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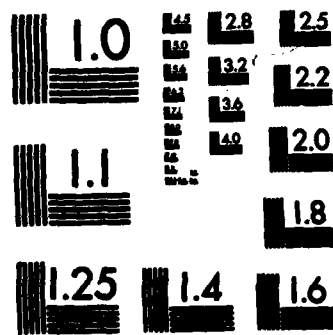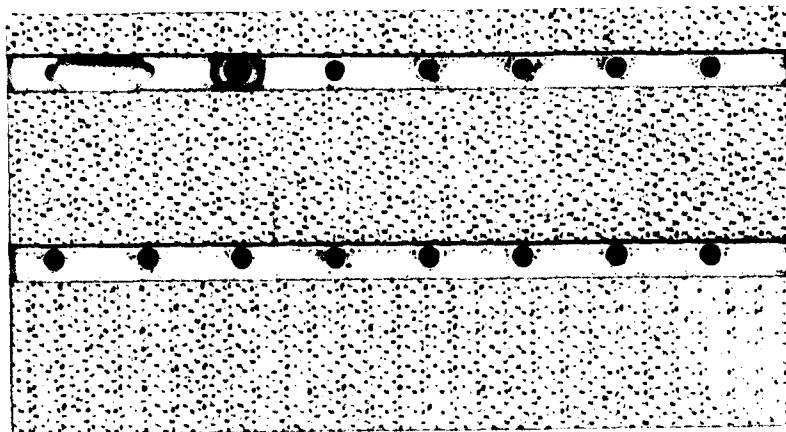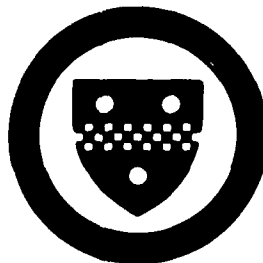UNCLASSIFIED   TR-82-28 AFOSR-TR-83-0018 F49620-82-K-0001 F/G 12/1   NL

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ROBUSTNESS PROPERTIES OF THE F-TEST AND
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ROBUSTNESS PROPERTIES OF THE F-TEST AND BEST LINEAR UNBIASED ESTIMATORS IN LINEAR MODELS

by

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ABSTRACT

Consider a linear model $Y = X\beta + \sigma\varepsilon$, $E(\varepsilon) = 0$, $E(\varepsilon\varepsilon') = I_n$ with β, σ unknown. For the problem of testing the linear hypothesis $C\beta = \gamma$, $\mathrm{im}(C') \subseteq \mathrm{im}(X')$, Ghosh and Sinha (1980) proved that the properties of the usual F-test being LRT and UMPI (under a suitable group of transformations) remain valid for specific non-normal families. In this paper it is shown that both criterion and inference robustness of the F-test hold under the assumption $\varepsilon \sim q(\varepsilon'\varepsilon)$, q convex and isotonic. This result is similar to a robustness property of Hotelling's T^2-test proved by Kariya (1981). Finally it is proved that the Best Linear Unbiased Estimator (BLUE) of any estimable function $C\beta$ is more concentrated around $C\beta$ than any other unbiased estimator of $C\beta$ under the assumption that ε is spherically distributed.

Key words: Linear model, linear hypothesis, spherical distribution, estimable function, LRT, UMPI, BLUE.

1. INTRODUCTION

We consider the linear model

$$Y = X\beta + \sigma\epsilon, \qquad (1.1)$$

where Y is $n \times 1$, X is $n \times k$, β is $k \times 1$ and $\sigma > 0$. It is assumed that Rank $(X) = r \leq k < n$ and that

$$E\epsilon = 0, \ \text{Cov} \ \epsilon = E(\epsilon\epsilon') = I_n. \qquad (1.2)$$

Then

$$EY = X\beta, \ \text{Cov} \ Y = \sigma^2 I_n. \qquad (1.3)$$

We denote by $\text{im}(B)$ the column-space of a matrix B.

We want to consider linear hypotheses of the kind $H_0: C\beta = \delta$ where C is a $s \times k$-matrix, Rank $(C) = s$. In order that these hypotheses are testable it is necessary that $\text{im}(C') \subseteq \text{im}(X')$, i.e., $C = TX$ for some T. It is well-known that the standard F-test is both LRT (Likelihood-Ratio-Test) and UMPI (uniformly most powerful invariant) under the normality of the vector ϵ with respect to a suitable group of affine transformations. Ghosh and Sinha (1980) proved that both of these properties of the F-test remain valid even under typical non-normal distributions of the errors, namely normal (scale) mixtures which include multivariate t-distributions. The purpose of this paper is to prove an even stronger result. It is shown that the F-test is both LRT and UMPI under the distributional assumption

$$\epsilon \sim q(\epsilon'\epsilon), \quad q \hat{\uparrow}, \text{ convex} \qquad (1.4)$$

($\hat{\uparrow}$ describes an isotonic function). The results are similar to a robustness property proved for Hotelling's T^2-test by Kariya (1981).

In section 2 we present a canonical form of the problem and section 3 deals with the UMPI property of the F-test. To obtain the distribution of a maximal invariant, certain results due to Dawid (1977), (see also Dempster (1969)) and Wijsman's representation theorem (Wijsman, 1967) are used. The property of the F-test being LRT and the distribution of the F-statistics under (1.4) are indicated in section 4. Finally, in section 5 it is shown for a more general linear model with spherically distributed error that the BLUE (Best Linear Unbiased Estimator) of an estimable function $C\beta$ is more concentrated around $C\beta$ than any other unbiased estimator (robustness property of the BLUE).

2. A CANONICAL FORM

According to our assumption rank $(X) = r \leq k < n$. Then there exists (see, e.g., Rao (1973)) a quasi-orthogonal matrix P_1 of order $k \times n$ such tha $P_1 P_1' = I_k$, and $k \times k$-diagonal matrix Λ and an orthogonal $k \times k$-matrix Q such that the singular value decomposition

$$X = P_1' \Lambda Q \qquad (2.1)$$

holds. Here $\Lambda^2 = \text{diag}(\lambda_1^2, \ldots, \lambda_r^2, 0, \ldots, 0)$ and the λ_i^2 are the positive eigenvalues of $X'X$. Supplement P_1 by P_2, a matrix of

order $n - k \times n$ such that $P = (P_1' : P_2')'$ is an orthogonal $n \times n$-matrix. Let us now consider the transformation

$$Y \rightarrow \begin{cases} Y_{(1)} = P_1 Y = P_1 X \beta + \sigma P_1 \epsilon = \Lambda \gamma + \sigma \epsilon_1^* \\ Y_{(2)} = P_2 Y = \sigma P_2 \epsilon = \sigma \epsilon_1^* \, , \end{cases} \qquad (2.2)$$

where

$$\epsilon_1^* = P_1 \epsilon, \quad \epsilon_2^* = P_2 \epsilon, \quad \gamma = Q\beta = (\gamma_1, \ldots, \gamma_k)' . \qquad (2.3)$$

Note that $\Lambda \gamma = \text{diag}(\lambda_1 \gamma_1, \ldots, \lambda_r \gamma_r, 0, \ldots, 0)$. We denote the $n-r$ components of the vector $Y = (Y_{(1)}', Y_{(2)}')'$ which have mean zero by $u_1, \ldots, u_{n-r}$. The vector of the first r components of $Y_{(1)}'$ which have possibly a non-zero mean, is denoted by $Y_{(11)}$. Since $C = TX$, the equation $C\beta = \delta$ is equivalent to $TX\beta = TP_1' \Lambda Q\beta = TP_1' \Lambda \gamma = \delta$. Now let us split up P_1 as $P_1 = (P_{11}, P_{12})$, where P_{11} is $n \times r$ and P_{12} is $n \times k-r$. Let $T_1 = T P_{11}'$. Then clearly rank $(T_1) = s$ and $TX\beta = \delta$ is equivalent to $T_1 \text{diag}(\lambda_1 \gamma_1, \ldots, \lambda_r \gamma_r) = \delta$. Let T_2 be a $r-s \times r$-matrix such that Rank $(T_2) = r-s$ and $T_2' T_1 = 0$. We now employ the following transformation:

$$Y_{(11)} \rightarrow \begin{cases} (T_1 T_1')^{-1}(T_1 Y_{(11)} - \delta) = (v_1, \ldots, v_s)' \\ (T_2 T_2')^{-1}(T_2 Y_{(11)}) = (w_1, \ldots, w_{r-s})' . \end{cases} \qquad (2.4)$$

Combining (2.2) and (2.4) we can conclude the following:

$$v = (v_1, \ldots, v_s)' = \theta + \sigma \epsilon_1 \qquad (2.5)$$

$$w = (w_1, \ldots, w_{r-s})' = \eta + \sigma \epsilon_2$$

$$u = (u_1, \ldots, u_{n-r})' = \sigma \epsilon_3 \, ,$$

with $\epsilon = (\epsilon_1', \epsilon_2', \epsilon_3')'$, $E\epsilon = 0$, $E\epsilon\epsilon' = I_n$, $\epsilon \sim q(\epsilon'\epsilon)$, q convex, $\dagger$ and $H_0: A\beta = \delta$ is equivalent to $H_0: \theta = 0$, η and σ^2 being nuisance para-

meters. This is the canonical form of the problem. It may be
noted that Ghosh and Sinha (1980) made use of this canonical form
without explicitly stating how it can be obtained. The above
analysis spells out clearly the necessary steps leading from (1.1)
to (2.3). Under the assumption of normality of ε it follows
clearly that the F-test based on $F = (v'v)/(u'u)$ is LRT and UMPI
(under a suitable group of transformations).

3. UMPI - PROPERTY OF THE F-TEST

Without any loss of generality we treat the problem in the
canonical form (2.5). As in Ghosh and Sinha (1980) obviously the
testing problem remains invariant under the group G of trans-
formations

$$v \rightarrow c\, P_s v, \quad w \rightarrow w+b, \quad u \rightarrow c\, P_{n-r} u, \qquad (3.1)$$

where c runs over the non-zero reals, b runs over $\mathbb{R}^{r-s}$,
and P_k runs over p_k, the group of k×k orthogonal transformations
(k=s and n-r, respectively). As is well-known a maximal invariant
under G is $T_0 = (v'v)/(u'u)$ and a maximal invariant parameter is
$\delta = (\theta'\theta)/\sigma^2$. To obtain a formal expression of the distribution
function of T_0, we first note that the joint distribution of v
and u has the density (Dawid (1977), Dempster (1969))

$$f(v,u|\theta,\sigma^2) = \sigma^{-(n-r+s)}\, q([(v-\theta)'(v-\theta)+u'u]/\sigma^2), \quad q \text{ convex} \hat{\uparrow}. (3.2)$$

To obtain the distribution of T_0 from (3.2) we apply Wijsman's
(1967) representation theorem which we state below as a lemma in
a suitable form:

3.1 Lemma. Let $f(z)$ be the pdf of $z = (v', u')'$ and let $T_0 = t(z)$ be a maximal invariant under the group G of transformations (3.1): $v \rightarrow c\, P_s v$, $u \rightarrow c\, P_{n-r} u$. Let, moreover, $P_\delta^{T_0}$ be the distribution induced by T_0 under $\delta = (\theta' \theta)/\sigma^2$. Then the pdf of T with respect to $P_0^{T_0}$ evaluated at $T_0 = t(z)$ is given by

$$\frac{dP_\delta^{T_0}}{dP_0^{T_0}} (t(z)) = f_{T_0}(t(z)|\delta) \tag{3.3}$$

$$= (\int_G f(gz|\theta,\sigma^2)|c|^{n-r-s} d\nu(g))/(\int_G f(g \cdot z|0,\sigma^2)|c|^{n-r-s} d\nu(g))$$

where ν is a left invariant measure on G and $g = (c, P_s, P_{n-r})$, $gz = (c\, P_s v, c\, P_{n-r} u)$.

Conditions under which (3.3) is valid are stated in Wijsman (1967). It is easy to verify that these conditions are satisfied in our case [see Kariya (1978, 1981)]. We choose $\nu = \nu_1 \times \nu_2 \times \nu_3$ where $d\nu_1(c) = dc /|c|$ and ν_2 and ν_3 are the left invariant probability measures on P_s and P_{n-r}, respectively. Lemma 3.1 applied to our setup (3.2) yields the following.

3.2 Lemma. The pdf of T_0, $f_{T_0}(t|\delta)$ is evaluated as

$$(\int_{-\infty}^{\infty} \int_{P_s} q(c^2 - 2c[(v' P_s \eta)/(v'v + \eta'\eta)^{\frac{1}{2}}] + \eta'\eta) d\nu_2(P_s)|c|^{n-r+s-1} dc)$$

$$/(\int_{-\infty}^{\infty} q(c^2)|c|^{n-r+s-1} dc), \tag{3.4}$$

where $\eta = \theta/\sigma$ and $\eta'\eta = \delta$.

Proof. From (3.2) and the action of the transformation g on $z = (v', u')'$ we get

$$f(g \cdot z | \theta, \sigma^2) = \sigma^{-(n-r+s)} q([(cP_s v - \theta)'(cP_s v - \theta) + c^2 u'u]/\sigma^2)$$

$$= \sigma^{-(n-r+s)} q([c^2(v'v + u'u) - 2cv'P_s'\theta + \theta'\theta]/\sigma^2)$$

$$= \sigma^{-(n-r+s)} q((c^*)^2 - 2c^*(v'v + u'u)^{-\frac{1}{2}} v'P_s'\eta + \eta'\eta)$$

where $c^* = [c(v'v + u'u)^{\frac{1}{2}}]/\sigma$.

From (3.3) the numerator of $f_{T_0}(t|\delta)$ is obtained as

$$\int_{-\infty}^{\infty} \int_{P_{r-s}} \int_{P_s} q((c^*)^2 - 2c^* v'P_s\eta(v'v+u'u)^{-\frac{1}{2}} + \eta'\eta) \quad (3.5)$$

$$|c^*|^{n-r+s-1}(v'v+u'u)^{-(n-r+s)/2} dc^* d\nu_2(P_s) d\nu_3(P_{r-s})$$

whereas the denominator of (3.3) obtained from (3.5) by putting $\eta = 0$, is given by

$$\int_{-\infty}^{\infty} \int_{P_{r-s}} \int_{P_s} q((c^*)^2) |c^*|^{n-r+s-1}(v'v+u'u)^{-(n-r+s)/2} dc^* d\nu_2(P_s) d\nu_3(P_{r-s})$$

$$(3.6)$$

Since the integrand in (3.5) does not depend on P_{r-s} and the integrand in (3.6) does not depend on P_s and P_{r-s} and the integral $\int_{r-s} d\nu_3(P_{r-s})$ equals one, together with $\int_{P_s} d\nu_2(P_s) = 1$, by taking the ratio of (3.5) to (3.6), the lemma follows. Q.E.D.

We are now in a position to prove our main result.

3.3 Theorem. Under the model (2.3), the F-test based on T_0 is UMPI with respect to the group G of transformations (3.1).

Proof. Using the convexity of q we will show that $f_{T_0}(t|\delta)$ for fixed $\delta > 0$ is isotonic in t ($\uparrow t$), thereby proving the theorem.

Let $t^* = v(v'v+u'u)^{-\frac{1}{2}}$ implying that $f_{T_0}(t|\delta)$ evidently depends on t^*. We first show that $f_{T_0}(t|\delta)|_{t^*} = f_{T_0}(t|\delta)|_{0(s)t^*}$ for

all orthogonal matrices $O(s) \in P_s$. This will mean that f really depends on $(t^*)'t^*$, i.e., on t. Replacing t^* by $O(s)t^*$ we get the numerator of $f_{T_0}(t|\delta)$ in (3.4) as

$$\int_{-\infty}^{\infty} \int_{P_s} q(c^2 - 2c \ v'(O(s))'P_s \ \eta \ (v'v + u'u)^{-\frac{1}{2}} + \eta'\eta) |c|^{n-r+s-1} d\nu_2(P_s) dc.$$

Since $(O(s))'P_s$ is orthogonal and $\nu_2(P_s) = \nu_2((O(s))'P_s)$ due to the left-invariance property of the measure ν the claim follows.

Denoting the numerator of $f_{T_0}(t|\delta)$ by $H(t^*|\delta)$, then in view of $H(t^*|\delta) = H(-t^*|\delta)$ it follows from the convexity of q that for any $\alpha \in [0,1]$:

$$H(-t^*|\delta) = H(t^*|\delta) = \alpha \ H(t^*|\delta) + (1-\alpha) \ H(-t^*|\delta) \tag{3.7}$$

$$= \int_{-\infty}^{\infty} \int_{P_s} [\alpha \ q(c^2 - 2c(t^*)' \ P_s \ \eta + \eta'\eta)$$

$$+ (1-\alpha) \ q(c^2 + 2c((t^*)'P_s \ \eta + \eta'\eta)] |c|^{n-r+s-1} d\nu_2(P_s) dc$$

$$\geq \int_{-\infty}^{\infty} \int_{P_s} [q(c^2 - 2c(2\alpha-1)(t^*)'P_s \ \eta + \eta'\eta)] |c|^{n-r+s-1} d\nu_2(P_s) dc$$

$$= H((2\alpha-1)t^*|\delta) = H(-(2\alpha-1)t^*|\delta).$$

Since $H(t^*|\delta)$ depends only on $(t^*)'t^*$ and $(2\alpha-1)^2 \leq 1$ it follows indeed that $f_{T_0}(t|\delta)$ is isotonic. Q.E.D.

4. LRT - PROPERTY OF THE F-TEST AND ITS DISTRIBUTION

Since $q \uparrow$, it follows that the maximum likelihood estimators of θ, η are given by $\hat{\theta} = v, \hat{\eta} = w$, yielding

$$\sup_{\theta, \eta} L(\theta, \eta, \sigma^2 | v, w, u) = \sigma^{-n} q([u'u]/\sigma^2). \qquad (4.1)$$

In order to maximize the right hand side of (4.1) with respect to σ it is easily seen that

$$\sup_{\sigma} \sigma^{-n} q([u'u]/\sigma^2) = (u'u)^{-n/2} K \qquad (4.2)$$

where $K = \sup_{\sigma} \sigma^{-n} q(\sigma^{-2})$.

Similarly the supremum of the likelihood function under $H_0: \theta = 0$ is obtained as

$$\sup_{\eta, \sigma} \sigma^{-n} q([v'v + (w-\eta)'(w-\eta)]/\sigma^2) = (u'u + v'v)^{-n/2} K. \qquad (4.3)$$

Hence the LRT is equivalent to the F-test.

That the F-statistic $F = (v'v)(n-r)/(u'u)s$ is distributed as a central F-variable under H_0 whatever q may be follows from the results due to Dawid (1977).

5. A ROBUSTNESS PROPERTY OF THE BEST LINEAR UNBIASED ESTIMATOR

Consider the linear model

$$y = X\beta + U\varepsilon, \quad E(\varepsilon) = 0, \quad E(\varepsilon\varepsilon') = I_m, \qquad (5.1)$$

where X is a given $n \times k$ and U is a given $n \times m$ - matrix, Rank $(U) = m$. Then

$$Ey = X\beta, \quad \text{Cov } y = E(y-Ey)(y-Ey)' = UU' = :Q. \qquad (5.2)$$

Let $C\beta$ be any estimable function of β, i.e., $C = TX$ for some T. A best linear unbiased estimator of $C\beta$ is any linear function G such that $GX = C$ and $GQa = 0$ if $X'a = 0$.

In the sequel we will assume that ε is spherically distributed. This means that $T\varepsilon$ has the same distribution as ε for all orthogonal $m \times m$ matrices T. If ε has a density this is equivalent to the requirement that the density is a function of $\varepsilon'\varepsilon$. This follows from the fact that $\varepsilon'\varepsilon$ is a maximal invariant with respect to the group of orthogonal transformations.

We will show that any BLUE Gy of $C\beta$ is more concentrated around $C\beta$ than any other unbiased estimator Ly of $C\beta$. More precisely:

5.1 <u>Theorem</u>. Let the model (5.1) be given and let ε be spherically distributed. Then for any $c \varepsilon R$ and any n.n.d. matrix A of appropriate order

$$P((Gy-C\beta)' A(Gy-C\beta) \le c^2) \ge \tag{5.3}$$
$$P((Ly-C\beta)' A(Ly-C\beta) \le c^2),$$

where Gy is any BLUE of $C\beta$ and Ly is any unbiased estimator of $C\beta$.

<u>Proof</u>. For any unbiased estimator Ly of $C\beta$ we get $Ly = LX\beta + LU\varepsilon = C\beta + LU\varepsilon$. Therefore the two probabilities which have to be compared are:

$$P((\varepsilon'U'G' AGU\varepsilon) \le c^2) \text{ and } P((\varepsilon'U'L' ALU\varepsilon) \le c^2). \tag{5.4}$$

We prove that for any L_0 the non-zero eigenvalues of $U'L_0'A L_0 U$ and $A^{\frac{1}{2}} L_0 U U' L_0' A^{\frac{1}{2}} = A^{\frac{1}{2}} L_0 Q L_0' A^{\frac{1}{2}}$ coincide. Indeed if $U'L_0 A L_0 Ux = \lambda x$, $x \ne 0$, then $\lambda A^{\frac{1}{2}}L_0 Ux = A^{\frac{1}{2}} L_0 UU' L_0 A^{\frac{1}{2}}A^{\frac{1}{2}}L_0 Ux = A^{\frac{1}{2}}L_0 Q L_0'A^{\frac{1}{2}}(A^{\frac{1}{2}}L_0 Ux)$ implying that λ is an eigenvalue of $A^{\frac{1}{2}}L_0 Q L_0'A^{\frac{1}{2}}$ unless $A^{\frac{1}{2}}L_0 Ux = 0$

in which case $\lambda = 0$ in view of $x \neq 0$. In the same way it follows from $A^{\frac{1}{2}} L_0 Q L_0' A^{\frac{1}{2}} y = \mu y$ by premultiplying with $U' L_0' A^{\frac{1}{2}}$ that μ is an eigenvalue of $U' L_0' A L_0 U$ unless $U' L_0' A^{\frac{1}{2}} y = 0$ in which case again $\mu = 0$.

Since $A^{\frac{1}{2}} G Q G' A^{\frac{1}{2}} \leq A^{\frac{1}{2}} L Q L' A^{\frac{1}{2}}$ by the BLUE-property it follows from the Courant-Fisher min-max-Theorem (see, e.g., Bellman (1970), p. 115-117) that all the eigenvalues of $A^{\frac{1}{2}} G Q G' A^{\frac{1}{2}}$ are smaller than the corresponding eigenvalues of $A^{\frac{1}{2}} L Q L' A^{\frac{1}{2}}$ if ordered according to decreasing magnitude. This would also hold for $U' G' A G U$ and $U' L' A L U$ if we can show that the dimension of the null-space of $U' L' A U$ cannot be greater than the dimension of the null-space of $U' G' A G U$. Now $U' G A U x = 0$ is equivalent to $A G U x = 0$. This can happen either if $x = U' b$, $X' b = 0$ or $U x = X \beta$ and $A C \beta = A T X \beta = 0$. If $U x = X \beta$ and $T X \beta = C \beta \in \text{im}(A)$ does not vanish unless $C \beta = 0$. But in this case $A L U x = A L X \beta = C \beta$ does not vanish, too. Therefore $\dim(\text{im}(ALU)') \geq \dim(U^{-1}(\text{im} X) \cap (TU)^{-1}(\text{im} A)) = \dim(\text{im}(U' G A G U))$. This implies the required condition about the null-space. Now let T be an orthogonal transformation such that $U' G' A G U = T \text{diag}(\lambda_1, \ldots, \lambda_n) T'$ and similarly T_1 be an orthogonal transformation such that $T_1 \text{diag}(\lambda_1^*, \ldots, \lambda_m^*) T_1' = U' L' A L U$. By spherical symmetry $T' \epsilon$ and $T_1' \epsilon$ have the same distribution as ϵ. Therefore

$$P(\epsilon' U' G' A G U \leq c^2) = P(\epsilon' \text{diag}(\lambda_1, \ldots, \lambda_m) \epsilon \leq c^2) \tag{5.5}$$

and

$$P(\epsilon' U' L' A L U \epsilon \leq c^2) = P(\epsilon' \text{diag}(\lambda_1^*, \ldots, \lambda_m^*) \epsilon \leq c^2). \tag{5.6}$$

Since $\lambda_i \leq \lambda_i^*$, $i = 1, 2, \ldots, m$, it follows that ϵ' diag $(\lambda_1, \ldots, \lambda_m)\epsilon \leq$ ϵ' diag $(\lambda_1^*, \ldots, \lambda_m^*)\epsilon$ and from this evidently (5.3) is obtained.

Q.E.D.

5.2 Remark. A weak version of this result was recently presented by Ali and Ponnapalli, (1982).

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Consider a linear model $Y = X\beta + \sigma\epsilon$, $E(\epsilon) = 0$, $E(\epsilon\epsilon') = I_n$ with β, σ unknown. For the problem of testing the linear hypothesis $C\beta = \delta$, $im(C') \subseteq im(X')$, Ghosh and Sinha (1980) proved that the properties of the usual F-test being LRT and UMPI (under a suitable group of transformations) remain valid for specific non-normal families. In this paper it is shown that both criterion and inference robustness of the F-test hold under the assumption $\epsilon \sim q(\epsilon'\epsilon)$, q convex and isotonic.

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This result is similar to a robustness property of Hotelling's T^2-test proved by Kariya (1981). Finally it is proved that the Best Linear Unbiased Estimator (BLUE) of any estimable function $C\beta$ is more concentrated around $C\beta$ than any other unbiased estimator of $C\beta$ under the assumption that ε is spherically distributed.

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