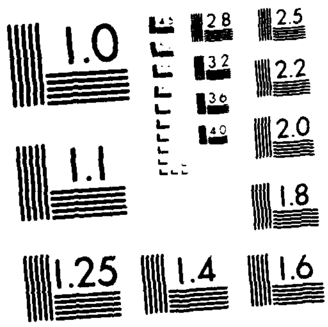
AD-A125 584 MULTIPARAMETER ACOUSTIC IMAGING IN THE BORN APPROXIMATION(U) UTAH UNIV SALT LAKE CITY DEPT OF MATHEMATICS C H WILCOX AUG 82 N00014-76-C-0276 1/1

UNCLASSIFIED F/G 12/1 NL

END
DATE
FILMED
4-83
DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963-A

MULTIPARAMETER ACOUSTIC IMAGING

IN THE BORN APPROXIMATION

Calvin H. Wilcox

Technical Summary Report #44
August 1982

MULTIPARAMETER ACOUSTIC IMAGING

IN THE BORN APPROXIMATION

Calvin H. Wilcox

Technical Summary Report #44
August 1982

Prepared with partial support from

Office of Naval Research, Contract No. N00014-76-C-0276.
American Cancer Society, Grant No. PDT-110B.
National Cancer Institute, Grant No. 1R01-CA 29728.

DTIC
ELECTE
MAR 1 4 1983
S D
B

DISTRIBUTION STATEMENT A
Approved for public release;
Distribution Unlimited

Contents

Accession For
NTIS GRA&I
DTIC TAB
A

DTIC
COPY
INSPECTED
2

Abstract.

Samples of biological tissue are modelled as inhomogeneous fluids with density $\rho(x)$ and sound speed $c(x)$ at point x. The samples are contained in the sphere $|x| \leq \delta$ and it is assumed that $\rho(x) \equiv \rho_0 = 1$ and $c(x) \equiv c_0 = 1$ for $|x| \geq \delta$, and $|\gamma_n(x)| << 1$, $|\gamma_\rho(x)| << 1$ and $|\nabla\gamma_\rho(x)| << 1$ where $\gamma_\rho(x) = \rho(x) - 1$ and $\gamma_n(x) = c^{-2}(x) - 1$. The samples are insonified by plane pulses $s(x \cdot \theta_0 - t)$ where $|\theta_0| = 1$ and the scattered pulse is shown to have the form $|x|^{-1}\, e_s(|x| - t,\theta,\theta_0)$ in the far field, where $x = |x|\theta$. The response $e_s(\tau,\theta,\theta_0)$ is measurable. The goal of the work is to construct the sample parameters γ_n and γ_ρ from $e_s(\tau,\theta,\theta_0)$ for suitable choices of s, θ and θ_0.

In the limiting case of constant density: $\gamma_\rho(x) \equiv 0$ it is shown that

$$\gamma_n(x) = \frac{4}{\pi} \int_{S^2} e_\delta(2x \cdot \theta,\theta,-\theta)\, d\theta$$

where δ represents the Dirac δ and S^2 is the unit sphere $|\theta| = 1$. Analogous formulas, based on two sets of measurements, are derived for the case of variable $c(x)$ and $\rho(x)$.

1. Introduction.

Computer-based acoustic imaging techniques have been studied intensively during the last decade [6,8,9]*. Typical techniques involve irradiating a sample with prescribed sound fields, measuring the resulting scattered fields and applying a computational algorithm to the scattering data to produce maps of such sample parameters as density, sound speeds and perhaps others. These techniques have important applications to medical ultrasonic imaging where the sample is a living organism, to non-destructive evaluation where the sample is a manufactured item such as a metal casting or ceramic object and to geophysical prospecting where the sample is a portion of the earth's crust.

This paper treats a problem of medical ultrasonic imaging. The sample is modelled as an inhomogeneous fluid which is characterized by a variable density $\rho(x)$ and sound speed $c(x)$. The use of a fluid model is motivated by the fact that in biological tissues, other than bone, acoustic shear waves are not observed.

In acoustic imaging, the scale of the smallest structures that can be resolved is of the order of the smallest wavelength employed. A typical sound speed in biological tissue is $c = 1500$ m/sec. Thus for a wavelength of $\lambda = 1$ mm. $= 10^{-3}$ m., a frequency of $f = c/\lambda = 1.5 \times 10^6$ hertz $= 1.5$ megahertz is needed. This is in the high ultrasonic range.

The acoustic field in an inhomogeneous fluid with density $\rho(x)$ and sound speed $c(x)$ may be characterized by a real valued function

---

*Numbers in square brackets denote references from the list at the end of the paper.

$u(t,x)$ that satisfies the scalar wave equation

$$\frac{\partial^2 u}{\partial t^2} - c^2(x)\ \rho(x)\ \nabla \cdot \left\{\frac{1}{\rho(x)}\ \nabla u\right\} = 0 \tag{1.1}$$

where t is a time coordinate and $x = (x_1, x_2, x_3) \in R^3$ denotes a triple of rectangular coordinates in space. $u(t,x)$ may be interpreted as the acoustic pressure; that is, the difference between the instantaneous pressure and the equilibrium pressure in the fluid. It is directly measurable. Derivations of (1.1) from the principles of fluid dynamics may be found in a number of books and monographs; see, e.g. [1,2,3,7].

It will be assumed here that the sample to be imaged is contained in the ball $B(0,\delta) = \{x : |x| \leq \delta\}$, and that

$$\rho(x) \equiv \rho_0 = 1 \text{ and } c(x) \equiv c_0 = 1 \text{ outside } B(0,\delta), \tag{1.2}$$

where ρ_0 and c_0 are the constant parameters of the ambient fluid. The conditions $\rho_0 = 1$, $c_0 = 1$ are a convenient normalization that can always be achieved by a suitable choice of units.

The deviations of the sample parameters from those of the ambient fluid will be measured by the parameters

$$\gamma_\rho(x) = \rho(x) - 1 \tag{1.3}$$

and

$$\gamma_n(x) = c^{-2}(x) - 1 = n^2(x) - 1 \tag{1.4}$$

where $n(x) = c^{-1}(x)$ is the index of refraction. The acoustic imaging method developed below is based on the Born approximation to solutions of (1.1). The conditions for the validity of the approximation are

$$(1.5) \qquad |\gamma_n(x)| << 1, \ |\gamma_\rho(x)| << 1 \text{ and } |\nabla\gamma_\rho(x)| << 1.$$

The purpose of an ultrasonic imaging technique for (1.1) is to construct accurate maps of the functions $\rho(x)$ and $c(x)$ by applying a computational algorithm to the measured data of suitable scattering experiments. An explicit method for solving this two parameter imaging problem is derived below under the weak scattering hypothesis (1.5). The basic scattering experiment is the scattering by the sample of a plane wave

$$(1.6) \qquad u_0(t,x) = s(x \cdot \theta_0 - t)$$

where θ_0 is a fixed unit vector, or point on the unit sphere $S^2 = \{x : |x| = 1\} \subset R^3$, and $s(\tau)$ is a prescribed wave profile.

The imaging method developed below is based on the author's theory of asymptotic wave functions as developed in [14,15,16]. In the present context the theory states that if $u(t,x)$ is the total field due to the interaction of the pulse (1.6) with the sample, and if

$$(1.7) \qquad u^{sc}(t,x) = u(t,x) - u_0(t,x)$$

is the scattered field then u^{sc} has the far field form

$$(1.8) \qquad u^{sc}(t,x) = |x|^{-1} e_s(|x| - t,\theta,\theta_0) + o(1),$$

where $x = |x|\theta$, $\theta \in S^2$ and the error term $o(1)$ tends to zero when $t \to \infty$.

The imaging algorithm developed below is based on the echo profile function $e_s(\tau,\theta,\theta_0)$. The results take their simplest form when

$s(\tau) = \delta(\tau)$, the Dirac δ-function. Of course, an ideal infinitely sharp pulse cannot be realized in practice. However, good approximations can be generated. Alternatively, one can obtain the response $e_\delta(\tau,\theta,\theta_0)$ by electronic filtering of the response $e_s(\tau,\theta,\theta_0)$ to a more realistic pulse profile $s(\tau)$.

The imaging method takes a particularly simple form in the constant density case ($\gamma_\rho(x) \equiv 0$) when there is only one parameter to be imaged. In this case it will be shown that, in the Born approximation,

$$\gamma_n(x) = \frac{4}{\pi} \int_{S^2} e_\delta(2x \cdot \theta,\theta,-\theta)\, d\theta \tag{1.9}$$

where $d\theta$ is the element of area (solid angle) on S^2. Thus γ_n is obtained by integrating the back scattered echoes $e_\delta(\tau,\theta,-\theta)$ over all directions. In the general case a second set of measurements is needed to determine the parameters γ_ρ and γ_n.

A formula equivalent to (1.9) was derived by S. J. Norton and M. Linzer [9] who obtained it as a limiting case of an imaging method based on near field measurements (see [9, p. 215, (81)]). More recently, J. H. Rose and J. M. Richardson [11] have given without proof an analogue of (1.9) for the imaging of inhomogeneities in isotropic elastic solids. In their work they also formulate analogues of (1.8) for elastic solids and discuss their applications to multiparameter imaging.

The two goals of this paper can now be formulated. The first goal is to show how the author's theory of asymptotic wave functions, cited above, and the Born approximation can be used to provide a simple and rigorous derivation of a functional relation between the echo waveform $e_\delta(\tau,\theta,\theta_0)$ and the Radon transforms of γ_n and γ_ρ. The second goal is to use this relation to derive an explicit imaging algorithm for the two parameter case. It will be seen from the analysis that the same method is applicable to electromagnetic imaging, and to acoustic imaging of solids where more than two parameters are to be imaged.

The remainder of the paper is organized as follows. §2 presents a brief discussion of the facts concerning the scattering of time-harmonic plane waves that are needed to analyze pulse mode scattering. This theory is similar to, and simpler than, the theory of scattering by bounded obstacles developed in [14,16]. §3 presents the Born approximation to the time-harmonic scattered fields and the scattering amplitude. §4 develops the functional relation between the scattering amplitude and the Radon transforms of γ_n and γ_ρ. §5 reviews the theory of pulse mode scattering as a boundary value problem and the associated theory of asymptotic wave functions. The final §6 develops the acoustic imaging method for the two parameter problem. For clarity the known one parameter formula (1.9) is derived first. Then the method is extended to the general two parameter case. The section ends with a brief discussion of the numerical implementation of the method.

2. The Scattering of Time-Harmonic Plane Waves.

A time-harmonic plane wave propagating in the direction $\theta_0 \in S^2$ in the ambient fluid may be characterized by the complex wave function

$$(2.1) \qquad w_0(x,\omega\theta_0) = (2\pi)^{-3/2} e^{i\omega\theta_0 \cdot x}$$

where $\omega/2\pi$ is the wave frequency and the time factor $e^{-i\omega t}$ has been suppressed. The amplitude factor $(2\pi)^{-3/2}$ is a normalization that is included to facilitate the application below of the results of [16].

If the scatterer is irradiated by the field (2.1) the resulting time-harmonic field

$$(2.2) \qquad w(x,\omega\theta_0) = w_0(x,\omega\theta_0) + w^{sc}(x,\omega\theta_0)$$

is uniquely characterized by the properties

$$(2.3) \qquad c^2(x)\ \rho(x)\ \nabla \cdot \left(\frac{1}{\rho(x)} \nabla w\right) + \omega^2 w = 0$$

for all $x \in R^3$ and

$$(2.4) \qquad \frac{\partial w^{sc}}{\partial |x|} - i\omega w^{sc} = O(|x|^{-2}), \quad |x| \to \infty .$$

Equation (2.3) is just the wave equation (1.1) for $w(x,\omega\theta_0)\, e^{-i\omega t}$. (2.4) is the Sommerfeld outgoing radiation condition. (2.3) and (2.4) will be shown to imply that w^{sc} has the far field form

$$(2.5) \qquad w^{sc}(x,\omega\theta_0) = \frac{e^{i\omega|x|}}{4\pi|x|} T(\omega\theta,\omega\theta_0) + O(|x|^{-2}), \quad |x| \to \infty,$$

where $x = |x|\theta$. The coefficient $T(p,p')$ is called the scattering amplitude of the scatterer. It is known to satisfy the reciprocity law

$$(2.6)\qquad T(p,p') = T(-p',-p).$$

$T(p,p')$ plays a key role in the theory of acoustic imaging presented below.

The wave field $w(x,p)$ and its scattering amplitude can be constructed by solving an integral equation. To derive it note that

$$(2.7)\qquad \nabla\cdot\left(\frac{1}{\rho}\nabla w\right) = \frac{1}{\rho}\Delta w - \frac{1}{\rho^2}\nabla\rho\cdot\nabla w .$$

Thus (2.3) may be written

$$(2.8)\qquad \Delta w - (\nabla \ln \rho(x))\cdot\nabla w + \frac{\omega^2}{c^2(x)} w = 0 .$$

This can be treated by perturbation theory by introducing the parameters γ_n and γ_ρ.

Note that by (1.3)

$$(2.9)\qquad \nabla \ln \rho = \nabla \ln (1+\gamma_\rho) = (1+\gamma_\rho)^{-1}\nabla\gamma_\rho .$$

Hence (2.8) is equivalent to

$$(2.10)\qquad \Delta w + \omega^2 w = -\omega^2\gamma_n(x)w + (1+\gamma_\rho(x))^{-1}\nabla\gamma_\rho(x)\cdot\nabla w$$

by (1.4) and (2.9). Recall that by hypothesis

$$(2.11)\qquad \Gamma \equiv \operatorname{supp}\gamma_n \cup \operatorname{supp}\gamma_\rho \subset B(0,\delta)$$

where $\operatorname{supp}\gamma_n$ denotes the support of γ_n (the smallest closed set outside of which $\gamma_n(x) = 0$). Application of Green's theorem to $w(x,p)$ and the outgoing Green's function for the ambient fluid

$$(2.12) \qquad G(x-x',p) = \frac{e^{i|p||x-x'|}}{4\pi|x-x'|}$$

gives, after a standard calculation,

$$(2.13) \qquad \begin{aligned} w(x,p) &= w_0(x,p) \\ &+ \int_\Gamma G(x-x',p)\{|p|^2\gamma_n(x')\, w(x',p) - (1+\gamma_\rho(x'))^{-1}\, \nabla\gamma_\rho(x')\cdot\nabla w(x',p)\}\, dx' \end{aligned}$$

where $dx' = dx_1'\, dx_2'\, dx_3'$. If x is restricted to Γ then (2.13) is an integro-differential equation for $w(x,p)|\Gamma$, the restriction of $w(x,p)$ to Γ. It may be transformed into a pure integral equation by an integration by parts in the last term. Solution of this integral equation by standard techniques provides a construction of $w(x,p)|\Gamma$. The continuation of $w(x,p)|\Gamma$ to all of R^3 is then provided by (2.13).

A verification of (2.5) and a construction of $T(p,p')$ are also provided by (2.13). Indeed, if $x = |x|\theta$ then

$$(2.14) \qquad |x - x'| = |x| - x'\cdot\theta + \mathcal{O}(|x|^{-1}),\ |x| \to \infty,$$

uniformly for all $x' \in \Gamma$. Thus

$$(2.15) \qquad G(x - x',p) = \frac{e^{i|p||x|}}{4\pi|x|} e^{-i|p|\theta\cdot x'} + \mathcal{O}(|x|^{-2}),\ |x| \to \infty,$$

uniformly for $x' \in \Gamma$. Substitution of (2.15) into (2.13) gives (2.5) with

$$T(\omega\theta,\omega\theta_0) = \int_\Gamma e^{-i\omega\theta\cdot x} \{\omega^2\gamma_n(x)\ w(x,\omega\theta_0) - (1+\gamma_\rho(x))^{-1}\nabla\gamma_\rho(x)\cdot\nabla w(x,\omega\theta_0)\}\ dx \tag{2.16}$$

Thus T can be calculated from the parameters γ_n, γ_ρ and the field $w(x,p)$ inside the scatterer.

3. Weak Scatterers and the Born Approximation.

When the scatterers are weak in the sense of conditions (1.5) the integro-differential equation (2.13) for $w(x,p)$ can be solved by iteration. On dropping terms of orders higher than the first in γ_n, γ_ρ and $\nabla\gamma_\rho$ one obtains the Born approximation to $w^{sc}(x,p)$. It is given by

$$w^{sc}(x,p) = \int_\Gamma G(x-x',p) \{|p|^2\gamma_n(x') \, w_0(x',p) - \nabla\gamma_\rho(x') \cdot \nabla w_0(x',p)\} \, dx'$$

(3.1)

$$= (2\pi)^{-3/2} \int_\Gamma G(x-x',p) \, e^{ip\cdot x'} \{|p|^2\gamma_n(x') - ip \cdot \nabla\gamma_\rho(x')\} \, dx'.$$

The corresponding Born approximation to the scattering amplitude is

(3.2) $$T(\omega\theta,\omega\theta_0) = (2\pi)^{-3/2} \int_\Gamma e^{-i\omega(\theta-\theta_0)\cdot x} \{\omega^2\gamma_n(x) - i\omega\theta_0 \cdot \nabla\gamma_\rho(x)\} \, dx.$$

An integration by parts in the last term gives the alternative representation

(3.3) $$T(\omega\theta,\omega\theta_0) = \frac{\omega^2}{(2\pi)^{3/2}} \int_\Gamma e^{-i\omega(\theta-\theta_0)\cdot x} \{\gamma_n(x) + (\theta \cdot \theta_0 - 1) \, \gamma_\rho(x)\} \, dx.$$

The notation

(3.4) $$\gamma_\mu(x) = \gamma_n(x) + (\mu - 1) \, \gamma_\rho(x)$$

will be used in what follows. Equation (3.3) can then be written concisely as

$$T(\omega\theta,\omega\theta_0) = \omega^2 \, \hat{\gamma}_{\theta\cdot\theta_0}(\omega(\theta - \theta_0)) \tag{3.5}$$

where $\hat{\gamma}$ is the usual Fourier transform, defined by

$$\hat{\gamma}(p) = (2\pi)^{-3/2} \int_{R^3} e^{-ip\cdot x} \, \gamma(x) \, dx. \tag{3.6}$$

In the remainder of this paper the approximation (3.5) will be used for the scattering amplitude.

4. The Born Approximation and the Radon Transform.

The Radon transform of a function $\gamma(x)$ with compact support is the function $\tilde{\gamma} : R \times S^2 \to R$ defined by [5,10]

$$(4.1) \qquad \tilde{\gamma}(s,\eta) = \int_{x\cdot\eta=s} \gamma(x)\, dS$$

where $(s,\eta) \in R \times S^2$. Equation (4.1) means that $\tilde{\gamma}(s,\eta)$ is the integral of $\gamma(x)$ over the plane with equation $x \cdot \eta = s$ and surface element dS. An alternative notation is

$$(4.2) \qquad \tilde{\gamma}(s,\eta) = \int_{R^2} \gamma(s\eta + x^{\perp})\, dx^{\perp}$$

where $x^{\perp}$ is a point in the plane through the origin that has normal η and surface element $dx^{\perp}$. Note that $\tilde{\gamma}(s,\eta)$ also has compact support: supp $\gamma \subset B(0,\delta) \Rightarrow \tilde{\gamma}(s,\eta) = 0$ for $|s| > \delta$.

In this section a known relation between the Fourier and Radon transforms is derived and used to relate the Born approximation (3.5) to the Radon transform of $\gamma_{\theta\cdot\theta_0}$. This relation and the known inverse Radon transform provide the basis for the imaging method developed in §6.

To begin, consider the Fourier integral

$$(4.3) \qquad (2\pi)^{3/2}\, \hat{\gamma}(\omega(\theta - \theta_0)) = \int_{R^3} e^{-i\omega(\theta-\theta_0)\cdot x}\, \gamma(x)\, dx$$

where ω, θ and θ_0 are fixed, $\omega \neq 0$ and $\theta \neq \theta_0$. Introduce new variables

(4.4) $$\eta = \frac{\theta-\theta_0}{|\theta-\theta_0|} \in S^2,$$

(4.5) $$\tau = (\theta - \theta_0) \cdot x,$$

and

(4.6) $$s = x \cdot \eta = \frac{\tau}{|\theta-\theta_0|} .$$

Then

(4.7) $$x = (x \cdot \eta)\eta + x^{\perp} = s\eta + x^{\perp} ,$$

where $x^{\perp}$ is in the plane orthogonal to η, and a rotation of coordinates in (4.3) gives

$$(2\pi)^{3/2}\,\hat{\gamma}(\omega(\theta - \theta_0)) = \int_{R^3} e^{-i\omega\tau}\,\gamma(s\eta + x^{\perp})\,ds\,dx^{\perp}$$

(4.8) $$= \int_{-\infty}^{\infty} e^{-i\omega\tau} \left(\int_{R^2} \gamma(s\eta + x^{\perp})\,dx^{\perp}\right) ds$$

$$= |\theta - \theta_0|^{-1} \int_{-\infty}^{\infty} e^{-i\omega\tau}\,\tilde{\gamma}\left(\frac{\tau}{|\theta-\theta_0|}, \frac{\theta-\theta_0}{|\theta-\theta_0|}\right) d\tau.$$

The last integral is a one dimensional Fourier transform. Hence multiplying it by ω^2 is equivalent to the operation $-\partial^2/\partial\tau^2$ on the integrand. Thus

(4.9) $$(2\pi)^{3/2}\,\omega^2\hat{\gamma}(\omega(\theta-\theta_0)) = -|\theta-\theta_0|^{-3} \int_{-\infty}^{\infty} e^{-i\omega\tau}\,\tilde{\gamma}''\left(\frac{\tau}{|\theta-\theta_0|}, \frac{\theta-\theta_0}{|\theta-\theta_0|}\right) d\tau$$

where $\tilde{\gamma}''(s,\eta)$ denotes the second s derivative of $\tilde{\gamma}(s,\eta)$. Now the function

(4.10) $$h_\gamma(s,\eta) = -\frac{1}{8\pi^2}\tilde{\gamma}''(s,\eta)$$

occurs in the inverse Radon transform. In fact, the inverse is given by [5]

(4.11) $$\gamma(x) = \int_{S^2} h_\gamma(x\cdot\eta,\eta)\,d\eta.$$

Thus it is natural to rewrite (4.9) as

(4.12) $$\omega^2\,\hat{\gamma}(\omega(\theta-\theta_0)) = \frac{2(2\pi)^{1/2}}{|\theta-\theta_0|^3}\int_{-\infty}^{\infty} e^{-i\omega\tau}\,h_\gamma\left(\frac{\tau}{|\theta-\theta_0|},\frac{\theta-\theta_0}{|\theta-\theta_0|}\right)d\tau.$$

Applying (4.12) to the Born approximation (3.5) gives the relation

(4.13) $$T(\omega\theta,\omega\theta_0) = \frac{2(2\pi)^{1/2}}{|\theta-\theta_0|^3}\int_{-\infty}^{\infty} e^{-i\omega\tau}\,h_{\theta\cdot\theta_0}\left(\frac{\tau}{|\theta-\theta_0|},\frac{\theta-\theta_0}{|\theta-\theta_0|}\right)d\tau$$

where, for brevity, the notation

(4.14) $$h_{\theta\cdot\theta_0} \equiv h_{\gamma_{\theta\cdot\theta_0}} = \gamma_n + (\theta\cdot\theta_0 - 1)\gamma_\rho$$

has been introduced.

The scattering amplitude can be obtained from far field measurements; see (2.5). Hence, the relation (4.13) is a natural starting point for an imaging method. The Fourier transform of (4.13) is

(4.15) $$\frac{1}{(2\pi)^{1/2}}\int_{-\infty}^{\infty} e^{i\tau\omega}\,T(\omega\theta,\omega\theta_0)\,d\omega = \frac{4\pi}{|\theta-\theta_0|^3}\,h_{\theta\cdot\theta_0}\left(\frac{\tau}{|\theta-\theta_0|},\frac{\theta-\theta_0}{|\theta-\theta_0|}\right).$$

Suitable choices of τ, θ and θ_0 in this relation, with $\theta\cdot\theta_0 = \mu$ fixed, give $h_\mu(s,\eta)$. Then the inversion formula (4.11) gives γ_μ. By doing this for two values of μ one can deduce γ_n and γ_ρ from (3.4). This program is carried out in §6 below. Moreover, it is shown that the values of the Fourier integral on the left-hand side of (4.15) can be obtained directly from pulse mode scattering measurements. This is developed in the next section.

5. <u>Pulse Mode Scattering</u>.

The problem of the scattering by the sample of plane wave pulses

$$u_0(t,x) = s(x \cdot \theta_0 - t) \tag{5.1}$$

is formulated and solved in this section. The pulse profile $s(\tau)$ is assumed to be a prescribed function with compact support:

$$\operatorname{supp} s \subset [a,b] \ . \tag{5.2}$$

Thus for any fixed value of t one has

$$\operatorname{supp} u_0(t,\cdot) \subset \{x : a + t \le x \cdot \theta_0 \le b + t\} \tag{5.3}$$

and the pulse (5.1) does not interact with the sample, which is contained in $B(0,\delta)$, before the time $t_0 = -b - \delta$: see Figure 1.

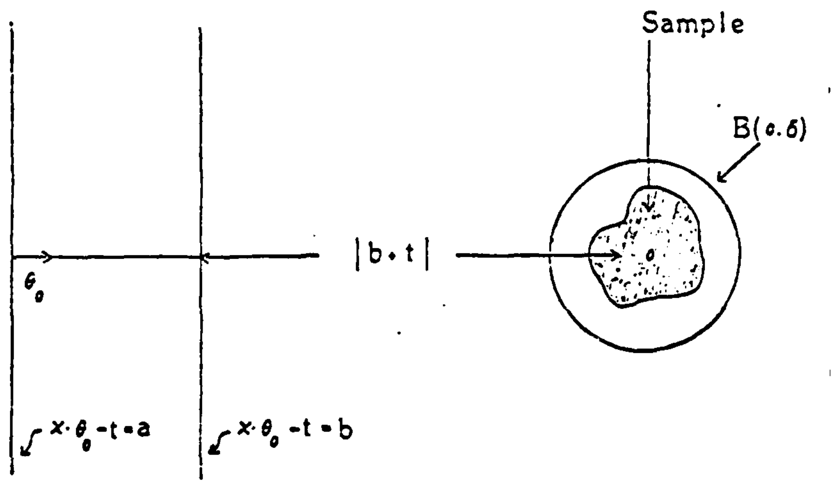


Figure 1. Incident pulse u_0 before interaction with the sample.

Therefore, the total acoustic pressure field u(t,x) due to the scattering of the pulse (5.1) satisfies

(5.4) $$u(t,x) \equiv u_0(t,x) \text{ for all } t \le t_0 \text{ and } x \in R^3.$$

It follows that u(t,x) is a solution of an initial value problem for the wave equation (1.1):

(5.5) $$\frac{\partial^2 u}{\partial t^2} - c^2(x)\,\rho(x)\,\nabla\cdot\left\{\frac{1}{\rho(x)}\,\nabla u\right\} = 0 \text{ for all } t > t_0 \text{ and } x \in R^3,$$

(5.6) $$u(t_0,x) = f(x) \text{ and } \frac{\partial u(t_0,x)}{\partial t} = g(x) \text{ for all } x \in R^3$$

where

(5.7) $$\begin{cases} f(x) = u_0(t_0,x) = s(x\cdot\theta_0 - t_0), \text{ and} \\ g(x) = \dfrac{\partial u_0(t_0,x)}{\partial t} = -s'(x\cdot\theta_0 - t_0)\ . \end{cases}$$

The theory of the initial value problem (5.5), (5.6) is analogous to, and simpler than, the theory of initial-boundary value problems for the wave equation, as developed in [14,15,16]. The theory will be outlined here without proofs. Details may be found in the references cited.

The problem (5.5), (5.6) is most simply discussed in terms of the scattered field

(5.8) $$u^{sc}(t,x) = u(t,x) - u_0(t,x).$$

It is a solution of the problem

(5.9) $$\frac{\partial^2 u^{sc}}{\partial t^2} - c^2(x)\,\rho(x)\,\nabla\cdot\left\{\frac{1}{\rho(x)}\,\nabla u^{sc}\right\} = F(t,x) \text{ for } t > t_0,\ x \in R^3,$$

$$(5.10) \qquad u^{sc}(t_0,x) = 0 \text{ and } \frac{\partial u^{sc}(t_0,x)}{\partial t} = 0 \text{ for } x \in R^3$$

where F(t,x) is defined by

$$(5.11) \qquad F(t,x) = -\frac{\partial^2 u_0(t,x)}{\partial t^2} + c^2(x)\ \rho(x)\ \nabla \cdot \left(\frac{1}{\rho(x)} \nabla u_0(t,x)\right)$$

for all $t \geq t_0$ and $x \in R^3$. Note that (5.3) and the assumption that $\rho(x) = 1$, $c(x) = 1$ outside $B(0,\delta)$ imply that F has compact support in space-time. More precisely,

$$(5.12) \qquad \text{supp } F \subset [-b-\delta, -a+\delta] \times B(0,\delta) \ .$$

A simple approach to the initial value problem (5.9), (5.10) may be based on the theory of the operator

$$(5.13) \qquad Au = -c^2(x)\ \rho(x)\ \nabla \cdot \left(\frac{1}{\rho(x)} \nabla u\right)$$

in the Hilbert space

$$(5.14) \qquad \mathcal{H} = L_2(R^3, c^{-2}(x)\ \rho^{-1}(x)\ dx)$$

with scalar product

$$(5.15) \qquad (u,v) = \int_{R^3} \overline{u(x)}\ v(x)\ c^{-2}(x)\ \rho^{-1}(x)\ dx \ .$$

The theory may be based on Kato's theory of sesquilinear forms in Hilbert spaces [4]; see [17]. It follows that if the domain of A is defined by

$$(5.16)\qquad D(A) = \mathcal{H} \cap \left\{u : \nabla u \text{ and } \nabla\cdot\left(\frac{1}{\rho}\nabla u\right) \text{ are in } \mathcal{H}\right\}$$

then A is a selfadjoint non-negative operator in $\mathcal{H}$. The problem (5.9), (5.10) may then be reformulated as the Hilbert space problem

$$(5.17)\qquad \frac{d^2u^{sc}}{dt^2} + Au^{sc} = F(t,\cdot) \text{ for } t > t_0,$$

$$(5.18)\qquad u^{sc}(t_0) = 0 \text{ and } \frac{du^{sc}(t_0)}{dt} = 0 .$$

The formal solution is given by the Duhamel integral

$$(5.19)\qquad u^{sc}(t,\cdot) = \int_{t_0}^{t} \{A^{-1/2} \sin (t-\tau) A^{1/2}\} F(\tau,\cdot)\, d\tau .$$

A rigorous interpretation of (5.19) may be based on the spectral theorem for A. In particular, u^{sc} is a "strict solution with finite energy" in the sense of [12] provided that $t \to F(t,\cdot) \in \mathcal{H}$ is continuous. If $\rho(x)$, $c(x)$ and $s(\tau)$ are smooth functions then known regularity results [13] imply that u^{sc} is a classical solution.

It will be convenient here to represent $u^{sc}(t,x)$ as

$$(5.20)\qquad u^{sc}(t,x) = \text{Re}\,\{v^{sc}(t,x)\}$$

where v^{sc} is the complex valued wave function defined by

$$(5.21)\qquad v^{sc}(t,\cdot) = i\, A^{-1/2} e^{-itA^{1/2}} \int_{t_0}^{t} e^{i\tau A^{1/2}} F(\tau,\cdot)\, d\tau .$$

Note that, since $F(t,\cdot) = 0$ for $t \geq t_1 = -a + \delta$, one has

$$(5.22)\qquad v^{sc}(t,\cdot) = e^{-itA^{1/2}} h \text{ for } t \geq t_1$$

where

$$h = i\, A^{-1/2} \int_{t_0}^{t_1} e^{i\tau A^{1/2}} F(\tau,\cdot)\, d\tau \; . \tag{5.23}$$

The asymptotic behavior for $t \to \infty$ of wave functions of the form (5.22) was calculated in [14,16] for solutions of the d'Alembert equation in exterior domains. By the same methods it can be shown that if $x = |x|\theta$ then

$$v^{sc}(t,x) = |x|^{-1} F(|x| - t, \theta) + \mathcal{O}(1) \tag{5.24}$$

where

$$\int_{-\infty}^{\infty} \int_{S^2} |F(\tau,\theta)|^2 \, d\theta d\tau < \infty \tag{5.25}$$

and $\mathcal{O}(1) \to 0$ in $\mathcal{H}$ when $t \to \infty$. The function

$$v_\infty^{sc}(t,x) = |x|^{-1} F(|x| - t, \theta) \tag{5.26}$$

is called the asymptotic wave function for v^{sc}.

The results (5.24), (5.25) are based on the fact that if

$$\left.\begin{cases} w_+(x,p) = w(x,p) \\ w_-(x,p) = \overline{w_+(x,-p)} = \overline{w(x,-p)} \end{cases}\right\} \text{ for } x \in R^3,\ p \in R^3, \tag{5.27}$$

where $w(x,p)$ is the time-harmonic field of §2, then each of the families $\{w_+(x,p) : p \in R^3\}$ and $\{w_-(x,p) : p \in R^3\}$ is a complete family of generalized eigenfunctions for the operator A. In fact, if

$$\hat{h}_{\pm}(p) = \int_{R^3} \overline{w_{\pm}(x,p)}\; h(x)\; c^{-2}(x)\; \rho^{-1}(x)\; dx \tag{5.28}$$

then the wave profile $F(\tau,\theta)$ is given by

$$F(\tau,\theta) = \frac{1}{(2\pi)^{1/2}} \int_0^{\infty} e^{i\tau\omega}\; \hat{h}_{-}(\omega\theta)(-i\omega)\; d\omega\;. \tag{5.29}$$

In the special case in which F is given by (5.11) with $u_0(t,x) = s(x \cdot \theta_0 - t)$, one has

$$F(\tau,\theta) = \int_0^{\infty} e^{i\tau\omega}\; T(\omega\theta,\omega\theta_0)\; \hat{s}(\omega)\; d\omega \tag{5.30}$$

where $T(\omega\theta,\omega\theta_0)$ is the scattering amplitude of §2 and

$$\hat{s}(\omega) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} e^{-i\omega\tau}\; s(\tau)\; d\tau\;. \tag{5.31}$$

These results may be verified by the method of [14] and [16].

By taking the real part of (5.24) one gets the asymptotic form of $u^{sc}(t,x)$ for large t:

$$u^{sc}(t,x) = |x|^{-1}\; e_s(|x|-t,\theta,\theta_0) + o(1) \tag{5.32}$$

where $e_s(\tau,\theta,\theta_0) = \mathrm{Re}\,\{F(\tau,\theta)\}$ and $o(1) \to 0$ in $\mathcal{H}$ when $t \to \infty$. By (5.30) the wave profile e_s is given by

$$\begin{aligned} e_s(\tau,\theta,\theta_0) &= \mathrm{Re}\left\{\int_0^{\infty} e^{i\tau\omega}\; T(\omega\theta,\omega\theta_0)\; \hat{s}(\omega)\; d\omega\right\} \\ &= \frac{1}{2}\int_{-\infty}^{\infty} e^{i\tau\omega}\; T(\omega\theta,\omega\theta_0)\; \hat{s}(\omega)\; d\omega\;. \end{aligned} \tag{5.33}$$

The last equation follows from the assumption that $s(\tau)$ is real, whence $\hat{s}(-\tau) = \overline{\hat{s}(\tau)}$, and the analogous property of the scattering amplitude:

$$T(-\omega\theta,-\omega\theta_0) = \overline{T(\omega\theta,\omega\theta_0)} \ . \tag{5.34}$$

Relation (5.32) implies that the pulse echo profile $e_s(\tau,\theta,\theta_0)$ may be obtained from far field measurements:

$$e_s(\tau,\theta,\theta_0) \approx |x| \ u^{sc}(|x|-\tau,|x|\theta), \tag{5.35}$$

with an error that tends to zero when $t \to \infty$, or equivalently when $|x| \to \infty$. This result and the relation (5.33) are used in §6 to construct an imaging method.

6. Pulse Mode Imaging.

In this section the Born approximation (4.13) and the representation (5.33) for the pulse echo profile are combined to obtain an explicit relation between e_s and the function $\gamma_{\theta\cdot\theta_0}$.

First, note that (4.13) guarantees that, in the Born approximation, $T(\omega\theta,\omega\theta_0)$ is the Fourier transform of the well-behaved function of τ given by (4.15). It follows from (4.15), (5.33) and the convolution theorem

$$(6.1) \qquad \int_{-\infty}^{\infty} f(\tau')\, g(\tau-\tau')\, d\tau' = \int_{-\infty}^{\infty} e^{i\tau\omega}\, \hat{f}(\omega)\, \hat{g}(\omega)\, d\omega$$

that

$$(6.2) \qquad e_s(\tau,\theta,\theta_0) = \frac{2\pi}{|\theta-\theta_0|^3} \int_{-\infty}^{\infty} s(\tau')\, h_{\theta\cdot\theta_0}\left(\frac{\tau-\tau'}{|\theta-\theta_0|}, \frac{\theta-\theta_0}{|\theta-\theta_0|}\right) d\tau' .$$

Note that on taking $s(\tau) = \delta(\tau)$, the Dirac delta, one gets

$$(6.3) \qquad e_\delta(\tau,\theta,\theta_0) = \frac{2\pi}{|\theta-\theta_0|^3}\, h_{\theta\cdot\theta_0}\left(\frac{\tau}{|\theta-\theta_0|}, \frac{\theta-\theta_0}{|\theta-\theta_0|}\right) .$$

Relation (6.3) provides the basis for the imaging method of this paper. The pulse echo profile e_δ will be regarded as obtainable from scattering measurements. Equation (6.2) shows that good approximations to e_δ can be obtained with incident pulses $s(\tau)$ that are smooth approximations to $\delta(\tau)$. Moreover, as mentioned in the introduction, e_δ can be obtained by suitable filtering of the echoes from other profiles $s(\tau)$.

The imaging of the parameters γ_n and γ_ρ will be based on (6.3) and the inverse Radon transform (4.11). To see how (6.3) can be used let $\Omega \subset S^2$ and suppose that mappings

$$\text{(6.4)} \qquad \begin{cases} \theta \ : \ \Omega \to S^2, & \eta \to \theta(\eta), \\ \theta_0 \ : \ \Omega \to S^2, & \eta \to \theta_0(\eta), \end{cases}$$

can be found such that

$$\text{(6.5)} \qquad \theta(\eta) \cdot \theta_0(\eta) = \mu = \text{const. for } \eta \in \Omega,$$

and

$$\text{(6.6)} \qquad \theta(\eta) - \theta_0(\eta) = c_\mu \, \eta \text{ for } \eta \in \Omega$$

where

$$\text{(6.7)} \qquad c_\mu = |\theta(\eta) - \theta_0(\eta)| = \sqrt{2(1-\mu)} \text{ for } \eta \in \Omega \ .$$

Making the substitutions $\theta \to \theta(\eta)$, $\theta_0 \to \theta_0(\eta)$ and $\tau \to c_\mu \tau$ in (6.3) gives

$$\text{(6.8)} \qquad h_\mu(\tau,\eta) = \frac{c_\mu^3}{2\pi} \, e_\delta(c_\mu \tau, \theta(\eta), \theta_0(\eta)) \text{ for } \eta \in \Omega \ ,$$

and hence

$$\text{(6.9)} \qquad \int_\Omega h_\mu(x \cdot \eta, \eta) \, d\eta = \frac{c_\mu^3}{2\pi} \int_\Omega e_\delta(c_\mu x \cdot \eta, \theta(\eta), \theta_0(\eta)) \, d\eta \ .$$

In particular, if $\Omega = S^2$ then (4.11) implies that (6.9) provides an explicit construction of $\gamma_\mu(x)$ from the scattering data. However, mappings $\theta(\eta)$, $\theta_0(\eta)$ may only exist on proper subdomains $\Omega \subset S^2$. In any case, if a decomposition

$$S^2 = \Omega_1 \cup \Omega_2 \cup \cdots \cup \Omega_N \tag{6.10}$$

and corresponding mappings

$$\begin{cases} \theta^j : \Omega_j \to S^2 \ , \\ \theta_r^j : \Omega_j \to S^2 \ , \end{cases} \tag{6.11}$$

satisfying (6.5) - (6.7) can be found, then one has

$$\begin{aligned} \gamma_\mu(x) &= \int_{S^2} h_\mu(x \cdot \eta, \eta) \, d\eta \\ &= \sum_{j=1}^{N} \int_{\Omega_j} h_\mu(x \cdot \eta, \eta) \, d\eta \\ &= \frac{c_\mu^3}{2\pi} \sum_{j=1}^{N} \int_{\Omega_j} e_\delta(c_\mu x \cdot \eta, \theta^j(\eta), \theta^j(\eta)) \, d\eta \ . \end{aligned} \tag{6.12}$$

If this can be done for two distinct values of μ, say μ_1 and u_2, then γ_{μ_1} and γ_{μ_2} can be computed and γ_n, γ_ρ can be found from the equations

$$\begin{cases} \gamma_{\mu_1} = \gamma_n + (\mu_1 - 1)\gamma_\rho \ , \\ \gamma_{\mu_2} = \gamma_n + (\mu_2 - 1)\gamma_\rho \ . \end{cases} \tag{6.13}$$

The section is concluded with a description of a particular method for carrying out this program.

<u>Back Scattering</u>. Back scattering is characterized by the condition $\theta = -\theta_0$. This may be realized in the context of equations

(6.4) - (6.7) by defining $\Omega = S^2$, and

$$\left\{\begin{aligned} \theta(\eta) &= \eta \\ \theta_0(\eta) &= -\eta \end{aligned}\right. \quad \text{for all } \eta \in S^2 \; . \tag{6.14}$$

Then (6.5) - (6.7) hold with $\mu = -1$ and $c_\mu = c_{-1} = 2$. Hence (6.9) with $\Omega = S^2$ and (4.11) give the relation

$$\gamma_{-1}(x) = \frac{4}{\pi} \int_{S^2} e_\delta(2x \cdot \eta, \eta, -\eta)\, d\eta \; . \tag{6.15}$$

In the special case of negligible density variations, $\gamma_\rho(x) \equiv 0$, one has $\gamma_{-1} = \gamma_n$ and (6.15) is the one parameter imaging formula (1.9) of the introduction.

Orthogonal Scattering. Orthogonal scattering is characterized by the condition $\theta \cdot \theta_0 = 0$; i.e., $\mu = 0$. To obtain fields $\theta(\eta)$, $\theta_0(\eta)$ that satisfy (6.4) - (6.7) with $\mu = 0$, fix a geographical coordinate system in S^2 with colatitude α and longitude β and define

$$\theta = (\cos\beta \sin\alpha, \sin\beta \sin\alpha, \cos\alpha) \tag{6.16}$$

and

$$\begin{aligned} \theta_0 = \frac{\partial\theta}{\partial\alpha} &= (\cos\beta\cos\alpha, \sin\beta\cos\alpha, -\sin\alpha) \\ &= \left(\cos\beta\sin\left(\alpha + \tfrac{\pi}{2}\right), \sin\beta\sin\left(\alpha + \tfrac{\pi}{2}\right), \cos\left(\alpha + \tfrac{\pi}{2}\right)\right) \; . \end{aligned} \tag{6.17}$$

It is clear from the identity $\theta \cdot \theta = 1$ that $\theta \cdot \theta_0 = 0$. Moreover, a short calculation gives, see Figure 2:

$$(6.18)\quad \eta = \frac{1}{\sqrt{2}}(\theta - \theta_0) = \left(\cos\beta \sin\left(\alpha - \frac{\pi}{4}\right),\ \sin\beta \sin\left(\alpha - \frac{\pi}{4}\right),\ \cos\left(\alpha - \frac{\pi}{4}\right)\right)\ .$$

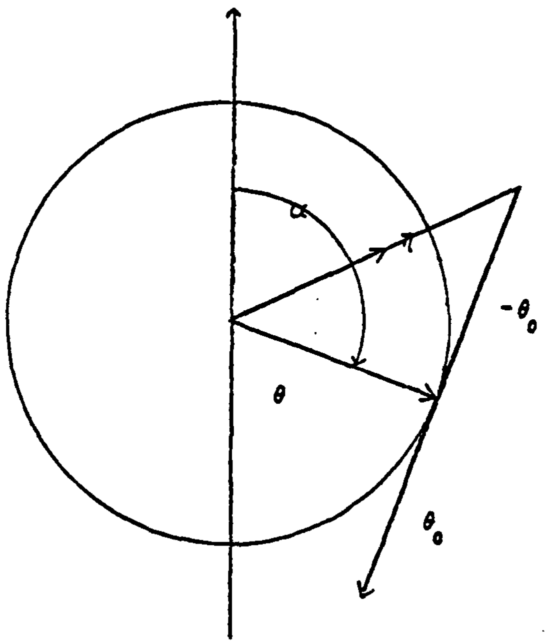


Figure 2. Graphical definition of θ, θ_0 and η.

For the remainder of the discussion it will be convenient to make the change of parameter $\alpha \rightarrow \alpha + \pi/4$ and define

$$(6.19)\begin{cases} \eta = (\cos\beta \sin\alpha,\ \sin\beta \sin\alpha,\ \cos\alpha), \\ \theta(\eta) = \left(\cos\beta \sin\left(\alpha + \frac{\pi}{4}\right),\ \sin\beta \sin\left(\alpha + \frac{\pi}{4}\right),\ \cos\left(\alpha + \frac{\pi}{4}\right)\right), \\ \theta_0(\eta) = \left(\cos\beta \sin\left(\alpha + \frac{3\pi}{4}\right),\ \sin\beta \sin\left(\alpha + \frac{3\pi}{4}\right),\ \cos\left(\alpha + \frac{3\pi}{4}\right)\right), \end{cases}$$

where

$$(6.20)\qquad 0 \le \alpha \le \pi \text{ and } 0 \le \beta \le 2\pi.$$

With these choices η ranges over the entire unit sphere; i.e., $\Omega = S^2$ in

(6.4). The coordinates α, β are regular on S^2 except at the north and south poles: $\alpha = 0$ and $\alpha = \pi$. It follows that $\theta(\eta)$ and $\theta_0(\eta)$ are also regular except at these points where they have discontinuities. In principle, these discontinuities cause no problem in evaluating the inverse Radon transform. Hence, (4.11) and (6.9) with $\Omega = S^2$ give

$$\gamma_0(x) = \frac{\sqrt{2}}{\pi} \int_{S^2} e_\delta(\sqrt{2}\, x \cdot \eta, \theta(\eta), \theta_0(\eta))\, d\eta \,, \tag{6.21}$$

where $\theta(\eta)$ and $\theta_0(\eta)$ are defined by (6.19). Of course, if one wishes to avoid discontinuous fields then the decomposition (6.10) - (6.12) may be used with different geographical coordinate systems for each component Ω_j.

Equations (6.15) and (6.21) define an imaging method for the two parameter problem because

$$\begin{cases} \gamma_\rho = \gamma_0 - \gamma_{-1}, \text{ and} \\ \gamma_n = 2\gamma_0 - \gamma_{-1} \,. \end{cases} \tag{6.22}$$

Of course, in practice the integrals in (6.15) and (6.21) will be approximated by means of a numerical quadrature method. If the quadrature formula is

$$\int_{S^2} F(\eta)\, d\eta = \sum_{k=1}^{M} F(\eta_k)\, \Delta\eta_k \tag{6.23}$$

where $\eta_1, \eta_2, \cdots, \eta_M \in S^2$ and $\Delta\eta_k$ are suitable weights then the algorithm for computing γ_{-1} and γ_0 is

$$\gamma_{-1}(x) = \frac{4}{\pi} \sum_{k=1}^{M} e_\delta(2x \cdot \eta_k, \eta_k, -\eta_k)\, \Delta\eta_k \tag{6.24}$$

and

$$\gamma_0(x) = \frac{\sqrt{2}}{\pi} \sum_{k=1}^{M} e_\delta(\sqrt{2}\, x \cdot \eta_k, \theta(\eta_k), \theta_0(\eta_k))\, \Delta\eta_k \, . \tag{6.25}$$

References

1. Brekhovskikh, L. M., Waves in Layered Media, Academic Press, 1960.

2. Chernov, L. A., Wave Propagation in a Random Medium, New York, McGraw-Hill, 1969.

3. Friedlander, F. G., Sound Pulses, Cambridge, Cambridge Univ. Press, 1958.

4. Kato, T., Perturbation theory of Linear Operators, New York, Springer, 1966.

5. Ludwig, D., The Radon transform on Euclidean space, Comm. Pure Appl. Math. 19, 49-82 (1966).

6. Metherell, A., Ed., Acoustic Imaging, Vol. 8, New York, Plenum Press, 1978.

7. Morse, P. M., and K. U. Ingard, Theoretical Acoustics, New York, McGraw-Hill, 1968.

8. Mueller, R. K., M. Kaveh and G. Wade, Reconstructive tomography and applications to ultrasonics, Proc. IEEE, V. 67, 567-587 (1979).

9. Norton, S. J., and M. Linzer, Ultrasonic reflectivity imaging in three dimensions: exact inverse scattering solutions for plane, cylindrical and spherical apertures, IEEE Trans. on Biomedical Engr., V. BME-28, 202-220 (1981).

10. Radon, J., Über die Bestimmung von Funktionen durch ihre Integralwerte längs gewisser Mannigfaltigkeiten, Berichte Sächsische Akademie der Wissenschaften, 69, 262-277 (1917).

11. Rose, J. H., and J. M. Richardson, Time domain Born approximation (preprint 1982, submitted to J. Nondestructive Eval.).

12. Wilcox, C. H., Initial-boundary value problems for linear hyperbolic partial differential equations of the second order, Arch. Rational Mech. Anal., 10, 361-400 (1962).

13. Wilcox, C. H., Regularity theorems for solutions of initial-boundary value problems for hyperbolic equations of the second order, Notices AMS, (January 1965).

14. Wilcox, C. H., Scattering Theory for the d'Alembert Equation in Exterior Domains, Springer Lecture Notes in Mathematics, V. 442 (1975).

15. Wilcox, C. H., Spectral and asymptotic analysis of acoustic wave propagation, in Boundary Value Problems for Linear Evolution Partial Differential Equations, H. G. Garnir, Ed., Dordrecht, D. Reidel Publ. Co., 1977.

References (cont.)

16. Wilcox, C. H., Sonar echo analysis, Math. Meth. in the Applied Sci., V. 1, 70-88 (1978).

17. Wilcox, C. H., Sound Propagation in Stratified Fluids, monograph 1982.

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER 44	2. GOVT ACCESSION NO. A125 584
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle) Multiparameter Acoustic Imaging in the Born Approximation	5. TYPE OF REPORT & PERIOD COVERED Technical Summary Rept.
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Calvin H. Wilcox	8. CONTRACT OR GRANT NUMBER(s) N00014-76-C-0276
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Mathematics University of Utah Salt Lake City, UT 84112	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS NR-041-370
11. CONTROLLING OFFICE NAME AND ADDRESS Office of Naval Research, Code 411 MA Arlington, VA 22217	12. REPORT DATE
	13. NUMBER OF PAGES
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report) Unclassified
	15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release, distribution unlimited.

17. DISTRIBUTION ST ENT (of · abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY TES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Acoustic imaging
Ultrasound
Born approximation

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Abstract.

Samples of biological tissue are modelled as inhomogeneous fluids with density $\rho(x)$ and sound speed $c(x)$ at point x. The samples are contained in the sphere $|x| \leq \delta$ and it is assumed that $\rho(x) \equiv \rho_0 = 1$ and $c(x) \equiv c_0 = 1$ for $|x| \geq \delta$, and $|\gamma_n(x)| << 1$, $|\gamma_\rho(x)| << 1$ and $|\nabla\gamma_\rho(x)| << 1$ where $\gamma_\rho(x) = \rho(x) - 1$ and $\gamma_n(x) = c^{-2}(x) - 1$. The samples are insonified by plane pulses $s(x \cdot \theta_0 - t)$ where $|\theta_0| = 1$ and the scattered pulse is shown to have the form $|x|^{-1}\, e_s(|x| - t, \theta, \theta_0)$ in the far field, where $x = |x|\theta$. The response $e_s(\tau,\theta,\theta_0)$ is measurable. The goal of the work is to construct the sample parameters γ_n and γ_ρ from $e_s(\tau,\theta,\theta_0)$ for suitable choices of s, θ and θ_0.

In the limiting case of constant density: $\gamma_\rho(x) \equiv 0$ it is shown that

$$\gamma_n(x) = \frac{4}{\pi} \int_{S^2} e_\delta(2x \cdot \theta, \theta, -\theta)\, d\theta$$

where δ represents the Dirac δ and S^2 is the unit sphere $|\theta| = 1$. Analogous formulas, based on two sets of measurements, are derived for the case of variable $c(x)$ and $\rho(x)$.

END

DATE
FILMED

4-83

DTIC