  model in the turbulent core to treat flow of supercritical carbon dioxide with significant density variation, but a van Driest mixing length model was employed near the wall. They found the predicted wall temperature distributions to be the reverse of observed trends and surmised that the reason was failure to allow for modification of the turbulence behavior due to buoyant forces.

Daniels and Browne [1981] compared five turbulence models, varying in complexity from a simple mixing length model to a "two-equation" model, to measurements of surface heat flux in air flow through a turbine blade cascade at conditions representative of gas turbine practice. The wall-to-inlet temperature ratio was 0.7, approximately. No advantages of complex models were seen for the suction surface; on the pressure surface the two-equation model was qualitatively better but quantitatively still poor. Since the complex models required much more computer time and were difficult to modify, Daniels and Browne doubted their value as a design tool.

The surface renewal approach, proposed by Higbie [1935], Danckwerts [1951] and Hanratty [1956] and applied exhaustively by Thomas, attempts to represent some features observed in the viscous layer qualitatively by transient solutions,  $T(y, \theta)$  and  $u(y, \theta)$ . Thomas, Rajagopal and Chung [1974] developed a version to account for variable conductivity and viscosity in liquid flows, but the technique does not appear to have been used for property variation



in gases.

Typically, one-dimensional transient versions of the momentum and energy equations are solved as for a suddenly accelerated flat plate. The solutions are terminated at the end of a "contact time" and are then averaged over some contact time distribution to give mean values of  $u^+(y^+)$  and  $t^+(y^+)$  and, consequently,  $Nu$  and  $f$ . Boundary and initial conditions, necessary for the solutions, introduce several control parameters. Ultimately,  $Nu$  and  $f$  are coupled in the result so a knowledge of the correct value of  $f$  is needed to predict  $Nu$ ; for the present topic, this is a weakness in the method since the effect of property variation on  $f$  is not known in advance. The idea of representing the sweep-ejection process of the viscous layer [Corino and Brodkey, 1969], at least qualitatively, has a certain attractiveness but the method needs further development before it can be used for predictions in cases where there is no foreknowledge of the results.

Large eddy simulation (LES) is a relatively new approach to the calculation of turbulent flows [Deardorff, 1970; Schuman, Grötzbach and Kleiser, 1979]. The basic idea comes from the experimental observations that the large scale structure of turbulent flows varies greatly for different geometries and is therefore difficult to model in a general way, and that the small-scale turbulence structures are apparently close to being universal in character and hence amenable to modeling. In LES one calculates the large-scale structures by a three-dimensional time-dependent computation using essentially the full Navier-Stokes equations and incorporates simpler modeling for the small-scale turbulence. The boundary conditions employed are usually spacewise periodic in the streamwise and lateral directions. Such calculations require a great deal of computer time and storage so they are not generally practical for direct use in an engineering design. However, they can provide insight into the behavior of turbulence in some simple flows, such as fully

established channel flows. By including a governing equation for thermal energy in the calculations one can also account for temperature fluctuations and heat transfer; this has been done by Grötzbach, who has also made calculations for secondary flows in partly roughened channels, buoyancy and liquid metal flows in both plain channels and annuli [Schuman, Grötzbach and Kleiser, 1979]. In these computations the boundary conditions were modified by wall functions based on assumed logarithmic profiles and thereby the dynamics of the viscous region for simple shear layers were essentially ignored.

The first LES calculation of turbulent channel flow that computed the flow in the immediate neighborhood of the wall rather than modeling it was apparently that of Moin, Reynolds and Ferziger [1978]. The recent report of Moin and Kim [1982] extended that study to a finer numerical grid in order to provide better details of the smallest structures in the wall region. Using 516,096 grid points in a calculation on the ILLIAC IV computer, they obtained realistic predictions of a fully-developed, turbulent, duct flow at  $Re_{D_h} \approx 5 \times 10^4$ . In agreement with experimental observations, it was found that the computed flow pattern in the wall region was characterized by coherent structures of low and high speed streaks alternating in the spanwise direction [Kline et al., 1967].

It should be possible to apply LES modeling to a few problems involving large temperature differences by accounting for property variation due to the temperature dependence. For example, one could extend a calculation like that of Grötzbach or of Moin and Kim to a channel flow with one wall at a much higher temperature than the other wall, with both constant in space. This is a situation which leads to an invariant behavior downstream in the streamwise direction and, therefore, would be adaptable to the boundary conditions employed by these investigators. Such a simulation could evaluate the effect of property variation on wall functions or on mixing length models

for the immediate wall region provided that the thermal boundary condition was a constant wall temperature. One could also envision solving the problem with a specified wall heat flux on one surface and a specified wall temperature on the other surface, leading to essentially the same fully established situation downstream. However, practical applications with specified wall heat flux distributions usually lead to an acceleration of the flow such that streamwise periodic boundary conditions would be inappropriate; further, the resulting wall temperature increases rapidly in the streamwise direction so that the properties of the viscous layer would be continuously changing in that direction. These difficulties might be surmounted by applying modified periodic boundary conditions as used by Short [1977] and Faas and McEligot [1979] for velocity and temperature distributions in laminar flow over spacewise periodic geometries, such as artificially fabricated roughness elements on the wall of a duct.

#### IV. Low-Reynolds-number turbulent flows and laminarization

##### IV.1. Preliminary comments

One can expect a turbulent flow eventually to become laminar when heated because, as mentioned in the Introduction, the change in viscosity causes the Reynolds number to decrease continuously. After it is reduced below some transition Reynolds number, one would expect the turbulent processes to decay. Indeed, the experiments of Sibulkin [1962] and the current studies of Champagne at the University of Arizona investigate these laminarization phenomena by an expansion in internal diameter, thereby reducing the Reynolds number. However, for heated gas flows with a constant cross-sectional area, it has been observed that heat transfer parameters can approach those predicted by laminar analyses even though the local Reynolds

number is still well above the usual value for a transition Reynolds number in a circular tube [McEligot, 1963; Bankston, 1965].

Further, any strongly heated internal gas flow will laminarize in the sense of Patel and Head [1968] - a thickening of the viscous sublayer - but that effect is handled as standard practice in empirical and numerical relations for variable property, turbulent gas flow. It must be emphasized that in the present context neither of these situations is implied. Instead, as an operational definition, laminarization is considered a process where wall parameters approach the appropriate laminar predictions at local Reynolds numbers where turbulent flow is normally expected to occur. The range where laminarization occurs can be extremely important since a sudden decrease in heat exchanger performance may occur in this region. In fact, Coon [1968] reported a pair of experiments intended to be at identical conditions - the same heating rate and flow rate - where one remained turbulent and the other evidently laminarized.

Since the apparent laminarization or approach thereto typically occurs at low Reynolds numbers (3000 to 20,000) it is important to understand the limiting conditions in this range, i.e., the behavior the flow would show at the same local Reynolds number if unheated. As the constant property behavior in this range differs from the expectations based on understanding of high Reynolds number duct flows, this chapter will first consider the evidence available for low-Reynolds-number flows at heating rates sufficiently low that the constant properties idealization is reasonably valid.

#### IV.2. Low-Reynolds-number, turbulent, constant-property flow

In developing their empirical correlation for Reynolds numbers as low as about 2500, Dittus and Boelter [1930] demonstrated agreement with the relation they proposed. On the other hand, when the semi-empirical analyses of Martinelli [1947], Deissler and Eian [1952] and others are extended into the low-Reynolds-number, turbulent flow region, the predictions show increases above a straight line extrapolation, which would correspond to the Dittus-Boelter correlation. These predictions have been based (1) on the assumption of a so-called universal velocity profile, developed for high Reynolds number turbulent flow, (2) on approximations of the heat flux and shear stress profiles and (3) on the assumption of equal eddy diffusivities. Comparison to the data of McEligot [1963] and Leung, Kays and W.C. Reynolds [1962] shows the Dittus-Boelter correlation to be a better description in the range  $3000 \lesssim Re \lesssim 20,000$ . The analyses continuously diverge from the data as the Reynolds number is lowered.

McEligot [1963] based the radial heat flux distribution on the velocity profile used by Martinelli and found a slight improvement over the latter's predictions. Deissler [1952] proposed a model to allow for variation of  $Pr_t$  with dependence on the Peclet number; this dependency was evaluated by forcing the theory to agree with heat transfer data for gases while maintaining the "universal velocity profile" as the basic flow description. (More recent data have shown that  $Pr_t$  does not vary significantly, but the velocity profile does.)

Measurements of actual velocity profiles at Reynolds numbers between 3000 and 15,000 have been made by Nikuradse [1932], Deissler [1950] and Senecal [1951] in circular tubes. Of these works only that by Senecal shows use of a method to eliminate the effects of the probe on the flow pattern; his data also extend well into the viscous layer. For flow in the low Reynolds

number turbulent range, Senecal's velocity profiles, taken at a sufficient axial distance to have an intermittancy factor of approximately unity [Rotta, 1956], are higher in the region  $5 < y^+ < 30$  than predicted by the universal velocity profiles used by Martinelli and others. This difference would cause an increase in the convective heat transfer resistance and, thus, a lowering of Nusselt numbers below the predictions of Martinelli. In turbulent boundary layers at  $425 < Re_{\theta} < 600$ , White [1981] also found that the so-called universal velocity profile was no longer valid. Likewise, he concluded that in that range the viscous layer no longer maintains a characteristically small size but grows into a "super sublayer" which approaches ten percent of the boundary layer thickness, greatly exceeding conventional predictions.

In order to improve predictions of heat transfer in this range, McEligot, Ormand and Perkins [1966] developed an improved prediction of the velocity profile by adopting a simple turbulence model, one with a completely laminar sublayer,  $y_l$ , to agree with the measured data of Senecal. A mixing length distribution was used for the turbulent core, subject to the constraint that the overall continuity equation was satisfied in accordance with the Blasius relationship for friction factor [Knudsen and Katz, 1958]. Reynolds analogy was assumed for  $Pr_t$ . As shown by Figure 11, these simple requirements lead to excellent agreement with the measurements for fully developed flow.

Subsequently, H.C. Reynolds [1968] obtained additional adiabatic velocity profile measurements and modified Reichardt's [1951] eddy diffusivity model to agree. H.C. Reynolds, Swearingen and McEligot [1969] applied this model in the manner of Sparrow, Hallman and Siegel [1957], to predict the thermal entry development and verified the predictions by experiments. The velocity profiles have also been confirmed by the measurements of fully developed pipe flow by Patel and Head [1969].

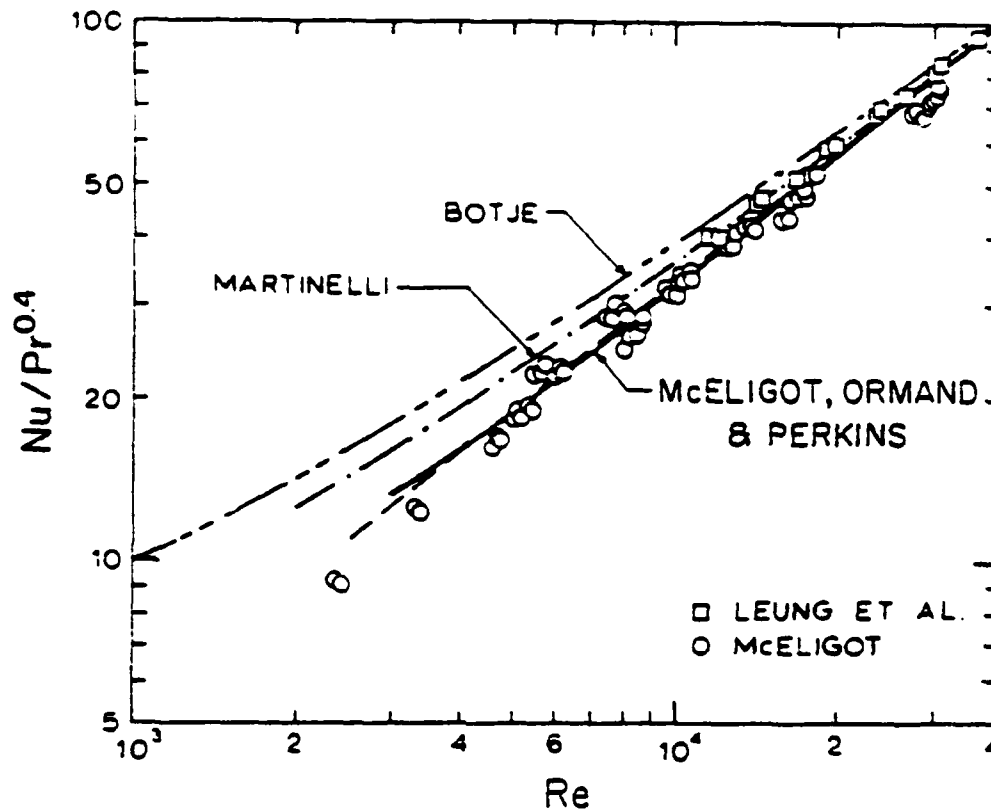


Figure 11. Heat transfer in fully-developed, low-Reynolds number, turbulent flow. Constant properties,  $Pr \approx 0.7$ . [McEligot, Ormand and Perkins, ASME J. Heat Transfer, 1966].

Wilson and Azad [1975] extended the eddy diffusivity approach of Reynolds, McEligot and Davenport [1968] by adding an intermittancy function. They modified the function for the sublayer thickness slightly and applied a least squares method with the data of Patel and Head to evaluate the constants. By these means they developed a single prediction technique for mean velocity profiles and friction factors in fully developed laminar, transitional and turbulent Reynolds number ranges and found good agreement with data for  $100 < Re < 500,000$ .

#### IV.3. Typical behavior during laminarization by heating

Laminar and transitional flow in a square duct was tackled by Lowdermilk, Weiland and Livingood [1954] but they obtained average parameters, rather than local values, and at the time the phenomena of laminarization was not recognized. In the first report of flow laminarizing due to heating, the present author [1963] speculated that the cause of the unexpected reduction in heat transfer parameters was due to a thickening of the viscous sublayer beyond the value for an asymptotic flow. Perkins and Worsoe-Schmidt [1965] extended the measurements to more extreme property variation and Bankston [1965, 1970], obtained a wide range of systematic measurements in a particularly long tube.

The difficulty which can occur with laminarization was graphically demonstrated in Fig. 1 which showed the data of Coon [1968]. After approximate agreement with the behavior in the immediate thermal entry, when predicted by a correlation accounting for fluid property variation, the wall temperatures can become much higher than predicted as the flow proceeds further downstream.

The axial development can be described graphically in a number of



ways. Useful for an overview is the technique of Bankston [1970] who plots the local Stanton number versus the local Reynolds number with the predictions for fully developed, constant property conditions used as guides. In Fig. 12 illustrative measurements by Bankston [1965] are plotted in this manner. These data were obtained for hydrogen precooled to liquid nitrogen temperature before electrical heating in a circular tube about 420 diameters long. Four runs at varying entry Reynolds numbers and varying heating rates are shown. Since the local Reynolds number decreases from heating as  $x$  increases, the downstream progression of the data points is from right to left on the figure. One sees first the very high values of Stanton number associated with the immediate thermal entry region; these serve also to identify the entering Reynolds number approximately. Three classes of experiments are evident in the figure:

- (1) Temperatures near the end of the tube are sufficiently high that  $Re_b$  decreases below 2000 or so and laminar results are expected, e.g., run 81, shown with circles.
- (2) Heating causes an expected reduction of heat transfer parameters to below the constant property prediction for turbulent flow near the start of heating, then the downstream result converges back towards the turbulent prediction as the temperature ratio decreases in the downstream direction, e.g., runs 103 and 106, shown by squares and crossed circles, respectively.
- (3) Although  $Re_b$  is well above 2000 at the exit of the tube, results converge to the laminar prediction downstream, e.g., run 131, triangles.

Shown also on Fig. 12 by solid lines are predictions using the van Driest turbulence model as described by Bankston and McEligot [1970]. For

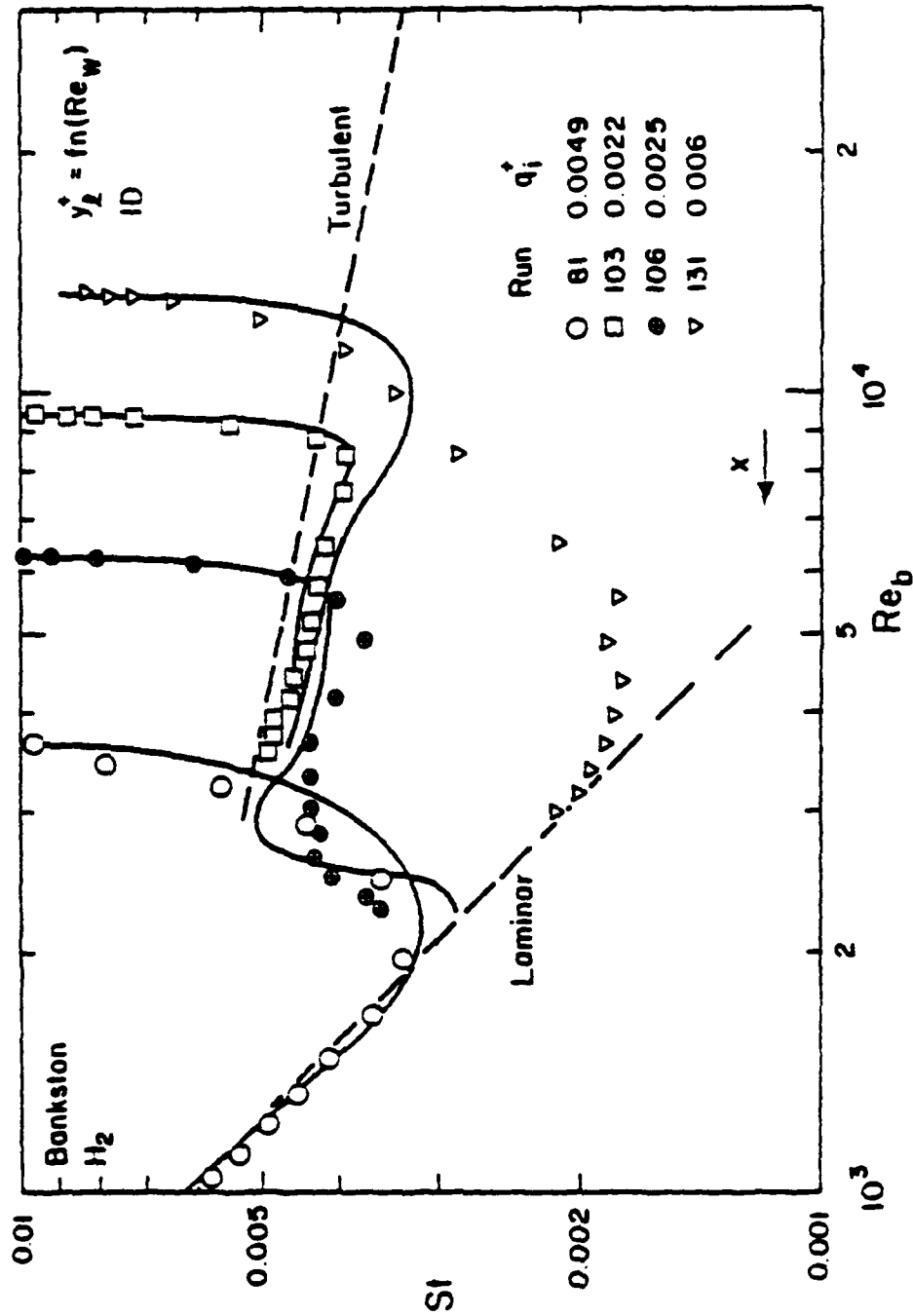


Figure 12. Comparison of heat transfer predictions and measurements for turbulent and laminarizing flows in tubes. McEligot and Bankston, ASME paper 69-III-52, 1969.

the most part, predictions shown for classes (1) and (2) seem adequate, but for run 131 they correspond to turbulent behavior throughout (class 2) while the data clearly established the situation as a case of effectively complete laminarization.

Alternate views are presented in Fig. 13 with data for strongly heated flow in a square duct [Perkins, Schade and McEligot, 1973]. Again the axial variation is shown, this time for a sequence of runs at a relatively low entering Reynolds number ( $Re_1 \approx 4000$ ) and the downstream progression is the normal left to right. The third subfigure compares the data to the variable properties, empirical correlation recommended by Battista and Perkins [1970] for heating in a square tube; thus, agreement with this correlation (a value of unity) would imply a normal turbulent run. Examining the immediate thermal entry on the lower subfigure, one sees that the data converged towards a Leveque approximation, i.e.,  $Nu \sim (x^+)^{-1/3}$ , regardless of heating rate. The same trend had been demonstrated previously for turbulent flow in circular tubes by the numerical analyses of Bankston and McEligot [1970]. In contrast, laminar flow predictions for the effect of property variation show a distinct dependence on heating rate in the immediate thermal entry [Worsoe-Schmidt and Leppert, 1965]. Thus, this presentation demonstrates a normally turbulent entering flow. As seen in the third subfigure, the data at the lower heating rate do not differ significantly from the turbulent prediction after the thermal entry, while the other heating rates successively diverge. The lower subfigure shows them to eventually approach the prediction for laminar constant properties behavior.

Since the turbulent profile in the adiabatic entry is rather blunt, even at low Reynolds numbers, one could suggest that a solution for laminar flow with a uniform inlet velocity profile would provide reasonable predictions for the axial variation of laminarizing flows. However, the solution for a

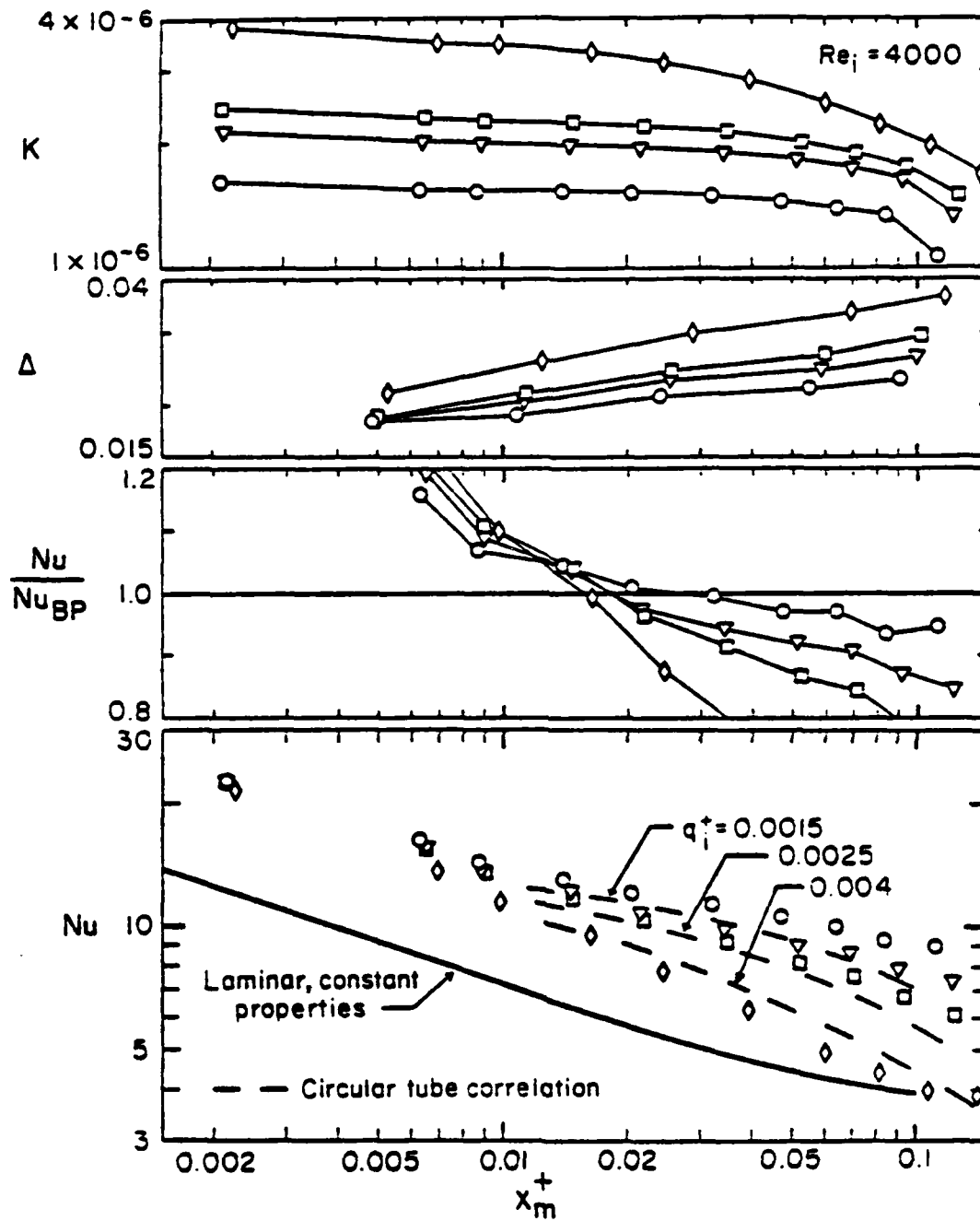


Figure 13. Axial behavior of integral parameters in turbulent and laminarizing flows. [Perkins, Schade and McEligot, Int. J. Heat Mass Transfer, 1973].

uniform inlet velocity profile in laminar flow in a circular tube with strong heating [Bankston and McEligot, 1970] underpredicts the data shown in Fig. 13 significantly and the comparable solution for a square tube would be expected to be slightly lower. Thus, for these data there presumably was still significant turbulent transport of energy in the thermal entry region.

In his early work on flow laminarizing due to heating, the present author [1963] developed a crude empirical correlation for heat transfer during an axial transition from turbulent to laminar flow in circular tubes. Under the assumptions of constant Prandtl number, specific heat and heat flux, this correlation may be transformed to the form

$$Nu \approx 0.021 Re_i^{0.8} Pr^{0.4} [1 + K_i Re_i x/D_h]^{-2n} \quad (IV-1)$$

where  $n$  represents the exponent in a power law approximation of the viscosity-temperature relationship and  $K_i$  is a measure of the fluid acceleration to be discussed later. This relationship was developed from data for gas entering at room temperature and for moderate axial distances. It is plotted as the dashed lines on the lowest part of Fig. 13. One sees that for data which do not agree with the turbulent correlation, the trends are predicted well and the magnitudes are reasonable. Obviously, this correlation should not be used beyond the range of conditions for which it was developed; in particular, ridiculous values would be calculated at axial distances that are either too short or too long.

Data do not appear to be available for the velocity profile development for laminarizing flows with dominant forced convection for some of those same reasons mentioned earlier relative to turbulent flow at higher Reynolds numbers. By using a thermocouple probe in the same configuration as a hot wire sensor for a boundary layer, K.R. Perkins and McEligot [1975]

have determined the mean temperature distributions, radially and axially, for a series of experiments ranging from turbulent to laminarizing. The results at an axial position of about 25 diameters are presented in Fig. 14 for a sequence of typical runs with successively more severe conditions (lowering inlet Reynolds number and increasing heating rate). The shapes and trends of these normalized temperature profiles bear distinct similarities to velocity profiles,  $u^+(y^+)$ , observed in external flows laminarizing due to acceleration [Blackwelder and Kovasznay, 1972]. It is seen that the centerline value of  $y^+$  decreases as the Reynolds number is reduced, as expected in adiabatic flows, and it also decreases further as the heating rate is raised. At low  $y^+$  each run converges with the "linear" sublayer prediction ( $u^+ \approx y^+$ ) with the exception of a few points in the immediate vicinity of the wall. All show a possible turbulent core where  $t^+$  varies as  $\ln y^+$ . Two or three runs appear to show a constant temperature near the centerline, implying that the thermal boundary layer may not fill the tube completely at this station; this observation will be discussed later in comparison to numerical predictions. These data do seem to confirm the qualitative explanation of McEligot [1963] and the numerical conclusion of McEligot and Bankston [1969] that thickening of the viscous sublayer provides a plausible explanation of laminarization.

#### IV.4. Criteria for laminarization by heating

As noted above, the consequence of laminarization is a substantial reduction of heat transfer coefficients and an increase in wall temperature. The danger is obvious for situations where the heating rate is being controlled. (For thermal protection of the walls of a rocket nozzle, on the other hand, such a reduction would be an advantage.) Until turbulence models are developed to predict the necessary parameters through the axial laminarization process reliably, the engineer needs design criteria to predict when it

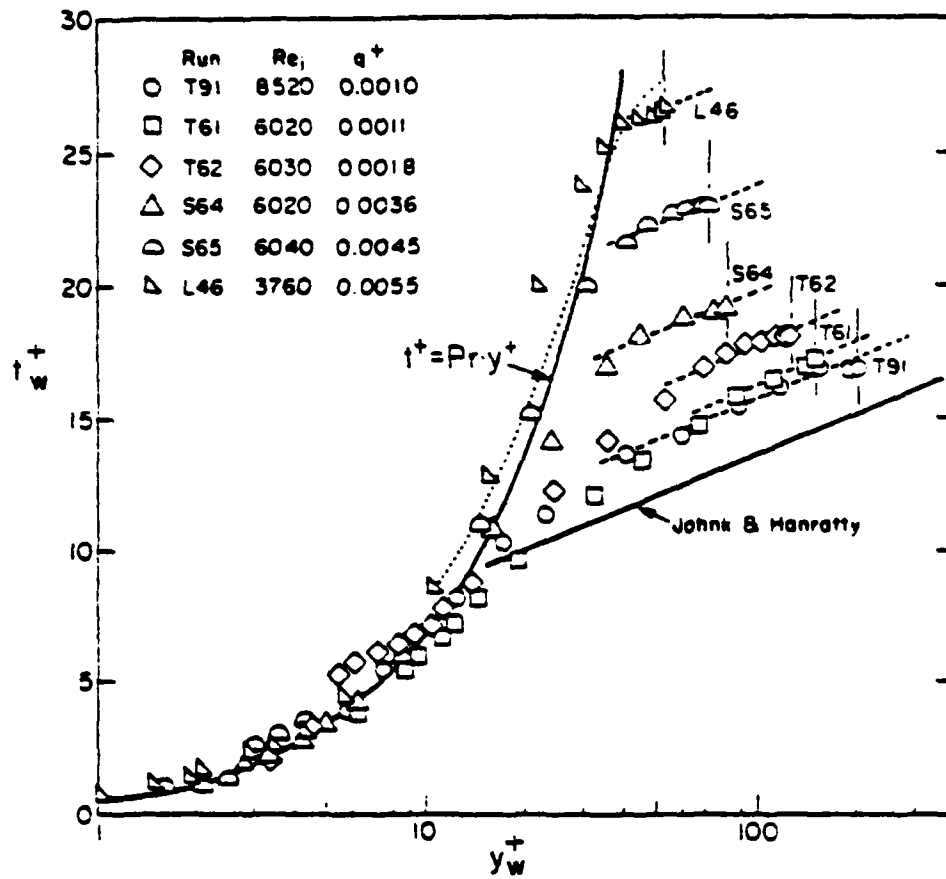


Figure 14. Mean temperature profiles in quasi-developed flows; turbulent, sub-turbulent and laminarizing. [Perkins and McEligot, ASME J. Heat Transfer, 1975].

is imminent. Approximate criteria have been developed from experiments which have been conducted with constant wall heat flux.

McEligot [1963] examined such flows and estimated their flow regimes as turbulent or transitional (laminarizing) from agreement with his turbulent and transitional correlations. The transitional runs were "strongly" heated ones in which the unheated entry flow was clearly in the turbulent range but which gave downstream results diverging from the turbulent, variable properties correlations. He presented the estimated regimes as graphical functions of the non-dimensional heating parameter,  $q_1^+$ , and the entering Reynolds number  $Re_1$ . Thus, the application of his graphical criterion is constrained to comparable boundary conditions. With additional data from the work of Perkins and Worsoe-Schmidt [1965] an alternate presentation of the regimes was developed in terms of  $Re_{wm}(x)$  and the temperature ratio by McEligot, Ormand and Perkins [1966]. Bankston, Sibbitt and Skoglund [1966] devised still another classification scheme.

In studies of heat transfer to flows accelerating due to a change in cross-sectional area, as through a nozzle, Moretti and Kays [1963] found that when a non-dimensional acceleration parameter

$$K = \frac{\gamma}{u_\infty^2} \frac{du_\infty}{dx} \quad (IV-2)$$

was less than  $3 \times 10^{-6}$  the flow apparently remained turbulent, while for higher values it was likely to show evidence of laminar results. In personal discussion, Bankston [1966] noted that he had found that some of his transitory data had values of  $K$  of the same order of magnitude as those of Moretti and Kays. McEligot, Coon and Perkins [1970] examined this suggestion further and demonstrated that the definitions of  $K$  and the non-dimensional heating param-



ters and Reynolds numbers can be interrelated for the situation where the Mach number is low and the heating rate is high so that  $(dp/p) \ll (dT/T)$  and suitable approximations are made. They show that  $K$  can be represented as

$$K \approx \frac{4q^+}{Re_i} \left[ 1 + 4q^+ \frac{x}{D} \right]^{a-d-1} \quad (IV-3)$$

based on entering fluid properties for a constant wall heat flux and

$$K \approx \frac{8Q^+}{Re_b^2 Pr} \quad (IV-4)$$

based on local bulk properties for use with variable wall heat flux. The exponents  $a$  and  $d$  represent power law approximations to the property variation of viscosity and specific heat, respectively. If  $K$  is evaluated at the start of heating it is simply  $4q^+/Re_i$ , i.e., constructed of the parameters suggested by McEligot [1963]. Independently, Larsen, Lord and Farman [1970] derived a comparable approach. By examination of a range of data from Stanford and the University of Arizona, McEligot, Coon and Perkins concluded that the occurrence of a transition from turbulent to laminar flow could be approximately predicted for both internal and external flow experiments by the use of the same parameter  $K_{trans}$ , and that the order of magnitude is the same for both heating in a tube and acceleration in a nozzle.

For the upward of carbon dioxide at supercritical pressures, Hall and Jackson [1969] noticed that "deterioration" in the heat transfer coefficient (apparently a version of laminarization) occurred when the heat flux exceeded a critical value. Based on a phenomenological description of the turbulence behavior in the fluid, a criterion for the occurrence of the phenomenon was tentatively proposed as

$$\frac{Gr'}{Re_c^{2.7}} \left( \frac{\nu_w}{\nu_c} \right)^3 \left( \frac{\rho_w}{\rho_c} \right)^{1/2} \gtrsim 10^{-4}$$

with  $Gr' = \left( \frac{\rho_c - \rho_w}{\rho_c} \right) \frac{g D^3}{\nu_w^2}$  and  $Re_c = \frac{u_c D}{\nu_c}$ .

With a qualitative discussion they also pointed out that the behavior was comparable to that of an accelerated boundary layer as treated by Moretti and Kays [1965].

A number of wall parameters could serve as indicators of laminarization. In many practical applications, the heat flux varies considerably in the axial direction so the local behavior and local criteria become important. Ultimately, the designer is interested in knowing when a laminarizing run does diverge from the turbulent predictions, how the heat transfer parameters vary during the turbulent-to-laminar transition and when the wall parameters will finally exhibit laminar behavior. Perkins, Schade and McEligot [1973] examined the axial trends of a number of integral parameters in their data from flows laminarizing in square tubes. The search was not exhaustive but, of those examined, a wall shear stress parameter

$$\Delta = \frac{4\mu_w}{D_h \sqrt{\rho_w g_c \tau_w}} = \frac{4}{D_h} \quad (IV-5)$$

appeared to be a better local indicator of incipient laminarization than the acceleration parameter  $K$ . Incipient laminarization was indicated when a value of  $\Delta$  of the order of 0.02 was exceeded.

Using the correlation of Taylor [1967] in conjunction with the Blasius

relation for friction factor, one can transform this criterion ( $\Delta \approx 0.02$ ) to the form

$$Re_w^{7/8} \left( \frac{T_w}{T_b} \right)^{1/4} \gtrsim 1000 \quad (IV-6)$$

For isothermal flow, this relationship reduces to the requirement that  $Re > 2700$  to be turbulent, in fair agreement with the observation of Taylor for flows with strong property variation. However, as the temperature ratio is increased, this relationship would allow a turbulent flow at a lower value of  $Re_{wm}$ . On the other hand, Taylor suggested a constant value and McEligot, Ormand and Perkins [1966] would require an increasing value from their experiments. Thus, there are unresolved differences in the criteria suggested by different authors. While these differences in detail exist, in general it appears that laminarization can be expected when the acceleration parameter  $K$  approaches the order of  $2 \times 10^{-6}$ . The process itself is a continuous phenomenon along a heated tube, so it is likely that no single parameter can describe the likelihood of laminarization exactly since both the local profiles and the previous history of the flow upstream can be expected to affect the results.

Narasimha and Sreenivasan [1979] have pointed out in their review that a wide variety of different detection techniques have been applied in order to recognize phenomena like laminarization, e.g., comparison of velocity profiles, measurement of turbulent bursting frequencies, comparison to predictions of wall parameters, etc. In general, only one technique has been applied in a particular experiment, so it is also likely that different numerical values for laminarization criteria will be ascertained in experiments which are otherwise the same. Thus, for example, there seems to be no definitive evidence that failure of the predictions or correlations for flow with property variation necessarily corresponds to a modification in the bursting frequency or other pointwise indicators of turbulence in the flow. Bankston [1970] did

include a hot wire anemometer on the centerline in some of his experiments and obtained limited indications that there was a decay in turbulent activity as the flow proceeded downstream; Ogawa et al. [1982] have recently conducted comparable experiments. For unheated duct flow accelerating due to converging side walls, Chambers, Murphy and McEligot [1982] have demonstrated that the reduction in turbulent bursting frequencies observed with a wall shear stress sensor corresponded to disagreement between numerical predictions and observations. For moderate accelerations causing disagreement between predictions and pressure measurements, other indicators from the temporal signals were found not to be affected significantly; the shape of the conditionally averaged signal did not change noticeably until high acceleration parameters were reached, approaching those for primarily laminar flow. Comparable direct comparisons of detection techniques do not seem to have been conducted yet for flows laminarizing due to strong heating.

#### IV.5. Predictions

Although it has been argued [Jones and Launder, 1972] that mixing length models have been extended as far as would be profitable, their simplicity and broad range of application make them logical for use for many engineering applications. Advanced turbulence models have developed quite rapidly and they are likely to provide more realistic approximations of very complicated turbulence processes in the future, but there is still difficulty in predicting complex turbulent flows [Bradshaw, 1976]. Further, the computer time required for the use of advanced turbulence models can still be prohibitively expensive for many design applications.

McEligot and Bankston [1969] attempted the application of mixing length models to laminarizing situations and found moderate success. The following discussion describes their approach.

For heat transfer to gases flowing turbulently at low Reynolds numbers and constant property conditions, Reynolds, Swearingen and McEligot [1969] had successfully modified Reichardt's eddy diffusivity hypothesis to provide accurate predictions for low-Reynolds-number flow. By correlating the quantity  $y_1^+(Re)$ , which controls the viscous sublayer thickness, in order to provide reasonable calculations for the friction factor, they also found velocity profiles and local Nusselt numbers were predicted adequately. But when Bankston and McEligot [1970] tested the Reichardt model against data at strong heating rates and high Reynolds numbers, they found agreement to be poor. Better agreement was found with mixing length models. Accordingly, McEligot and Bankston adjusted the constant in the van Driest model to become a function of Reynolds number,  $A^+(Re)$ , since constant property predictions from the van Driest model become poor in the low-Reynolds number range. The results of that version, with the Reynolds number evaluated with wall properties, have been shown already in Fig. 12. Predictions were reasonable except for those where evidence of significant laminarization appeared at  $Re_i$  significantly above the normal  $Re_{trans}$ . The model was then modified further to overcome this deficiency.

The effect of upstream history was introduced and evaluation of the function  $A^+(Re)$  was conducted. Studies of apparent laminarization by accelerating, unheated gas streams had shown that in the velocity profiles the viscous region extended further from the wall than normally expected [Lauder, 1964]. For such flows the Reynolds number increases in the streamwise direction. When turbulent processes existing at an upstream location are convected to the station of interest, then the usual measured quantities will appear more viscous than expected since the level of "turbulent activity" is less further upstream. For example, turbulent "bursts" [Corino and Brodkey, 1969] arriving at a local station have been convected from an earlier position with a lower mean wall shear stress driving them. Retention of the upstream

"history" effects predicts more laminar-like flows locally in converging sections; on the other hand, for strongly heated flows the reverse would be expected since the Reynolds number decreases in the streamwise direction.

As noted earlier, application of the program of Bankston and McEligot [1970] to run 140 of Perkins and Worsoe-Schmidt [1965] shows good agreement in general, but the predicted wall temperature variation increased more rapidly than the measurements in the immediate thermal entry and was lower downstream. Since only conditions at the local station had been used in the turbulence model, the model was essentially one-dimensional ( $f_n(y)$ ), i. e., its use implied the existence of some state of fully developed turbulence. The following treatment accounting for upstream history, overcame these difficulties to a significant extent.

In an integral method of solution Nash and MacDonald [1967] had introduced rate equations for integral parameters closely related to the development of the turbulent shear stress in order to include dependence on upstream turbulence quantities. The general form may be written

$$L \frac{dC_D}{dx} = K [C_{D, \text{equil}} - C_D] \quad (\text{IV-7})$$

where  $L$  is an appropriate length scale for the boundary layer and  $C_D$  is an "average turbulent shear stress." McEligot and Bankston wrote their version of the rate equation with mixing length  $l$  as the dependent variable and hypothesized the rate of readjustment to vary with the pointwise shear stress and inversely with the viscosity, as follows

$$\frac{\partial l(x, y)}{\partial x} = C \frac{\sqrt{\tau(y)/\rho(y)}}{\nu(y)} [l_{fd}(x, y) - l(x, y)] \quad (\text{IV-8})$$

The quantity  $l_{fd}$  implies the mixing length which would exist at the point under the same conditions if a fully developed situation were possible. Malik and Pletcher [1978] adopted a comparable approach. Large values of  $C$  represent rapid readjustment. Small values allow essentially no adjustment from the mixing length profile existing at the start of heating. Nash and MacDonald had checked flows in both favorable and adverse pressure gradients and had found different values of  $C_D$  appropriate to each. They justified this observation on the basis that turbulent advection plays a large role in the decay of the turbulent shear stress towards an equilibrium value, but that advection is small when growing. Use of  $C = 0.001$  when  $x > l_{fd}$  and  $C = 0.0004$  when  $l_{fd} > x$  gave encouraging improvement in  $T_w(x)$  for run 140 from the experiments of Perkins and Wörsoe-Schmidt [1965].

Another subtle problem becomes apparent in attempting to extend a high-Reynolds-number turbulence model to the low-Reynolds-number range important in conditions with laminarization due to strong heating. It is not evident which would be the appropriate definition of the Reynolds number to use in the function  $A^+(Re)$ . With constant properties all definitions are equivalent. At high Reynolds numbers,  $A^+(Re)$  is approximately constant so the question is less important.

Since it was thought that the viscous region might be thickening in the strongly heated flows, it was hypothesized that the behavior of the viscous region would be much more strongly dependent on fluid properties in the vicinity of the wall than on the bulk properties, which are more representative of the conditions in the turbulent core. Thus, it was expected to be preferable to relate  $A^+(Re)$  to wall conditions, hence wall Reynolds numbers. Two definitions are common:  $Re_w$  and  $Re_{wm}$ , defined in section 1.3. The former characterizes the ratio of bulk momentum flux to wall shear stress. On the other hand,  $Re_{wm}$  might be considered to relate a characteristic velocity

to the transverse velocity of propagation of axial velocity disturbances near the wall. The latter definition is approximately comparable to  $Re = u_{\infty} \delta \nu_w$  for external flows. One can see that the operational effect of the choice of definitions is to cause thicker viscous sublayers and higher wall temperatures to be predicted when employing  $Re_{wm}$  since  $A^+(Re)$  increases as  $Re$  decreases.

The several possibilities of turbulence models resulting were compared to the laminarizing data from run 44 by Coon [1968] which was clearly of class (3) described above. All models chosen predicted substantial thickening of the viscous sublayer (compared to isothermal flow) near the thermal entrance. However, the two based on  $Re_w$  appeared to revert to turbulent behavior whether or not axial delay was included via the rate equation (IV-8) for the "history" effects. The two predictions based on  $A^+(Re_{wm})$  did demonstrate laminarization. Even with the two-dimensional delay model the viscous region rapidly approached the center of the tube. (Here the viscous sublayer thickness was defined as the location where  $\epsilon_m = \nu$ .) While the laminarization predicted was a bit more rapid than the data showed, agreement approached ten percent. McEligot and Bankston concluded that such severe thickening of the viscous sublayer was necessary in order to obtain reasonable numerical results under conditions of these data.

Further comparisons to the laminarizing data of Coon and other investigators failed to reveal any universal agreement between the data and a particular version of these simple turbulence models. In most cases the model employing  $A^+(Re_{wm})$  led to conservative results for heating, i.e., wall temperatures would be overpredicted, and in most cases the model with  $A^+(Re_w)$  led to the other extreme. McEligot and Bankston concluded their exploratory study, relating numerical solutions of the governing equations to empiricism for a situation of laminarizing, low-Reynolds-number, turbulent



gas flows, with the observation that the predictions of such behavior are strongly dependent on the description of the turbulent transport model and the manner in which the variables within the model are evaluated. In later, unpublished studies Bankston and McEligot modified this strongly empirical, mixing length approach further to improve agreement with data over a wider range of conditions.

One difficulty in such empirical adjustment of a turbulence model is that the experimental uncertainties in the data [Kline and McClintock, 1953] have not been presented with the data. Typically, the percentage uncertainties and related errors in the non-dimensional results increase as the Reynolds number is reduced into the range where laminarization may occur. The analyst should guard against adjusting his empirical constants and functions to agree with erroneous data: a careful determination of the expected experimental uncertainties by the experimenter can help the analyst significantly. The requirement by the ASME Journal of Fluids Engineering in this regard is to be applauded.

The data examined by McEligot and Bankston, and others more recently, came from experiments conducted in tubes with diameters less than 13 mm (1/2 inch). Consequently, internal profile measurements were not feasible and analysts have been constrained to comparison to integral wall parameters. Later, Steiner [1971] and Carr, Conner and Buhr [1973] obtained mean profile measurements in larger tubes. The diameters of their tubes were sufficiently large that the buoyancy forces probably dominated the flow; consequently, their results may be useful for testing turbulence models which have been developed to handle mixed or natural convection. Steiner has not documented the inlet and boundary conditions in sufficient detail to use in testing numerical predictions but such data are available in the thesis by Conner.

To provide internal data, K.R. Perkins and McEligot [1975] con-

ducted measurements to obtain mean axial and radial distributions for turbulent and laminarizing flows with dominant forced convection including careful measurements of the thermal boundary conditions and inlet conditions. They were unable to obtain mean velocity profiles, but did obtain mean temperature profiles with a thermocouple probe; their downstream data have been presented in the previous section. Perkins and McEligot examined the model proposed by McEligot and Bankston [1969] further by comparison to their measurements of developing temperature profiles. The numerical method was the program of Bankston and McEligot [1970] extended to include a body force term in the x momentum equation,

$$\bar{\rho} \bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} - 2 \bar{\rho} \bar{v} \frac{\partial \bar{u}}{\partial \bar{r}} = \frac{d\bar{p}}{d\bar{x}} + \frac{4}{Re_i} \frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} (\bar{r} \bar{\mu}_{eff} \frac{\partial \bar{u}}{\partial \bar{r}}) + \frac{Gr^*}{Re_i^2} \bar{\rho} \quad (IV-9)$$

to account for buoyancy effects (overbars represent non-dimensional values). In the 2.5 cm tube of the experiments the buoyancy forces were not dominant, but at the highest heating rates and lowest Reynolds numbers they were not completely negligible.

Three turbulence models were tested: a van Driest-wall model, a modified van Driest version and a so-called laminar model. The van Driest-wall model was used to illustrate turbulent behavior as expected with property variation at higher Reynolds numbers. The modified van Driest model was that which evolved from McEligot and Bankston [1969] and is presented in Table I. Reynolds analogy was assumed to evaluate the effective thermal conductivity in both cases. The third model accounted only for molecular transport as a Newtonian fluid and suppressed any turbulent contribution to the effective viscosity and effective thermal conductivity; however, the entering turbulent velocity profile was employed as the initial condition. This model was meant to represent a situation of "instantaneous" laminarization at the

Table I. Modified van Driest model for low-Reynolds-number flow in tubes

$$l_{fd} = Ky \left[ 1 - \exp \left\{ - \frac{y \sqrt{g_c \tau_w / \rho_w}}{A^+(Re_{wm} \nu_w)} \right\} \right]$$

$$K = 0.4$$

$$C = 0.001 \text{ for } l_{fd}(x, y) < l(x, y)$$

$$= 0.0004 \text{ for } l_{fd}(x, y) \geq l(x, y)$$

for equation (IV-8)

$Re_{wm}$	$A^+(Re_{wm})$
$< 2,225$	$10^4$
2,225	1250
2,560	64
3,000	54
4,000	47
9,000	37
14,000	34
44,000	30
80,000	28.5
500,000	27

initiation of strong heating. The experimental runs had been categorized as turbulent, laminarizing or an intermediate condition, called sub-turbulent, by comparison to appropriate correlations of the heat transfer parameters. Comparisons of temperature profiles were conducted at distances from 2.9 to 25.3 diameters beyond the start of heating. For the runs identified as turbulent, both the van Driest-wall model and the modified van Driest version agreed with the data reasonably well, with the latter being slightly better in predicting the temperatures near the wall. As would be expected the predictions of the laminar model differed significantly from the data with the predicted thermal boundary layer reaching only halfway to the center of the tube before the exit.

One run identified as laminarizing was presented in detail. The measurements showed a thick thermal boundary layer which filled the tube within 9 to 14 diameters. The thermal resistance was spread across the tube downstream as in laminar flow rather than being concentrated near the wall as in turbulent flow. Application of the modified van Driest model to this run yielded results which were in close agreement with the measurements throughout the tube. The laminar model also agreed closely though slightly diverging from the data near the wall as calculations proceeded in the downstream direction. At the first station,  $x/D = 2.9$ , both models predicted the same temperature distribution since the thermal boundary layer appeared to be still within the viscous layer; thereafter a small difference appeared due to the small turbulent transport still implied by the modified van Driest model at low local Reynolds numbers. It was apparent that, even at the high heating rate of that run, laminarization did not occur abruptly and the inherent delay in decay of the turbulent behavior resulted in a slightly lower wall temperature and better agreement than predicted by the laminar model.

Comparisons to data from a sub-turbulent run are demonstrated in Fig. 15. These conditions could be called intermediate, laminarescent or

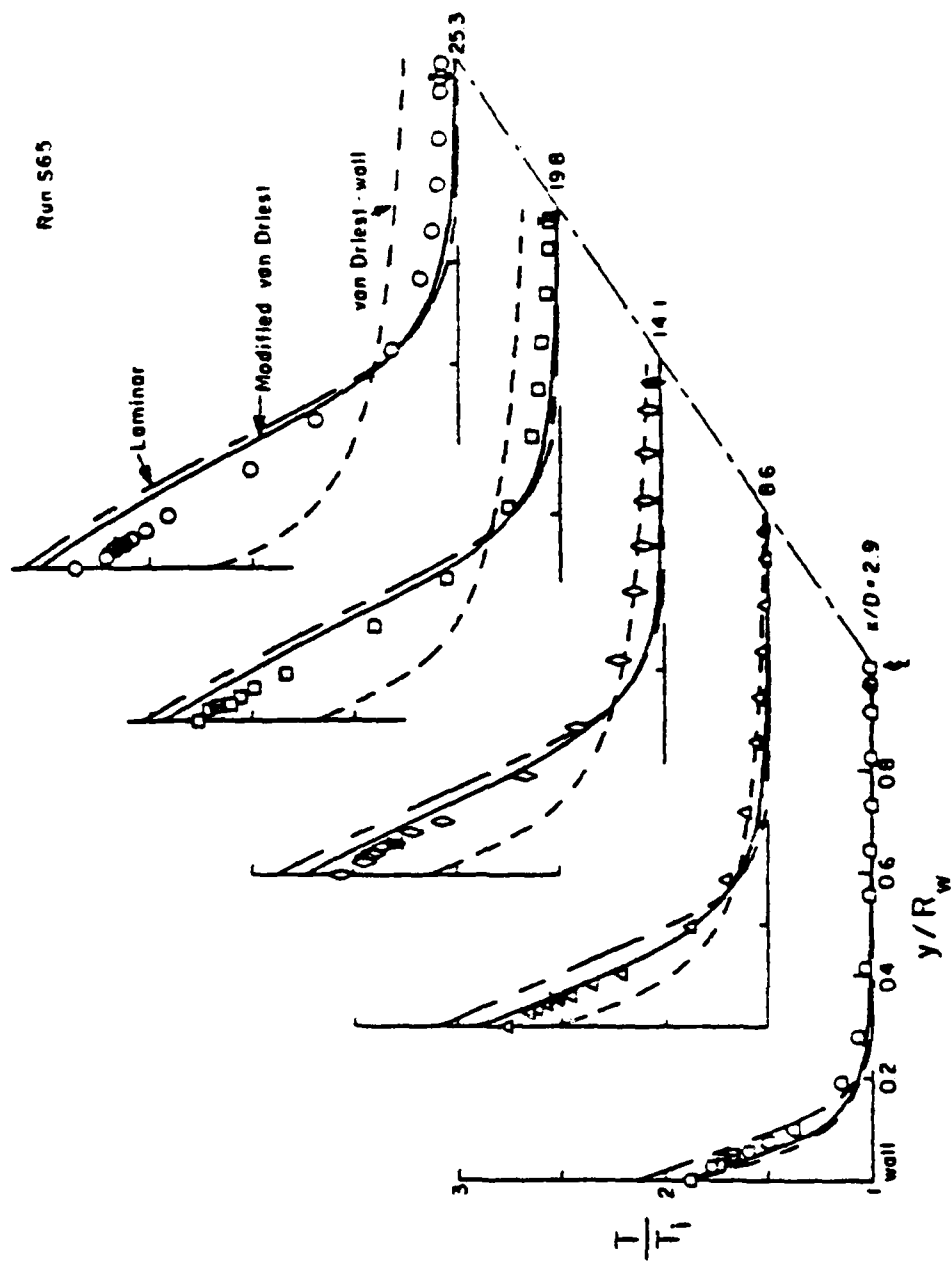


Figure 15. Development of mean temperature profile in sub-turbulent flow. Perkins and McEligot, ASME J. Heat Transfer, 1975).

even transitional in other terminology. At the first station the data showed the thermal boundary layer to be slightly thicker than for a turbulent run at the same entering Reynolds number (i.e., lower heating rate) and the profiles still appeared as a thermal boundary layer at the exit. However, the center-line temperature exceeded the inlet temperature showing that the thermal boundary layer did fill the cross section within about 14 diameters. For these conditions, the modified van Driest model and the laminar model provided predictions which differed from each other near the entrance but approached the same values successively downstream. The data corresponded to the modified model at the entry then after about eight diameters they began to diverge from the predictions near the wall. Near the exit both models over-predicted the temperatures in the important wall region. In comparison to the laminar model, the numerical results from the modified van Driest model imply slightly faster thermal boundary layer growth due to turbulent transport, but neither model showed the thermal boundary layer penetrating to the center-line as the data did. The modified model apparently predicted essentially laminar transport for much of the tube while the data indicated that turbulent transport persisted to a greater extent.

Perkins and McEligot [1975] concluded that simple modifications of the van Driest mixing length model, in conjunction with Reynolds analogy, predicted the trends of local Nusselt numbers and mean temperature profiles quite well for air flows laminarizing due to heating. Magnitudes were predicted closely for turbulent flows and flows laminarizing rapidly, but improvement of the turbulence model is desirable for the sub-turbulent or slowly laminarizing flows. Data for mean velocity profiles are not yet available to examine the solutions of the momentum equation but comparison to the axial pressure distributions did provide confidence that the successful turbulence models did not sacrifice the solutions of the momentum equation for the benefit of the thermal solution.

The advanced turbulence model of Jones and Launder [1972], a  $k-\epsilon$  version, was developed to treat comparable unheated flows which were laminarizing due to acceleration. The basic  $k-\epsilon$  model was extended into the viscous layer by adding the viscous terms to the governing equations and introducing additional constants and functions to represent the behavior of these terms for low turbulence Reynolds numbers. The first node for calculations was located well within the linear layer so that solutions were obtained through the viscous layer which is usually avoided in advanced turbulence models. In the mid-1970's, Launder and Amman derived a comparable version of the  $k-\epsilon$  model to account for transport property variation in the viscous region [personal communication]. Final results of the study are not yet available, but Launder has provided details of the governing equations and empirical functions to A.M. Shehata at the University of Arizona. Shehata has extended the model to axisymmetric coordinates for application to laminarization in a circular tube by heating and has added the model to the program of Bankston and McEligot [1970]. His study is still in progress and results likewise are not yet available.

H. Kawamura [1979a] has conducted predictions for laminarization of heated gas flows using three turbulence models for low Reynolds numbers:  $k-k_L$ ,  $k-\epsilon$  and  $k-W$ . Unfortunately, the details are only written in Japanese. (however, a companion paper [1979b] provides some information in English). Predictions were compared to the high Reynolds number data from Laufer ( $Re = 50,000$ ); all three models represented the Reynolds stress reasonably well near the wall and the  $k-\epsilon$  model described the peak in turbulence kinetic energy best. When compared to runs 119 and 122 of Perkins and Worsoe-Schmidt [1965] with significant property variation, the  $k-k_L$  model followed the data; there does not seem to be a comparison to the most extreme case, run 140.

Kawamura chose the  $k-k_L$  approach for further predictions. The

governing equation for turbulence kinetic energy (k) is

$$\rho \bar{u} \frac{\partial k}{\partial x} + \rho \bar{v} \frac{\partial k}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} \left[ \left( \frac{\mu_t}{Pr_k} + \mu \right) r \frac{\partial k}{\partial r} \right] + \mu_t \left( \frac{\partial \bar{u}}{\partial r} \right)^2 - c_D \rho \frac{k^{3/2}}{L} - \frac{\mu}{y} \frac{\partial k}{\partial y} \quad (IV-10)$$

For the product of turbulence kinetic energy and dissipation length scale (kL) the governing equation is

$$\begin{aligned} \rho \bar{u} \frac{\partial (kL)}{\partial x} + \rho \bar{v} \frac{\partial (kL)}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} \left[ \left( \frac{\mu_t}{Pr_L} + \mu \right) r \frac{\partial (kL)}{\partial r} \right] - c_1 \mu_t L \left( \frac{\partial \bar{u}}{\partial r} \right)^2 \\ - c_2 c_D \rho k^{3/2} + c_3 \mu_t \frac{2L^3}{y^2} \left( \frac{\partial \bar{u}}{\partial r} \right)^2 - c_4 \mu \frac{L}{y} \left[ \frac{\partial k}{\partial y} \right] \end{aligned} \quad (IV-11)$$

with the turbulent viscosity written as

$$\mu_t = c_\mu f_\mu \rho \sqrt{k} L \quad (IV-12)$$

Empirical constants  $c_i$  and the function  $f_\mu$  are presented in the paper.

Predictions for this model were further compared to data of Bankston [1970] and Coon [1968] with good results. The agreement with local Nusselt number for Coon's runs 41a and 41b seems particularly impressive; these were two runs which were intended to be at the same conditions but they gave differing results, 41a reverting to a turbulent flow and 41b appearing to laminarize. Kawamura's predictions showed the turbulence kinetic energy increasing in the axial direction for run 41a and decaying almost to zero for run 41b, thereby indicating a substantial thickening of the viscous sublayer. The level of agreement achieved for these complicated flows seems quite remarkable. His paper warrants translation into English in order to be available to a wider audience than possible in Japan alone. The predictions of Kawamura have also



been effective in predicting heat transfer parameters during laminarization and retransition in two ducts of differing spacing, joined by a gradual geometric transition [Tanaka, et al., 1982].

#### V. Concluding remarks

Predictions for the effects of gas property variation in laminar flow can be conducted reasonably reliably with existing numerical techniques provided that all pertinent phenomena are included in the calculation; otherwise alternate techniques may be necessary. Since heat transfer coefficients are usually small in laminar flow of gas by forced convection, the designer should be alert for the effects - and possible dominance - of buoyancy forces, thermal radiation and thermal conduction through the walls, supports, etc. The numerical calculations of Worsoe-Schmidt and Leppert [1965] give some guidance as to when natural convection can become important and order-of-magnitude comparisons of approximate thermal resistances can help avoid studying the wrong problem in other cases.

For gaseous turbulent flows the viscous sublayer is most important, particularly the range  $5 \lesssim y^+ \lesssim 30$ . In the linear layer,  $y^+ \lesssim 5$ , the thermal resistance can be predicted closely since molecular transport dominates; in the turbulent core the thermal resistance is small due to the effectiveness of the turbulent transport. Thus, heat transfer results are most sensitive to uncertainties in the region where molecular and turbulent transport are of the same order of magnitude.

For heat transfer in the thermal entry region - under the idealizations of constant properties, fully developed velocity profiles, and constant wall heat flux - use of the van Driest mixing length model and Reynolds

analogy in the viscous layer yields predictions for  $0.3 \lesssim Pr \lesssim 0.7$  in substantial agreement with measurements. In other words, the turbulent Prandtl number can be assumed to be unity in this region if used with the van Driest model. For  $0.3 \lesssim Pr \lesssim 0.7$  the resulting dependence of the Nusselt number on the Prandtl number is approximately as  $Pr^{0.6}$ , in agreement with Kays [1966] and Petukhov [1970] but in conflict with earlier analogies by Colburn and others which use  $Pr^{1/3}$  or  $Pr^{0.4}$ .

Based on convenience and reasonable accuracy, a few empirical correlations can be recommended to the designer for turbulent flows in circular tubes at moderate heat transfer rates:

Constant wall heat flux (heating)  $T_w/T_b \gtrsim 2.5$ ,  $0.3 < Pr \lesssim 0.7$

$$Nu_b = 0.021 Re_b^{0.8} Pr_b^{0.55} [(T_w/T_b)^{-0.4} + 0.85 D/x]$$

Constant wall temperature (cooling),  $T_w/T_b \gtrsim 0.12$ ,  $Pr \approx 0.7$

$$Nu_b = 0.022 Re_b^{0.8} Pr_b^{0.4} \text{ (quasi-developed)}$$

For variable wall boundary conditions or more severe heat transfer rates, a numerical solution is recommended; the author has found a simple turbulence model, consisting of Reynolds analogy and a modified van Driest mixing length, usually to be adequate.

With significant property variation due to large temperature ratios, it is easier to model turbulent flows with a constant wall temperature as the boundary condition than those with a specified wall heat flux. With a constant wall temperature, the fluid properties in the viscous region do not change substantially in the axial direction. With a constant or increasing wall heat flux, the transport properties in the viscous region change continuously and rapidly for strong heating rates as one progresses downstream; this situation can lead to concern for the "history" of the turbulence quantities in the viscous layer. Since most current versions of higher order turbulence models employ a wall function in order to locate the first calculation point at  $y^+ \approx 50$ , they cannot be expected to provide reliable predictions in this case without further

analytical development. The current situation is that for turbulent and laminarizing flows the analyst can match existing experimental data by adjustment of his model (either simple or advanced) and then can employ the model for predictions over the same range of conditions for which the experiments exist. However, one cannot necessarily predict in advance the behavior for situations where measurements are not yet available, particularly as other phenomena become important and interact with the mean flow and, perhaps, with the turbulence model.

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#### Footnotes

1. In the present review, it is assumed that the reader is familiar with forced convective heat transfer as presented by Kays and Crawford [1980] or an equivalent text.

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