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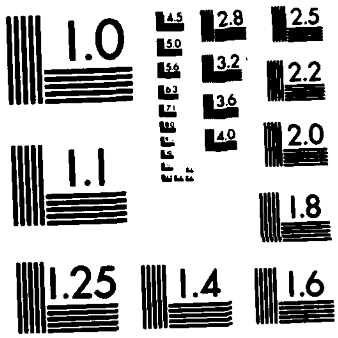

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NUMERICAL ANALYSIS IN FRACTURE MECHANICS

by

A. S. Kobayashi

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NUMERICAL ANALYSIS IN FRACTURE MECHANICS

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ABSTRACT

Recent developments in four numerical techniques in structural mechanics, which are used to extract fracture parameters for linear elastic, nonlinear and dynamic fracture mechanics, are reviewed. Primary emphasis is placed on the finite element methods for determining two- and three-dimensional (2-D and 3-D) stress intensity factors in linear elastic fracture mechanics. Crack opening displacements (COD) and J-integrals for 2-D, stable crack growth, ductile fracture, and use of elastic finite element method in its generation mode for obtaining dynamic elastic fracture parameters are discussed. The second topic is the finite difference method for analyzing the elasto-dynamic and elastic-plastic dynamic states in fracturing 2- and 3-D problems. The use of a super finite difference code to study dynamic ductile fracture using the void growth and coalescence model is discussed. The third topic is the boundary element method which has evolved into a practical tool for numerical analysis in 3-D linear elastic fracture mechanics. The final topic is the updated alternating technique, which was merged with a 3-D finite element code and together with a break-through in its analytical formulation, has become a cost-effective numerical technique in solving part and complete elliptical crack problems in 3-D linear elastic fracture mechanics. Comparisons between the J-integral of a 3-point bend specimen, the stress intensity factor for a surface flaw specimen and the dynamic stress intensity factor of a fracturing dynamic tear test specimen obtained by various investigators are made.

INTRODUCTION

Successful applications of linear elastic fracture mechanics (LEFM) in numerous postmortem analyses of failures in aerospace structures of the early 60's and its expanded role in design synthesis in the 70's required precise knowledge of the stress intensity factors associated with cracks. Such stress intensity factors of two-dimensional (2-D) and three-dimensional (3-D) cracks, which are subjected to complex loading conditions can only be obtained through numerical techniques. As a result, numerical analysis of fracture mechanics problems became the most active branche of structural mechanics in the 1970's [1]. One of the first numerical solutions in fracture problems, however, is the finite difference elastic-plastic result of Jacobs in 1950 [2] which was followed by others in the 1960's [3 - 5]. Swedlow et al [6], on the other hand, used finite element method to study a similar problem. In essence, these numerical results on the elasto-plastic states in cracked plates were ahead of their time since no plausible ductile fracture criterion was available in the early 1960's.

During this period of soul searching, other specialized numerical techniques, which were specially designed to compute mode I (opening mode) SIF in 2-D crack problems, emerged. Such techniques include extensive studies using the boundary collocation method [7], series expansion of a complex mapping function [8] and the method of Laurent series expansion [9]. The numerical solutions, which were generated by these special techniques, are still valid today and are listed in stress intensity factor handbooks and are liberally quoted in literature as well. These techniques, however, failed to generate subsequent supporters and thus will not be discussed in this paper.

In contrast, the overwhelming popularity of finite element method together with the growing acceptance of linear elastic fracture mechanics in structural mechanics in the late 1960's provided the impetus for an orderly development in the use of finite element method for determining stress intensity factors for 2-D linear elastic fracture mechanics [10]. The explosive developments in finite element method approaches to linear elastic fracture mechanics and also to nonlinear as well as dynamic fracture mechanics of the 1970's are documented in several review papers and special conference proceedings [1, 11 - 16]. Limited reviews of available linear elastic fracture mechanics computer software for fracture mechanics are given in References [17, 18].

Review papers covering the other three topics of finite difference method, boundary element method and alternating technique are few and scarce. The boundary element method, however, has attracted a large core of users and its applications to fracture mechanics have been presented at numerous conferences.

The purpose of this paper is to review the above four numerical techniques in fracture mechanics, with particular emphasis on development of finite element method following the period covered in Reference [1].

FINITE ELEMENT METHOD

The above mentioned popularity in the use of finite element method in every aspect of fracture mechanics has resulted in technical papers too numerous to be included in this review. Thorough reviews on the applications of finite element techniques to 2-D static and quasi-static problems in fracture mechanics through the 1970's have appeared in References [1, 11 - 15]. The historical and important developments of this era will not be repeated as this paper will concentrate on the 3-D static and 2-D nonlinear and dynamic analyses which emerged during and after this period.

A. 3-D Static LEFM Singularity Element

Although 3-D finite element method codes are available commercially, the mode I stress intensity factor for a seemingly simple surface flaw in a uniaxial tension plate requires inordinate amount of computer time. The densely packed 3-D constant strain quadrilaterials along the curved crack front [19] for proper modeling of the $1/\sqrt{r}$ stress singularity results in an inefficient use of computer time. Although crack opening displacements (COD) was used to improve the accuracy of mode I stress intensity factor, K_I, the accuracy of such brute force computation remains in doubt. The 3-D counterpart [20] of the virtual crack extension method [21] attempted to increase, by computing the local strain energy change for small crack tip displacements in conventional displacement elements, the solution accuracy without excessive number of finite elements.

a. Singularity Element

Computational efficiency can also be improved by incorporating the $1/\sqrt{r}$ strain singularity in the displacement elements. Raju and Newman [22] used such singularity element and reduced the effect of interelement displacement discontinuity by surrounding the crack tip with two layers of "square-root" elements. A series of nodal forces adjacent to the crack tip was then used to compute K_I. The multitude of 3-D problems analyzed by this procedure include the surface flaw problems [23] in pressurized cylinders.

b. Collapsed Quarter-Point Isoparametric Element

The popularity [1] of Barsoum's collapsed quarter-point isoparametric element [24] is due to its simplicity in execution which does not require special subroutines to available 3-D finite element method codes. These elements have the correct $1/\sqrt{r}$ singularity and together with proper stress intensity factor extraction procedure, will yield stress intensity factor of sufficient accuracy along the crack front. While many use the crack opening displacement procedures or the crack-tip stress formula to compute the stress intensity factor in 3-D problems, few procedures are developed specifically for 3-D applications. In the following, two such procedures are described.

Ingraffea [26] has shown that for collapsed 20-node isoparametric elements surrounding the crack front of Figure 1, the three modes of stress intensity factors can be written in terms of the mapped curvilinear coordinates of ξ, η and ζ as

$$K_I = \frac{E}{4(1-\nu^2)}\sqrt{\frac{\pi}{2L_1}}\Big[2v_B - v_C + 2v_E - v_F + v_D - 2v_{B'} + v_{C'} - 2v_{E'} + v_{F'} - v_{D'} + \frac{1}{2}\eta(-4v_B + v_C + 4v_E - v_F + 4v_{B'} - v_{C'} - 4v_{E'} + v_{F'}) + \frac{1}{2}\eta^2(v_F + v_C - 2v_D - v_{F'} - v_{C'} + 2v_{D'})\Big] \quad (1)$$

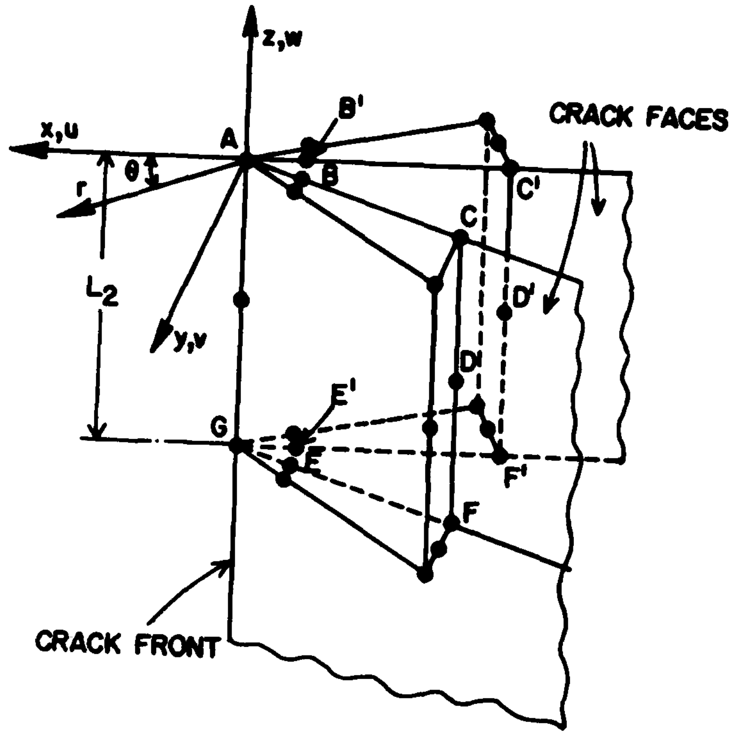

Fig. (1) - Quarter-point Wedge Elements

$$K_{II} = \frac{E}{4(1-\nu^2)} \sqrt{\frac{\pi}{2L_1}} \Big[2u_B - u_C + 2u_E - u_F + u_D - 2u_{B'} + u_{C'} - 2u_{E'} + u_{F'} - u_{D'} + \frac{1}{2}\eta(-4u_B + u_C + 4u_E - u_F + 4u_{B'} - u_{C'} - 4u_{E'} + u_{F'}) + \frac{1}{2}\eta^2 (u_F + u_C - 2u_D - u_{F'} - u_{C'} + 2u_{D'}) \Big] \tag{2}$$

$$K_{III} = \frac{E}{4(1+\nu)}\sqrt{\frac{\pi}{2L_1}}\Big[2w_B - w_C + 2w_E - w_F + w_D - 2w_{B'} + w_{C'} - 2w_{E'}$$

$$+ w_{F'} - w_{D'} + \frac{1}{2}\eta(1 - 4w_B + w_C + 4w_E - w_F + 4w_{B'} - w_{C'} - 4w_{E'} + w_{F'})$$

$$+ \frac{1}{2}\eta^2\,(w_F + w_C - 2w_D - w_{F'} - w_{C'} + 2w_{D'})\Big] \quad (3)$$

where E and ν are the modulus of elasticity and Poisson's ratio, respectively. The three stress intensity factors at η distance along the crack front can be computed by the crack surface displacements of u, v and w on the collapsed element face of $\xi = -1$. In terms of the physical distance along the z coordinate

$$\eta = -(\,2z/L_2 + 1\,) \quad (4)$$

where L_2 is the element length along the crack front as shown in Figure 1. The quadratic variation in displacement field in the original 20-node element also results in a quadratic variation in stress intensity factor along each crack front element.

The accuracy of the stress intensity factor extraction procedure is demonstrated by analyzing the embedded elliptical crack problem solved analytically by Green and Sneddon [27]. Because of symmetry, one-eighth of an elliptical crack in a cube composed of 23 elements and 141 nodes was analyzed using SAP IV [28]. The maximum errors in the numerical results, when compared against the theoretical results [27], were 5% and 7% for a circular crack and an elliptical crack with an aspect ratio of 1.5, respectively. At the tip of the minor axis where the SIF is maximum in the elliptical crack, the error was 2%.

c. Finite Element Hybrid Method

Since a review on the use of finite element hybrid method in fracture mechanics appeared in Reference [1], substantial progress has been made in improving its computational efficiency. Hybrid formulation is now restricted to finite elements surrounding the crack front with the crack-tip singularity being preserved through assumed $1/\sqrt{r}$ stress or $\sqrt{r}$ displacement field. Computational efficiency is achieved by the general purpose finite element code which models the bulk of the boundary value problem. The effect of inter-element discontinuity between the singular and conventional elements is minimized through the hybrid formulation. For 2-D problems, the hybrid method can be formulated such that one element, which has the assumed $1/\sqrt{r}$ singularity and also an assumed compatible boundary displacements, completely encompasses the crack tip [29]. For 3-D problems, the crack front becomes one of the boundaries of the several hybrid elements which surround the crack tip. The assumed stress hybrid method is based on an assumed equilibriating stress field, which contains the proper crack-tip singularity, and on independently assumed boundary displacements [30]. In terms of the local coordinates of n, z and t, the singular stress and corresponding displacements fields are represented as

$$\begin{Bmatrix} \sigma_n \\ \sigma_z \\ \sigma_t \\ \tau_{nz} \\ \tau_{zt} \\ \tau_{nt} \end{Bmatrix} = \frac{1}{\sqrt{2\pi r}} \begin{Bmatrix} K_I \begin{Bmatrix} \cos\frac{\theta}{2}(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}) \\ \cos\frac{\theta}{2}(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}) \\ 2\nu\cos\frac{\theta}{2} \\ \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} \end{Bmatrix} + K_{II} \begin{Bmatrix} -\sin\frac{\theta}{2}(2+\cos\frac{\theta}{2}\cos\frac{3\theta}{2}) \\ \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} \\ -2\nu\sin\frac{\theta}{2} \\ \cos\frac{\theta}{2}(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}) \end{Bmatrix} \\ K_{III} \begin{Bmatrix} \cos\frac{\theta}{2} \\ -\sin\frac{\theta}{2} \end{Bmatrix} \end{Bmatrix} \tag{5}$$

$$\begin{Bmatrix} u_n \\ u_z \\ u_t \end{Bmatrix} = \frac{1}{G}\sqrt{\frac{2r}{\pi}} \begin{Bmatrix} \frac{K_I}{8} \begin{Bmatrix} (5-8\nu)\cos\frac{\theta}{2} - \cos\frac{3\theta}{2} \\ (7-8\nu)\sin\frac{\theta}{2} - \sin\frac{3\theta}{2} \end{Bmatrix} + \frac{K_{II}}{8} \begin{Bmatrix} (9-8\nu)\sin\frac{\theta}{2} + \sin\frac{3\theta}{2} \\ (-3+8\nu)\cos\frac{\theta}{2} - \cos\frac{3\theta}{2} \end{Bmatrix} \\ K_{III}\sin\frac{\theta}{2} \end{Bmatrix} \tag{6}$$

where r and θ are the local polar coordinates in the plane normal to the crack front. While 2-D finite element method codes can incorporate higher order terms in r, such as the constant, $\sqrt{r}$, r ... terms [31], corresponding series forms of equations (3) and (4) are not available. The assumed stress in the crack tip hybrid element is thus represented as

$$\sigma_{ij} = \sum_m P_{s_{ijm}} (1/\sqrt{r},\theta)\, K_m + \sum_n P_{r_{ijn}} (x,y,z)\beta_n \tag{7}$$

where the first term of the above is equation (4) and the second term is a regular polynomial of x, y and z. For a traction free crack, this second term must satisfy the homogeneous equilibrium equations and the traction free boundary conditions on the crack surface. The assumed element boundary displacements contain equation (7) for all element boundaries which intersect with the crack front and satisfy inter-element compatibility. When these hybrid elements are merged into a general purpose 3-D code, equation (7) will introduce as unknowns, the three stress intensity factors of K_m (m= 1,2 or 3) in addition to the unknown generalized nodal displacement of $\underset{\sim}{q}$ as

$$\begin{bmatrix} \underset{\sim}{K}_{rr} & \underset{\sim}{K}_{rs} \\ \underset{\sim}{K}_{rs}^{T} & \underset{\sim}{K}_{ss} \end{bmatrix} \begin{Bmatrix} \underset{\sim}{q} \\ \underset{\sim}{K}_{m} \end{Bmatrix} = \begin{Bmatrix} Q \\ \underset{\sim}{0} \end{Bmatrix} \tag{8}$$

$\underset{\sim}{q}$ and K_m are thus obtained by solving equation (8).

Pian and Moriya [32] used a twelve node, assumed stress hybrid, half element, as shown in Figure 2, to determine the distribution of stress

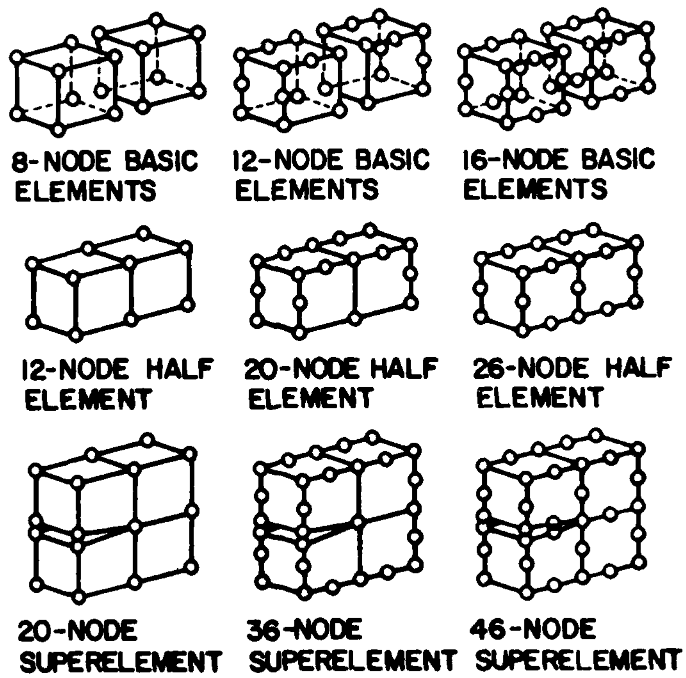

Fig. (2) Three-Dimensional Hybrid Elements

intensity factor along a semi-circular surface crack in a tension specimen. Because of four way symmetry, only one quadrant of the specimen was analyzed with 284 nodes and 852 degrees of freedom. The numerical results, when compared with Smith's results [33] obtained by the alternating technique, were within 12% at the free surface and coincided at the maximum crack depth.

The assumed displacement hybrid method is based on an assumed displacement field, which includes equation (6) with K_m, and independently assumed element boundary displacements and boundary tractions. The general format of the final element equation is identical to equation (8) where K_m is treated also as three unknowns along the the crack front.

Atluri and Katherisean [34] used static condensation to produce a 20 node quadratic isoparametric super-element, as shown in Figure 3, based on the

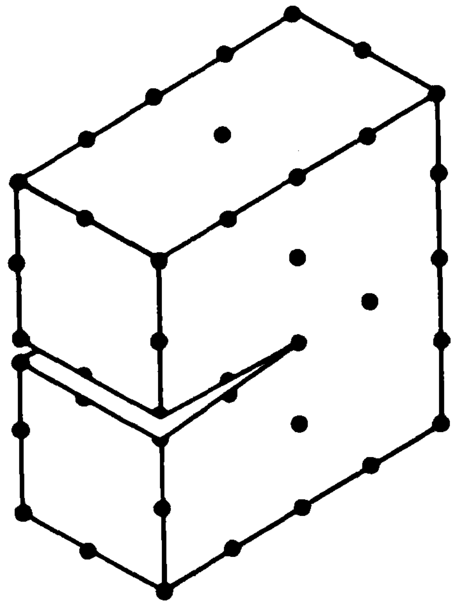

Fig. (3) Super Singular Element

assumed displacement hybrid method. Using this super-element in a general 3-D finite element code, they analyzed the surface flaw problem with 280 finite elements and 4815 degrees of freedom. The results are discussed in the last section entitled "Benchmark Solutions" of this paper.

B. *2-D* Nonlinear Singularity Element

Since the early use [6] of constant strain elements for elastic-plastic finite element method analysis of 2-D fracture problems, this simple element is still used today with success, primarily due to the moderate or lack of stress singularity at the plastically yielded crack tip. Kanninen et al [35] used such finite element method code in both its generation and application modes to study stable crack growth and instability of A533-B steel and 2219-T87 aluminum, center cracked and compact specimens. Loadline displacement and crack length measurements were input into generation-mode calculations and the applied load among others were output for evaluation. Figure 4 shows the computed and measured applied load versus loadline displacement relation of the steel compact specimen. Other uses of the conventional element include

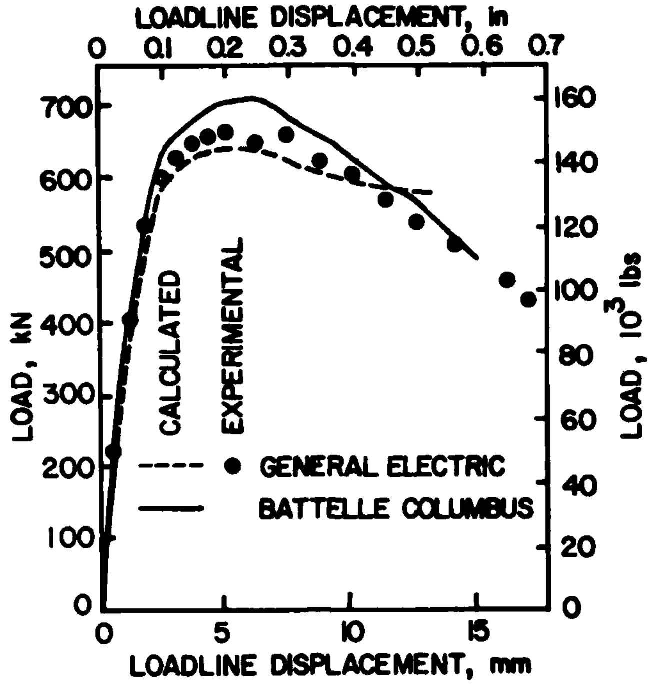

Fig. (4) Load-displacement Curve for A533B Steel CT Specimens

the updated Lagrangian finite-deformation, finite-element analysis by McMeeking and Parks [36] who studied the influence of crack-tip blunting on the J-based characterization of the crack tip region.

For a material with a strain hardening index of n, the strain field of the dominant singularity at the crack tip under deformation theory of plasticity is [37, 38]

$$\sigma_{ij} = \sigma_0 \left(\frac{EJ}{\sigma_0^2 r}\right)^{1/(n+1)} \tilde{\sigma}_{ij}(\theta,n) \tag{9}$$

$$\sigma_{ij} = \frac{\sigma_0}{E}\left(\frac{EJ}{\sigma_0^2 r}\right)^{n/(n+1)} \tilde{\varepsilon}_{ij}(\theta,n) \tag{10}$$

where J is the J-integral as per Rice [39], $\tilde{\sigma}_{ij}$ and $\tilde{\varepsilon}_{ij}$ are functions of θ and n, and σ_0 is the yield stress.

Early uses of the above singluar stress and strain fields include that of Hilton [40] who used deformation theory of plasticity to study ductile crack initiation under monotonically increasing load. Shih [40], on the other hand, constructed a circular crack tip element with the dominant stress and strain singularities and studied the changes in elastic-plastic boundaries with variations in n values under small scale yielding. Atluri et al [42] incorporated the above stress and strain singularities into the hybrid-displacement

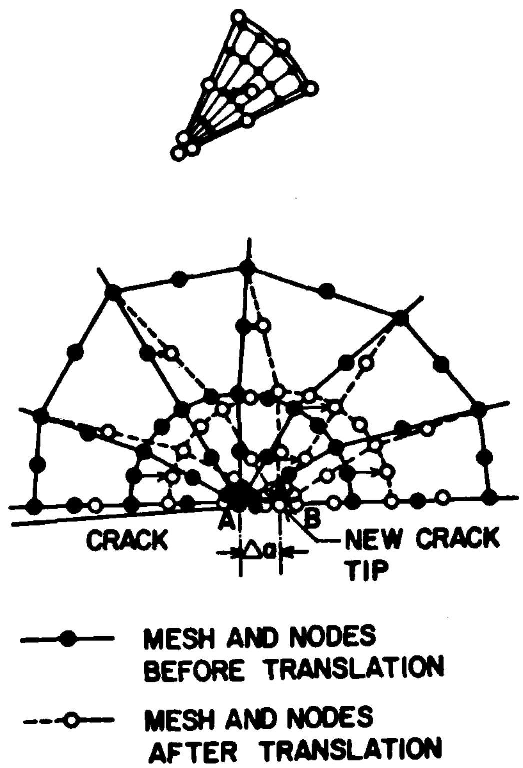

Fig. (5) Translation of Singular Element

finite elements surrounding the crack tip. The virtual work equation for an incremental crack growth, Δa, as shown in Figure 5, was used to simulate crack closure and opening under a cyclic loading condition with a single overload as shown in Figure 6. The penalty function and superposititon method used by Yagawa et al [43] is similar in formulation where the penalty coefficients are stiffness coefficients which optimize the potential function under nonlinear constraints.

For an ideally plastic material, $n = \infty$ in equations(9) and (10) yield $\sigma_{11} = \sigma_o$ and ϵ_{11} with an 1/r singularity. Barsoum [44] showed that the same triangular quarter-point element [24] of elastic analysis will possess the 1/r strain singularity when the condensed crack tip nodes are allowed to slide. Shih et al [45] used this quarter point element to model crack tip blunting and growth. As shown in Figure 7, crack-tip blunting is modeled by separating the condensed crack tip nodes and crack extension is modeled by sequential shifting of the crack tip node. The crack tip opening displacement (CTOA) and crack tip displacement can be determined directly from Figure 7(c).

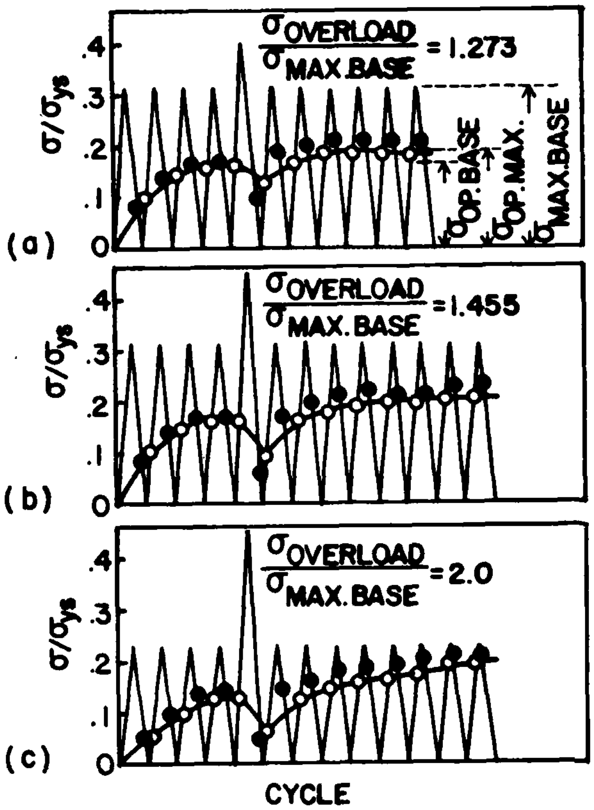

Fig. (6) Crack Closure and Opening Stress under Single Overload

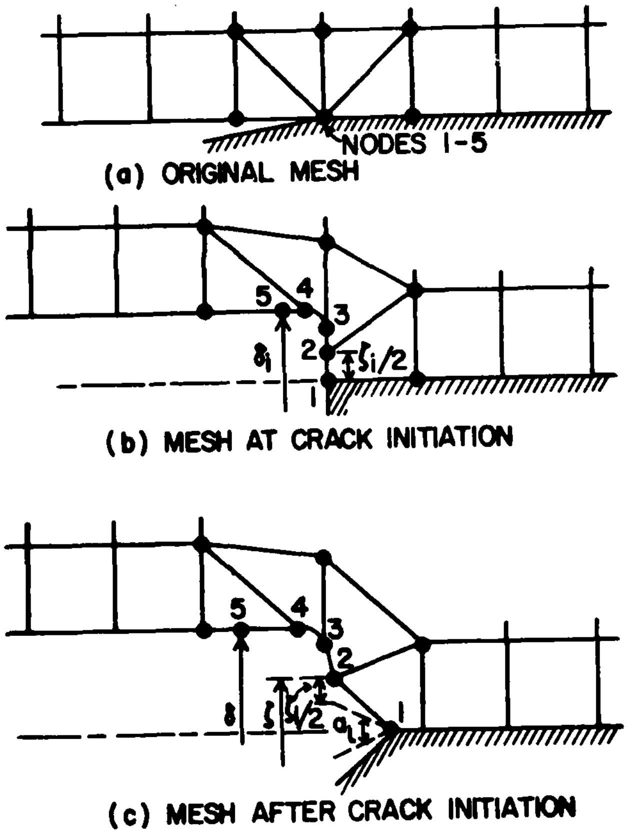

Fig. (7) Crack Tip Blunting and Growth

C. 3-D Non-linear Singularity Element

The singularity element used in 3-D elastic-plastic analysis of cracks apparently is limited to Barsoum's triangular quarter-point element. Although this element contains only a 1/r singularity, Benzley [46] showed heuristically that when used with a stress-strain relation governed by a power hardening law, this element provided the correct singular states of strain and stress interior of the element. deLorenzi [47] used 556 of such 20-noded isoparametric elements in an 8300 degree-of-freedom system to study the elastic-plastic behavior of a surface flaw in the beltline region of a pressurized reactor vessel. The semi-elliptical crack front was surrounded with triangular quarter-point elements and a Ramberg-Osgood power hardening law for stress-strain relation was used. COD at the symmetry plane, as shown in Figure 8, demonstrated the need for a 3-D elastic-plastic analysis since the plane strain (2-D) approximation clearly overestimated the severity of the flaw.

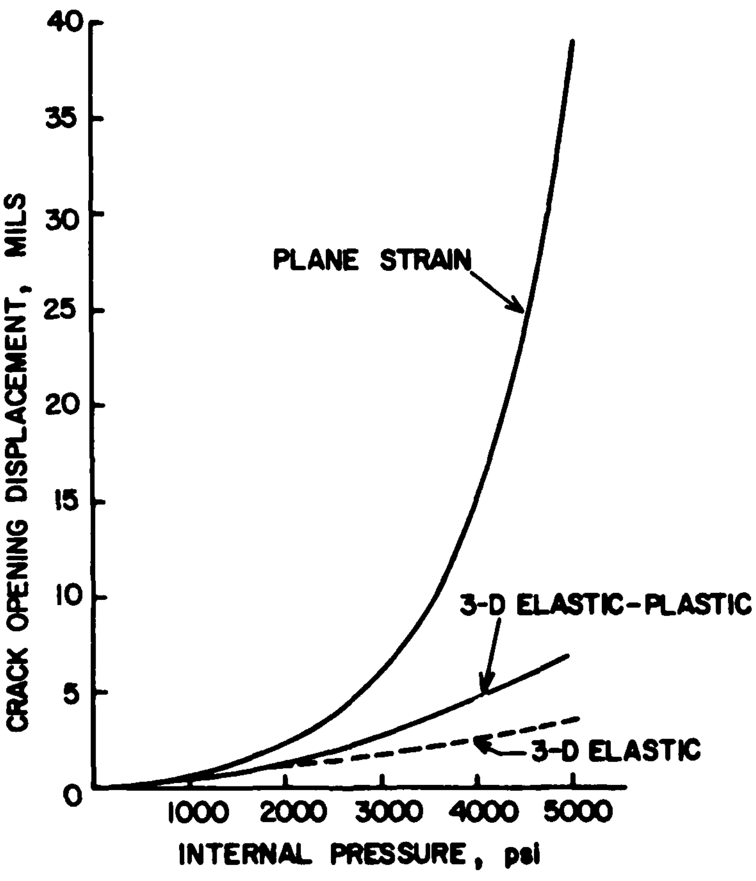

Fig. (8) COD of a Surface Flaw

D. 2-D Dynamic Singularity Element

Dynamic fracture analysis has also been conducted successfully with conventional elements during the past five years. Such analysis include the extensive code verifications by Kobayashi et al [48, 49] and the recent work by Jung et al [50]. Accuracy of such simplified finite element method codes can be improved by using a proper crack node release mechanism which was the subject of considerable debate in the late 1970's [51 - 53]. This writer's experiences, however, indicate that the simple node release mechanism of linearly decreasing crack-tip nodal force yield dynamic stress intensity factor which are in good agreement with those obtained by photoelasticity [54]. Yagawa et al [55], on the otherhand, represented the crack surface traction, acting on the extending crack suface, as a Lagrange multiplier, and optimized the dissipated surface energy during crack extension.

While singularity elements in dynamic finite element method was used earlier by Anderson et al [56] and Aoki et al [57], the most successful use of such element is by Atluri et al who incorporated the $1/\sqrt{r}$ singularity in the displacement hybrid crack tip element [58, 59]. Figure 9 shows the Atluri's

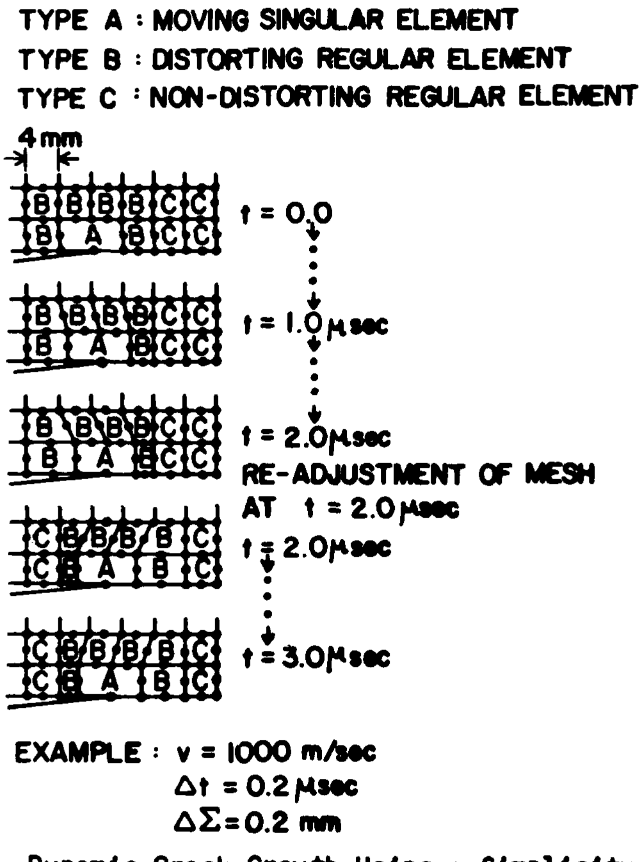

Fig. (9) Dynamic Crack Growth Using a Simplicity Element

singular element, which moves with the crack tip and retains its shape, and the continually distorting regular elements surrounding the crack tip. Periodic mesh readjustment is necessary with crack extension when this procedure is used. Similar approach without the distorting regular elements was used by Gunther et al [60]. Figure 10 shows good agreement between the numerically generated dynamic stress intensity factor by Atluri et al and the theoretical solution by Broberg [61].

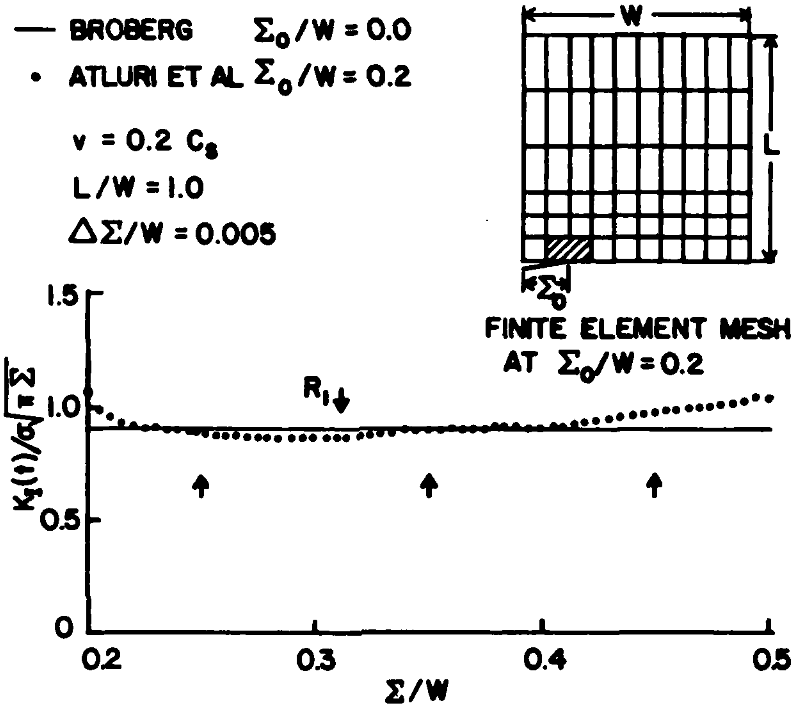

Fig. (10) Dynamic Stress Intensity Factor of Broberg Crack

Literature is void with 3-D dynamic finite element analysis, due in part to the enormity in computational requirements, but also due to the lack of definitive experimental observations on 3-D dynamic crack extension. Such dynamic crack extension history is necessary to execute a dynamic finite element code in its generation mode. 3-D dynamic analysis by the propagation mode, on the otherhand, requires apriori a 3-D dynamic crack propagation law which is equally missing at this time.

FINITE DIFFERENCE METHOD

As mentioned in the Introduction, finite difference method predates the now popular finite element codes in its application to fracture problems. Subsequent development of the finite difference codes were taken over by weapon researchers and re-emerged as vastly superior general purpose codes which could solve 3-D problems ranging from dynamic plasticity to gas dynamics. For example, the HEMP 3-D code [62] is an explicit finite difference code which does not require large computer storage. Crack tip singularity is thus handled by swamping the crack region with large number of zones and no known attempt has been made to incorporate crack tip singularities into the computation. Such simplified model results in enormous computer time which normally cannot be executed outside of special laboratories. The code can also handle ductile fracture with relative ease due to the reduced severity in stress singularity.

A. Static Analysis

Historically, the available general purpose finite difference codes were designed to solve complex dynamic problems and thus no static finite difference programs for analyzing static fracture problems exist to date. In an overkill attempt to demonstrate the versatility of such supercodes, Chen [63] showed that the static stress intensity factor can be obtained from a simple average of the stress waves set up in the crack-tip region when high artificial viscosity is inserted to damp out the stress waves in the 3-D supercodes. A similar converge scheme using dynamic relaxation was used by Shmuely et al [64] to obtain the static SIF in a finite thickness, central crack tension plate.

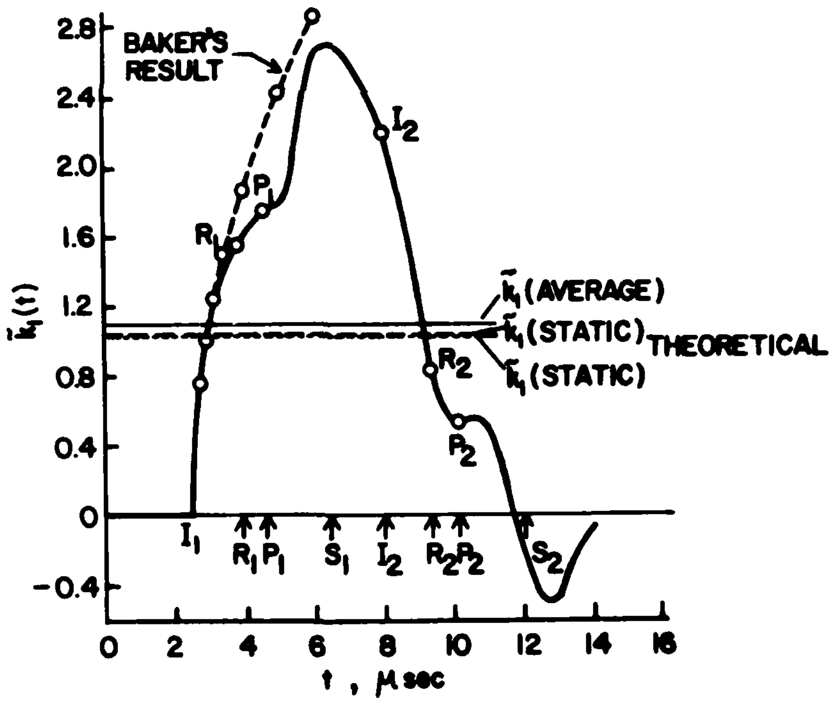

Fig. (11) Dynamic Stress Intensity Factor in an Impulse Loaded Plate

B. Dynamic Analysis

For elastic analysis, a simple extrapolation scheme is used to extract stress intensity factors from the numerically determined crack tip stresses

[63]. Figure 11 shows the dynamic stress intensity factor in a center cracked plate subjected to sudden tension loading. Also shown for comparison is Baker's solution [65] for a pressurized semi-infinite crack which suddenly appeared in an infinite plate. Complex fracture problems, such as an internally or externally surface flawed cylinder subjected to sudden pressurization has also been solved [66] by the HEMP code. The advantage of such supercode, however, lies in its ability to analyze elastic-plastic dynamic fracture problems, such as a notched bar subjected to sudden tensile loading [66, 67] and a Charpy V-notched specimen [68]. In these ductile fracture analyses, a void growth and coalescence criterion [68] was used to predict the onset and propagation of a ductile crack.

Special purpose finite difference codes have been used to analyze dynamic fracture problems but the earlier analyses [70, 71] did not focus on viable dynamic fracture parameters, primarly due to the undeveloped state of science in dynamic fracture at that time. Finite difference method was used by Popelar et al [72] to study the dynamic elastic response of an internally cracked cylinder subjected to impulse loading. The dynamic stress intensity factor was determined by the energy release rate calculated from the displacements at the nodes in the vicinity of the propagating crack tip. Shmuely et al [73] incorporated a moving substructure, as shown in Figure 12 with the proper $1/\sqrt{r}$

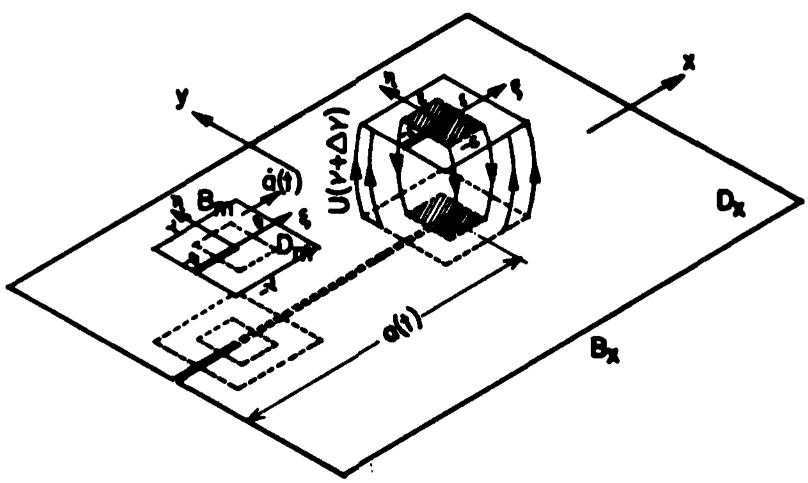

Fig. (12) Stationary and Moving Grid

singularity in their finite difference program. One half reduction in computing time in static analysis and improved accuracy in energy release rate computation in dynamic analysis are claimed. As a straightforward application of a dynamic finite difference shell code, Emery et al [74] studied the axial cracking of a pressurized crack with coupled depressurization calculation. This analysis was extended to large scale yielding with critical CTOA as a dynamic fracture criterion [75].

BOUNDARY ELEMENT METHOD

While the boundary element method, which was earlier referred to as the boundary integral equation method, as a stress analysis tool dates back to the mid 1960's [76], its application towards solving fracture mechanics problems was pioneered by Cruse [77, 78] in the early 1970's. Boundary equation method requires only the discretization of the boundary of the structure in contrast to the domain discretization required by finite element method. Thus boundary equation method is not suitable for analyzing elastic-plastic and elasto-dynamic fracture problems although some recent work [79, 80] suggests that such use of boundary equation method may not be far. On the other hand, the reduced system of equation makes it suitable for solving 3-D linear elastic fracture mechanics problems although it yields a fully populated, nonsymmetric system of equations. Also, for an extending crack, boundary equation method requires only the recomputation of nodes along the crack surface while finite element method requires complete remeshing around the original and extended crack. Recent applications of boundary element method to fracture mechanics has been reviewed by Cruse [81].

A. 2-D Boundary Element Meythod

Since boundary element method emerged as a numerical tool during the period of successful application of 2-D finite element method to elasto-static problems in linear elastic fracture mechanics, its use in 2-D linear elastic fracture mechanics did not grow until a special Green's function approach [78, 82] was developed. Utilizing the eleastic Green's function for an infinite plate with a crack and an interior point load, the stress intensity factors can be represented by the following set of path independent integrals enclosing a crack tip as

$$(K_I, K_{II}) = -\int_B R_i^{I,II}(z_Q)u_i(Q)ds + \int_B L_i^{I,II}(z_Q)t_i(Q)ds \tag{11}$$

where u_i (Q) and t_i (Q) refer to the displacements and surface tractions at boundary point Q. Details of functional, $R_i^{I,II}$ and $L_i^{I,II}$, are given in [83]. By taking advantage oif symmetry, Cruse used only seven boundary points to obtain K_I for a central crack fracture specimen with 6.4 CPU seconds.

Blandford et al [84] eliminated the complex arithmetic involved in the above by using traction singular quarter-point boundary elements along each side of the crack tip. The procedure is the boundary element method counterpart of Barsoum's finite element method procedure [24] where the midpoint in the isoparametric quadratic boundary element is shifted to the quarter point. This totally numerical procedure eliminates the non-uniqueness of flat crack modeling [82] and the matrix singularity problem [85]. Atkinson et al [86] used the path-independent F_1 and M integrals [87] to determine K_{III} in an edge cracked square torsion bar with a quadratically varying shear modulus.

B. 3-D Boundary Element Method

Since the first applications of boundary element method to 3-D linear elastic fracture mechanics in the early 1970's [85], significant inprovements has been made in the computational algorithm. The use of quadratic isoparametric boundary element elements with quarter point nodes [89] provided accurate COD's which were used to extract the K_I values. Recent solutions to 3-D linear elastic fracture mechanics include the surface flaw solutions in a tension plate [90] and in a pressurized cylinder [91] by Heliot et al and crack growth studies of surface flaw by Cruse [92].

ALTERNATING TECHNIQUE

Application of the alternating technique to 3-D linear elastic fracture mechanics was first introduced by Smith et al [93] who solved the semi-circular surface flaw problem. The procedure was extended to an elliptical crack by Shah et al [94] who determined K_I of an embedded elliptical crack near a free surface. Later, Smith et al [95] extended the solution procedure to K_{II} and K_{III} determination of an elliptical crack. The alternating techniques of those days were relatively inaccurate due to the limited curve fitting capabilities of the third order polynomial of the elliptical crack pressure and of the modeling of the surrounding finite geometry. A major breakthrough in the latter was made by Browning et al [96] and Kullgren et al [97] who used a 3-D finite element code to model the surrounding finite geometry. Grandt [98] and Barrachin et al [99] used this procedure to analyze surface flaw problems and in particular, the well studied surface flaw at a hole.

Another significant improvement was made with the derivation of the complete analytical solution [100] for an embedded elliptical crack in an infinite solid and subjected to modes I, II and III crack tip deformation. Nishioka et al used twelve terms of a fifth order polynomial of this analytical solution together with a standard 3-D finite element code to analyze among other, the surface flaw problem [101] and the internally and externally flawed pressurized cylinders [102]. Figure 13 shows the reduction in residual stress with alternating cycles of iteration for analyzing an externally flawed pressurized cylinder. Figure 14 shows the resultant stress intensity factor which is compared with those of [103, 104]. The finite element method portion of this alternating technique used 96 20-noded isoparametric elements with 1815 degrees of freedom and the total CPU time was about 1000 seconds with a CYBER 74.

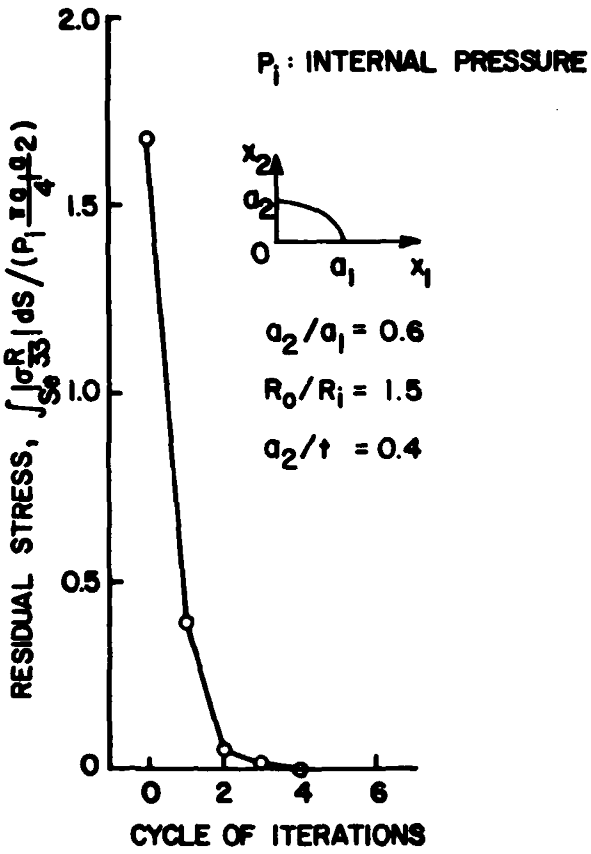

Fig. 13 Residual Stress on Crack Surface

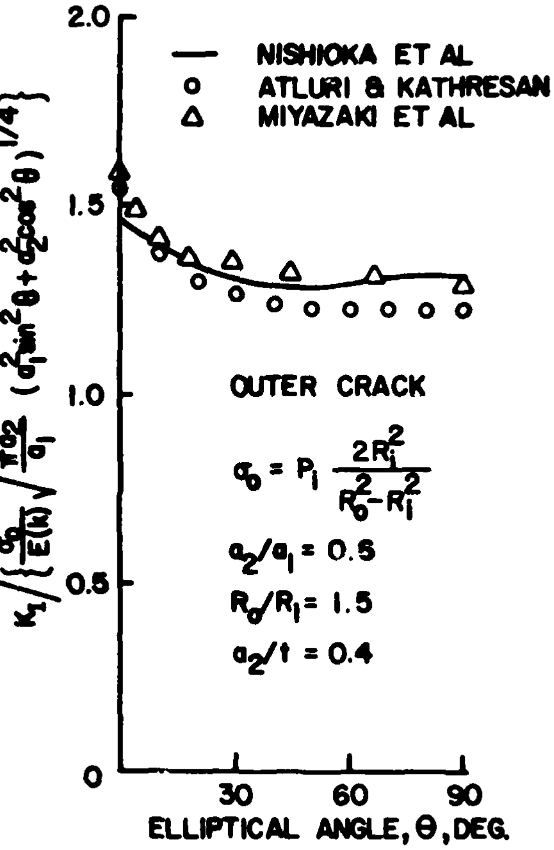

Fig. (14) Stress Intensity Factors for an Outer Surface Flaw in a Pressurized Cylinder

BENCHMARK SOLUTIONS

In most cases, numerical techniques are validated by comparing the numerical results with known theoretical solutions. By nature, these theoretical problems in fracture mechanics are simple in geometry and uncomplicated in loading and thus agreement between the numerical and theoretical solutions are bound to be good. Theoretical solutions for more realistic fracture problems do not exist and a consensus between various numerical solutions does not guarantee their correctness. Lacking other means of comparative study, benchmark solutions which are well defined boundary and initial value problems, are used to eliminate any ambiguity in problem definition and to assure that all numerical solutions relate to the same problem. Three such benchmark problems are discussed in the following.

A. 2-D Elastic-Plastic Crack Problem

In 1975, ASTM Committee E24.01.09 undertook a task to compare numerical solutions to elastic-plastic plane strain problems. A three-point fracture toughness test specimen with a uniaxial stress-strain relation of A533B steel was analyzed by 10 respondents and the edited and assembled solutions were presented by Wilson [105]. Figure 15 shows the average J-value, which was

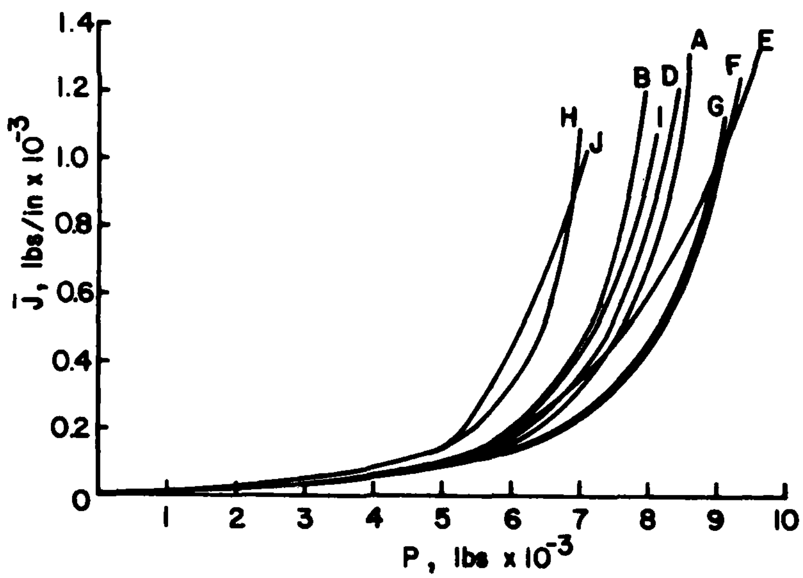

Fig. (15) J versus Applied Load

computed over several integration paths by each investigator, with increasing applied load. While substantial progress has been made in the elastic-plastic codes in the ensuing six years, the wide differences in the results obtained in the late 1970's are still indicative of the lack of consensus for valid 2-D elastic-plastic codes as well as for basic physical laws, such as the constitutive relations under plastic flow under severe strain gradients.

B. 3-D LEFM Problem

The surface flaw problem is an outgrowth of a 1976 workshop at Battelle Columbus Laboratories at which time several 3-D benchmark problems were designated for numerical analysis. The numerical results for the semi-elliptical, surface flaw plates in tension and bending were subsequently assembled and edited by McGowan [106]. Figure 16 shows the normalized stress intensity factors by six investigators where moderate differences are seen.

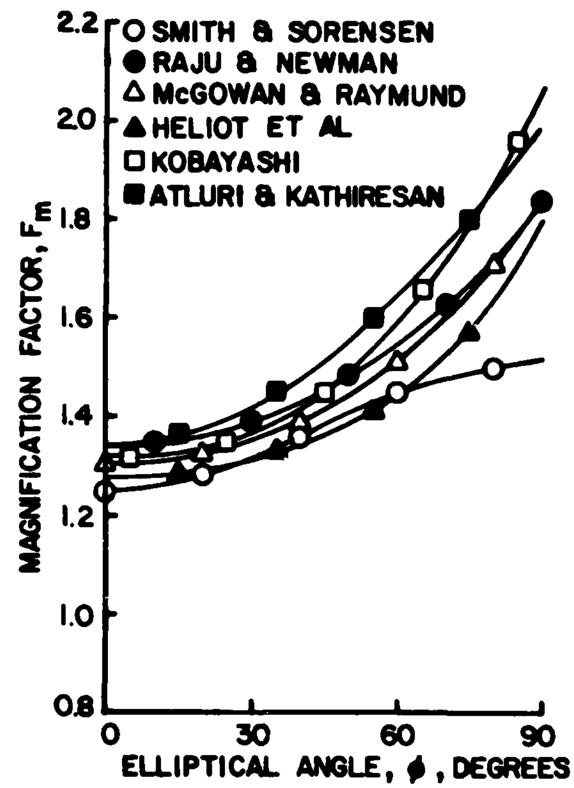

Fig. (16) Stress Intensity Factor for a Surface-Flawed Tension Plate

C. 2-D Dynamic Crack Problem

While no formal benchmark problem in 2-D dynamic fracture was ever designated for round robin studies, an informal comparative study was made between the investigators at the Georgia Institute of Technology, Battelle Columbus Laboratories and the University of Washington. Dynamic crack propagation in an A533B steel, dynamic tear test (DTT) specimen was analyzed by generation calculation using the crack velocity data provided in [107]. Figure 17 shows the dynamic SIF computed by the three investigators. All three elastic results are in good agreement with each other.

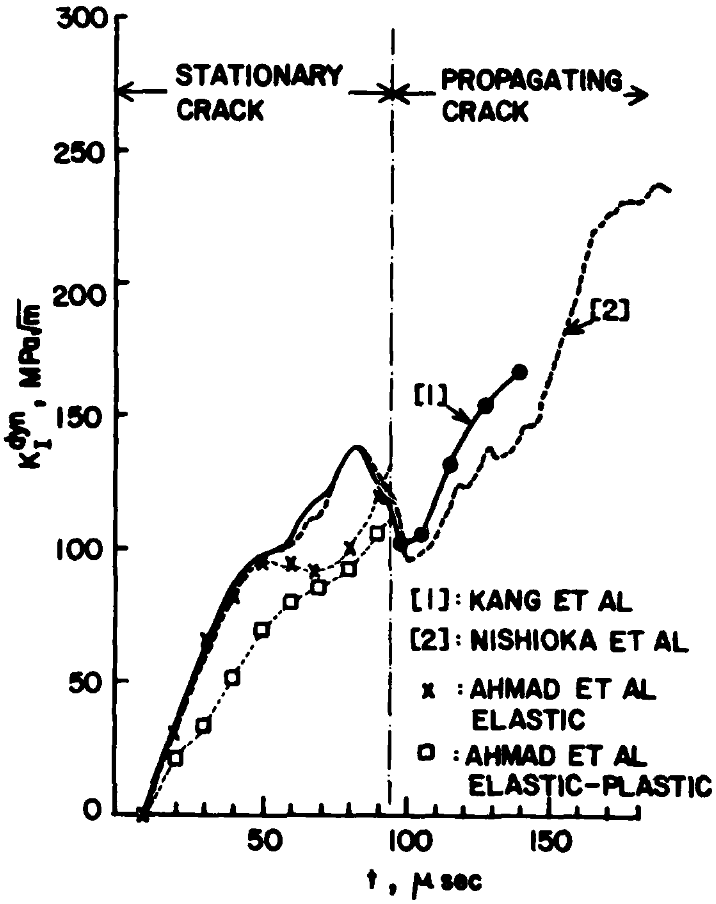

Fig. (17) Dynamic Stress Intensity Factor of a Dynamic Tear Test Specimen

CONCLUDING REMARKS

The concluding remarks in this paper could have been taken from Reference [1] since the four cited and the numerous uncited numerical techniques in fracture mechanics continue to develop at equal or even higher pace since Reference [1] was written. Also, the efficiency in numerical techniques continues to improve despite the rapid increase in the complexity of the problems studied.

Despite this explosive rate of development, coordinated efforts to evaluate the old as well as new numerical codes through benchmark problems are scarce. In order to properly assess the numerous codes available today, an international effort in code verification is badly needed at this time.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Recent developments in four numerical techniques in structural mechanics, which are used to extract fracture parameters for linear elastic, nonlinear and dynamic fracture mechanics, are reviewed. Primary emphasis is placed on the finite element methods for determining two- and three-dimensional (2-D and 3-D) stress intensity factors in linear elastic fracture mechanics. Crack opening displacements (COD) and J-integrals for 2-D, stable crack growth,

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ductile fracture, and use of elastic finite element method in its generation mode for obtaining dynamic elastic fracture parameters are discussed. The second topic is the finite difference method for analyzing the elasto-dynamic and elastic-plastic dynamic states in fracturing 2- and 3-D problems. The use of a super finite difference code to study dynamic ductile fracture using the void growth and coalescence model is discussed. The third topic is the boundary element method which has evolved into a practical tool for numerical analysis in 3-D linear elastic fracture mechanics. The final topic is the updated alternating technique, which was merged with a 3-D finite element code and together with a break-through in its analytical formulation, has become a cost-effective numerical technique in solving part and complete elliptical crack problems in 3-D linear elastic fracture mechanics. Comparisons between the J-integral of a 3-point bend specimen, the stress intensity factor for a surface flaw specimen and the dynamic stress intensity factor of a fracturing dynamic tear test specimen obtained by various investigators are made.

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