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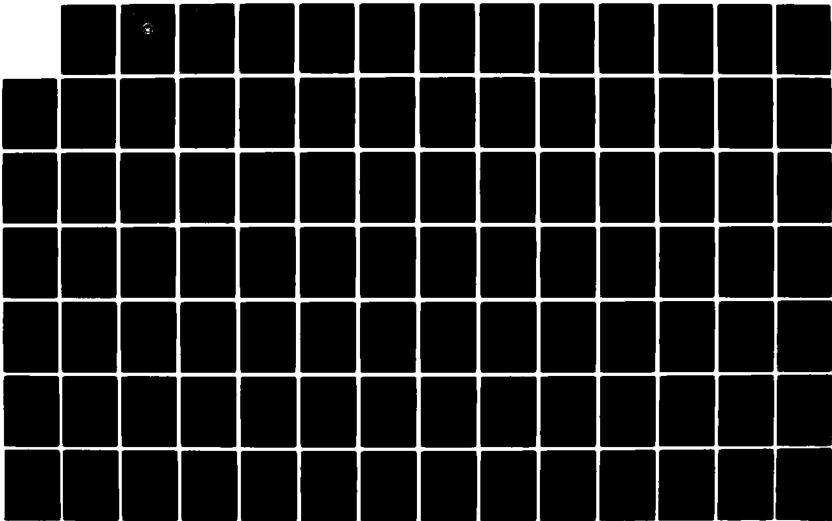

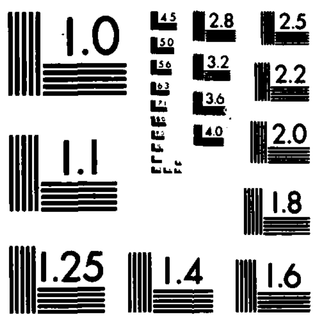

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THESIS

PROBABILISTIC DESIGN USING
NUMERICAL OPTIMIZATION

by

James Harris Hopper III

October 1982

Thesis Advisor: G. N. Vanderplaats

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#20 - ABSTRACT - (Continued)

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Probabilistic Design Using Numerical Optimization

by

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Submitted in partial fulfillment of the requirements for the degree of

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ABSTRACT

A FORTRAN program has been developed to allow for the use of probabilistic design methods in the numerical optimization process. The program was written as a set of subroutines for COPES (Control Program For Engineering Synthesis). COPES maximizes or minimizes a numerically defined objective function subject to a set of inequality constraints using the optimization program CONMIN (A Fortran Program for Constrained Function Minimization). The program developed here allows for the use of both the normal and lognormal distribution models. Design examples are presented to demonstrate the program capabilities. User instructions are provided for inclusion in the COPES user's manual.

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I. INTRODUCTION

In the traditional deterministic approach in engineering design the variables are treated as single valued numbers. These variables, whether dimensions, material properties, loads, etc., are actually statistical in nature. By the use of safety factors designers protect against these variations by usually over designing. A more logical approach is to take into account the statistical data known about each variable and design for a certain reliability. By considering the statistical nature of each variable we should be able to better predict reliability and performance.

Numerical optimization has proven to be a very powerful tool in engineering design. Virtually all design problems require minimization or maximization of some objective. For the design to be acceptable, it must also satisfy a certain set of specified requirements called constraints. If these constraints are specified as probabilities of failure the result would be a probabilistic design.

The purpose of this research was to test the applicability of combining probabilistic design concepts with those of numerical optimization by: (1) developing a pilot computer code to calculate probabilities of failure, (2) incorporate this computer code with COPES/CONMIN in order to perform probabilistic optimization, and (3) test using numerous examples.

It is assumed throughout this discussion that the reader is familiar with the use of the COPES/CONMIN optimization program [Ref. 1], and [Ref. 2].

II. NUMERICAL OPTIMIZATION

The numerical optimization problem considered here is stated as follows: Find the set of n design variables contained in the vector $\underline{X}$ which will

$$\text{Minimize} \quad F(\underline{X}) \tag{1}$$

Subject to:

$$g_j(\underline{X}) \leq 0 \qquad j = 1,m \tag{2}$$

$$x_i^{\ell} \leq x_i \leq x_i^{\mu} \qquad i = 1,n \tag{3}$$

The components, x_i, of $\underline{X}$ are referred to as design variables which are changed to improve the design. The function $F(\underline{X})$ is called the objective. Inequality constraints, $g_j(\underline{X})$, are the response limits imposed on the design. There are n design variables and m inequality constraints. The lower and upper bounds, x_i^{ℓ} and x_i^{μ} are limits imposed on the design variables to insure a practical result.

For a deterministic design COPES [Ref. 1], determines the constraints and objective as follows. Consider the design of a single bar undergoing uniaxial tension. If we wish to minimize the weight, the objective,

$$F(\underline{X}) = \rho A L \tag{4}$$

where

ρ = the specific weight;

A = the area of the bar;

L = the length of the bar.

The design variable is the area, A. If the stress in the bar is S and the stress limit imposed is Sy we desire that S be less than or equal to Sy so the constraint becomes

$$S/Sy - 1 \leq 0 \tag{5}$$

In this investigation a computer subroutine was developed to provide constraints based on probabilities of failure.

If the allowed probability of failure is PF and the actual probability of failure was calculated to be Pf, then the constraint would be

$$Pf/PF - 1 \leq 0 \tag{6}$$

III. PROBABILISTIC DESIGN

A. RANDOM VARIABLES

If one has a large population of n elements for which some parameter x_i, i=1,n is determined, the mean value of the population is

$$\mu_x = \frac{1}{n} \sum_{i=1}^{n} (x_i) \qquad (7)$$

The variance is

$$V_x = \frac{1}{n} \sum_{i=1}^{n} (x_i - \mu_x)^2 \qquad (8)$$

The standard deviation is

$$\sigma_x = \sqrt{V_x} \qquad (9)$$

The standard deviation is a measure of the amount of variability of the population data. An additional measure of the variability is the dimensionless coefficient of variation.

$$C_x = \frac{\sigma_x}{\mu_x} \qquad (10)$$

In engineering design the coefficient of variation is frequently used.

B. FUNCTIONS OF SEVERAL VARIABLES

If Y is a function of n variables, the standard deviation of the function Y can be approximated as follows:

$$\text{If} \quad Y = f(x_i;\ i = 1,n)$$

Assuming that all x_i are independent random variables then according to [Ref. 3, p. 59]

$$\sigma_Y \approx \left[\sum_{i=1}^{n} \left(\frac{\partial Y}{\partial x_i}\right)^2 \sigma_{x_i}^2\right]^{1/2} \tag{11}$$

C. STATISTICAL MODELS

1. Normal Distribution

The normal distribution model is one of the most widely used. The normal density function is given by

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma_x} \exp\left[\frac{-(x - \mu_x)^2}{2\sigma_x^2}\right] \tag{12}$$

where:

$f_X(x)$ = the probability density function of the random variable X;

σ_x = standard deviation of the random variable X;

μ_x = mean of the random variable X.

The magnitude of the standard deviation determines the dispersion of the distribution. A large standard deviation will result in a wide bell curve and a small standard deviation will result in a narrow one. The curve is symmetric about the mean. Approximately 68.26% of all samples will lie within plus or minus one standard deviation of the mean value. The standardized form of the normal density function [Ref. 4: p. 194], can be expressed as

$$\Phi_Y(y) = \frac{1}{\sqrt{2\pi}} \exp\left[\frac{-y^2}{2}\right] \tag{13}$$

where

$$y = \frac{x - \mu_X}{\sigma_X} \tag{14}$$

$\Phi_Y(y)$ = the standardized form of the single variate normal density function.

The single variate cumulative normal distribution function is given by

$$F_X(x) = \int_{-\infty}^{X} f_X(x)\,dx \tag{15}$$

where:

$F_X(x)$ = the probability that the random variable X is equal to or less than the specific value x.

Or, using the expression,

$$y = \frac{x - \mu_x}{\sigma_x},$$

Equation (15) can be rewritten as, [Ref. 4: p. 195]

$$F_X(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{(x-\mu_x)/\sigma_x} \exp\left[\frac{-y^2}{2}\right] dy \qquad (16)$$

or

$$F_X(x) = \Phi_Y\left[\frac{x - \mu_x}{\sigma_x}\right] \qquad (17)$$

2. Lognormal Distribution

If ln(X) has a normal distribution then X is said to have a lognormal distribution. The lognormal density function, [Ref. 4: p. 196], is given by

$$f_X(x) = \frac{1}{\sqrt{2\pi}\hat{\sigma}_x} \frac{1}{x} \exp\left[\frac{-(\ln(x) - \hat{X})^2}{2\hat{\sigma}_x^2}\right]; \quad x > 0 \qquad (18)$$

where:

$\hat{X}$ = mean of ln(X);

$\hat{\sigma}_x$ = standard deviation of ln(X).

The cumulative lognormal distribution function, [Ref. 4: p. 196], is given by

$$F_X(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\ln(x)-\hat{X}}{\hat{\sigma}_x}} \exp[\frac{-y^2}{2}]dy; \; x > 0 \quad (19)$$

or

$$F_X(x) = \Phi_Y[\frac{\ln(x)-\hat{X}}{\hat{\sigma}_x}] \quad (20)$$

The mean, μ_x, and the variance V_x, of X are

$$\mu_x = \exp[\hat{X} + \frac{\hat{\sigma}_x^2}{2}] \quad (21)$$

$$V_x = \exp[2\hat{X} + \hat{\sigma}_x^2][\exp(\hat{\sigma}_x^2) - 1] \quad (22)$$

Solving for $\hat{X}$ and $\hat{\sigma}_x$

$$\hat{X} = \ln(\mu_x^2) - \frac{1}{2}\ln(V_x + \mu_x^2) \quad (23)$$

$$\hat{\sigma}_x = \sqrt{-\ln(\mu_x^2) + \ln[V_x + \mu_x^2]} \quad (24)$$

D. PROBABILITY OF FAILURE

Consider the constant area bar in Figure 3.1 under load P. If the stress is the bar is S and the stress limit in

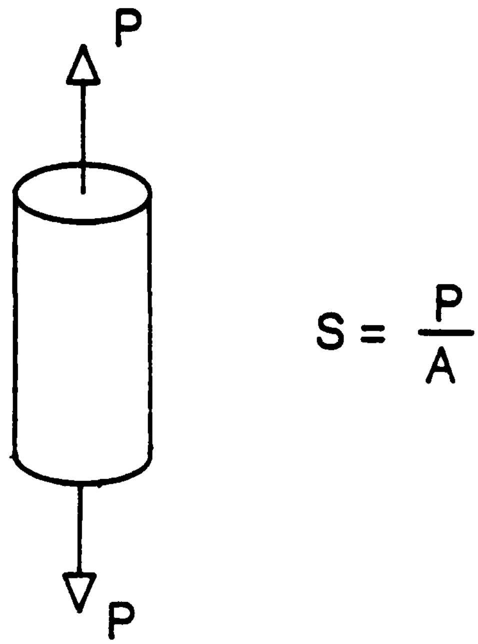


Figure 3.1. BAR UNDER UNIAXIAL LOAD

tension for the material is Sy, then the probability of failure of the bar is the probability that the stress S will be greater than the limit Sy. The method for determining the probability of failure in the bar for both the normal and the lognormal formats is presented below.

1. Normal Distribution

If both S and Sy are random variables which follow a normal distribution then the probability of failure is the probability the S is greater than Sy. The failure function, [Ref. 5: p. 8-24]

$$Z = (Sy - S) \tag{25}$$

is also normally distributed. The probability of failure is the probability that the failure function is less than or equal to 0.0 or

$$Pf = P(Z \leq 0) \qquad (26)$$

In the standard form

$$Pf = \Phi_\beta[\frac{-\mu_z}{\sigma_z}] \qquad (27)$$

Where, by the algebra of functions, [Ref. 3]

$$\mu_z = \mu_{Sy} - \mu_S \qquad (28)$$

and

$$\sigma_Z = \sqrt{\sigma_{Sy}^2 + \sigma_S^2} \qquad (29)$$

The safety index, [Ref. 5: p. 8-24], is defined as

$$\beta = \frac{\mu_z}{\sigma_z} = \frac{1}{C_z} \qquad (30)$$

The probability of failure is then

$$Pf = \Phi_\beta[-\beta] \qquad (31)$$

or the area under the unit normal density function from β to ∞.

$$Pf = \int_{\beta}^{\infty} \frac{1}{\sqrt{2\pi}} \exp[\frac{-\beta^2}{2}] d\beta \tag{32}$$

2. Lognormal Distribution

If the same two variables S and Sy follow a logrithetic normal distribution then the probability of failure is expressed by, [Ref. 5: p. 8-27]

$$Pf = P(Sy \leq S) = P(\ln(Sy/S) \leq 0) \tag{33}$$

Letting

$$Z = \ln(Sy/S) \tag{34}$$

then

$$Pf = P(Z \leq 0) \tag{35}$$

or

$$Pf = \Phi_{\beta}[-\beta] \tag{36}$$

The safety index, [Ref. 5: p. 8-29] is

$$\beta = \frac{\ln(\tilde{S}y/\tilde{S})}{\sqrt{\ln[(1+C_{Sy}^2)(1+C_S^2)]}} \tag{37}$$

where the $\tilde{S}y$ and $\tilde{S}$ are median values and

$$\tilde{S}y = \frac{\mu_{Sy}}{\sqrt{1+C_{Sy}^2}} \tag{38}$$

$$\tilde{S} = \frac{\mu_s}{\sqrt{1+C_S^2}} \tag{39}$$

where

$$C_{Sy} = \frac{\sigma_{Sy}}{\mu_{Sy}} \tag{40}$$

$$C_S = \frac{\sigma_S}{\mu_S} \tag{41}$$

β will be normally distributed and the probability of failure is

$$Pf = \Phi_\beta\left[\frac{-\ln(\tilde{S}y/\tilde{S})}{\sqrt{\ln[(1+C_{Sy}^2)(1+C_S^2)]}}\right] \tag{42}$$

The probability that S is greater than Sy is then determined in the same manner as the normally distributed case, by integrating the unit normal density function from β to ∞.

IV. PROBABILISTIC OPTIMIZATION

The objective of probabilistic optimization is to minimize or maximize the mean value of a given function subject to a set of constraints based on allowed probabilities of failure.

A. CONSTRAINTS

The computer subroutine developed in this investigation, COPE19, calculates the constraints as described in Chapter II. Given a constrained variable in terms of a desired probability of failure, one needs only to calculate the actual probability of failure to determine the constraint value.

Consider the cantilevered beam in Figure 4.1. A probability of failure, PF, is desired for stress in bending. The mean values and coefficients of variation for the yield strength, Sy, the length L, the load P, and the dimensions B and H are given. The stress in bending

$$SB = \frac{6PL}{BH^2} \tag{43}$$

The standard deviation of the bending stress

$$\sigma_{SB} \simeq \sqrt{\left(\frac{\partial SB}{\partial P}\sigma_P\right)^2+\left(\frac{\partial SB}{\partial L}\sigma_L\right)^2+\left(\frac{\partial SB}{\partial H}\sigma_H\right)^2+\left(\frac{\partial SB}{\partial B}\sigma_B\right)^2} \tag{44}$$

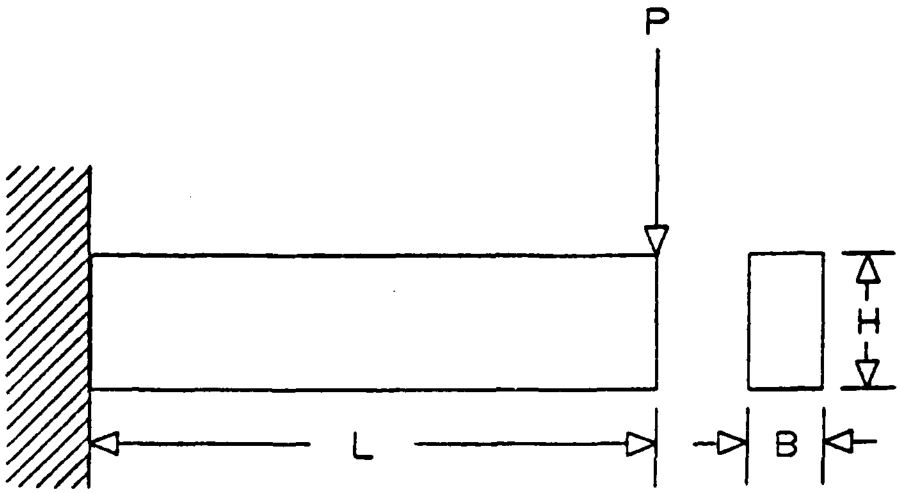


Figure 4.1. CANTILEVERED BEAM

The safety index

$$\beta = \frac{Sy - SB}{\sqrt{\sigma_{Sy}^2 + \sigma_{SB}^2}} \qquad (45)$$

The calculated probability of failure

$$Pf = \Phi_\beta[-\beta] \qquad (46)$$

The constraint will be

$$G = \frac{Pf}{PF} - 1 \qquad (47)$$

In the computer program developed, COPE19, the required partial derivatives of the constrained variables with respect to the variables in the expression are calculated by finite difference methods.

B. OPTIMIZATION

Consider the three bar truss in Figure 4.2. The geometry is specified and there are two independent load conditions. The design task is to determine the areas required for each bar that will yield a minimum mean value of the structure weight while keeping the stress in any member less than the yield stress.

THREE BAR TRUSS

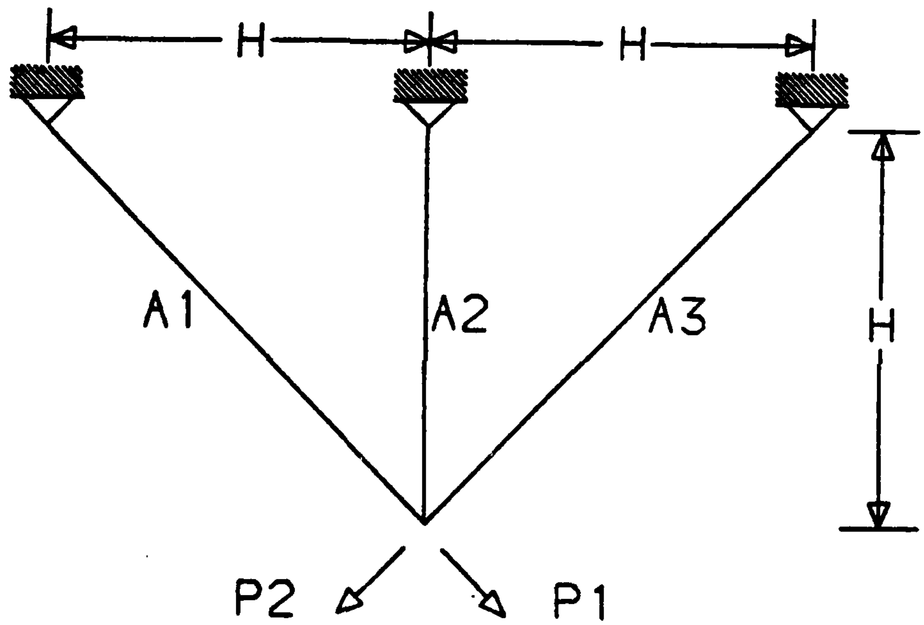


Figure 4.2. THREE BAR TRUSS

Minimize: $W = \rho H(\sqrt{2}A1 + A2 + \sqrt{2}A3)$

Subject to: Sy compression $\leq SIG_{ij} \leq$ Sy tension

where SIG_{ij} is the stress in member i under load condition j.

Given: Material ρ = .1 lb./cu. in.

Geometry H = 10.0 in.

Loads P1 = P2 = 20,000 lb.

Stress limits: Sy compression = -15,000 psi

Sy tension = 20,000 psi

Maintain symmetry A1 = A3

Beginning with the design A1 = A2 = A3 = 1.0 sq. in. and optimizing using COPES/CONMIN (deterministic) the following results are obtained:

W = 2.632 lb.

A1 = A3 = .7796 sq. in.

A2 = .4275 sq. in.

The critical constraints are SIG11 and SIG32, which are equal. The design space is shown in Figure 4.3. As no safety factor was used, the above deterministic design

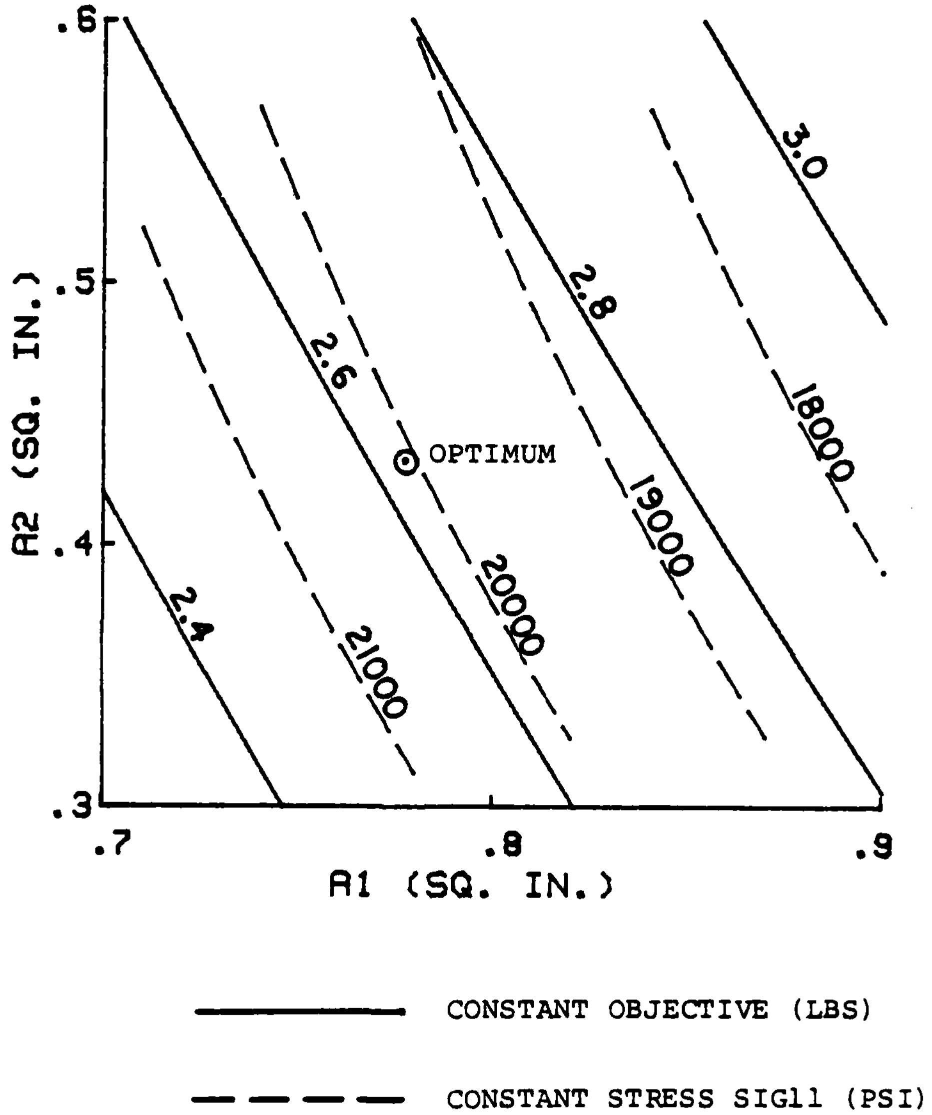


Figure 4.3. THREE BAR TRUSS DESIGN SPACE (DETERMINISTIC)

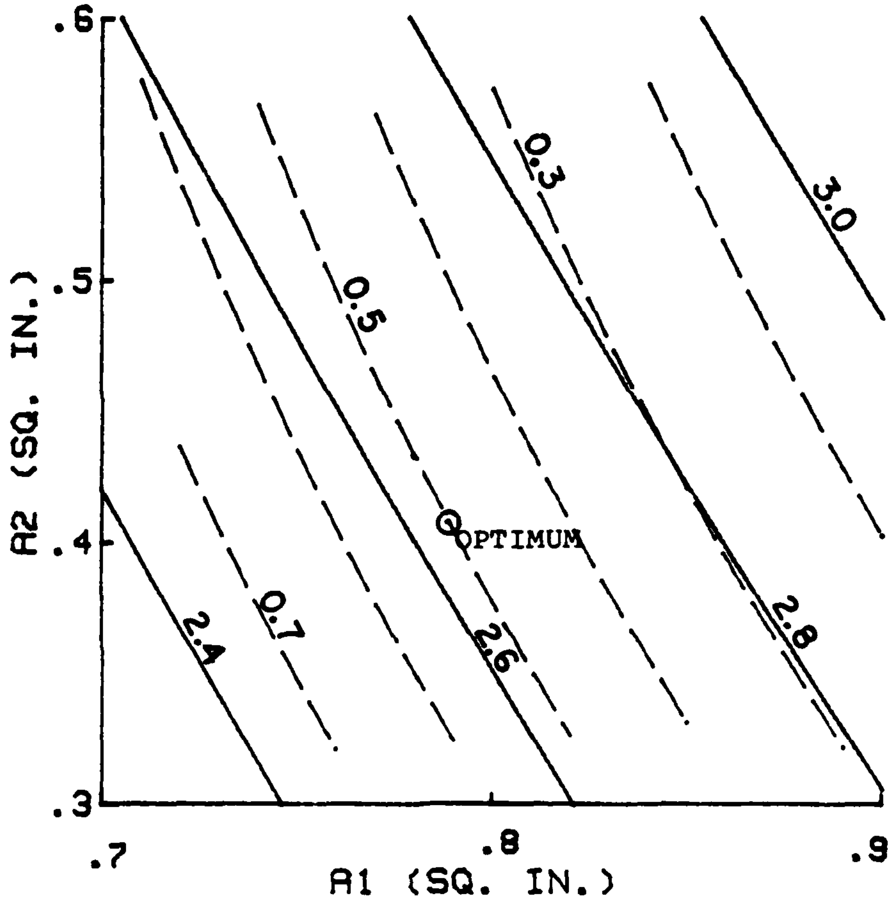
corresponds to a probability of failure of 0.5 or a 50% chance that SIG11 and SIG32 will be greater than the stress limit Sy. Utilizing the subroutine COPE19 to provide probabilistic based constraints and assuming that the coefficients of variation of all the variables are 0.1, the following results are obtained when designing for a maximum probability of failure of 0.5 in each member; assuming a normal distribution,

W = 2.639 lb.

A1 = A3 = .7883 sq. in.

A2 = .4094 sq. in.

The design space is shown in Figure 4.4. The lines of constant probability of failure for SIG11 are shown. As one would expect the line of constant 0.5 probability of failure for SIG11 in Figure 4.4 is the same as the constant 20,000 psi line for SIG11 in Figure 4.3. The reverse is also true. If the three bar truss is optimized for a maximum probability of failure of 0.4 the resulting safety factor for SIG11 is 1.03 or 19,361 psi and the minimum weight is 2.726 lb. Using 19,361 psi as the limit of stress for a deterministic optimization yields approxiamtely the same minimum weight. In both cases the feasible design space is the same with the line of constant 0.4 probability of failure and constant 19,361 psi lying on top of each other.



CONSTANT OBJECTIVE (LBS)

CONSTANT PROBABILITY OF FAILURE SIG11

Figure 4.4. THREE BAR TRUSS DESIGN SPACE (PROBABILISTIC)

Table I shows the results of minimizing the weight of the three bar truss for various probabilities of failure. Table II provides the deterministic optimum designs for the equivalent factor of safety. The difference in designs is within the numerical accuracy of the optimization.

TABLE I

THREE BAR TRUSS PROBABILISTIC DESIGNS

PROBABILITY OF FAILURE	WEIGHT LBS.	A1 = A3 SQ. IN.	A2 SQ. IN.	SIG11 PSI	SAFETY FACTOR
.5	2.6390	.78827	.40938	20000	1.00
.4	2.7260	.81606	.41785	19361	1.03
.3	2.8238	.84478	.43438	18691	1.07
.2	2.9430	.86887	.48544	17940	1.11
.1	3.1208	.92482	.50500	16914	1.18
.01	3.6099	1.0490	.64280	14640	1.37
.001	4.0459	1.1777	.71493	13060	1.53
.0001	4.4770	1.3212	.73994	11792	1.70

TABLE II

THREE BAR TRUSS DETERMINISTIC DESIGNS

SAFETY FACTOR	WEIGHT LBS.	A1 = A3 SQ. IN.	A2 SQ. IN.	SIG11 PSI
1.00	2.6326	.77962	.42752	20051
1.03	2.7212	.80145	.45437	19402
1.07	2.8200	.82773	.47889	18726
1.11	2.9322	.87045	.47014	18002
1.18	3.1180	.91845	.52000	16933
1.37	3.5936	1.0807	.53693	14688
1.53	4.0258	1.2033	.62227	13110
1.70	4.4800	1.3379	.69573	11781

It should be noted that the correspondence of the probability of failure and the safety factor shown here is unique to this problem, in which all C = 0.1. In general nonlinear optimization, this one to one relationship cannot be assured.

C. EFFECTS OF COEFFICIENTS OF VARIATION ON THE DESIGN SPACE

As the coefficients of variation of the variables used in the design subroutine become smaller, the bands in the design space for the constrained values between Pf = 1.0 and Pf = 0.0 becomes narrower. Figure 4.5 shows the design space for the three bar truss in the vicinity of the optimum for the case where all variables have coefficients of variation, C = 0.1. Figure 4.6 shows the same space for the case where all coefficients of variation, C = 0.001. In both figures the lines of constant objective and probabilities of failure for the critical constrained variable, SIG11, are shown. The line of constant Pf = 0.5 is the same for both but the band between Pf = 0.9 and Pf = 0.1 is much narrower for the case where C = 0.001.

This phenomenon can cause numerical difficulties in the optimization process. Premature termination of optimization can occur as the optimizer may be unable to move down the resulting 'narrow valley'. The absolute and relative termination criteria of CONMIN may have been satisfied but the true optimum might not have been reached.

If the initial design is a considerable distance from the narrow band of changing probabilities of failure, CONMIN may obtain zero gradients for the constraints which would

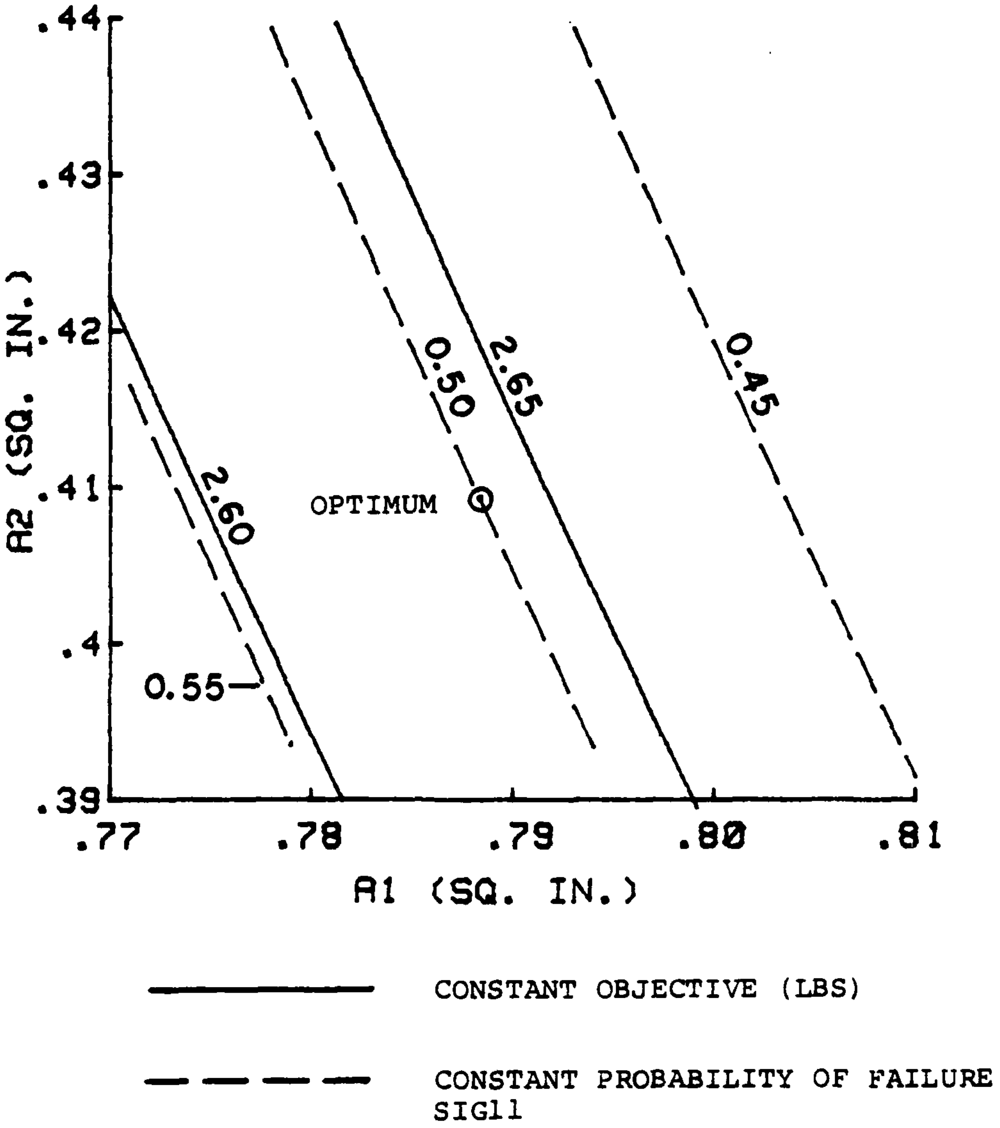
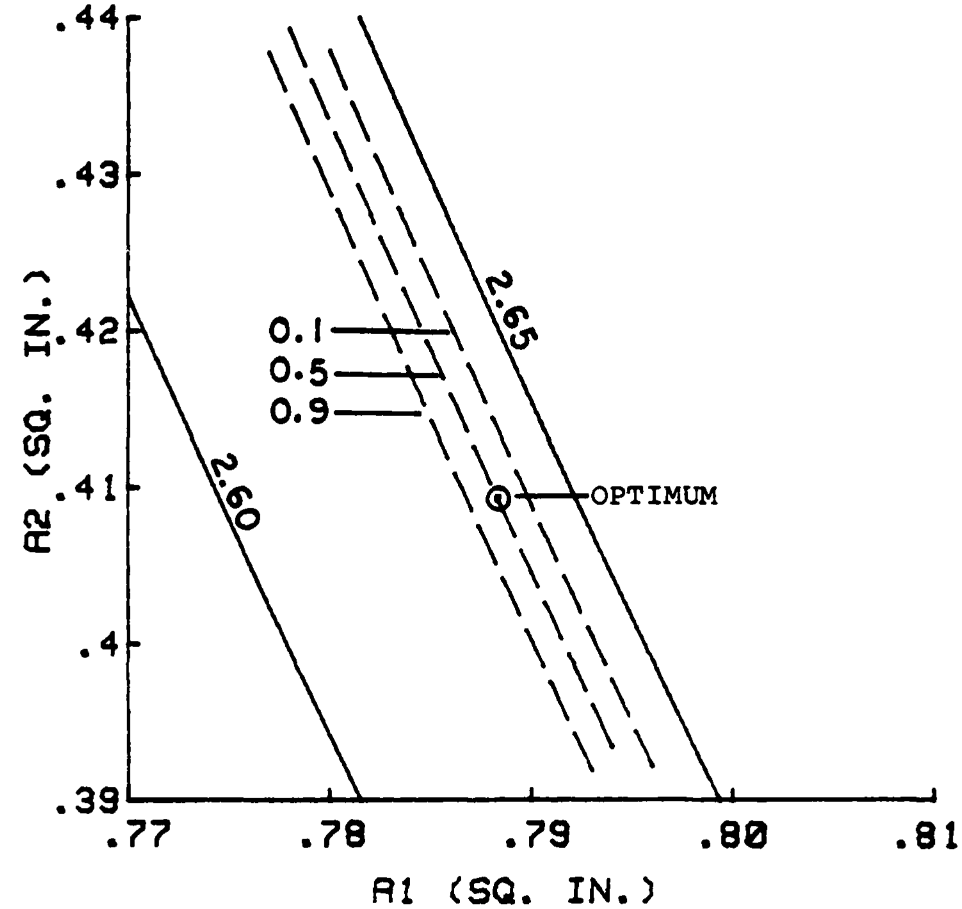


Figure 4.5. THREE BAR TRUSS DESIGN SPACE (C = 0.1)



CONSTANT OBJECTIVE (LBS)

CONSTANT PROBABILITY OF FAILURE SIG11

Figure 4.6. THREE BAR TRUSS DESIGN SPACE (C = 0.001)

also cause termination of the optimization process. A routine to attempt to prevent this was included in COPE19. The cumulative distribution curve for the normal format is shown in Figure 4.7. The numerical integration routine used to calculate the area under the unit normal distribution curve yields a probability of failure, Pf of 1.0000000 for a β of -5.2999821 and a Pf of 0.0000000 for a β of 5.5000086. It was modified to "widen" the zone between Pf = 1.0 and Pf = 0. A polynomial (a line equation) was placed at each end of the cumulative distribution routine to yield a Pf of 1.0 at β equals -25.0, decreasing to a Pf of 0.9999866 at -4.2 and a Pf of 0.00001335144 at β equals 4.2 decreasing to a Pf of 0.0 at 25.0. This is shown in Figure 4.8. This provides the same optimization results within the accuracy of the program, but avoids the problem of zero gradients of the constrained variables.

An additional approximation was used to determine the probability of failure when the coefficients of variation concerned are zero or nearly so. The probability that X is greater than Y is, in the standard format

$$\Phi_\beta[-\beta] = \Phi_\beta[-\frac{\mu_z}{\sigma_z}] \qquad (48)$$

where

$$\mu_z = \mu_y - \mu_x \qquad (49)$$

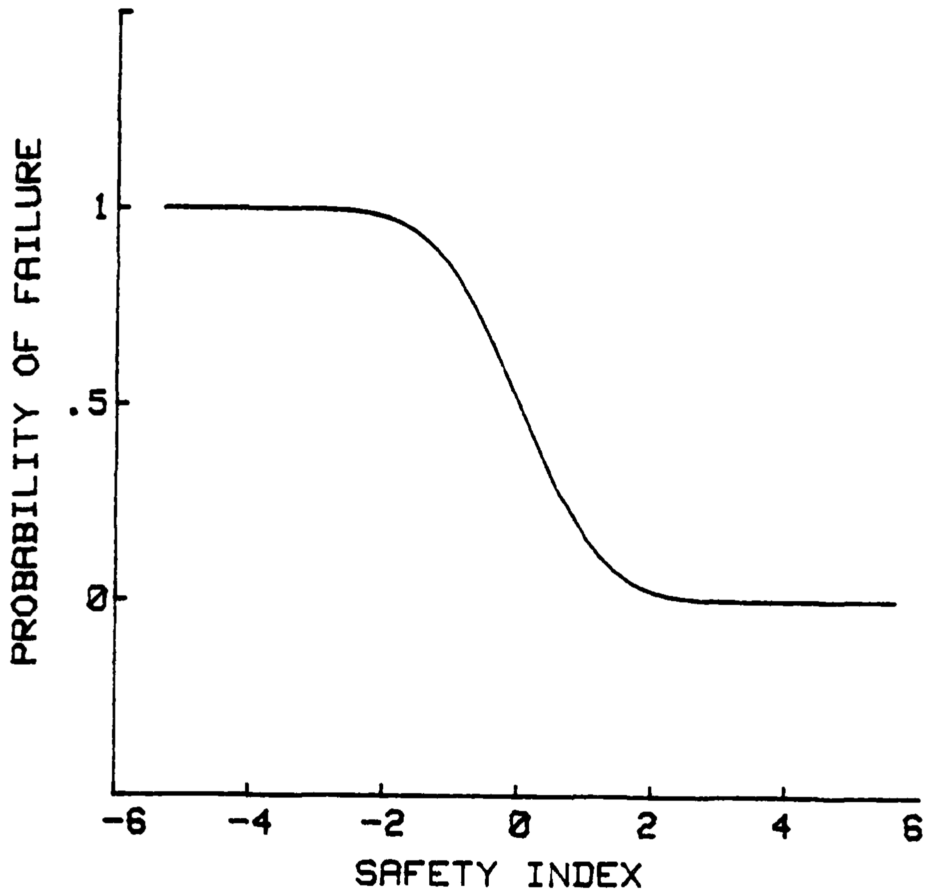


Figure 4.7. CUMULATIVE NORMAL DISTRIBUTION CURVE

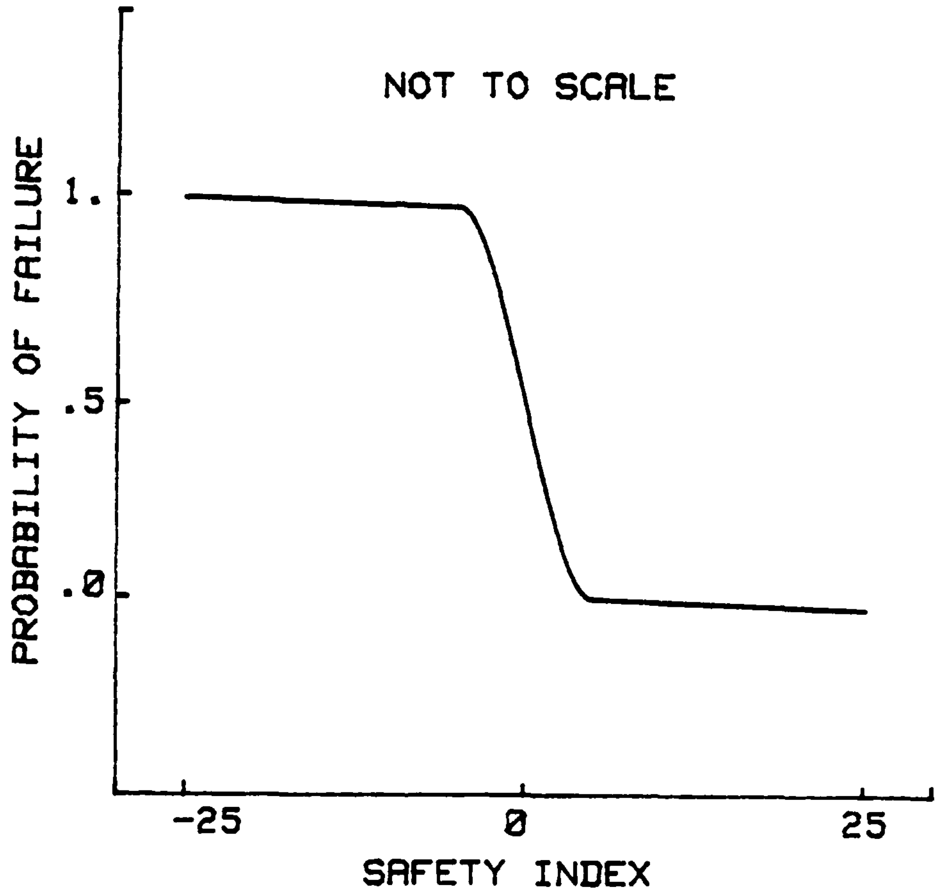


Figure 4.8. MODIFIED CUMULATIVE NORMAL DISTRIBUTION CURVE

and

$$\sigma_z = \sqrt{\sigma_x^2 + \sigma_y^2} \tag{50}$$

Mathematically, if $\sigma_z = 0$, then one of three situations can occur.

1. $X = Y$, $\beta = 0$, then $Pf = 0.5$;

2. $X > Y$, $\beta \rightarrow -\infty$, then $Pf = 1.0$;

3. $X < Y$, $\beta \rightarrow \infty$, then $Pf = 0.0$.

This is shown graphically in Figure 4.9. In order to provide CONMIN with a smooth function with which to work, a polynomial was used to connect the $Pf = 1.0$ and $Pf = 0.0$ in Figure 4.9. This is shown in Figure 4.10. The interpolating polynomial is used when $\bar{\beta}$, the modified safety index, is within one unit of 0. Where $\bar{\beta}$ is defined to be

$$\bar{\beta} = \frac{Y - X}{\sqrt{Y^2 + X^2}} \tag{51}$$

A similar approximation was developed for the lognormal model.

Experience has shown that these modifications dramatically improve the numerical stability of the optimization process without significantly affecting the mathematical result.

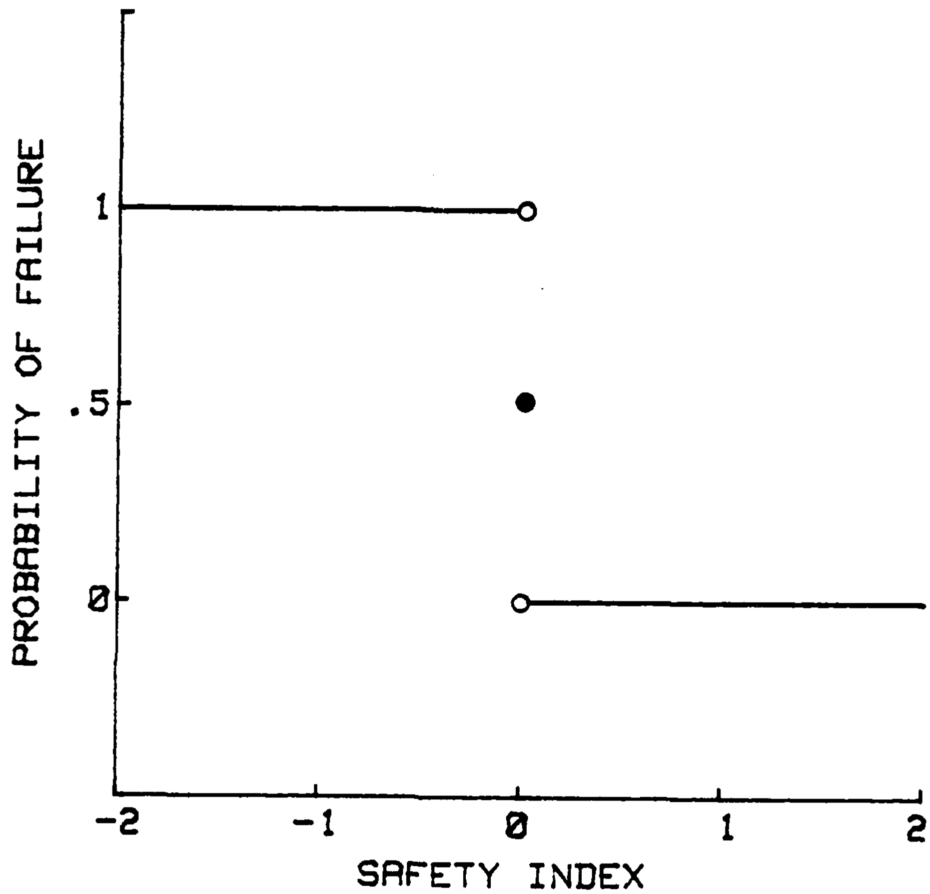


Figure 4.9. CUMULATIVE NORMAL DISTRIBUTION CURVE (C = 0.0)

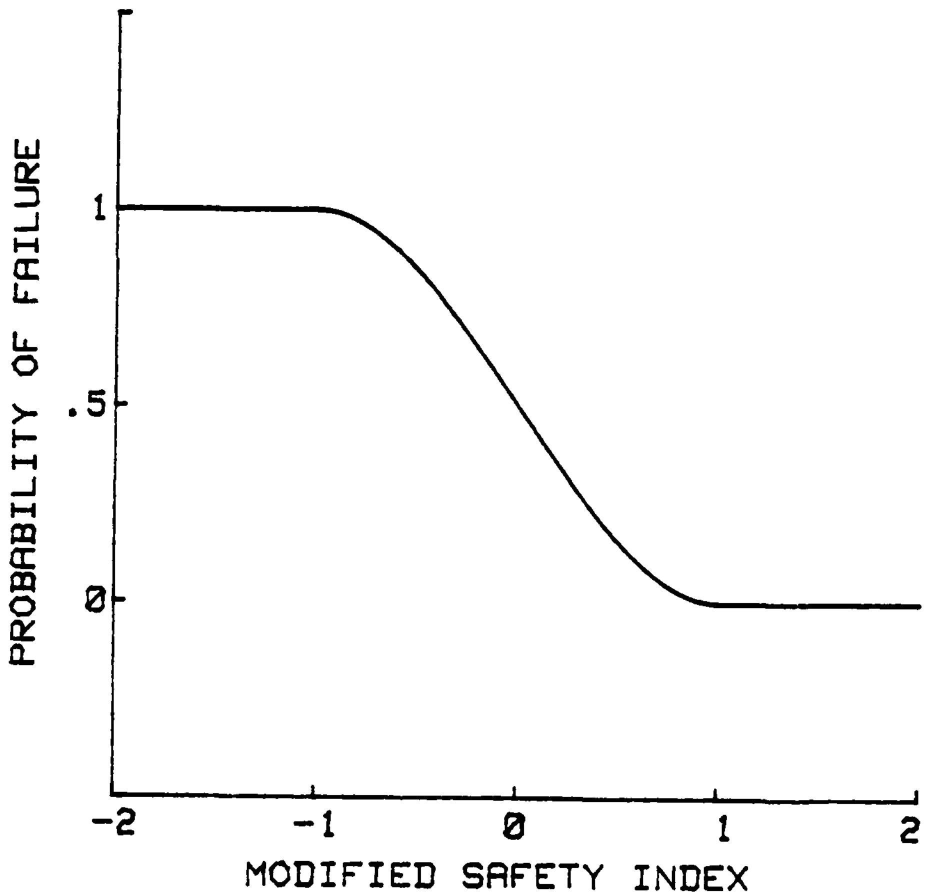


Figure 4.10. MODIFIED CUMULATIVE NORMAL DISTRIBUTION CURVE (C = 0.0)

V. COMPUTER PROGRAM

A. COPES INPUT DATA

Standard deterministic optimization requires that COPES data blocks A-0 and V be used. For probabilistic optimization the same data is entered in these blocks with the exception of NCALC and IPROB in data block B. Here an NCALC = 7 would be entered to indicate probabilistic optimization was to be performed and an IPROB = 1 or 2 to indicate which probability model is to be used. Additional data blocks are required for probabilistic optimization. Appendix A contains detailed input instructions for these data blocks as well as an example problem including sample input data.

B. OPTIMIZATION DATA FLOW

The interaction between COPES, CONMIN, and the analysis subroutine will be the same for probabilistic optimization as it is in standard deterministic optimization with one exception. In the optimization process, each time COPES requires the determination of a constraint vector it will call COPE 19. COPE19 will call the analysis subroutine as many times as necessary to determine the required partial derivatives by finite difference steps. Once these are obtained, the constraint vector is determined and provided to COPES. A flow diagram for probabilistic optimization is provided in Figure 5.1.

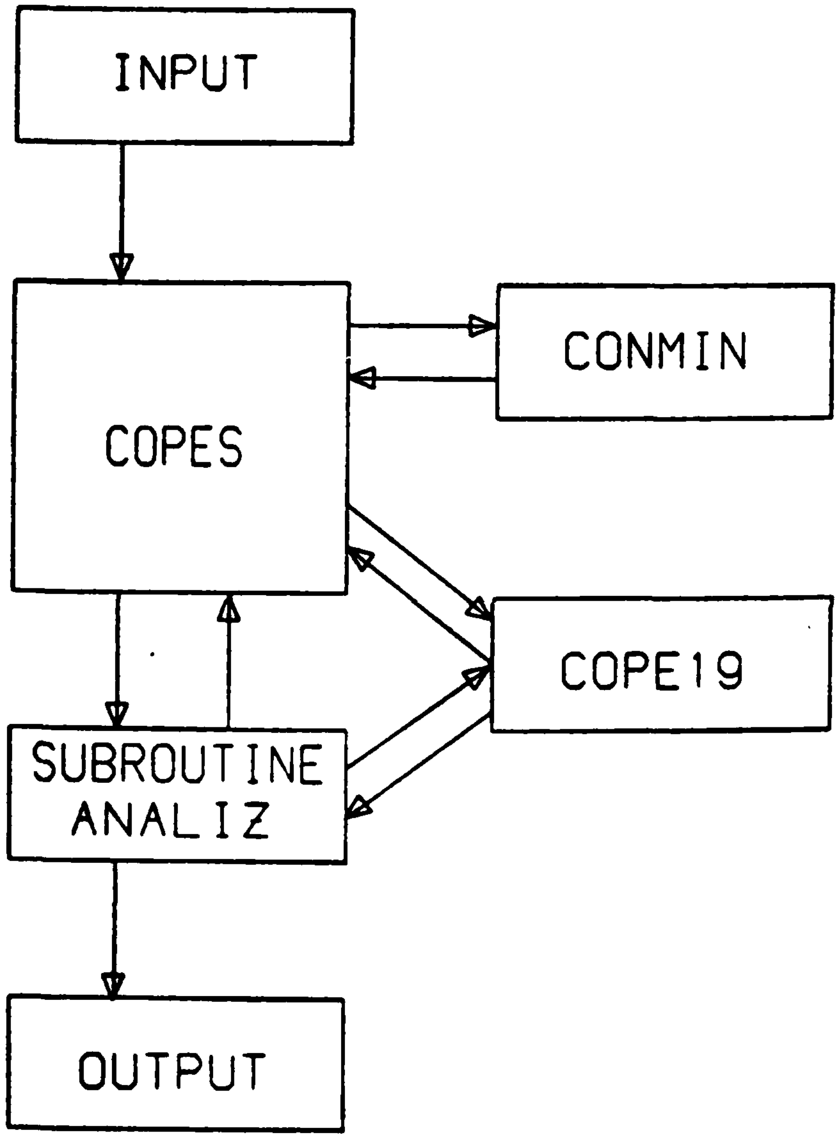


Figure 5.1. PROBABILISTIC OPTIMIZATION FLOW DIAGRAM

C. COPE19 SUBROUTINE

The main computer subroutine developed in this investigation is COPE19. Three additional subroutines, COPE20 through COPE22, were also developed to perform specific tasks required by COPE19. The operation and flow of these subroutines are discussed below.

1. Input

The following is provided to COPE19:

a. The COPES control parameter ICALC.

b. Which probability model to use, either normal or lognormal.

c. The number of probability variables which make up the constraint equations (IVAR).

d. The global locations in the ANALIZ subroutine common block of each IVAR.

e. Coefficient of variation of each IVAR, assumed constant throughout the optimization.

f. The number of constrained variables.

g. The global location in the ANALIZ subroutine common block of each of the constrained variables.

h. The upper and lower limits imposed on each of the constrained variables.

i. The allowed probability of failure at the upper and lower limits of each constrained variable.

j. The coefficient of variation of the constrained variable limits.

k. The variable MGRAD which determines how often the constrained variable gradients are calculated.

2. Determination of Constraint Vector

When COPES requires a constraint vector for a given design it will call COPE19 with ICALC = 2. COPE19 will then calculate the partial derivatives of each constrained variable with respect to the variables which make up the constraint equations. The subroutine COPE20 controls the finite difference calculations. In the example in Section IV.A, the cantilevered beam, it would calculate:

$$\frac{\partial SB}{\partial L} = \frac{6P}{BH^2} \tag{52}$$

$$\frac{\partial SB}{\partial P} = \frac{6L}{BH^2} \tag{53}$$

$$\frac{\partial SB}{\partial H} = -\frac{12PL}{BH^3} \tag{54}$$

$$\frac{\partial SB}{\partial B} = -\frac{6PL}{B^2H^2} \tag{55}$$

COPE19 will then calculate the standard deviation (Equation 44) of the constrained variable, in this case SB, as discussed in Section III.B and Section IV.A. Now the safety index is calculated and subroutine COPE21 called to determine the probability of failure. When required, COPE21 will call subroutine COPE22 to perform numerical integration of the normal density function. The constraint value to be stored in the G vector is then determined (Equation 6). The above would be performed for each constrained variable and the resulting G vector provided to COPES.

D. REDUCTION OF FUNCTION EVALUATIONS

In order to determine the gradients of the constrained variables finite difference steps are taken resulting in a significant increase in the number of analyses performed in the optimization process. In many problems these gradients remain essentially the same throughout the optimization. Therefore, it is sometimes possible to reduce the frequency with which the gradients are calculated and achieve approximately the same result. The input variable code LGRAD is included as an option in probabilistic optimization to allow this choice. LGRAD has the following meanings:

- 0: Calculate gradients each time COPE19 is called.
- 1: Calculate gradients at the beginning of each CONMIN iteration.
- 2: Calculate gradients only at the beginning of optimization.

Table III demonstrates the result of various LGRAD selections when designing the three bar truss for an allowed probability of failure of 0.50, where all coefficients of variations, C = 0.1.

Probabilistic optimization was performed using the three LGRAD values on numerous additional test cases. In almost

TABLE III

REDUCTION OF FUNCTION EVALUATIONS: THREE BAR TRUSS DESIGN

	LGRAD	WEIGHT LBS.	A1 = A3 SQ. IN.	A2 SQ. IN.	NUMBER OF FUNCT EVALS
Deterministic		2.6326	.77962	.42752	35
Probabilistic	0	2.6390	.78827	.40938	343
Probabilistic	1	2.6389	.77386	.45011	244
Probabilistic	2	2.6379	.78972	.40427	56

every case the optimization results were essentially the same for LGRAD = 0 and 1. The composite driveshaft design in Chapter V provides an excellent example showing the function evaluation reductions when performing a relatively complex design. Table IV demonstrates the results of reducing the frequency of constrained variable gradient calculations when designing the composite driveshaft for minimum weight, using the normal distribution model. For this particular problem, calculation of constrained variable gradients only at the beginning of optimization was insufficient to obtain results near the optimum.

TABLE IV

REDUCTION OF FUNCTION EVALUATIONS: DRIVESHAFT DESIGN

LGRAD	WEIGHT LBS.	NUMBER OF FUNCTION EVALUATIONS
0	9.0203	4011
1	9.0205	1344
2	27.216	89

VI. NUMERICAL EXAMPLES

The following examples are presented to demonstrate the capabilities of the computer subroutines developed in this investigation. In both cases an existing ANALIZ subroutine was used after minor modifications. For these two shaft designs the subroutine from [Ref. 6], was used. The ANALIZ subroutine was altered to include the required probability variables in the global common block and to remove the factor of safety calculations which were originally used as constraints.

In each example a similar deterministic design was performed to give the reader a better feel for the results. The deterministic designs are not intended to duplicate the probabilistic design results.

The mean values used in the designs are from [Ref. 6]. The coefficients of variation are reasonable assumed values used for demonstration purposes.

A. STEEL DRIVESHAFT

Design, for minimum weight, a steel driveshaft to transmit 150.0 horsepower at 300.0 RPM. The loading is presented in Figure 6.1. The shaft is designed against failure in strain, in torsional buckling, vibration frequency and a maximum deflection of 0.05 inches.

The design variables are the shaft thickness and the shaft inside diameter. The thickness is to be maintained

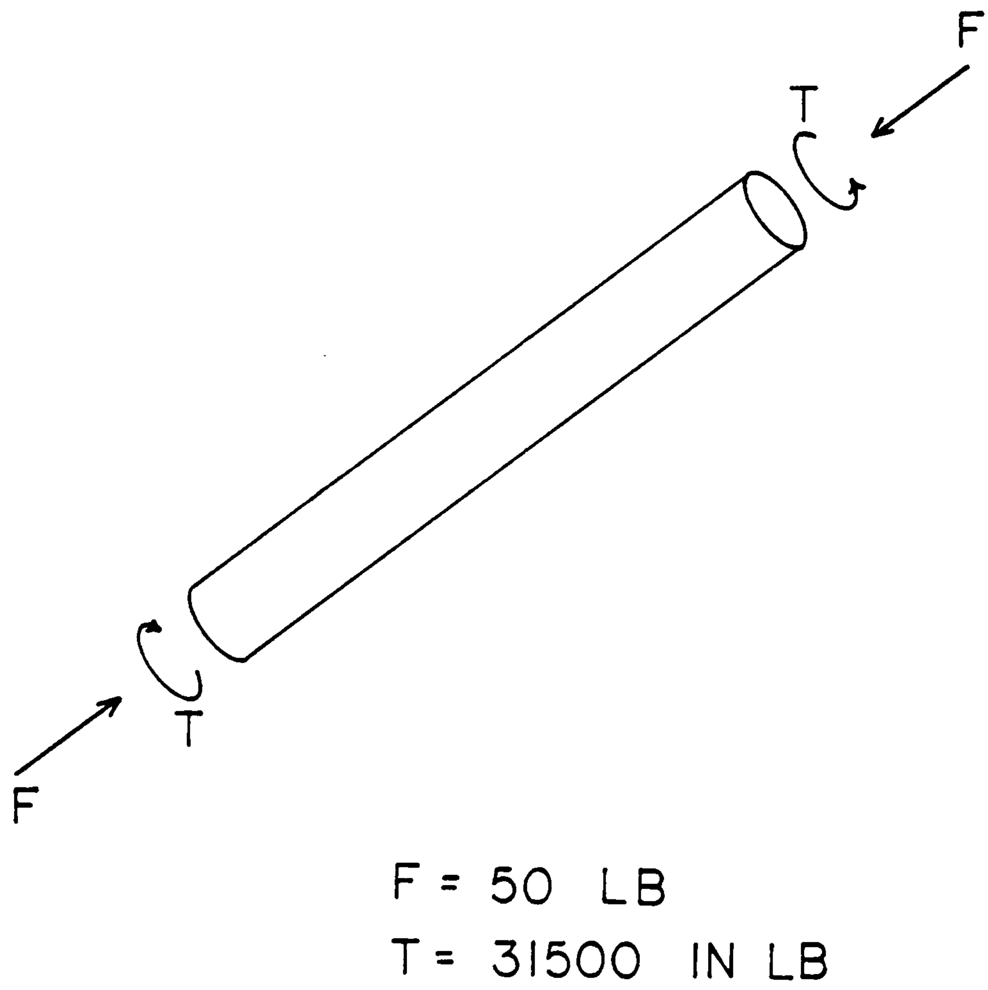


Figure 6.1. DRIVESHAFT LOADING

between 0.01 and 2.0 inches and the inside diameter between 0.5 inches and 5.0 inches. The initial design is an inside diameter of 2.0 inches and a thickness of 1.0 inch. The remaining data for the analysis is presented in Table V.

TABLE V

INPUT DATA FOR STEEL DRIVESHAFT

	MEAN VALUE		COEFFICIENT OF VARIATION
HORSEPOWER	150.	HP	.08
SPEED	300.	RPM	.08
AXIAL LOAD	50.	LB.	.18
LENGTH	120.	IN.	.015
YOUNGS MODULUS	30,000,000	PSI	.038
SHEAR MODULUS	11,538,000	PSI	.042
POISSONS RATIO	.3		.026
SPECIFIC WEIGHT	.282	LB/CU. IN.	.01
STRAIN LIMIT	.001	IN./IN.	.01
MAX DEFLECTION	.05	IN.	.05
THICKNESS			.015
INSIDE DIAMETER			.015
FAILURE DUE TO:			
TORSIONAL BUCKLING			.21
FREQUENCY			.06

Allowing for maximum probabilities of failure of 0.01 in strain, torsional buckling, deflection and vibration frequency; a minimum weight of 54.15 lbs is achieved for the normal model and 52.88 lbs for the lognormal model. The initial design summary is presented in Table VI and the final designs summaries and COPES optimization results in Tables VII through XII.

Designing for a factor of safety against failure of 2.0 for strain, torsional buckling, vibration frequency, and

TABLE VI

STEEL DRIVESHAFT INITIAL DESIGN SUMMARY

```
STEEL DRIVESHAFT OUTPUT

     NUMBER OF PLYS               =     1
     NUMBER OF MATERIAL TYPES     =     1
     NUMBER OF LOAD CONDITIONS    =     1
     ECCENTRICITY                 =     0.0

     DIMENSIONS
     PLY   THICKNESS    PERCENT   DIAMETER    THETA
                 INSIDE 0.20000E+01
      1 0.10000E+01   100.00    0.40000E+01   0.0

     STIFFNESS
     AE = 0.31071E+09
     EI = 0.38838E+09
     GJ = 0.27187E+09

LOADS:
L.C.       T           M          F          H.P.         RPM
 1   0.31513E+05   0.0    0.50000E+02 0.15000E+03 0.30000E+03

LOAD CONDITION    1
PLY     EPL        S.F.    EPT      S.F.        EPLT        S.F.
1 0.16092E-06  100.00   0.0      100.00    0.23182E-03    5.61

     CRITICAL SPEED       = 0.15543E+04
     MAXIMUM DEFLECTION   = 0.19195E-01
     WEIGHT               = 0.31893E+03
     VOLUME               = 0.11310E+04
```

TABLE VII

STEEL DRIVESHAFT COPES STANDARD OUTPUT: NORMAL

OPTIMIZATION RESULTS

OBJECTIVE FUNCTION
GLOBAL LOCATION 3 FUNCTION VALUE 0.54154E+02

DESIGN VARIABLES

ID	D. V. NO.	GLOBAL VAR. NO.	LOWER BOUND	VALUE	UPPER BOUND
1	1	11	0.10000E-01	0.99883E-01	0.20000E+01
2	2	2	0.50000E+00	0.50000E+01	0.50000E+01

DESIGN CONSTRAINTS

DETERMINISTIC

ID	GLOBAL VAR. NO.	LOWER BOUND	MEAN VALUE	UPPER BOUND
1	51	-0.10000E-02	0.94774E-06	0.10000E-02
3	52	-0.10000E-02	0.0	0.10000E-02
5	53	-0.10000E-02	0.68213E-03	0.10000E-02
7	54	0.31513E+05	0.24340E+06	0.11000E+16
8	56	0.30000E+03	0.25068E+04	0.11000E+16
9	57	-0.11000E+16	0.72075E-02	0.50000E-01

TABLE VIII

STEEL DRIVESHAFT COPES PROBABILISTIC OUTPUT: NORMAL

PROBABILISTIC

ID	MEAN VALUE	STANDARD DEVIATION	COEFFICIENT OF VARIATION
1	0.94774E-06	0.1753E-06	0.1753E-01
3	0.0	0.0	0.0
5	0.68213E-03	0.6535E-04	0.9581E-01
7	0.24340E+06	0.1364E+05	0.5606E-01
8	0.25068E+04	0.9634E+02	0.3843E-01
9	0.72076E-02	0.5707E-03	0.7918E-01

	PROBABILITY OF FAILURE AT LOWER BOUND		PROBABILITY OF FAILURE AT UPPER BOUND	
ID	ALLOWED	CALCULATED	ALLOWED	CALCULATED
1	0.1000E-01	0.0	0.1000E-01	0.0
3	0.1000E-01	0.0	0.1000E-01	0.0
5	0.1000E-01	0.0	0.1000E-01	0.1000E-01
7	0.1000E-01	0.0	0.1000E+01	0.0
8	0.1000E-01	0.0	0.1000E+01	0.0
9	0.1000E+01	0.0	0.1000E-01	0.0

TABLE IX

STEEL DRIVESHAFT DESIGN SUMMARY: NORMAL

```
STEEL DRIVESHAFT OUTPUT

      NUMBER OF PLYS              =     1
      NUMBER OF MATERIAL TYPES    =     1
      NUMBER OF LOAD CONDITIONS   =     1
      ECCENTRICITY                =     0.0

      DIMENSIONS
      PLY    THICKNESS    PERCENT    DIAMETER      THETA
                   INSIDE 0.50000E+01
       1 0.99883E-01   100.00    0.51998E+01     0.0

      STIFFNESS
      AE = 0.52757E+08
      EI = 0.17158E+09
      GJ = 0.12011E+09


LOADS:
L.C.       T             M           F            H.P.            RPM
 1 0.31513E+05  0.0     0.50000E+02  0.15000E+03  0.30000E+03


LOAD CONDITION     1
PLY      EPL         S.F.       EPT          S.F.       EPLT          S.F.
1   0.94774E-06  100.00   0.0          100.00   0.68213E-03    1.91

      CRITICAL SPEED       = 0.25068E+04
      MAXIMUM DEFLECTION   = 0.72076E-02
      WEIGHT               = 0.54154E+02
      VOLUME               = 0.19204E+03
```

TABLE X

STEEL DRIVESHAFT COPES STANDARD OUTPUT: LOGNORMAL

OPTIMIZATION RESULTS

OBJECTIVE FUNCTION
GLOBAL LOCATION 3 FUNCTION VALUE 0.52882E+02

DESIGN VARIABLES

ID	D. V. NO.	GLOBAL VAR. NO.	LOWER BOUND	VALUE	UPPER BOUND
1	1	11	0.10000E-01	0.97581E-01	0.20000E+01
2	2	2	0.50000E+00	0.50000E+01	0.50000E+01

DESIGN CONSTRAINTS

DETERMINISTIC

ID	GLOBAL VAR. NO.	LOWER BOUND	MEAN VALUE	UPPER BOUND
1	51	-0.10000E-02	0.97054E-06	0.10000E-02
3	52	-0.10000E-02	0.0	0.10000E-02
5	53	-0.10000E-02	0.69856E-03	0.10000E-02
7	54	0.31513E+05	0.22961E+06	0.11000E+16
8	56	0.30000E+03	0.25056E+04	0.11000E+16
9	57	-0.11000E+16	0.72144E-02	0.50000E-01

TABLE XI

STEEL DRIVESHAFT COPES PROBABILISTIC OUTPUT: LOGNORMAL

PROBABILISTIC

ID	MEAN VALUE	STANDARD DEVIATION	COEFFICIENT OF VARIATION
1	0.97054E-06	0.1796E-06	0.1796E-01
3	0.0	0.0	0.0
5	0.69856E-03	0.6692E-04	0.9580E-01
7	0.22961E+06	0.1288E+05	0.5611E-01
8	0.25056E+04	0.9630E+02	0.3843E-01
9	0.72144E-02	0.5713E-03	0.7918E-01

	PROBABILITY OF FAILURE AT LOWER BOUND		PROBABILITY OF FAILURE AT UPPER BOUND	
ID	ALLOWED	CALCULATED	ALLOWED	CALCULATED
1	0.1000E-01	0.0	0.1000E-01	0.0
3	0.1000E-01	0.0	0.1000E-01	0.0
5	0.1000E-01	0.0	0.1000E-01	0.9993E-02
7	0.1000E-01	0.0	0.1000E+01	0.0
8	0.1000E-01	0.0	0.1000E+01	0.0
9	0.1000E+01	0.0	0.1000E-01	0.0

TABLE XII

STEEL DRIVESHAFT DESIGN SUMMARY: LOGNORMAL

STEEL DRIVESHAFT OUTPUT

```
     NUMBER OF PLYS               =    1
     NUMBER OF MATERIAL TYPES     =    1
     NUMBER OF LOAD CONDITIONS    =    1
     ECCENTRICITY                 =    0.0

     DIMENSIONS
     PLY   THICKNESS    PERCENT    DIAMETER     THETA
                 INSIDE 0.50000E+01
      1 0.97581E-01   100.00    0.51952E+01    0.0

     STIFFNESS
     AE = 0.51518E+08
     EI = 0.16740E+09
     GJ = 0.11718E+09


LOADS:
L.C.         T           M          F            H.P.          RPM
 1   0.31513E+05   0.0    0.50000E+02  0.15000E+03  0.30000E+03


LOAD CONDITION     1
PLY       EPL       S.F.       EPT       S.F.        EPLT         S.F.
1   0.97054E-06  100.00    0.0       100.00    0.69856E-03    1.86

     CRITICAL SPEED       = 0.25056E+04
     MAXIMUM DEFLECTION   = 0.72144E-02
     WEIGHT               = 0.52882E+02
     VOLUME               = 0.18752E+03
```

deflection an optimum of 73.78 lbs is obtained. The COPES optimization results and final design summary are contained in Tables XIII and XIV.

B. COMPOSITE DRIVESHAFT

Design, for minimum weight a 120 in. long, four ply graphite epoxy driveshaft to transmit 150.0 horsepower at 300 RPM. Figure 6.1 shows the shaft loading condition. The shaft is designed against failure in transverse, logitudinal, and shear strain; torsional buckling, vibration frequency and a maximum deflection of 0.05 inches. Table XV contains the initial design.

The design variables are the thickness of each ply, the orientation of each ply and the inside diameter. Additionally, the thickness of ply 2 and ply 3 must remain equal to each other and the orientation of ply 3 equals the negative of ply 2. The following additional constraints are placed on the design variables: The inside diameter must remain between 0.5 inches and 5.0 inches and the thickness of any given ply between 0.01 and 0.5 inches. Ply 2 is allowed to vary between 0.0 and 90.0 degrees. The orientation of plys 1 and 4 remain constant. The remaining input data is presented in Table XVI.

Allowing for maximum probabilities of failure of 0.01 in strain, torsional buckling, deflection, and vibration frequency; an optimum weight of 9.02 lbs is achieved using the normal distribution model and 10.28 lbs for the lognormal

TABLE XIII

STEEL DRIVESHAFT COPES OUTPUT: DETERMINISTIC

OPTIMIZATION RESULTS

OBJECTIVE FUNCTION
GLOBAL LOCATION 3 FUNCTION VALUE 0.73783E+02

DESIGN VARIABLES

ID	D. V. NO.	GLOBAL VAR. NO.	LOWER BOUND	VALUE	UPPER BOUND
1	1	11	0.10000E-01	0.13515E+00	0.20000E+01
2	2	2	0.50000E+00	0.50000E+01	0.50000E+01

DESIGN CONSTRAINTS

DETERMINISTIC

ID	GLOBAL VAR. NO.	LOWER BOUND	MEAN VALUE	UPPER BOUND
1	51	-0.50000E-03	0.39561E-06	0.50000E-03
3	52	-0.50000E-03	0.0	0.50000E-03
5	53	-0.50000E-03	0.50035E-03	0.50000E-03
7	54	0.63025E+05	0.51838E+06	0.11000E+16
8	56	0.60000E+03	0.25247E+04	0.11000E+16
9	57	-0.11000E+16	0.71044E-02	0.20000E-01

TABLE XIV

STEEL DRIVESHAFT DESIGN SUMMARY: DETERMINISTIC

```
STEEL DRIVESHAFT OUTPUT

     NUMBER OF PLYS              =     1
     NUMBER OF MATERIAL TYPES    =     1
     NUMBER OF LOAD CONDITIONS   =     1
     ECCENTRICITY                =     0.0

     DIMENSIONS
     PLY    THICKNESS    PERCENT    DIAMETER      THETA
                  INSIDE 0.50000E+01
      1 0.13515E+00   100.00    0.52703E+01    0.0

     STIFFNESS
     AE = 0.71879E+08
     EI = 0.23709E+09
     GJ = 0.16597E+09

LOADS:
L.C.       T              M          F              H.P.           RPM
 1   0.31513E+05   0.0    0.50000E+02   0.15000E+03   0.30000E+03

LOAD CONDITION      1
PLY      EPL         S.F.      EPT        S.F.       EPLT          S.F.
1   0.69561E-06  100.00   0.0        100.00    0.50035E-03    2.60

     CRITICAL SPEED       = 0.25247E+04
     MAXIMUM DEFLECTION   = 0.71044E-02
     WEIGHT               = 0.73783E+02
     VOLUME               = 0.26164E+03
```

TABLE XV

COMPOSITE DRIVESHAFT INITIAL DESIGN

	THICKNESS IN.	ORIENTATION DEGREES
PLY 1	.25	0.0
PLY 2	.25	20.0
PLY 3	.25	-20.0
PLY 4	.25	90.0

INSIDE DIAMETER 2.0 IN.

TABLE XVI

INPUT DATA FOR COMPOSITE DRIVESHAFT

	MEAN VALUE		COEFFICIENT OF VARIATION
HORSEPOWER	150.	HP	.08
SPEED	300.	RPM	.08
LENGTH	120.	IN.	.015
AXIAL LOAD	50.	LB.	.18
LONGITUDINAL MODULUS	21,000,000	PSI	.14
TRANSVERSE MODULUS	1,700,000	PSI	.14
SHEAR MODULUS	650,000	PSI	.14
MAJOR POSSIONS RATIO	.21		.08
STRAIN LIMITS:			
LONGITUDINAL COMPRESSIVE	-.00857	IN./IN.	.15
LONGITUDINAL TENSILE	.00857	IN./IN.	.15
TRANSVERSE COMPRESSIVE	-.0176	IN./IN.	.15
TRANSVERSE TENSILE	.00471	IN./IN.	.15
MAX SHEAR	.0184	IN./IN.	.15
SPECIFIC WEIGHT	.056	LB/CU. IN.	.05
MAX DEFLECTION	.05	IN.	.05
PLY ORIENTATION			.05
PLY THICKNESS			.015
INSIDE DIAMETER			.015
FAILURE DUE TO:			
TORSIONAL BUCKLING			.20
FREQUENCY			.06

model. The ANALIZ subroutine design summary for the initial design is presented in Table XVII. Tables XVIII through XXIII contain the design summaries and final COPES optimization results.

Designing for factors of safety against failure of 2.0 for strain, torsional buckling, deflection, and vibration frequency an optimum weight of 10.23 lbs is obtained. Tables XXIV and XXV present the deterministic design results.

TABLE XVII

COMPOSITE DRIVESHAFT INITIAL DESIGN SUMMARY

```
COMPOSITE DRIVESHAFT OUTPUT

     NUMBER OF PLYS              =     4
     NUMBER OF MATERIAL TYPES    =     1
     NUMBER OF LOAD CONDITIONS   =     1
     ECCENTRICITY                =     0.0

     DIMENSIONS
     PLY    THICKNESS    PERCENT     DIAMETER     THETA
                   INSIDE 0.20000E+01
      1 0.25000E+00     25.00     0.25000E+01     0.0
      2 0.25000E+00     25.00     0.30000E+01     0.20000E+02
      3 0.25000E+00     25.00     0.35000E+01    -0.20000E+02
      4 0.25000E+00     25.00     0.40000E+01     0.90000E+02

     STIFFNESS
     AE = 0.12143E+09
     EI = 0.12426E+09
     GJ = 0.37225E+08


LOADS:
L.C.        T          M          F             H.P.          RPM
 1   0.31513E+05   0.0    0.50000E+02   0.15000E+03   0.30000E+03

LOAD CONDITION      1
PLY      EPL        S.F.        EPT          S.F.        EPLT        S.F.
1   0.41176E-06  100.00   0.0            100.00   0.10582E-02  17.39
2   0.40847E-03   20.98  -0.40806E-03     43.13   0.97246E-03  18.92
3  -0.47576E-03   18.01   0.47617E-03      9.89   0.11351E-02  16.21
4   0.53159E-09  100.00   0.41123E-06    100.00  -0.16931E-02  10.87

     CRITICAL SPEED     = 0.19725E+04
     MAXIMUM DEFLECTION = 0.11746E-01
     WEIGHT             = 0.63334E+02
     VOLUME             = 0.11310E+04
```

TABLE XVIII

COMPOSITE DRIVESHAFT COPES STANDARD OUTPUT: NORMAL

OPTIMIZATION RESULTS

OBJECTIVE FUNCTION
GLOBAL LOCATION 3 FUNCTION VALUE 0.90203E+01

DESIGN VARIABLES

ID	D. V. NO.	GLOBAL VAR. NO.	LOWER BOUND	VALUE	UPPER BOUND
1	1	11	0.10000E-01	0.10000E-01	0.50000E+00
2	2	12	0.10000E-01	0.10212E+00	0.50000E+00
3	2	13	0.10000E-01	0.10212E+00	0.50000E+00
4	3	14	0.10000E-01	0.10000E-01	0.50000E+00
5	4	32	0.0	0.34968E+02	0.90000E+02
6	4	33	0.0	-0.34968E+02	-0.90000E+02
7	5	2	0.50000E+00	0.16811E+01	0.50000E+01

DESIGN CONSTRAINTS

DETERMINISTIC

ID	GLOBAL VAR. NO.	LOWER BOUND	MEAN VALUE	UPPER BOUND
1	51	-0.85700E-02	0.35785E-05	0.85700E-02
3	52	-0.17600E-01	0.0	0.47100E-02
5	53	-0.18400E-01	0.47679E-02	0.18400E-01
7	54	-0.85700E-02	0.25105E-02	0.85700E-02
9	55	-0.17600E-01	-0.25069E-02	0.47100E-02
11	56	-0.18400E-01	0.18288E-02	0.18400E-01
13	57	-0.85700E-02	-0.27745E-02	0.85700E-02
15	58	-0.17600E-01	0.27781E-02	0.47100E-02
17	59	-0.18400E-01	0.20319E-02	0.18400E-01
19	60	-0.85700E-02	0.18741E-08	0.85700E-02
21	61	-0.17600E-01	0.35766E-05	0.47100E-02
23	62	-0.18400E-01	-0.59688E-02	0.18400E-01
25	63	0.31513E+05	0.55160E+05	0.11000E+16
26	65	0.30000E+03	0.11715E+04	0.11000E+16
27	66	-0.11000E+16	0.34815E-01	0.50000E-01

TABLE XIX

COMPOSITE DRIVESHAFT COPES PROBABILISTIC OUTPUT: NORMAL

PROBABILISTIC

ID	MEAN VALUE	STANDARD DEVIATION	COEFFICIENT OF VARIATION
1	0.35785E-05	0.4929E-06	0.4929E-01
3	0.0	0.0	0.0
5	0.47679E-02	0.7317E-03	0.1535E+00
7	0.25105E-02	0.3838E-03	0.1529E+00
9	-0.25069E-02	0.3834E-03	0.1530E+00
11	0.18288E-02	0.4363E-03	0.2386E+00
13	-0.27745E-02	0.4221E-03	0.1521E+00
15	0.27781E-02	0.4225E-03	0.1521E+00
17	0.20319E-02	0.4938E-03	0.2430E+00
19	0.18741E-08	0.4687E-03	0.1000E+02
21	0.35766E-05	0.4687E-03	0.1000E+02
23	-0.59688E-02	0.9181E-03	0.1538E+00
25	0.55160E+05	0.7849E+04	0.1423E+00
26	0.11715E+04	0.9463E+02	0.8078E-01
27	0.34815E-01	0.6032E-02	0.1733E+00

	PROBABILITY OF FAILURE AT LOWER BOUND		PROBABILITY OF FAILURE AT UPPER BOUND	
ID	ALLOWED	CALCULATED	ALLOWED	CALCULATED
1	0.1000E-01	0.0	0.1000E-01	0.0
3	0.1000E-01	0.0	0.1000E-01	0.0
5	0.1000E-01	0.0	0.1000E-01	0.9537E-06
7	0.1000E-01	0.0	0.1000E-01	0.3338E-05
9	0.1000E-01	0.0	0.1000E-01	0.0
11	0.1000E-01	0.0	0.1000E-01	0.0
13	0.1000E-01	0.9537E-05	0.1000E-01	0.0
15	0.1000E-01	0.0	0.1000E-01	0.9468E-02
17	0.1000E-01	0.0	0.1000E-01	0.0
19	0.1000E-01	0.0	0.1000E-01	0.0
21	0.1000E-01	0.0	0.1000E-01	0.0
23	0.1000E-01	0.1001E-04	0.1000E-01	0.0
25	0.1000E-01	0.9410E-02	0.1000E+01	0.0
26	0.1000E-01	0.0	0.1000E+01	0.0
27	0.1000E+01	0.0	0.1000E-01	0.1002E-01

TABLE XX

COMPOSITE DRIVESHAFT DESIGN SUMMARY: NORMAL

COMPOSITE DRIVESHAFT OUTPUT

NUMBER OF PLYS = 4
NUMBER OF MATERIAL TYPES = 1
NUMBER OF LOAD CONDITIONS = 1
ECCENTRICITY = 0.0

DIMENSIONS

PLY	THICKNESS	PERCENT	DIAMETER	THETA
	INSIDE		0.16811E+01	
1	0.10000E-01	4.46	0.17011E+01	0.0
2	0.10212E+00	45.54	0.19054E+01	0.34968E+02
3	0.10212E+00	45.54	0.21095E+01	-0.34968E+02
4	0.10000E-01	4.46	0.21295E+01	0.90000E+02

STIFFNESS
AE = 0.13972E+08
EI = 0.63116E+07
GJ = 0.56217E+07

LOADS:

L.C.	T	M	F	H.P.	RPM
1	0.31513E+05	0.0	0.500000E+02	0.15000E+03	0.30000E+03

LOAD CONDITION 1

PLY	EPL	S.F.	EPT	S.F.	EPLT	S.F.
1	0.35785E-05	100.00	0.0	100.00	0.47679E-02	3.86
2	0.25105E-02	3.41	-0.25069E-02	7.02	0.18288E-02	10.06
3	-0.27745E-02	3.09	0.27781E-02	1.70	0.20319E-02	9.06
4	0.18741E-08	100.00	0.35766E-05	100.00	-0.59688E-02	3.08

CRITICAL SPEED = 0.11715E+04
MAXIMUM DEFLECTION = 0.34815E-01
WEIGHT = 0.90203E+01
VOLUME = 0.16[illegible]8E+03

TABLE XXI

COMPOSITE DRIVESHAFT COPES STANDARD OUTPUT: LOGNORMAL

OPTIMIZATION RESULTS

OBJECTIVE FUNCTION
GLOBAL LOCATION 3 FUNCTION VALUE 0.10281E+02

DESIGN VARIABLES

ID	D. V. NO.	GLOBAL VAR. NO.	LOWER BOUND	VALUE	UPPER BOUND
1	1	11	0.10000E-01	0.19474E-01	0.50000E+00
2	2	12	0.10000E-01	0.92916E-01	0.50000E+00
3	2	13	0.10000E-01	0.92916E-01	0.50000E+00
4	3	14	0.10000E-01	0.10000E-01	0.50000E+00
5	4	32	0.0	0.41392E+02	0.90000E+02
6	4	33	0.0	-0.41392E+02	-0.90000E+02
7	5	2	0.50000E+00	0.20465E+01	0.50000E+01

DESIGN CONSTRAINTS

DETERMINISTIC

ID	GLOBAL VAR. NO.	LOWER BOUND	MEAN VALUE	UPPER BOUND
1	51	-0.85700E-02	0.37986E-05	0.85700E-02
3	52	-0.17600E-01	0.0	0.47100E-02
5	53	-0.18400E-01	0.34484E-02	0.18400E-01
7	54	-0.85700E-02	0.18651E-02	0.85700E-02
9	55	-0.17600E-01	-0.18613E-02	0.47100E-02
11	56	-0.18400E-01	0.46804E-03	0.18400E-01
13	57	-0.85700E-02	-0.20132E-02	0.85700E-02
15	58	-0.17600E-01	0.20170E-02	0.47100E-02
17	59	-0.18400E-01	0.51418E-03	0.18400E-01
19	60	-0.85700E-02	0.12861E-08	0.85700E-02
21	61	-0.17600E-01	0.37974E-05	0.47100E-02
23	62	-0.18400E-01	-0.40960E-02	0.18400E-01
25	63	0.31513E+05	0.54975E+05	0.11000E+16
26	65	0.30000E+03	0.12564E+04	0.11000E+16
27	56	-0.11000E+16	0.29994E-01	0.50000E-01

TABLE XXII

COMPOSITE DRIVESHAFT COPES PROBABILISTIC OUTPUT: LOGNORMAL

PROBABILISTIC

ID	MEAN VALUE	STANDARD DEVIATION	COEFFICIENT OF VARIATION
1	0.37986E-05	0.5215E-06	0.5215E-01
3	0.0	0.0	0.0
5	0.34484E-02	0.5257E-03	0.1524E+00
7	0.18651E-02	0.2842E-03	0.1524E+00
9	-0.18613E-02	0.2838E-03	0.1525E+00
11	0.46804E-03	0.2815E-03	0.6015E+00
13	-0.20132E-02	0.3069E-03	0.1524E+00
15	0.20170E-02	0.3073E-03	0.1524E+00
17	0.51418E-03	0.3062E-03	0.5956E+00
19	0.12861E-08	0.3216E-03	0.1000E+02
21	0.37974E-05	0.3216E-03	0.1000E+02
23	-0.40960E-02	0.6256E-03	0.1527E+00
25	0.54975E+05	0.7813E+04	0.1421E+00
26	0.12564E+04	0.1012E+03	0.8058E-01
27	0.29994E-01	0.5135E-02	0.1712E+00

	PROBABILITY OF FAILURE AT LOWER BOUND		PROBABILITY OF FAILURE AT UPPER BOUND	
ID	ALLOWED	CALCULATED	ALLOWED	CALCULATED
1	0.1000E-01	0.0	0.1000E-01	0.0
3	0.1000E-01	0.0	0.1000E-01	0.0
5	0.1000E-01	0.0	0.1000E-01	0.0
7	0.1000E-01	0.0	0.1000E-01	0.0
9	0.1000E-01	0.0	0.1000E-01	0.0
11	0.1000E-01	0.0	0.1000E-01	0.0
13	0.1000E-01	0.0	0.1000E-01	0.0
15	0.1000E-01	0.0	0.1000E-01	0.3338E-04
17	0.1000E-01	0.0	0.1000E-01	0.0
19	0.1000E-01	0.0	0.1000E-01	0.5313E-02
21	0.1000E-01	0.0	0.1000E-01	0.9957E-02
23	0.1000E-01	0.0	0.1000E-01	0.0
25	0.1000E-01	0.1000E-01	0.1000E+01	0.0
26	0.1000E-01	0.0	0.1000E+01	0.0
27	0.1000E+01	0.0	0.1000E-01	0.1544E-02

TABLE XXIII

COMPOSITE DRIVESHAFT DESIGN SUMMARY: LOGNORMAL

COMPOSITE DRIVESHAFT OUTPUT

NUMBER OF PLYS = 4
NUMBER OF MATERIAL TYPES = 1
NUMBER OF LOAD CONDITIONS = 1
ECCENTRICITY = 0.0

DIMENSIONS

INSIDE 0.20465E+01

PLY	THICKNESS	PERCENT	DIAMETER	THETA
1	0.19474E-01	9.04	0.20855E+01	0.0
2	0.92916E-01	43.16	0.22713E+01	0.41392E+02
3	0.92916E-01	43.16	0.24571E+01	-0.41392E+02
4	0.10000E-01	4.64	0.24771E+01	0.90000E+02

STIFFNESS
AE = 0.13163E+08
EI = 0.82517E+07
GJ = 0.95289E+07

LOADS:

L.C.	T	M	F	H.P.	RPM
1	0.31513E+05	0.0	0.500000E+02	0.15000E+03	0.30000E+03

LOAD CONDITION 1

PLY	EPL	S.F.	EPT	S.F.	EPLT	S.F.
1	0.37986E-05	100.00	0.0	100.00	0.34484E-02	5.34
2	0.18651E-02	4.59	-0.18613E-02	9.46	0.46804E-03	39.31
3	-0.20132E-02	4.26	0.20170E-02	2.34	0.51418E-03	35.79
4	0.12861E-08	100.00	0.37974E-05	100.00	-0.40960E-02	4.49

CRITICAL SPEED = 0.12564E+04
MAXIMUM DEFLECTION = 0.29994E-01
WEIGHT = 0.10281E+02
VOLUME = 0.18359E+03

TABLE XXIV

COMPOSITE DRIVESHAFT COPES OUTPUT: DETERMINISTIC

OPTIMIZATION RESULTS

OBJECTIVE FUNCTION
GLOBAL LOCATION 3 FUNCTION VALUE 0.10231E+02

DESIGN VARIABLES

ID	D. V. NO.	GLOBAL VAR. NO.	LOWER BOUND	VALUE	UPPER BOUND
1	1	11	0.10000E-01	0.10000E-01	0.50000E+00
2	2	12	0.10000E-01	0.10697E+00	0.50000E+00
3	2	13	0.10000E-01	0.10697E+00	0.50000E+00
4	3	14	0.10000E-01	0.10000E-01	0.50000E+00
5	4	32	0.0	0.30524E+02	0.90000E+02
6	4	33	0.0	-0.30524E+02	-0.90000E+02
7	5	2	0.50000E+00	0.18376E+01	0.50000E+01

DESIGN CONSTRAINTS

DETERMINISTIC

ID	GLOBAL VAR. NO.	LOWER BOUND	MEAN VALUE	UPPER BOUND
1	51	-0.42850E-02	0.26961E-05	0.42850E-02
3	52	-0.88000E-02	0.0	0.23550E-02
5	53	-0.92000E-02	0.43686E-02	0.92000E-02
7	54	-0.42850E-02	0.21334E-02	0.42850E-02
9	55	-0.88000E-02	-0.21307E-02	0.23550E-02
11	56	-0.92000E-02	0.23560E-02	0.92000E-02
13	57	-0.42850E-02	-0.23496E-02	0.42850E-02
15	58	-0.88000E-02	0.23522E-02	0.23550E-02
17	59	-0.92000E-02	0.26043E-02	0.92000E-02
19	60	-0.42850E-02	0.17024E-08	0.42850E-02
21	61	-0.88000E-02	0.26944E-05	0.23550E-02
23	62	-0.92000E-02	-0.54219E-02	0.92000E-02
25	63	0.63026E+05	0.64108E+05	0.11000E+16
26	65	0.60000E+03	0.13824E+04	0.11000E+16
27	66	-0.11000E+16	0.24515E-01	0.25000E-01

TABLE XXV

COMPOSITE DRIVESHAFT DESIGN SUMMARY: DETERMINISTIC

COMPOSITE DRIVESHAFT OUTPUT

NUMBER OF PLYS	=	4
NUMBER OF MATERIAL TYPES	=	1
NUMBER OF LOAD CONDITIONS	=	1
ECCENTRICITY	=	0.0

DIMENSIONS

PLY	THICKNESS	PERCENT	DIAMETER	THETA
	INSIDE	0.18376E+01		
1	0.10000E-01	4.27	0.18576E+01	0.0
2	0.10697E+00	45.73	0.20715E+01	0.30524E+02
3	0.10697E+00	45.73	0.22855E+01	-0.30524E+02
4	0.10000E-01	4.27	0.23055E+01	0.90000E+02

STIFFNESS

AE = 0.18546E+08

EI = 0.99273E+07

GJ = 0.66999E+07

LOADS:

L.C.	T	M	F	H.P.	RPM
1	0.31513E+05	0.0	0.500000E+02	0.15000E+03	0.30000E+03

LOAD CONDITION 1

PLY	EPL	S.F.	EPT	S.F.	EPLT	S.F.
1	0.26961E-05	100.00	0.0	100.00	0.43586E-02	4.21
2	0.21334E-02	4.02	-0.21307E-02	8.26	0.23560E-02	7.81
3	-0.23496E-02	3.65	0.23522E-02	2.00	0.26043E-02	7.07
4	0.17024E-08	100.00	0.26944E-05	100.00	-0.54219E-02	3.39

CRITICAL SPEED	=	0.13824E+04
MAXIMUM DEFLECTION	=	0.24515E-01
WEIGHT	=	0.10231E+02
VOLUME	=	0.18270E+03

VII. CONCLUSIONS

Numerical optimization using probabilistic design techniques provides an effective method for designing structures based on allowed component reliability.

Existing ANALIZ subroutines can be easily modified to perform probabilistic optimization and as demonstrated, complex designs can be accomplished.

The major drawback to the program developed is the large increase in function evaluations as compared to the standard deterministic optimization. This disadvantage can be partially offset by reducing the frequency in which the constrained variable gradients are calculated.

VIII. RECOMMENDATIONS

A method of scaling the variables should be investigated to reduce ill-conditioning in the optimization process. This is a possible area to pursue in order to alleviate problems caused by very small coefficients of variation.

Further efforts should be undertaken to accomplish the reduction of function evaluations while maintaining suitable optimization results. The altering of the frequency in which constrained variable gradients are calculated appears to be the most promising approach.

In order to reduce function evaluations it is recommended that COPES be modified to allow the user to supply mathematical expressions for the partial derivatives of the constrained variables via the ANALIZ subroutine. These could be used instead of the finite different gradients calculated by COPE19/COPE20. Modifications to allow the user to supply precalculated gradients would also be helpful to this end. The sophisticated user can accomplish this now by using CONMIN directly in conjunction with COPE19, COPE21, and COPE22.

It is further recommended that additional work be undertaken to modify COPES to allow for the use of the following options with probabilistic design: A) two variable function space; B) sensitivity analysis; C) optimum sensitivity. An additional worthwhile modification would be to provide for the calculation in COPE19 and subsequent output of the standard deviation of the objective function mean value.

This investigation dealt only with component reliability in the design process. Work should be undertaken to ascertain the potential for the inclusion of system reliability considerations in the optimization process.

APPENDIX A

COPES MANUAL ADDENDUM

This appendix is intended as an addendum to the COPES Manual, [Ref. 1].

I. INTRODUCTION

The purpose of this document is to provide user instructions for performing numerical optimization using probabilistic design techniques. Four subroutines have been included in COPES (Control Program for Engineering Synthesis) to provide this additional design tool.

This discussion describes the capabilities of probabilistic optimization using the COPES/CONMIN program. A simple design example is first presented to demonstrate the program capabilities. Guidelines are given for writing analysis codes. The data organization is outlined and sample data is presented.

This publication was written to serve as an addendum to the COPES Manual; "COPES--A Fortran Program for Engineering Synthesis," L. E. Madsen and G. N. Vanderplaats, NPS69-81-003, Naval Postgraduate School, Monterey, California, March 1982.

II. DESIGN EXAMPLE

It is required to design the cantilevered beam shown in Figure 1. The objective is to find the minimum mean volume of material which will support the concentrated load and maintain an allowed probability of failure of 10%.

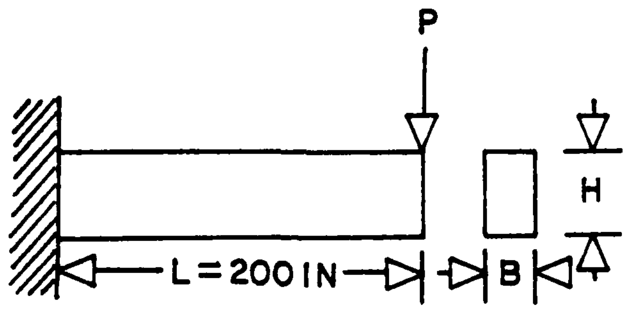


Figure 1. CANTILEVERED BEAM

That is,

$$\text{Minimize volume} = B * H * L \tag{1}$$

The mean bending stress in the beam must not exceed its limit which is 20,000 psi which has a coefficient of variation of 0.07;

$$\text{BSTRES} = \frac{Mc}{I} = \frac{6PL}{BH^2} \leq 20{,}000 \tag{2}$$

The mean shear stress must not exceed its limit of 10,000 psi which has a coefficient of variation of 0.05;

$$\text{SHRSTR} = \frac{3P}{2A} = \frac{3P}{2BH} \leq 10{,}000 \tag{3}$$

and the deflection under the load must not exceed one inch, coefficient of variation equals 0.02;

$$\text{DELTA} = \frac{PL^3}{3EI} = \frac{4PL^3}{EBH^3} \leq 1.0 \tag{4}$$

Additionally, geometric limits are imposed on the mean dimensions so that;

$$0.5 \leq B \leq 5.0 \tag{5}$$

$$1.0 < H \leq 20.0 \tag{6}$$

$$\frac{H}{B} \leq 10.0 \tag{7}$$

The manufacturing procedure is such that all dimensions have a coefficient of variation of 0.01. The mean value of the dead load is 10,000 lb. with a coefficient of variation of 0.07. The Young's modulus mean value is 30.E+06 psi with a 0.06 coefficient of variation.

The ANALIZ subroutine on page 11 of the COPES manual is used for the analysis. Only one modification is needed.

The variable P, for the load, is added to the global common block.

The COPES data used for the standard optimization, pages 60 and 61 of the COPES manual is modified to perform probabilistic optimization. NCALC in data block B is changed to 7 to indicate probabilistic optimization is to be performed and IPROB equals 1 is entered in column eight so the normal distribution model will be used. Three additional data blocks are required, U1, U2 and U3.

DATA BLOCK U1:

There are five probability variables which make up the constraint equations 2, 3, 4 and 7. They are B, H, E, AL, and P; so IVAR equals 5. It is desired that gradients of the constrained variables only be calculated at the beginning of each CONMIN iteration, therefore LGRAD equals 1 is entered in column two.

```
$ DATA BLOCK U1
5,1
```

DATA BLOCK U2:

The global location of the probability variables and their corresponding coefficients of variation are entered in this block.

```
$ DATA BLOCK U2
1,.01         width, B
2,.01         height, H
```

10,.05	load, P
8,.06	Young's mod, E
9,.01	length, AL

DATA BLOCK U3:

This block contains the allowed probability of failure and coefficient of variation at both the upper and lower bound of each constrained variable. All four constrained variables have upper bounds, but are unbounded on the lower end.

```
$ DATA BLOCK U3
1.,.07,.1,.07       BSTRES
1.,.05,.1,.05       SHRSTR
1.,.02,.1,.02       DELTA
1.,.01,.1,.01       H/B
```

The allowed probability of failure at the upper bound and the coefficient of variation are listed in columns three and four. Because there are no lower bounds the resulting probability of failure will be zero. These first two columns will be ignored during probability of failure calculations in COPES.

Coupling the ANALIZ subroutine to COPES yields the following optimum design:

CANTILEVERED BEAM

AL	=	200.0
P	=	0.10000E+05
E	=	0.30000E+08
B	=	1.92
H	=	18.75
VOL	=	7261.54
BSTRES	=	0.17802E+05
SHRSTR	=	0.41727E+03
DELTA	=	0.84382E+00
H/B	=	9.78

The COPES output for this example is provided in Figure 2.

III. PROGRAMMING GUIDELINES

The programming guidelines presented in Chapter II of the COPES manual all apply in performing probabilistic optimization. One additional rule must be followed when writing the analysis subroutine: All variables in the constraint equations which have probabilistic distributions must be included in the global common block of the analysis subroutine.

IV. DATA BLOCKS

Input instructions for the probabilistic optimization data blocks are presented in the following five pages.

DATA BLOCK B

DESCRIPTION: Program Control Parameters.

FORMAT AND EXAMPLE

1	2	3	4	5	6	7	8	FORMAT
NCALC	NDV	NSV	N2VAR	NXAPRX	IPNPUT	IPBDG	IPROB	8I10
2	2	0	0	0	0	0	1	

FIELD CONTENTS

1 NCALC: Calculation Control

0 - Read input and stop. Data of blocks A, B and V required. Remaining data is optional.

1 - One cycle through program. The same as executing ANALIZ stand-alone. Data of blocks A, B and V is required. remaining data is optional.

2 - Optimization. Data of blocks A, B and V is required. Remaining data is optional.

3 - Sensitivity analysis. Data of blocks A, B, P, Q and V is required. Remaining data is optional.

4 - Two variable function space. Data of blocks A, B and R-V required. Remaining data is optional.

5 - Optimum sensitivity. Data of blocks A-K and V is required. Remaining data is optional.

6 - Optimization using approximation techniques. Data of blocks A-O and V is required. Remaining data is optional.

7 - Probabilistic optimization. Data of blocks A-I, U1-U3 and V is required. Remaining data is optional.

2 NDV: Number of independent design variables in optimization.

3 NSV: Number of variables on which sensitivity analysis will be performed.

4 N2VAR: Number of objective functions in a two variable function space study.

5 NXAPRX: Number of X-variables for approximate analysis/optimization.

6 IPNPUT: Input print control.

0 - Print card images of data plus formatted print of input data.

1 - Formatted print only of input data.

2 - No print of input data.

7 IPDBG: Debug print control.

8 IPROB: Probability model for probabilistic optimization. Used only with NCALC = 0, 1 and 7.

1 - Normal

2 - Lognormal

DATA BLOCK U1 OMIT IF IPROB = 0 IN BLOCK B

DESCRIPTION: Probability Variables and Gradient information.

FORMAT AND EXAMPLE

1	2		FORMAT
IVAR	LGRAD		2I10
5	1		

FIELD		CONTENTS
1	IVAR:	Total number of probability variables in ANALIZ constraint equations.
2	LGRAD:	Frequency for COPES to calculate gradients of constrained variables with respect to each IVAR.

0 - Each time constraint vector required.

1 - Beginning of each CONMIN iteration.

2 - Beginning of optimization only.

DATA BLOCK U2   OMIT IF IPROB = 0 IN DATA BLOCK B

DESCRIPTION: Probability Variable Information.

FORMAT AND EXAMPLE

1	2		FORMAT
IVLOC	CVAR		I10,F10
1	.01		

NOTE: Read one card for each of the IVAR probability variables.

FIELD	CONTENTS
1	IVLOC: Global location of probability variable.
2	CVAR: Coefficient of variation of probability variable.

DATA BLOCK U3   OMIT IF IPROB = 0 IN BLOCK B

DESCRIPTION: Probability Information for Constraint bounds.

FORMAT AND EXAMPLE

1	2	3	4		FORMAT
PPF	CA	PPF	CA		4F10
1.0	0.	.1	.01		

NOTE: Read one card for each constrained variable in data block I. The first card corresponds to the first constrained variable in data block I and the second card to the second, etc.
CA is the coefficient of variation associated with the bound given in data block I.

FIELD   CONTENTS

1 - PPF: Lower bound allowed probability of failure. Enter 1.0 if corresponding BL in data block I is less than -1.0E+15.

2 - CA: Lower bound coefficient of variation. Enter .0 if corresponding BL in data block I is less than -1.0E+15.

3 - PPF: Upper bound allowed probability of failure. Enter 1.0 if corresponding BU in data block I is greater than 1.0E+15.

4 - CA: Upper bound coefficient of variation. Enter .0 if corresponding BU in data block I is greater than 1.0E+15.

V. NOTES TO THE USER

1. The probabilistic optimization feature is not compatible with the following COPES options:

   A. sensitivity analysis

   B. two variable function space

   C. optimum sensitivity

   D. approximate optimization

2. It is not recommended that one attempt to design for allowed probabilities of failure of less than 0.00001 or greater than .99999 due to modifications made to "widen" the cumulative normal distribution curve.

3. The optimization process is very sensitive to the coefficients of variation of the probability variables which make up the constraint equations. Coefficients of variation of less than 0.0001 may result in erroneous results if the coefficients of variation of some of the constraint limits are also less than 0.0001.

4. In some cases the initial design will be such that the optimization process terminates unsatisfactorily due to the calculation of zero constraint gradients. Termination will occur either due to an inability to achieve a feasible design or CONMIN will reach the maximum number of iterations no matter how large ITMAX is made. This probably will be due to the fact that the probabilities for failure calculated are either 1.0 or 0.0 and the finite difference steps are insufficient to reach the region along the cumulative

normal distribution curve between Pf = 1.0 and Pf = 0.0. This could be caused by an initial design which is a great distance from the optimum or by very small coefficients of variation. If this happens, perform a deterministic optimization to achieve a design in the vicinity in the design space desired and use this result for the initial design in probabilistic optimization.

5. In order to determine probabilities of failure, COPES calculates the gradients of the constrained variables with respect to the probability variables which make up the constraint equations. This is done by finite difference steps resulting in numerous calls to the ANALIZ subroutine. In many problems these gradients remain essentially the same so the user is given the ability to determine the frequency in which these gradients are calculated. This provides for a significant reduction in the number of function evaluations.

```
CCCCCCC   CCOOOOC   PPPPPPP   EEEEEEE   SSSSSSS
C         C     C   P     P   E         S
C         C     C   P     P   E         S
C         C     C   PPPPPPP   EEEE      SSSSSSS
C         C     C   P         E               S
C         C     C   P         E               S
CCCCCCC   COOOOOC   P         EEEEEEE   SSSSSSS


        C O N T R O L   P R O G R A M
                    F O R
   E N G I N E E R I N G   S Y N T H E S I S

                  T I T L E
CANTILEVERED BEAM ANALYSIS AND DESIGN


CARD IMAGES OF CONTROL DATA

CARD                  IMAGE

  1)  $ DATA BLOCK A
  2)  CANTILEVERED BEAM ANALYSIS AND DESIGN
  3)  $ DATA BLOCK B
  4)  $     NCALC       NDV       NSV     N2VAR    NXAPRX    IPNPUT     IPDBG     IPROB
  5)            7         2         0         0         0         0         0         1
  6)  $ DATA BLOCK C - DEFAULT ALL BUT PRINT CONTROL
  7)  $    IPRINT
  8)            1         0         0         0         0         0         0
```

Figure 2

```
$ DATA BLOCK D - ALL DEFAULTS
         0.
         0.
$ DATA BLOCK E
$     NCVTOT          IOBJ      SGNOPT
           0             3        -1.0
$ DATA BLOCK F
$        VLB           VUB
$ WIDTH, B
          .5            5.
$ HEIGHT, H
          1.           20.
$ DATA BLOCK G
$      NDSGN         IDSGN
$ WIDTH, B
           1             1
$ HEIGHT, H
           2             2
$ DATA BLOCK H
$      NCONS
           4
$ DATA BLOCK I
$       ICON          JCON
$         BL         SCAL1          BU
$ CONSTRAINT ON BSTRES
           4             4
    -1.0+20             0.      20000.
$ CONSTRAINT ON SHRSTR
           5             5
    -1.0+20             0.      10000.
$ CONSTRAINT ON DELTA
           6             6
    -1.0+20             0.          1.
$ CONSTRAINT ON H/B
           7             7
    -1.0+20             0.         10.
```

```
$ DATA BLOCKS J-U ARE NOT REQUIRED
$ DATA BLOCK U1
$       IVAR     LGRAD
          5         1
$ DATA BLOCK U2
$      IVLOC          CVAR
$ WIDTH, B
          1          .01
$ HEIGHT, H
          2          .01
$ LOAD, L
         10          .05
$ YOUNGS MOD, E
          8          .06
$ LENGTH, AL
          9          .01
$ DATA BLOCK U3
$          LOWER BOUND          UPPER BOUND
$         PPF          CA       PPF          CA
$ BSTRES
         1.0          .07        .1          .07
$ SHRSTR
         1.0          .05        .1          .05
$ DELTA
         1.0          .02        .1          .02
$ H/E
         1.0          .01        .1          .01
$ DATA BLOCK V
$ REQUIRED END CARD
END
```

```
TITLE:
CANTILEVEREC BEAM ANALYSIS AND DESIGN

CONTROL PARAMETERS:
CALCULATICN CONTROL,                     NCALC =     7
NUMBER OF GLOBAL DESIGN VARIABLES,         NDV =     2
NUMBER OF SENSITIVITY VARIABLES,           NSV =     0
NUMBER OF FUNCTICNS IN TWO-SPACE,        N2VAR =     0
NUMBER OF APPROXIMATING VAR.            NXAPRX =     0
INPUT INFORMATION PRINT CODE,           IPNPUT =     0
DEBUG PRINT CODE,                        IPDBG =     0
PROBABILISTIC DESIGN CODE,               IPRCB =     1

CALCULATICN CONTROL, NCALC
VALUE    MEANING
  1      SINGLE ANALYSIS
  2      OPTIMIZATION
  3      SENSITIVITY
  4      TWC-VARIABLE FUNCTION SPACE
  5      OPTIMUM SENSITIVITY
  6      APPRCXIMATE OPTIMIZATION
  7      PRCBABLISTIC OPTIMIZATION


* * CPTIMIZATION INFORMATION

GLOBAL VARIABLE NUMBER OF OBJECTIVE              =    3
MULTIPLIER (NEGATIVE INDICATES MINIMIZATION) = -0.1000E+01

CONMIN PARAMETERS (IF ZERO, CONMIN DEFAULT WILL OVER-RIDE)

IPRINT  ITMAX   ICNDIR   NSCAL   ITRM   LINOBJ  NACMX1  NFDG
  1       C       0        0       0      0       4       0

 FDCH            FDCHM            CT              CTMIN
 0.0             0.0              0.0             0.0

 CTL             CTLMIN           THETA           PHI
 0.C             0.0              0.0             0.0

 DELFUN          DABFUN           ALPHAX          ABOBJ1
 0.C             0.0              0.0             0.0
```

DESIGN VARIABLE INFORMATION
NON-ZERO INITIAL VALUE WILL OVER-RIDE MODULE INPUT

D. V. NO.	LOWER BOUND	UPPER BOUND	INITIAL VALUE	SCALE
1	0.50000E+00	0.50000E+01	0.0	0.0
2	0.10000E+01	0.20000E+02	0.0	0.0

DESIGN VARIABLES

ID	D. V. NO.	GLOBAL VAR. NO.	MULTIPLYING FACTOR
1	1	1	0.10000E+01
2	2	2	0.10000E+01

CONSTRAINT INFORMATION

THERE ARE 4 CONSTRAINT SETS

ID	GLOBAL VAR. 1	GLOBAL VAR. 2	LINEAR ID	LOWER BOUND	NORMALIZATION FACTOR	UPPER BOUND	NORMALIZATION FACTOR
1	4	4	0	-0.11000E+16	0.11000E+16	0.20000E+05	0.20000E+05
2	5	5	0	-0.11000E+16	0.11000E+16	0.10000E+05	0.10000E+05
3	6	6	0	-0.11000E+16	0.11000E+16	0.10000E+01	0.10000E+01
4	7	7	0	-0.11000E+16	0.11000E+16	0.10000E+02	0.10000E+02

TOTAL NUMBER OF CONSTRAINED PARAMETERS = 4

* * INPUT DATA FOR PROBABILISTIC DESIGN

NORMAL DISTRIBUTION

TOTAL NUMBER OF PROBABILITY VARIABLES IN CONSTRAINT EQUATIONS = 5

VARIABLE ID	GLOBAL LOCATION	COEFFICIENT OF VARIATION
1	1	0.10000E-01
2	2	0.10000E-01
3	10	0.50000E-01
4	8	0.60000E-01
5	9	0.10000E-01

CONSTRAINED PARAMETER	ALLOWED PROBABILITY OF FAILURE		COEFFICIENT OF VARATION	
ID	LOWER BOUND	UPPER BOUND	LOWER BOUND	UPPER BOUND
1	0.10000E+01	0.10000E+00	0.70000E-01	0.70000E-01
2	0.10000E+01	0.10000E+00	0.50000E-01	0.50000E-01
3	0.10000E+01	0.10000E+00	0.20000E-01	0.20000E-01
4	0.10000E+01	0.10000E+00	0.10000E-01	0.10000E-01

* * ESTIMATED DATA STORAGE REQUIREMENTS

REAL			INTEGER		
INPUT	EXECUTION	AVAILABLE	INPUT	EXECUTION	AVAILABLE
58	212	10000	26	26	1000

```
CANTILEVERED BEAM

AL       =   200.00
P        = 0.10000E+05
E        = 0.30000E+08

B        =     2.50
H        =    10.00


CANTILEVERED BEAM

AL       =   200.00
P        = 0.10000E+05
E        = 0.30000E+08

B        =     2.50
H        =    10.00

VOL      = 5000.00

BSTRES   = 0.48000E+05
SHRSTR   = 0.60000E+03
DELTA    = 0.42667E+01
H/B      =     4.00
```

```
* * * * * * * * * * * * * * * * * * * * * * * * * * *
*                                                   *
*                    C O N M I N                    *
*                                                   *
*               FORTRAN PROGRAM FOR                 *
*                                                   *
*        CONSTRAINED FUNCTION MINIMIZATION          *
*                                                   *
* * * * * * * * * * * * * * * * * * * * * * * * * * *

INITIAL FUNCTION INFORMATION

OBJ =    0.500000E+04

DECISION VARIABLES (X-VECTOR)
  1)    0.25000E+01  0.10000E+02

CONSTRAINT VALUES (G-VECTOR)
  1)    0.89999E+01 -0.99996E+00  0.89999E+01 -0.10000E+01

FINAL OPTIMIZATION INFORMATION

OBJ =    0.718959E+04

DECISION VARIABLES (X-VECTOR)
  1)    0.19170E+01  0.18752E+02

CONSTRAINT VALUES (G-VECTOR)
  1)   -0.18656E-02 -0.99996E+00 -0.77823E+00  0.21935E-04

THERE ARE    2 ACTIVE CONSTRAINTS
CONSTRAINT NUMBERS ARE
    1    4

THERE ARE    0 VIOLATED CONSTRAINTS

THERE ARE    0 ACTIVE SIDE CONSTRAINTS

TERMINATION CRITERION
     ABS(1-OBJ(I-1)/OBJ(I)) LESS THAN DELFUN FOR  3 ITERATIONS
     ABS(OBJ(I)-OBJ(I-1))   LESS THAN DABFUN FOR  3 ITERATIONS

NUMBER OF ITERATIONS =   17
OBJECTIVE FUNCTION WAS EVALUATED            75  TIMES
CONSTRAINT FUNCTIONS WERE EVALUATED         75  TIMES
```

AD-A125 232 PROBABILISTIC DESIGN USING NUMERICAL OPTIMIZATION(U) NAVAL POSTGRADUATE SCHOOL MONTEREY CA J H HOPPER OCT 82

2/2

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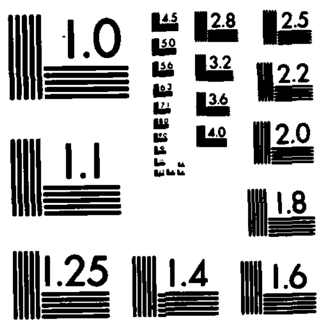

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

CPTIMIZATICN RESULTS

OBJECTIVE FUNCTICN
GLOBAL LOCATION 3 FUNCTION VALUE 0.72615E+04

DESIGN VARIABLES

IC	D. V. NO.	GLOBAL VAR. NO.	LOWER BOUND	VALUE	UPPER BOUND
1	1	1	0.50000E+00	0.19170E+01	0.50000E+01
2	2	2	0.10000E+01	0.18752E+02	0.20000E+02

DESIGN CONSTRAINTS

DETERMINISTIC

ID	GLOBAL VAR. NC.	LOWER BOUND	MEAN VALUE	UPPER BOUND
1	4	-0.11000E+16	0.17802E+05	0.20000E+05
2	5	-0.11000E+16	0.41727E+03	0.10000E+05
3	6	-0.11000E+16	0.84383E+00	0.10000E+01
4	7	-0.11000E+16	0.97820E+01	0.10000E+02

PRCBABILISTIC

ID	MEAN VALUE	STANDARD DEVIATION	COEFFICIENT OF VARIATION
1	0.17802E+05	0.9889E+03	0.5555E-01
2	C.41727E+03	0.2167E+02	0.5193E-01
3	C.84383E+00	0.7505E-01	0.8894E-01
4	C.97820E+01	0.1377E+00	0.1407E-01

	PRCBABILITY OF FAILURE AT LOWER BOUND		PROBABILITY OF FAILURE AT UPPER BOUND	
ID	ALLCWED	CALCULATED	ALLOWED	CALCULATED
1	0.100CE+01	0.0	0.1000E+00	0.9981E-01
2	0.100CE+C1	0.0	0.1000E+00	0.0
3	C.100CE+01	0.0	0.1000E+00	0.2218E-01
4	0.1000E+01	0.0	0.1000E+00	0.1000E+00

```
CANTILEVERED BEAM

AL     =   200.00
P      = 0.10000E+05
E      = 0.30000E+08

B      =     1.92
H      =    18.75

VOL    = 7261.48

BSTRES = 0.17802E+05
SHRSTR = 0.41727E+03
DELTA  = 0.84383E+00
H/B    =     9.78


PROGRAM CALLS TO ANALIZ

   ICALC   CALLS
     1       1
     2     256
     3       2
```

LIST OF REFERENCES

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