



ł

 .

• •

MICROCOPY RESOLUTION TEST CHART NATIONAL BUREAU OF STANDARDS-1963-A

Final Report

on

Research Conducted Under the Office of Naval Research ONR Contract N00014-77-C-0743

APPROVED FOR PUBLIC RELEASE DISTRIBUTION UNLIMITED

Ralph L. Disney, Principal Investigator Gordon Professor Dept. of IEOR VPI & SU Blacksburg, Virginia 24061

Accession For	
NTIS GRA&I	
By Distribution/	
Availability Codes	
Dist	Avail and/or Special

March 1982



Final Report for ONR Contract NO0014-77-C-0743

<u>Major Accomplishments</u>. Two major areas of research were opened up under this contract. One was the initiation of research into approximations for stochastic processes occuring in queueing networks. The other was the initiation of research into sojourn times in queueing networks.

In the former problem conditions have been established for necessary, sufficient and necessary and sufficient conditions for a Markov renewal process to be equivalent to a renewal process. Markov renewal processes were targetted for special study since previous research, in part under ONR sponsorship, has shown that these processes occur quite frequently in queueing network flow processes.

The latter problem was new for us and new for research in queueing network theory. Prior to our involvement sojourn time problems had been studied in nearly all of the classical queueing models. However, during 1977-1979 three papers appeared purporting to solve the sojourn time problem in queueing networks. We showed that each of these papers was in error. From this we lauched into a study of the problem which continues. Resume of Majcr Accomplishments. a) Equivalence of Markov renewal Processes and Renewal Processes.

Based on a considerable number of results obtained from previous research of ours (much of which was supported under previous ONR contracts) it appears that a major candidate for studying flow processes in queueing networks are Markov renewal processes. Such processes are rather difficult to work with if one is to obtain numerical results. We ask, can one approximate the Markov-renewal processes with renewal processes? Our initial approach was to ask under what conditions is a Markov renewal process equivalent to a renewal process. "Equivalent" here means that the intervals in the Markov renewal process are i.i.d. random variables and independent of the state of the Markov renewal process. The following results are given in Tech. reports 7801, 8001 and 8015.

Necessary, sufficient and necessary and sufficient conditions were found for such equivalence. Several queueing systems whose flows are known to be Markov renewal processes were studied to see how these results were applied to those systems. In the process we found that we could give an almost trivial proof to the known theorem that says M/M/1 queues have Poisson (i.e., renewal) departure processes. The converse to that theorem comes trivially from the given necessary and sufficient conditions.

Simple birth-death queues were investigated to prove the known theorem that if $\lambda_n = \lambda$ then the departure process is a renewal process. In doing this we were able to provide a converse to the result which to the best of our knowledge is new.

A queueing model with feedback was investigated where it was shown that the input process is never equivalent to a renewal process. This result is in agreement with that found in Tech. report 7922.

Finally, surprisingly, it was found that one could produce Markov renewal processes that were equivalent to at least two renewal processes. The question of conditions for uniqueness is, thus, still open.

These results were used by Foley in his study of queues with delayed feedback to investigate properties of departure p cesses from his queues. His work was not supported under this contract.

Chandramohan in his work which is still under investigation has asked

2

the question, if one decomposes a random point process under what conditions are the subprocesses uncorrelated. He has investigated the decomposition of marked point processes in general and Markov renewal and Poisson processes in particular under decomposition rules that may depend on the process being decomposed. Among his results is one showing that if a renewal process is decomposed by a Bernoulli switch then the subprocesses are uncorrelated if and only if all three processes are Poisson. This theorem is well known in point process theory if one requires independence of the subprocesses. His result is the only one of which we are aware that uses the easier to verify condition of 0-correlation. He further shows that if a Poisson process is decomposed by a Markov switch then the subprocesses are uncorrelated if and only if the Markov switch is a Bernoulli switch. This is a partial converse to the first result. Finally, several queueing systems are investigated to exhibit correlation properties between flows in queueing networks. This type of study seems to be new to queueing theory and can be important to any statistical study of such networks.

b) Sojourn Times.

ŧ

In response to an inquiry given to us by a journal editor, we studied a three node, acyclic, Jackson queueing network (Tech. report 7901). We were able to show dependencies among nodal sojourn times.

In Tech. report 8016 Kiessler prepared a carefully considered simulation of the network studied in Tech. report 7901. He showed that for large sample sizes (1000,5000) over all parameter settings studied one could not verify statistically that the node 1-node 3 correlations in the sojourn times of a customer were different from 0.

Further, Kiessler showed that if one assumed that the sojourn times were independent (in spite of the results of Tech. report 7901), the

3

distribution function of the total sojourn time of a customer and the simulated total sojourn time (in which the assumed independence is not present) were nearly identical and could not be distinguished by a Kolmogorov-Smirnov test of good-of-fit.

In an attempt to investigate this seeming anomaly we undertook to investigate the same Kiessler network analytically. The results of this investigation are reported in Tech. report 8203. The difficulty of an analytic investigation of the problem eventually shows itself in the need to determine the departure process from a non-stationary M-server queue which has a pre-load. Our funding ran out before we could investigate this problem further. Research is presently continuing, unfunded, on the problem.

In Tech. report 8102 we investigate another sojourn time problem. This one is associated with the queueing system reported in Tech. report 7922. Here we can get a complete characterization of the sojourn time problem and see that computational problems come from the need to compute the semi-Markov kernel associated with a Markov renewal process which includes the sojourn time.

<u>Conclusions</u>. These two topics of approximating random processes and sojourn times in networks of queues both are suggesting a need for an indepth study of approximation methods in queueing networks. We now know a considerable amount about flow processes in these networks. We now have many qualitative theorems. A next step then is to provide algorithms for computing values of use (other than expected values where algorithms do exist.)

However, we are still in need of research into queueing network theory. These networks are ubiquitous. There is still much to be learned

about them.

Other Accomplishments. In addition to the above areas of major accomplishments we have engaged in several other tasks. In particular we were invited by the American Mathematical Society to prepare a paper on queueing network theory which was given at their national meeting in Duluth, 1979. Subsequent requests were received to give the same paper for the Institute of Mathematical Statistics (Vail, Colorado, 1981) and for the AORS XX program held in October, 1981 at Ft. Lee, Virginia. This paper (Tech. report 7923) has since been published in <u>Operations Research</u>: <u>Mathematics and Models</u>, Proceedings of Symposia in Applied Mathematics, vol. 25 (1981), American Mathematical Society, Providence, Rhode Island.

The paper "A Tutorial on Markov Renewal Theory, Semi-regenerative Processes and Their Applications" (Tech. report 8007) was prepared at the invitation of O.R.S.A./T.I.M.S. for delivery at their national meeting held in Colorado Springs, Colorado, November 1980. It will appear in a collection of other tutorials to appear in the summer 1982.

We have been invited to present a major series of lectures at Johns Hopkins University, June, 1982, on the topic of queueing network theory and its applications. (See Institute of Mathematical Statistics Bulletin, 11 (March, 1982) p. 86.) These lectures will be expanded to a book and will appear in 1983 or 1984.

We have been invited to present a paper to the I.E.E.E. national meeting, October, 1982 to be held at Virginia Beach, Virginia on large scale systems theory.

All of these papers and talks and the background research work was supported wholly or partly under this ONR contract.

<u>Documentation</u>. Enclosed is a list of Technical Memoranda produced wholly or in part under this ONR contract. Each of these has been previously distributed to the appropriate mailing list.

Items 7801, 8001, and 8015 are concerned with the necessary, sufficient and necessary and sufficient conditions for a Markov renewal process to be equivalent to a renewal process. Item 8015 has been submitted to <u>Operations Research</u> for publication and is presently in review.

Items 7901, 7906, 7922, 8016, 8102, 8106, and 8203 are all concerned with aspects of the sojourn time problem in queueing networks. 7901 has been published in <u>Management Science</u>, October, 1980. 7922 was published in <u>Naval Research Logistics Quarterly</u>, December, 1980. 8102 has appeared in <u>Naval Research Logistics Quarterly</u>, December, 1981. 8106 has been submitted to <u>Advances in Applied Probability</u> and accepted subject to revisions which are underway. 8203 has been recently completed and formed the basis of a talk given to a research audience at Bell Labs.

Items 7923 and 8007 are discussed above.

Item 8008 is a revision of work completed under a previous ONR contract. It was rewritten and presented at the Symposium on Point Processes and Queueing Theory, Karpacz, Poland, January, 1980. It will appear in <u>Zastosowania</u> Matematyki - <u>Applicationes Mathematicae</u>.

Item 8009 was written to correct an error in previously published work that was unfunded at the time of writing. This correction was supported under this ONR contract.

<u>Personnel</u>. Under this contract, the following people were partially supported for varying amounts of time since 1978.

a) Research Personnel

1. Mr. Jagadeesh Chandramohan - graduate student, Department of Industrial Engineering and Operations Research, Virginia Polytechnic Institute and State University. Ph.D. degree expected June, 1982. Tech. report on his research due by September, 1982.

Dr. Ralph L. Disney - principal investigator, Charles O. Gordon
Professor of Industrial Engineering and Operations Research, Virginia
Polytechnic Institute and State University.

3. Dr. Robert D. Foley - presently Asst. Professor, Department of Industrial Engineering and Operations Research, Virginia Polytechnic Institute and State University.

4. Mr. Dale Fox - graduate student, Department of Industrial Engineering and Operations Research, Virginia Polytechnic Institute and State University. M.S., completed, June, 1978. Presently in Ph.D. program, Department of Industrial Engineering, Purdue University.

5. Mr. Peter Kiessler - graduate student, Department of Industrial Engineering and Operations Research, Virginia Polytechnic Institute and State University. M.S., completed, December, 1980. Presently Ph.D. candidate, Department of Industrial Engineering and Operations Research, Virginia Polytechnic Institute and State University. Degree expected December, 1982.

6. Dr. Burton Simon - presently at Bell Laboratories, Holmdel, New Jersey.

b) Secretarial Support

1. Ms. Janet Chapman

2. Ms. Iyone Gillis

3. Ms. Paula Kirk

Tech. reports from the Department of IEOR, Virginia Polytechnic Institute and State University, Applied Probability Series, funded in whole or in part under this contract.

7801 Equivalences Between Markov Renewal Processes, Burton Simon.

- 7901 Some Results on Sojourn Times in Acyclic Jackson Networks, B. Simon and R. D. Foley.
- 7906 Markov Processes with Imbedded Markov Chains Having the Same Stationary Distribution, Robert D. Foley.
- 7922 The M/G/1 Queue with Instantaneous Bernoulli Feedback, Ralph L. Disney, Donald C. McNickle and Burton Simon.
- 7923 Queueing Networks, Ralph L. Disney, revised August, 1980.
- 8001 Equivalent Markov Renewal Processes, Burton Simon.
- 8007 A Tutorial on Markov Renewal Theory, Semi-Regenerative Processes, and Their Applications, Ralph L. Disney.
- 8008 The Superposition of Two Independent Markov Renewal Processes, W. Peter Cherry and Ralph L. Disney.
- 8009 A Correction Note on "Two Finite M/M/l Queues in Tandom: A Matrix Solution for the Steady State", Ralph L. Disney and Jagadeesh Chandramohan.
- 8015 Some Conditions for the Equivalence Between Markov Renewal Processes and Renewal Processes, Burton Simon and Ralph L. Disney.
- 8016 A Simulation Analysis of Sojourn Times in a Jackson Network, Peter C. Kiessler.
- 8102 A Note on Sojourn Times in M/G/1 Queues with Instantaneous, Bernoulli Feedback, Ralph L. Disney.
- 8106 Queues with Delayed Feedback, Robert D. Foley and Ralph L. Disney.
- 8203 The Sojourn Time in a Three Node, Acyclic, Jackson Queueing Network, Peter Kiessler and Ralph L. Disney.