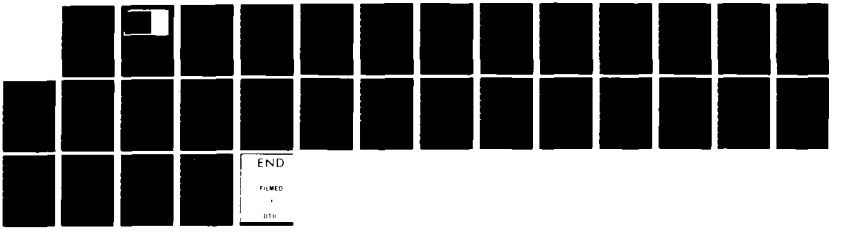
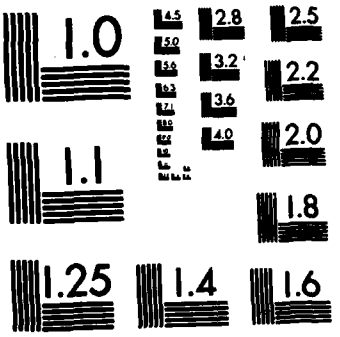
AD-A125 247 THERMOELASTIC REPPONSE TO A SHORT LASER PULSE(U) 1/1
WISCONSIN UNIV-MADISON MATHEMATICS RESEARCH CENTER
J C STRIKWERDA ET AL. NOV 82 MRC-TSR-2451
UNCLASSIFIED DAAG29-80-C-0041 F/G 20/5 NL





MRC Technical Summary Report #2451

THERMOELASTIC RESPONSE TO A SHORT LASER PULSE

John C. Strikwerda
and Andrew M. Scott

Mathematics Research Center
University of Wisconsin—Madison
610 Walnut Street
Madison, Wisconsin 53706

November 1982

(Received August 9, 1982)

Approved for public release
Distribution unlimited

Sponsored by

U. S. Army Research Office
P. O. Box 12211
Research Triangle Park
North Carolina 27709

UNIVERSITY OF WISCONSIN-MADISON
MATHEMATICS RESEARCH CENTER

THERMOELASTIC RESPONSE TO A SHORT LASER PULSE

John C. Strikwerda and Andrew M. Scott*

Technical Summary Report #2451
November 1982

ABSTRACT

The authors

We consider a one-dimensional model of the thermoelastic response to laser heating by a pulse of very short duration. We treat the linear equations of thermoelasticity by perturbation methods using the pulse duration as the perturbation parameter. The perturbation analysis separates the problem into two time regimes, the short time scale, which is of the same order as the pulse duration, and the long time scale, which is much longer than the pulse duration. Convenient analytical expressions are obtained for the temperature change and displacement in both time regimes.

AMS (MOS) Subject Classifications: 35Q20, 73C25

Key Words: Laser, Thermoelasticity, Perturbation Analysis

Work Unit Number 2 (Physical Mathematics)

A

*Computer Sciences Department, University of Wisconsin-Madison, Madison, Wisconsin 53706

Sponsored by the United States Army under Contract No. DAAG29-80-C-0041.

SIGNIFICANCE AND EXPLANATION

In this paper we study the effects caused by heating a material with a single short laser pulse. We consider only a one-dimensional model but consider both temperature changes and elastic changes, which are coupled together. By use of perturbation analysis we can separate the problem into two time regimes, the short time scale, which is on the order of the pulse duration, and the long time scale, which is much longer than the pulse duration. Convenient expressions are given for evaluating the temperature changes and displacement.

THERMOELASTIC RESPONSE TO A SHORT LASER PULSE

John C. Strikwerda and Andrew M. Scott*

1. INTRODUCTION.

In this paper we study the temperature change and displacements induced in a solid material by a short laser pulse. The determination of the thermoelastic response of solids to laser radiation is an important problem for many applications, especially as laser pulses become increasingly powerful and of shorter duration.

We apply perturbation techniques to the equations of linear thermoelasticity using the pulse duration as the perturbation parameter. This approach has the advantage of yielding relatively simple expressions for the temperature change and displacement induced by a laser pulse of short duration.

In other work on this problem Bechtel (1975) and Ready (1965) considered the heating caused by laser pulses but did not consider displacement effects. Dunbar (1981) considered the effects of temperature on displacements but neglected the coupling effect of displacements on temperature. He also included the modified Fourier law of heat conduction. (Dunbar's results are in error due to an inappropriate application of his initial conditions.) All of the above used a Green's function approach to obtain explicit, though complicated, formulae for the temperature field. The perturbation method used here has the advantage of giving simpler expressions for the temperature and displacements. Moreover, we treat the full equations of linear thermoelasticity in one-dimension with the modified Fourier law. The method is limited by the requirement that the pulse duration be short. The method can be extended to two and three dimensional problems and to include nonlinear effects, however, the resulting equations will then have to be solved by numerical methods.

*Computer Sciences Department, University of Wisconsin-Madison, Madison, Wisconsin 53706

Sponsored by the United States Army under Contract No. DAAG29-80-C-0041.

Nayfeh (1977) has treated a problem similar to the one considered in this paper by the method of perturbation expansions. He considered an instantaneous heat source distributed on the surface of the material. We compare our results with Nayfeh's in Section 5.

The equations describing the thermoelastic response in one dimension to laser heating are

$$(1.1)\qquad \left(1 + \tau_0 \frac{\partial}{\partial \bar{t}}\right)\left(\rho c_v \frac{\partial \bar{T}}{\partial \bar{t}} + (3\lambda + 2\mu)\gamma \bar{T}_0 \frac{\partial^2 \bar{u}}{\partial \bar{x}\partial \bar{t}}\right) - K \frac{\partial^2 \bar{T}}{\partial \bar{x}^2} = \left(1 + \tau_0 \frac{\partial}{\partial \bar{t}}\right)\bar{A}(\bar{t}/b)e^{-a\bar{x}}$$

$$(1.2)\qquad \rho \frac{\partial^2 \bar{u}}{\partial \bar{t}^2} - (\lambda + 2\mu) \frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + (3\lambda + 2\mu)\gamma \frac{\partial \bar{T}}{\partial \bar{x}} = 0$$

where $\bar{T}$ is the temperature change measured from the mean state $\bar{T}_0$ and $\bar{u}$ is the displacement. The solid material occupies the region with $\bar{x} \geq 0$. The normal stress is given by

$$(1.3)\qquad \bar{\Sigma} = (\lambda + 2\mu) \frac{\partial \bar{u}}{\partial \bar{x}} - (3\lambda + 2\mu)\gamma \bar{T} .$$

These equations are those derived by Lord and Shulman (1967) with the addition of the term involving $\bar{A}$ which represents the laser heating. (We have ignored the speed of propagation of the laser pulse which is sufficiently fast to be regarded as infinite.) The parameter b is a measure of the pulse width. The exponential damping of the laser heating in space is the result of the attenuation of electromagnetic radiation in a conductor. The parameter τ_0 is the relaxation time for the modified Fourier law for which the heat flux Q satisfies

$$(1.4)\qquad Q + \tau_0 \frac{\partial Q}{\partial \bar{t}} = -K \operatorname{grad} \bar{T}$$

where K is the thermal conductivity of the material. The other parameters in equations (1.1), (1.2) and (1.3) are the Lamé elastic constants λ and μ, the density ρ, the specific heat at constant volume c_v, and the coefficient of thermal expansion γ. The bars on the several variables indicate that these are dimensional quantities.

We now non-dimensionalize the equations. The natural length scale for this problem is the attenuation length α^{-1}, so define $x = \bar{x}/x_{ref}$ where x_{ref} is α^{-1}. There are several possible time scales and we choose the time scale of heat conduction as the reference time,

$$t_{ref} = \rho c_v x_{ref}^2/K .$$

The perturbation parameter is to be β which is b/t_{ref}, the non-dimensional pulse width. The relaxation time is non-dimensionalized as

$$\tau = \tau_0/(t_{ref}\beta) = \tau_0/b .$$

The pulse magnitude is normalized to have a non-dimensional integral equal to unity. Therefore we define A_{ref} by

$$\int_{-\infty}^{\infty} \bar{A}(\bar{t}/b)d\bar{t} = A_{ref}t_{ref}$$

then

$$\bar{A}(\bar{t}/b) = A_{ref}\frac{1}{\beta}A(t/\beta)$$

and thus

$$\int_{-\infty}^{\infty} A(s)ds = 1 . \tag{1.5}$$

The reference temperature and displacement are chosen as

$$T_{ref} = A_{ref}(x_{ref})^2/K$$

and

$$u_{ref} = \beta^2 T_{ref}\gamma(3\lambda + 2\mu)t_{ref}^2/(\rho x_{ref}) .$$

The natural reference velocity is

$$vel_{ref} = x_{ref}/(\beta t_{ref}) .$$

We thus obtain for the non-dimensionalized equations

$$\left(1 + \beta\tau\frac{\partial}{\partial t}\right)\left(\frac{\partial T}{\partial t} + d^2\frac{\partial^2 u}{\partial x \partial t}\right) - \frac{\partial^2 T}{\partial x^2} = \left(1 + \beta\tau\frac{\partial}{\partial t}\right)\frac{1}{\beta}A(t/\beta)e^{-x} \tag{1.6}$$

$$\beta^2\frac{\partial^2 u}{\partial t^2} - c^2\frac{\partial^2 u}{\partial x^2} + \frac{\partial T}{\partial x} = 0 \tag{1.7}$$

where

$$c = \left(\frac{\lambda + 2\mu}{\rho}\right)^{1/2} / \text{vel}_{\text{ref}}$$

and

$$d = \frac{(3\lambda + 2\mu)\gamma}{\rho} \left(\frac{\bar{T}_0}{c_v}\right)^{1/2} / \text{vel}_{\text{ref}} .$$

The non-dimensional stress is

$$\Sigma = \frac{\partial u}{\partial x} - \frac{1}{c^2} T . \tag{1.9}$$

For many common materials the non-dimensional parameters c and d are of order zero in β, that is,

$$\beta << c, \; d << \beta^{-1} .$$

The relaxation parameter τ plays an unimportant role in the analysis and satisfies

$$0 \leq \beta\tau << 1 .$$

The justification of our treatment of these equations and the non-dimensionalization we have used rests on the above observations and on the reasonableness of the results.

The boundary conditions for equations (1.6) and (1.7) are the conditions of no radiative heat loss

$$\frac{\partial T}{\partial x} = 0 \quad \text{at} \quad x = 0 , \tag{1.10}$$

and no stress at the wall

$$\frac{\partial u}{\partial x} - \frac{1}{c^2} T = 0 \quad \text{at} \quad x = 0 . \tag{1.11}$$

The initial conditions are that $T(t,x)$ and $u(t,x)$ and all their derivatives vanish as t approaches negative infinity.

We will solve equations (1.6)-(1.11) using perturbation expansions with β as the perturbation parameter. Since β is on the order of 10^{-10} for many problems, excellent results can be obtained with only one term.

The paper is organized in the following manner. In section 2 we derive the perturbation expansion for the short time scale, that is times on the order of the pulse width, and in section 3 we consider the long time scale behavior. In section 4 we solve for the boundary layer correction at the material surface which is needed to complete the

solution considered in section 2. In section 5 we consider some explicit examples and compare our results with those of Bechtel (1975), Ready (1965), and Nayfeh (1977).

2. THE SOLUTION ON THE SHORT TIME SCALE

In this section we consider the solution of equations (1.6) and (1.7) for times on the order of β, the pulse width. We rescale the time variable using

$$t_1 = t/\beta \tag{2.1}$$

obtaining the equations

$$\left(\tau \frac{\partial}{\partial t_1} + 1\right)\left(\frac{\partial T_1}{\partial t_1} - A(t_1)e^{-x} + d^2 \frac{\partial^2 u_1}{\partial t_1 \partial x}\right) = \beta \frac{\partial^2 T_1}{\partial x^2} \tag{2.2}$$

$$\frac{\partial^2 u_1}{\partial t_1^2} - c^2 \frac{\partial^2 u_1}{\partial x^2} + \frac{\partial T_1}{\partial x} = 0 \tag{2.3}$$

with

$$\frac{\partial T_1}{\partial x} = 0 \quad \text{and} \quad \frac{\partial u_1}{\partial x} - c^{-2} T_1 = 0 \quad \text{at} \quad x = 0 \tag{2.4}$$

and

$$\frac{\partial^k T_1}{\partial t^k}, \frac{\partial^k u_1}{\partial t^k} \to 0 \quad \text{as} \quad t \to -\infty \quad \text{for} \quad k \geq 0 . \tag{2.5}$$

We seek solutions to these equations in the form of power series in the perturbation parameter β i.e.

$$T = T_1^0(t_1,x) + \beta T_1^1(t_1,x) + \beta^2 T_1^2(t_1,x) + \ldots$$

$$u = u_1^0(t_1,x) + \beta u_1^1(t_1,x) + \beta^2 u_1^2(t_1,x) + \ldots .$$

Substituting these expansions into equations (2.2) and (2.3) and equating like powers of β gives the following system of equations

$$\left(\tau \frac{\partial}{\partial t_1} + 1\right)\left(\frac{\partial T_1^0}{\partial t_1} - A(t_1)e^{-x} + d^2 \frac{\partial^2 u_1^0}{\partial x \partial t_1}\right) = 0 \tag{2.6}$$

$$\frac{\partial^2 u_1^k}{\partial t_1^2} - c^2 \frac{\partial^2 u_1^k}{\partial x^2} + \frac{\partial T_1^k}{\partial x} = 0, \quad \text{for} \quad k \geq 0 \tag{2.7}$$

and

$$(2.8)\qquad \left(\tau \frac{\partial}{\partial t_1} + 1\right)\left(\frac{\partial T_1^k}{\partial t_1} + d^2 \frac{\partial^2 u_1^k}{\partial x \partial t_1}\right) = \frac{\partial^2 T_1^{k-1}}{\partial x^2}, \quad k > 0 .$$

Equation (2.6) can be integrated to

$$\frac{\partial T_1^0}{\partial t_1} - A(t_1)e^{-x} + d^2 \frac{\partial^2 u_1^0}{\partial t_1 \partial x} = C(x)e^{-t_1/\tau}$$

for some function $C(x)$ The initial conditions (2.6) imply that C is identically zero. Another integration in t and using (2.6) again gives

$$(2.9)\qquad T_1^0(t_1,x) = e^{-x}\int_{-\infty}^{t_1} A(s)ds - d^2 \frac{\partial u_1^0}{\partial x}(t_1,x) .$$

Differentiating (2.9) with respect to x and substituting in (2.7) for $k = 0$ gives

$$(2.10)\qquad \frac{\partial^2 u_1^0}{\partial t_1^2} - (c^2 + d^2)\frac{\partial^2 u_1^0}{\partial x^2} = e^{-x}\int_{-\infty}^{t_1} A(s)ds$$

with

$$(2.11)\qquad \frac{\partial u_1^0}{\partial x} = \frac{1}{(c^2 + d^2)}\int_{-\infty}^{t_1} A(s)ds \quad \text{at} \quad x = 0 .$$

The solution to (2.10) and (2.11) satisfying the initial condition (2.5) is

$$(2.12)\qquad u_1^0(t_1,x) = e^{-x}\int_{-\infty}^{t_1} a^{-2}(\cosh a(t_1 - s) - 1)\, A(s)ds$$

$$- \int_{-\infty}^{t_1} a^{-2}\sinh a(t_1 - s)\, A(s - x/a)ds$$

where $a = (c^2 + d^2)^{1/2}$. From (2.9) we obtain

$$(2.13)\qquad T_1^0(t_1,x) = c^2/a^2 e^{-x}\int_{-\infty}^{t_1} A(s)ds + {}^2/a^2 \int_{-\infty}^{t_1} \cos^{\text{-}} {}_1(t_1-s)(e^{-x}A(s) - A(s-x/a))ds .$$

Formulas (2.12) and (2.13) can be rewritten as

$$u_1^0(t_1,x) = -a^{-2}e^{-x}\int_{-\infty}^{t_1} A(s)ds + a^{-2}\cosh(x)e^{-at_1}\int_{-\infty}^{t_1-x/a} e^{as}A(s)ds \tag{2.12'}$$

$$+ a^{-2}e^{-x}\int_{t_1-x/a}^{t_1} \cosh a(t_1 - s)\, A(s)ds$$

and

$$T_1^0(t_1,x) = c^2/a^2e^{-x}\int_{-\infty}^{t_1} A(s)ds - d^2/a^2 \sinh(x)e^{-at_1}\int_{-\infty}^{t_1-x/a} e^{as}A(s)ds \tag{2.13'}$$

$$+ d^2/a^2e^{-x}\int_{t_1-x/a}^{t_1} \cosh a(t_1 - s)\, A(s)ds\ .$$

Since $A(t_1) \to 0$ as $t \to +\infty$ we obtain from (2.12') and (2.13')

$$\lim_{t_1\to+\infty} u_1^0(t_1,x) = -a^{-2}e^{-x}\int_{-\infty}^{\infty} A(s)ds = -a^{-2}e^{-x} \tag{2.14}$$

by (1.5) and

$$\lim_{t_1\to+\infty} T_1^0(t_1,x) = c^2/a^2e^{-x} = \frac{c^2}{c^2 + d^2}\, e^{-x}\ . \tag{2.15}$$

We also obtain from the last two terms in (2.12') and (2.13') the limiting form of the compression wave induced by the laser heating. We have for large t_1 and x

$$u_1^0(t_1,x) \sim \frac{1}{2a^2}\, f(t_1 - x/a) \tag{2.16}$$

and

$$T_1^0(t_1,x) \sim \frac{d^2}{2a^2}\, f'(t_1 - x/a) \tag{2.17}$$

where

$$f(r) = e^{-ar} \int_{-\infty}^{r} e^{as} A(s)ds + e^{ar} \int_{r}^{\infty} e^{-as} A(s)ds ,$$

$$= \int_{-\infty}^{\infty} e^{-a|r-s|} A(s)ds .$$

Note that the temperature change associated with the compression wave is first positive then negative as the wave passes any point.

The functions T_1^k do not satisfy the boundary condition (1.10). A boundary-layer expansion is needed to satisfy this boundary condition and this expansion is the topic of section 4.

3. THE SOLUTION ON THE LONG-TIME SCALE

In this section we consider the thermoelastic response on the long time scale, that is for times much larger than the laser pulse width. The equations to be solved are (1.6) and (1.7) which we rewrite as

$$\frac{\partial T}{\partial t} - \frac{\partial^2 T}{\partial x^2} + d^2 \frac{\partial^2 u}{\partial x \partial t} + \beta\tau \left(\frac{\partial^2 T}{\partial t^2} + d^2 \frac{\partial^3 u}{\partial x \partial t^2}\right) = \frac{1}{\beta} \left(A(t/\beta) + \tau A'(t/\beta)\right) e^{-x} , \tag{3.1}$$

$$- c^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial T}{\partial x} + \beta^2 \frac{\partial^2 u}{\partial t^2} = 0 , \tag{3.2}$$

with the initial conditions that T and u vanish as t approaches negative infinity, and the boundary conditions (1.10) and (1.11). We will obtain the solution as a perturbation expansion in the variable β. However, since the right-hand side of (3.1) has a singularity at $\beta = 0$, the terms of the perturbation expansion will be distributions and we must consider the weak form of (3.1) with regard to derivatives in t.

First, let us consider equation (3.2). Writing

$$T(t,x) \sim \sum_{j=0}^{\infty} \beta^j T^j(t,x)$$

and

$$u(t,x) \sim \sum_{j=0}^{\infty} \beta^j u^j(t,x)$$

we obtain

$$- c^2 \frac{\partial^2 u^j}{\partial x^2} + \frac{\partial T^j}{\partial x} = - \frac{\partial^2 u^{j-2}}{\partial t^2} \qquad j = 0,1,\ldots, \tag{3.3}$$

where $u^{-2} = u^{-1} = 0$.

When j is 0 and 1, we have, by integrating,

$$- c^2 \frac{\partial u^0}{\partial x} + T^0 = 0$$

$$\text{and} \qquad - c^2 \frac{\partial u^1}{\partial x} + T^1 = 0 \tag{3.4}$$

which satisfy both the no stress boundary condition (1.11) at $x = 0$, and the decay of u and T as $x \to \infty$. Thus to within $O(\beta^2)$ there is no stress on the long time scale.

The weak form of (3.1) which we consider is

$$\int_{-\infty}^{\infty} \left(-\phi'(t)T - \phi(t)\frac{\partial^2 T}{\partial x^2} - d^2\phi'(t)\frac{\partial u}{\partial x} + \beta\tau\phi''(t)\left(T + d^2\frac{\partial u}{\partial x}\right)\right)dt$$

(3.5)

$$= \frac{1}{\beta}\int_{-\infty}^{\infty} (\phi(t) - \beta\tau\phi'(t))A(t/\beta)dt\, e^{-x}$$

for all $\phi \in C_0^\infty(\mathbb{R})$. To continue we must expand the right hand side of (3.5) as a series in β. Making the change of variable $t = \beta s$ we have

$$\frac{1}{\beta}\int_{-\infty}^{\infty} (\phi(t) - \beta\tau\phi'(t))A(t/\beta)dt$$

$$= \int_{-\infty}^{\infty} (\phi(\beta s) - \beta\tau\phi'(\beta s))A(s)ds\ .$$

Now for any positive integer M we will obtain an expansion of the above integral with error $O(\beta^M)$. First choose R so that

$$\left|\int_{|s|>R} s^k A(s)ds\right| \le \beta^{M-k} \quad \text{for } k = 0,\ldots,M\ .$$

Then, by the Taylor series expansion of $\phi(\beta s)$,

$$\int_{-\infty}^{\infty} \phi(\beta s)A(s)ds = \int_{-R}^{R} \phi(\beta s)A(s)ds + \int_{|s|>R} \phi(\beta s)A(s)ds$$

$$= \sum_{j=0}^{M-1} \int_{-R}^{R} \frac{1}{j!}\phi^{(j)}(0)(\beta s)^j A(s)ds$$

$$+ \beta^M \int_{-R}^{R} \frac{A(s)}{(M-1)!}\int_0^s \phi^{(M)}(\beta s')(s - s')^{M-1}ds'ds + \int_{|s|>R} \phi(\beta s)A(s)ds$$

$$= \sum_{j=0}^{M-1} \beta^j \frac{\phi^{(j)}(0)}{j!}\int_{-\infty}^{\infty} s^j A(s)ds + O(\beta^M)\|\phi\|_{\infty,M}$$

where $\|\phi\|_{\infty,M} = \sup_{s} \sup_{0\le j\le M} |\phi^{(j)}(s)|$. For convenience define

$$a_j = \frac{1}{j!} \int_{-\infty}^{\infty} s^j A(s)\,ds \qquad j = 0,1,\ldots \tag{3.6}$$

with $a_{-1} = 0$. By the normalization of $A(t)$ we have that a_0 is 1.

Therefore,

$$\frac{1}{\beta} \int_{-\infty}^{\infty} \phi(t) A\left(\frac{t}{\beta}\right) dt = \sum_{j=0}^{M-1} \beta^j a_j \phi^{(j)}(0) + O(\beta^M)\|\phi\|_{\infty,M}$$

and similarly

$$\int_{-\infty}^{\infty} \phi'(t) A(t/\beta)\,dt = \sum_{j=1}^{M-1} \beta^j a_{j-1} \phi^{(j)}(0) + O(\beta^M)\|\phi\|_{\infty,M}$$

Thus the right-hand side of (3.5) is

$$\sum_{j=0}^{M-1} \beta^j (a_j - \tau a_{j-1}) \phi^{(j)}(0) + O(\beta^M)\|\phi\|_{\infty,M} .$$

Now expanding the left-hand side of (3.5) in powers of β we have for $j = 0$

$$\int_{-\infty}^{\infty} -\phi' T^0 - \phi \frac{\partial^2 T^0}{\partial x^2} - d^2 \phi' \frac{\partial u^0}{\partial x}\,dt = \phi(0) e^{-x}$$

and using (3.4) we obtain

$$\int_{-\infty}^{\infty} -\phi'\left(1 + \frac{d^2}{c^2}\right) T^0 - \phi \frac{\partial^2 T^0}{\partial x^2}\,dt = \phi(0) e^{-x}$$

Since the right-hand side of this equation depends on $\phi(t)$ only at $t = 0$ we have that T^0 is differentiable for all t other than $t = 0$. Integrating by parts then gives us

$$\int_{-\infty}^{\infty} \phi(t)\left\{\left(1 + \frac{d^2}{c^2}\right) \frac{\partial T^0}{\partial t} - \frac{\partial^2 T^0}{\partial x^2}\right\} dt + \phi(0)[T^0(0+,x) - T^0(0-,x)] = \phi(0) e^{-x} .$$

Thus T^0 is a solution to the heat equation for $t > 0$ and $t < 0$, with the homogeneous boundary condition (1.10). Since T^0 vanishes as $t \to -\infty$ the uniqueness of the initial boundary value problem implies that T^0 is identically zero for $t < 0$. Therefore for $t > 0$, T^0 satisfies

(3.7) $$\frac{\partial T^0}{\partial t} - k\frac{\partial^2 T^0}{\partial x^2} = 0$$

with $T^0(0,x) = ke^{-x}$ and $\frac{\partial T^0}{\partial x}(t,0) = 0$ where $k = c^2/(c^2 + d^2)$.

Note that the initial condition for T^0 is the same as the limit of T_1^0 as t_1 becomes infinite for the short scale (2.15). Similarly for u^0, from (3.4) and the initial value for T^0 we have

(3.8) $$u^0(0,x) = -\frac{1}{c^2 + d^2}e^{-x},$$

which agrees with (2.14). The explicit solution to (3.7) is

(3.9) $$T^0(t,x) = \sqrt{\frac{k}{4\pi t}}\int_0^\infty (e^{-(x-y)^2/4kt} + e^{-(x+y)^2/4kt})e^{-y}dy$$

$$= \frac{k}{2}e^{kt}(e^{-x}(1 - \operatorname{erf}(\sqrt{kt} - x/\sqrt{4kt})) + e^{x}(1 - \operatorname{erf}(\sqrt{kt} + x/\sqrt{4kt}))),$$

and so

(3.10) $$u^0(t,x) = \frac{1}{c^2 + d^2}\left(-1 + \frac{1}{2}\int_0^\infty (\operatorname{erf}((x - y)/\sqrt{4kt}) + \operatorname{erf}((x + y)/\sqrt{4kt}))e^{-y}dy\right).$$

From (3.4) it is seen that u^0 is determined only up to a function of t. In deriving (3.10) we have used the condition that u^0 vanishes as x becomes larger. From (3.10) we see that

$$u^0(t,x) \to -1/(c^2 + d^2) \quad \text{as} \quad t \to \infty.$$

This non-zero limit for the displacement is the result of neglecting any restraining forces on the material which would restore it to its original position. The equation for T^1, using (3.4), is

$$\int_{-\infty}^{\infty} (-\phi' T^1 - \phi k\frac{\partial^2 T^1}{\partial x^2} + \phi'' \tau T^0)dt = \phi'(0)k(a_1 - \tau a_0)e^{-x}.$$

As before T^1 is zero for $t < 0$. $T^1(t,x)$ can be written as the sum of a distribution and a smooth function $\tilde{T}^1(t,x)$. We have

$$T^1(t,x) = -ka_1\delta(t)e^{-x} + \tilde{T}^1(t,x)$$

where $\delta(t)$ is the usual Dirac delta function and $\tilde{T}^1(t,x)$ satisfies

$$\frac{\partial \tilde{T}^1}{\partial t} - k\frac{\partial^2 \tilde{T}^1}{\partial x^2} + \tau\frac{\partial^2 T^0}{\partial t^2} = 0$$

with $\tilde{T}^1(0,x) = -k(a_1 k + \tau)e^{-x}$ and $\frac{\partial \tilde{T}^1}{\partial x}(t,0) = 0$. The equation for $u^1(t,x)$ from (3.4) is

$$u^1(t,x) = \frac{1}{c^2 + d^2} ka_1 \delta(t)e^{-x} - \frac{1}{c^2}\int_x^\infty \tilde{T}^1(t,y)dy \ .$$

In a similar way one may derive expressions for T^2 and u^2, however, for most applications the first two terms should suffice.

4. THE BOUNDARY CORRECTION ON THE SHORT TIME SCALE.

In this section we calculate the correction terms which are needed to satisfy the radiation boundary condition (1.10) for the short time scale. As in section 2 the time variable is t_1, i.e. t/β, and we write the temperature function as

$$T(t_1,x,\beta) = T_1(t_1,x,\beta) + \beta^{1/2}\,\hat{T}_1(t_1,x\beta^{-1/2},\beta)\ . \tag{4.1}$$

Setting $x_1 = x\beta^{-1/2}$, we have by (1.10)

$$\frac{\partial T_1}{\partial x}(t_1,0,\beta) + \frac{\partial \hat{T}_1}{\partial x_1}(t_1,0,\beta) = 0\ . \tag{4.2}$$

$\hat{T}_1$ satisfies the same initial conditions as T_1, that is $\hat{T}_1$ and all its derivatives vanish as t_1 tends to negative infinity.

To obtain meaningful equations for $\hat{T}_1$ and $\hat{u}_1$ we set

$$u(t_1,x,\beta) = u_1(t_1,x,\beta) + \beta\hat{u}_1(t_1,x_1,\beta) \tag{4.3}$$

and, from (2.2) and (2.3), the equations for $\hat{T}_1$ and $\hat{u}_1$ are

$$\tau\frac{\partial^2\hat{T}_1}{\partial t_1^2} + \frac{\partial\hat{T}_1}{\partial t_1} + d^2\frac{\partial^2\hat{u}_1}{\partial t_1\,\partial x_1} - \frac{\partial^2\hat{T}_1}{\partial x_1^2} = 0 \tag{4.4}$$

$$\beta\frac{\partial^2\hat{u}_1}{\partial t_1^2} - c^2\frac{\partial^2\hat{u}_1}{\partial x_1^2} + \frac{\partial\hat{T}_1}{\partial x_1} = 0\ . \tag{4.5}$$

The system (4.4) and (4.5) with the boundary condition (4.2) and the initial conditions determine $\hat{T}_1$ and $\hat{u}_1$. We again solve these equations using perturbation expansions in β. We write

$$\begin{aligned}\hat{T}_1(t_1,x_1,\beta) &\sim \sum_{k=0}^{\infty}\beta^k\hat{T}_1^k(t_1,x_1)\\ \hat{u}_1(t_1,x_1,\beta) &\sim \sum_{k=0}^{\infty}\beta^k\hat{u}_1^k(t_1,x_1)\ .\end{aligned} \tag{4.6}$$

The equations for $\hat{T}^0$ and $\hat{u}^0$ are

$$\tau \frac{\partial^2 \hat{T}_1^0}{\partial t_1^2} + \frac{\partial \hat{T}_1^0}{\partial t_1} + d^2 \frac{\partial^2 \hat{u}_1^0}{\partial t_1 \partial x_1} + \tau d^2 \frac{\partial^3 \hat{u}_1^0}{\partial t_1^2 \partial x_1} - \frac{\partial^2 \hat{T}_1^0}{\partial x_1^2} = 0$$

and

$$- c^2 \frac{\partial^2 \hat{u}_1^0}{\partial x_1^2} + \frac{\partial \hat{T}_1^0}{\partial x_1} = 0 \ .$$

Integrating this last equation, we have

$$- c^2 \frac{\partial \hat{u}_1^0}{\partial x_1} + \hat{T}_1^0 = 0 \ ,$$

which shows that there is no correction to the stress to the first order in β. We then have

$$\tau \frac{\partial^2 \hat{T}_1^0}{\partial t_1^2} + \frac{\partial \hat{T}_1^0}{\partial t_1} - k \frac{\partial^2 \hat{T}_1^0}{\partial x_1^2} = 0 \tag{4.7}$$

where, as in section 3, $k = c^2/(c^2 + d^2)$. The boundary condition for $\hat{T}_1^0$ is, from (4.2) and (2.13),

$$\frac{\partial \hat{T}_1^0}{\partial x_1}(t_1,0) = k \int_{-\infty}^{t_1} A(s) + (1 - k)e^{-at_1} \int_{-\infty}^{t_1} e^{as} A(s)ds = B^0(t_1) \ . \tag{4.8}$$

The solution to (4.7) and (4.8) is

$$\hat{T}_1^0(t_1,x_1) = -c_1 \int_{-\infty}^{t_1 - x_1/c_1} e^{-(t_1 - s)/2\tau} I_0\left(\frac{1}{2\tau}\left((t_1 - s)^2 - \left(\frac{x_1}{c_1}\right)^2\right)^{1/2}\right) B^0(s)ds \ .$$

where c_1 is $\sqrt{k/\tau}$ and I_0 is the modified Bessel function of order zero. The formula for the displacement correction is

$$\hat{u}_1^0(t_1,x_1) = \left(\frac{c_1}{c}\right)^2 \int_{-\infty}^{t_1 - x_1/c_1} e^{-(t_1 - s)/2\tau} U(t_1 - s, x_1/c) B^0(s)ds$$

where

$$U(\sigma,\xi) = \int_{\xi}^{\sigma} I_0\left(\frac{1}{2\tau}(\sigma^2 - \eta^2)^{1/2}\right)d\eta .$$

For the case of $\tau = 0$, these formulas are

$$\hat{T}_1^0(t_1,x_1) = -\int_{-\infty}^{t_1} \frac{e^{-x_1^2/4k(t_1 - s)}}{\sqrt{4k\pi(t_1 - s)}} B^0(s)ds$$

and

$$\hat{u}_1^0(t_1,x_1) = \frac{1}{2a^2}\int_{-\infty}^{t_1} (1 - \operatorname{erf}(x_1/\sqrt{4k(t_1 - s)}))B^0(s)ds .$$

Note that the effect of the correction is to decrease the temperature at the surface by $O(\beta^{1/2})$ and to decrease the magnitude of the displacement by $O(\beta)$. This is as expected since the correction accounts for the effects of radiation.

5. EXAMPLES AND CONCLUSIONS

We now present some explicit results of the preceding analysis. We consider two pulse shapes, the first pulse has the shape of a Gaussian

$$A(s) = \pi^{-1/2} e^{-s^2} \tag{5.1}$$

and the second has an exponential decay

$$A(s) = \begin{cases} 0 & s \leq 0 , \\ se^{-s} & s > 0 . \end{cases} \tag{5.2}$$

The analysis of sections 2 and 3 shows that one may take $A(s)$ to be any function with unit integral satisfying

1) $\int_{-\infty}^{\infty} s^k A(s)ds < \infty \quad k = 0,1,\ldots$

2) $e^{at} \int_{-\infty}^{t} e^{-as} A(s)ds \to 0 \quad \text{as} \quad t \to -\infty,$

where $a = (c^2 + d^2)^{1/2}$.

Graphs of T_1^0 and u_1^0, the short time scale temperature change and displacement, are shown in figures 1 to 5 for several values of x as functions of t_1. The values of T_1^0 and u_1^0 are computed using equations (2.12) and (2.13) for $x = 0,1,2,$ etc. The values of t_1 run up to 10.0 in each figure.

Figures 1a and 1b show the temperature change and displacement for the Gaussian pulse (5.1) for values of $c = 1.0$ and $d = 1.0$. Figures 2a and 2b show similar results for the exponential pulse (5.2) with $c = 1.0$ and $d = 1.0$. Comparing figures 1 and 2 one can see the effect on the temperature and displacement of the different rates of energy deposition due to the different pulse shapes. The exponential pulse (5.2) deposits energy more slowly than does the Gaussian pulse (5.1) and therefore the temperature and displacement approach their limiting values more slowly. Figures 3a and 3b show results for the Gaussian pulse (5.1) for $c = 0.7$ and $d = 1.0$. Figures 4a and 4b display the temperature change and

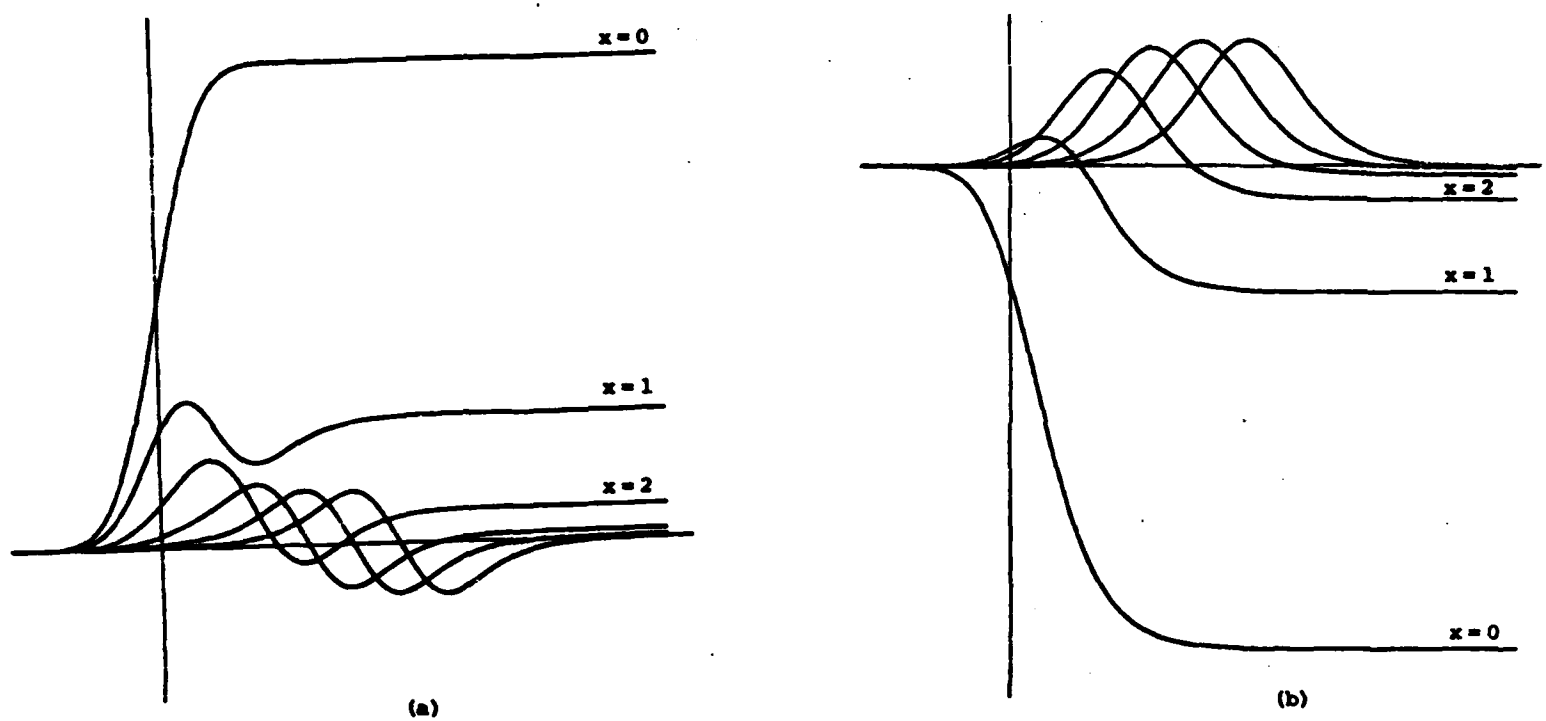


Figure 1. The temperature change (a) and displacement (b) on the short time scale for a Gaussian pulse for $x = 0(1)5$ and t_1 less than 10, $c = 1.0$ and $d = 1.0$.

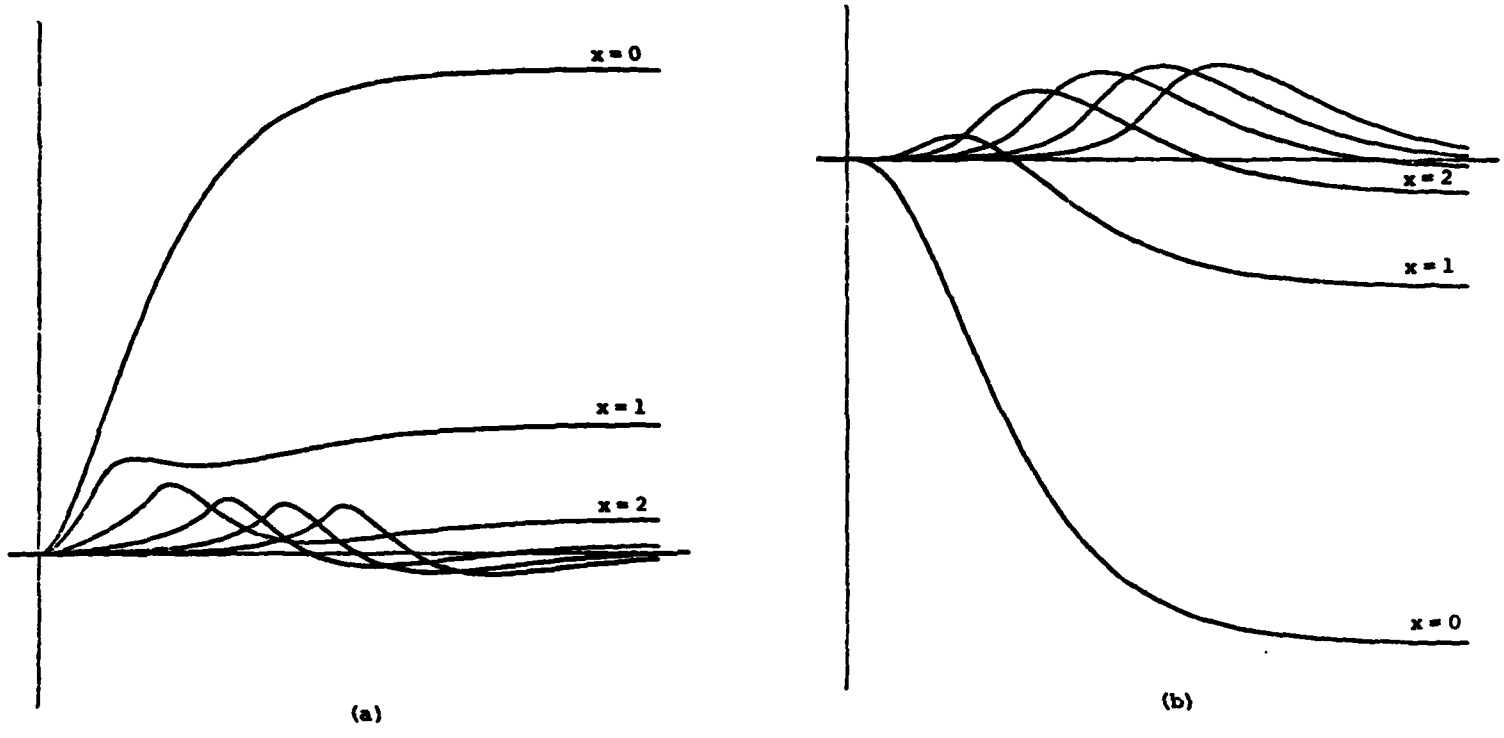


Figure 2. The temperature change (a) and displacement (b) on the short time scale for an exponential pulse for $x = 0(1)5$ and t_1 less than 10, $c = 1.0$ and $d = 1.0$.

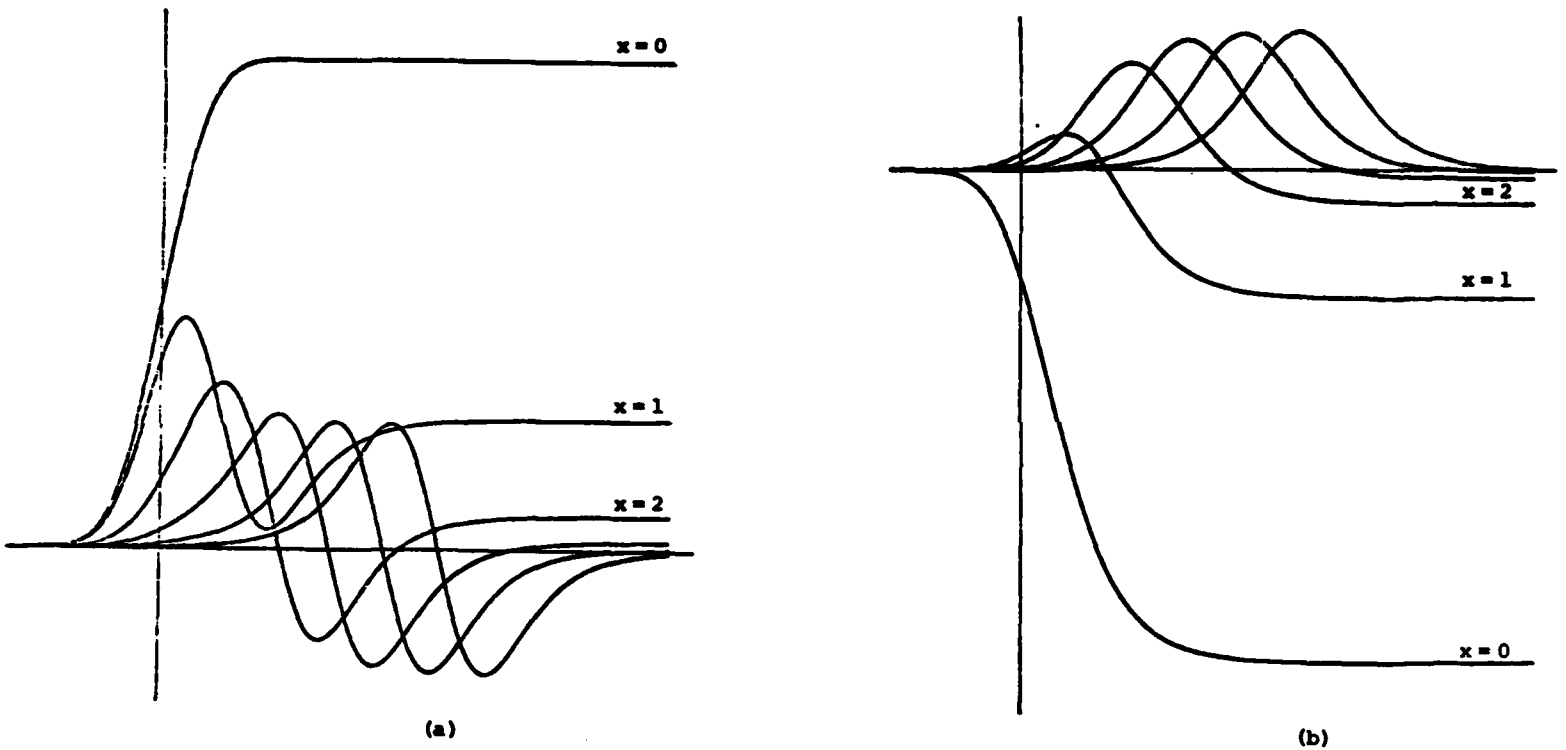


Figure 3. The temperature change (a) and displacement (b) on the short time scale for a Gaussian pulse for $x = 0(1)5$ and t_1 less than 10, $c = 0.7$ and $d = 1.0$.

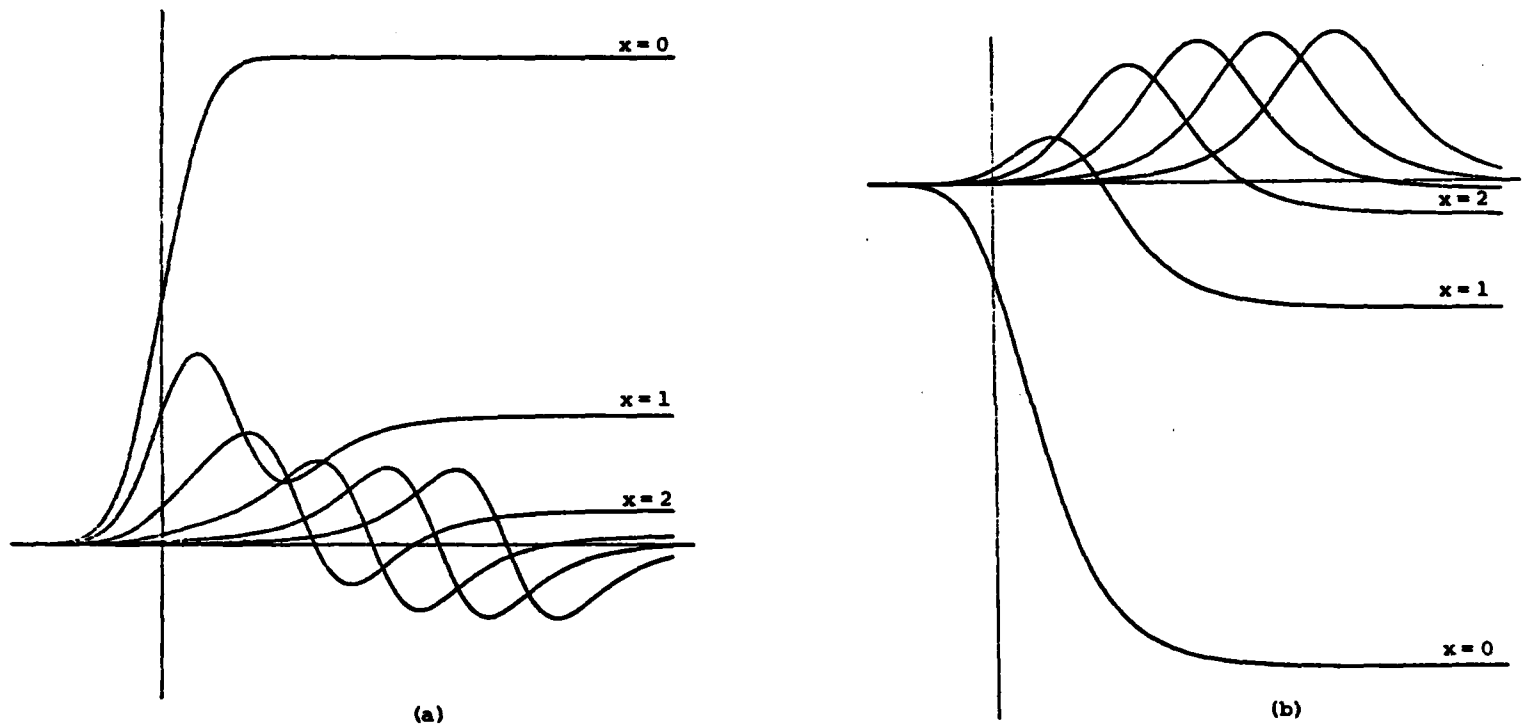


Figure 4. The temperature change (a) and displacement (b) on the short time scale for a Gaussian pulse for $x = 0(1)5$ and t_1 less than 10, $c = 0.7$ and $d = 0.7$.

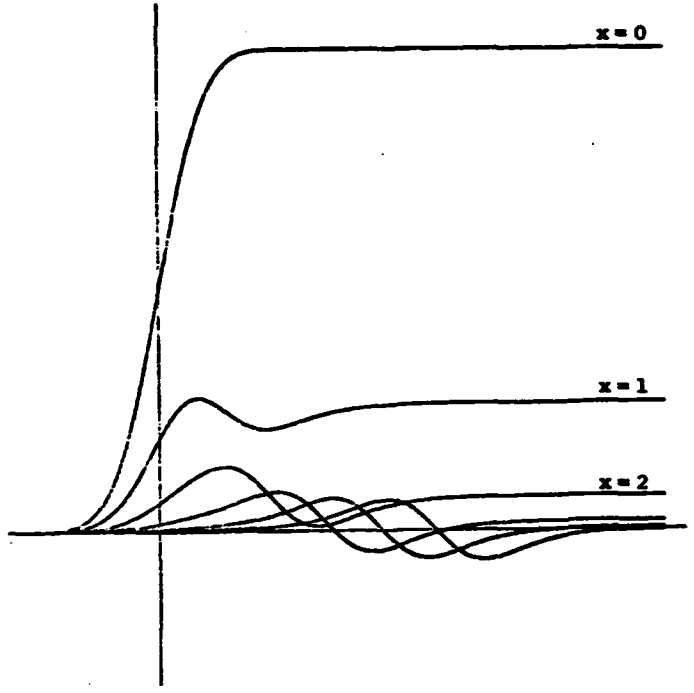


Figure 5. The temperature change on the short time scale for a Gaussian pulse for x = 0(1)5 and t_1 less than 10, c = 1.0 and d = 0.7.

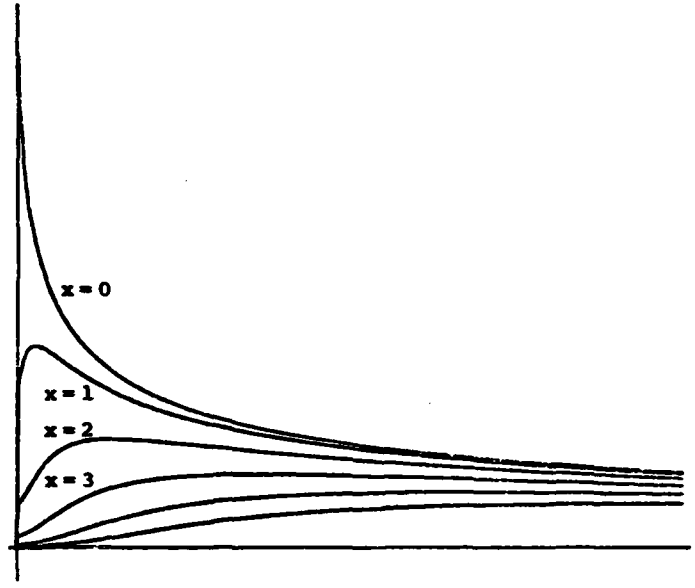


Figure 6. The temperature change on the long time scale for x = 0(1)5 and t less than 40, c = 1.0 and d = 1.0.

displacement for the Gaussian pulse (5.1) with $c = 0.7$ and $d = 0.7$. Figure 5 shows the temperature change for the Gaussian pulse (5.1) with $c = 1.0$ and $d = 0.7$. The displacement corresponding to figure 5 is the same as figure 3b. In each case the limit of the temperature for large t, is $c^2(c^2 + d^2)^{-1}e^{-x}$ and the limit of the displacement is $-(c^2 + d^2)^{-1}e^{-x}$.

One clearly sees in these figures the beginning of the compression wave given by (2.16) and (2.17). The negative temperatures associated with the compression wave are a consequence of the strong thermoelastic coupling and comparative insignificance of diffusion on the short time scale. Similar behavior is not seen in Nayfeh's results (Nayfeh 1977) since in that work the thermoelastic coupling coefficient, i.e. d, is used as the perturbation parameter.

The different figures also show the effect of the parameters c and d on the relative magnitudes of temperature and displacement of the compression wave as compared to the temperature and displacement on the surface. Further, figures 1, 2, and 4 illustrate the decrease in speed of the compression wave as c and d are decreased. The temperature on the surface is independent of c and d except through a multiplicative factor, therefore because of our scaling, the temperature curves for $x = 0$ are identical in figures 1, 2, 4 and 5.

Figure 6 shows the long term behavior of the temperature for values of c and d of 1.0, hence k is 0.5. Equation (3.9) is plotted for values of x of 0, 1.0, 2.0, etc. and t ranges up to 40.

The results displayed in these figures agree qualitatively with the results of Bechtel (1975) and Ready (1965). Moreover, the final results are, we believe, easier to compute and more informative than previous results since the perturbation analysis emphasizes the dominant effects.

The original system of equations (1.1) and (1.2) has two propagation speeds associated with it when the relaxation time is non-zero. In our analysis these show up on two different scales. On short time scale, as discussed in section 2, the effects propagate with the non-dimensional velocity $a = (c^2 + d^2)^{1/2}$, and in the boundary correction for

the short time discussed in section 4 the thermal disturbances propagate with speed $c_1\beta^{-1/2} = c(a^2\tau\beta)^{1/2}$. Moreover, the disturbances propagating with the faster speed $c_1\beta^{-1/2}$ die out rapidly and those propagating with the slower speed do not die out. These results agree with those of Nayfeh (1977) and are compatible with the results of Norwood and Warren (1969). On the long time scale of section 3, there is no propagation as such since diffusion is the dominant phenomena.

REFERENCES

Bechtel, J. H. 1975. Heating of solid targets with laser pulses, Jrnl. of Applied Physics, 46, p. 1585-1593.

Dunbar, M. 1981. Thermal shocking an elastic half-space with a laser pulse, Thesis, University of Wisconsin-Madison.

Lord, H. W. and Shulman, Y. 1967. A generalized dynamical theory of thermoelasticity, J. Mech. Phys. Solids, 15, p. 299-309.

Nayfeh, A. H. 1977. Propagation of thermoelastic disturbances in non-Fourier solids, AIAA Jrnl., 15, p. 957-960.

Norwood, F. R. and Warren, W. E. 1969. Wave propagation in the generalized dynamical theory of thermoelasticity, Quart. Jrnl. Mech. and Appl. Math., 22, p. 283-290.

Ready, J. F. 1965. Effects due to absorption of laser radiation, Jrnl. Appl. Physics, 36, p. 462-468.

JCS/AMS/ed

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 2451	2. GOVT ACCESSION NO. AD-A125 24	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Thermoelastic Response to a Short Laser Pulse		5. TYPE OF REPORT & PERIOD COVERED Summary Report - no specific reporting period 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) John C. Strikwerda and Andrew M. Scott		8. CONTRACT OR GRANT NUMBER(s) DAAG29-80-C-0041
9. PERFORMING ORGANIZATION NAME AND ADDRESS Mathematics Research Center, University of Wisconsin 610 Walnut Street Madison, Wisconsin 53706		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Work Unit Number 2 - Physical Mathematics
11. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Research Office P.O. Box 12211 Research Triangle Park, North Carolina 27709		12. REPORT DATE November 1982 13. NUMBER OF PAGES 26
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Laser, Thermoelasticity, Perturbation Analysis

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

We consider a one-dimensional model of the thermoelastic response to laser heating by a pulse of very short duration. We treat the linear equations of thermoelasticity by perturbation methods using the pulse duration as the perturbation parameter. The perturbation analysis separates the problem into two time regimes, the short time scale, which is of the same order as the pulse duration, and the long time scale, which is much longer than the pulse duration. Convenient analytical expressions are obtained for the temperature change and displacement in both time regimes.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 68 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)