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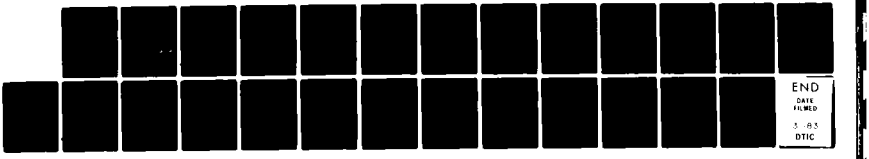

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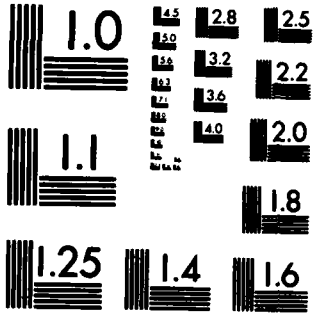

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CRACK GROWTH UNDER PLANE STRESS CONDITIONS IN AN ELASTIC PERFECTLY-PLASTIC MATERIAL

by

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ABSTRACT

Mode-I crack growth under conditions of generalized plane stress has been investigated. It has been assumed that near the plane of the crack in the loading zone, the simple stress components corresponding to a central fan field maintain validity up to the elastic-plastic boundary. By the use of expansions of the particle velocities in the coordinate y, and by matching of the relevant stress components and particle velocities to the dominant terms of appropriate elastic fields at the elastic-plastic boundary, a complete solution has been obtained for $\varepsilon_{(y)}$ in the plane of the crack. The solution is valid from the propagating crack tip up to the moving elastic-plastic boundary. A self-similar crack nucleating at a point and steady-state crack propagation have been considered as special cases.

1. INTRODUCTION

The asymptotic structure of quasi-static near-tip fields of stress and deformation for a growing crack in an elastic perfectly-plastic material has been discussed by several authors. A detailed discussion of near-tip fields has recently been given by Rice (1982), who reviewed the earlier contributions and retrieved them as special cases of a general formulation for materials of arbitrary yield condition and associated flow rule. In general, the analytical near-tip results must be supplemented by numerical calculations to obtain certain arbitrary functions that enter in the asymptotic considerations of the near-tip fields.

In this paper analytic expressions have been obtained for the strain in the plane of the crack, which are valid from the propagating crack tip to the moving elastic-plastic boundary. The analytical approach employs expansions of the particle velocities in powers of y (the distance from the plane of the crack), and yields ordinary differential equations with respect to x for the coefficients. The arbitrary functions that enter in integrating these equations have been obtained by matching the fields in the plastic loading zone to the dominant terms of the corresponding elastic fields at the elastic-plastic boundary.

The method is first demonstrated for anti-plane strain, and the results of Rice (1968) are recovered. Next, the case of generalized plane stress has been considered in detail. It has been assumed that the stress components for a centered fan field (see e.g. Hutchinson,

1968), which satisfy the yield condition and the equilibrium equations, are valid up to the elastic-plastic boundary (at least in the plane of the crack). An explicit expression has been obtained for the total strain $\varepsilon_y(x,0,t)$. Self-similar crack growth fields and steady-state fields have been considered as special cases. The results of this paper are particularly suited for use in conjunction with a critical strain criterion.

2. Governing Equations

The x_3-axis of a stationary coordinate system is parallel to the crack front, and x_1 points in the direction of crack growth. The position of the crack tip is defined by $x_1 = a(t)$. A moving coordinate system, x,y,z is centered at the crack tip, with its axes parallel to the x_1,x_2 and x_3 axes. Relative to the moving coordinate system we also define polar coordinates r,θ, with $\theta = 0$ coinciding with the positive x direction. The geometry is shown in Fig. 1.

Relative to the stationary coordinates the equilibrium equations are

$$\partial_i \sigma_{ij} = 0 \tag{2.1}$$

where $\sigma_{ij} = \sigma_{ji}$ is the stress tensor (i,j,k = 1,2,3), $\partial_i = \partial/\partial x_i$, and the summation convention applies.

The Huber-Mises yield criterion may be stated as

$$\frac{1}{2} s_{ij} s_{ij} = k^2 \tag{2.2}$$

where s_{ij} is the deviatoric stress tensor and k is the yield stress in pure shear.

The rate of deformation is defined by

$$D_{ij} = \dot{\varepsilon}_{ij} = \frac{1}{2}(\partial_j v_i + \partial_i v_j) \tag{2.3}$$

Here ε_{ij} is the (infinitesimal) strain, the superposed dot denotes time rate at a fixed material point, v_i is the velocity field, and

$v_i = \dot{u}_i$, where u_i is the displacement field. The total strain rates are defined by

$$D_{ij} = D^e_{ij} + D^p_{ij} , \tag{2.4}$$

where the elastic strain rates are related to the stress rates by

$$D^e_{ij} = \frac{1}{E}[(1+\nu)\dot{\sigma}_{ij} - \nu\delta_{ij}\dot{\sigma}_{kk}] , \tag{2.5}$$

and the plastic strain rates are

$$D^p_{ij} = \dot{\Lambda} s_{ij} \tag{2.6}$$

In (2.5)-(2.6), E and ν are Young's modulus and Poisson's ratio, respectively and $\dot{\Lambda}$ is a positive function of time and the spatial coordinates.

The spatial derivatives with respect to the cartesian coordinates in the stationary and moving coordinate systems are directly related by

$$\partial_1 = \partial_x , \quad \partial_2 = \partial_y , \quad \partial_3 = \partial_z \tag{2.7}$$

In the moving coordinate system the material time derivative is

$$(\dot{}) = \partial_t - \dot{a}\partial_x \tag{2.8}$$

where $\dot{a} = da/dt$ is the speed of the crack tip.

3. Anti-plane Strain

In the moving coordinate system the only non-vanishing displacement component is w(x,y,t). The non-vanishing stress components are $\sigma_{xz} = \sigma_{zx}$ and $\sigma_{yz} = \sigma_{zy}$. The Huber-Mises yield criterion (2.2) reduces to

$$(\sigma_{xz})^2 + (\sigma_{yz})^2 = k^2 \tag{3.1}$$

In the loading region ahead of the crack tip, the stresses may be written as

$$\sigma_{yz} = k\cos\theta\ , \qquad \sigma_{xz} = -k\sin\theta \tag{3.2a,b}$$

It may be verified that (3.2a,b) not only satisfy (3.1), but also the equilibrium equations (2.1).

For small values of θ (i.e., $y/x << 1$) the plastic fields in the loading zone will be matched to the dominant terms of the elastic fields. In polar coordinates R,ψ centered at point E the dominant terms of the elastic solution on the elastic side of the elastic-plastic boundary are taken in the general form

$$w = \left(\frac{R}{2\pi}\right)^{\frac{1}{2}} \frac{2}{\mu} K_{III} \sin\frac{1}{2}\psi\ , \quad \mu = \text{shear modulus} \tag{3.3}$$

$$\sigma_{Rz} = \left(\frac{1}{2\pi R}\right)^{\frac{1}{2}} K_{III} \sin\frac{1}{2}\psi \tag{3.4}$$

$$\sigma_{\psi z} = \left(\frac{1}{2\pi P}\right)^{\frac{1}{2}} K_{III} \cos\frac{1}{2}\psi \tag{3.5}$$

It should be noted that the center of the elastic field is <u>not</u> taken to coincide with the crack tip. The center is located at a moving point E, whose position is defined by $x_1 = e(t)$. The geometry is shown in Fig. 2.

From the condition that the elastic field should just reach the yield condition (3.1) at the elastic-plastic boundary, we obtain by the use of (3.4)-(3.5)

$$\left(\frac{1}{2\pi R_p}\right)^{\frac{1}{2}} K_{III} = k \tag{3.6}$$

where $R = R_p$ defines the radius of curvature of the elastic-plastic boundary, at least for small values of ψ. Another condition is that σ_{Rz} (i.e. the shear stress component in the R,ψ system) should be continuous at the elastic-plastic boundary. We find by using (3.2a,b):

$$\sigma_{Rz} = \sigma_{yz}\cos\psi + \sigma_{xz}\sin\psi = k\,\sin(\psi-\theta) \tag{3.7}$$

Thus, by the use of (3.4) and (3.6)

$$\sin\tfrac{1}{2}\psi = \sin(\psi-\theta) \quad , \text{ i.e., } \theta = \tfrac{1}{2}\psi \tag{3.8a,b}$$

For <u>small</u> θ and ψ this result implies that the center of the elastic field E is located at

$$x_E = \frac{1}{2}\,x_p, \text{ and thus } e(t) = a(t) + \frac{1}{2}\,x_p(t), \quad R_p(t) = \frac{1}{2}\,x_p(t) \ , \tag{3.9a,b,c}$$

where $x_p(t)$ defines the x-coordinate of the elastic-plastic boundary in the plane $y = 0$. Thus, just as for the case of a stationary crack, the elastic field is centered halfway between the crack tip and the elastic-plastic boundary.

The third condition requires that the particle velocity is continuous at the elastic-plastic boundary. In the moving coordinates R,ψ, the material time derivative may be written as

$$(\dot{}) = \partial_t - \dot{e}\,(\cos\psi\ \partial_R - \frac{\sin\psi}{R}\ \partial_\psi) \tag{3.10}$$

where $\dot{e}$ is the velocity in the x_1-direction of the center E of the elastic field. By applying the operator (3.10) to the expression for

w given by (3.3), we obtain at $R = R_p$

$$\dot{w}_p = \frac{k}{\mu} (\dot{R}_p + \dot{e}) \sin\frac{1}{2}\psi \tag{3.11}$$

where the relation (3.6) has been used. Since $\dot{e} = \dot{a} + \frac{1}{2} \dot{x}_p$ and $\dot{R}_p = \frac{1}{2} \dot{x}_p$, Eq.(3.11) further reduces to

$$\dot{w}_p = \frac{k}{\mu} (\dot{a} + \dot{x}_p) \sin\theta \tag{3.12}$$

where (3.8b) has also been used.

For quasi-static growth of cracks in anti-plane strain, the particle velocity in the loading region, has been derived by Rice (1968) in terms of the position of the elastic-plastic boundary. The corresponding expression for the total strain in the plane of the crack in between the crack tip and the elastic-plastic boundary depends only on a(t) and $x_p(t)$. This interesting result suggests that it should be possible to obtain the strain independently from a closed system of equations valid only near y = 0. We will show the derivation of the relevant equations primarily as a preliminary to the corresponding analysis for the more complicated plane stress case.

As point of departure we take the following expansions for $y/x << 1$:

$$\dot{w}(x,y,t) = \dot{w}_1(x,t)\, y + \dot{w}_3(x,t)\, y^3 + O(y^5) \tag{3.13}$$

$$\dot{\Lambda}(x,y,t) = \dot{\Lambda}_o(x,t) + \dot{\Lambda}_2(x,t)\, y^2 + O(y^4) \tag{3.14}$$

The strain rates follow from (3.13) as

$$D_{xz} = \frac{1}{2}\frac{\partial \dot{w}_1}{\partial x}\, y + O(y^3) \tag{3.15}$$

$$D_{yz} = \frac{1}{2}\,\dot{w}_1 + \frac{3}{2}\,\dot{w}_3\, y^2 + O(y^4) \tag{3.16}$$

From Eqs.(2.4)-(2.6) we find by using (3.2a,b)

$$D_{xz} = -\frac{k}{2\mu}\frac{\dot{a}}{r}\sin\theta\cos\theta - \dot{\Lambda}k\sin\theta \tag{3.17}$$

$$D_{yz} = -\frac{k}{2\mu}\frac{\dot{a}}{r}\sin^2\theta + \dot{\Lambda}k\cos\theta \tag{3.18}$$

For small $\theta \sim y/x << 1$, equality of terms of order unity in (3.16) and (3.18) yields

$$\dot{\Lambda}_o k = \frac{1}{2}\,\dot{w}_1 \tag{3.19}$$

Equality of terms of order y in (3.15) and (3.17) gives

$$-\frac{1}{2}\frac{k}{\mu}\frac{\dot{a}}{x^2} - \dot{\Lambda}_o\frac{k}{x} = \frac{1}{2}\frac{\partial \dot{w}_1}{\partial x} \tag{3.20}$$

By combining (3.19) and (3.20) we then find

$$\frac{\partial \dot{w}_1}{\partial x} + \frac{\dot{w}_1}{x} = -\frac{k}{\mu}\frac{\dot{a}}{x^2} \tag{3.21}$$

Integration of (3.21) yields

$$\dot{w}_1 = -\frac{k}{\mu}\frac{\dot{a}}{x}\,\ell n\left(\frac{x}{x_p}\right) + \frac{A(t)}{x} \tag{3.22}$$

where $x = x_p(t)$ is the length of the plastic zone in the plane of the crack. Evidently, $A(t)(y/x)$ is the particle velocity at the elastic plastic boundary near y=0. For small values of θ it follows from (3.12) that $A(t) = (k/\mu)(\dot{a} + \dot{x}_p)$. Integration of (3.22) over time in the fixed coordinates yields for $t \geq t_p$:

$$\frac{\partial w}{\partial y}(x_1,0,t) = \frac{k}{\mu} + \frac{k}{\mu}\int_{t_p}^{t} \frac{1}{x_1-a(t)} \left\{\left[1 - \ell n \frac{x_1-a(t)}{x_p(t)}\right]\frac{da}{dt} + \frac{dx_p}{dt}\right\} dt \qquad (3.23)$$

where t_p is the time that the elastic-plastic boundary arrives at position x_1, i.e., $a(t_p) + x_p(t_p) = x_1$ and k/μ is the elastic strain at the elastic-plastic boundary.

Several interesting observations based on (3.23) have been made by Rice (1968). As discussed by Rice a criterion of critical plastic strain γ_p at a distance ρ_s ahead of the crack tip (McClintock and Irwin, 1965) may be employed in conjunction with (3.23) to compute the required $x_p(t)$ for quasistatic extension of the crack over a distance a(t). We note that the corresponding value of $R_p(t)$ is given by (3.9c), while the required external load subsequently follows from (3.6).

4. Generalized Plane Stress

For generalized plane stress the Huber-Mises yield criterion (2.2) becomes

$$\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\sigma_{xy}^2 = 3k^2 \tag{4.1}$$

In the loading region ahead of the crack tip expressions for the stresses which satisfy both (4.1) and the equilibrium equations are

$$\sigma_x = k\cos^3\theta, \quad \sigma_y = k(2\cos^3\theta + 3\sin^2\theta\cos\theta), \quad \sigma_{xy} = -k\sin^3\theta \tag{4.2a,b,c}$$

For small values of θ (i.e., $y/x \ll 1$) the field in the plastic loading zone will be matched at the elastic-plastic boundary to the dominant terms of a corresponding elastic field. For the elastic field we do, however, not take the field for a crack, but rather that for a notch with $\frac{1}{2}\rho$ as radius of curvature at its tip. In polar coordinates R,ψ, the appropriate Mode-I stress fields are given by Creager and Paris (1967) as

$$\sigma_x = \left(\frac{1}{2\pi R}\right)^{\frac{1}{2}} K_I\{\cos\tfrac{1}{2}\psi\,[1-\sin\tfrac{1}{2}\psi\,\sin\tfrac{3}{2}\psi] - \frac{\rho}{2R}\cos\tfrac{3}{2}\psi\} \tag{4.3}$$

$$\sigma_y = \left(\frac{1}{2\pi R}\right)^{\frac{1}{2}} K_I\{\cos\tfrac{1}{2}\psi\,[1+\sin\tfrac{1}{2}\psi\,\sin\tfrac{3}{2}\psi] + \frac{\rho}{2R}\cos\tfrac{3}{2}\psi\} \tag{4.4}$$

$$\sigma_{xy} = \left(\frac{1}{2\pi R}\right)^{\frac{1}{2}} K_I\{\sin\tfrac{1}{2}\psi\,\cos\tfrac{1}{2}\psi\,\cos\tfrac{3}{2}\psi - \frac{\rho}{2R}\sin\tfrac{3}{2}\psi\} \tag{4.5}$$

Note that the tip of the notch, which is not the tip of the crack nor the elastic-plastic boundary, is a distance $\frac{1}{2}\rho$ from the origin E, as shown in Fig. 3. Just as for the Mode-III case, the center of the elastic field E, whose position is defined by $x_1 = e(t)$, $y_1 = 0$, is located in between the crack tip and the elastic-plastic boundary

defined by $x=x_p(t)$. For generalized plane stress, the displacements corresponding to (4.3)-(4.5) are

$$u = \left(\frac{R}{2\pi}\right)^{\frac{1}{2}} \frac{1}{2\mu} K_I\{\cos\tfrac{1}{2}\psi\ [\kappa-1 + 2\sin^2\tfrac{1}{2}\psi] + \frac{\rho}{R}\cos\tfrac{1}{2}\psi\} \tag{4.6}$$

$$v = \left(\frac{R}{2\pi}\right)^{\frac{1}{2}} \frac{1}{2\mu} K_I\{\sin\tfrac{1}{2}\psi\ [\kappa+1 - 2\cos^2\tfrac{1}{2}\psi] + \frac{\rho}{R}\sin\tfrac{1}{2}\psi\} \tag{4.7}$$

where $\kappa = (3-\nu)/(1+\nu)$.

From the condition that the elastic field should just reach the yield condition at the elastic-plastic boundary, we obtain by the use of (4.1) and (4.3)-(4.5)

$$\left[\left(\frac{1}{2\pi R_p}\right)^{\frac{1}{2}} K_I\right]^2 \left[1+3\left(\frac{\rho}{2R_p}\right)^2\right] = 3k^2 \tag{4.8}$$

where $R = R_p$ at the elastic-plastic boundary, at least for small values of y. Another condition is that σ_x should be continuous at the elastic-plastic boundary on y=0. By the use of (4.2a) and (4.3) we find

$$k = \left(\frac{1}{2\pi R_p}\right)^{\frac{1}{2}} K_I \left(1 - \frac{\rho}{2R_p}\right) \tag{4.9}$$

From (4.8) and (4.9) it follows that

$$\frac{\rho}{R_p} = \frac{2}{3} \tag{4.10}$$

Hence (4.9) yields

$$\left(\frac{1}{2\pi R_p}\right)^{\frac{1}{2}} K_I = \frac{3}{2}k \tag{4.11}$$

Equations (4.8) and (4.11) show why we have taken elastic fields for a notch rather than for a crack. For an elastic crack-tip field the conditions of reaching the yield condition at the elastic-plastic

boundary would conflict with the condition of continuity of σ_x, as can be checked by setting $\rho \equiv 0$ in (4.8) and (4.9).

For small values of θ and ψ we next consider the continuity of $\sigma_{R\psi}$ and σ_R (i.e., the shear stress and the radial stress in the R,ψ system). We use

$$\sigma_{R\psi} = (\cos^2\psi - \sin^2\psi)\sigma_{xy} + (\sigma_y - \sigma_x)\sin\psi\cos\psi \tag{4.12}$$

and

$$\sigma_R = 2\sigma_{xy}\sin\psi\cos\psi + \sigma_x\cos^2\psi + \sigma_y\sin^2\psi \ , \tag{4.13}$$

in conjunction with (4.2a,b,c) and (4.3)-(4.5). It may be verified that $\sigma_{R\psi}$ is continuous to first order in ψ. The stress σ_R is continuous to order unity by virtue of Eq.(4.11). Collecting terms to order θ^2 and ψ^2 yields the relation

$$\theta = \gamma\psi \ , \text{ where } \gamma = \sqrt{1/2}. \tag{4.14}$$

This result implies that for small values of θ and ψ the center of the elastic field E is located at

$$x_E = (1-\gamma)x_p, \quad \text{and thus} \quad R_p = \gamma x_p \tag{4.15a,b}$$

For $y/x << 1$ it follows from (4.2a,b,c) that

$$\sigma_x = k[1 - \frac{3}{2}(\frac{y}{x})^2] + O(\frac{y}{x})^4 \tag{4.16}$$

$$\sigma_y = 2k + O(\frac{y}{x})^4 \tag{4.17}$$

$$\tau_{xy} = -k(\frac{y}{x})^3 + O(\frac{y}{x})^5 \tag{4.18}$$

From the stress-strain relations (2.4)-(2.6) we find by using (4.16)-(4.18)

$$D_x = -\dot{a}(t)\frac{k}{E}\frac{3}{x}\left(\frac{y}{x}\right)^2 - \dot{\Lambda}k\left(\frac{y}{x}\right)^2 + O\left(\frac{y}{x}\right)^4 \tag{4.19}$$

$$D_y = \dot{a}(t)\frac{k\nu}{E}\frac{3}{x}\left(\frac{y}{x}\right)^2 + \dot{\Lambda}k\left[1 + \frac{1}{2}\left(\frac{y}{x}\right)^2\right] + O\left(\frac{y}{x}\right)^4 \tag{4.20}$$

$$D_{xy} = -\dot{a}(t)\frac{(1+\nu)k}{E}\frac{3}{x}\left(\frac{y}{x}\right)^3 - \dot{\Lambda}k\left(\frac{y}{x}\right)^3 + O\left(\frac{y}{x}\right)^5 \tag{4.21}$$

Now consider the following expansions for $y/x << 1$

$$\dot{u}(x,y,t) = \dot{u}_o(x,t) + \dot{u}_2(x,t)y^2 + O(y^4) \tag{4.22}$$

$$\dot{v}(x,y,t) = \dot{v}_1(x,t)y + O(y^3) \tag{4.23}$$

$$\dot{\Lambda}(x,y,t) = \dot{\Lambda}_o(x,t) + \dot{\Lambda}_2(x,t)y^2 + O(y^4) \tag{4.24}$$

The strain rates follow from (4.22)-(4.23) as

$$D_x = \frac{\partial \dot{u}_o}{\partial x} + \frac{\partial \dot{u}_2}{\partial x}y^2 \tag{4.25}$$

$$D_y = \dot{v}_1 \tag{4.26}$$

$$D_{xy} = \left(2\dot{u}_2 + \frac{\partial \dot{v}_1}{\partial x}\right)y \tag{4.27}$$

By equating terms of the same order in (4.19)-(4.21) to the corresponding ones in (4.22)-(4.24) we obtain

$$\frac{\partial \dot{u}_o}{\partial x} = 0\ ,\ \frac{\partial \dot{u}_2}{\partial x} = -\ 3\ \frac{k}{E}\frac{\dot{a}}{x^3} - \frac{1}{x^2}\dot{\Lambda}_o k \tag{4.28a,b}$$

$$\dot{v}_1 = \dot{\Lambda}_o k \tag{4.29}$$

$$2\dot{u}_2 + \frac{\partial \dot{v}_1}{\partial x} = 0 \tag{4.30}$$

By combining (4.28), (4.29) and (4.30) we obtain

$$\frac{1}{2}\frac{\partial^2 v_1}{\partial x^2} - \frac{\dot{v}_1}{x^2} = 3\frac{k}{E}\frac{\dot{a}}{x^3} \tag{4.31}$$

The general solution to (4.31) is

$$\dot{v}_1 = \frac{k}{E}\{-2\frac{\dot{a}}{x}\ln(\frac{x}{x_p}) + \frac{B(t)}{x} + C(t)x^2\}\ , \tag{4.32}$$

and (4.30) subsequently yields

$$\dot{u}_2 = \frac{k}{E}\{\frac{\dot{a}}{x^2}[1-\ln(\frac{x}{x_p})] + \frac{1}{2}\frac{B(t)}{x^2} - C(t)x\}, \tag{4.33}$$

while it follows from (4.28a) that

$$\dot{u}_o = (k/E)\ D(t)$$

The functions B(t), C(t) and D(t) must be determined from continuity of the particle velocity at the elastic-plastic boundary.

By the use of (3.10) we obtain from (4.6) and (4.6) at the elastic-plastic boundary

$$\dot{u}_p = (k/E)[U_o(t) + U_2(t)\theta^2] + O(\theta^4) \tag{4.35}$$

$$\dot{v}_p = (k/E)V_1(t)\theta + O(\theta^3), \tag{4.36}$$

where (4.10), (4.11) and (4.14) have been used. In (4.35)-(4.36)

$$U_o(t) = \frac{3}{4}[(\kappa\gamma - \frac{1}{2}\kappa - \frac{1}{3}\gamma + \frac{5}{6})\dot{x}_p - \frac{1}{2}(\kappa - \frac{5}{3})\dot{a}](E/\mu) \tag{4.37}$$

$$U_2(t) = \frac{3}{64}\frac{1}{\gamma^2}[(-2\gamma - 2\kappa\gamma + \kappa + 5)\dot{x}_p + (\kappa+5)\dot{a}](E/\mu) \tag{4.38}$$

$$V_1(t) = \frac{3}{16}\frac{1}{\gamma}(\kappa+1)(\dot{x}_p+\dot{a})(E/\mu) \tag{4.39}$$

By comparing (4.32)-(4.34) to (4.35)-(4.36) we conclude

$$B(t) = B_1\dot{a}(t) + B_2\dot{x}_p(t) \tag{4.40}$$

$$C(t) = [C_1\dot{a}(t) + C_2\dot{x}_p(t)]/[x_p(t)]^3, \tag{4.41}$$

where

$$B_1 = \frac{1}{32}\frac{1}{\gamma^2}[\kappa+5 + 4\gamma(\kappa+1)](E/\mu) - \frac{2}{3} \tag{4.42}$$

$$B_2 = \frac{1}{32}\frac{1}{\gamma^2}[\kappa+5 + 2\gamma(\kappa+1)](E/\mu) \tag{4.43}$$

$$C_1 = \frac{1}{32}\frac{1}{\gamma^2}[-(\kappa+5)+ 2\gamma(\kappa+1)](E/\mu) + \frac{2}{3} \tag{4.44}$$

$$C_2 = \frac{1}{32}\frac{1}{\gamma^2}[-(\kappa+5) + 4\gamma(\kappa+1)](E/\mu) \tag{4.45}$$

In the plane of the crack we have $\varepsilon_y = v_1$. At the elastic plastic boundary (4.7) yields for small y

$$(\varepsilon_y)_{PB} = \frac{3}{8}\frac{k}{\mu}\frac{1}{\gamma}(\kappa - \frac{1}{3}) \tag{4.46}$$

where (4.11), (4.14) and $\theta \sim y/x$ have been used. In the stationary coordinate system, (4.32) can now be integrated to yield the total strain as

$$\begin{aligned}\varepsilon_y(x_1,t) = (\varepsilon_y)_{PB} &+ \frac{k}{E}\left[\ln\frac{x_1-a(t)}{x_p(t)}\right]^2 - \frac{k}{E}(B_1 + B_2\,\dot{x}_p/\dot{a})\ln\frac{x_1-a(t)}{x_p(t)} \\ &+ \frac{k}{E}\int_{t_p}^{t}\{[2\frac{\dot{x}_p}{x_p} + B_2\frac{d}{dt}(\dot{x}_p/\dot{a})]\ln\frac{x_1-a(t)}{x_p(t)} - (B_1+B_2\,\dot{x}_p/\dot{a})\frac{\dot{x}_p}{x_p}\}\,dt \\ &+ \frac{k}{E}\int_{t_p}^{t}[C_1\dot{a} + C_2\dot{x}_p]\frac{[x_1-a(t)]^2}{[x_p(t)]^3}\,dt\end{aligned} \tag{4.47}$$

Here t_p is the time that the elastic-plastic boundary arrives at position x_1, i.e., $a(t_p) + x_p(t_p) = x_1$.

For a given external load, we presumably know K_I in terms of the increase in "crack length" $a(t) + (1-\gamma)x_p(t)$. A relation between $x_p(t)$ and $a(t)$ can subsequently be obtained by the use of (4.15b) and (4.11). Hence, in principle, $a(t)$ is the only unknown quantity in (4.47). An equation for $a(t)$ and $\dot{a}(t)$ can be obtained from (4.47) by the use of the critical strain criterion for crack propagation. This criterion stipulates that crack growth will proceed when a critical strain level ε_{cr} is maintained for ε_y, in the plane of the crack at a characteristic distance Δ ahead of the crack tip. It appears, however, that it will be very difficult to solve $a(t)$ and $\dot{a}(t)$ from the integral equation that can be extracted from (4.47).

It is of interest to note that $\dot{v}_1$ as defined by (4.32), (4.40) and (4.41) contains terms proportional to $\dot{a}(t)$ and terms proportional to $\dot{x}_p(t)$. The former correspond to crack propagation, while the latter are equally valid for a stationary crack under monotonic loading. By setting $\dot{a}(t) \equiv 0$, we obtain

$$\frac{\partial v_1}{\partial t} = \frac{k}{E}\left\{\frac{B_2}{x} + \frac{C_2 x^2}{[x_p(t)]^3}\right\}\frac{dx_p}{dt}\,, \tag{4.48}$$

This result shows that for a stationary crack and plane stress, the strain ε_y has a singularity of order $1/x$ at the crack tip, in agreement with the results of Rice (1973).

The solution for ε_y in the plane of the crack simplifies considerably for two special cases. The first of these is the case of a self-similar field. A self-similar solution materializes when $a(t)$ is proportional to $x_p(t)$,

$$a(t) = A_p\, x_p(t) \tag{4.49}$$

Substitution of (4.49) into (4.47) and subsequent introduction of the new variable

$$s = [x_1 - a(t)]/x_p(t) = [x_1 - A_p x_p(t)]/x_p(t) \tag{4.50}$$

reduces (4.47) to

$$\varepsilon_y(x/x_p) = (\varepsilon_y)_{PB} + \frac{k}{E}\left[\ln\left(\frac{x}{x_p(t)}\right)\right]^2 - \frac{k}{E}(B_1+B_2/A_p)\ln\left(\frac{x}{(x_p(t)}\right)$$

$$- \frac{k}{E}\int_1^{x/x_p} [2\ln(s) - (B_1+B_2/A_p)]\,\frac{ds}{s+A_p} - \frac{k}{E}\int_1^{x/x_p} (C_1A_p+C_2)\,\frac{s^2 ds}{s+A_p} \tag{4.51}$$

where we have used that $x = x_1 - a(t)$. It is noted that the strain ε_y comes out to depend only on the self-similar variable $x/x_p(t)$.

For the second special case all fields are assumed to be time-invariant to an observer traveling with the crack tip. This is the steady-state case when ε_y depends on $x = x_1 - a(t)$ only. Now we have that $\dot{a} = \dot{x}_p = \text{constant} = c_F$, and $(\dot{}) = -c_F\partial_x$. Equation (4.32) becomes

$$\frac{dv_1}{dx} = \frac{k}{E}\left\{\frac{2}{x}\ln\left(\frac{x}{x_p}\right) - \frac{1}{x}(B_1+B_2) - (C_1+C_2)x^2/x_p^3\right\}, \tag{4.52}$$

where B_1, B_2, C_1 and C_2 are defined by (4.42)-(4.45), and x_p = constant. Equation (4.52) may be integrated to yield

$$\varepsilon_y(x) = (\varepsilon_y)_{PB} + \frac{k}{E}\{[\ell n(\frac{x}{x_p})]^2 - (B_1+B_2)\ell n(\frac{x}{x_p}) - \frac{1}{3}(C_1+C_2)[(\frac{x}{x_p})^3 - 1]\} \tag{4.53}$$

Equation (4.53) offers better possibilities for the application of a critical strain criterion. If $\varepsilon_y = \varepsilon_{cr}$ at $x = \Delta$, (4.53) can be solved for the constant value of x_p. The corresponding external load can subsequently be computed by the use of (4.15b) and (4.11).

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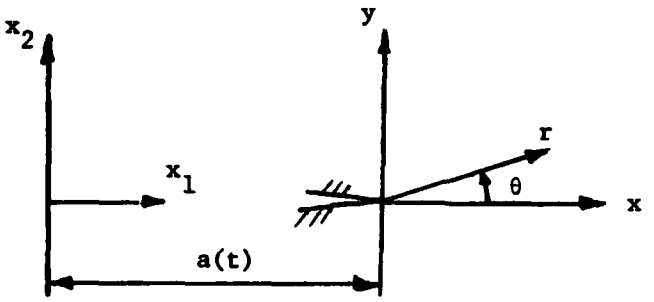

Fig. 1: Moving crack tip

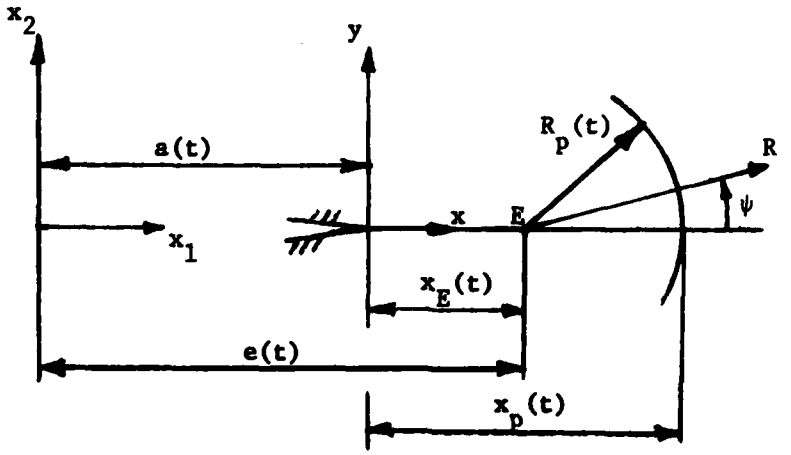

Fig. 2: Center of elastic field E and elastic-plastic boundary

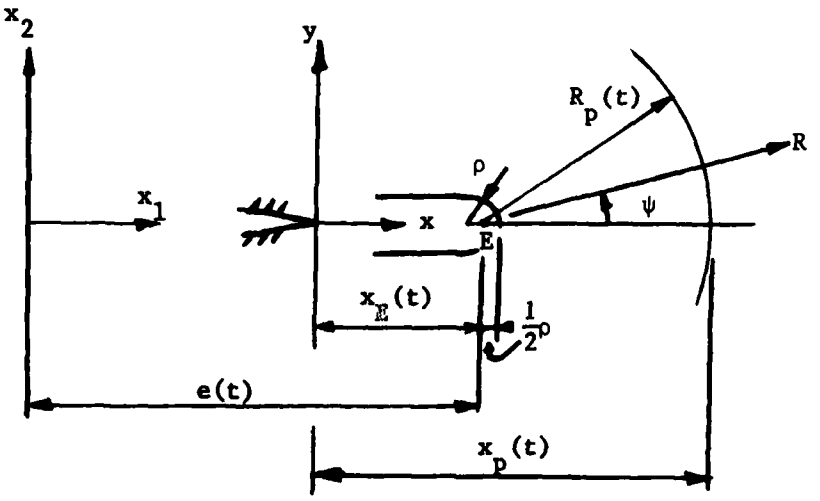

Fig. 3: Center of elastic field E, for slender notch of tip radius ρ

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Mode-I crack growth under conditions of generalized plane stress has been investigated. It has been assumed that near the plane of the crack in the loading zone, the simple stress components corresponding to a central fan field maintain validity up to the elastic-plastic boundary. By the use of expansions of the particle velocities in the coordinate y, and by matching of the relevant stress components and particle velocities to the dominant terms of appropriate elastic fields at the elastic-plastic boundary, a complete solution has been obtained for σ_y in the plane of the crack. The solution is valid from the

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propagating crack tip up to the moving elastic-plastic boundary. A self-similar crack nucleating at a point and steady-state crack propagation have been considered as special cases.

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