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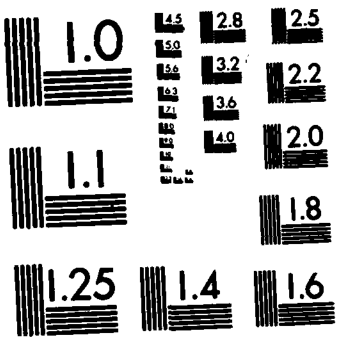

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Magneto Gas Dynamics Model
of the
Inter-Electrode Flow
in a
Pulsed Plasma Thruster

Final

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A set of basic equations necessary to model the acceleration of the plasma produced by a millipound pulsed plasma thruster is presented and discussed. Various simplifying assumptions and approximations typically made in the derivation of magnetogasdynamic equations are examined in light of available experimental data. In particular, it is shown that the mean free path and Debye shielding length are appropriate to the assumption of a neutral continuum with pressure and Lorentz force dominating the acceleration. The neglect of displacement current is also justified and a case is made for the

assumption of scalar rather than tensor thermal and electrical conductivities and the neglect of viscous effects.

The possibility of describing the flow with a model of fewer than three dimensions is discussed and a drastically simplified one-dimensional model is presented, along with a discussion of its limitations and shortcomings.

A finite difference algorithm for the one-dimensional model is presented, but difficulties, apparently arising from incorrect boundary conditions prevented meaningful numerical experimentation. Additional investigation of this problem has been undertaken through a program at the Air Force Rocket Propulsion Laboratory, and the results will appear in an AFRPL report.

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ABSTRACT

A set of basic equations necessary to model the acceleration of the plasma produced by a millipound pulsed plasma thruster is presented and discussed. Various simplifiying assumptions and approximations typically made in the derivation of magnetogasdynamic equations are examined in light of available experimental data. In particular, it is shown that the mean free path and Debye shielding length are appropriate to the assumption of a neutral continuum with pressure and Lorentz force dominating the acceleration. The neglect of dispalcement current is also justified and a case is made for the assumption of scalar rather than tensor thermal and electrical conductivities and the neglect of viscous effects.

The possibility of describing the flow with a model of fewer than three dimensions is discussed and a drastically simplified one-dimensional model is presented, along with a discussion of its limitations and shortcomings.

A finite difference algorithm for the one-dimensional model is presented, but difficulties, apparently arising from incorrect boundary conditions prevented meaningful numerical experimentation.

I. INTRODUCTION

The Teflon PPT has been around for some fifteen years. Small (approximately one micropound thrust) versions have been proved feasible on satellites. Larger versions are currently being used, with one to be monitored for contamination on a satellite to be launched in the fall of 1982. The thruster of primary concern in this report is the much larger (one millipound thrust) model which is still in the process of being developed.

The principles of PPT operation are described in several of the references (see refs 1, 2) and only a very brief outline is given here for completeness. Basically a PPT consists of a pair of bar electrodes connected by a capacitor bank with external power supply, and a feed system which forces the teflon "fuel" into position between the electrodes. This is shown schematically in Figure 1. The capacitor is charged between firings, and the directly coupled electrodes are charged as well. A minute amount of material is introduced into the electrode gap allowing an arc discharge. The high temperature arc across the face of the teflon ablates, dissociates and ionizes a small quantity of teflon (producing a carbon-fluorine plasma) which supports the continuing discharge. The current flow induces a high magnetic field and reacts with that field to accelerate the plasma to velocities of several thousand meters per second, thus providing the thrust.

The very high exhaust velocity produces one of the PPT's most promising features, a specific impulse several times larger than that of chemical rockets. The exhaust itself poses problems, however. A serious concern of the satellite designers is the contamination potential of the exhaust plume. The plume of the PPT is known to contain ionized carbon and flourine primarily, but additional small quantities of heavy molecules (metals and metal oxides stripped from the electrodes and insulators as well as flourocarbons) probably also exist. It is important to determine how much of what exhaust products may reach sensitive satellite surfaces.

The obvious potential of the PPT for long life, dependable, accurate, and very efficient attitude control and station keeping functions, and its apparent potential as a contamination source, have been the subjects of a considerable amount of research. Reference 1 describes much of the work pertinent to the present study, and that material will not be reviewed in detail here, except to note that references 2 and 3 present attempts to model the internal and external plasma flow respectively. These models were adopted as the basis of the investigation presented here, and will be discussed in some detail below. The internal model was developed for a thruster with an entirely different feed system and so is inapplicable to the present device in some respects.

Important new experimental data are now available from investigations performed at the Jet Propulsion Laboratory and the Arnold Engineering Development Center (references 4, 5 and 6) which have enabled additional critical evaluation of the models. This information is discussed below in conjunction with its consequences with regard to various aspect of the model.

2. THRUSTER OPERATION

Figure 1 shows the millipound thruster. The teflon bars are forced in against stops machined on the copper anode. The igniter is fired, releasing a minute quantity of ions and electrons into the space between the charged electrodes which then begin to discharge. The discharge initiates the ablation, dissociation and ionization of the face of the teflon bars. The plasma thus formed supports the continuing current discharge which interacts with the induced magnetic field to accelerate the plasma.

The complete electrical circuit (consisting of capacitor bank, electrodes, and plasmoid, plus connections) behaves as a series R, L, C circuit. That is, the current exhibits damped sinusoidal variation as shown in Figure 2. The contribution of the plasma to the total circuit resistance and inductance should be variable since the amount of plasma in the interelectrode gap changes with time. (The contribution to the capacitance is negligible.) As shown in the figure, however, the data agree remarkably well with the simple damped sine wave obtained by assuming constant circuit inductance and resistance. The coefficients shown in the figure were determined by matching the data to the equation and these can be used to determine the circuit parameters (see reference 6)

$$C = 3.2(10^{-8})\text{farad}$$

$$R = 1.3(10^{-2})\text{ ohm}$$

$$L = 9.7(10^{-8})\text{ Henry}$$

It is possible that, at least during the first cycle (during which nearly all the plasma is formed), the plasma components of resistance and inductance are small compared to those of the fixed portion of the circuit, so that the totals are essentially constant. During the second current cycle very little additional teflon is ablated, and even less is ionized; thus the resistance is significantly increased and the damping is strongly enhanced. Similar data taken with smaller capacitors installed show this latter effect more strongly.

If one assumes that the amount of teflon ablated per half-cycle of current discharge is proportional to the integral of the square of the current over that half-cycle (which is related to the energy consumed) one finds that some 92.1% is ablated during the first half-cycle, and 7.3% in the next half-cycle, leaving less than 1% for the remainder of the discharge. As will be discussed in Section 4.1, below, such a drastic difference in plasma densities makes it unlikely that a single theoretical model can be developed to describe the entire process.

Further support for this assertion was sought by analyzing the (unpublished) data obtained by discharging the thruster through an aluminum shorting bar across the electrodes. This test was run in the atmosphere, and the current history is again closely approximated by a damped sine wave. The parameters calculated for this circuit (with the shorting bar replacing the plasma) are

$$C = 3.2\ (10^{-8})\ \text{farad}$$

$$R = 1.4\ (10^{-2})\ \text{ohm}$$

$$L = 2.1\ (10^{-7})\ \text{Henry}$$

The fact that the resistance agrees well with that obtained from the plasma forming discharge strengthens the assertion that the resistance of the plasma itself is small. The significant (factor of two) increase in the inductance may be due primarily to the increase in physical size of the current loop. (The shorting bar was located at the very end of the electrodes).

The field vectors in Figure 1 indicate the direction of the major field components during the first half-cycle of the discharge. It should be noted that the z-component of the magnetic field vector changes sign somewhere between the origin and the leading edge of the plasmoid. In the second half-cycle the J and B vectors reverse, but the acceleration vector does not. Reference 6 provides field data for the millipound thruster which appear to agree qualitatively with those for the micropound thruster given in Reference 2.

3. MODEL DEVELOPMENT - OVERVIEW

A complete theoretical model of the PPT operation and plume can be partitioned into an internal model which would describe the circuit discharge, teflon chemistry and plasma acceleration, and an external model which would describe the plume expansion beyond the ends of the electrodes. The internal model must realistically represent the physics of the plasma flow using appropriate field equations for the fluid flow parameters (densities, velocities, pressures, temperatures) and the electromagnetic field variables (electric field, magnetic field, current density, etc.). Also required are constitutive equations which describe the nature of the medium (transport coefficients, etc.), equations to model the formation of the plasmoid (ablation, ionization), and boundary conditions. The external model will also deal with plasma flow. However, since the plasmoid expands rapidly into a vacuum, the density quickly becomes so low that the continuum equations of the internal model are no longer appropriate. Thus, a different approach is needed. (This may also be true for the interelectrode flow of all but the first one or two half-cycles of the discharge.) In addition, if the model is to be used in conjunction with ground tests, chamber effects must be considered.

4. MODEL DEVELOPMENT - INTERNAL MODEL

4.1 Field Equations

Basically, the plasma flow variables are governed by the time dependent conservation equations (mass, momentum, energy) and the Maxwell electromagnetic field equations. Before a detailed discussion of these, however, some attention should be paid to the medium (i.e., the nature of the plasmoid).

The plasma produced by the PPT discharge is a complex medium which, although the subject of several investigations, has yet to be fully characterized or understood. The principal constituents are carbon, flourine, and flourocarbons which originate in the teflon. These atoms and molecules may be neutral or ionized. In addition, elements stripped from the electrodes and other parts of the thruster may contribute.

Reference 6 presents spectroscopic data (in the UV and VUV range) and identifies at least the following emitters in the plasmoid at the exit plane.

C^{o}, C^{+}, C^{++}, F^{o}, F^{++}, Cu^{+}, Al^{+}

Previous studies have reported additional ions. The copper is apparently from the electrodes while the aluminum probably originates as an oxide in the ceramic insulator which protects the sides of the teflon bars from the hot plasma.

Several observations can be made from the spectroscopic data. First, the radiation is extremely intense, especially in the VUV range where it is estimated to be on the order of 10^6 times the solar VUV flux. An estimate of the temperature is also given to be about 40,000 K.

The spectra exhibit sharp, well defined peaks characte istic of atomic rather than molecluar emission although flourocarbons do emit in the 2300-3300 Å range (reference

6). This would seem to indicate that the plasma environment is too severe for the heavy molecules to survive, or that dissociation is complete.

The study also presents data obtained by monitoring the temporal history of several individual emission lines at the thruster exit plane. All of the lines monitored exhibit a strong pulse between eleven and nineteen microseconds into the cycle and a second much smaller (by as much as a factor of ten), pulse between thirty and thiry five microseconds. The species so monitored include aluminum, neutral and ionized carbon, and flourine. All pulses are quite similar in character and timing.

Several deductions about the nature of the plasma can be made from the above observations. The most important is that the flow is dominated by collision effects. That this must be the case is evidenced by the fact that atoms with vastly different (even zero) charge to mass ratio pass the exit plane at the same time, and apparently with quite similar velocities. In addition, this information implies that, although the plasmoid is highly complex mixture of species, a model which considers it to be a three, or even one, component fluid may be adequate to give reasonable accuracy.

A three component model might consist of neutrals, ions, and electrons, as suggested in reference 2. There it was proposed that an average molecular weight of 16.67 be used for the ions and neutrals (this being the weighted average for two flourines to one carbon), and that the number densities of ions and elections are equal (assuming charge neutrality and single ionization).

In this author's opinion, we know both too much and too little to adopt this model. First, it is apparent that single ionization is not a good assumption. Also, it is quite likely that the average molecular weights of the ion gas and the neutral gas are not equal, since the ionization potentials of carbon and flourine are quite different. On the other hand, sufficient data are not available to enable one to predict with any confidence the energy transfer between the various gas components. Reference 2 presents much pertinent data, such as dissociation and ionization energies, as well as a model for ionization rates, etc. Of importance here is the fact that the total energy required to ablate, dissociate and ionize the total mass of teflon is a small fraction of that available, or that required to accelerate the plasma to such high velocities. Since reaction rates, collision cross sections, etc., are not known, it seems inappropriate to include such refinements in a model at this time. In fact, no data are available with which to compare the output of such a model. It thus would seem preferable, as well as much simpler, to model the plasma as a single conducting gas. In this case, the conservation equations consist of one mass, one to three momentum (depending on dimensionality of the model), and one energy equation.

There are, of course, inaccuracies built in to such a one fluid model. For example, doppler shift spectroscopy measurements (references 6 and 7) seem to indicate somewhat different velocities for different species. These data, however, are not entirely consistent and are difficult to interpret. Another anomaly is demonstrated by the data taken at different positions relative to the thruster axis (Reference 5). These show considerable variation in the intensity of certain emission lines with position. Again, interpretation is difficult.

Some approximate calculations have been performed to check the validity of the assertions above. The mean free path in the plasma is essentially proportional to the inverse of the number density, n, whereas the Debye length is proportional to the square root of temperature/number density. Total collision cross sections of carbon and or flourine atoms are not available but assumed diameters on the order of 10^{-10} m give a mean free path λ of about $10^{19}/n$ m. The Debye length is given by

$$\lambda_D = \epsilon_0 K T / n_i e^{2\ 1/2}$$

where ϵ_0 is the free space permittivity, K is Boltzman's constant, T is temperature, n_i is the ion number density and e is the electron charge. This formula yields.

$$\lambda_D = 69(T/n)^{1/2} \text{ meters}$$

with T in Kelvin.

To approximate the number density in the plasmoid one can refer to photographic evidence. At 11 sec after initiation the plasmoid just about fills the interelectrode volume, and about half of the first half-cycle energy has been deposited. Assuming, then, that about .7mg of material with an average molecular weight of 16.67 occupies a volume of $1.5(10^{-4})$ m^3 yields approximately $2(10^{23})$ for the number density of the first plasmoid. These approximations then yield a mean free path λ of $5(10^{-5})$ m. The additional assumptions that the plasmoid is about 20% ionized yields a Debye length of $8(10^{-8})$ m. The first figure alows the use of continuum equations as λ is small compared to physical dimensions, and since $\lambda_D \ll \lambda$ the assumption of charge neutraility is also valid.

It must be noted, however, that the above computations are valid only for the first plasmoid (that produced by the first half-cycle of the discharge). As the ablated mass appears to decrease by an order of magnitude with each successive half-cycle the mean free path will increase accordingly, and certainly by the third half-cycle the continuum hypothesis breaks down. The Debye length also increases, but inversely as the root of the density. Thus a model developed to describe the first plasmoid may not be accurate for the second one and certainly will not apply to any subsequent emissions. Unfortunately, from the viewpoint of contamination analysis, it may well be that the latter portion of the thruster exhaust is of prominent importance.

For the reasons given above it is suggested that the one fluid model be developed further. For such a model the governing equations respresent conservation of mass, energy, and three components of momentum, coupled with Maxwell's equations of electromagnetics. These can be written as:

$$\rho_t + \mathrm{div}\,(\rho\,\underline{v}) = 0$$

$$e_t + \underline{v} \cdot \mathrm{grad}\ e = \text{conduction} + \text{dissipation effects}$$

$$\rho\,\underline{v}_t + \rho\underline{v} \cdot \mathrm{grad}\ v + \mathrm{grad}\ p = \text{Lorentz body force} + \text{viscous forces}$$

$$\mathrm{div}\ \underline{E} = 0$$

$$\mathrm{Curl}\ \underline{E} = -\underline{B}_t$$

$$\mathrm{div}\ B = 0$$

$$\mathrm{curl}\ \underline{B} = \mu(\underline{J} - \epsilon_o\,\underline{E}_t)$$

Here ρ = gas density

$\underline{v}$ = gas mean velocity

t = time and $(\ )_t$ represents partial differentiation w.r.t.t

e = energy per unit mass

p = gas pressure

$\underline{E}$ = electric field intensity

$\underline{B}$ = magnetic flux density

$\underline{J}$ = current density

μ = magnetic permeability

ϵ_0 = vacuum permittivity.

To these equations must be coupled constitutive equations which further describe the medium itself. These are discussed in the next section.

4.2 Constitutive Equations

In the most general one fluid plasma models with strong magnetic fields present the constitutive equations are tensor relationships. That is, such plasmas are non-isotropic, with the direction of the magnetic flux density vector $\underline{B}$ being a "preferred direction" in the sense that material properties in this direction differ from those in perpendicular directions. The properties involved include both electric and thermal conductivities and the viscosity. These control the right hand sides of the momentum and energy equations above and, therefore, some preliminary analysis is in order at this time.

Any computation of the plasma transport coefficients is, at best, an estimate, but, since no experimental data exist for a plasma resembling that produced by the PPT, one is required to make such an estimate. Reference 8 presents a theory for computing transport coefficients for a two component plasma (singly charged ions and electrons). In particular, this theory allows one to estimate the different components of the transport tensors.

The strength of the magnetic field is conveniently described by the cyclotron frequency for the ions

$$\Omega_i = eB/m_i$$

where e is the ionic charge and m_i the mass. This is the frequency at which a single free ion would orbit. For a singly charged ion of atomic mass number 16.67 (mass = 2.88(10^{-26}) kg) and a field of about .5 Telsa (which is appropriate for the first plasmoid) one obtains

$$\Omega_i = 3\,(10^6)\ \text{sec}^{-1}$$

A measure of the collision effect is a collision time which can be estimated from

$$\tau_i = \frac{40\pi\epsilon_0 m_i^{1/2}(KT)^{3/2}}{n_i e^4 \log\Lambda}$$

where $\Lambda = 1.24\,(10^7)\, T^{3/2} n^{-1/2}$

For a temperature of 40,000 $^{\circ}$K and a number density of 10^{23} the above yields a collision time of about 10^{-9} sec. Since the orbital frequency $\Omega_i = 3(10^6)\ \text{sec}^{-1}$, it is apparent that an ion would experience many collisions per orbit. This calculation substantiates the assertion above that the process is collision dominated, and also implies that a scalar approximation for the transport coefficients is appropriate. Since the collisions are randominizing events, the "preference" for the direction parallel to the magnetic field is lost. When an ion begins to react to the presence of the field it suffers a collision and is redirected.

It still remains to provide some model for the magnitudes of the scalar transport coefficients. Theoretical estimates based on kinetic theory are available, but all such theories presume plasmas of a much less complex composition than the one produced by the PPT. Thus any model chosen will import a serious uncertainty on the calculations, unless it is found that the coefficeints are of such a magnitude that variations do not alter the flow in a significant fashion. Otherwise, some experimental verification of the models will be needed.

Reference 8 provides some formulae for transport coefficients, based on a two-component plasmas (i.e., highly ionized), as follows

$$\sigma = .74\,\frac{ne^2\tau_e}{m_e} \qquad \text{= electrical conductivity}$$

$$\kappa = 2.9\,\frac{nK^2T\tau_e}{m_e} \qquad \text{= thermal conductivity}$$

$$\eta = .5\,nKT\tau_i \qquad \text{= viscosity}$$

where τ_e is the collision time for the electrons and is equal to $(Me/M_i)^{1/2}\tau_i$. Using these formulae with numerical values appropriate to the first plasmoid yields the following order of magnitude estimates

$\sigma = 10^4$ mhos/m

$\kappa = 10$ J/sec °K

$\eta = 10^{-8}$ N·S/m^2

To examine the validity of these estimates one must rely on some very rough comparisons with available data. For example, in the case of conductivity, if it assumed that the plasmoid provides a 20cm^2 cross section for current flow between the electrodes which are about 8cm apart, the above conductivity yields a total resistance of about $4(10^{-3})\,\Omega$. This is about one-fourth of the total circuit resistance as determined by the current discharge measurements (see section 2.0 above), which is probably high, but not completely unreasonable according to the earlier assertion that the plasma resistance is small.

The viscosity above, along with appropriate values of velocity (10,000 m/s) and density (3×10^{-3}kg/m^2) gives a Reynolds number

$$Re = 10^8$$

which implies that viscous forces are negligibly small except within an extremely thin boundary layer along the electrodes. It is thus proposed to ignore viscous terms in the model. The only experimental verification of this assumption is that photographs of the plasmoid seem to indicate fairly uniform velocity profiles which would not occur under the action of strong viscous effects.

With nothing better than a rough global estimate of temperature, no similar conclusions may be drawn concerning heat transfer. In fact, considering the intensity of both the UV emission and visible emission from the plasmoid; it is possible that radiation is more important in heat transfer than thermal conduction.

Thus, at this point, the choice of transport coefficient models is not entirely clear. A reasonable approach would seem to be to use the above expressions for electrical and thermal conductivity and to ignore viscous effects. (Note that, unlike oridinary gases, plasmas conduct heat primarily through the electrons and therefore thermal conduction and viscous diffusion are not coupled by a Prandtl number near unity).

One further simplification of the model equations is usually made in magnetodydrodynamic theory, namely the neglect of the displacement current, $\epsilon_0 \underline{E}_t$, compared with the conduction current $\underline{J}$. The magnitudes of the variables in the first PPT plasmoid are completely compatible with this simplification (which is quite significant), but some caution is required. In the vacuum ahead of and behind the plasmoid there can be no conduction current. The electric and magnetic fields, however, do not vanish (or become uniform). Disturbances propagate with the velocity of light, which is much higher than either the ion velocity or the Afven wave propagation in the plasma. Thus, the MHD equations can be used within the plasmoid, but the computation domain must contain only that region in space so occupied. The boundaries of the computations domain thus move and must be relocated at each time step. The boundary conditions applied at these boundaries must consist of matching the field variables across the plasmoid/vacuum interface.

Applying the simplifications above to the general equations, eliminating the electric field using the generalized Ohm's law with scalar conductivity

$$\underline{E} = \underline{B} \times \underline{V} + \underline{J}/\sigma$$

and eliminating the current density using

$$\underline{J} = \frac{1}{\mu} \text{curl } \underline{B}$$

(which is a result of neglecting displacement current) produces a familiar set of equations

$$\rho_t + \text{div}(\rho \underline{V}) = 0$$

$$\rho \underline{v}_t + \rho \underline{v} \cdot \text{grad } \underline{v} + \text{grad } \underline{v} \text{ grad } p = (\underline{E} + \underline{V} + \underline{B}) \times \underline{B}$$

$$\rho (e + \underline{v} \cdot \underline{v}/2)_t + \rho \underline{v} \cdot \text{grad } (e + \underline{v} \cdot \underline{v}/2) \underline{v}) + \text{div } (p \underline{v}) = \sigma (\underline{E} + \underline{V} \times \underline{B}) \cdot \underline{E} + T_{xx}$$

To these must be added the algebraic equations of state

$$p = \rho RT$$

$$e = C_p T$$

and either the above expressions for σ and κ or some other model. The state equations here assume a perfect gas, and assuming a monatomic gas with atomic weight 16.67 gives $R = 500$, $C_p = 2.5R$.

4.3 One Dimensional Model

This model should provide a reasonable approximation to reality. For the purpose of the present study, the model was simplified still further for numerical implementation. First, the dimensionality of the model was reduced to one by neglecting variations in the y and z directions and considering only the x-component of $\underline{V}$, the y - component of $\underline{J}$, and the Z-component of $\underline{B}$. It is apparent that, while these are indeed the dominant components of these vectors, the one dimensional model can provide only a general description of the process. Because of this limitation and the uncertainty in the transport coefficients and , the dissipation (Joule heating) and heat conduction terms were dropped and temperature was taken to be constant. The one-dimensional, isothermal model is governed by the equations

$$\rho_t + (\rho u)x = S$$

$$(\rho u)_t + p_x = \frac{1}{\mu} BB_x$$

$$B_t + (uB)_x = 0$$

$$p = \rho RT (T \text{ constant})$$

Here the source term S must be introduced since the addition of plasma due to the ablation and ionization of teflon occurs over the first few centimeters in the x-direction. In the full three-dimensional model this material addition would appear as a boundary condition rather than in the model equations.

The source function is one of the weakest points in the model development (whether as here or as a boundary condition). The rate of ablation of the teflon must depend on the energy flux at the surface. Unfortunately, energy transfer in the plasma is not well understood. Conductivity models have not been verified experimentally, temperatures have not been measured, radiation has not been measured in the IR range, etc. In addition, ablation rates are known only at much lower temperatures. Thus, the ablation model is at this time very uncertain. For the purpose of testing the one dimensional isothermal model, it was assumed that the source term in the mass conservation equation is proportional to the square of the current density at a particular value of x, as long as $x<4.5$cm (the extent of the teflon surface). Even here the proportionality factor is a matter of conjecture and numerical experimentation.

It should be noted here that the idea of coupling the ablation rate to the total current flow at any time (as proposed in reference 2) is not appropriate, especially for the side feed thruster currently under consideration; the ablation is a much more local phenomenon as the teflon fuel cannot "see" the bulk of the plasma. Thus, even if radiation is the primary heat transfer mechanism, only that plasma located in the narrow "combustion chamber" between the teflon bars can contribute, while after a few microseconds the bulk of the current flow is well out between the electrodes. In fact, according to the current flow data of reference 6 the current flow extends well past the electrodes as the plasmoid emerges from the thruster.

The ablation model suggested here effectively assumes that ablation, dissociation, heating, and ionization times are short enough to be neglected, and that the actual power source which accomplishes this is that produced by the current flow. In a two dimensional flow model a similar device could be used (assuming uniformity in the z direction) but in a full three dimensional model ablation would take place on a boundary surface. In the latter case additional investigation into exactly what portion of the plasma contributes to the local heat transfer would be required.

A drastically simplified set of equations has been proposed to describe the plasma motion and the magnetic field. One consequence of the simplifications made can be examined without recourse to numerical implementation of the model (which is impossible without a set of initial and boundary conditions). The neglect of the displacement current reduces Faraday's equation to

$$\frac{1}{\mu} \text{curl } \underline{B} = \underline{J}$$

as discussed above, and allows the elimination of $\underline{J}$ from the equations. The reduction of the system to the one-dimensional model produces

$$J = -\frac{1}{\mu} B_x$$

where J and B are the y and z components respectively. In fact, although symmetry through the z=0 plane implies that B_l (the x-component of $\underline{B}$) is zero it cannot be true that $B_l|_z = 0$ unless the medium is unbounded in the z-direction (as the one-dimensional model implies). Comparison of current flow and magetic field data from reference 6 indicates that the current density is not proportional to the magnetic field gradient along the axis as the model implies (see Figure 3). This could be result of the failure of the one-dimensional simplification or the fact that the prob-s used to measure the current and field disturbed the flow. It is not believed that the displacement current could be responsible.

4.4 Numerical Model

An explicit, forward time step, finite difference algorith was chosen to numerically test the simple model developed above. Although a characteristics method or an implicit finite difference method might be more appropriate for the simple model, these would be very difficult to generalize to the multidimensional dissipative model. The specific algorithm chosen was a Lax-Wendroff type, similar to that presented in reference 2. The equations are written in conservation form

$$w_t + f_x = 0$$

where w is a component of the variables vector (, u, B) and the f components are functions of the variables. A single time step is taken in two parts, and central differences are used for the x derivatives. Thus, if w_n^j represents the variable w at the nth grid space and jth time step, the equations can be written:

$$W_{n+1/2}^{j+1/2} = 1/2(W_{n+1}^{j} + W_{n}^{j}) - \frac{\Delta t}{2\Delta x}\ (f_{n+1}^{j} - f_{n}^{j})$$

$$f_{n+1/2}^{j+1/2} = f(w_{n+1/2}^{j+1/2})$$

$$W_{n}^{j+1} = W_{n}^{j} - \frac{\Delta t}{\Delta x}\ (f_{n+1/2}^{j+1/2} - f_{n-1/2}^{j+1/2})$$

This procedure must, of course, be modified for the density equation to include the source term in which B_x is replaced by a central difference.

Schemes such as this are known to be numerically unstable if the time step is longer than the time required for information to progate over one grid space. Thus, the local sound speeds and Alfvén wave speeds must be computed at each time step, and the next Δt chosen accordingly. Here Δt was taken to be be

$$\Delta t \leq \frac{\Delta x}{\left|u_n^j\right| + \left[RT + B_n^{j\,2}/\mu \rho_n^j\right]^{1/2}}$$

This algorithm produces a solution in only a triangular sort of domain in the x,t-plane, as shown in Figure 4, since it provides no value for w^{j+1} at either the first or last x grid point. These values must be determined by appropriate boundary conditions. The analysis of boundary conditions was not a part of this project, but is included in a subsequent study conducted at the Air Force Rocket Propulsion Laboratory.

A computer program based on this algorithm and mathematical model was tested and behaved reasonably. The model appears to be stable, but low densities cause very short time steps (since the Alfvén speed varies inversely with density). The results produced depend strongly on the initial spatial variations assumed, as expected, and all that can be concluded here is that initial conditions which seem to be compatible with the experimental data for a particular time give rise to later conditions which are altered in the correct direction.

Some attempt to impose boundary conditions was made, but without success. The boundary conditions chosen caused very steep gradients of the variables at the boundaries and these did trigger numerical instabilities, even if the time steps were shortened. The proper handling of the boundary conditions is obviously crucial to the model's application, and these conditions, especially those at the moving plasma front, are non-trivial.

5. CONCLUSIONS

The multidimensional model presented above, with scalar conductivities and zero viscosity, is believed to provide a reasonable description of the acceleration of the first plasmoid ejected by a firing of the PPT. The same model may not, however, be accurate for the second or later plasmoids. The one-dimensional isothermal model which was implemented numerically should provide a crude approximation if boundary and initial conditions are handled properly, and this should be used in additional studies of these boundary conditions.

Considerable additional study will be required before a truly predictive internal model, capable of providing input for a plume model, is obtained. The work presented here is merely a small step in the process. The present model, even in the one-dimensional form, may, however, prove valuable for thruster performance studies, since nearly all of the thrust must be produced by the first plasmoid.

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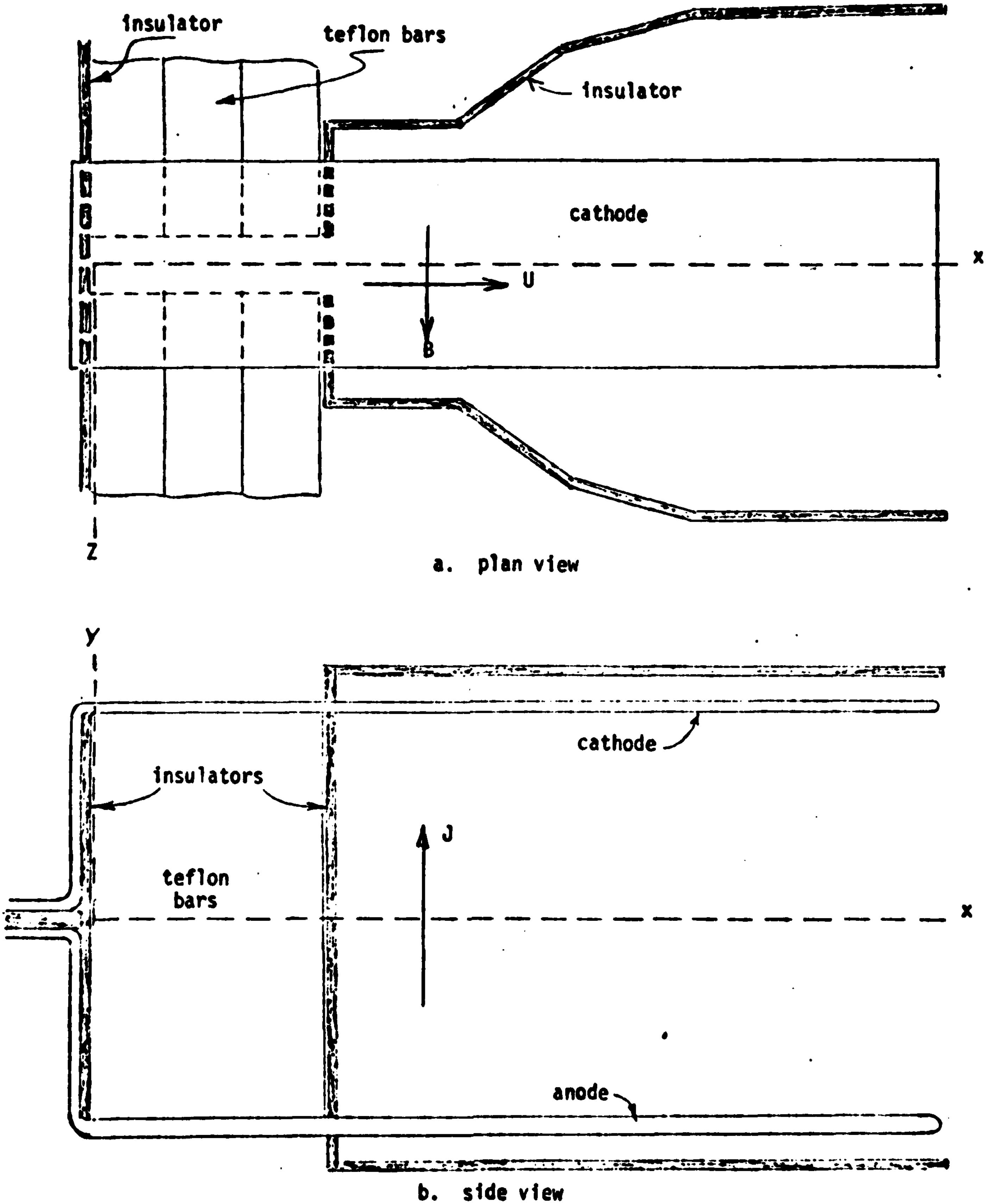

Figure 1. Schematic of PPT

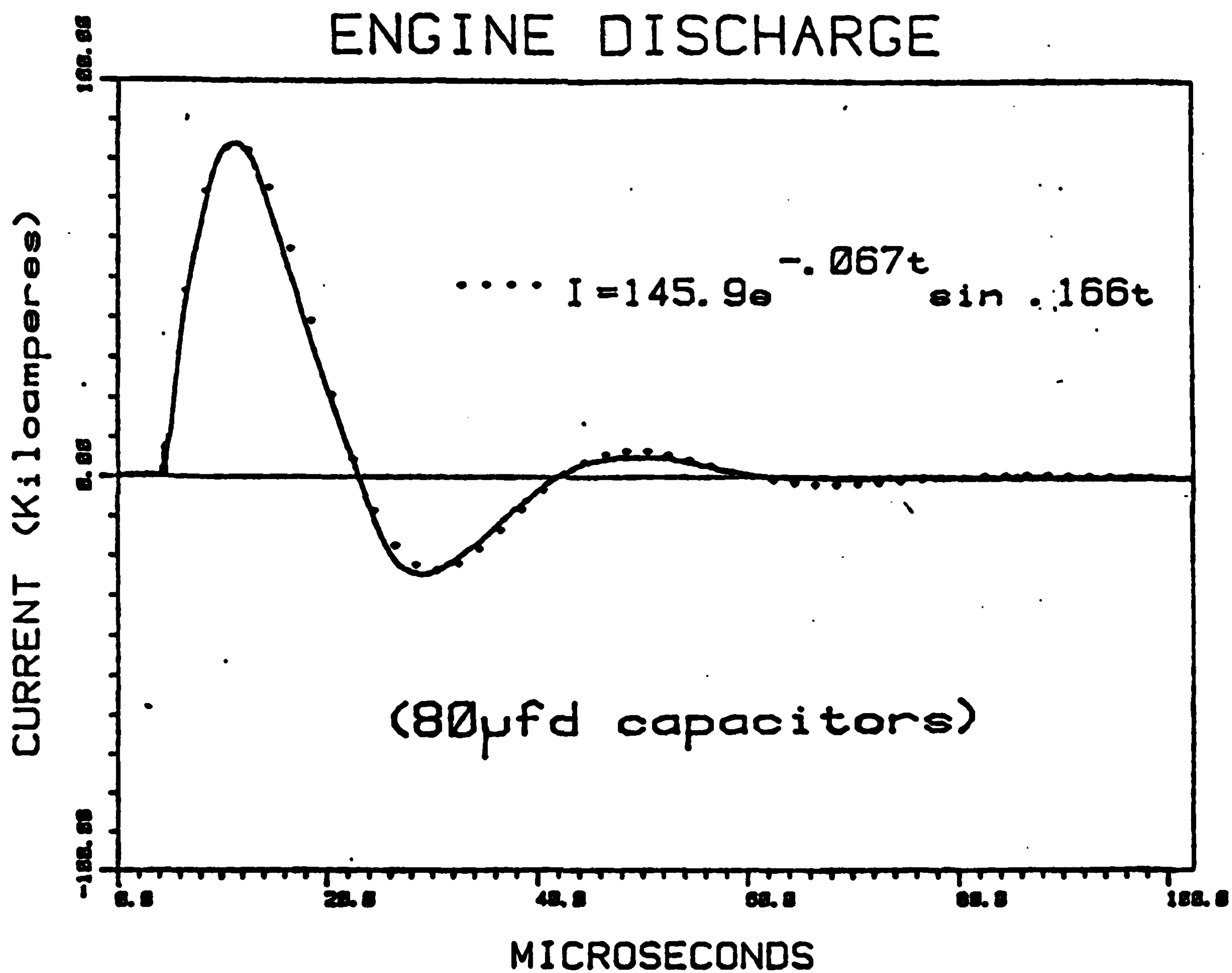

Figure 2. Circuit Current

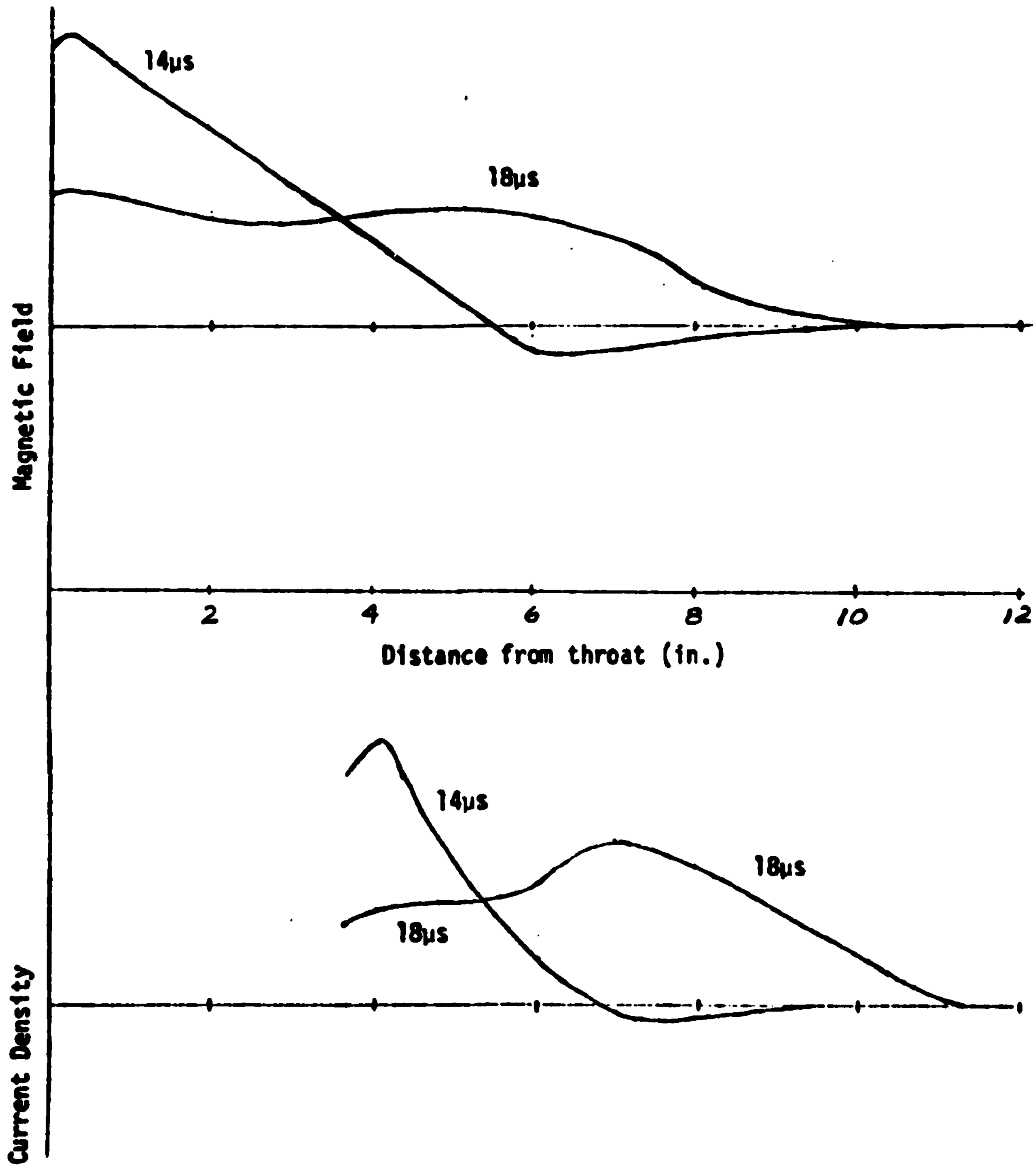

Figure 3. Cross-plotted data from reference 6

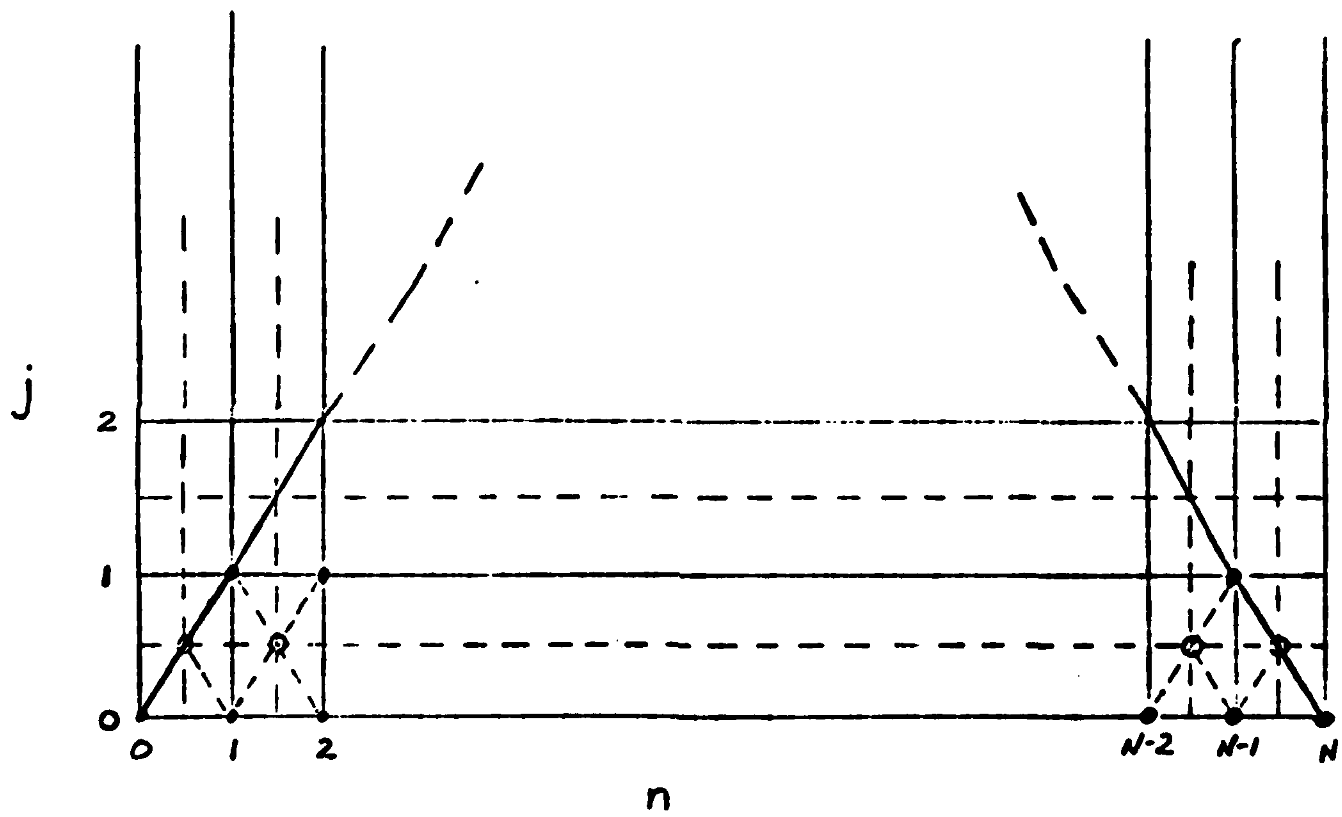

Figure 4. Solution Domain of Numerical Algorithm

END