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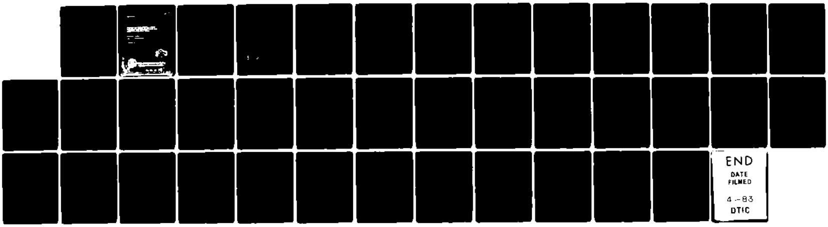

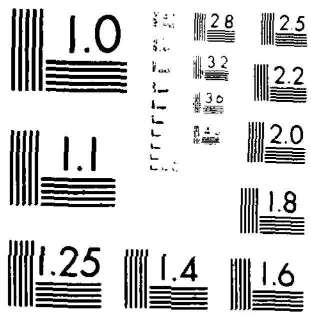

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

An approximate method was developed to determine the optimum inclination at which to launch an additional satellite to improve the determination of the earth's gravity field from satellite observations. In an example where data was assumed to be available at orbital inclinations of 65°, 90°, 108°, and 115°, the optimum inclination of a new satellite was found to be 95° although a nearby equivalent could be obtained at inclinations of 85° or below 55°.

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FOREWORD

In the determination of the spherical harmonics of the earth's gravity field on the basis of observations of artificial earth satellites, the separation of the various terms in the series is highly dependent on the distribution of the orbital inclinations of the satellites used in the solution. On occasion, one wishes to know at what orbital inclinations it would be most useful to launch an additional satellite to improve the gravity solution based on available data. A complete computer simulation to conduct such a study is prohibitively expensive. This report describes a computer program to approximate the simulation and presents an example of such a study.

Released by:

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O. F. BRAXTON, Head
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INTRODUCTION

The coefficients of the expansion of the earth's gravity field of degree, ℓ, and order, m, are frequently divided into three classes

(a) zonal coefficients, for which m = 0

(b) resonance coefficients, for which

$$m \sim \frac{k \times \text{period of earth's rotation}}{\text{orbit period}} \tag{1}$$

where k = 1, 2

(c) the remaining coefficients

Zonal and resonance coefficients constitute a relatively small proportion of the total set of coefficients; and, since they have very large effects on the motion of artificial satellites, they are relatively easy to determine to the accuracy desired for non-satellite applications such as the computation of a gravity anomaly or geoid height on the earth's satellite. Therefore, in this study attention was focused on "the remaining coefficients", although resonance coefficients were included as a special case in the computer program designed to study non-zonal gravity coefficients.

The orbital perturbations due to gravity coefficients are given by Kaula.[1] To zeroth order in orbital eccentricity the perturbation in mean anomaly is

$$\Delta M_{\ell mpq} = \frac{\mu a_e^{\ell}\, 2\,(\ell + 1)\, F_{\ell mp}\, \overline{S}_{\ell mpq}}{n a^{\ell+3} \frac{d}{dt}\phi} \tag{2}$$

[1] Kaula, W. M., *Theory of Satellite Geodesy*, Blaisdell Publishing Co., 1966.

(3) where $\phi = (\ell-2p)\omega + (\ell-2p)\ M + m(\Omega-\theta)$

$\overline{S}_{\ell mpq}$ = the integral of $S_{\ell mpq}$

$$S_{\ell mpq} = \begin{bmatrix} C_{\ell m} \\ -S_{\ell m} \end{bmatrix}^{\ell-m \text{ even}}_{\ell-m \text{ odd}} \cos\phi$$

$$+ \begin{bmatrix} S_{\ell m} \\ C_{\ell m} \end{bmatrix}^{\ell-m \text{ even}}_{\ell-m \text{ odd}} \sin\phi$$

(4)
$$F_{\ell mp}(i) = \sum_t \frac{(2\ell-2t)!}{t!(\ell-t)!(2-m-2t)!2^{2\ell-2t}} \sin^{\ell-m-2t} i$$

$$\times \sum_{s=o}^{m} \binom{m}{s} \cos^s i \sum_c \binom{\ell-m-2t+s}{c} \binom{m-s}{p-t-c} (-1)^{c-k}$$

μ = central gravitational constant

a_e = earth's semi-major axis

n = satellite mean motion

a = satellite orbit semi-major axis

ω = argument of perigee

M = mean anomaly

Ω = right ascension

θ = longitude of Greenwich

$t \leq p$

k = integer $[(\ell-m)/p]$

$0 < t < \begin{cases} p \\ k \end{cases}$

c = all values for which binomial coefficients ( ) are non-zero

q = 0 for low eccentricity

The formulation and study discussed in this report is based totally on the evaluation of this perturbation for reasons given below.

FORMULATION

The effect on mean anomaly, ΔM, is a maximum when the divisor, $\frac{d}{dt}\phi = (\dot{\Omega}-\dot{\theta}) \quad [(\ell-2p)\ \frac{\dot{\omega}}{\dot{\Omega}-\dot{\theta}} + (\ell-2p)\ \frac{\dot{M}}{\dot{\Omega}-\dot{\theta}} + m]$, is a minimum. The first term is small compared to the other two since the ratio of perigee rate to the motion of longitude of the node, $\dot{\omega}/(\dot{\Omega} - \dot{\theta})$, is small. The divisor is minimized either for $p = \ell/2$ for any m or for $p = \frac{\ell+1}{2}$ when equation (1) is satisfied. Consideration of ϕ shows that the first condition gives rise to the "m-daily" effects while the second gives rise to the resonance effects: For $p = \ell/2$ and even degree coefficients, the effect is multiplied by sines and cosines of $\phi = m(\Omega - \theta)$, (neglecting the term in ω) so that the effects have m-daily periods of (the earth's rotational period/m); for $p = \ell/2$ and odd degree coefficients $\phi = M + m\ (\Omega - \theta)$, so that the effect is fundamentally at the orbit period but modulated at the m-daily period. For $p = \frac{\ell+1}{2}$, and m odd, $\phi = m\omega - M$, which is the phase of the resonance frequency between $m\omega$ and M; for m even, $\phi = M + (m\omega - M)$ so that the resonance frequency modulates the effect which is fundamentally at the orbit frequency.

The computer program developed to study the effect of satellite inclination on non-zonal gravity parameter solutions is based on the assumption that such solutions are primarily dependent on the determination of the phase and amplitude of these m-daily and resonance effects on mean anomaly. For strict m-daily (even degree) and strict resonance (odd degree) terms, this assumption is reasonable at low orders because the effects on mean anomaly are an order of magnitude larger than those on other orbital elements as a result of the squaring of the small divisor. For the effects modulated at the m-daily (odd degree) period or resonance (even degree) period and for higher orders or higher eccentricities, the effects on other orbital elements are comparable to those on mean anomaly. For these cases, the assumption is conservative but should provide a guide as to the relative usefulness of an additional satellite at various inclinations.

It is assumed that the determination of the phase of the effect serves to separate the "C" and "S" coefficients of a given degree and order so that only the amplitude of the effect need be evaluated. It is assumed that the modulated and unmodulated effects can be separated so that odd and even degrees are treated separately. The problem then is to form normal equations by evaluating the partials of the amplitude of the mean anomaly with respect to either odd or even degree coefficients of a given order which are given by equation (2) with

$$\bar{S}_{\ell mpq} \Big/ \frac{d}{dT}\psi = \frac{86400\ T\ a_e\ 10^{-5}}{2\pi \left[\frac{1436\ (\ell-2p)}{T} + m\right]^2} \left[\frac{2\ (\ell-m)!\ (2\ell+1)}{(\ell+m)!}\right]^{1/2}$$

where T is the orbit period in minutes, a_e is the earth's radius in minutes, $q = \ell/2$ for diurnal effects and $q = (\ell + 1)/2$ for resonance effects. The 10^{-5} factor expresses the result as 10^{-8} units in normalized coefficients per meter accuracy in the determination of the amplitude of the effect (that is, the square root of the observation weight is included in the partial derivative). For more exact calculation of resonance effects, the 1436 minutes should be replaced by the period of earth's rotation relative to the line of nodes of the satellite; but the approximation does not affect separation of coefficients.

The normal matrices were made for ten degrees at each order, once for odd degree coefficients and again for even coefficients. The equations are formed for a given set of N orbital inclinations plus an (N + 1) inclination which is varied over the range of inclinations of interest. The eigenvalues of the normal matrices are computed and their square roots are plotted versus the inclination of the (N + 1) ST satellite. Since the inverse of the square root of the eigenvalue is the standard deviation of a linear combination of normalized gravity coefficients, the optimum orbital inclination for a new satellite is that inclination for which the eigenvalues are a maximum.

EXAMPLES

Satellites with Doppler transmitters have been launched at about a dozen orbital inclinations. However, many of the satellites were launched many years ago before satellite oscillators and ground equipment reached its present state of development. Modern data have been obtained at four orbital inclinations, 63.7°, 90°, 108°, and 115°. The program described was executed to determine what satellite inclination would contribute most to the improvement in the gravity coefficients determined by the set of five satellites. Since five satellites would determine five sets of even or five sets of odd coefficients at most under the assumptions used, only the 3rd, 4th and 5th eigenvalues of the solutions were plotted, even though 10 even order and 10 odd degree coefficients were computed for each order. Results for orders 1-4 and 17 based on m-daily effects and orders 13 and 14 for resonance effects are given in Appendix A for even degree coefficients and in Appendix B for odd degree coefficients.

The scale on the right-hand side gives the standard deviation of a linear combination of normalized coefficients in units of 10^{-8} per meter accuracy in the determination of the amplitude of the m-daily or resonance effect. For the even degree low order coefficients, there is significant improvement in the accuracy of the 4th or 5th eigenvalue with the addition of data from a satellite with an inclination as little as 1 or 2 degrees separated from the inclinations and from 180° minus the inclinations of the first four satellites. For the seventeenth order coefficients and for non-resonance odd degree coefficients, the standard deviations in the coefficients per meter accuracy of determination of the amplitude of the m-daily effects are equal to or larger than the expected size of the coefficients. However, the results are pessimistic due to the neglect of perturbations to orbital elements other than mean anomaly, due to neglect of information arising from orbit eccentricity, and because averaging large quantities of data will yield accuracies in amplitude better than one meter. In view of the pessimism, the graphs lead to the same conclusion as those for even degree low order coefficients.

Although the charts show there is a wide selection of orbital inclinations which would improve the four satellite solutions, the determination of the amplitude and phase of the m-daily effect can best be done for high inclination satellites because more ground stations will observe higher inclination satellites, and because high latitude stations will observe more passes per day. Therefore the optimum inclination for an additional satellite under the assumptions of this example would be near either 85° or 95°.

Since 96.6° is the inclination used by LANDSAT type satellites, a further experiment was run in which it was assumed that data were available from five satellites at inclinations of 63.7, 90.0, 96.6, 108.0, and 115.0°. Appendix B gives the accuracy to be expected in first through fourth order coefficients as a function of the inclination of a sixth satellite. Only two of the graphs show sensitivity because for a six satellite solution, the sixth eigenvalue should have been added to the plot. The first and third order even degree results do show sensitivity because for these coefficients the 63.7 degree inclination and the 65° retrograde satellite give nearly redundant information.

SUMMARY

A computer program was developed to help assess the sensitivity of solutions for gravity coefficients from satellite observations to the orbital inclination of the satellites observed. Sample cases indicate significant improvement in accuracy of the gravity field can be achieved by the addition of observations on satellites at inclinations separated by about 2 degrees.

APPENDIX A

EIGENVALUES OF EVEN DEGREE GRAVITY COEFFICIENTS FOR 5 SATELLITE SOLUTIONS

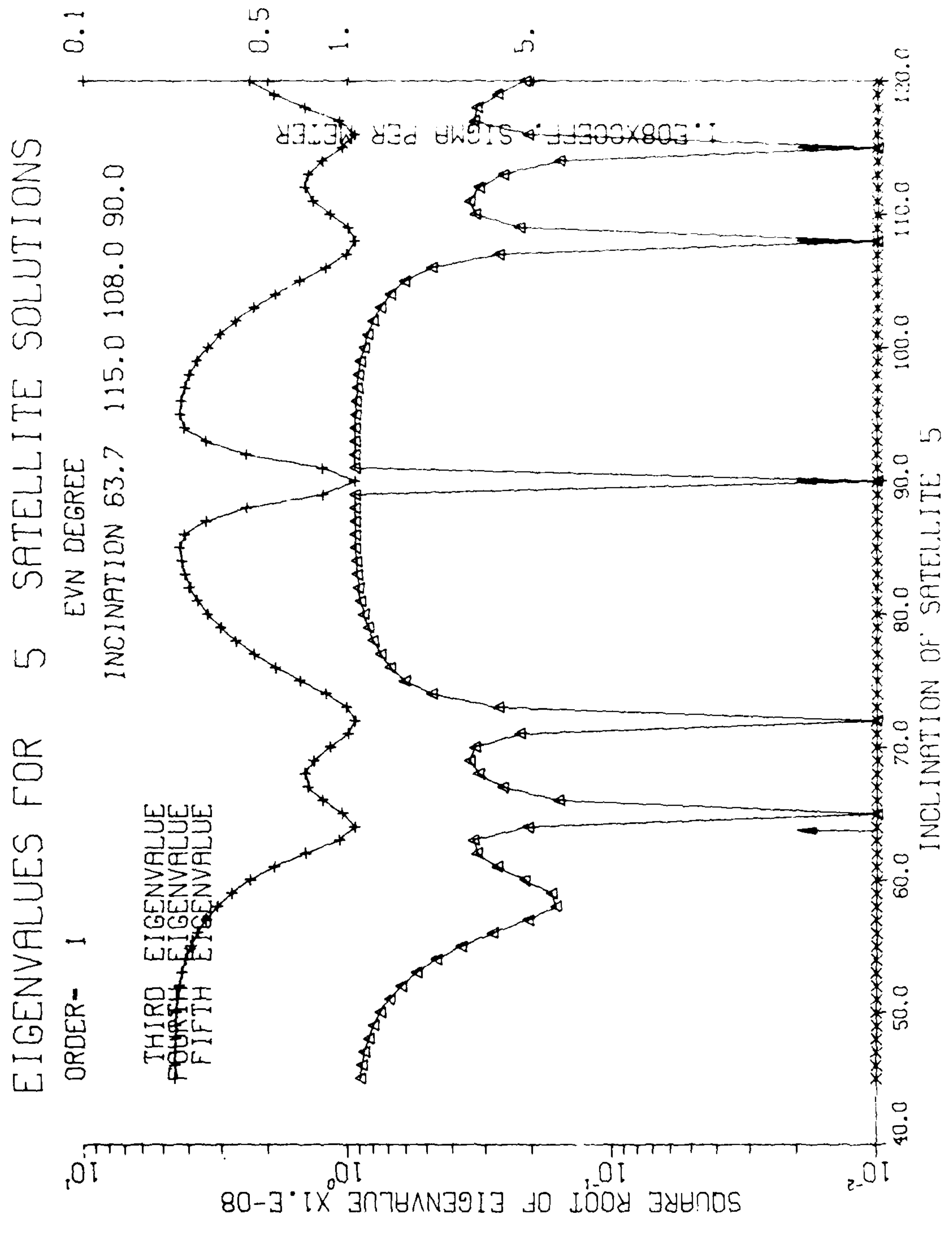

FIGURE A-1. FIRST ORDER EVEN DEGREE EIGENVALUES FOR 5 SATELLITE SOLUTIONS

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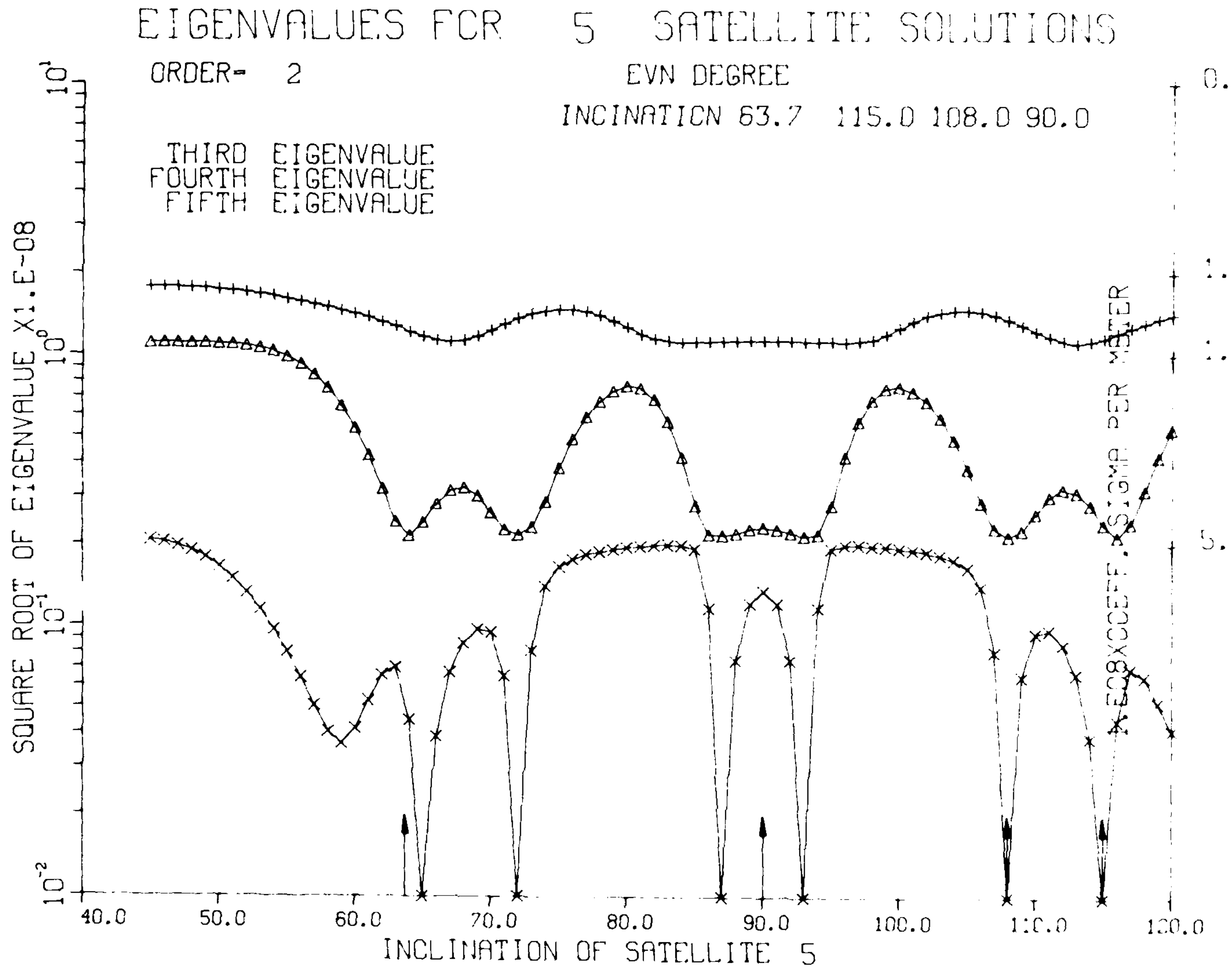

FIGURE A-2. SECOND ORDER EVEN DEGREE EIGENVALUES FOR 5 SATELLITE SOLUTIONS

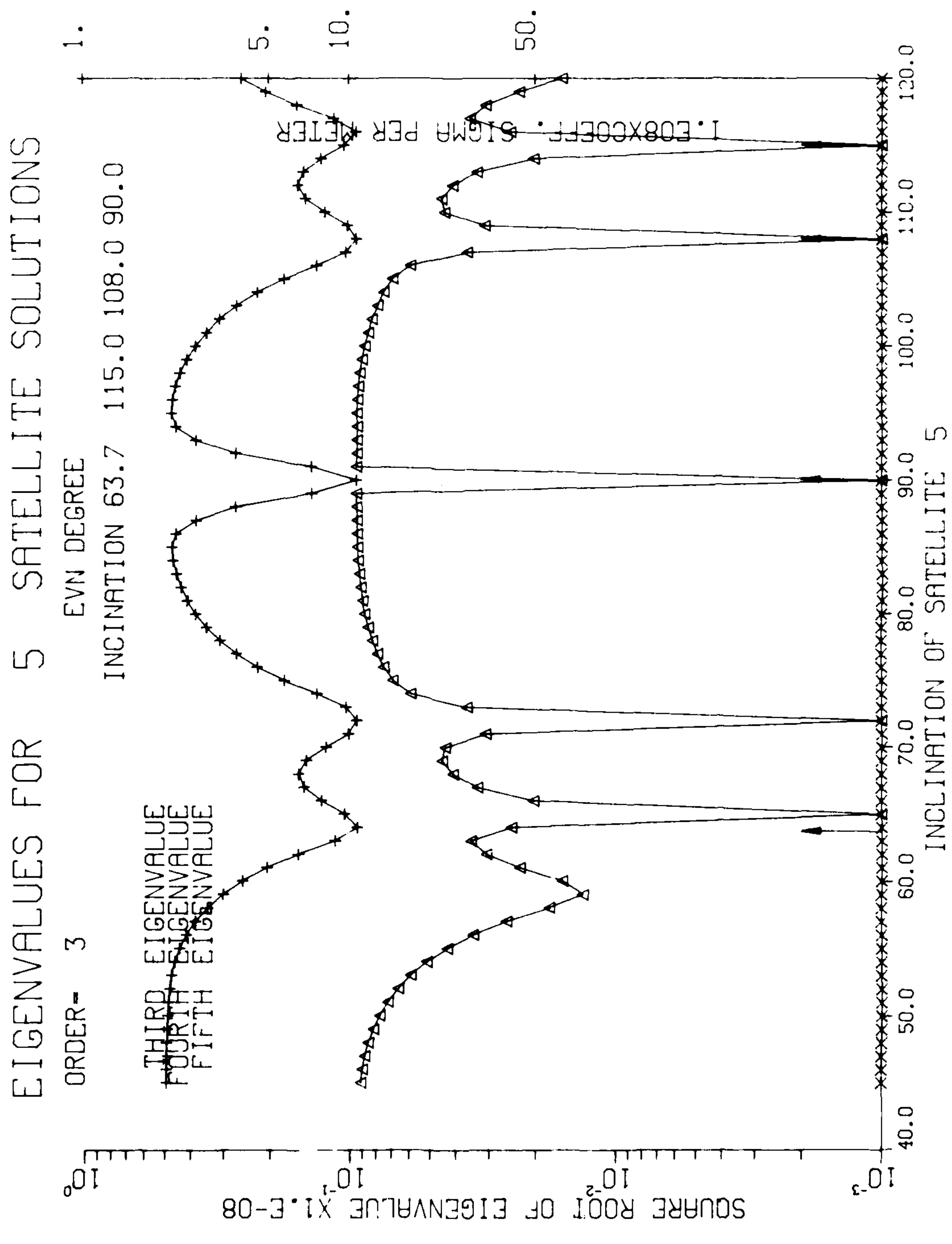

FIGURE A-3. THIRD ORDER EVEN DEGREE EIGENVALUES FOR 5 SATELLITE SOLUTIONS

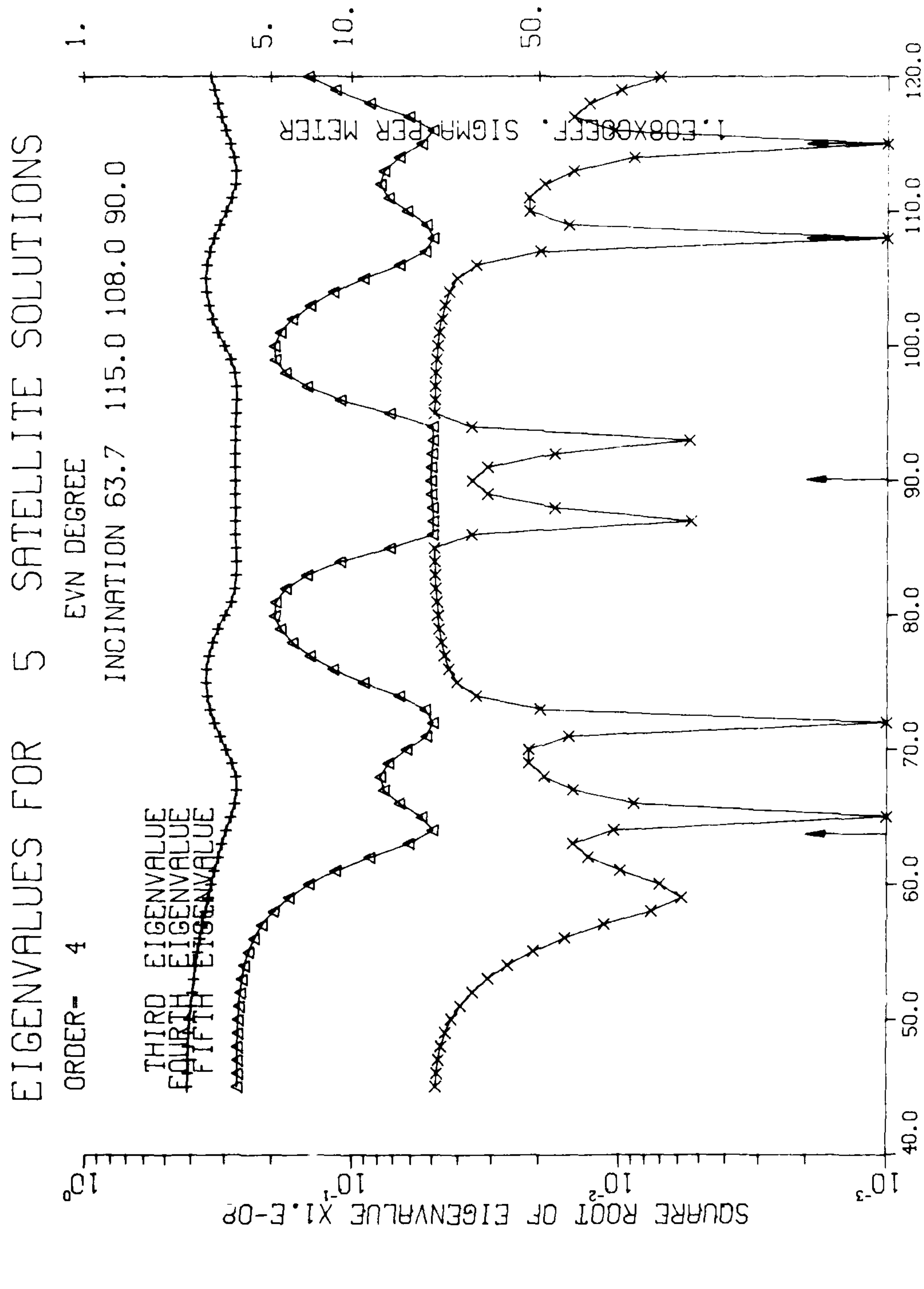

FIGURE A-4. FOURTH ORDER EVEN DEGREE EIGENVALUES FOR 5 SATELLITE SOLUTIONS

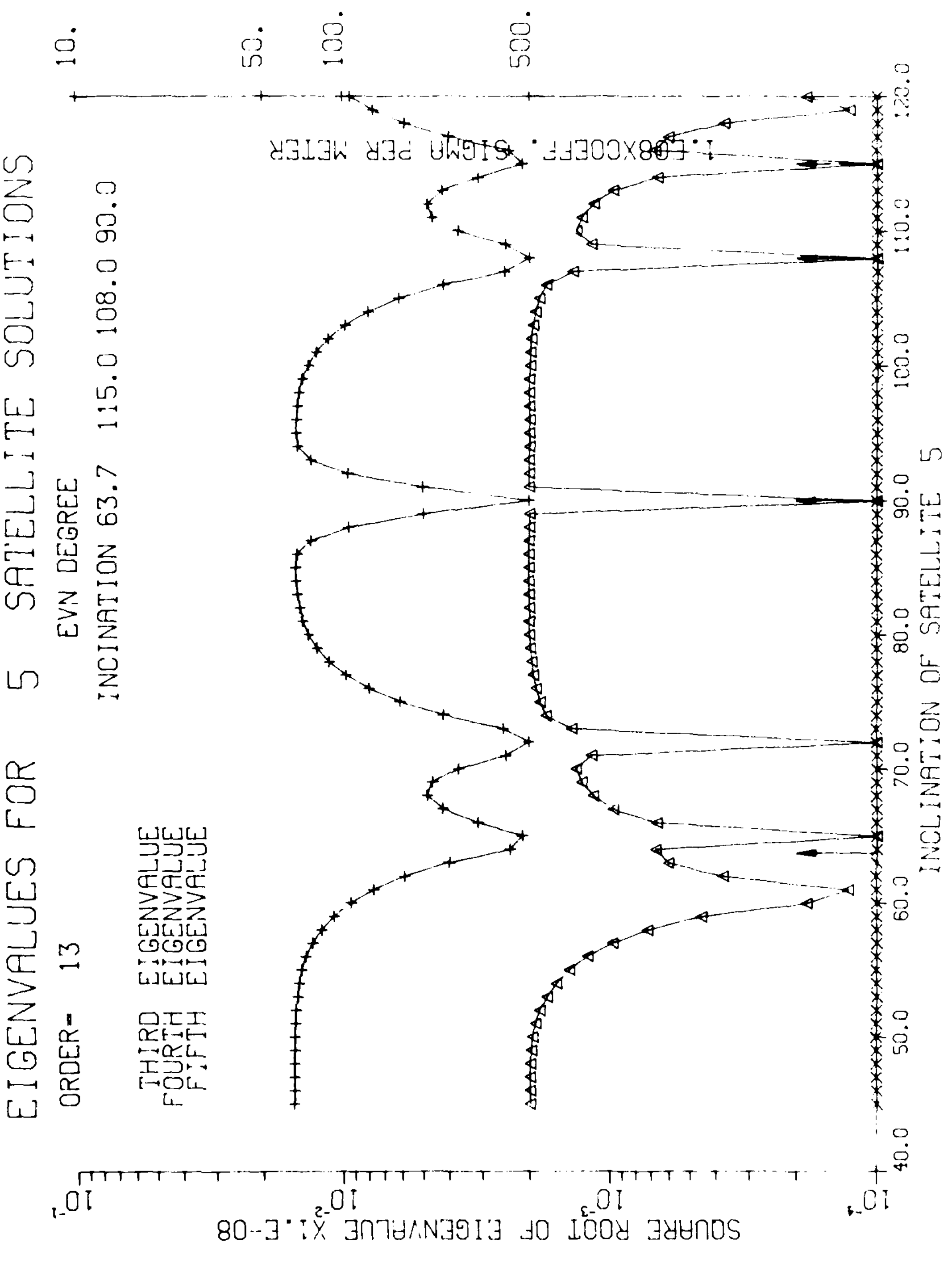

FIGURE A-5. THIRTEENTH ORDER EVEN DEGREE EIGENVALUES FOR 5 SATELLITE SOLUTIONS

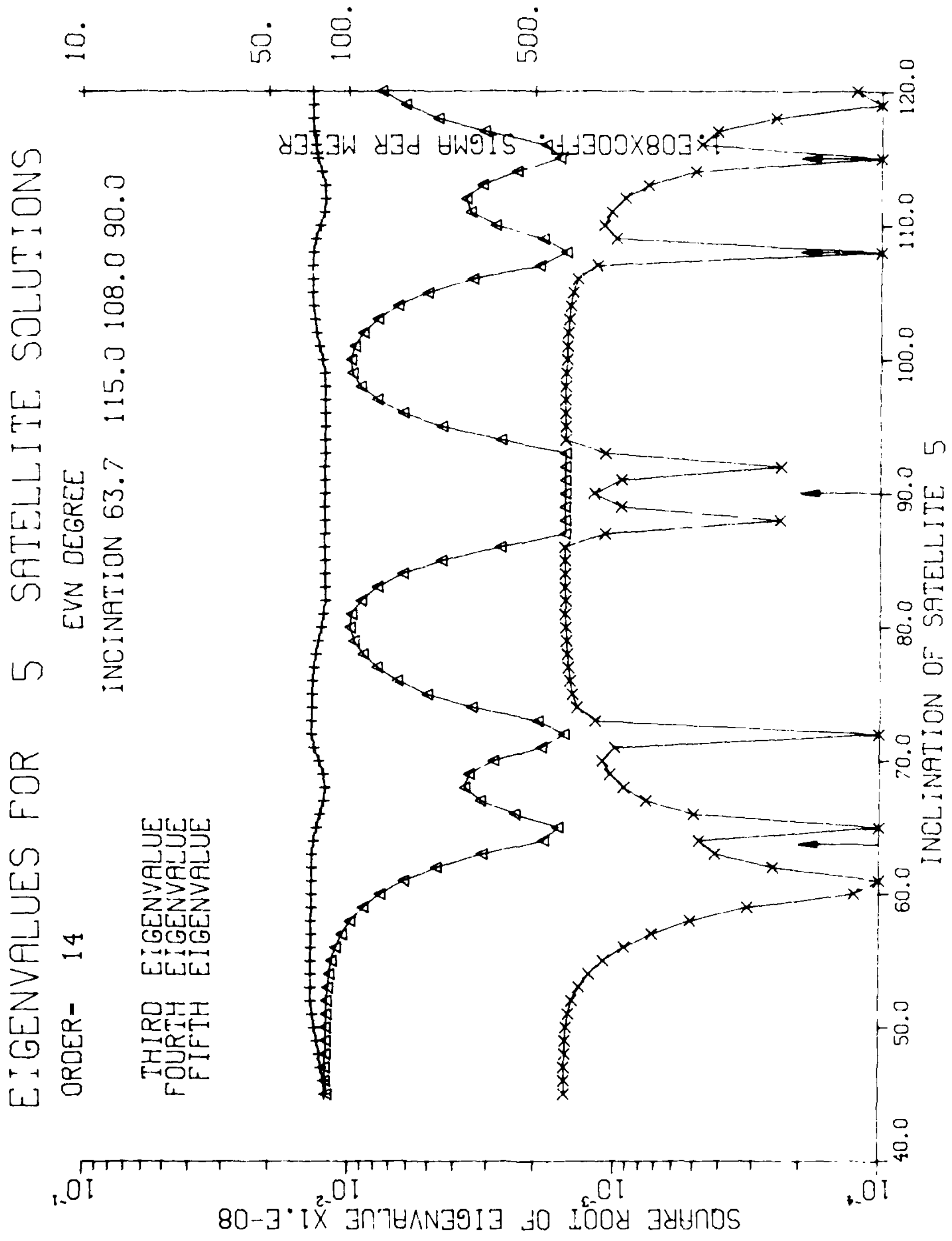

FIGURE A-6. FOURTEENTH ORDER EVEN DEGREE EIGENVALUES FOR 5 SATELLITE SOLUTIONS

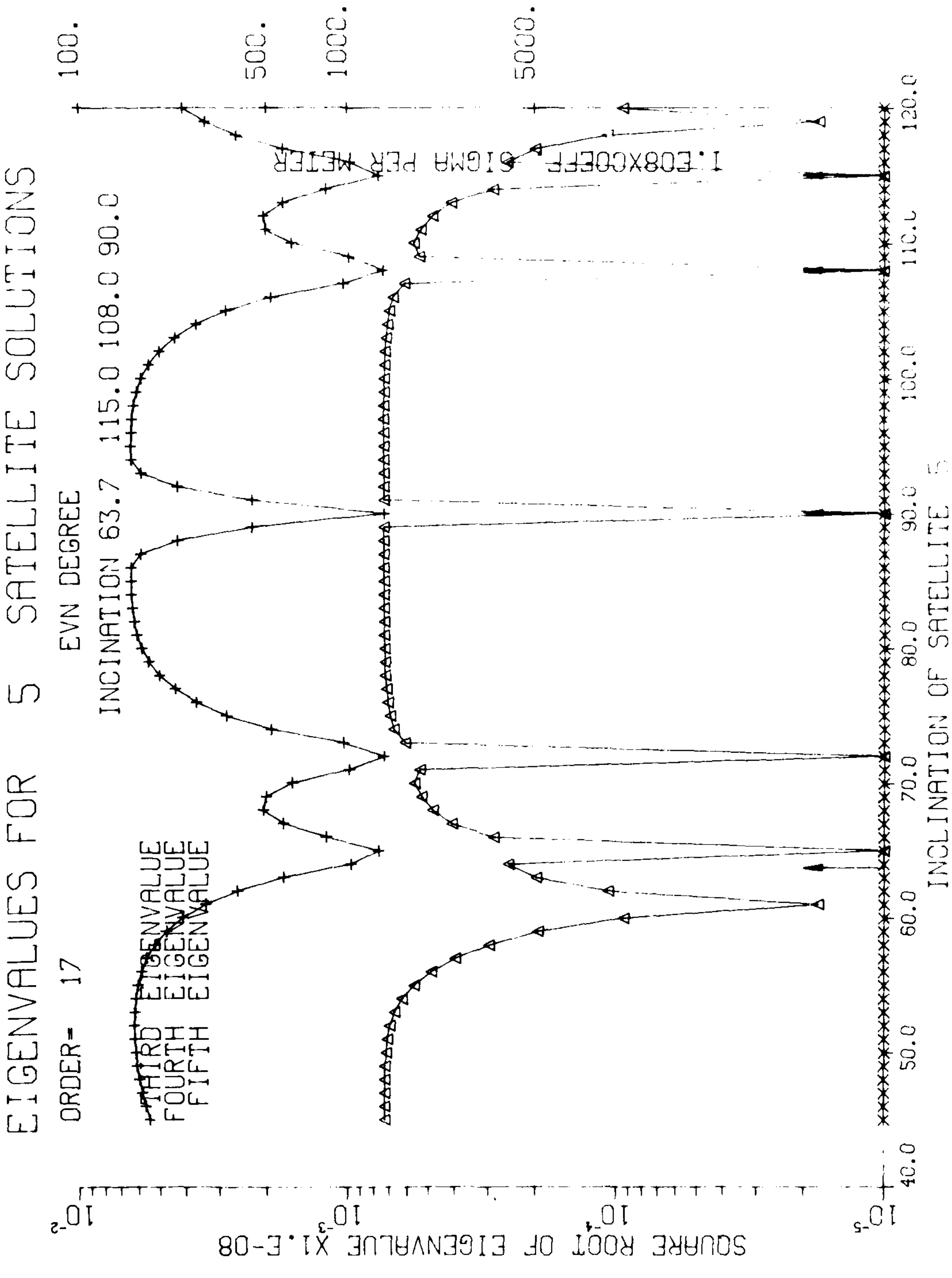

FIGURE A-7. SEVENTEENTH ORDER EVEN DEGREE EIGENVALUES FOR 5 SATELLITE SOLUTIONS

APPENDIX B

EIGENVALUES OF ODD DEGREE GRAVITY COEFFICIENTS FOR 5 SATELLITE SOLUTIONS

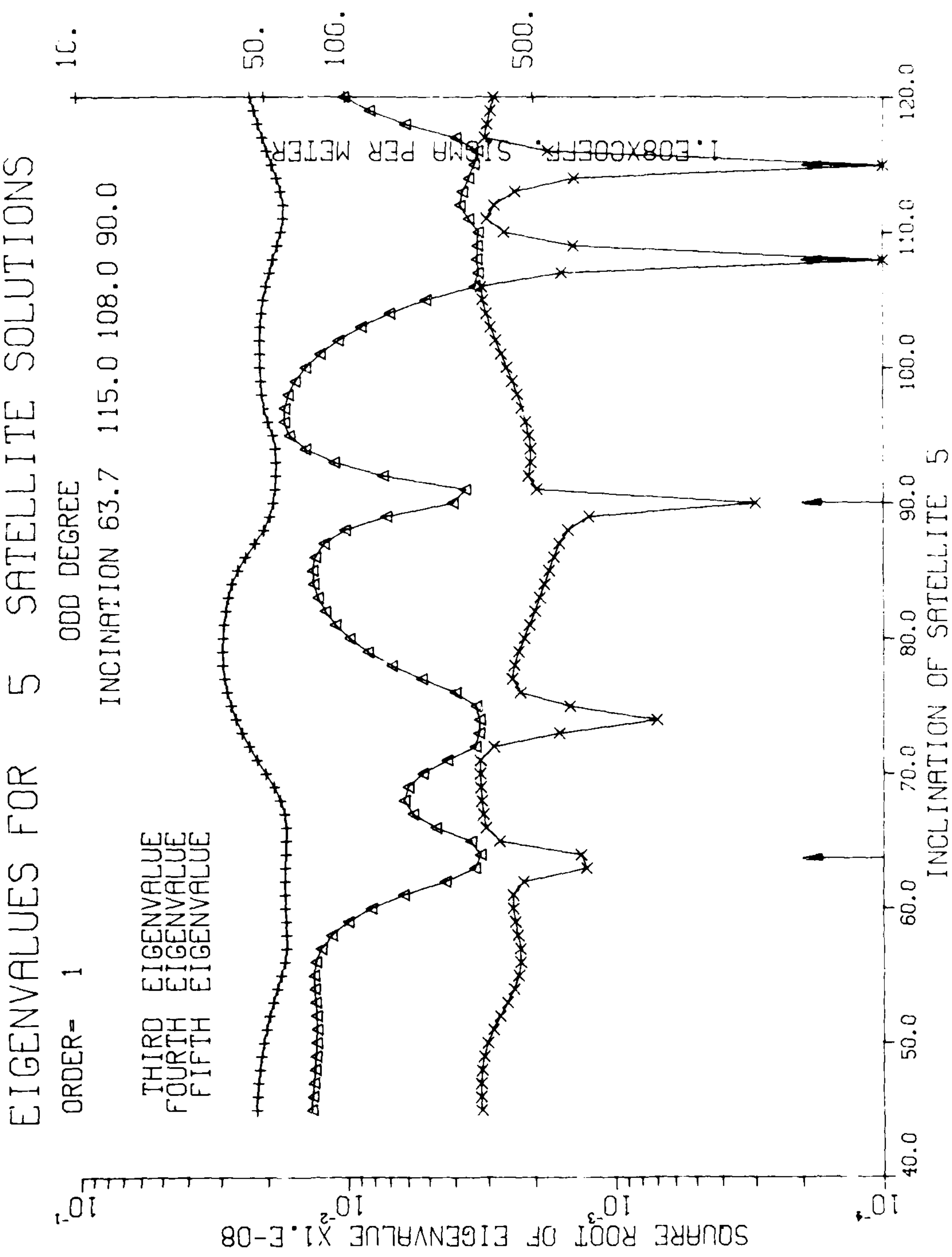

FIGURE B-1. FIRST ORDER ODD DEGREE EIGENVALUES FOR 5 SATELLITE SOLUTIONS

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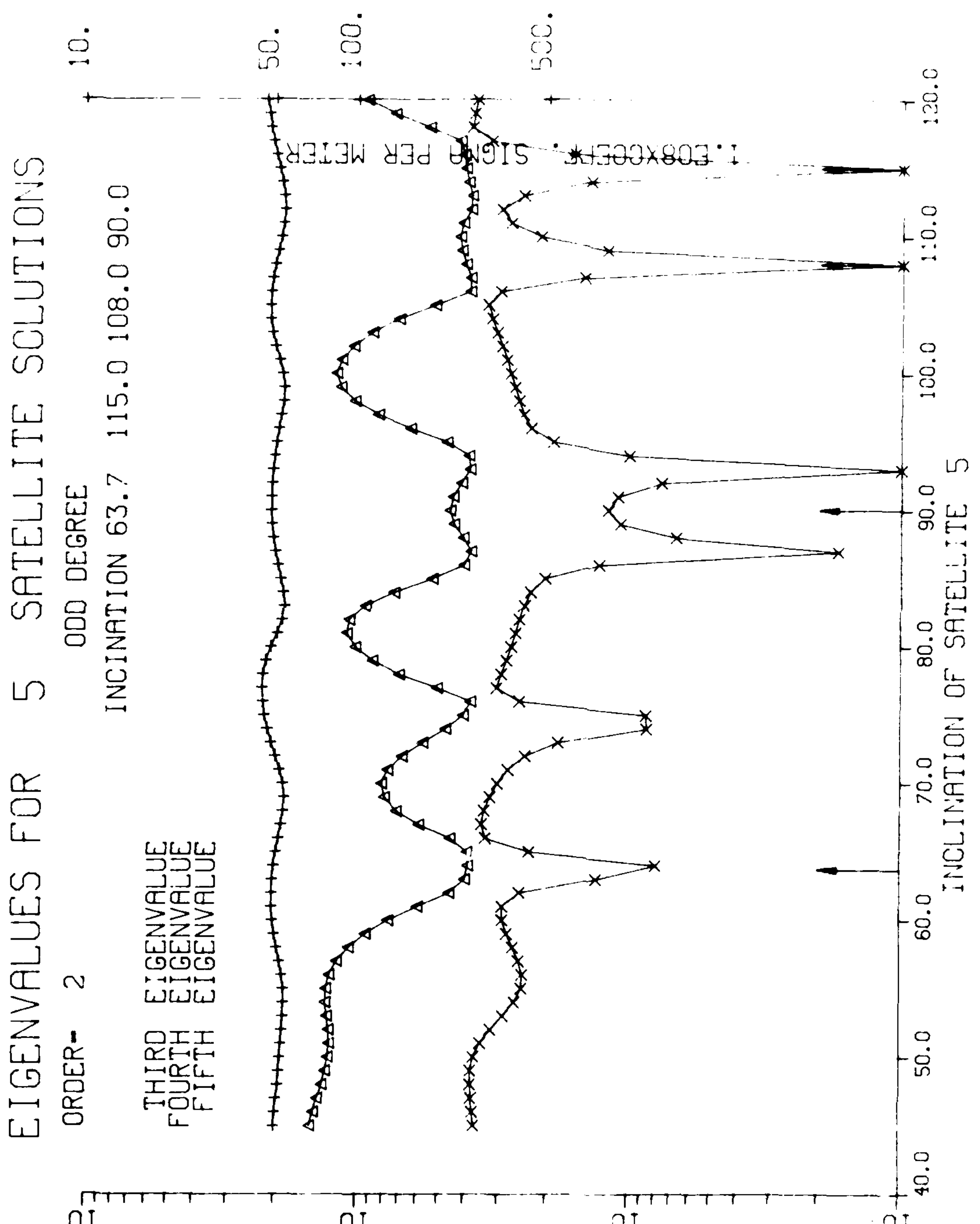

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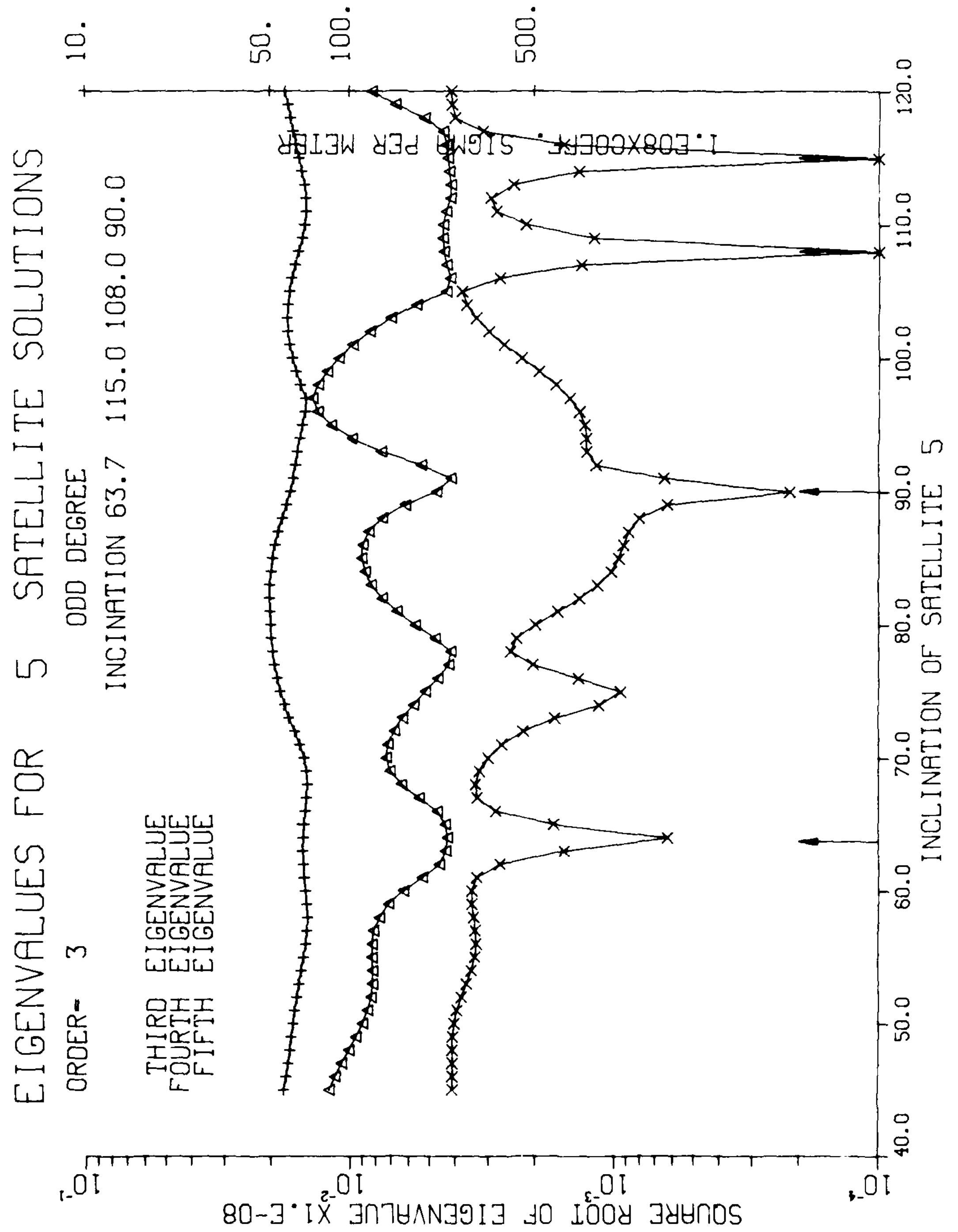
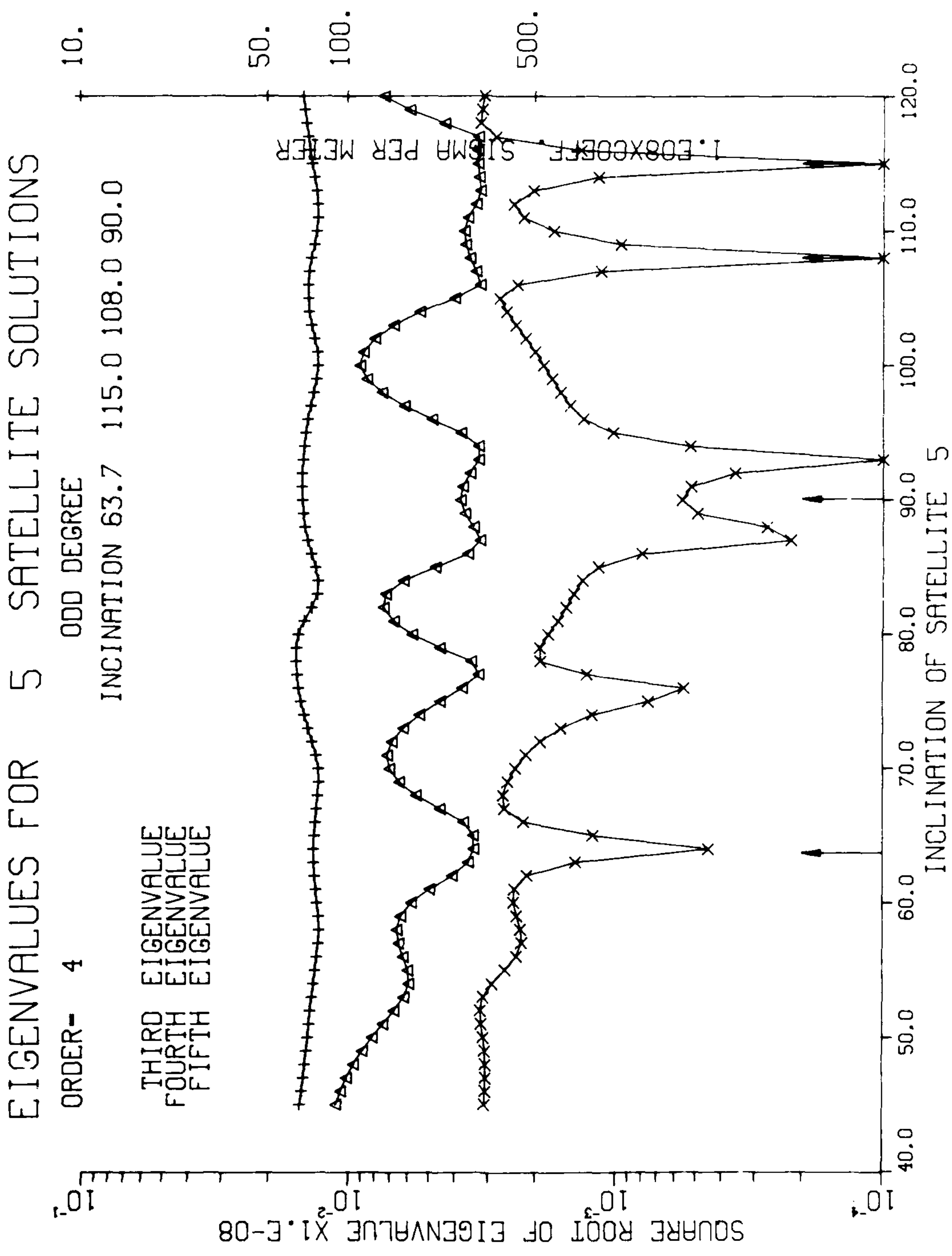


FIGURE B-3. THIRD ORDER ODD DEGREE EIGENVALUES FOR 5 SATELLITE SOLUTIONS



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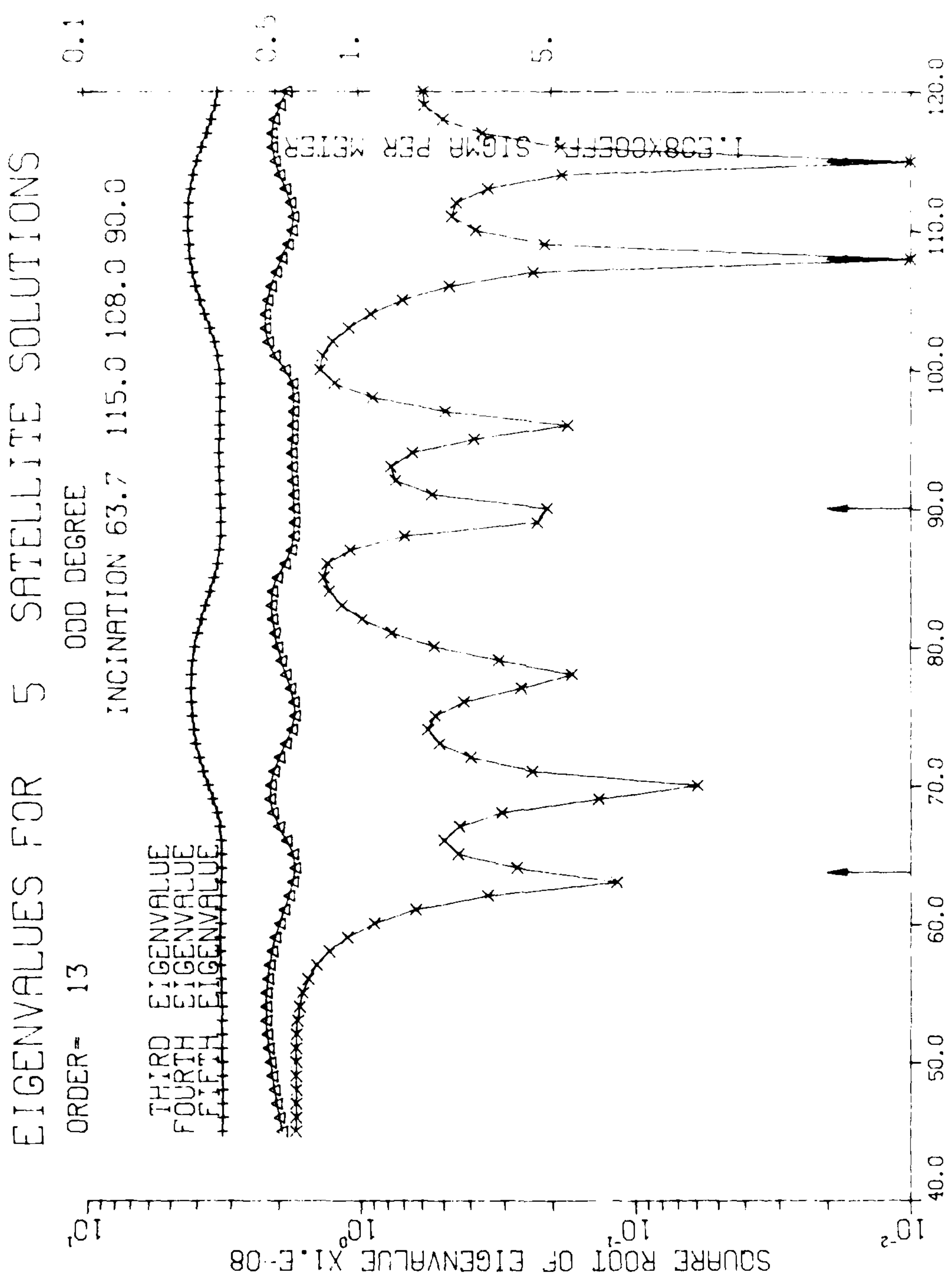

FIGURE B-5. THIRTEENTH ORDER ODD DEGREE EIGENVALUES FOR 5 SATELLITE SOLUTIONS

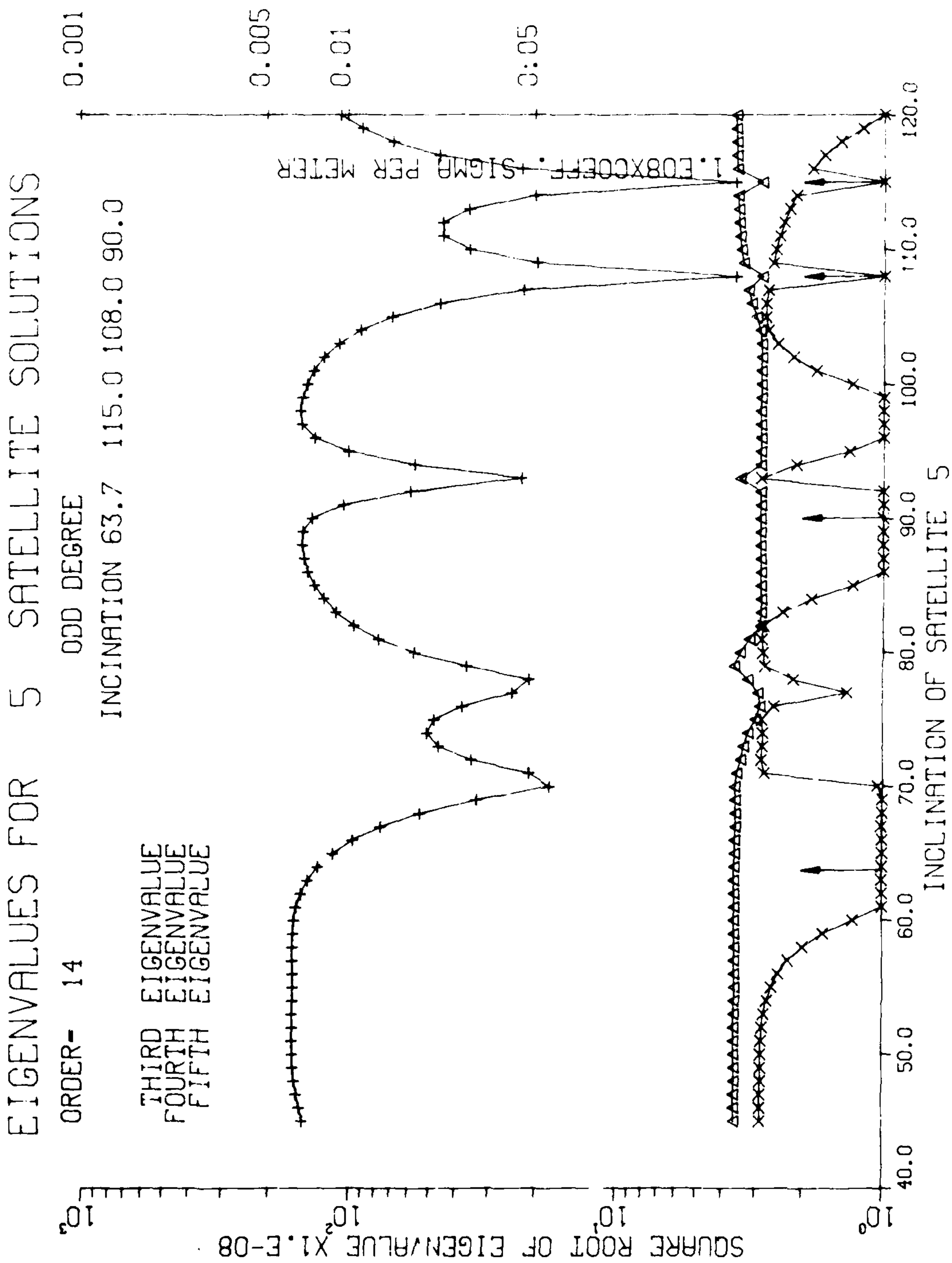

FIGURE B-6. FOURTEENTH ORDER ODD DEGREE EIGENVALUES FOR 5 SATELLITE SOLUTIONS

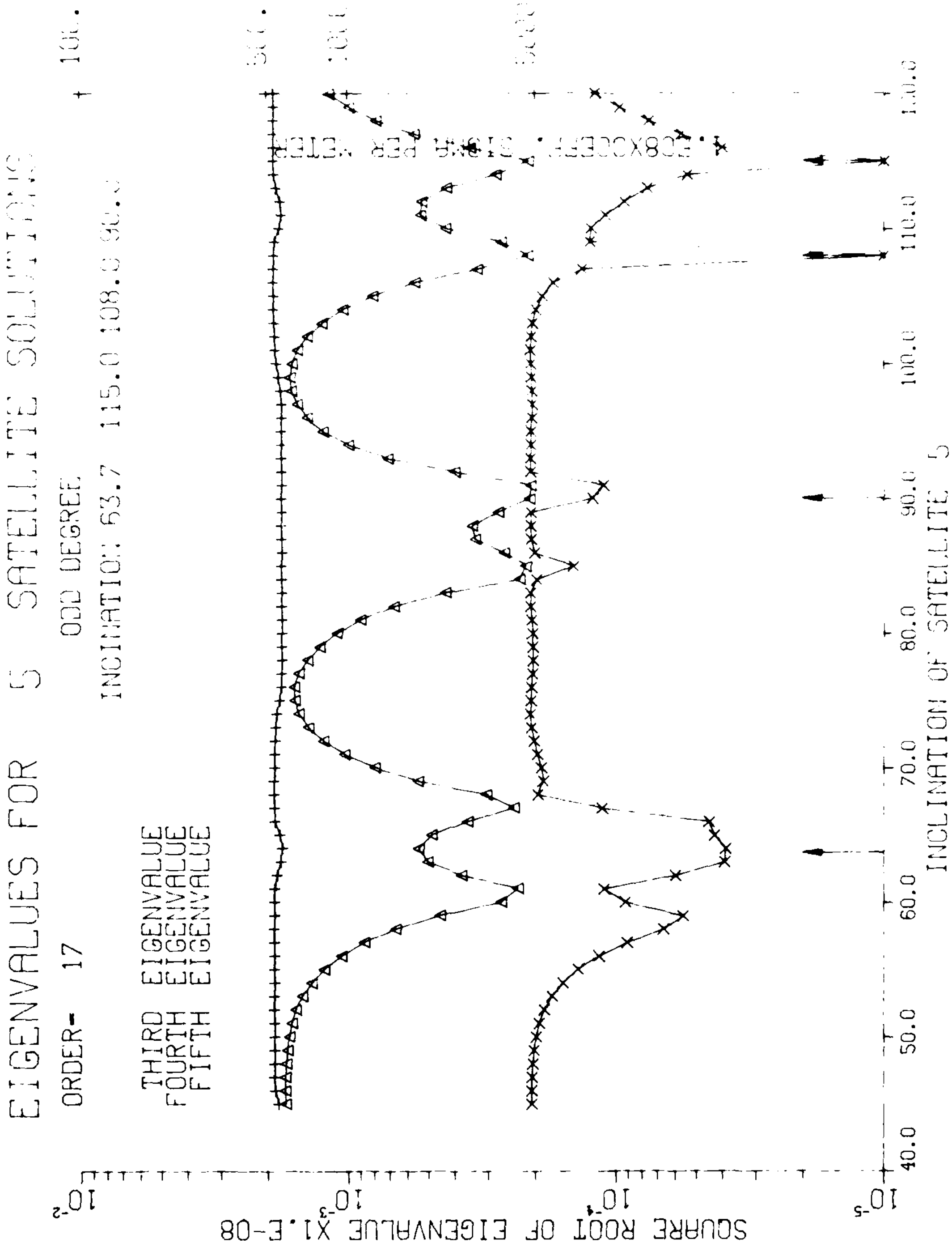

FIGURE B-7. SEVENTEENTH ORDER ODD DEGREE EIGENVALUES FOR 5 SATELLITE SOLUTIONS

APPENDIX C

EIGENVALUES FOR 6 SATELLITE SOLUTIONS

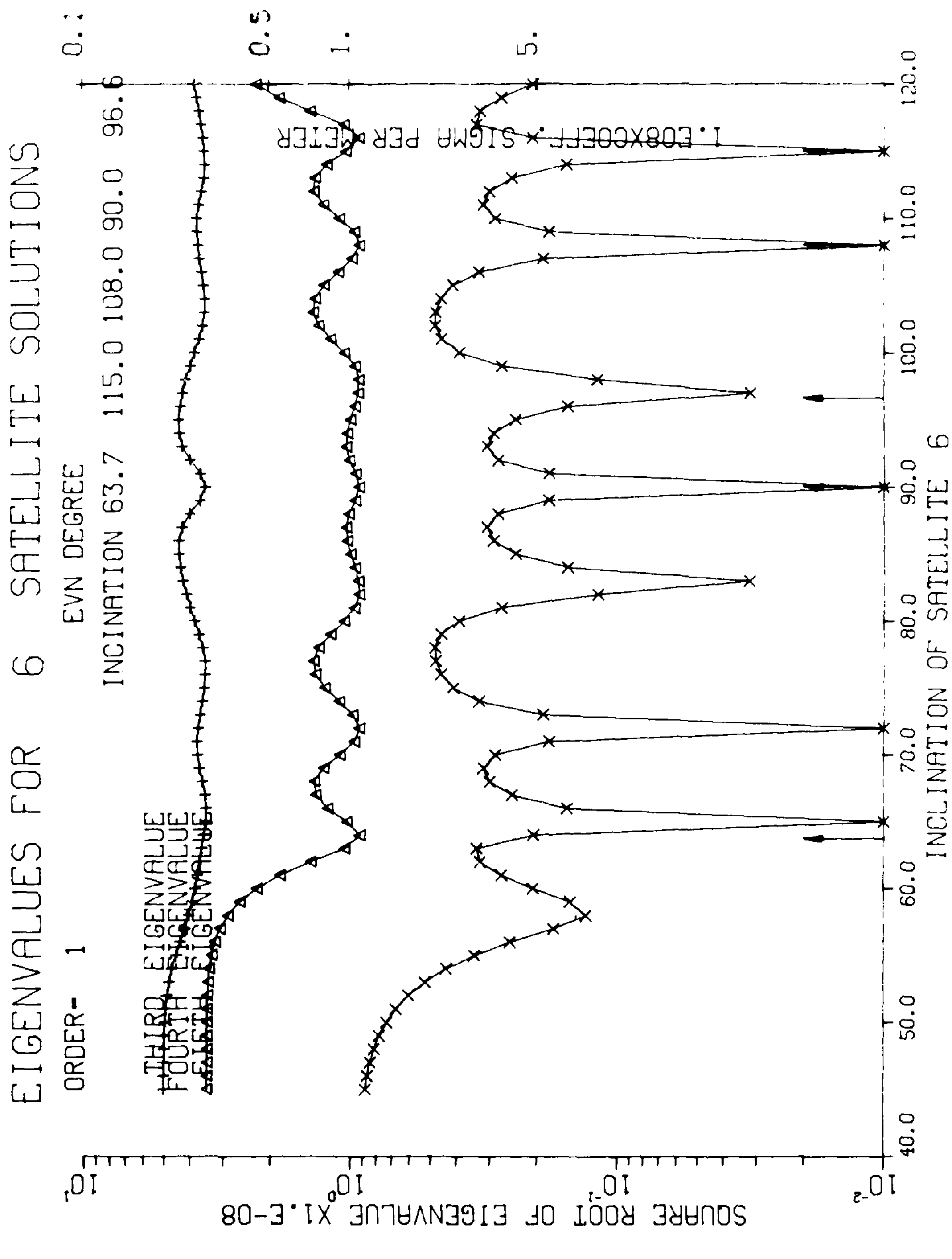

FIGURE C-1. FIRST ORDER EVEN DEGREE EIGENVALUES FOR 6 SATELLITE SOLUTIONS

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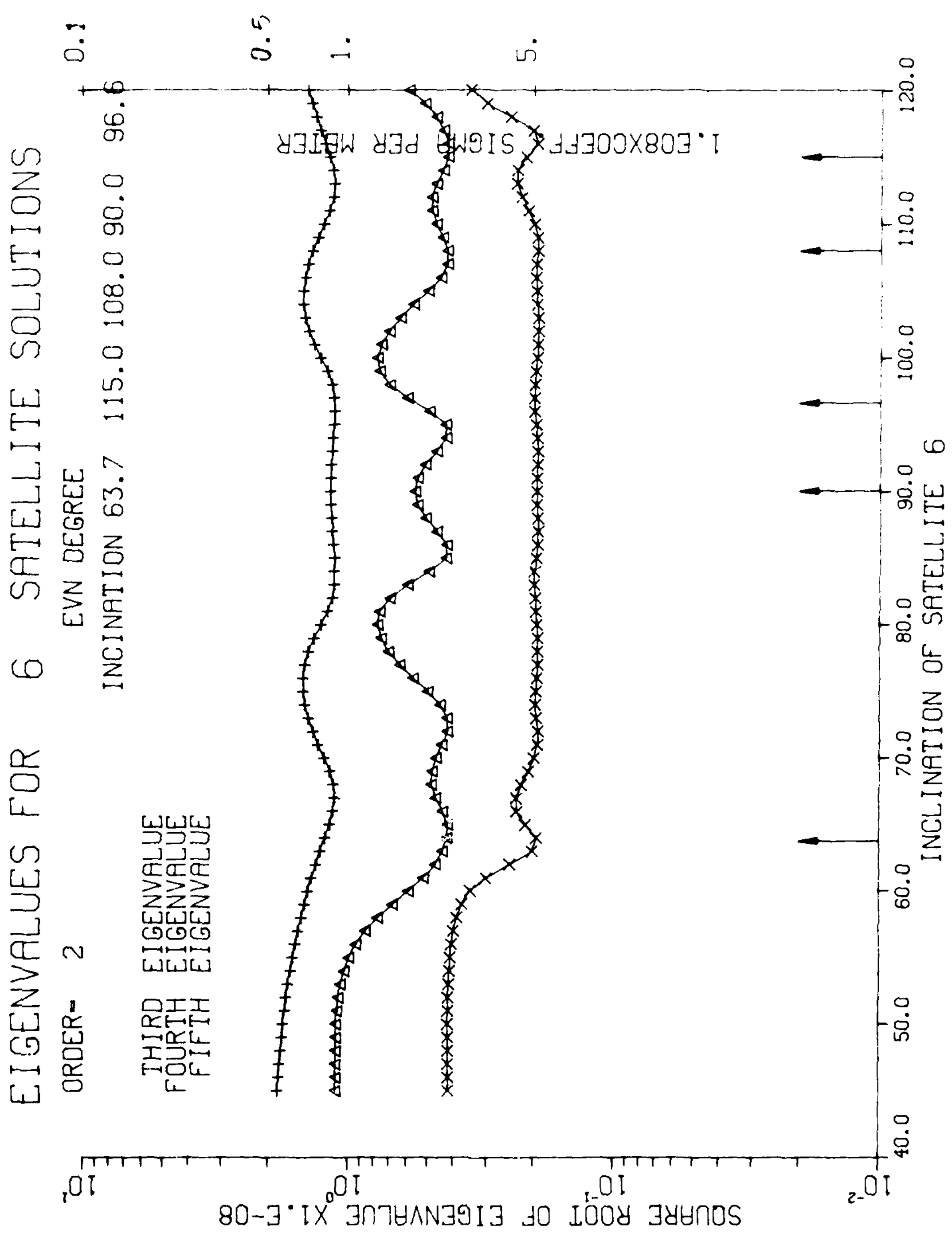


FIGURE C-2. SECOND ORDER EVEN DEGREE EIGENVALUES FOR 6 SATELLITE SOLUTIONS

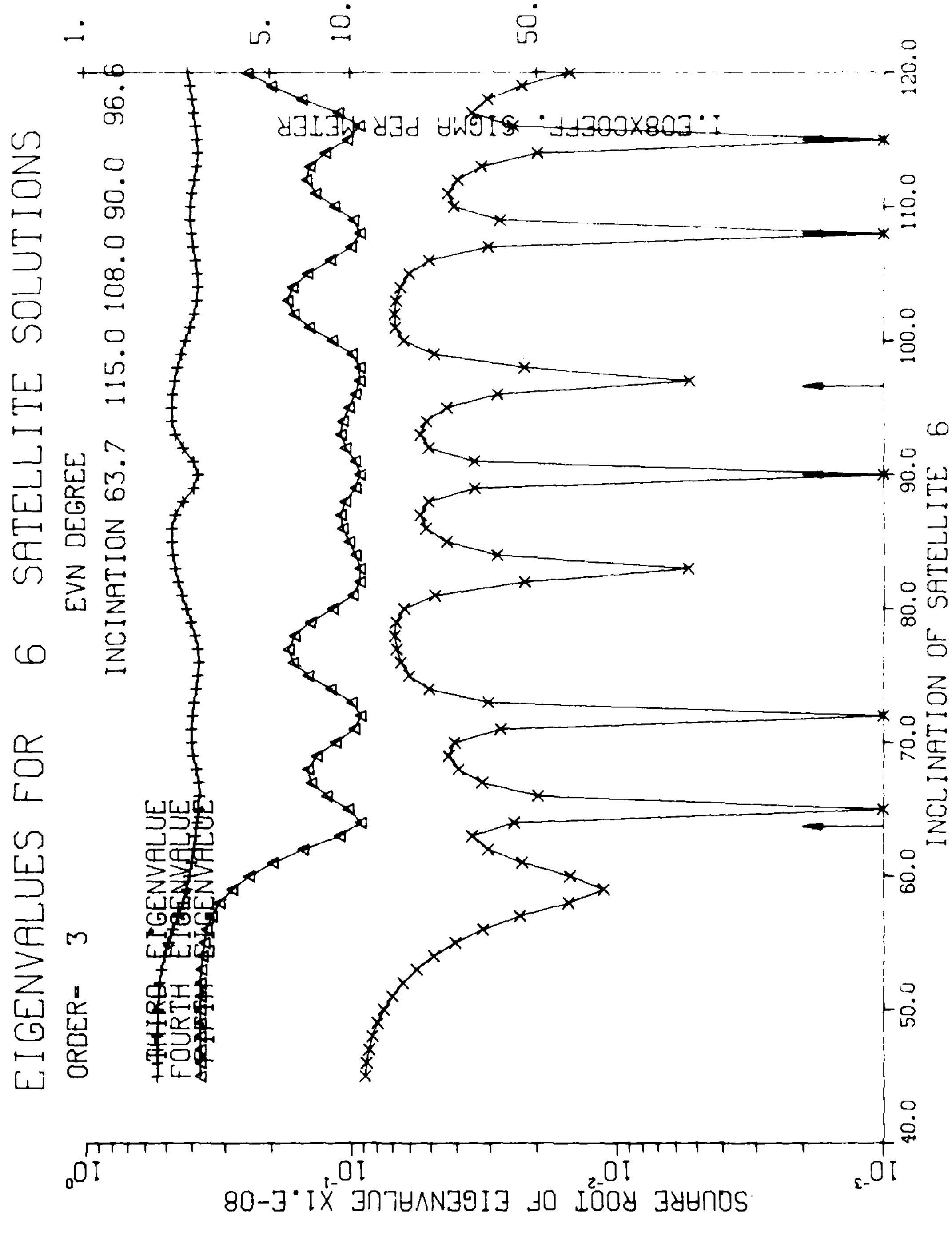


FIGURE C-3. THIRD ORDER EVEN DEGREE EIGENVALUES FOR 6 SATELLITE SOLUTIONS

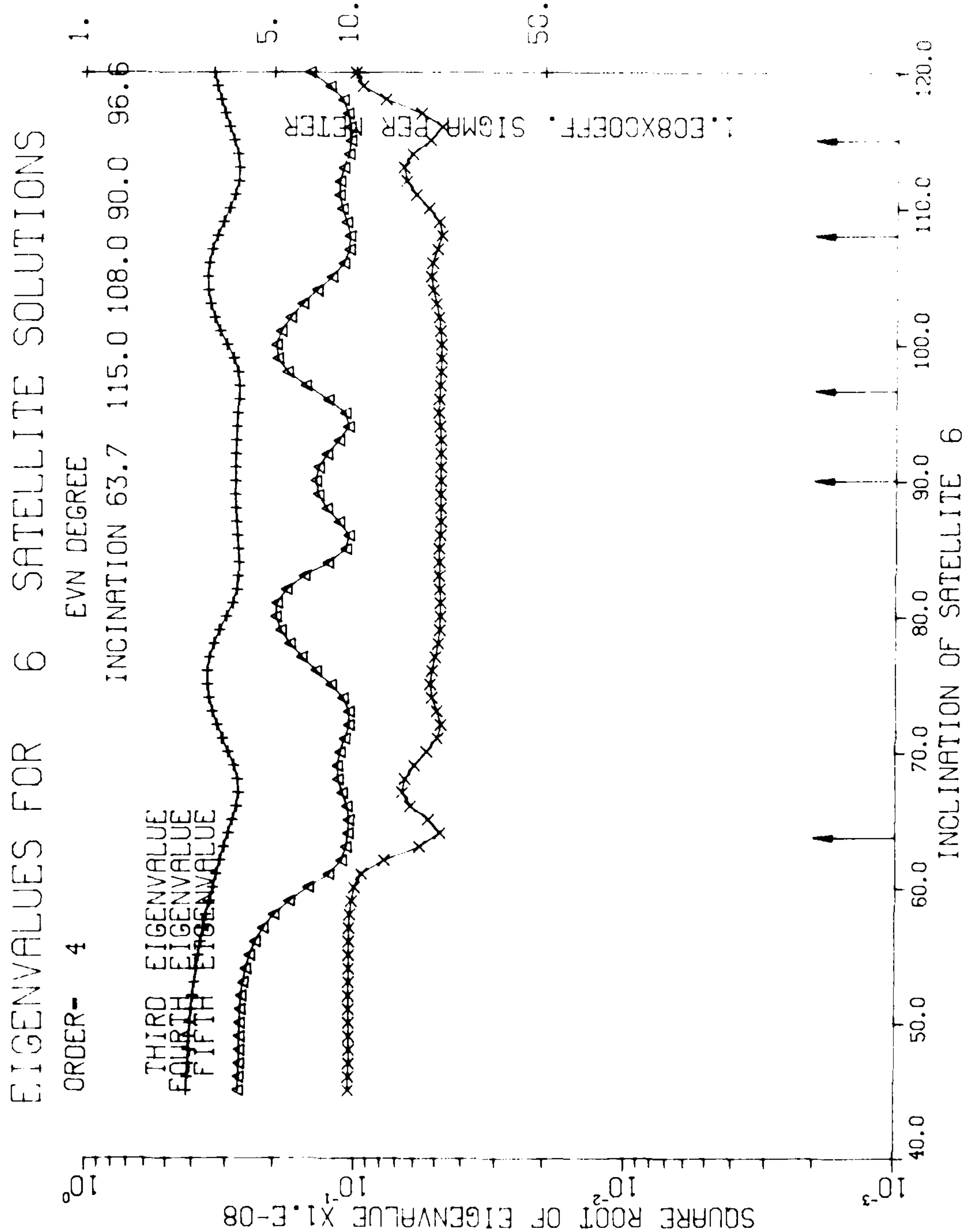


FIGURE C-4. FOURTH ORDER EVEN DEGREE EIGENVALUES FOR 6 SATELLITE SOLUTIONS

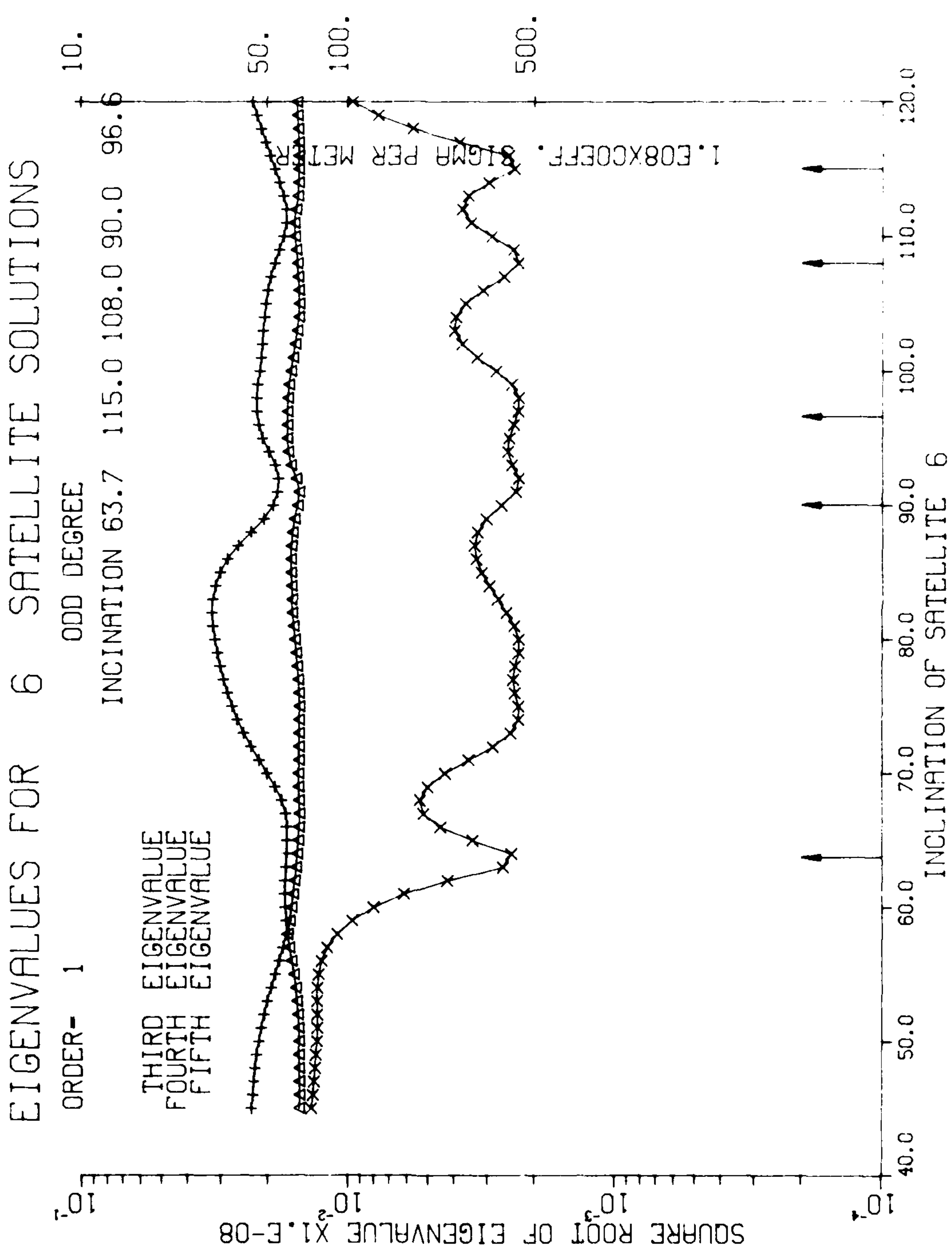


FIGURE C-5. FIRST ORDER ODD DEGREE EIGENVALUES FOR 6 SATELLITE SOLUTIONS

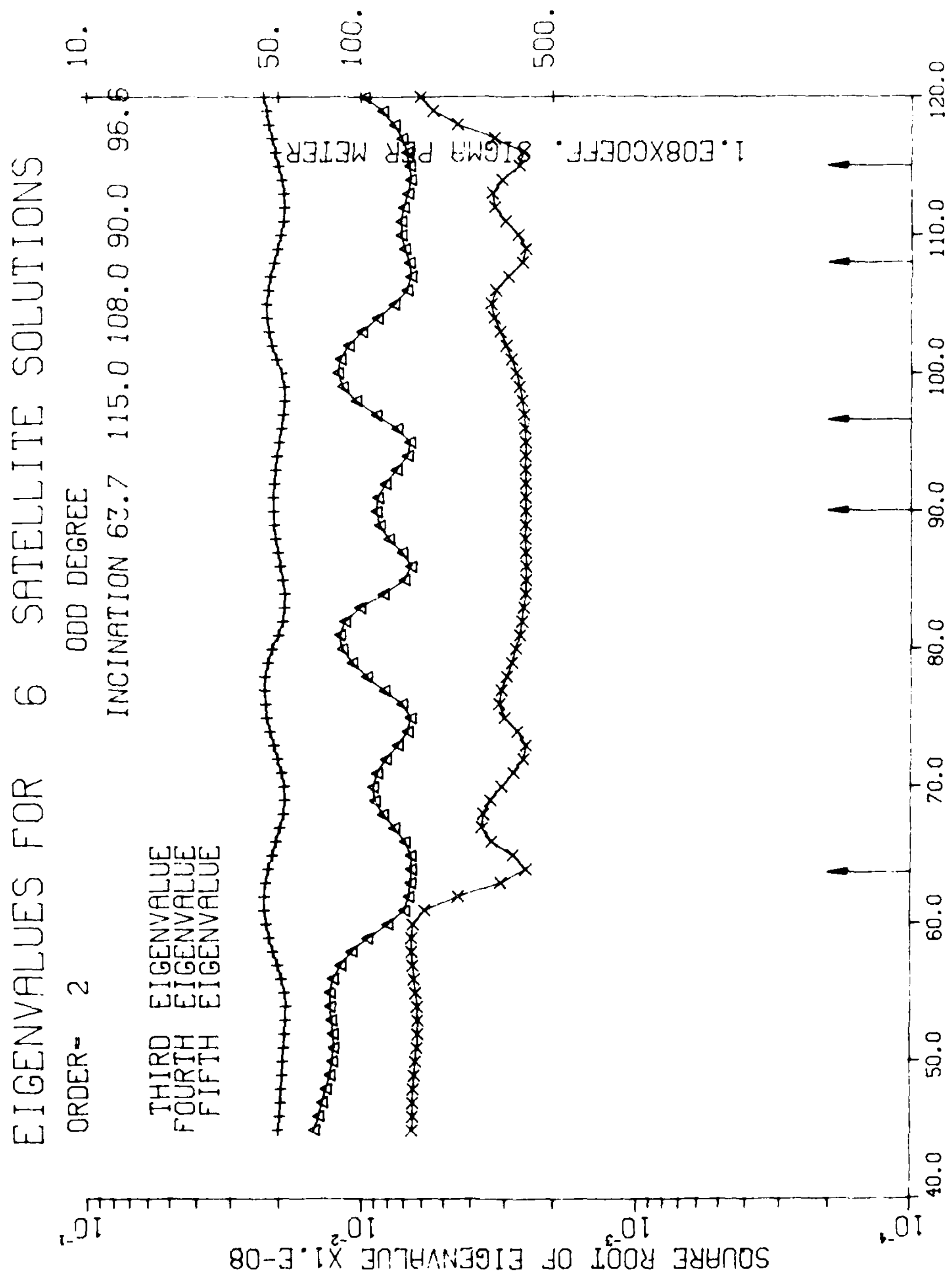


FIGURE C-6. SECOND ORDER ODD DEGREE EIGENVALUES FOR 6 SATELLITE SOLUTIONS

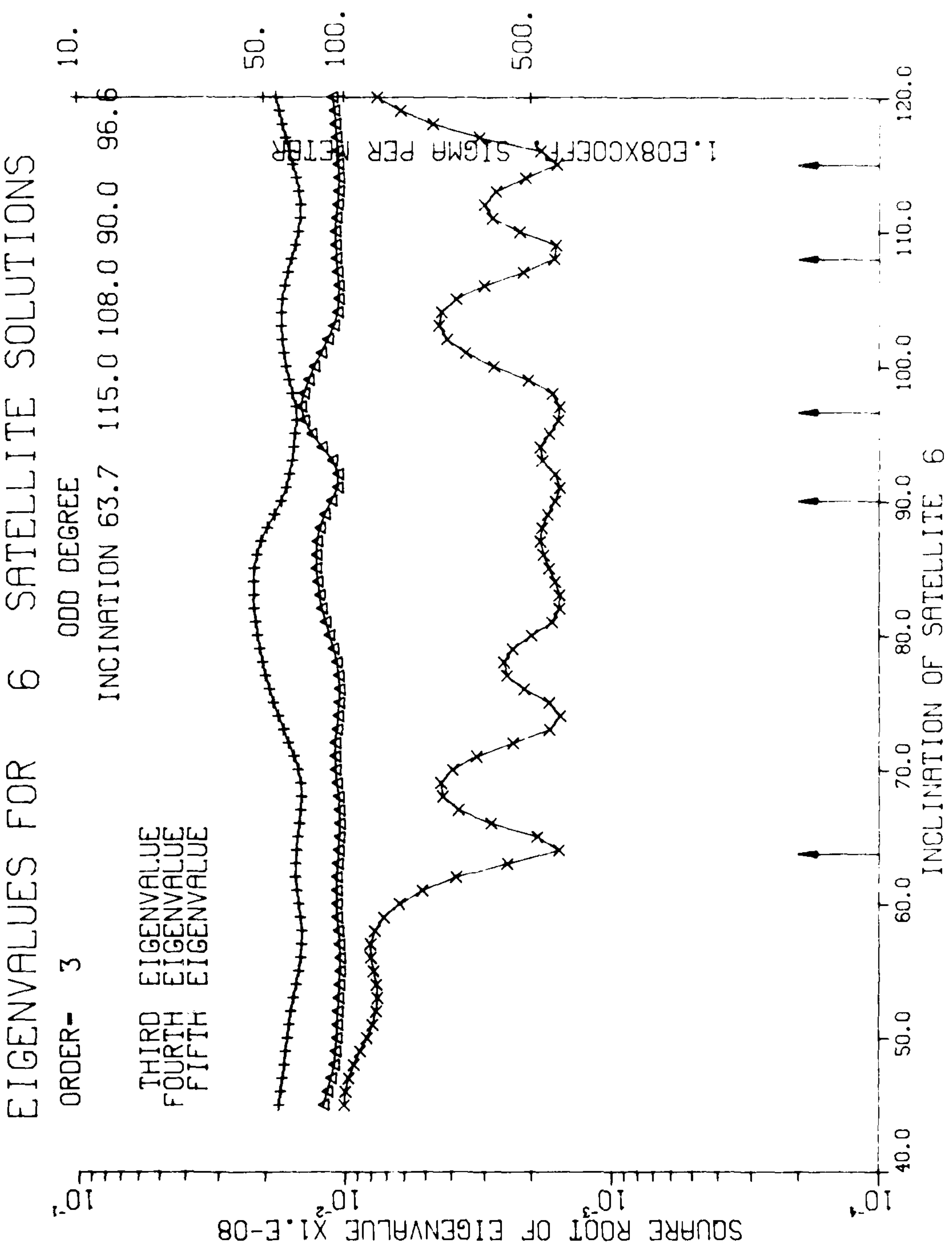
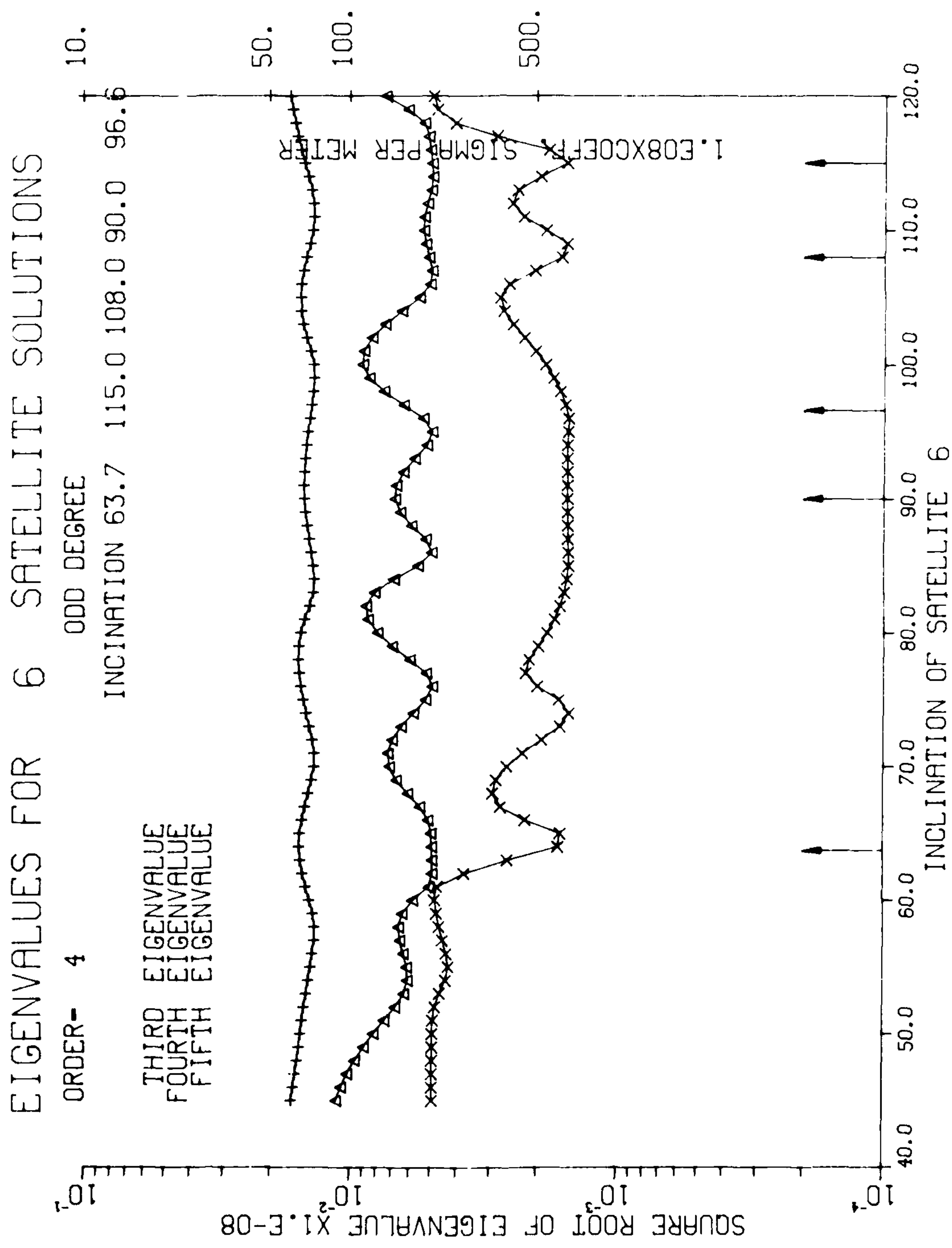


FIGURE C-7. THIRD ORDER ODD DEGREE EIGENVALUES FOR 6 SATELLITE SOLUTIONS



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