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DYNAMIC ANALYSIS OF THE 75MM ADMAG GUN SYSTEM

Prepared by

S&D Dynamics
755 New York Ave.
Huntington, NY 11743

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US ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
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A detailed description of the finite element (lumped parameter) analytical simulation model of the 75mm ADMAG gun system, developed for the purpose of simulating the dynamic response of the physical system to single shot and burst mode firings, is herein presented. Model output consisting of natural frequencies, normal mode shapes and system response to a firing (including time-dependent trunnion and elevation link loads, and muzzle linear and angular displacements, velocities and accelerations) is presented. Model predictions for right and left horizontal, vertical and lateral trunnion loads and the

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) elevation link load are correlated with experimental data obtained from a ten-round sample of 75mm APFSDS firings.

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1. INTRODUCTION

This final report has been prepared by S&D Dynamics, Inc. under Contract No. DAAK11-78-C-0134 to the U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD.

The objectives of this report are to present (i) a detailed description and documentation of the finite element (lumped parameter) analytical simulation model of the 75mm ADMAG gun system devised and developed by S&D Dynamics, Inc. for the purpose of simulating the dynamic response of the physical gun system to single shot and burst mode firings, (ii) output obtained from the model (including natural frequencies, normal mode shapes and system response to a firing), and (iii) an assessment of model validation via correlation of model output with test data.

The detailed development of the finite element (lumped parameter) analytical simulation model is presented in Section 2. The formulation and computer solution of the associated eigenvalue problem (which prescribes the natural frequencies and normal mode shapes of the system) is presented in Section 3. The formulation and computer solution of the forced motion problem (which prescribes the response of the system to a firing) is presented in Section 4. Correlation of model output with test data is presented in Section 5. Conclusions and recommendations are presented in Section 6. References are presented in Section 7.

2. FINITE ELEMENT (LUMPED PARAMETER) MODEL

A schematic (provided by ARES, Inc.) of the 75mm ADMAG gun system is as depicted in Figure 1. The finite element (lumped parameter) representation herein devised and adopted for the purpose of simulating the dynamic response of the physical gun system to single shot and burst mode firings is as depicted in Figure 2. The underlying rationale for adopting this type of representation is based on the ability to simulate the dynamic characteristics of an element of a physical structure by its inertia and stiffness properties, as depicted in Figure 3. In addition, the resulting mathematical formulation associated with such a representation readily lends itself to determining the inherent dynamic characteristics of the physical system (i.e., its natural frequencies and normal mode shapes), as well as the dynamic response of the system when subjected to applied loads (as in single shot and burst mode firings).

Referring to Figure 2, the physical gun system depicted in Figure 1 has been replaced by a lumped parameter representation consisting of nineteen elastically connected mass points. Each mass point is permitted six degrees-of-freedom (three independent displacements and three independent rotations), resulting in a 114 degree-of-freedom model. A qualitative description of the relation between the physical system and model is as follows:

(i) m_1, m_2, and a portion of m_3 denote the mass points associated with the receiver (including gun pod and side plates);

(ii) a portion of m_3, and m_4 thru m_7 denote the mass points associated with the barrel support-tube;

(iii) m_8 and m_9 denote the mass points associated with the breech;

(iv) m_{10} denotes the mass point associated with the chamber;

(v) m_{11} thru m_{19} denote the mass points associated with the gun tube;

(vi) $k_{i,j}$ represents the stiffness matrix between mass points i and j;

(vii) $k_{2,g}$ represents the stiffness matrix between the receiver mass point m_2 and ground (i.e., earth -- when the gun is mounted in a test fixture; the turret -- when the gun is mounted on the vehicle);

(viii) $S':(x',y',z')$ denotes an earth-fixed coordinate system; $S:(x,y,z)$ denotes a typical local coordinate system associated with a typical element of the structure; the relation between S and S' is as depicted;

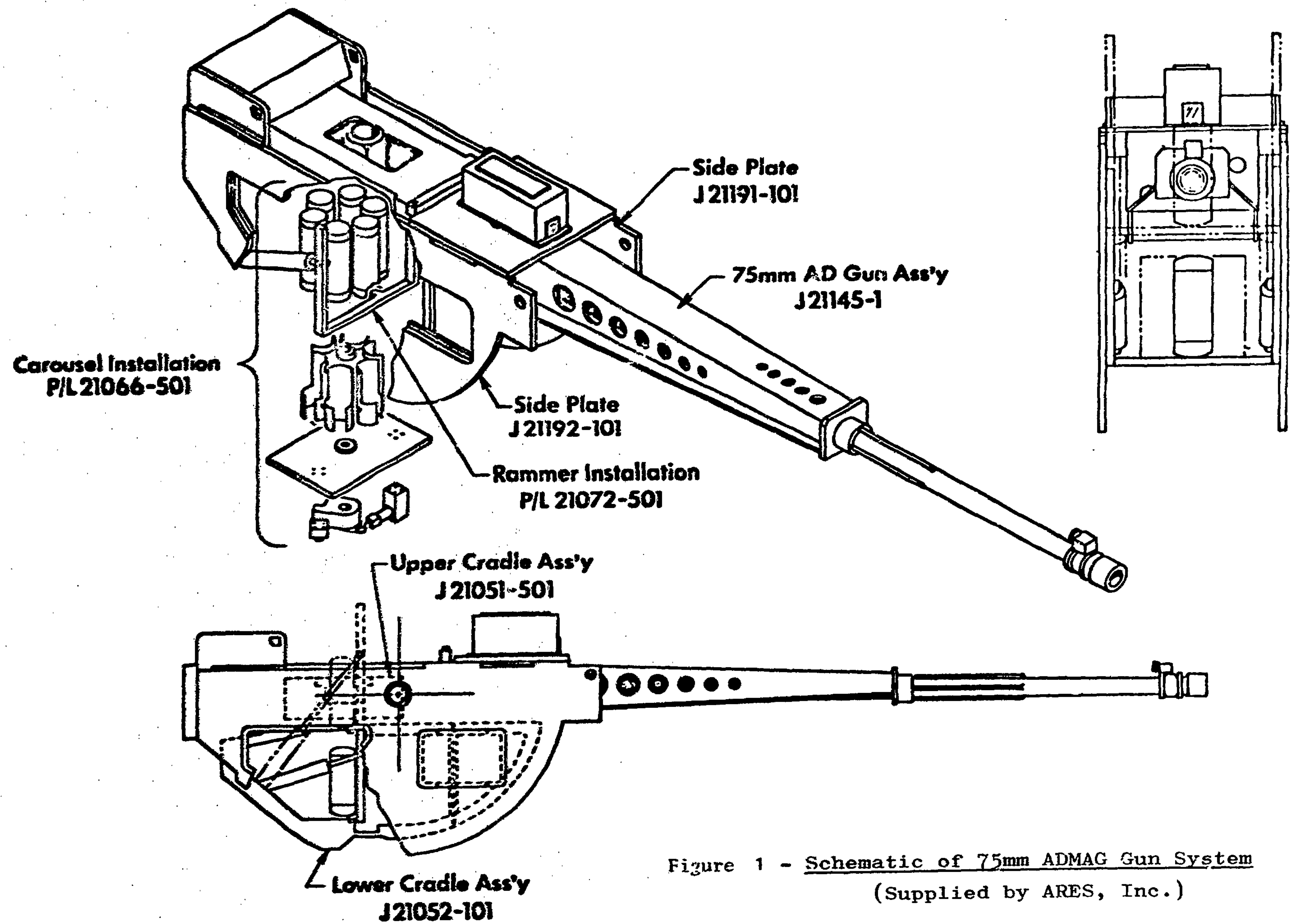


Figure 1 - Schematic of 75mm ADMAG Gun System
(Supplied by ARES, Inc.)

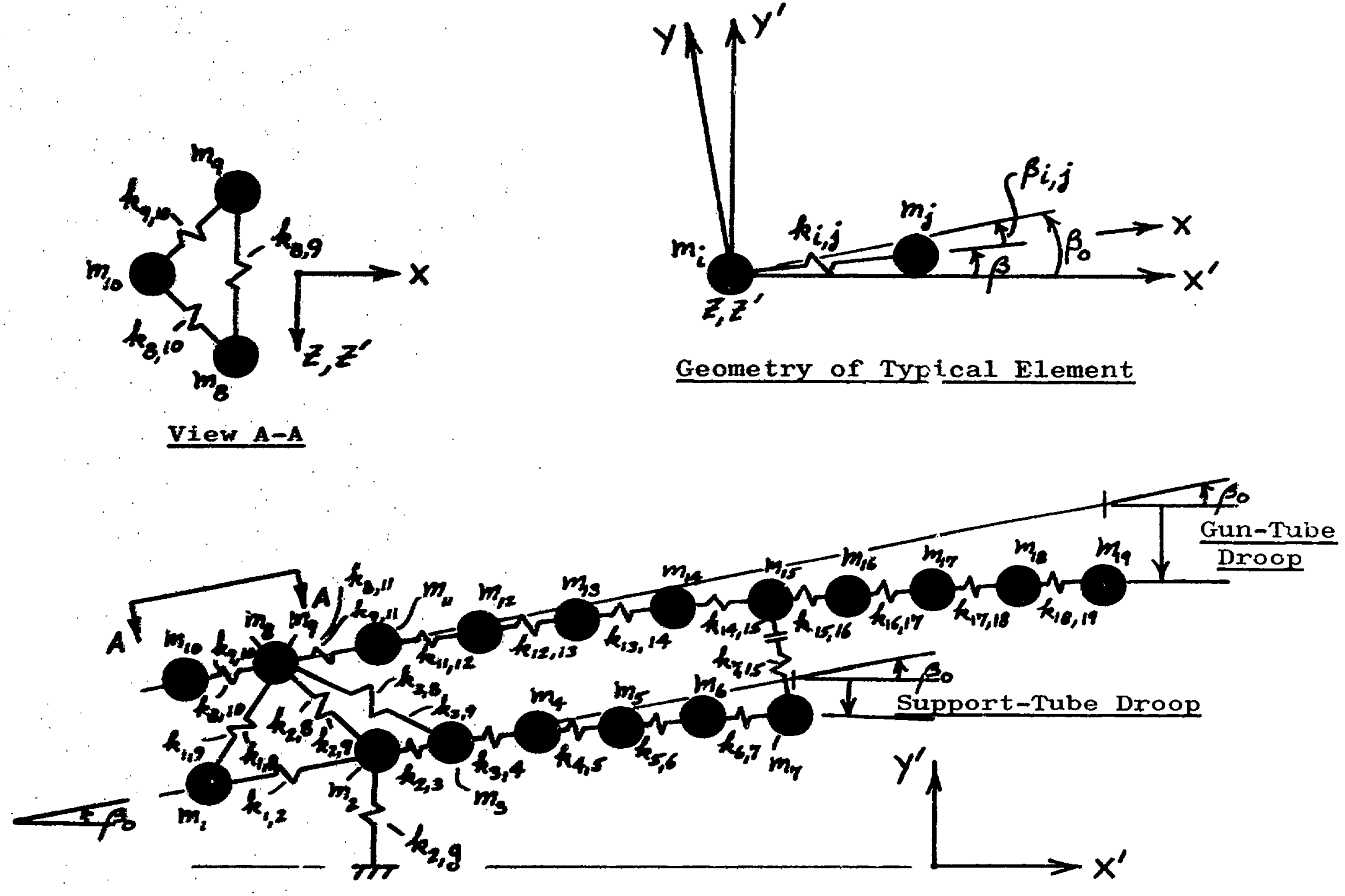


Figure 2 - Lumped Parameter Model of ADMAG Gun System

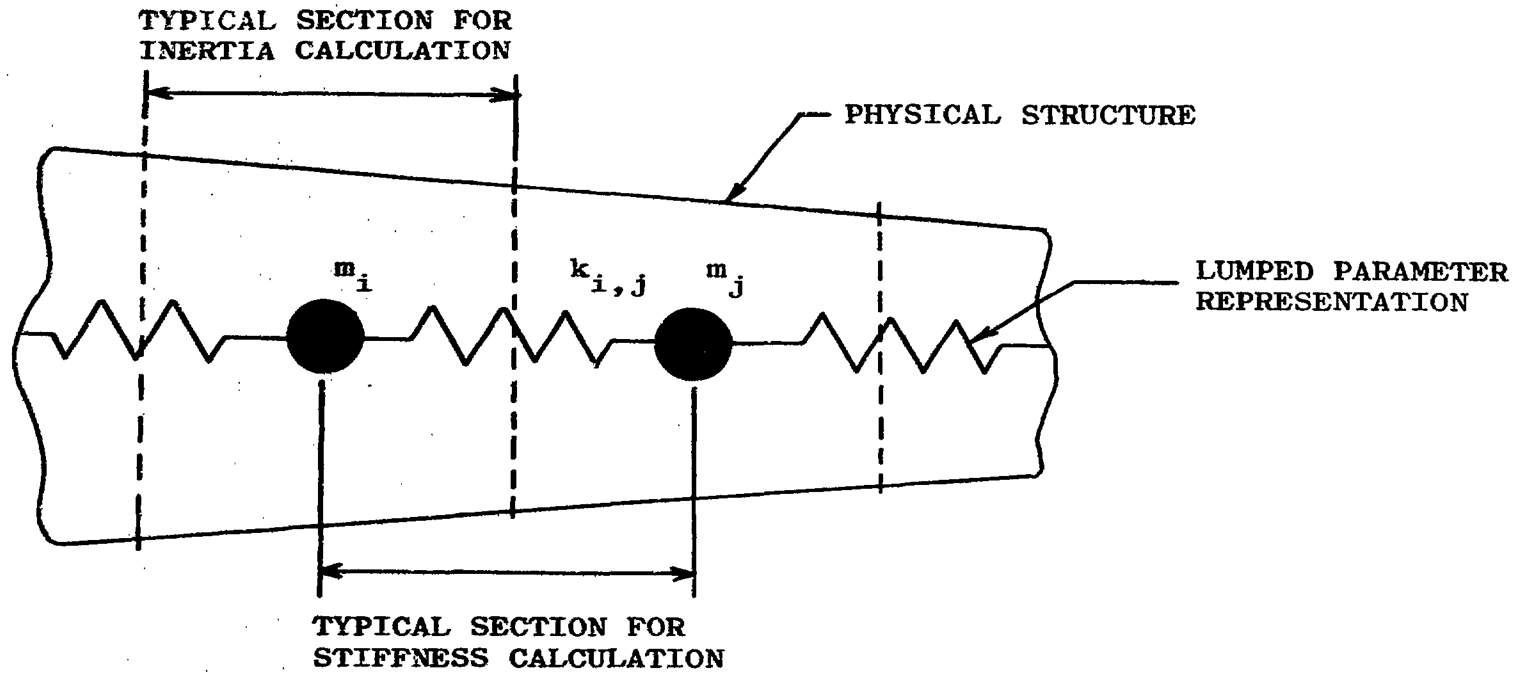


Figure 3 - TYPICAL LUMPED PARAMETER REPRESENTATION OF PHYSICAL STRUCTURE

(ix) β_o denotes the angle of gun elevation; $\beta_{i,j}$ denotes the rotation of an element between mass points i and j due to gun tube droop, manufacturing eccentricities, etc.; $\beta = (\beta_o - \beta_{i,j})$ denotes the resultant orientation of S relative to S' for each element of the system.

In order to determine the dynamic response characteristics of the physical gun system via analysis of the model depicted in Figure 2, we must first quantify the relations cited above. To achieve such quantification, we formulate the stiffness and inertia matrices associated with the model in terms of the mechanical, physical and geometrical properties of the actual gun system.

Allowing, as previously noted, for six degrees-of-freedom per mass point, each of the nineteen mass points depicted in Figure 2 represents a 6 x 6 inertia matrix, while each of the twenty-seven elastic connections (twenty-six between mass points and one to ground) represents a 12 x 12 stiffness matrix. The elements entering each of these matrices are of course determined based on the mechanical, physical and geometrical properties of the actual gun system, as implied in Figure 3.

To obtain the single, overall, system stiffness matrix, and the single, overall, system inertia matrix, which characterizes the dynamic response of the model depicted in Figure 2, each of the individual stiffness and inertia matrices are first defined relative to the local coordinate system associated with the physical structural element they represent. Each matrix is then transformed, via an appropriate coordinate transformation, from local to global (earth-fixed) coordinates. Appropriate partitioning and restructuring of the individual transformed matrices renders the overall (114 x 114 element) system stiffness and inertia matrices. The detailed analyses associated with the formulation, transformation, partitioning and restructuring of the individual matrices into the overall system stiffness and inertia matrices are presented in Sections 2.1 and 2.2.

To complete the description of the finite element (lumped parameter) model, initial gun tube curvature (including droop in the vertical plane, manufacturing eccentricities, etc.) is prescribed, based on measurements performed at Yuma Proving Ground, in Section 2.3. Numerical evaluation of the elements entering the stiffness and inertia matrices is presented, based on detailed drawings of the gun system provided by ARES, Inc., in Section 2.4.

2.1 Formulation of System Stiffness Matrix

Treating the receiver, gun tube and support tube as elastic beams, and allowing for bending, shear, axial and torsional deformations, the generalized stiffness matrix presented in Figure 4, and corresponding to the dynamic equilibrium equation presented in Figure 5, is applicable to a typical beam segment between any two mass points m_i and m_j, within each of the above mentioned gun system sub-structures, and with respect to its own local coordinate system S.

$$
k_{i,j} = \left[\begin{array}{cccccc|cccccc}
k_{1\,1} & 0 & 0 & 0 & 0 & 0 & -k_{1\,1} & 0 & 0 & 0 & 0 & 0 \\
0 & k_{2\,2} & 0 & 0 & 0 & k_{2\,6} & 0 & -k_{2\,2} & 0 & 0 & 0 & k_{2\,6} \\
0 & 0 & k_{3\,3} & 0 & -k_{3\,5} & 0 & 0 & 0 & -k_{3\,3} & 0 & -k_{3\,5} & 0 \\
0 & 0 & 0 & k_{4\,4} & 0 & 0 & 0 & 0 & 0 & -k_{4\,4} & 0 & 0 \\
0 & 0 & -k_{3\,5} & 0 & k_{5\,5} & 0 & 0 & 0 & k_{3\,5} & 0 & k_{5\,11} & 0 \\
0 & k_{2\,6} & 0 & 0 & 0 & k_{6\,6} & 0 & -k_{2\,6} & 0 & 0 & 0 & k_{6\,12} \\
\hline
-k_{1\,1} & 0 & 0 & 0 & 0 & 0 & k_{1\,1} & 0 & 0 & 0 & 0 & 0 \\
0 & -k_{2\,2} & 0 & 0 & 0 & -k_{2\,6} & 0 & k_{2\,2} & 0 & 0 & 0 & -k_{2\,6} \\
0 & 0 & -k_{3\,3} & 0 & k_{3\,5} & 0 & 0 & 0 & k_{3\,3} & 0 & k_{3\,5} & 0 \\
0 & 0 & 0 & -k_{4\,4} & 0 & 0 & 0 & 0 & 0 & k_{4\,4} & 0 & 0 \\
0 & 0 & -k_{3\,5} & 0 & k_{5\,11} & 0 & 0 & 0 & k_{3\,5} & 0 & k_{5\,5} & 0 \\
0 & k_{2\,6} & 0 & 0 & 0 & k_{6\,12} & 0 & -k_{2\,6} & 0 & 0 & 0 & k_{6\,6}
\end{array}\right]
$$

Figure 4 - Stiffness Matrix (w.r.t. S) for Typical Beam Segment Between Mass Points i and j

$$
k_{i,j} \begin{bmatrix} x_i \\ y_i \\ z_i \\ \theta_{x_i} \\ \theta_{y_i} \\ \theta_{z_i} \\ x_j \\ y_j \\ z_j \\ \theta_{x_j} \\ \theta_{y_j} \\ \theta_{z_j} \end{bmatrix} = \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ M_{x_i} \\ M_{y_i} \\ M_{z_i} \\ X_j \\ Y_j \\ Z_j \\ M_{x_j} \\ M_{y_j} \\ M_{z_j} \end{bmatrix}
$$

Figure 5 - Typical Dynamic Equilibrium Equation (w.r.t. S) for Elastically Connected Mass Points i and j

Referring to Figure 4, the elements contained within the generalized stiffness matrix $k_{i,j}$ are defined in terms of the mechanical, physical and geometrical properties of the beam segment by modifying the classical elastic beam stiffness expressions to account for shear, as follows (Ref. 7):

$$
\left.
\begin{array}{ll}
k_{1\,1} = \left[\dfrac{A\,E}{L}\right]_{i,j} & k_{4\,4} = \left[\dfrac{D_t}{L}\right]_{i,j} \\[2ex]
k_{2\,2} = \left[\dfrac{12\,E\,I_{zz}}{L^3}\left(\dfrac{1}{\epsilon_y}\right)\right]_{i,j} & k_{5\,5} = \left[\dfrac{4\,E\,I_{yy}}{L}\left(\dfrac{\delta_z}{\epsilon_z}\right)\right]_{i,j} \\[2ex]
k_{2\,6} = \left[\dfrac{6\,E\,I_{zz}}{L^2}\left(\dfrac{1}{\epsilon_y}\right)\right]_{i,j} & k_{5\,11} = \left[\dfrac{2\,E\,I_{yy}}{L}\left(\dfrac{\eta_z}{\epsilon_z}\right)\right]_{i,j} \\[2ex]
k_{3\,3} = \left[\dfrac{12\,E\,I_{yy}}{L^3}\left(\dfrac{1}{\epsilon_z}\right)\right]_{i,j} & k_{6\,6} = \left[\dfrac{4\,E\,I_{zz}}{L}\left(\dfrac{\delta_y}{\epsilon_y}\right)\right]_{i,j} \\[2ex]
k_{3\,5} = \left[\dfrac{6\,E\,I_{yy}}{L^2}\left(\dfrac{1}{\epsilon_z}\right)\right]_{i,j} & k_{6\,12} = \left[\dfrac{2\,E\,I_{zz}}{L}\left(\dfrac{\eta_y}{\epsilon_y}\right)\right]_{i,j}
\end{array}
\right\} \quad (1)
$$

where,

$$
\begin{array}{ll}
\epsilon_y = (1 + 12\,\alpha_y\,\lambda_z) & \alpha_y = \sigma_{xy\,max} / \sigma_{xy\,avg} \\
\delta_y = (1 + 3\,\alpha_y\,\lambda_z) & \alpha_z = \sigma_{xz\,max} / \sigma_{xz\,avg} \\
\eta_y = (1 - 6\,\alpha_y\,\lambda_z) & \lambda_y = E\,I_{yy} / G\,A\,L^2 \\
\epsilon_z = (1 + 12\,\alpha_z\,\lambda_y) & \lambda_z = E\,I_{zz} / G\,A\,L^2 \\
\delta_z = (1 + 3\,\alpha_z\,\lambda_y) & I_{yy} = \int_A z^2\,dA \\
\eta_z = (1 - 6\,\alpha_z\,\lambda_y) & I_{zz} = \int_A y^2\,dA
\end{array}
$$

and

E, G -- respectively denote the beam segment elastic modulus in bending and shear

A, L -- respectively denote the beam segment cross-sectional area and length

D_t -- denotes the beam segment torsional stiffness

Referring to Figure 5

x_i, y_i, z_i	--	denote displacements of mass point i along the respective coordinates of S
Θ_{x_i}, Θ_{y_i}, Θ_{z_i}	--	denote rotations of mass point i about the respective coordinates of S
X_i, Y_i, Z_i	--	denote forces (including inertia) applied to mass point i along the respective coordinates of S
M_{x_i}, M_{y_i}, M_{z_i}	--	denote moments (including rotary inertia) applied to mass point i about the respective coordinates of S

The generalized stiffness matrix $k_{i,j}$ presented in Figure 4 is applicable to a typical beam segment of the receiver, gun tube or support tube with respect to its own local coordinate system, S. However, since the final stiffness matrix of the overall system is to be written with respect to the earth-fixed coordinate system S', each $k_{i,j}$ matrix for each beam segment of each sub-structure must be transformed to S' prior to generating the stiffness matrix of the overall system.

Letting $L_{i,j}$ denote the transformation matrix from S to S' for a typical beam segment between mass points i and j of a typical sub-structure, it may be shown that

$$k'_{i,j} = L_{i,j}\, k_{i,j}\, L^{-1}_{i,j} \tag{2}$$

where $k'_{i,j}$ denotes the desired transformed stiffness matrix, and, for beam segments of the sub-structure under consideration, $L_{i,j}$ is as presented in Figure 6, wherein

$$\left.\begin{aligned} L_{1\,1} &= \cos\beta \\ L_{1\,2} &= \sin\beta \\ L_{3\,3} &= 1 \\ \beta &= \beta_o - \beta_{i,j} \end{aligned}\right\} \tag{3}$$

Performing the indicated matrix operations, there results the desired transformed stiffness matrix $k'_{i,j}$ as presented in Figure 7; corresponding to the dynamic equilibrium equation presented in Figure 8.

$$
L_{i,j} = \left[\begin{array}{cccccc|cccccc}
L_{11} & -L_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
L_{12} & L_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & L_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & L_{11} & -L_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & L_{12} & L_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & L_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & L_{11} & -L_{12} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & L_{12} & L_{11} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_{11} & -L_{12} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_{12} & L_{11} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_{33}
\end{array}\right]
$$

Figure 6 - Transformation Matrix From S to S'

$$
k'_{i,j} = \left[\begin{array}{cccccc|cccccc}
k'_{11} & k'_{12} & 0 & 0 & 0 & -k'_{16} & -k'_{11} & -k'_{12} & 0 & 0 & 0 & -k'_{16} \\
k'_{12} & k'_{22} & 0 & 0 & 0 & k'_{26} & -k'_{12} & -k'_{22} & 0 & 0 & 0 & k'_{26} \\
0 & 0 & k'_{33} & k'_{34} & -k'_{35} & 0 & 0 & 0 & -k'_{33} & k'_{34} & -k'_{35} & 0 \\
0 & 0 & k'_{34} & k'_{44} & k'_{45} & 0 & 0 & 0 & -k'_{34} & k'_{4\,10} & k'_{4\,11} & 0 \\
0 & 0 & -k'_{35} & k'_{45} & k'_{55} & 0 & 0 & 0 & k'_{35} & k'_{4\,11} & k'_{5\,11} & 0 \\
-k'_{16} & k'_{26} & 0 & 0 & 0 & k'_{66} & k'_{16} & -k'_{26} & 0 & 0 & 0 & k'_{6\,12} \\
\hline
-k'_{11} & -k'_{12} & 0 & 0 & 0 & k'_{16} & k'_{11} & k'_{12} & 0 & 0 & 0 & k'_{16} \\
-k'_{12} & -k'_{22} & 0 & 0 & 0 & -k'_{26} & k'_{12} & k'_{22} & 0 & 0 & 0 & -k'_{26} \\
0 & 0 & -k'_{33} & -k'_{34} & k'_{35} & 0 & 0 & 0 & k'_{33} & -k'_{34} & k'_{35} & 0 \\
0 & 0 & k'_{34} & k'_{4\,10} & k'_{4\,11} & 0 & 0 & 0 & -k'_{34} & k'_{44} & k'_{45} & 0 \\
0 & 0 & -k'_{35} & k'_{4\,11} & k'_{5\,11} & 0 & 0 & 0 & k'_{35} & k'_{45} & k'_{55} & 0 \\
-k'_{16} & k'_{26} & 0 & 0 & 0 & k'_{6\,12} & k'_{16} & -k'_{26} & 0 & 0 & 0 & k'_{66}
\end{array}\right]
$$

Figure 7 - Stiffness Matrix (w.r.t. S') for Typical Beam Segment Between Mass Points i and j

$$
k'_{i,j}
\begin{bmatrix}
x'_i \\ y'_i \\ z'_i \\ \theta_{x'_i} \\ \theta_{y'_i} \\ \theta_{z'_i} \\ x'_j \\ y'_j \\ z'_j \\ \theta_{x'_j} \\ \theta_{y'_j} \\ \theta_{z'_j}
\end{bmatrix}
=
\begin{bmatrix}
X'_i \\ Y'_i \\ Z'_i \\ M_{x'_i} \\ M_{y'_i} \\ M_{z'_i} \\ X'_j \\ Y'_j \\ Z'_j \\ M_{x'_j} \\ M_{y'_j} \\ M_{z'_j}
\end{bmatrix}
$$

Figure 8 - Typical Dynamic Equilibrium Equation (w.r.t. S') for Elastically Connected Mass Points i and j

Referring to Figure 7

$$
\left.
\begin{array}{l|l}
k'_{1\,1} = k_{1\,1}\,\mathrm{Cos}^2\beta + k_{2\,2}\,\mathrm{Sin}^2\beta & k'_{4\,4} = k'_{4\,4}\,\mathrm{Cos}^2\beta + k_{5\,5}\,\mathrm{Sin}^2\beta \\
k'_{1\,2} = (k_{1\,1} - k_{2\,2})\,\mathrm{Sin}\beta\,\mathrm{Cos}\beta & k'_{4\,5} = (k_{4\,4} - k_{5\,5})\,\mathrm{Sin}\beta\,\mathrm{Cos}\beta \\
k'_{1\,6} = k_{2\,6}\,\mathrm{Sin}\beta & k'_{4\,10} = -\,k_{4\,4}\,\mathrm{Cos}^2\beta + k_{5\,11}\,\mathrm{Sin}^2\beta \\
k'_{2\,2} = k_{1\,1}\,\mathrm{Sin}^2\beta + k_{2\,2}\,\mathrm{Cos}^2\beta & k'_{4\,11} = -\,(k_{4\,4} + k_{5\,11})\,\mathrm{Sin}\beta\,\mathrm{Cos}\beta \\
k'_{2\,6} = k_{2\,6}\,\mathrm{Cos}\beta & k'_{5\,5} = k_{4\,4}\,\mathrm{Sin}^2\beta + k_{5\,5}\,\mathrm{Cos}^2\beta \\
k'_{3\,3} = k_{3\,3} & k'_{5\,11} = -\,k_{4\,4}\,\mathrm{Sin}^2\beta + k_{5\,11}\,\mathrm{Cos}^2\beta \\
k'_{3\,4} = k_{3\,5}\,\mathrm{Sin}\beta & k'_{6\,6} = k_{6\,6} \\
k'_{3\,5} = k_{3\,5}\,\mathrm{Cos}\beta & k'_{6\,12} = k_{6\,12}
\end{array}
\right\} \quad (4)
$$

where the $k'_{i\,j}$'s are as defined in equation (1), and β is as defined in equation (3).

Referring to Figure 8

x'_i, y'_i, z'_i -- denote displacements of mass point i along respective coordinates of S'

Θ_{x_i}', Θ_{y_i}', Θ_{z_i}' -- denote rotations of mass point i about the respective coordinates of S'

X'_i, Y'_i, Z'_i -- denote forces (including inertia) applied to mass point i along the respective coordinates of S'

M_{x_i}', M_{y_i}', M_{z_i}' -- denote moments (including rotary inertia) applied to mass point i about the respective coordinates of S'

Referring to Figure 2, the generalized stiffness matrix $k'_{i,j}$ presented in Figure 7, defines the stiffness matrices $k'_{i,i+1}$ for i = 1 thru 6, and 11 thru 18. The remaining stiffness matrices to be defined are $k'_{i,j}$ for i = 1 thru 3, with j = 8,9; i = 8, 9 with j = 10, 11; $k'_{8,9}$, $k'_{7,13}$ and $k'_{2,8}$.

The sub-structure of the breech may also be treated as an elastic beam. However, noting the relative orientation of the breech mass points m_8 and m_9, as depicted in Figure 2, and requiring that they satisfy the dynamic equilibrium equation as presented in Figure 5 with i = 8 and j = 9, the elements of $k_{i,j}$ presented in Figure 4, and defined in equation (1), must be rearranged as presented in Figure 9.

Noting that $\beta = \beta_o$ for the breech, and performing the matrix operations indicated in equation (2), there results the desired stiffness matrix $k'_{8,9}$ as presented in Figure 10; corresponding to the dynamic equilibrium equation presented in Figure 8 with i = 8 and j = 9.

Referring to Figure 10

$$\left.\begin{aligned}
k'_{1\,1} &= k_{2\,2}\,\text{Sin}^2\beta_o + k_{3\,3}\,\text{Cos}^2\beta_o \\
k'_{1\,2} &= -(k_{2\,2} - k_{3\,3})\,\text{Sin}\,\beta_o\,\text{Cos}\,\beta_o \\
k'_{1\,4} &= -(k_{2\,6} - k_{3\,5})\,\text{Sin}\,\beta_o\,\text{Cos}\,\beta_o \\
k'_{1\,5} &= -k_{2\,6}\,\text{Sin}^2\beta_o - k_{3\,5}\,\text{Cos}^2\beta_o \\
k'_{2\,2} &= k_{2\,2}\,\text{Cos}^2\beta_o + k_{3\,3}\,\text{Sin}^2\beta_o \\
k'_{2\,4} &= k_{2\,6}\,\text{Cos}^2\beta_o + k_{3\,5}\,\text{Sin}^2\beta_o \\
k'_{3\,3} &= k_{1\,1} \\
k'_{4\,4} &= k_{5\,5}\,\text{Sin}^2\beta_o + k_{6\,6}\,\text{Cos}^2\beta_o \\
k'_{4\,5} &= -(k_{5\,5} - k_{6\,6})\,\text{Sin}\,\beta_o\,\text{Cos}\,\beta_o \\
k'_{4\,10} &= k_{5\,11}\,\text{Sin}^2\beta_o + k_{6\,12}\,\text{Cos}^2\beta_o \\
k'_{4\,11} &= -(k_{5\,11} - k_{6\,12})\,\text{Sin}\,\beta_o\,\text{Cos}\,\beta_o \\
k'_{5\,5} &= k_{5\,5}\,\text{Cos}^2\beta_o + k_{6\,6}\,\text{Sin}^2\beta_o \\
k'_{5\,11} &= k_{5\,11}\,\text{Cos}^2\beta_o + k_{6\,12}\,\text{Sin}^2\beta_o \\
k'_{6\,6} &= k_{4\,4}
\end{aligned}\right\} \qquad (5)$$

$$
k_{8,9} = \left[\begin{array}{cccccc|cccccc}
k_{3\,3} & 0 & 0 & 0 & -k_{3\,5} & 0 & -k_{3\,3} & 0 & 0 & 0 & -k_{3\,5} & 0 \\
0 & k_{2\,2} & 0 & k_{2\,6} & 0 & 0 & 0 & -k_{2\,2} & 0 & k_{2\,6} & 0 & 0 \\
0 & 0 & k_{1\,1} & 0 & 0 & 0 & 0 & 0 & -k_{1\,1} & 0 & 0 & 0 \\
0 & k_{2\,6} & 0 & k_{6\,6} & 0 & 0 & 0 & -k_{2\,6} & 0 & k_{6\,12} & 0 & 0 \\
-k_{3\,5} & 0 & 0 & 0 & k_{5\,5} & 0 & k_{3\,5} & 0 & 0 & 0 & k_{5\,11} & 0 \\
0 & 0 & 0 & 0 & 0 & k_{4\,4} & 0 & 0 & 0 & 0 & 0 & -k_{4\,4} \\
\hline
-k_{3\,3} & 0 & 0 & 0 & k_{3\,5} & 0 & k_{3\,3} & 0 & 0 & 0 & k_{3\,5} & 0 \\
0 & -k_{2\,2} & 0 & -k_{2\,6} & 0 & 0 & 0 & k_{2\,2} & 0 & -k_{2\,6} & 0 & 0 \\
0 & 0 & -k_{1\,1} & 0 & 0 & 0 & 0 & 0 & k_{1\,1} & 0 & 0 & 0 \\
0 & k_{2\,6} & 0 & k_{6\,12} & 0 & 0 & 0 & -k_{2\,6} & 0 & k_{6\,6} & 0 & 0 \\
-k_{3\,5} & 0 & 0 & 0 & k_{5\,11} & 0 & k_{3\,5} & 0 & 0 & 0 & k_{5\,5} & 0 \\
0 & 0 & 0 & 0 & 0 & -k_{4\,4} & 0 & 0 & 0 & 0 & 0 & k_{4\,4}
\end{array}\right]
$$

Figure 9 - Stiffness Matrix (w.r.t. S) Between Mass Points 8 and 9

$$
k'_{8,9} = \left[\begin{array}{cccccc|cccccc}
k'_{1\,1} & k'_{1\,2} & 0 & k'_{1\,4} & k'_{1\,5} & 0 & -k'_{1\,1} & -k'_{1\,2} & 0 & k'_{1\,4} & k'_{1\,5} & 0 \\
k'_{1\,2} & k'_{2\,2} & 0 & k'_{2\,4} & -k'_{1\,4} & 0 & -k'_{1\,2} & -k'_{2\,2} & 0 & k'_{2\,4} & -k'_{1\,4} & 0 \\
0 & 0 & k'_{3\,3} & 0 & 0 & 0 & 0 & 0 & -k'_{3\,3} & 0 & 0 & 0 \\
k'_{1\,4} & k'_{2\,4} & 0 & k'_{4\,4} & k'_{4\,5} & 0 & -k'_{1\,4} & -k'_{2\,4} & 0 & k'_{4\,10} & k'_{4\,11} & 0 \\
k'_{1\,5} & -k'_{1\,4} & 0 & k'_{4\,5} & k'_{5\,5} & 0 & -k'_{1\,5} & k'_{1\,4} & 0 & k'_{4\,11} & k'_{5\,11} & 0 \\
0 & 0 & 0 & 0 & 0 & k'_{6\,6} & 0 & 0 & 0 & 0 & 0 & -k'_{6\,6} \\
\hline
-k'_{1\,1} & -k'_{1\,2} & 0 & -k'_{1\,4} & -k'_{1\,5} & 0 & k'_{1\,1} & k'_{1\,2} & 0 & -k'_{1\,4} & -k'_{1\,5} & 0 \\
-k'_{1\,2} & -k'_{2\,2} & 0 & -k'_{2\,4} & k'_{1\,4} & 0 & k'_{1\,2} & k'_{2\,2} & 0 & -k'_{2\,4} & k'_{1\,4} & 0 \\
0 & 0 & -k'_{3\,3} & 0 & 0 & 0 & 0 & 0 & k'_{3\,3} & 0 & 0 & 0 \\
k'_{1\,4} & k'_{2\,4} & 0 & k'_{4\,10} & k'_{4\,11} & 0 & -k'_{1\,4} & -k'_{2\,4} & 0 & k'_{4\,4} & k'_{4\,5} & 0 \\
k'_{1\,5} & -k'_{1\,4} & 0 & k'_{4\,11} & k'_{5\,11} & 0 & -k'_{1\,5} & k'_{1\,4} & 0 & k'_{4\,5} & k'_{5\,5} & 0 \\
0 & 0 & 0 & 0 & 0 & -k'_{6\,6} & 0 & 0 & 0 & 0 & 0 & k'_{6\,6}
\end{array}\right]
$$

Figure 10 - Stiffness Matrix (w.r.t. S') Between Mass Points 8 and 9

where the $k_{i\ j}$'s are as defined in equation (1).

Considering the relation between the breech and the receiver, the following are noted:

(i) the breech is physically contained within, and is supported by, the receiver;

(ii) the breech moves longitudinally relative to the receiver in accordance with the dynamics of the recoil/counter-recoil system;

(iii) at any equilibrium position during the recoil/counter-recoil cycle, additional longitudinal motion of the breech relative to the receiver is resisted by the elasticity of the recoil rods;

(iv) the breech mass points m_8 and m_9 lie in a plane which is perpendicular to the longitudinal axis of the receiver, and which is located, at any instant, between receiver mass points m_1 and m_2;

(v) the breech mass points m_8 and m_9 lie sufficiently close to their respective interfacial breech/receiver supports, and within sufficiently stiff structural members of the breech, to assume a rigid connection between m_8 and the right-hand breech/receiver interfacial support, and m_9 and the left-hand breech/receiver interfacial support.

In view of (i), (ii), (iv) and (v) above,

$$k_{1,8} = k_{1,9} = \tfrac{1}{2}\, k_{i,j} \tag{6}$$

where $k_{i,j}$ is as presented in Figure 4, but with $k_{1\ 1} = 0$.

Similarly,

$$k_{2,8} = k_{2,9} = \tfrac{1}{2}\, k_{i,j} \tag{7}$$

but with the algebraic sign reversed for those elements representing loads induced by rotations, and moments induced by displacements, and with the above noted restriction on $k_{1\ 1}$.

Performing the required matrix transformations

$$k'_{1,8} = k'_{1,9} = \tfrac{1}{2}\, k'_{i,j} \tag{8}$$

and, similarly

$$k'_{2,8} = k'_{2,9} = \tfrac{1}{2}\, k'_{i,j} \tag{9}$$

where $k'_{i,j}$ is as presented in Figure 7, but with the above noted restrictions and sign changes, and with $\beta = \beta_o$.

In view of (iii) above, the stiffness matrices $k_{3,8}$ and $k_{3,9}$ are as presented in Figure 11; with $k_{1\,1}$ as defined in equation (1) for each recoil rod.

Performing the required matrix transformations, $k'_{3,8}$ and $k'_{3,9}$ are as presented in Figure 12; with its elements as defined in equation (4) with $\beta = \beta_o$.

Hence, equations (8) and (9), in conjunction with Figure 12, prescribe the stiffness matrices $k'_{i,j}$ for i = 1 thru 3, with j = 8 and 9.

Treating mass point $m_{1\,1}$ as fixed to the breech, and noting that it is located along the breech centerline; which is perpendicular to the plane containing the breech mass points m_8 and m_9, it is seen that

$$k'_{8,11} = k'_{9,11} = \tfrac{1}{2}\, k'_{i,j} \tag{10}$$

where $k'_{i,j}$ is as prescribed in Figure 7; with its elements as defined in equation (4) with $\beta = \beta_o$.

Hence, equation (10) prescribes the stiffness matrices $k'_{8,11}$ and $k'_{9,11}$; corresponding to the dynamic equilibrium equation presented in Figure 8 for i = 8, j = 11, and for i = 9, j = 11.

In view of the physical nature of the structural connection between the breech and chamber, the stiffness matrices $k_{8,10}$ and $k_{9,10}$ are taken to be of the form presented in Figure 13; which implies that the connection behaves as a system of independent linear springs with respect to each of its six degrees-of-freedom. Considering the nature of the physical connection, and referring to Figure 13, the matrix elements $k_{1\,1}$, $k_{2\,2}$ and $k_{3\,3}$ are determined as direct compression elements acting on each side of the connection; the matrix elements $k_{4\,4}$ and $k_{5\,5}$ are respectively defined for each side of the connection as $r^2\,k_{2\,2}$ and $r^2\,k_{1\,1}$, where r denotes the distance (moment arm) from the chamber c.g. to the chamber/breech interfacial support; $k_{6\,6}$ is defined considering bending and shear of one chamber actuating arm on each side of the connection.

Performing the matrix operations indicated in equation (2), there results the stiffness matrices $k'_{8,10}$ and $k'_{9,10}$ as presented in Figure 14; corresponding to the dynamic equilibrium equation presented in Figure 8 for i = 8, j = 10, and for i = 9, j = 10.

$$k_{3,8} = k_{3,9} = \left[\begin{array}{cccccc|cccccc}
k_{11} & 0 & 0 & 0 & 0 & 0 & -k_{11} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
-k_{11} & 0 & 0 & 0 & 0 & 0 & k_{11} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]$$

Figure 11 - Stiffness Matrix (w.r.t. S) Between Mass Points 3 and 8, and 3 and 9

$$
k'_{3,8} = k'_{3,9} = \left[\begin{array}{cccccc|cccccc}
k'_{11} & k'_{12} & 0 & 0 & 0 & 0 & -k'_{11} & -k'_{12} & 0 & 0 & 0 & 0 \\
k'_{12} & k'_{22} & 0 & 0 & 0 & 0 & -k'_{12} & -k'_{22} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\;0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
-k'_{11} & -k'_{12} & 0 & 0 & 0 & 0 & k'_{11} & k'_{12} & 0 & 0 & 0 & 0 \\
-k'_{12} & -k'_{22} & 0 & 0 & 0 & 0 & k'_{12} & k'_{22} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
$$

Figure 12 - Stiffness Matrix (w.r.t. S') Between Mass Points 3 and 8, and 3 and 9

$$
k_{8,10} = k_{9,10} = \left[\begin{array}{cccccc|cccccc}
k_{1\,1} & 0 & 0 & 0 & 0 & 0 & -k_{1\,1} & 0 & 0 & 0 & 0 & 0 \\
0 & k_{2\,2} & 0 & 0 & 0 & 0 & 0 & -k_{2\,2} & 0 & 0 & 0 & 0 \\
0 & 0 & k_{3\,3} & 0 & 0 & 0 & 0 & 0 & -k_{3\,3} & 0 & 0 & 0 \\
0 & 0 & 0 & k_{4\,4} & 0 & 0 & 0 & 0 & 0 & -k_{4\,4} & 0 & 0 \\
0 & 0 & 0 & 0 & k_{5\,5} & 0 & 0 & 0 & 0 & 0 & -k_{5\,5} & 0 \\
0 & 0 & 0 & 0 & 0 & k_{6\,6} & 0 & 0 & 0 & 0 & 0 & -k_{6\,6} \\
\hline
-k_{1\,1} & 0 & 0 & 0 & 0 & 0 & k_{1\,1} & 0 & 0 & 0 & 0 & 0 \\
0 & -k_{2\,2} & 0 & 0 & 0 & 0 & 0 & k_{2\,2} & 0 & 0 & 0 & 0 \\
0 & 0 & -k_{3\,3} & 0 & 0 & 0 & 0 & 0 & k_{3\,3} & 0 & 0 & 0 \\
0 & 0 & 0 & -k_{4\,4} & 0 & 0 & 0 & 0 & 0 & k_{4\,4} & 0 & 0 \\
0 & 0 & 0 & 0 & -k_{5\,5} & 0 & 0 & 0 & 0 & 0 & k_{5\,5} & 0 \\
0 & 0 & 0 & 0 & 0 & -k_{6\,6} & 0 & 0 & 0 & 0 & 0 & k_{6\,6}
\end{array}\right]
$$

Figure 13 - Stiffness Matrix (w.r.t. S) Between Mass Points 8 and 10, and 9 and 10

$$
k'_{8,10} = k'_{9,10} = \left[\begin{array}{cccccc|cccccc}
k'_{11} & k'_{12} & 0 & 0 & 0 & 0 & -k'_{11} & -k'_{12} & 0 & 0 & 0 & 0 \\
k'_{12} & k'_{22} & 0 & 0 & 0 & 0 & -k'_{12} & -k'_{22} & 0 & 0 & 0 & 0 \\
0 & 0 & k'_{33} & 0 & 0 & 0 & 0 & 0 & -k'_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & k'_{44} & k'_{45} & 0 & 0 & 0 & 0 & -k'_{44} & -k'_{45} & 0 \\
0 & 0 & 0 & k'_{45} & k'_{55} & 0 & 0 & 0 & 0 & -k'_{45} & -k'_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & k'_{66} & 0 & 0 & 0 & 0 & 0 & -k'_{66} \\
\hline
-k'_{11} & -k'_{12} & 0 & 0 & 0 & 0 & k'_{11} & k'_{12} & 0 & 0 & 0 & 0 \\
-k'_{12} & -k'_{22} & 0 & 0 & 0 & 0 & k'_{12} & k'_{22} & 0 & 0 & 0 & 0 \\
0 & 0 & -k'_{33} & 0 & 0 & 0 & 0 & 0 & k'_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & -k'_{44} & -k'_{45} & 0 & 0 & 0 & 0 & k'_{44} & k'_{45} & 0 \\
0 & 0 & 0 & -k'_{45} & -k'_{55} & 0 & 0 & 0 & 0 & k'_{45} & k'_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & -k'_{66} & 0 & 0 & 0 & 0 & 0 & k'_{66}
\end{array}\right]
$$

Figure 14 - Stiffness Matrix (w.r.t. S') Between Mass Points 8 and 10, and 9 and 10

Noting that $\beta = \beta_o$, and referring to Figure 14

$$\left.\begin{aligned}
k'_{1\,1} &= k_{1\,1} \cos^2\beta_o + k_{2\,2} \sin^2\beta_o \\
k'_{1\,2} &= (k_{1\,1} - k_{2\,2}) \sin\beta_o \cos\beta_o \\
k'_{2\,2} &= k_{1\,1} \sin^2\beta_o + k_{2\,2} \cos^2\beta_o \\
k'_{3\,3} &= k_{3\,3} \\
k'_{4\,4} &= k_{4\,4} \cos^2\beta_o + k_{5\,5} \sin^2\beta_o \\
k'_{4\,5} &= (k_{4\,4} - k_{5\,5}) \sin\beta_o \cos\beta_o \\
k'_{5\,5} &= k_{4\,4} \sin^2\beta_o + k_{5\,5} \cos^2\beta_o \\
k'_{6\,6} &= k_{6\,6}
\end{aligned}\right\} \qquad (11)$$

where the $k_{i\,i}$'s (i = 1 thru 6) are determined as described above.

In a similar manner, the stiffness matrix $k_{7,15}$, representing the connection between the gun tube and support tube at the barrel guide interface depicted in Figure 15, is taken to be of the form presented in Figure 16; which is identical with the form presented in Figure 13 except that $k_{1\,1}$ has been set equal to zero to reflect the fact that the barrel guide does not resist longitudinal motion of the gun tube. Referring to Figure 16, the elements $k_{2\,2}$ and $k_{3\,3}$ are determined, assuming a rigid barrel guide, based on bending and shear (in the neighborhood of the barrel guide interface) of Spline #1 (or #3) and Spline #2 (or #4), respectively. It is noted that $k_{2\,2}$ equals $k_{3\,3}$. The element $k_{4\,4}$ is determined as $2r^2\, k_{2\,2}$, where r denotes the mean radius from the center of the barrel guide to the barrel guide/spline interface. The elements $k_{5\,5}$ and $k_{6\,6}$ are equal, and determined as $\ell^2\, k_{2\,2}$, where ℓ denotes the length of the barrel guide/spline interface.

Performing the matrix operations indicated in equation (2), there results the stiffness matrix $k'_{7,15}$, as presented in Figure 17; corresponding to the dynamic equilibrium equation presented in Figure 8 for i = 7, j = 15. It is noted that the form of the stiffness matrix presented in Figure 17 is identical with the form presented in Figure 14.

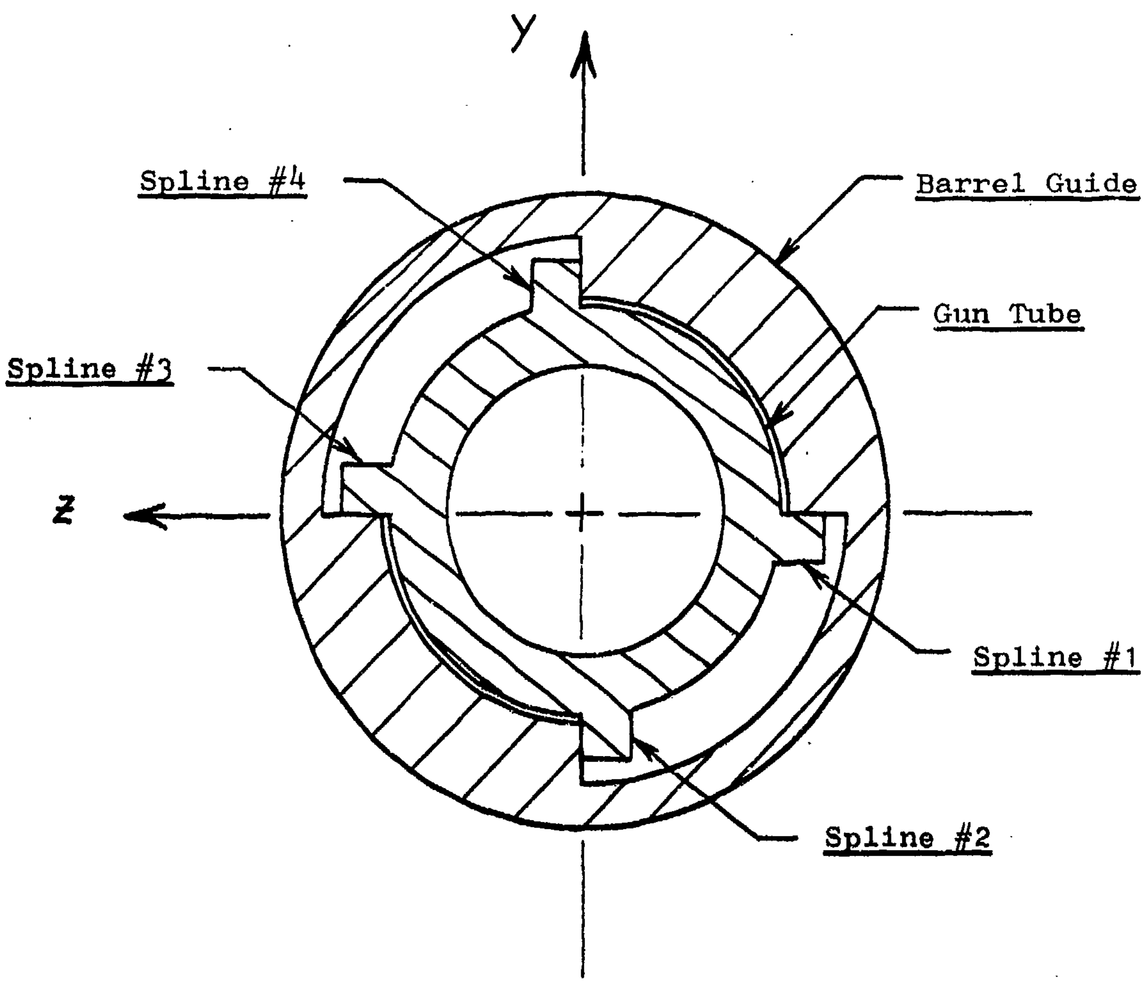


Figure 15 - Barrel Guide/Gun Tube Cross-Section

$$
k_{7,15} = \left[\begin{array}{cccccc|cccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & k_{2\,2} & 0 & 0 & 0 & 0 & 0 & -k_{2\,2} & 0 & 0 & 0 & 0 \\
0 & 0 & k_{3\,3} & 0 & 0 & 0 & 0 & 0 & -k_{3\,3} & 0 & 0 & 0 \\
0 & 0 & 0 & k_{4\,4} & 0 & 0 & 0 & 0 & 0 & -k_{4\,4} & 0 & 0 \\
0 & 0 & 0 & 0 & k_{5\,5} & 0 & 0 & 0 & 0 & 0 & -k_{5\,5} & 0 \\
0 & 0 & 0 & 0 & 0 & k_{6\,6} & 0 & 0 & 0 & 0 & 0 & -k_{6\,6} \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -k_{2\,2} & 0 & 0 & 0 & 0 & 0 & k_{2\,2} & 0 & 0 & 0 & 0 \\
0 & 0 & -k_{3\,3} & 0 & 0 & 0 & 0 & 0 & k_{3\,3} & 0 & 0 & 0 \\
0 & 0 & 0 & -k_{4\,4} & 0 & 0 & 0 & 0 & 0 & k_{4\,4} & 0 & 0 \\
0 & 0 & 0 & 0 & -k_{5\,5} & 0 & 0 & 0 & 0 & 0 & k_{5\,5} & 0 \\
0 & 0 & 0 & 0 & 0 & -k_{6\,6} & 0 & 0 & 0 & 0 & 0 & k_{6\,6}
\end{array}\right]
$$

Figure 16 - Stiffness Matrix (w.r.t. S) Between Mass Points 7 and 15

$$
k'_{7,15} = \left[\begin{array}{cccccc|cccccc}
k'_{11} & k'_{12} & 0 & 0 & 0 & 0 & -k'_{11} & -k'_{12} & 0 & 0 & 0 & 0 \\
k'_{12} & k'_{22} & 0 & 0 & 0 & 0 & -k'_{12} & -k'_{22} & 0 & 0 & 0 & 0 \\
0 & 0 & k'_{33} & 0 & 0 & 0 & 0 & 0 & -k'_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & k'_{44} & k'_{45} & 0 & 0 & 0 & 0 & -k'_{44} & -k'_{45} & 0 \\
0 & 0 & 0 & k'_{45} & k'_{55} & 0 & 0 & 0 & 0 & -k'_{45} & -k'_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & k'_{66} & 0 & 0 & 0 & 0 & 0 & -k'_{66} \\
\hline
-k'_{11} & -k'_{12} & 0 & 0 & 0 & 0 & k'_{11} & k'_{12} & 0 & 0 & 0 & 0 \\
-k'_{12} & -k'_{22} & 0 & 0 & 0 & 0 & k'_{12} & k'_{22} & 0 & 0 & 0 & 0 \\
0 & 0 & -k'_{33} & 0 & 0 & 0 & 0 & 0 & k'_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & -k'_{44} & -k'_{45} & 0 & 0 & 0 & 0 & k'_{44} & k'_{45} & 0 \\
0 & 0 & 0 & -k'_{45} & -k'_{55} & 0 & 0 & 0 & 0 & k'_{45} & k'_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & -k'_{66} & 0 & 0 & 0 & 0 & 0 & k'_{66}
\end{array}\right]
$$

Figure 17 - **Stiffness Matrix (w.r.t. S') Between Mass Points 7 and 15**

Referring to Figure 17

$$
\left.
\begin{array}{ll}
k'_{1\,1} = k_{2\,2} \sin^2\beta_o & k'_{4\,4} = k_{4\,4} \cos^2\beta_o + k_{5\,5} \sin^2\beta_o \\
k'_{1\,2} = -k_{2\,2} \sin\beta_o \cos\beta_o & k'_{4\,5} = (k_{4\,4} - k_{5\,5}) \sin\beta_o \cos\beta_o \\
k'_{2\,2} = k_{2\,2} \cos^2\beta_o & k'_{5\,5} = k_{4\,4} \sin^2\beta_o + k_{5\,5} \cos^2\beta_o \\
k'_{3\,3} = k_{3\,3} & k'_{6\,6} = k_{6\,6}
\end{array}
\right\} \quad (12)
$$

The preceding defines the stiffness characteristics of the individual structural elements of the gun system. The only remaining individual stiffness matrix to be defined is the matrix representing the connection between mass point 2 and ground, namely $k'_{2,g}$.

The matrix $k'_{2,g}$ represents the stiffness of a three-point connection between m_2 and ground; which manifests itself as six independent equivalent springs supporting m_2 and acting along and about each of the coordinates of S'. The elements of this 6 x 6 diagonal matrix are denoted, in accordance with the equilibrium equation for mass point 2, as $k_{x'}^{(2,g)}$, $k_{y'}^{(2,g)}$, $k_{z'}^{(2,g)}$, $k_{\theta_{x'}}^{(2,g)}$, $k_{\theta_{y'}}^{(2,g)}$ and $k_{\theta_{z'}}^{(2,g)}$. The numerical values entering the computation of these elements have been provided by BRL for the M240 artillery mount.

In order to combine the individual stiffness matrices defined above into a single stiffness matrix for the overall system, each individual matrix is first divided into four sub-matrices, as defined by the horizontal and vertical dashed lines presented in Figures 7, 10, 12, 14 and 17. Referring, for example, to Figure 7, which represents the connection between mass points i and j, the (6 x 6 element) submatrix contained in the upper left quadrant denotes the direct stiffness at mass point i (i.e., the forces and moments which must be applied at mass point i to sustain unit displacements and rotations at mass point i while holding mass poing j fixed); the (6 x 6 element) submatrix contained in the lower right quadrant denotes the direct stiffness at mass point j (i.e., the forces and moments which must be applied at mass point j to sustain unit displacements and rotations at mass point j while holding mass point i fixed); the (6 x 6 element) submatrix contained in the lower left quadrant denotes the transfer stiffness from mass point i to mass point j (i.e., the reaction forces and moments developed at mass point j due to imposed unit displacements and rotations at mass point i); and finally, the (6 x 6 element) submatrix contained in the upper right quadrant denotes the transfer stiffness from mass point j to mass point i (i.e., the reaction forces and moments developed at mass point i due to imposed unit displacements and rotations at mass point j). Applying this sub-division to each pertinent matrix, and noting that a given mass point may be common to more than one connection (e.g., mass point 8 is common to the connections 1-8, 2-8, 3-8, 8-9, 8-10 and 8-11), the stiffness matrix of the overall system is obtained, as depicted in Figure 18, and corresponding to the dynamic equilibrium equation presented in Figure 19, by summing corresponding elements of direct stiffness sub-matrices for common connection points, and appropriately locating transfer stiffness sub-matrices (Ref.8).

Letting the symbol $k'^{(i,j)}_{r\,s}$ denote the element located in row "r", column "s", of the individual (sub-structure) stiffness matrix $k'_{i,j}$, the elements $k'_{i\,j}$ (i,j = 1, 2, ..., 114) contained within the overall system stiffness matrix presented schematically in Figure 18, are presented, noting the required symmetry, in terms of the elements of the individual stiffness matrices, in Appendix A.

2.2 Formulation of System Inertia Matrix

Before presenting the details of the formulation of the system inertia matrix it is noteworthy to mention that either of two alternate approaches may be adopted. The first approach requires returning to the dynamic equilibrium equation associated with any two elastically connected mass points, as presented for example in Figure 5, expanding the column vector on the right-hand side so that the inertia loading is separated from the applied loading, and transforming the expanded equation in accordance with the matrix operations indicated in equation (2). This process is repeated for each matrix equation associated with each pair of elastically connected mass points. Furthermore, noting that in general a mass point is common to more than one elastic connection (e.g., referring to Figure 2 mass point 1 is common to the elastic connections 1-2, 1-8 and 1-9), the inertia and applied loads acting at each such mass point must be apportioned between its elastic connections such that upon adding the matrix equations for all of its connections there results a single matrix equation for each such mass point (with its full inertia and applied load acting). Performing the indicated addition, there results, in addition to the stiffness matrix of the overall system (as presented in Figure 18), the desired inertia matrix of the overall system. The second approach adopts the point of view that the stiffness matrix of the overall system is known (via other considerations), and requires only that the final form of the inertia matrix of the overall system be compatible (with respect to reference frame and displacement vector) with the system stiffness matrix. Although conceptually different, both approaches must of course render the same result, and since we have already developed the overall system stiffness matrix, this latter approach is adopted here.

The inertia matrix M_i associated with a typical mass point i, and written with respect to the local coordinate system S_i (taken here to coincide with the principal axes of the physical structural element represented by mass point i), is as presented in Figure 20; the corresponding dynamic equilibrium equation is presented in Figure 21.

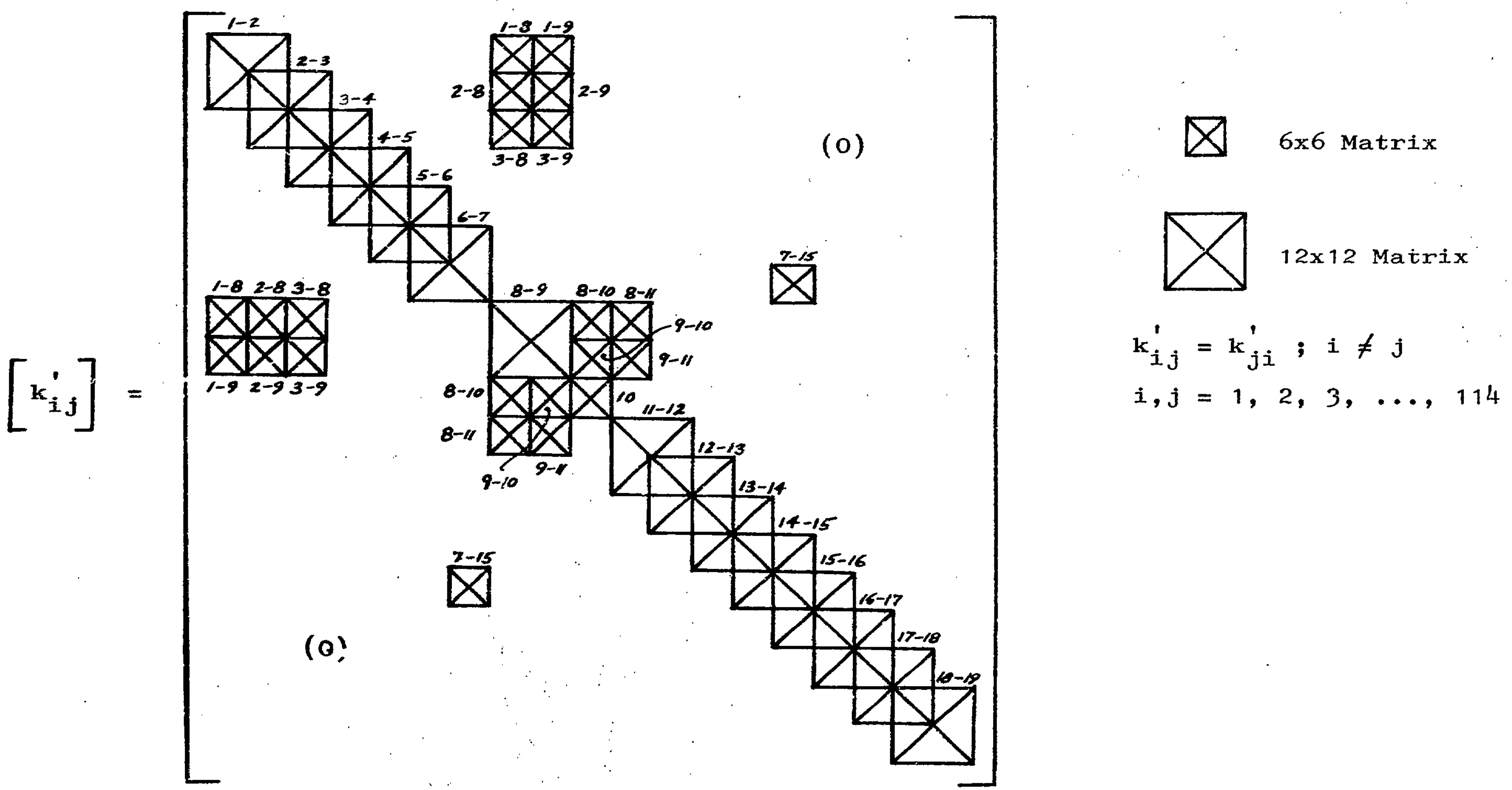


Figure 18 - Stiffness Matrix (w.r.t. S') of Overall System

$$
\begin{bmatrix} k'_{ij} \end{bmatrix}
\begin{bmatrix} x'_1 \\ y'_1 \\ z'_1 \\ \theta_{x'_1} \\ \theta_{y'_1} \\ \theta_{z'_1} \\ \cdot \\ \cdot \\ \cdot \\ x'_{19} \\ y'_{19} \\ z'_{19} \\ \theta_{x'_{19}} \\ \theta_{y'_{19}} \\ \theta_{z'_{19}} \end{bmatrix}
=
\begin{bmatrix} X'_1 \\ Y'_1 \\ Z'_1 \\ M_{x'_1} \\ M_{y'_1} \\ M_{z'_1} \\ \cdot \\ \cdot \\ \cdot \\ X'_{19} \\ Y'_{19} \\ Z'_{19} \\ M_{x'_{19}} \\ M_{y'_{19}} \\ M_{z'_{19}} \end{bmatrix}
$$

Figure 19 - Dynamic Equilibrium Equation (w.r.t. S') of Overall System

$$M_i = \begin{bmatrix} m_i & 0 & 0 & 0 & 0 & 0 \\ 0 & m_i & 0 & 0 & 0 & 0 \\ 0 & 0 & m_i & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{x_i} & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{y_i} & 0 \\ 0 & 0 & 0 & 0 & 0 & I_{z_i} \end{bmatrix}$$

Figure 20 - Inertia Matrix (w.r.t. $\underline{S}$) Associated With Mass Point i

$$
M_i \begin{bmatrix} \ddot{x}_i \\ \ddot{y}_i \\ \ddot{z}_i \\ \ddot{\theta}_{x_i} \\ \ddot{\theta}_{y_i} \\ \ddot{\theta}_{z_i} \end{bmatrix} = \begin{bmatrix} F_{x_i} \\ F_{y_i} \\ F_{z_i} \\ M_{x_i} \\ M_{y_i} \\ M_{z_i} \end{bmatrix}
$$

Figure 21 - Dynamic Equilibrium Equation (w.r.t. S) of Mass Point i

Referring to Figure 20

m_i -- denotes the mass associated with the physical structural element represented by mass point i

$I_{x_i}, I_{y_i}, I_{z_i}$ -- denote the principal mass moments of inertia about the respective principal axes (coordinates of S_i) of the physical structural element represented by mass point i

Referring to Figure 21

$\ddot{x}_i, \ddot{y}_i, \ddot{z}_i$ -- denote linear accelerations of mass point i along the respective coordinates of S_i

$\ddot{\theta}_{x_i}, \ddot{\theta}_{y_i}, \ddot{\theta}_{z_i}$ -- denote angular accelerations of mass point i about the respective coordinates of S_i

$F_{x_i}, F_{y_i}, F_{z_i}$ -- denote forces (excluding inertia; including linear elastic stiffness) applied to mass point i along the respective coordinates of S_i

$M_{x_i}, M_{y_i}, M_{z_i}$ -- denote moments (excluding rotary inertia; including angular elastic stiffness) applied to mass point i about the respective coordinate of S_i

Introducing the coordinate transformation L_i, from the local reference frame S_i to the earth-fixed reference frame S', it may be shown that the inertia matrix M_i' associated with mass point i, and written with respect to S', is obtained from the equation

$$M_i' = L_i M_i L_i^{-1} \qquad (13)$$

The matrix L_i is as prescribed in Figure 22, wherein β_i denotes the angle between x_i (of S_i) and x' (of S'). Comparing Figures 22 and 6 it is seen that L_i is identical with the sub-matrix contained within either the upper left or lower right quadrants of $L_{i,j}$; provided β in equation (3) is replaced by β_i. In addition, it is seen that equation (13) represents the counterpart to the transformation equation presented in equation (2).

Performing the matrix operations indicated in equation (13), there results the desired transformed inertia matrix M_i' as presented in Figure 23; the corresponding dynamic equilibrium equation is presented in Figure 24.

$$L_i = \begin{bmatrix} \cos\beta_i & -\sin\beta_i & 0 & 0 & 0 & 0 \\ \sin\beta_i & \cos\beta_i & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\beta_i & -\sin\beta_i & 0 \\ 0 & 0 & 0 & \sin\beta_i & \cos\beta_i & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Figure 22 - Transformation Matrix From S to S' for Mass Point i

$$M_i' = \begin{bmatrix} m_i & 0 & 0 & 0 & 0 & 0 \\ 0 & m_i & 0 & 0 & 0 & 0 \\ 0 & 0 & m_i & 0 & 0 & 0 \\ 0 & 0 & 0 & (I_{x_i} \cos^2\beta_i + I_{y_i} \sin^2\beta_i) & (I_{x_i} - I_{y_i}) \sin\beta_i \cos\beta_i & 0 \\ 0 & 0 & 0 & (I_{x_i} - I_{y_i}) \sin\beta_i \cos\beta_i & (I_{x_i} \sin^2\beta_i + I_{y_i} \cos^2\beta_i) & 0 \\ 0 & 0 & 0 & 0 & 0 & I_{z_i} \end{bmatrix}$$

Figure 23 - Inertia Matrix (w.r.t. S') Associated With Mass Point i

$$M_i' \begin{bmatrix} \ddot{x}_i' \\ \ddot{y}_i' \\ \ddot{z}_i' \\ \ddot{\theta}_{x_i}' \\ \ddot{\theta}_{y_i}' \\ \ddot{\theta}_{z_i}' \end{bmatrix} = \begin{bmatrix} F_{x_i}' \\ F_{y_i}' \\ F_{z_i}' \\ M_{x_i}' \\ M_{y_i}' \\ M_{z_i}' \end{bmatrix}$$

Figure 24 - Dynamic Equilibrium Equation (w.r.t. S') of Mass Point i

Referring to Figure 23

$\ddot{x}_i', \ddot{y}_i', \ddot{z}_i'$ -- denote linear accelerations of mass point i along the respective coordinates of S'

$\ddot{\theta}_{x_i}', \ddot{\theta}_{y_i}', \ddot{\theta}_{z_i}'$ -- denote angular accelerations of mass point i about the respective coordinates of S'

$F_{x_i}', F_{y_i}', F_{z_i}'$ -- denote forces (excluding inertia; including linear elastic stiffness) applied to mass point i along the respective coordinates of S'

$M_{x_i}', M_{y_i}', M_{z_i}'$ -- denote moments (excluding rotary inertia; including angular elastic stiffness) applied to mass point i about the respective coordinates of S'

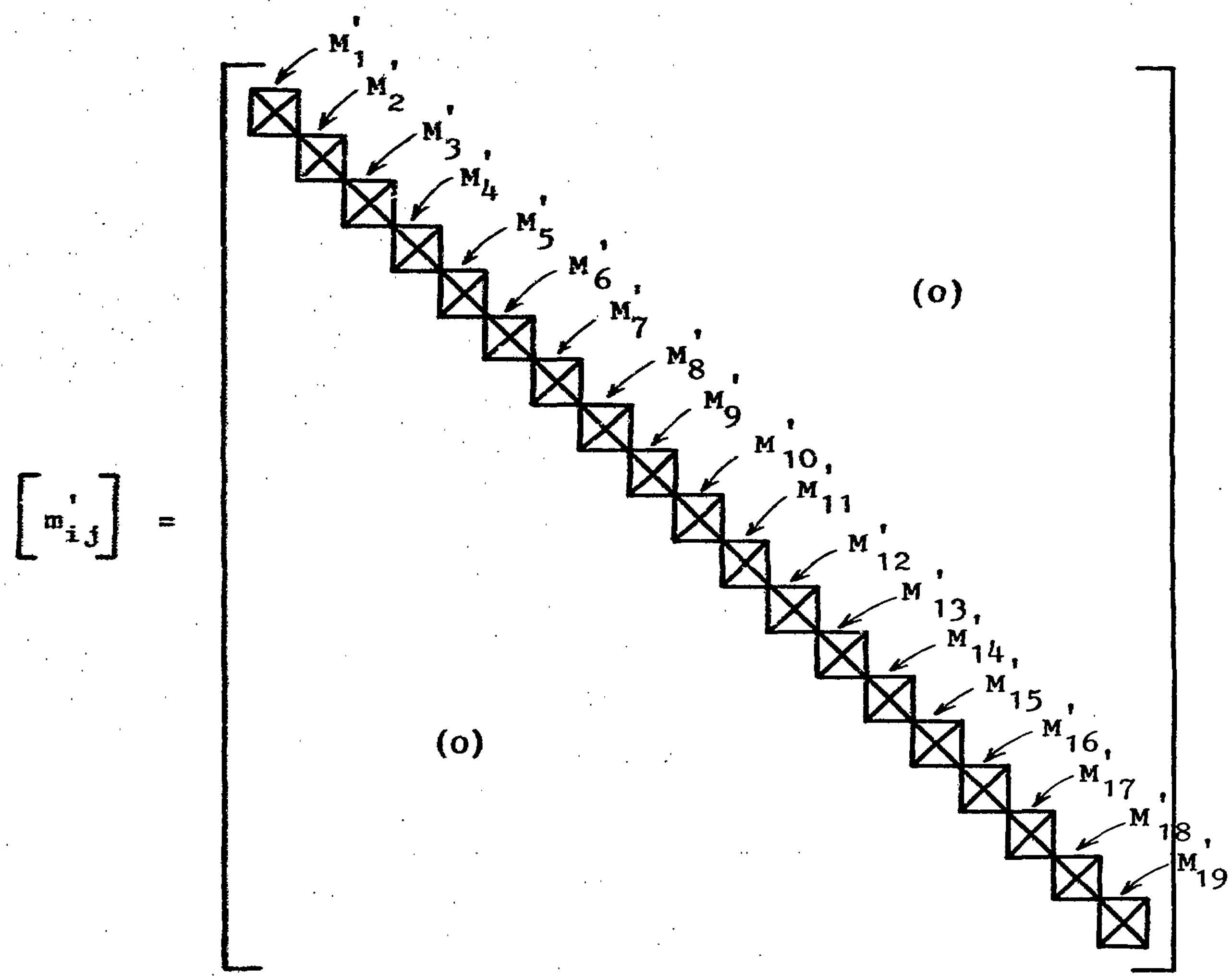
The transformed inertia matrix presented in Figure 23 for a typical mass point is directly applicable to each of the nineteen mass points of the lumped parameter model. Hence, the inertia matrix of the overall system is readily obtained with respect to the reference frame S', and in a form which is compatible with the previously developed system stiffness matrix, by the arrangement schematically depicted in Figure 25.

Noting the required symmetry of the inertia matrix of the overall system, and that for the system under consideration $\beta_i = \beta_o$ for i = 1 thru 3, and 8 thru 10, the elements m_{ij}' (i,j = 1, 2, ..., 114) are as presented in Appendix B.

For completeness of presentation, dynamic equilibrium of the system depicted in Figure 2, and compatible with the overall system stiffness and inertia matrices developed, is as prescribed in Figure 26; wherein the column vector on the right-hand side contains applied forces and moments only (including those which themselves are motion dependent). As an additional note, the equation presented in Figure 26 prescribes dynamic equilibrium based on relative elastic motions, but precludes relative rigid-body motions. Hence, to account for the motion of the breech relative to the receiver during the recoil/counter-recoil cycle, the instantaneous displacements and rotations of mass points 8 thru 19, obtained from the solution to this equation, must be augmented by the prescribed recoil/counter-recoil rigid-body motion. The additional accelerations arising as a consequence of recoil/counter-recoil motion must be incorporated as applied inertia loadings within the loading vector appearing on the right-hand side of the dynamic equilibrium equation.

2.3 Initial Gun Tube Curvature

In order to prescribe the β_i's and β_{ij}'s entering respectively the elements of the system inertia and stiffness matrices, and in addi-



 6x6 Matrix

$m'_{ij} = m'_{ji}\ ;\ i \neq j$

$i, j = 1, 2, 3, \ldots, 114$

Figure 25 - <u>Inertia Matrix (w.r.t. S') of Overall System</u>

$$\left[m'_{ij} \right] \begin{bmatrix} \ddot{x}'_1 \\ \ddot{y}'_1 \\ \ddot{z}'_1 \\ \ddot{\theta}_{x}{}'_1 \\ \ddot{\theta}_{y}{}'_1 \\ \ddot{\theta}_{z}{}'_1 \\ \cdot \\ \cdot \\ \cdot \\ \ddot{x}'_{19} \\ \ddot{y}'_{19} \\ \ddot{z}'_{19} \\ \ddot{\theta}_{x}{}'_{19} \\ \ddot{\theta}_{y}{}'_{19} \\ \ddot{\theta}_{z}{}'_{19} \end{bmatrix} + \left[k'_{ij} \right] \begin{bmatrix} x'_1 \\ y'_1 \\ z'_1 \\ \theta_{x}{}'_1 \\ \theta_{y}{}'_1 \\ \theta_{z}{}'_1 \\ \cdot \\ \cdot \\ \cdot \\ x'_{19} \\ y'_{19} \\ z'_{19} \\ \theta_{x}{}'_{19} \\ \theta_{y}{}'_{19} \\ \theta_{z}{}'_{19} \end{bmatrix} = \begin{bmatrix} X'_1 \\ Y'_1 \\ Z'_1 \\ M_{x}{}'_1 \\ M_{y}{}'_1 \\ M_{z}{}'_1 \\ \cdot \\ \cdot \\ \cdot \\ X'_{19} \\ Y'_{19} \\ Z'_{19} \\ M_{x}{}'_{19} \\ M_{y}{}'_{19} \\ M_{z}{}'_{19} \end{bmatrix}$$

Figure 26 - Dynamic Equilibrium Equation (w.r.t. S') of Overall System

tion, to properly account for the effects of propellant gas pressure and projectile loadings on the gun tube (which will become important in the sequel when we consider the response of the system to a firing), requires specification of the initial curvature of the gun tube (including droop in the vertical plane).

Raw data related to gun tube droop (including manufacturing and assembly eccentricities, thermal exposure, etc.) and static deflection characteristics of the 75mm ADMAG gun system mounted on the M240 artillery mount were generated at Yuma Proving Ground during the week of June 19, 1979. The data were generated using a surveyor's transit to establish a horizontal plane, and a height gage (calibrated at each measurement point using a 6" Johanson block) to measure vertical distance from selected points along the gun system to the established horizontal plane. The specific points at which measurements were taken coincide with the location of mass points in the lumped parameter model.

A detailed documentation of the raw data generated, and subsequent data reduction, is presented in Reference 1. This documentation is not repeated here; however, pertinent data, namely initial gun tube and support tube curvatures, and the corresponding β_i's and β_{ij}'s, have been extracted.

Initial gun tube curvature in the vertical (x-y) plane is presented graphically in Figure 27. The β_i's and β_{ij}'s are presented in tabular form in Tables 1 and 2 respectively. For completeness, Tables 1 and 2 include β_i and β_{ij} values for all elements of the gun system model. In connection with initial gun and support tube curvatures it is noted that measurements were performed for curvature in the vertical plane only; out of plane curvature has not been prescribed, and consequently, is not taken into account (although the model would readily accommodate such prescription). It is also noted that the initial elevation of the gun system about its trunnion axis (i.e., the angle β_o) was measured as 0.486°, as is reflected in Tables 1 and 2.

2.4 Numerical Evaluation of Stiffness and Inertia Matrix Elements

A detailed set of 75mm ADMAG gun system drawings was provided by ARES, Inc., Port Clinton, Ohio. The numerical data herein developed are based on these drawings.

Gun Tube -- the gun tube was simulated as depicted in Figure 28. Referring to Figure 28 and the relevant gun tube drawings, the mechanical, physical and geometrical parameters associated with the gun tube, and required for the stiffness element calculations as prescribed in equation (1), are presented in Table 3. Based on these parameters and equation (1), the stiffness elements associated with each stiffness matrix for each gun tube section (relative to the local coordinates of the section) are presented (in accordance with Figure 4) in Table 4. Combining the results presented in Table 4 with equation (4) and β_{ij} for the gun tube sections as prescribed in Table 2, there results the desired stiffness elements associated with each stiffness matrix for each gun tube section (relative to earth-fixed coordinates) as pre-

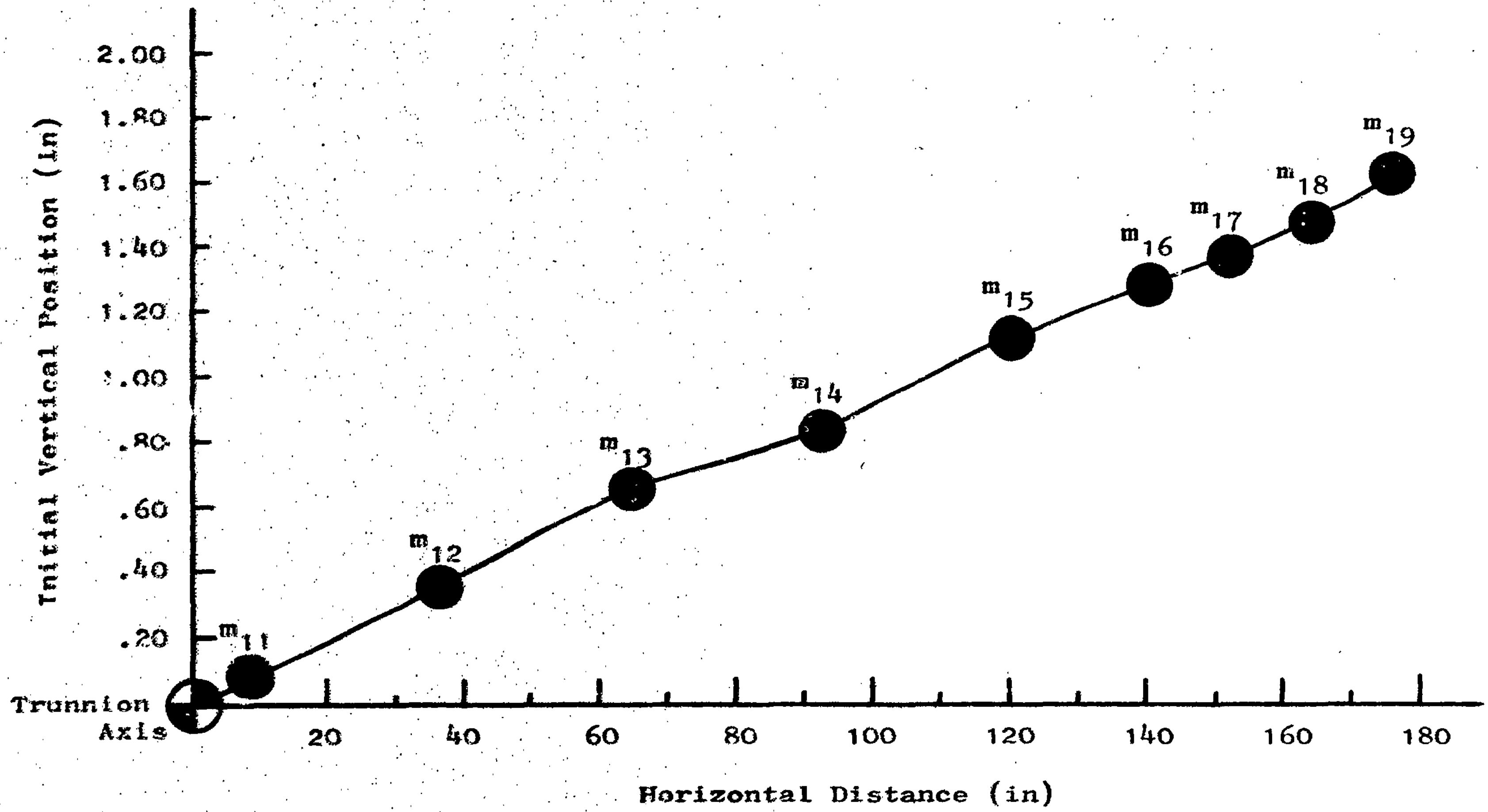


Figure 27 - Initial Gun Tube Curvature in Vertical Plane

Table 1 - β_i

Mass Point	$\beta_i^{*)}$ (deg)
1	.486
2	.486
3	.486
4	.526
5	.605
6	.901
7	1.157
8	.486
9	.486
10	.486
11	.525
12	.590
13	.485
14	.474
15	.538
16	.464
17	.453
18	.589
19	.717

*) Based on local slope of deflection curve

Table 2 - β_{ij}

Stiffness Section (i-j)	$\beta_{ij}^{*)}$ (deg)
1-2, 1-8, 1-9	.486
2-3, 2-8, 2-9	.486
3-4, 3-8, 3-9	.486
4-5	.565
5-6	.644
6-7	1.157
7-15	.486
8-9, 8-10, 8-11	.486
9-10, 9-11	.486
11-12	.563
12-13	.616
13-14	.353
14-15	.595
15-16	.481
16-17	.446
17-18	.460
18-19	.717

*) Based on mass point local deflection

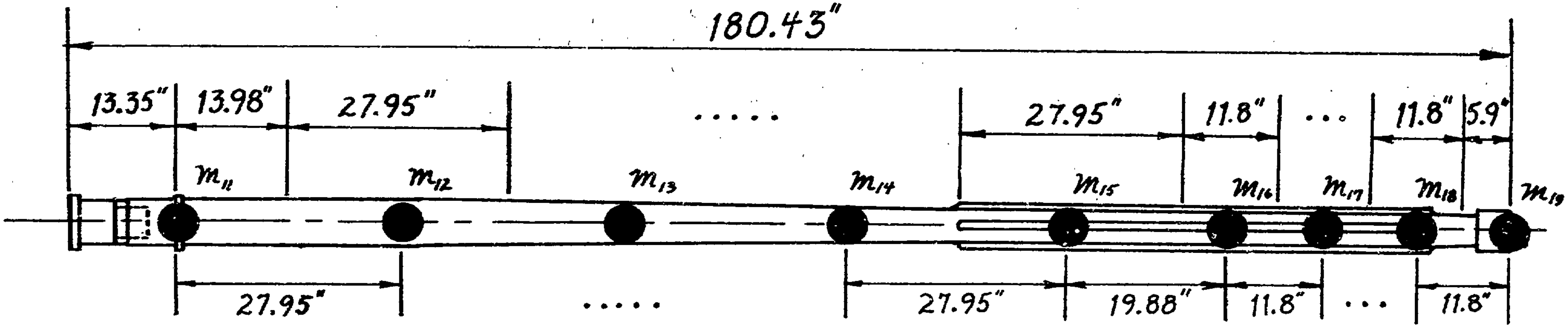


Figure 28 - Gun Tube Simulation

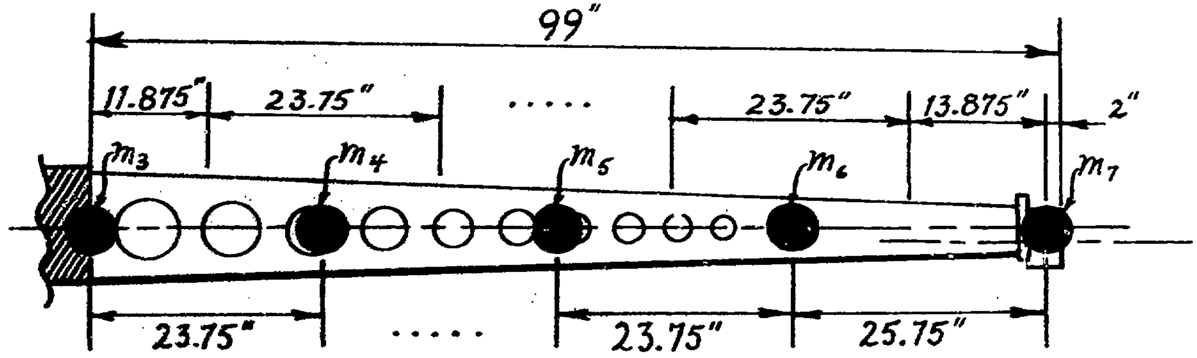


Figure 29 - Support Tube Simulation

Table 3 - Parameters Entering Gun Tube Stiffness Element Calculations

Parameter	Gun Tube Section							
	11-12	12-13	13-14	14-15	15-16	16-17	17-18	18-19
A (in^2)	21.43	18.87	13.02	8.943	8.795	8.795	8.795	7.606
D_t (10^6 lb-in^2)	1438	1174	664.3	386.6	377.8	377.8	377.8	309.4
E (10^6 lb/in^2)	30	30	30	30	30	30	30	30
G (10^6 lb/in^2)	12	12	12	12	12	12	12	12
I_{yy} (in^4)	59.90	48.90	27.68	16.11	15.74	15.74	15.74	12.89
I_{zz} (in^4)	59.90	48.90	27.68	16.11	15.74	15.74	15.74	12.89
L (in)	27.95	27.95	27.95	27.95	19.90	11.80	11.80	11.80
α_y	1.333	1.333	1.333	1.333	1.333	1.333	1.333	1.333
α_z	1.333	1.333	1.333	1.333	1.333	1.333	1.333	1.333

Table 4 - Gun Tube Stiffness Elements (w.r.t. S)

Stiffness Element*)	Gun Tube Section							
	11-12	12-13	13-14	14-15	15-16	16-17	17-18	18-19
$k_{1\,1}$ (lb/in)	23.00	20.25	13.97	9.599	13.26	22.36	22.36	19.34
$k_{2\,2}$ (lb/in)	0.8640	0.7118	0.4115	0.2432	0.7062	2.278	2.278	1.900
$k_{2\,6}$ (lb)	12.07	9.947	5.752	3.399	7.027	13.44	13.44	11.21
$k_{3\,3}$ (lb/in)	0.8640	0.7118	0.4115	0.2432	0.7062	2.278	2.278	1.900
$k_{3\,5}$ (lb)	12.07	9.947	5.752	3.399	7.027	13.44	13.44	11.21
$k_{4\,4}$ (in-lb)	51.43	41.99	23.77	13.833	18.98	32.01	32.01	26.22
$k_{5\,5}$ (in-lb)	233.0	245.7	110.1	64.79	93.64	119.3	119.3	98.89
$k_{5\,11}$ (in-lb)	104.5	86.53	50.67	30.21	46.24	39.28	39.28	33.35
$k_{6\,6}$ (in-lb)	233.0	245.7	110.1	64.79	93.64	119.3	119.3	98.89
$k_{6\,12}$ (in-lb)	104.5	86.53	50.67	30.21	46.24	39.28	39.28	33.35

*) All values are to be multiplied by 10^6

sented (in accordance with Figure 7) in Table 5. Gun tube inertia properties (written with respect to local coordinates) are presented (in accordance with Figure 20) in Table 6.

Support Tube -- the support tube was simulated as depicted in Figure 29. Referring to Figure 29 and the relevant support tube drawings, the mechanical, physical and geometrical parameters associated with the support tube, and required for the stiffness element calculations as prescribed in equation (1), are presented in Table 7. Based on these parameters and equation (1), the stiffness elements associated with each stiffness matrix for each support tube section (relative to local coordinates of the section) are presented (in accordance with Figure 4) in Table 8. Combining the results presented in Table 8 with equation (4) and β_{ij} for the support tube sections as prescribed in Table 2, there results the desired stiffness elements associated with each stiffness matrix for each support tube section (relative to earth-fixed coordinates) as presented (in accordance with Figure 7) in Table 9. Support tube inertia properties (written with respect to local coordinates) are presented (in accordance with Figure 20) in Table 10.

Receiver -- the receiver (including gun pod and side plates) was simulated as depicted in Figure 30. Referring to Figure 30 and the relevant receiver drawings, the mechanical, physical and geometrical parameters associated with the receiver, and required for the stiffness element calculations as prescribed in equation (1), are presented in Table 11. Based on these parameters and equation (1), the stiffness elements associated with each stiffness matrix for each receiver section (relative to local coordinates of the section) are presented (in accordance with Figure 4) in Table 12. Noting that $\beta = \beta_o$ for the receiver, there results, from Table 12 and equation (4), the desired stiffness matrix for each receiver section (relative to earth-fixed coordinates) as presented (in accordance with Figure 7) in Table 13. Receiver inertia properties (written with respect to local coordinates) are presented (in accordance with Figure 20) in Table 14.

Breech -- the breech was simulated as depicted in Figure 31. Referring to Figure 31 and the relevant breech drawings, the mechanical, physical and geometrical properties associated with the breech, and required for the stiffness element calculations as prescribed in equation (1), are presented in Table 15. Based on these parameters and equation (1), the stiffness elements associated with each stiffness matrix for each breech section (relative to local coordinates of the section) are presented (in accordance with Figure 9 for Section 8-9, and Figure 4 for Sections 8-11 and 9-11) in Table 16. Noting that $\beta = \beta_o$ for the breech, there results, from Table 16 and equation (5) for Section 8-9, and Table 16 and equation (4) for Sections 8-11 and 9-11, the desired stiffness matrix for each receiver section (relative to earth-fixed coordinates) as presented (in accordance with Figure 10 for Section 8-9, and Figure 7 for Sections 8-11 and 9-11) in Table 17. Breech inertia properties (written with respect to local coordinates) are presented (in accordance with Figure 20) in Table 18.

Table 5 - Gun Tube Stiffness Elements (w.r.t. S')

Stiffness Element*)	Gun Tube Support							
	11-12	12-13	13-14	14-15	15-16	16-17	17-18	18-19
$k'_{1\,1}$ (lb/in)	23.00	20.25	13.97	9.588	13.26	22.36	22.36	19.33
$k'_{1\,2}$ (lb/in)	0.2176	0.2101	0.08355	0.09711	0.1053	0.1562	0.1613	0.2181
$k'_{1\,6}$ (lb)	0.1187	0.1069	0.03543	0.03528	0.05896	0.1046	0.1079	0.1402
$k'_{2\,2}$ (lb/in)	0.8662	0.7117	0.4120	0.2442	0.7062	2.279	2.279	1.902
$k'_{2\,6}$ (lb)	12.07	9.946	5.751	3.399	7.027	13.44	13.44	11.21
$k'_{3\,3}$ (lb/in)	0.8640	0.7118	0.4115	0.2432	0.7062	2.278	2.278	1.900
$k'_{3\,4}$ (lb)	0.1187	0.1069	0.03543	0.03528	0.05896	0.1046	0.1079	0.1402
$k'_{3\,5}$ (lb)	12.07	9.946	5.751	3.399	7.027	13.44	13.44	11.21
$k'_{4\,4}$ (in-lb)	51.45	42.01	23.77	13.84	18.99	32.02	32.02	26.23

Table 5 - (Cont'd).

$k'_{4\ 5}$ (in-lb)	-1.785	-2.190	-0.5318	-0.5289	-0.6264	-0.6792	-0.7010	-0.9091
$k'_{4\ 10}$ (in-lb)	-51.42	-41.08	-23.77	-13.83	-18.98	-32.01	-32.01	-26.21
$k'_{4\ 11}$ (in-lb)	-1.532	-1.381	-0.4586	-0.4571	-0.5472	-0.5565	-0.5725	-0.7452
$k'_{5\ 5}$ (in-lb)	233.0	245.7	110.1	64.78	93.64	119.3	119.3	98.88
$k'_{5\ 11}$ (in-lb)	104.4	86.51	50.67	30.20	46.23	39.28	39.28	33.35
$k'_{6\ 6}$ (in-lb)	233.0	245.7	110.1	64.79	93.64	119.3	119.3	98.89
$k'_{6\ 12}$ (in-lb)	104.5	86.53	50.67	30.21	46.24	39.28	39.28	33.35

*) All values are to be multiplied by 10^6

Table 6 - Gun Tube Inertia Properties (w.r.t. S)

Mass Point	m ($lb\text{-}sec^2/in$)	I_x ($in\text{-}lb\text{-}sec^2$)	I_y ($in\text{-}lb\text{-}sec^2$)	I_z ($in\text{-}lb\text{-}sec^2$)
11	0.4374	2.521	29.39	29.45
12	0.4281	2.360	28.90	28.90
13	0.3251	1.530	21.92	21.92
14	0.2125	0.8137	14.23	14.23
15	0.1801	0.6625	12.05	12.05
16	0.07609	0.2798	1.022	1.022
17	0.07609	0.2798	1.022	1.022
18	0.07220	0.2585	0.9283	0.9283
19	0.04790	0.1935	0.5510	0.5510

Table 7 - Parameters Entering Support Tube Stiffness Element Calculations

Parameter	Support Tube Section			
	3-4	4-5	5-6	6-7
A (in^2)	9.09	7.65	6.84	5.80
D_t (10^6 lb-in^2)	2.02	1.56	1.50	1.34
E (10^6 lb/in^2)	30	30	30	30
G (10^6 lb/in^2)	12	12	12	12
I_{yy} (in^4)	223.7	162.2	123.4	90.85
I_{zz} (in^4)	185.4	115.2	68.41	33.98
L (in)	23.75	23.75	23.75	25.25
α_y	1.5	1.5	1.5	1.5
α_z	1.5	1.5	1.5	1.5

Table 8 - Support Tube Stiffness Elements (w.r.t. S)

Stiffness Element*)	Support Tube Section			
	3-4	4-5	5-6	6-7
$k_{1\,1}$ (lb/in)	11.48	9.663	8.640	7.484
$k_{2\,2}$ (lb/in)	1.896	1.406	1.023	0.6543
$k_{2\,6}$ (lb)	22.52	16.70	12.14	7.606
$k_{3\,3}$ (lb/in)	2.029	1.619	1.359	1.129
$k_{3\,5}$ (lb)	24.09	19.23	16.14	13.13
$k_{4\,4}$ (in-lb)	0.08509	0.06558	0.06316	0.05765
$k_{5\,5}$ (in-lb)	568.6	433.3	347.5	269.9
$k_{5\,11}$ (in-lb)	3.504	23.48	35.82	35.43
$k_{6\,6}$ (in-lb)	501.6	343.8	230.6	132.3
$k_{6\,12}$ (in-lb)	33.23	52.79	57.81	44.58

*) All values are to be multiplied by 10^6

Table 9 - Support Tube Stiffness Elements (w.r.t. S')

Stiffness Element*)	Support Tube Section			
	3-4	4-5	5-6	6-7
$k'_{1\,1}$ (lb/in)	11.48	9.662	8.639	7.484
$k'_{1\,2}$ (lb/in)	0.08129	0.08141	0.08561	0.1379
$k'_{1\,6}$ (lb)	0.1910	0.1647	0.1365	0.1536
$k'_{2\,2}$ (lb/in)	1.897	1.407	1.024	0.6571
$k'_{2\,6}$ (lb)	22.52	16.70	12.14	7.601
$k'_{3\,3}$ (lb/in)	2.029	1.619	1.359	1.129
$k'_{3\,4}$ (lb)	0.2043	0.1896	0.1814	0.2651
$k'_{3\,5}$ (lb)	24.09	19.23	16.14	13.13
$k'_{4\,4}$ (in-lb)	0.1249	0.1089	0.1084	0.1683

Table 9 - (Cont'd)

$k'_{4\ 5}$ (in-lb)	-4.821	-4.271	-3.905	-5.446
$k'_{4\ 10}$ (in-lb)	-0.08484	-0.06325	-0.05853	-0.04311
$k'_{4\ 11}$ (in-lb)	-0.03040	-0.2322	-0.4033	-0.7163
$k'_{5\ 5}$ (in-lb)	568.6	433.2	347.5	269.7
$k'_{5\ 11}$ (in-lb)	3.504	23.48	35.82	35.41
$k'_{6\ 6}$ (in-lb)	501.6	343.8	230.6	132.3
$k'_{6\ 12}$ (in-lb)	33.23	52.79	57.81	44.58

*) All values are to be multiplied by 10^6

Table 10 - Support Tube Inertia Properties (w.r.t. S)

Mass Point	m $(lb\text{-}sec^2/in)$	I_x $(in\text{-}lb\text{-}sec^2)$	I_y $(in\text{-}lb\text{-}sec^2)$	I_z $(in\text{-}lb\text{-}sec^2)$
3*)	0.06416	4.160	6.070	5.752
4	0.1446	5.865	10.10	9.351
5	0.1235	3.962	8.220	7.352
6	0.1121	2.749	7.157	5.841
7	0.1421	2.348	4.088	3.226

*) Does not include portion of receiver

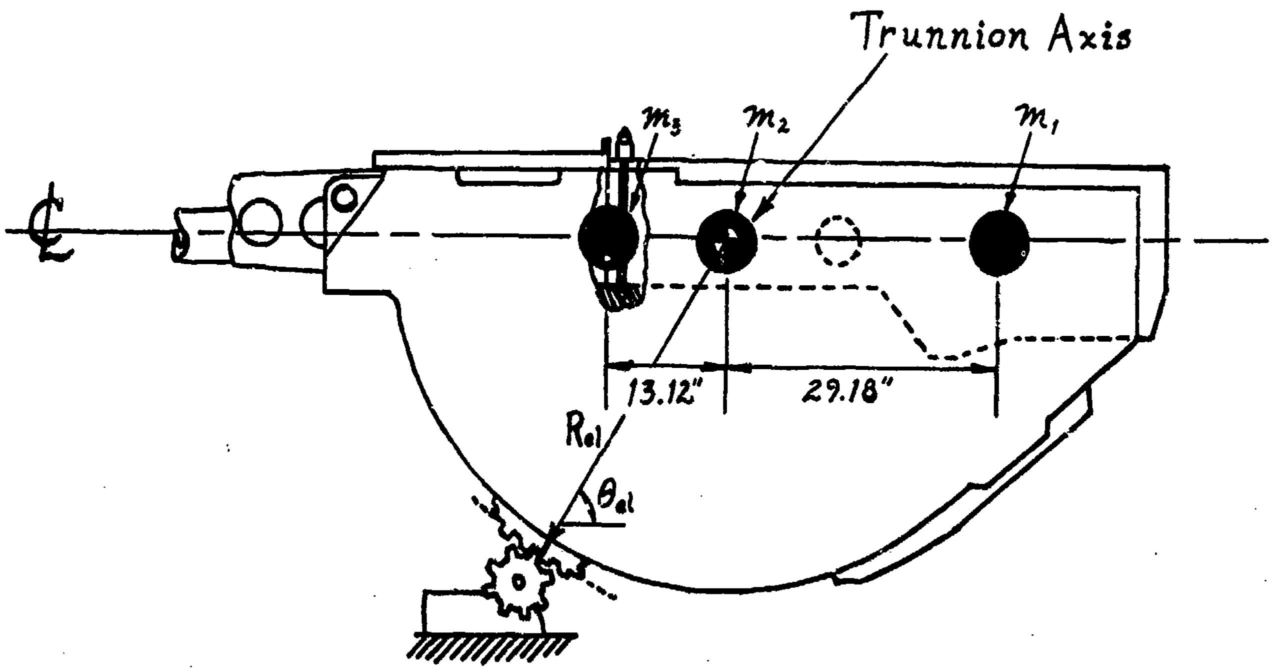


Figure 30 - Receiver Simulation

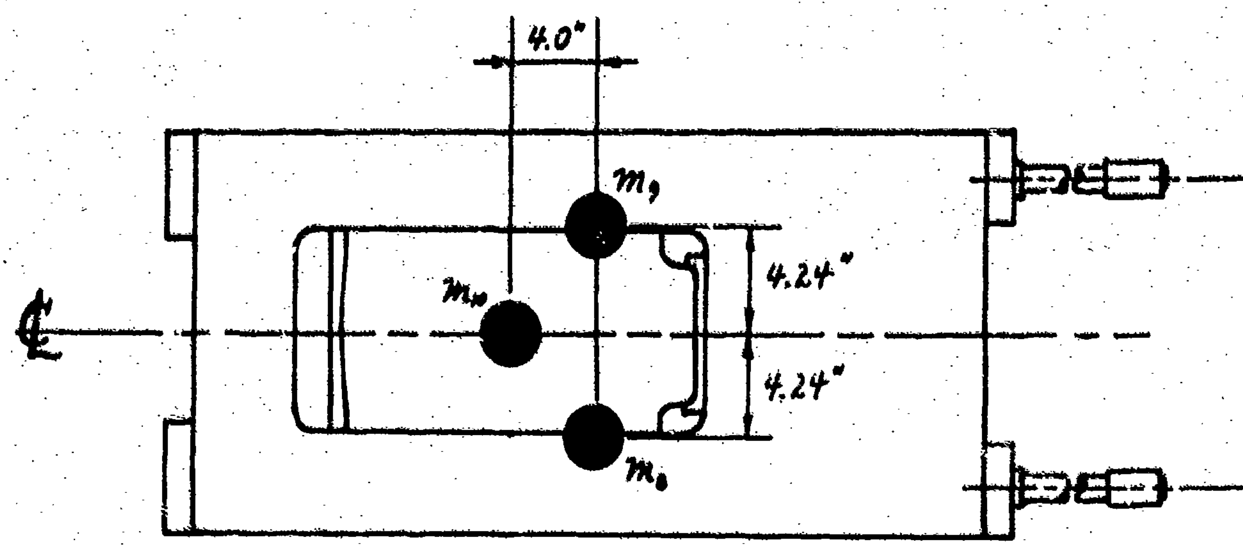


Figure 31 - Breech/Chamber Simulation

Table 11 - Parameters Entering Receiver Stiffness Element Calculations

Parameter	Receiver Section	
	1-2	2-3
A (in^2)	83.15 *)	154.7
D_t (10^6 lb-in^2)	96990 *)	210900
E (10^6 lb/in^2)	30	30
G (10^6 lb/in^2)	12	12
I_{yy} (in^4)	7971 *)	17480
I_{zz} (in^4)	10340 *)	24050
L (in)	29.18	13.00
α_y	1.5	1.5
α_z	1.5	1.5

*) Weighted Average

Table 12 - Receiver Stiffness (w.r.t. S)

Stiffness Element*)	Section	
	1-2	2-3
$k_{1\ 1}$ (lb/in)	85.48	357.0
$k_{2\ 2}$ (lb/in)	19.78	92.94
$k_{2\ 6}$ (lb)	288.7	604.1
$k_{3\ 3}$ (lb/in)	19.04	92.13
$k_{3\ 5}$ (lb)	277.8	598.8
$k_{4\ 4}$ (in-lb)	3324	16220
$k_{5\ 5}$ (in-lb)	12250	44240
$k_{5\ 11}$ (in-lb)	-4143	-36450
$k_{6\ 6}$ (in-lb)	14840	59430
$k_{6\ 12}$ (in-lb)	-6417	-51570

*) All values are to be multiplied by 10^6

Table 13 - Receiver Stiffness (w.r.t. S')

Stiffness Element*)	Section	
	1-2	2-3
$k'_{1\,1}$ (lb/in)	85.48	356.9
$k'_{1\,2}$ (lb/in)	0.5573	2.239
$k'_{1\,6}$ (lb)	2.449	5.125
$k'_{2\,2}$ (lb/in)	19.79	92.96
$k'_{2\,6}$ (lb)	288.7	604.1
$k'_{3\,3}$ (lb/in)	19.04	92.13
$k'_{3\,4}$ (lb)	2.356	5.079
$k'_{3\,5}$ (lb)	277.8	598.8
$k'_{4\,4}$ (in-lb)	3324	16230
$k'_{4\,5}$ (in-lb)	-75.70	-237.6
$k'_{4\,10}$ (in-lb)	-3324	-16230
$k'_{4\,11}$ (in-lb)	6.945	171.6
$k'_{5\,5}$ (in-lb)	12250	44240
$k'_{5\,11}$ (in-lb)	-4142	-36450
$k'_{6\,6}$ (in-lb)	14840	59430
$k'_{6\,12}$ (in-lb)	-6417	-51570

*) All values are to be multiplied by 10^6

Table 14 - Receiver Inertia Properties (w.r.t. S)

Mass Point	m ($lb\text{-}sec^2/in$)	I_x ($in\text{-}lb\text{-}sec^2$)	I_y ($in\text{-}lb\text{-}sec^2$)	I_z ($in\text{-}lb\text{-}sec^2$)
1	1.028	71.97	127.0	117.9
2*)	8.607	2841	3912	5448
3**)	0.2639	13.35	20.82	14.65

*) Includes five rounds of ammunition in carousel (0.0891 $lb\text{-}sec^2/in$ per round)

**) Does not include portion of support tube

Table 15 - Parameters Entering Breech Stiffness Element Calculations

Parameter	Breech Section		
	8-9	8-11	9-11
A (in^2)	77.95	92.61	92.61
D_t (10^6 $lb\text{-}in^2$)	3984	22420	22420
E (10^6 lb/in^2)	30	30	30
G (10^6 lb/in^2)	12	12	12
I_{yy} (in^4)	12380	1645	1645
I_{zz} (in^4)	611.1	533.2	533.2
L (in)	8.100	13.31	13.31
α_y	1.5	1.5	1.5
α_z	1.5	1.5	1.5

Table 16 - Breech Stiffness Properties (w.r.t. S)

Stiffness Element*)	Breech Section		
	8-9	8-11	9-11
$k_{1\ 1}$ (lb/in)	288.7	104.4	104.4
$k_{2\ 2}$ (lb/in)	64.91	16.53	16.53
$k_{2\ 6}$ (lb)	262.9	110.0	110.0
$k_{3\ 3}$ (lb/in)	76.29	22.78	22.78
$k_{3\ 5}$ (lb)	309.0	151.6	151.6
$k_{4\ 4}$ (in-lb)	491.9	842.2	842.2
$k_{5\ 5}$ (in-lb)	47090	2863	2863
$k_{5\ 11}$ (in-lb)	-44590	-844.8	-844.8
$k_{6\ 6}$ (in-lb)	3328	1333	1333
$k_{6\ 12}$ (in-lb)	-1199	131.3	131.3

*) All values are to be multiplied by 10^6

Table 17 - Breech Stiffness Properties (w.r.t. S')

Stiffness Element*)	Breech Section		
	8-9	8-11	9-11
$k'_{1\,1}$ (lb/in)	76.29	104.4	104.4
$k'_{1\,2}$ (lb/in)	0.09649	0.7450	0.7450
$k'_{1\,4}$ (lb)	0.3908	-	-
$k'_{1\,5}$ (lb)	-309.0	-	-
$k'_{1\,6}$ (lb)	-	0.9329	0.9329
$k'_{2\,2}$ (lb/in)	64.92	16.53	16.53
$k'_{2\,4}$ (lb)	262.9	-	-
$k'_{2\,6}$ (lb)	-	110.0	110.0
$k'_{3\,3}$ (lb/in)	288.7	22.78	22.78
$k'_{3\,4}$ (lb)	-	1.286	1.286

Table 17 - (Cont'd)

$k'_{3\ 5}$ (lb)	-	151.6	151.6
$k'_{4\ 4}$ (in-lb)	3331	842.3	842.3
$k'_{4\ 5}$ (in-lb)	-371.2	-17.14	-17.14
$k'_{4\ 10}$ (in-lb)	-1202	-842.2	-842.2
$k'_{4\ 11}$ (in-lb)	368.1	0.0221	0.0221
$k'_{5\ 5}$ (in-lb)	47090	2863	2863
$k'_{5\ 11}$ (in-lb)	-44590	-844.8	-844.8
$k'_{6\ 6}$ (in-lb)	491.9	1333	1333
$k'_{6\ 12}$ (in-lb)	-	131.3	131.3

*) All values are tc be multiplied by 10^6

Table 18 - Breech Inertia Properties (w.r.t. S)

Mass Point	m ($lb\text{-}sec^2/in$)	I_x ($in\text{-}lb\text{-}sec^2$)	I_y ($in\text{-}lb\text{-}sec^2$)	I_z ($in\text{-}lb\text{-}sec^2$)
m_8	0.8085	10.02	117.0	116.5
m_9	0.8085	10.02	117.0	116.5

Table 19 - Chamber Inertia Properties (w.r.t. S)

Mass Point	m ($lb\text{-}sec^2/in$)	I_x ($in\text{-}lb\text{-}sec^2$)	I_y ($in\text{-}lb\text{-}sec^2$)	I_z ($in\text{-}lb\text{-}sec^2$)
$m_{10}^{*)}$	0.5384	6.432	10.98	10.98

*) Includes one round of ammunition in the chamber (0.0891 $lb\text{-}sec^2/in$ per round).

Exclusive of the chamber and gun mount, the preceding numerically defines the major structural components of the gun system relative to the lumped parameter model depicted in Figure 2. The remaining quantities required to complete the definition are the chamber inertia matrix, the stiffness matrices associated with the connections between the chamber and breech, the breech and receiver, the gun tube and support tube at the barrel guide interface, and finally, the stiffness matrix of the gun mount.

Chamber inertia properties (written with respect to local coordinates) are presented (based on relevant chamber drawings and in accordance with Figure 20) in Table 19. The elements of the stiffness matrices representing the physical connections between the chamber and breech (relative to local coordinates) are presented (based on relevant structural drawings and in accordance with Figure 13) in Table 20. From the data presented in Table 20 and equation (11), there results the desired stiffness matrix for each chamber/breech connection (relative to earth-fixed coordinates) as presented (in accordance with Figure 14) in Table 21.

Referring to equations (6) and (7), Figures 4 and 11, and the relevant structural drawings, the elements of the stiffness matrices representing the physical connections between the breech and receiver (relative to local coordinates) are presented in Table 22. From the data presented in Table 22, equations (8) and (9), and Figures 7 and 12, there results the elements of the desired stiffness matrices for each breech/receiver connection (relative to earth-fixed coordinates) as presented in Table 23.

Referring to the schematic depicted in Figure 15, the corresponding relevant structural drawings, and Figure 16, the elements of the stiffness matrix representing the connection between the gun tube and support tube at the barrel guide interface (relative to local coordinates) are presented in Table 24. From the data presented in Table 24, equation (12), and Figure 17, there results the elements of the desired stiffness matrix representing the gun tube/support tube connection (relative to earth-fixed coordinates) as presented in Table 25.

The remaining stiffness matrix to be defined represents the gun mount. As previously described, the physical mount structure is represented (relative to the lumped parameter model depicted in Figure 2) as a three-point connection between m_2 and ground. Noting that local and earth-fixed coordinates coincide for this computation, the elements of the desired stiffness matrix are presented, based on data provided by BRL for the M240 artillery mount, in Table 26.

Substituting the appropriate numerical values obtained above for the elements of the individual stiffness and inertia matrices into the prescribed format presented in Appendices A and B, there results the elements of the overall system stiffness and inertia matrices presented respectively in Appendices C and D.

Table 20 - Chamber to Breech Stiffness (w.r.t. S)

Stiffness Element*)	Section	
	8-10	9-10
k_{11} (lb/in)	30.60	30.60
k_{22} (lb/in)	14.78	14.78
k_{33} (lb/in)	565.5	565.5
k_{44} (in-lb)	259.1	259.1
k_{55} (in-lb)	536.7	536.7
k_{66} (in-lb)	0.6834	0.6834

*) All values are to be multiplied by 10^6

Table 21 - Chamber to Breech Stiffness (w.r.t. S')

Stiffness Element*)	Section	
	8-10	9-10
k'_{11} (lb/in)	30.60	30.60
k'_{12} (lb/in)	0.1342	0.1342
k'_{22} (lb/in)	14.78	14.78
k'_{33} (lb/in)	565.5	565.5
k'_{44} (in-lb)	259.2	259.2
k'_{45} (in-lb)	-2.354	-2.354
k'_{55} (in-lb)	536.7	536.7
k'_{66} (in-lb)	0.6834	0.6834

*) All values are to be multiplied by 10^6

Table 22 - Breech/Receiver Stiffness*) (w.r.t. S)

Stiffness Element**)	Section					
	1-8	1-9	2-8	2-9	3-8	3-9
$k_{1\ 1}$ (lb/in)	0	0	0	0	0.8000	0.8000
$k_{2\ 2}$ (lb/in)	9.946	9.946	49.13	49.13	0	0
$k_{2\ 6}$ (lb)	108.7	108.7	-179.8	-179.8	0	0
$k_{3\ 3}$ (lb/in)	14.29	14.29	52.07	52.07	0	0
$k_{3\ 5}$ (lb)	156.2	156.2	-190.6	-190.6	0	0
$k_{4\ 4}$ (in-lb)	547.9	547.9	1636	1636	0	0
$k_{5\ 5}$ (in-lb)	4511	4511	9070	9070	0	0
$k_{5\ 11}$ (in-lb)	-1096	-1096	-7676	-7676	0	0
$k_{6\ 6}$ (in-lb)	2078	2078	3314	3314	0	0
$k_{6\ 12}$ (in-lb)	298.9	298.9	-1998	-1998	0	0

*) In-battery configuration

**) All values are to be multiplied by 10^6

Table 23 - Breech/Receiver Stiffness*) (w.r.t. S')

Stiffness Element**)	Section					
	1-8	1-9	2-8	2-9	3-8	3-9
k'_{11} (lb/in)	0.007156	0.007156	0.003535	0.003535	0.8000	0.8000
k'_{12} (lb/in)	-0.08436	-0.08436	-0.4167	-0.4167	0.006794	0.006794
k'_{16} (lb)	0.9221	0.9221	-1.525	-1.525	0	0
k'_{22} (lb/in)	9.945	9.945	49.13	49.13	0	0
k'_{26} (lb)	108.7	108.7	-179.8	-179.8	0	0
k'_{33} (lb/in)	14.29	14.29	52.07	52.07	0	0
k'_{34} (lb)	1.325	1.325	-1.616	-1.616	0	0
k'_{35} (lb)	156.2	156.2	-190.6	-190.6	0	0

Table 23 - (Cont'd)

$k'_{4\ 4}$ (in-lb)	548.3	548.3	1637	1637	0	0
$k'_{4\ 5}$ (in-lb)	-33.62	-33.62	-63.06	-63.06	0	0
$k'_{4\ 10}$ (in-lb)	-548.1	-548.1	-1637	-1637	0	0
$k'_{4\ 11}$ (in-lb)	4.651	4.651	51.23	51.23	0	0
$k'_{5\ 5}$ (in-lb)	4511	4511	9070	9070	0	0
$k'_{5\ 11}$ (in-lb)	-1096	-1096	-7675	-7675	0	0
$k'_{6\ 6}$ (in-lb)	2078	2078	3314	3314	0	0
$k'_{6\ 12}$ (in-lb)	298.9	298.9	-1998	-1998	0	0

*) In-battery configuration

**) All values are to be multiplied by 10^6

Table 24 - <u>Gun Tube to Support Tube Stiffness (w.r.t. S)</u>

Stiffness Element*)	Section 7-15
$k_{1\,1}$ (lb/in)	0
$k_{2\,2}$ (lb/in)	56.40
$k_{3\,3}$ (lb/in)	56.40
$k_{4\,4}$ (in-lb)	649.7
$k_{5\,5}$ (in-lb)	225.6
$k_{6\,6}$ (in-lb)	225.6

*) All values are to be multiplied by 10^6

Table 25 - <u>Gun Tube to Support Tube Stiffness (w.r.t. S')</u>

Stiffness Element*)	Section 7-15
$k'_{1\,1}$ (lb/in)	0.004058
$k'_{1\,2}$ (lb/in)	-0.4784
$k'_{2\,2}$ (lb/in)	56.40
$k'_{3\,3}$ (lb/in)	56.40
$k'_{4\,4}$ (in-lb)	649.7
$k'_{4\,5}$ (in-lb)	3.597
$k'_{5\,5}$ (in-lb)	225.6
$k'_{6\,6}$ (in-lb)	225.6

*) All values are to be multiplied by 10^6

Table 26 - Stiffness from m_2 to Ground (w.r.t. S')

Stiffness Element*)	Section 2-g
k_x' (lb/in)	51.04
k_y' (lb/in)	52.28
k_z' (lb/in)	0.628
k_{θ_x}' (in-lb)	12160
k_{θ_y}' (in-lb)	11870
k_{θ_z}'**) (in-lb)	100

*) All values are to be multiplied by 10^6

**) Nominal value -- includes mount sidewall flexure in neighborhood of elevation link

As a final note to the computations presented above, a summary of gun system weights is presented in Table 27. As is seen from this table, the computed weight of recoiling parts is 1550 lb; the computed total gun system weight is 5601 lb. These figures are to be compared with those prescribed by ARES, Inc., namely 1592 lb and 5622 lb respectively.

Table 27 - Summary of Calculated Gun System Weights

Item	Mass Point Representation	Weight (lb-sec²/in)	Weight (lb)
Receiver [1)]	m_1 thru m_3 [2)]	9.899	3825
Support Tube	m_3 [3)] thru m_7	0.5865	226.6
Breech	m_8 thru m_{10} [4)]	2.155	832.8
Gun Tube	m_{11} thru m_{19}	1.855	716.8
Recoiling Parts	m_8 thru m_{19}	4.01	1550
Gun System	m_1 thru m_{19}	14.50	5601

1) Includes gun pod with five rounds of ammunition in carousel

2) Does not include portion of support tube

3) Does not include portion of receiver

4) Includes one round of ammunition in chamber

3. FORMULATION AND SOLUTION OF EIGENVALUE PROBLEM (NATURAL FREQUENCIES AND NORMAL MODE SHAPES)

The 114 natural frequencies and corresponding normal mode shapes of the model depicted in Figure 2, and defined in the preceding section, are obtained from the solution to the eigenvalue problem

$$\left| [K] - \lambda_i [M] \right| \{X\}_i = [0] \quad ; \quad i = 1,2, \ldots, 114 \qquad (14)$$

where [K] denotes the overall system stiffness matrix depicted in Figure 18 and defined numerically in Appendix C, [M] denotes the overall system inertia matrix depicted in Figure 25 and defined numerically in Appendix D, λ_i denotes the i^{th} mode eigenvalue (equal to the square of the i^{th} mode natural frequency, ω_i), and $\{X\}_i$ denotes the corresponding i^{th} mode eigenfunction (equal to the i^{th} normal mode shape). It is noted that equation (14) has been obtained from the dynamic equilibrium equation presented in Figure 26 by setting the applied load vector equal to zero, the displacement vector equal to the i^{th} mode eigenfunction, and the acceleration vector equal to the i^{th} mode eigenvalue multiplied by the i^{th} mode eigenfunction.

In order to solve equation (14) for the desired natural frequencies and normal mode shapes, the computer program "ADMAGEG", consisting of a main control program and a package of appended IMSL (International Mathematical and Statistical Libraries) subroutines, was developed. The control program handles the input/output routines and the ordering of the eigenvalue solution beginning with the lowest natural frequency for the indicated number of modes desired in the output. The package of appended IMSL subroutines is accessed by a call to "EIGZF" in the main program, and solves for all 114 eigenvalues and eigenfunctions associated with equation (14). All necessary input is read directly from an appended data file. Initial output records function as a check to the stability and nature of the solution over the 114 normal modes. These records are followed by the natural frequency, generalized mass or inertia, and eigenfunction associated with each normal mode -- beginning with the mode with the lowest natural frequency and continuing (in ascending order) until all desired modes have been output. As a final check, the elements of the overall system stiffness and inertia matrices are output for comparison with their respective input values.

Since the eigenvalue solution forms (as will be shown in the sequel) the basis for the solution to the forced motion problem, it is of importance to determine the effect on the eiganvalue solution of breech displacement relative to the receiver (as experienced during the recoil/counter-recoil cycle). Consequently, employing the computer program "ADMAGEG", the natural frequencies and normal mode shapes of the 75mm ADMAG gun system mounted on the M240 artillery mount have been determined with the breech in its forward most (in-battery) position relative to the receiver, in its rearmost (sear) position relative to the receiver, and midway between the in-battery and sear positions. The importance of this study manifests itself in that if it is found that breech displacement relative to the receiver has a neglibible effect on the eigenvalue solution, then a single representative set of eigenvalues and eigenfunctions may be used throughout the forced motion solution. However, if it is found that the position of the breech relative to the receiver significantly affects the eigenvalue solution, then it would be necessary to generate several representative sets of eigenvalues and eigenfunctions for use, at different time intervals during the recoil/counter-recoil cycle, within the forced motion problem.

The first fifty modes of output associated with each of the above described eigenvalue problems have been obtained and compared. Based on this comparison it has been found that breech displacement relative to the receiver has a negligible effect on the eigenvalue solution. Hence, a single representative set of eigenvalues and eigenfunctions may be used within the forced motion problem throughout the entire recoil/counter-recoil cycle. The reader is referred to Reference 2 for documentation of this effort.

In addition to the above described study, Reference 2 also presents a study conducted for the purpose of determining the effect on the eigenvalue solution of restraining mount sidewall flexure in the neighborhood of the elevation link. Such flexure was evidenced during the Yuma Proving Ground firing tests, and is reflected by the value of k_{θ_z}, presented in Table 26. It was found that precluding mount sidewall flexure in the neighborhood of the elevation link inhibits the lowest normal mode which would otherwise be present. In fact, the lowest normal mode obtained precluding mount sidewall flexure coincides with the second normal mode obtained allowing for such flexure.

A variety of additional studies have been performed for the purpose of determining the effects on the eigenvalue solution of attaching weight to the muzzle of the gun tube (as for example, a laser mirror arrangement), adding ballast to the receiver, and introducing variations in the stiffness of the gun mount. The reader is referred to Reference 3 for discussion and documentation of these studies.

At the user's option, up to and including all 114 natural frequencies and corresponding normal mode shapes may be output from "ADMAGEG" for each eigenvalue problem considered. The number actually required in a given problem will of course depend upon the application. For example, considering the response of the gun system to a relatively low frequency forcing function acting at its base, as would be experienced if the gun were mounted on a slow moving vehicle with a relatively soft suspension system, it would be expected that the first few modes would suffice to describe the dynamic characteristics of the gun system. In contrast, considering the response of the gun system to the application of relatively high frequency forcing functions, as would be experienced during a firing, it would be expected that an accurate description of the dynamic characteristics of the gun system requires prescription of nearly half the number of available modes. In any event, it is noteworthy to bear in mind that as a general rule-of-thumb the accuracy with which a finite element (lumped parameter) model simulates the modal characteristics of a physical system diminishes as we exceed half the total number of degrees-of-freedom of the simulation model. Hence, for our case we can expect a reduction in the accuracy of the description of modes beyond the fifty-seventh.

For illustrative purposes, the solution obtained from "ADMAGEG" for the first ten natural frequencies and corresponding normal mode shapes of the 75mm ADMAG gun system mounted on the M240 artillery mount (with the gun in-battery, and allowing for mount sidewall flexure in the neighborhood of the elevation link, as previously described) is presented in Appendix E. As is seen from the data presented, the lowest natural frequency of the gun system is 11.77 cps; with a corresponding mode shape which predicts coupled axial deformation and in-plane bending. The second natural frequency is 16.11 cps; with a corresponding mode shape which predicts coupled torsional deformation and out-of-plane bending.

As a general observation, it is of interest to note that the normal mode shapes obtained predict coupled modes of deformation. Within a given mode, either axial deformation is coupled with in-plane bending, or torsional deformation is coupled with out-of-plane bending.

4. FORMULATION AND SOLUTION OF FORCED MOTION PROBLEM (DYNAMIC RESPONSE TO FIRING)

In accordance with the usual procedures of modal analysis, the solution to the forced motion problem (i.e., the response of the system depicted in Figure 2 to the applied and induced forces and moments due to single shot and burst mode firings) is obtained in terms of the eigenvalue solution by summing, with an appropriate time-dependent coefficient for each normal mode, the corresponding individual modal displacements and rotations (prescribed by the normal mode shapes) at each mass point, and superposing the rigid-body displacements of the breech relative to the receiver during the recoil/counter-recoil cycle.

Hence, the time-dependent response of the i^{th} mass point in the forced motion problem is assumed to be of the form

$$\left.\begin{aligned} x_i'(t) &= x_{o_i}'(t) + \sum_{k=1}^{N} q_k(t)\, x_i'^{(k)} & \Theta_{x_i}'(t) &= \sum_{k=1}^{N} q_k(t)\, \Theta_{x_i}'^{(k)} \\ y_i'(t) &= y_{o_i}'(t) + \sum_{k=1}^{N} q_k(t)\, y_i'^{(k)} & \Theta_{y_i}'(t) &= \sum_{k=1}^{N} q_k(t)\, \Theta_{y_i}'^{(k)} \\ z_i'(t) &= \sum_{k=1}^{N} q_k(t)\, z_i'^{(k)} & \Theta_{z_i}'(t) &= \sum_{k=1}^{N} q_k(t)\, \Theta_{z_i}'^{(k)} \end{aligned}\right\} \qquad (15)$$

where $x_i'(t)$, $y_i'(t)$, ..., $\Theta_{z_i}'(t)$ denote the (desired) time-dependent displacement and rotation components of the i^{th} mass point (i = 1 thru 19), along and about the respective axes of the earth-fixed reference frame, S'; $x_{o_i}'(t)$ and $y_{o_i}'(t)$ denote the (prescribed) time-dependent rigid-body displacement components of the i^{th} mass point (i = 8 thru 19) during the recoil/counter-recoil cycle, along the respective axes of S'; $x_i'^{(k)}$, $y_i'^{(k)}$, ..., $\Theta_{z_i}'^{(k)}$ denote the (known) time-independent displacement and rotation components of the i^{th} mass point (i = 1 thru 19) as prescribed by the k^{th} normal mode shape (k = 1 thru N); $q_k(t)$ denotes the (as yet unknown) time-dependent amplitude associated with the k^{th} normal mode; N denotes the number of normal modes considered ($1 \leq N \leq 114$).

Following the procedures of modal analysis, the quantity $q_k(t)$ is determined, for each of the N modes considered, from the second-order ordinary differential equation

$$\ddot{q}_k(t) + \omega_k^2\, q_k(t) = Q_k(t)/M_k \quad ; \quad k = 1, 2, \ldots, N \qquad (16)$$

where ω_k and M_k denote respectively the (known) natural frequency and generalized mass (or inertia) associated with the k^{th} normal mode, and $Q_k(t)$ denotes the (as yet unspecified) k^{th} mode generalized force (or moment).

The expression for the k^{th} mode generalized force (or moment) is given by

$$Q_k(t) = \sum_{i=1}^{19} X_i'(t)\, x_i'^{(k)} + \sum_{i=1}^{19} Y_i'(t)\, y_i'^{(k)} + \sum_{i=1}^{19} Z_i'(t)\, z_i'^{(k)} + \sum_{i=1}^{19} M_{x_i}'(t)\, \theta_{x_i}'^{(k)} + \sum_{i=1}^{19} M_{y_i}'(t)\, \theta_{y_i}'^{(k)} + \sum_{i=1}^{19} M_{z_i}'(t)\, \theta_{z_i}'^{(k)} \; ; \quad k = 1, 2, \ldots, N \tag{17}$$

where $X_i'(t)$, $Y_i'(t)$, ..., $M_{z_i}'(t)$ denote the (as yet unspecified) applied and induced time-dependent force and moment components (due to a firing) acting along and about the respective axes of S' at the i^{th} mass point (i = 1 thru 19), and $x_i'^{(k)}$, $y_i'^{(k)}$, ..., $\theta_{z_i}'^{(k)}$ are as previously defined.

4.1 Applied and Induced Loads and Moments During Firing

For the purposes of mathematical modeling, induced loads are herein defined as those loads arising from, and dependent upon, initial curvature of the gun tube, and dynamic elastic deformation of the gun system. Within the framework of the assumptions introduced, inertia loads associated with the rigid-body recoil/counter-recoil motion previously described, and the corresponding time-dependent loads generated within the recoil mechanism, are treated as applied loads.

The following assumptions are introduced in connection with both the applied and induced loads and moments during a firing:

(i) interior chamber pressure manifests itself as an applied load acting at the chamber centroid; moments arising as a consequence of breech and/or chamber eccentricities, and pressure variations within the chamber, are neglected;

(ii) at any instant during firing, the propellant gas pressure is assumed to vary linearly from the chamber to the base of the projectile;

(iii) subsequent to shot ejection, the interior pressure-time relationship at any point in the gun tube is prescribed by a simple exponential decay;

(iv) rigid-body recoil/counter-recoil acceleration (up to front bumper contact) is prescribed by the net unbalance between chamber pressure and recoil mechanism loads (specified by strip chart data supplied by ARES, Inc.) divided by the total mass of recoiling parts; subsequent to front bumper contact, the acceleration is derived numerically from breech strip chart displacement data;

(v) the effect of rigid-body displacement of recoiling parts relative to the receiver during recoil/counter-recoil motion on the natural frequencies and normal mode shapes of the system is negligible (as was discussed in Section 3); hence, the nominal eigenvalue solution presented in Section 3 prescribes the eigenvalues and eigenfunctions entering equations (15) thru (17);

(vi) for the purposes of determining projectile-induced loads acting on the gun tube, the projectile is treated as a point mass traveling along a deformable (elastic) beam element, with instantaneous displacement and velocity characteristics (relative to the gun tube axis) as prescribed by interior ballistics data;

(vii) induced loads dependent on projectile position within the gun tube manifest themselves as forces applied to the appropriate mass points; loads in this category include reaction forces generated against the projectile moving in a deformable gun tube, and loads arising from pressurization of the (curved) gun tube (i.e., the so-called "Bourdon" load); considering the relative action time of projectile passage, the induced moments generated as the projectile approaches, and passes, a given mass point, are considered sufficiently small to be neglected;

(viii) recoil mechanism loads are prescribed by the pressure-time history within the recoil accumulators; front bumper contact loads are prescribed by breech deceleration subsequent to front bumper contact; due to the physical configuration of the system, these loads are applied to mass points m_8 and m_9 within the breech, and to mass point m_3 within the receiver; the loads (and moments) acting at each of mass points m_8 and m_9 are attributable to the single accumulator on the corresponding side of the breech; the loads (and moments) acting at mass point m_3 are prescribed by the resultant force transmitted to the receiver through the recoil rods.

Applied Loads and Moments

Based on the assumptions introduced above, the applied loads and moments are as follows:

Interior Chamber Pressure Load -- under Assumption (i), the net force acting on the chamber (mass point m_{10}) at any instant, t, due to interior chamber pressure, and relative to the local coordinate system of the chamber, is given by

$$F_p(t) = - p_c(t)\, A_b \tag{18}$$

where $p_c(t)$ and A_b respectively denote the instantaneous chamber pressure and bore area.

Since the chamber is inclined to the earth-fixed reference frame S' by the elevation angle β_o, the loads applied to m_{10} (relative to S'), and due to propellant gas pressure, are

$$\left.\begin{aligned} X'_{10}(t)\Big|_p &= - p_c(t)\, A_b \cos\beta_o \\ Y'_{10}(t)\Big|_p &= - p_c(t)\, A_b \sin\beta_o \end{aligned}\right\} \tag{19}$$

Inertia Loads During Recoil/Counter-Recoil Motion -- rigid-body acceleration of the recoiling parts (mass points m_8 through m_{19} inclusive) relative to the earth-fixed reference frame, S', imposes inertia loads that must be reflected in the right-hand side of equation (17). Letting $a_r(t)$ denote the rigid-body acceleration of the recoiling mass points, at an inclination β_o to the earth-fixed coordinate system, the applied inertia loads (relative to S') are

$$\left.\begin{aligned} X'_i(t)\Big|_I &= - m_i\, a_r(t) \cos\beta_o \\ Y'_i(t)\Big|_I &= - m_i\, a_r(t) \sin\beta_o \end{aligned}\right\} \tag{20}$$

where i = 8 thru 19.

Under Assumption (iv), $a_r(t)$ is further specified as

$$a_r(t) = \begin{cases} \left[F_r(t) + F_p(t)\right]/M_r \; ; & 0 \le t < t_b \\ d^2x_r(t)/dt^2 \; ; & t \ge t_b \end{cases} \tag{21}$$

where $F_r(t)$ denotes the total load developed at any instant in the recoil accumulators, M_r denotes the total recoiling mass (i.e., $\sum_{i=8}^{19} m_i$), $x_r(t)$ denotes the instantaneous displacement of the breech relative to the receiver during the recoil/counter-recoil cycle (as prescribed by strip chart data), $F_p(t)$ is as defined in equation (18), and t_b denotes the time at which the breech impacts the front bumper.

Recoil/Counter-Recoil Mechanism Loads and Moments -- under Assumption (viii), and prior to front bumper contact, pressure is developed within the recoil accumulators in response to the build-up of propellant gas pressure and the motion of recoiling parts. Subsequent to front bumper contact, additional forces are generated which are coaxial with the recoil-rod axes. Hence, the net effective load acting on the breech at each instant during the recoil/counter-recoil cycle is expressed in terms of the forces located at, and directed along, each of the two recoil-rod axes. Corresponding reaction forces are transmitted through the recoil-rods to receiver mass point m_3.

Letting $F_8(t)$ denote the recoil accumulator load developed within the breech section represented by mass point m_8, and similarly, letting $F_9(t)$ denote the recoil accumulator load developed within the breech section represented by mass point m_9, the loads and moments acting at mass points m_8 and m_9, prior to front bumper contact, and relative to the earth-fixed reference frame, S', are for mass point m_8

$$\left.\begin{aligned} X_8'(t)\Big|_r &= F_8(t)\ \mathrm{Cos}\ \beta_o \\ Y_8'(t)\Big|_r &= F_8(t)\ \mathrm{Sin}\ \beta_o \\ M_{x_8}'(t)\Big|_r &= -\ \delta_z\ Y_8'(t)\Big|_r \\ M_{y_8}'(t)\Big|_r &= \delta_z\ X_8'(t)\Big|_r \\ M_{z_8}'(t)\Big|_r &= \delta_y\ F_8(t) \end{aligned}\right\} \qquad (22a)$$

and for mass point m_9

$$\left.\begin{aligned} X_9'(t)\Big|_r &= F_9(t)\ \mathrm{Cos}\ \beta_o \\ Y_9'(t)\Big|_r &= F_9(t)\ \mathrm{Sin}\ \beta_o \\ M_{x_9}'(t)\Big|_r &= \delta_z\ Y_9'(t)\Big|_r \\ M_{y_9}'(t)\Big|_r &= -\ \delta_z\ X_9'(t)\Big|_r \\ M_{z_9}'(t)\Big|_r &= -\ \delta_y\ F_9(t) \end{aligned}\right\} \qquad (22b)$$

where δ_y and δ_z denote respectively the vertical and lateral offsets of each recoil rod-axis from its associated breech mass point, as depicted in Figure 32.

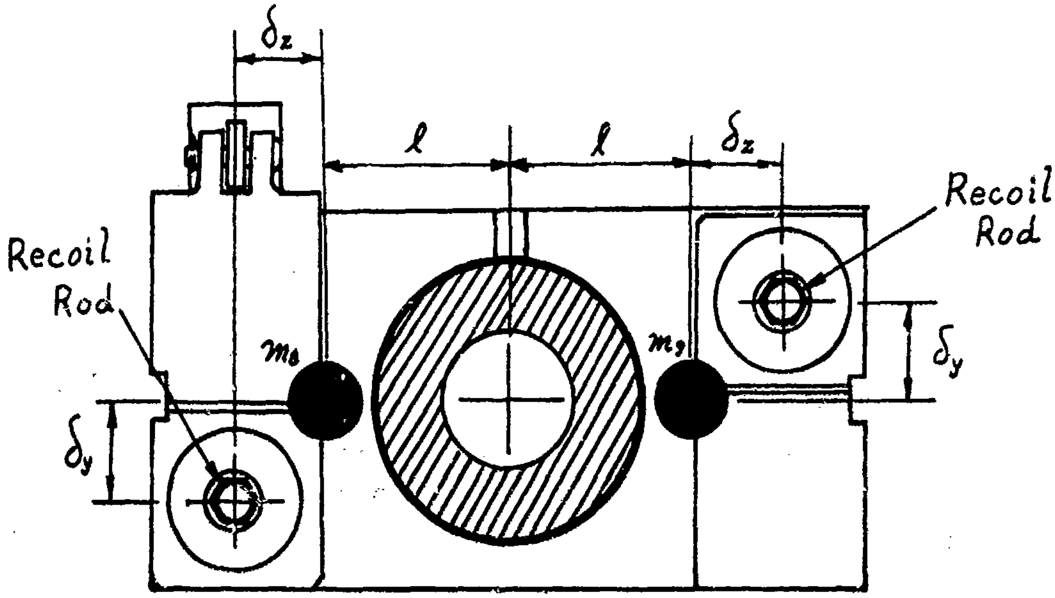


Figure 32 - Recoil Rod Offset

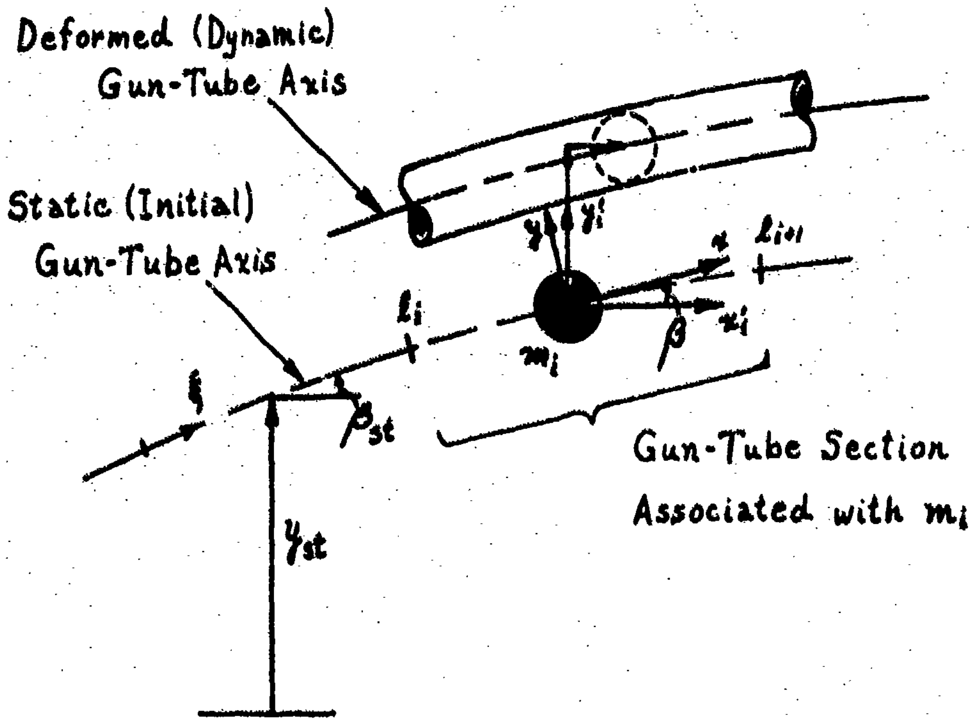


Figure 33 - Coordinate Relations for "Bourdon" and Projectile Induced Loads

The corresponding reaction loads and moments transmitted to receiver mass point m_3 are

$$\left.\begin{aligned}
X_3'(t)\Big|_r &= -\left[F_8(t) + F_9(t)\right] \cos\beta_o \\
Y_3'(t)\Big|_r &= -\left[F_8(t) + F_9(t)\right] \sin\beta_o \\
M_{x_3}'(t)\Big|_r &= -M_{x_8}' - M_{x_9}' + \left[F_8(t) - F_9(t)\right] \ell \sin\beta_o \\
M_{y_3}'(t)\Big|_r &= -M_{y_8}' - M_{y_9}' - \left[F_8(t) - F_9(t)\right] \ell \cos\beta_o \\
M_{z_3}'(t)\Big|_r &= -M_{z_8}' - M_{z_9}'
\end{aligned}\right\} \quad (23)$$

where ℓ denotes the horizontal distance depicted in Figure 32.

It is noted in connection with equations (22) and (23) that for the ideal case wherein the recoil accumulators develop equal pressures at each instant during the recoil/counter-recoil cycle, the breech mass points m_8 and m_9 are acted upon by equal loads, and equal but opposite moments; resulting in the transmission of a pure load to receiver mass point m_3.

The additional loads and moments generated at mass points m_8 and m_9 due to front bumper contact, and the additional reaction loads transmitted to receiver mass point m_3, are, in accordance with Assumption (viii), as defined above in equations (22) and (23) respectively, but with $F_8(t)$ and $F_9(t)$ each replaced by

$$F(t) = \tfrac{1}{2}\left[M_r \, d^2x_r(t)/dt^2 + F_p(t)\right] \qquad (24)$$

where M_r, $x_r(t)$ and $F_p(t)$ are as defined in equation (21).

It is noted that equation (24) prescribes an equal distribution of the front bumper contact load between each of the two recoil-rods. This prescription, which manifests itself via the transmission of a pure load to receiver mass point m_3, is predicated on the assumption of ideal breech/receiver alignment at the instant of front bumper contact. If it is found that this assumption is violated, then a reapportionment of this load would be necessary.

Induced Loads and Moments

Based on the assumptions introduced above, the induced loads and moments are as follows:

"Bourdon" Load -- the so-called "Bourdon" load (which is a misnomer as used here) is derived as a consequence of the internal pressurization of a curved gun tube. Within the framework of small displacement theory, the intensity (force per unit length) of this load, $f(\xi,t)$, which is directed along the instantaneous radius of curvature of the gun tube, is presented in Reference 4 in the form

$$f(\xi,t) = -p(\xi,t)\, A_b\, d^2u(\xi,t)/d\xi^2 \tag{25}$$

where ξ denotes longitudinal position measured along the initial orientation of the gun tube axis, A_b denotes the bore cross-sectional area, and $p(\xi,t)$ and $u(\xi,t)$ denote respectively the instantaneous internal pressure distribution and transverse displacement of the gun tube axis.

Under Assumption (ii) above, the internal pressure distribution, $p(\xi,t)$, is expressed as

$$p(\xi,t) = -A(t)\xi + B(t) \tag{26}$$

where,

$$\left.\begin{aligned} A(t) &= \left[p_c(t) - p_b(t)\right]/x_p(t) \\ B(t) &= p_c(t) \end{aligned}\right\} \tag{27}$$

and $p_c(t)$ and $p_b(t)$ denote respectively the instantaneous chamber and projectile-base pressures, and $x_p(t)$ denotes the instantaneous projectile location within the gun tube. Noting that for our purposes $p_b(t)$, $p_c(t)$ and $x_p(t)$ are specified by output from the BRL interior ballistics computer program, and that ξ is measured from the initial position of the projectile, equations (26) and (27) prescribe the desired internal pressure distribution.

To apply equation (25), consider the local coordinate system, S, associated with mass point m_i, as shown in Figure 2. Consider further the section of the gun tube associated with m_i (i.e., $\ell_i \leq \xi \leq \ell_{i+1}$) and subjected to an internal pressure distribution, as shown in Figure 33. Static and dynamic deformation of the gun tube will induce distributed loads in the y and z directions which are given, in accordance with equation (25), as

$$\left.\begin{aligned} f_y(\xi,t) &= -p(\xi,t)\, A_b\, d^2y/d\xi^2 \\ f_z(\xi,t) &= -p(\xi,t)\, A_b\, d^2z/d\xi^2 \end{aligned}\right\} \tag{28}$$

Substituting equations (26) and (27) into equation (28), and integrating along the length of the gun tube section under consideration, noting that for the purposes of integration and compatibility of equation (26) the origin of the local coordinate system associated with each mass point is translated to the initial projectile position, the "Bourdon" loads relative to S are

$$\left.\begin{aligned} Y_i(t) &= -A_b\left[p(\xi,t)\ dy/d\xi + A(t)\ y\right]_{\ell_i}^{\ell_{i+1}} \\ Z_i(t) &= -A_b\left[p(\xi,t)\ dz/d\xi + A(t)\ z\right]_{\ell_i}^{\ell_{i+1}} \end{aligned}\right\} \quad (29)$$

where $p(\xi,t)$ and $A(t)$ are as defined in equations (26) and (27).

The displacements y and z, and the slopes $dy/d\xi$ and $dz/d\xi$, entering equation (29) are relative to the local coordinate system, S. With reference to Figure 33, these quantities are defined in terms of displacements and rotations relative to the earth-fixed coordinate system, S', in accordance with the expressions

$$\left.\begin{aligned} y &= (y_{st} - x_i' \operatorname{Sin}\beta)/\operatorname{Cos}\beta + y_i' \operatorname{Cos}\beta - x_i' \operatorname{Sin}\beta \\ dy/d\xi &= \theta_{z_i}' + \beta_{st} - \beta \\ z &= z_i' \\ dz/d\xi &= \theta_{y_i}' \operatorname{Cos}\beta - \theta_{x_i}' \operatorname{Sin}\beta \end{aligned}\right\} \quad (30)$$

Resolution of the loads defined in equation (29) relative to S' prescribe the desired form of the "Bourdon" loads acting at mass point m_i

$$\left.\begin{aligned} X_i'(t)\Big|_b &= -Y_i(t) \operatorname{Sin}\beta \\ Y_i'(t)\Big|_b &= Y_i(t) \operatorname{Cos}\beta \\ Z_i'(t)\Big|_b &= Z_i(t) \end{aligned}\right\} \quad (31)$$

Projectile Load -- under Assumption (vi) above, it is shown in Reference 4 that the motion of the projectile (treated as a traveling point mass) induces a transverse load containing Coriolis, centrifugal and linear accelerations applied at its instantaneous location. If the projectile is at a location along the gun tube axis that is within the range associated with mass point m_i then, under Assumption (vii), the loads applied to m_i relative to the local coordinate system, S, are

$$\left.\begin{aligned} F_x &= -m_p\ g \operatorname{Sin}\beta \\ F_y &= -m_p\ (d^2y/dt^2 + 2\ V_p\ d^2y/dt\ dx + V_p^2\ d^2y/dx^2 + g \operatorname{Cos}\beta) \\ F_z &= -m_p\ (d^2z/dt^2 + 2\ V_p\ d^2z/dt\ dx + V_p^2\ d^2z/dx^2) \end{aligned}\right\} \quad (32)$$

where m_p, g and V_p denote respectively the projectile mass, gravitational acceleration and instantaneous projectile velocity.

Resolving the above loads into the earth-fixed reference frame, S', there results

$$\left.\begin{aligned} X_i'(t)\Big|_p &= m_p\,(d^2y/dt^2 + 2\,V_p\,d^2y/dt\,dx + V_p^2\,d^2y/dx^2)\,\mathrm{Sin}\,\beta \\ Y_i'(t)\Big|_p &= -\,m_p\left[(d^2y/dt^2 + 2\,V_p\,d^2y/dt\,dx + V_p^2\,d^2y/dx^2)\,\mathrm{Cos}\,\beta + g\right] \\ Z_i'(t)\Big|_p &= F_z \end{aligned}\right\} \quad (33)$$

4.2 Introduction of Damping

Up to this point we have not considered the effect of damping other than that which is inherent in the recoil mechanism, and which is reflected within the model via the prescribed rigid-body motion of recoiling parts. For our purposes, we adopt the generally accepted point of view that structural damping has little effect on the eigenvalue solution, and may be accounted for within the forced motion solution by adding the term $2\,\varepsilon_k\,\dot{q}_k(t)$ to the left-hand side of equation (16), where, for each normal mode, the scalar quantity ε_k denotes one-half the ratio of the system damping to inertia matrices. The quantity ε_k is related to the (conventional) logarithmic decrement of the motion, δ, via the relation

$$\varepsilon_k = \frac{\delta}{2\pi}\,\omega_k \quad (34)$$

Hence, equation (16) is replaced by its "damped" counterpart

$$\ddot{q}_k(t) + 2\,\varepsilon_k\,\dot{q}_k(t) + \omega_k^2\,q_k(t) = Q_k(t)/M_k \quad (35)$$

The reader is referred to References 5 and 6 for derivation, verification and documentation of the above.

Proceeding as indicated, a semi-empirical approach is adopted by requiring that the quantity δ be prescribed based on experimental evidence.

4.3 Computer Program

In order to solve equation (35) for the generalized coordinates, $q_k(t)$ $(k = 1, \ldots, N)$, and hence, specify the displacements, rotations and reaction forces of the system when subjected to a firing, the program "FORCFCN" was developed. The program consists of a main control program, two subroutines and three single-valued external functions.

The control program handles all input/output routines, overall assembly of the instantaneous applied and induced loads, formation of the corresponding generalized forces and moments, and finally, integration of the simultaneous equation set for the determination of the generalized coordinates, $q_k(t)$, and generalized velocities, $\dot{q}_k(t)$, via a fourth-order Runge-Kutta scheme. The two subroutines "BOURDON" and "PROJLD" are responsible for calculation of the instantaneous induced loads due to pressurization of the gun tube and projectile interaction, respectively. Communication with the main program (for overall assembly) is accomplished through COMMON blocks. The three external functions $\dot{x}_r(t)$, $F_r(t)$ and $a_r(t)$ return instantaneous rigid-body recoil displacement, total recoil mechanism load and rigid-body counter-recoil acceleration subsequent to front bumper contact, respectively. The functional representations of $x_r(t)$ and $a_r(t)$ were developed from piecewise polynominal least-square fits to the ARES, Inc. supplied experimental data. The functional representation of $F_r(t)$ was developed from the pressure-time history of the recoil mechanism, as supplied by ARES, Inc. In addition, the program recognizes the divergence of the physical gun mount design from its assumed ideal design (defined in Section 2.1) by computing and applying induced loads and moments to mass point m_2 to account for the disparity between left and right trunnion support stiffnesses, and by distributing the elevation link load (as a bearing load) between the left and right trunnion supports (in accordance with equilibrium requirements).

The necessary input data for execution of "FORCFCN" is entered through DATA statements in the body of the main program, and through an appended data file. The DATA statements specify the mass, static linear and angular orientation, and the boundaries of the domain for each of the recoiling mass points. In addition, the necessary geometry for calculation of applied and induced loads and moments, the average logarithmic damping decrement, as well as the necessary stiffnesses for the calculation of trunnion loads and moments, are supplied. The appended data file begins with one record containing integration and print control parameters, projectile mass, pressure decay factor, and the number and time interval of the interior ballistics tabular data. The interior ballistics data, adopted from BRL supplied computer output, follows with each record specifying chamber and projectile-base pressures, and projectile displacement and velocity for each unit time interval. The number of modes to be utilized by the program is specified next. The remaining portion of the data file is appended by merging with the appropriate part of the output file generated by the computer program "ADMAGEG" (described in Section 3). These records supply the specific required modal data beginning with the lowest mode.

The output file generated by "FORCFCN" consists of a tabular time-history of critical response parameters. In particular, muzzle displacements, rotations and linear and angular velocities and accelerations are output, as well as left and right horizontal, vertical and lateral trunnion loads, and the elevation link load. At the user's option, displacement, velocity, acceleration and load data could of course be output at any other point (or points) of interest along the gun system.

4.4 Solution for Nominal Case

Strip chart data for a "typical" 75mm APFSDS round (identified as Round No. 67) obtained from a ten-round sample of firings conducted at Yuma Proving Ground during June 1979, and corresponding interior ballistics data obtained as output from the BRL computer code, were used in conjunction with the first fifty modes obtained from the eigenvalue solution for the nominal case described in Section 3, and the experimentally determined, system average, logarithmic damping decrement $\delta = 0.239$, for the purpose of obtaining the solution to the forced motion problem described above. For illustrative purposes, the first fifty lines of computer output obtained for the trunnion and elevation link loads (in BRL gauge coordinates) and muzzle linear and angular displacements, velocities and accelerations are presented in Appendix F.

Since, in the final analysis, the validity of a theoretical model, such as the one herein developed, is established by demonstrating its ability to simulate reality, we reserve graphical presentation of the trunnion and elevation link loads, and muzzle motion parameters, for Section 5; wherein a detailed correlation of these data with experimental data is presented.

5. CORRELATION WITH EXPERIMENTAL DATA

Experimental data consisting of right and left horizontal, vertical and lateral trunnion gauge loads, elevation link loads and muzzle accelerations were recorded, digitized and presented graphically by BRL for a ten-round sample of 75mm APFSDS firings conducted at Yuma Proving Ground during June 1979. Muzzle accelerations were integrated to form muzzle velocities and displacements. For each recorded parameter BRL computed the mean and standard deviation across the population of recordings at each recording-time interval, and graphically presented the mean time-dependent response of each parameter superimposed on a bandwidth six standard deviations broad -- representing a spread of plus and minus three standard deviations from the mean. A detailed description of the experimental set-up, recording and data reduction procedures will be presented under separate cover by BRL.

To demonstrate the high degree of repeatibility and reliability of the load cell measurements, load cell data consisting of right and left horizontal, vertical and lateral trunnion gauge loads and the elevation link load are presented in Appendix G. Set A contains a description of these data during the first 1000 ms subsequent to firing. Set B presents an expanded view of each parameter during the first 200 ms. The solid curve represents the mean of the ten-round sample. The dotted curves represent plus and minus three standard deviations from the mean.

To access the validity of the simulation model herein developed, model output were generated for the right and left horizontal, vertical and lateral trunnion loads (in gauge coordinates) and the elevation link load, as described in Section 4.4. These data are presented, along with the corresponding experimental mean, in Figures 34 thru 47. Figures 34 thru 40, contained within Set A, present these data during the first 1000 ms subsequent to firing. Figures 41 thru 47, contained within Set B, present an expanded view of each parameter during the first 200 ms. The solid curve represents the simulation model prediction. The dotted curve represents the experimental mean of the ten-round sample. As is evidenced from the comparisons presented in these figures, and with the exception of lateral trunnion load data, excellent theoretical/experimental agreement has been achieved. Theoretical lateral trunnion load data differs from its experimental counterpart by a scale factor of 10^{-3} (a discrepancy which has not as yet been resolved).

Unfortunately, unlike the load cell data, muzzle accelerometer data are widely spread across the ten-round sample, leaving open the question of reliability of these measurements. In view of this situation, no attempt is made here to correlate model muzzle motion predictions with accelerometer data obtained from the Yuma firings. Such correlation has been reserved for future testing which is currently being supported by ARES, Inc., and which, since beyond the scope of this present contractual effort, will be reported upon under separate cover. Nevertheless, for the purpose of completeness, model output obtained for muzzle linear and angular displacements, velocities and accelerations are presented for the first 150 ms subsequent to firing in Appendix H.

Set A -- Figures 34 thru 40

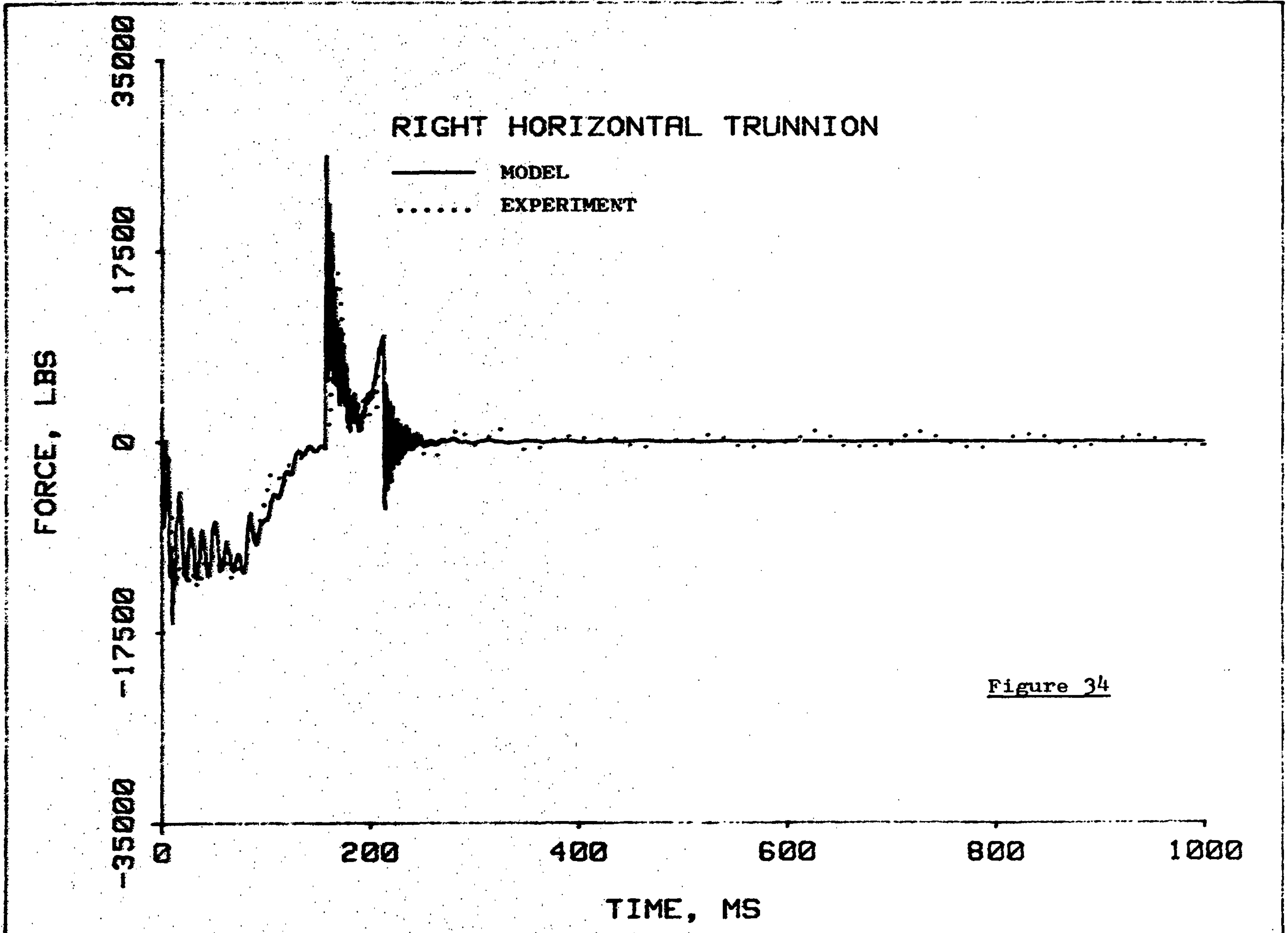
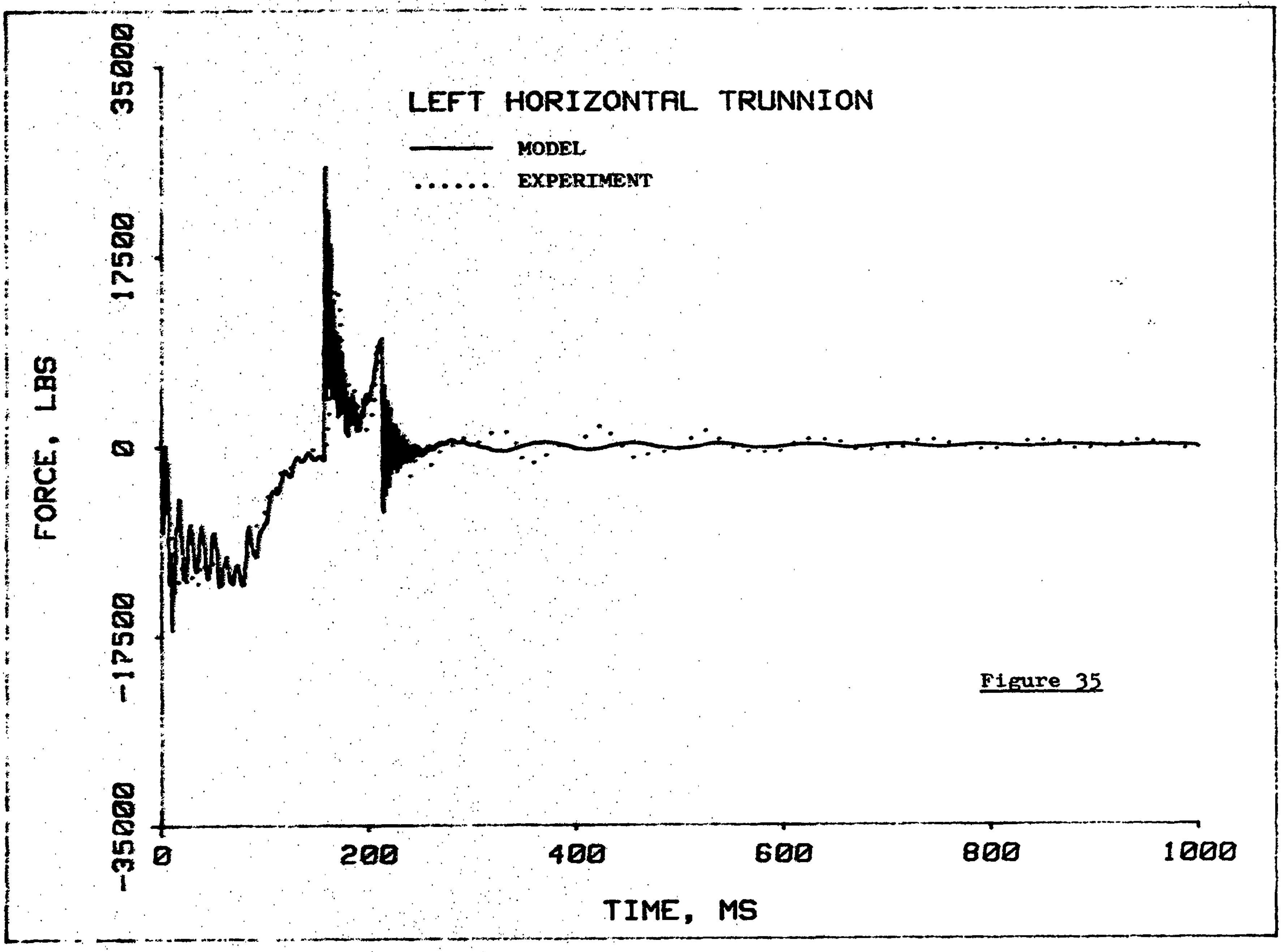
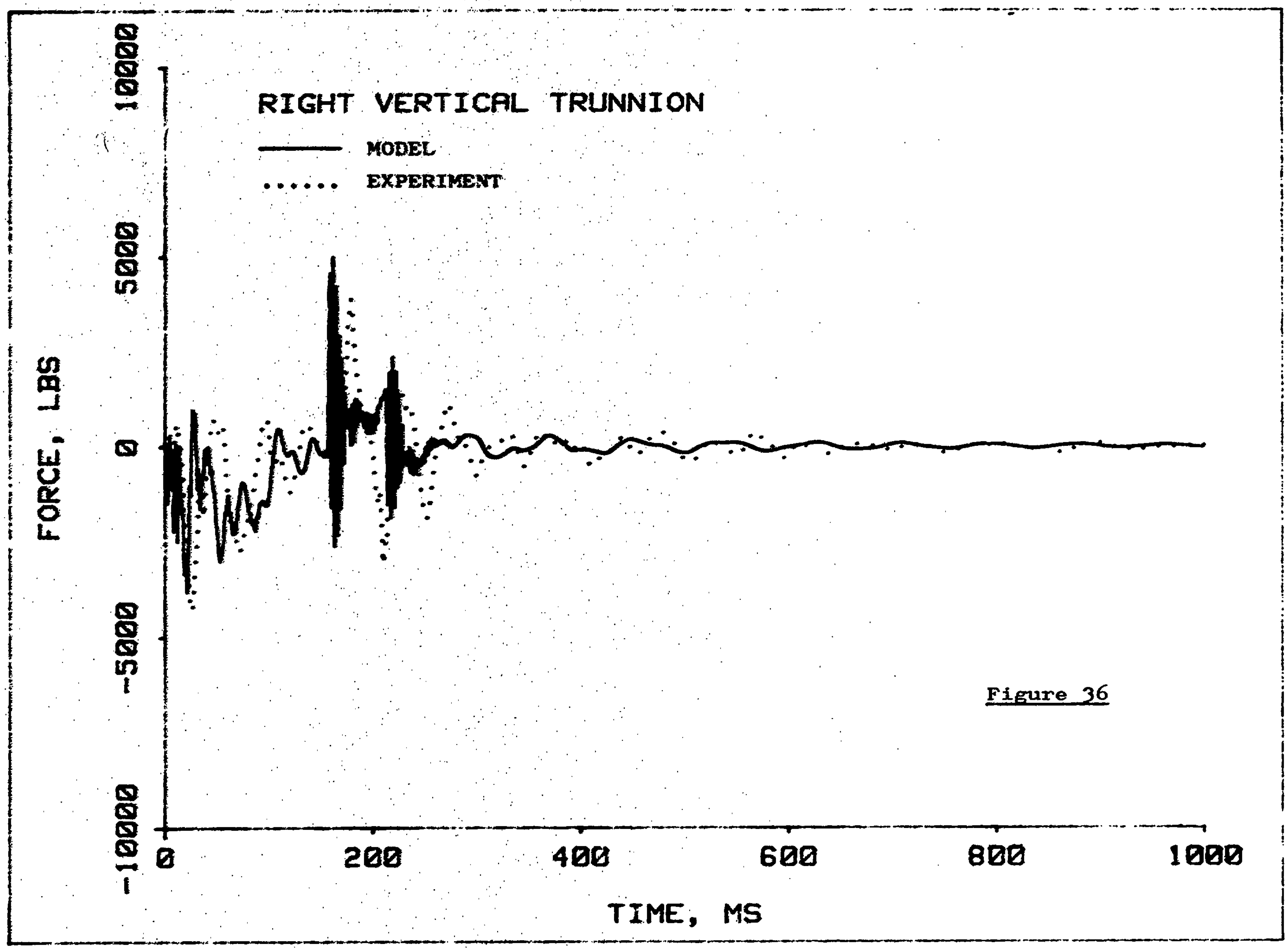

RIGHT HORIZONTAL TRUNNION
MODEL
EXPERIMENT
FORCE, LBS
35000
17500
0
-17500
-35000
0
200
400
600
800
1000
TIME, MS

Figure 34


LEFT HORIZONTAL TRUNNION
MODEL
EXPERIMENT
FORCE, LBS
35000
17500
0
-17500
-35000
0
200
400
600
800
1000
TIME, MS
Figure 35



RIGHT VERTICAL TRUNNION
MODEL
EXPERIMENT
FORCE, LBS
10000
5000
0
-5000
-10000
0
200
400
600
800
1000
TIME, MS

Figure 36

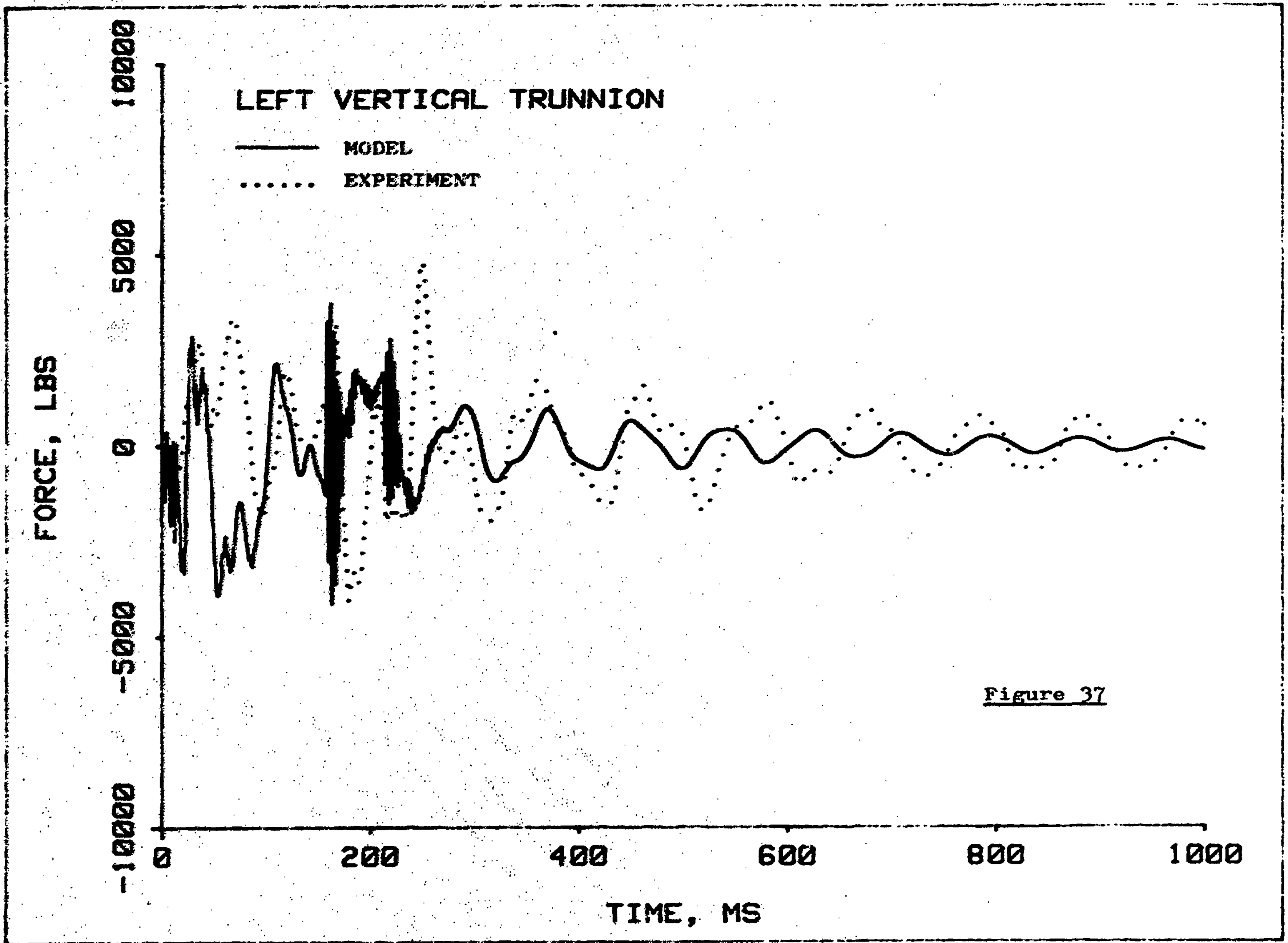

LEFT VERTICAL TRUNNION
MODEL
EXPERIMENT
FORCE, LBS
10000
5000
0
-5000
-10000
0
200
400
600
800
1000
TIME, MS

Figure 37

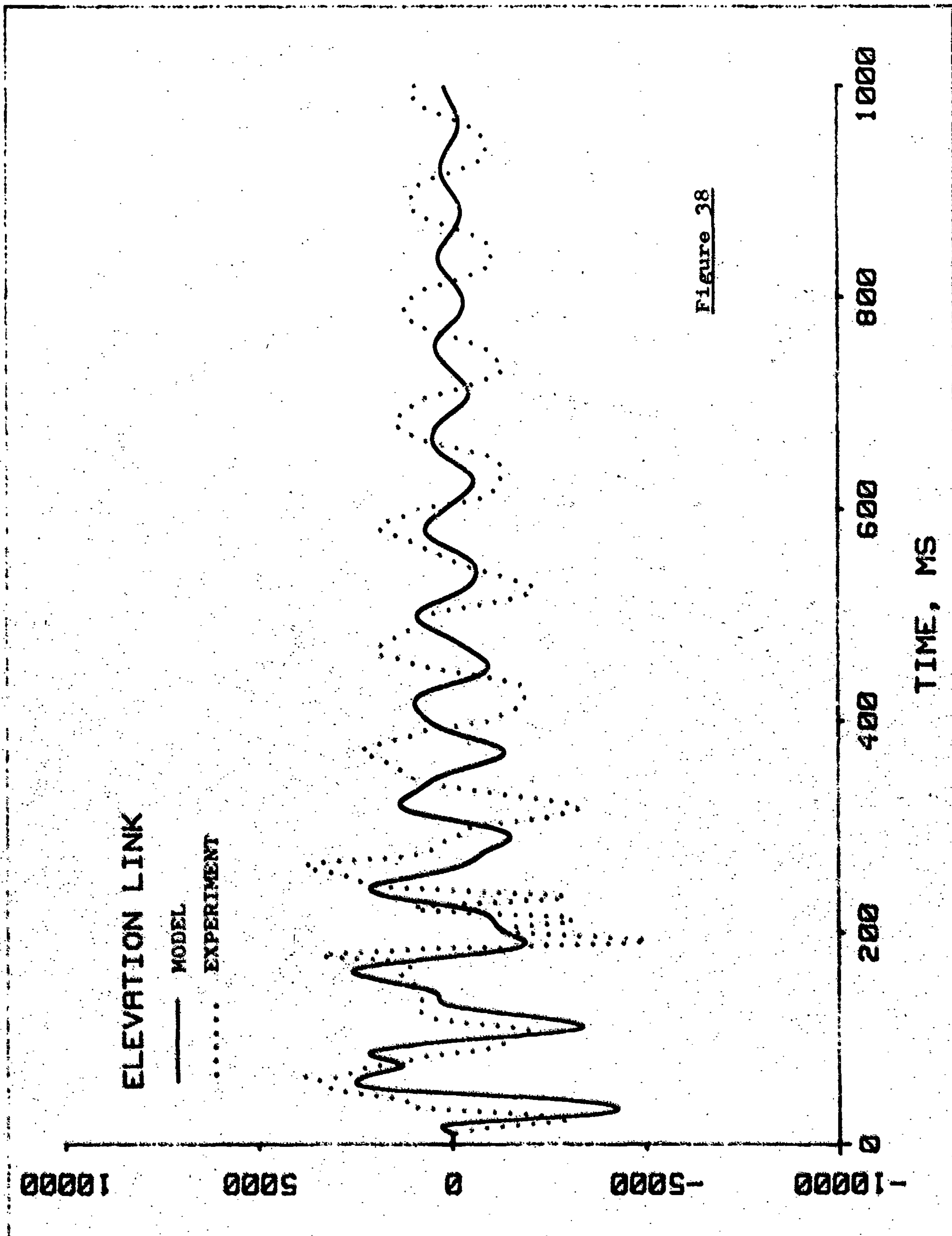
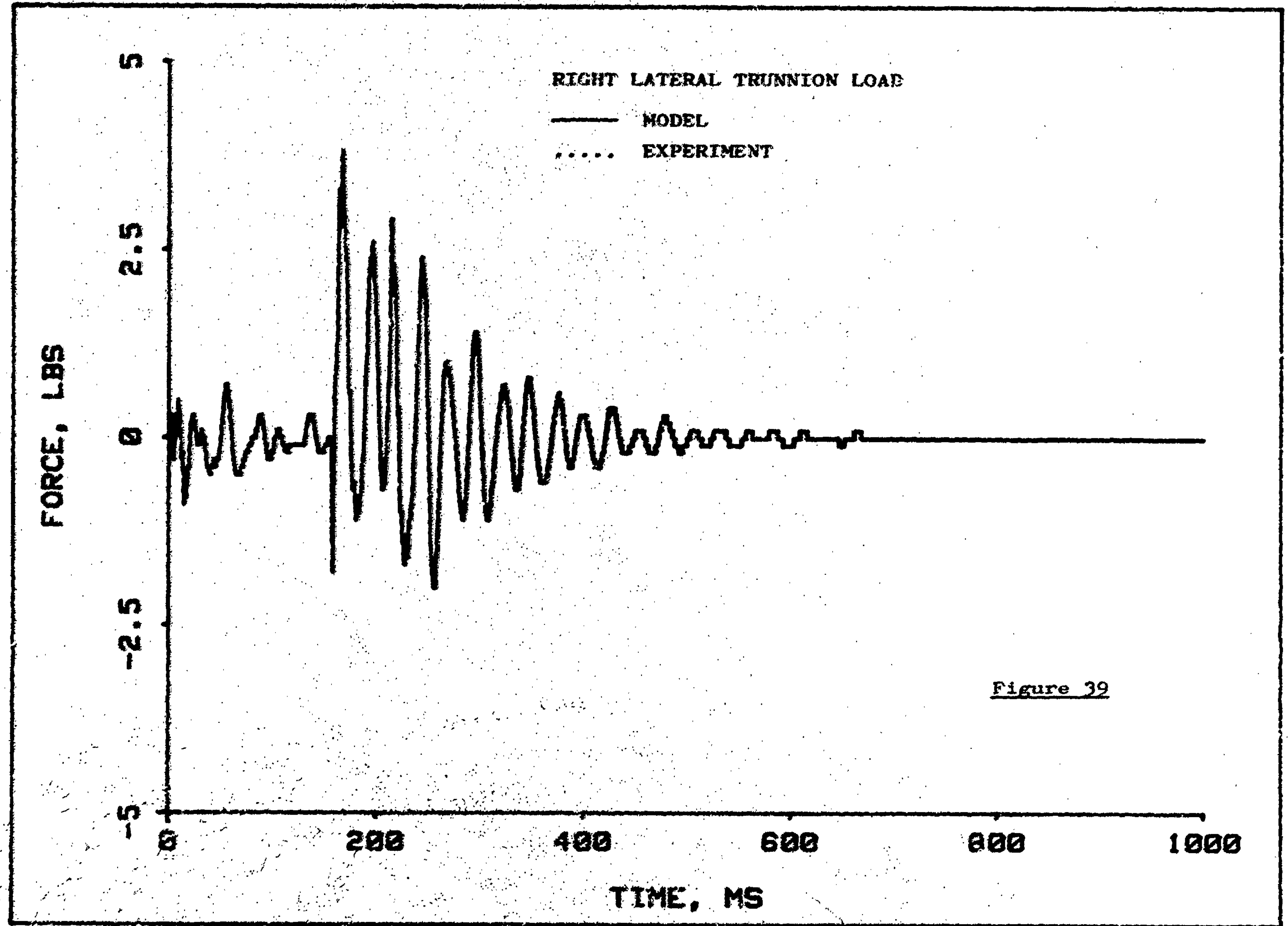

ELEVATION LINK
MODEL
EXPERIMENT
FORCE, LBS
10000
5000
0
-5000
-10000
TIME, MS
0
200
400
600
800
1000

Figure 38



RIGHT LATERAL TRUNNION LOAD
MODEL
EXPERIMENT
FORCE, LBS
5
2.5
0
-2.5
-5
0
200
400
600
800
1000
TIME, MS

Figure 39

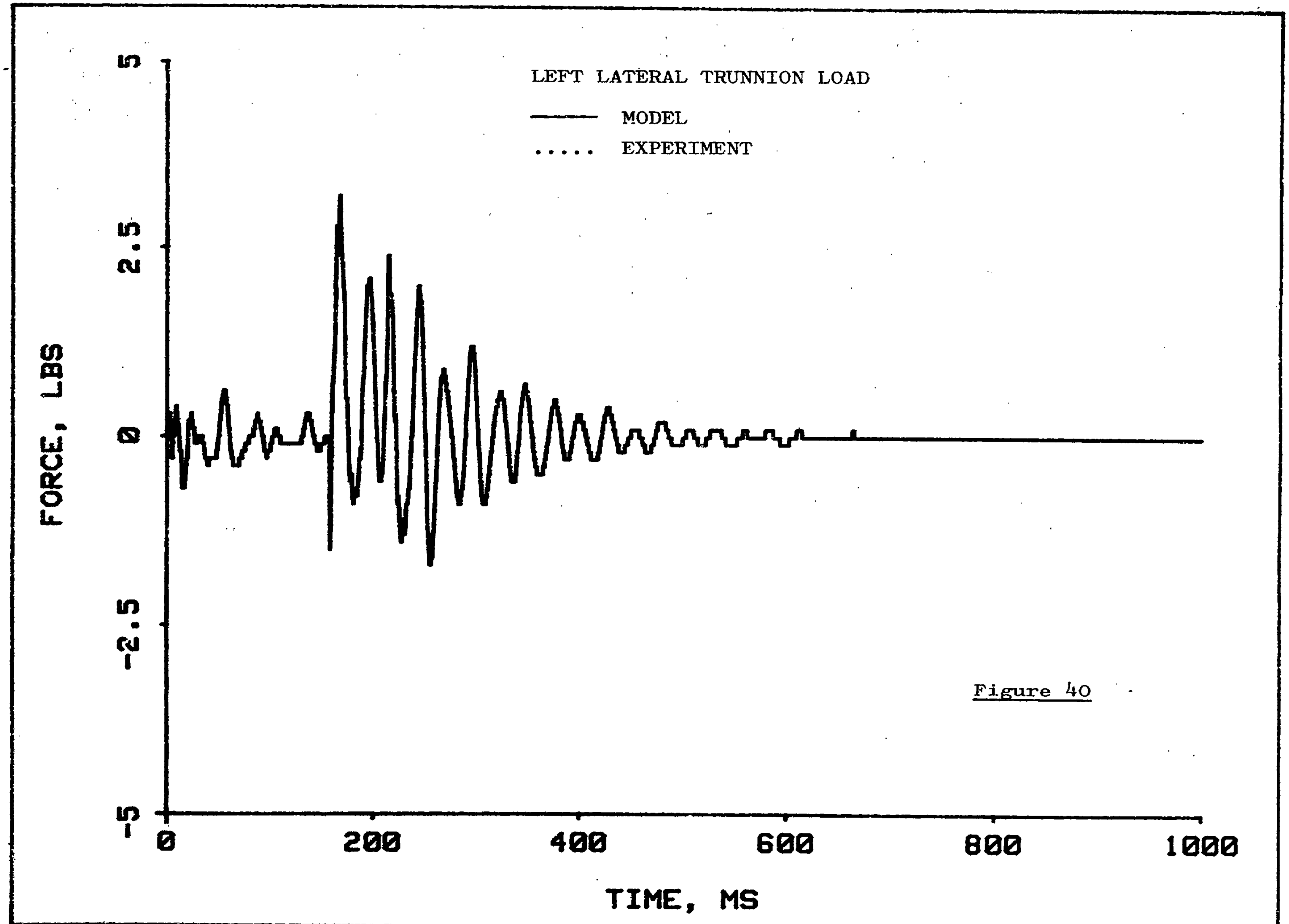

LEFT LATERAL TRUNNION LOAD
MODEL
EXPERIMENT
FORCE, LBS
5
2.5
0
-2.5
-5
0
200
400
600
800
1000
TIME, MS

Figure 40

Set B -- Figures 41 thru 47

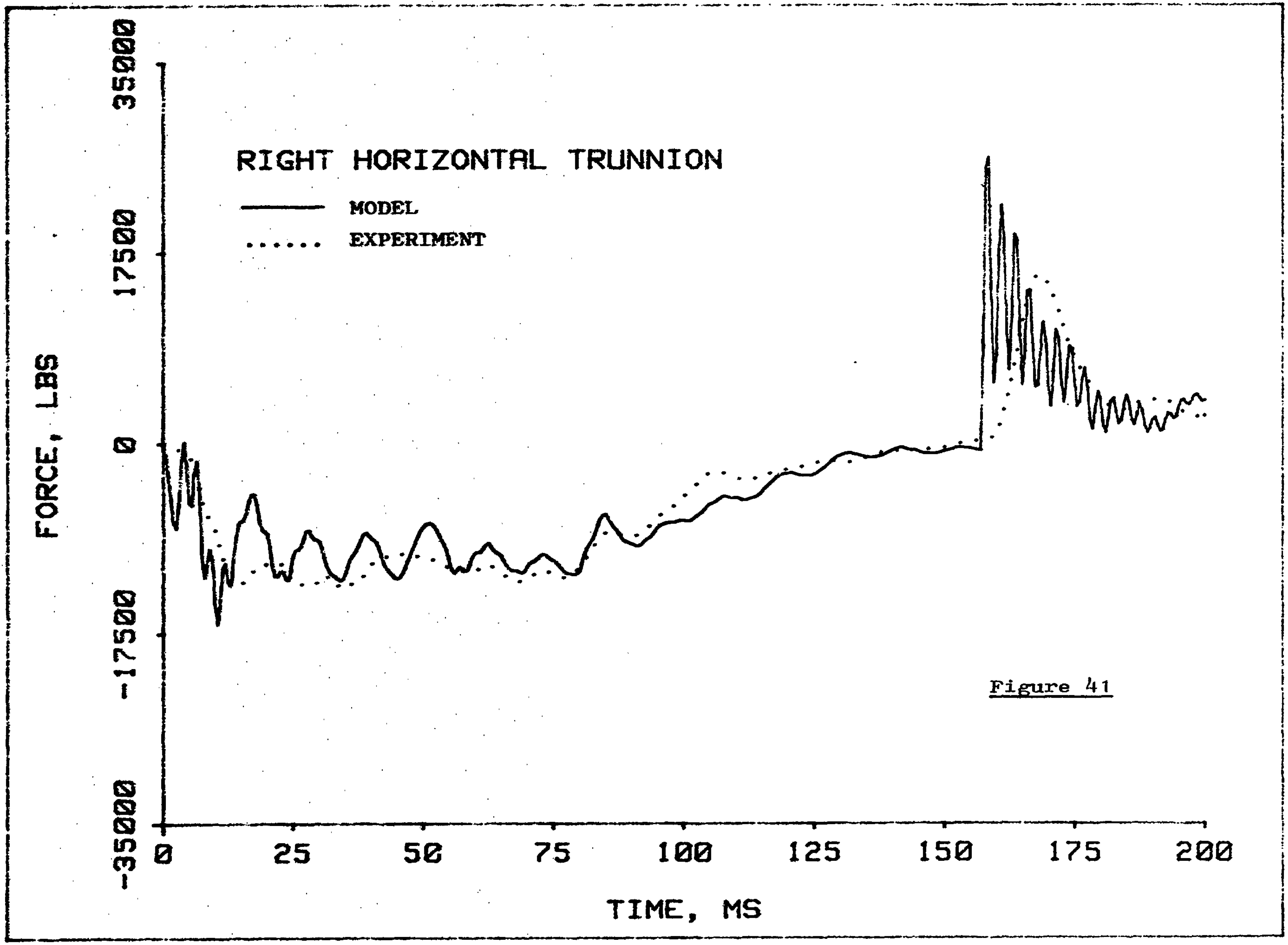
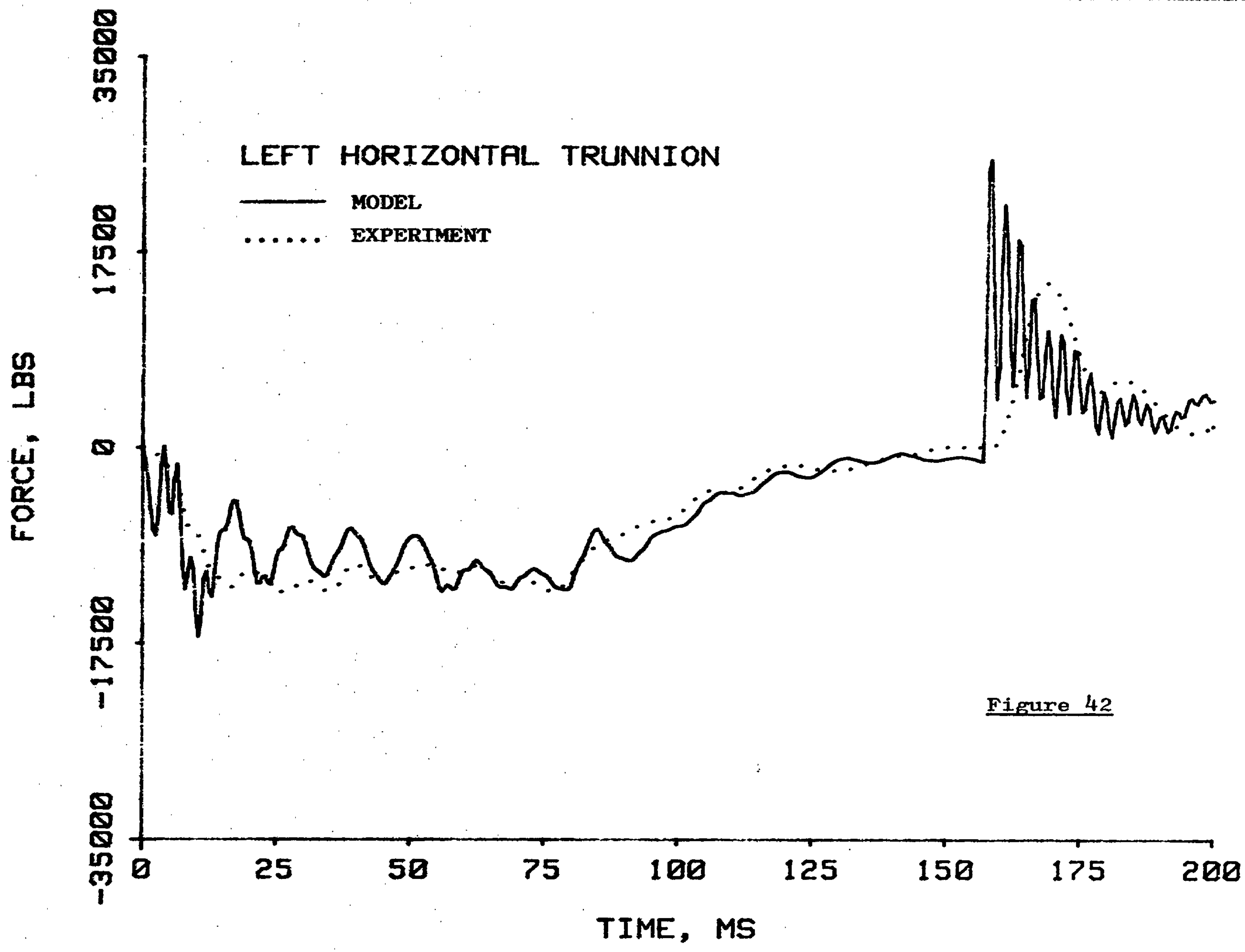

RIGHT HORIZONTAL TRUNNION
MODEL
EXPERIMENT
FORCE, LBS
35000
17500
0
-17500
-35000
0
25
50
75
100
125
150
175
200
TIME, MS

Figure 41



LEFT HORIZONTAL TRUNNION
MODEL
EXPERIMENT
FORCE, LBS
35000
17500
0
-17500
-35000
0
25
50
75
100
125
150
175
200
TIME, MS

Figure 42

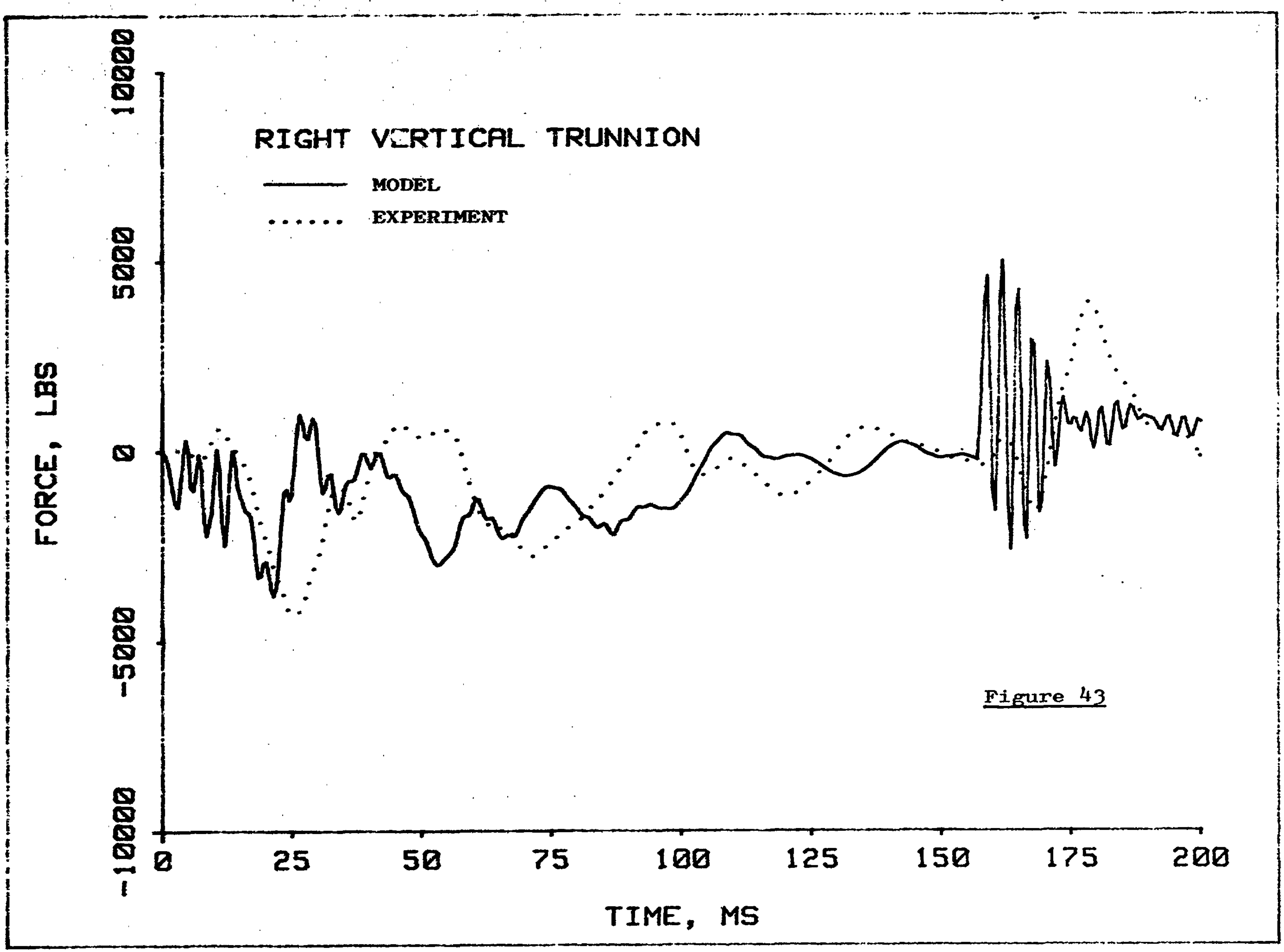

RIGHT V=RTICAL TRUNNION
MODEL
EXPERIMENT
FORCE, LBS
10000
5000
0
-5000
-10000
TIME, MS
0
25
50
75
100
125
150
175
200

Figure 43

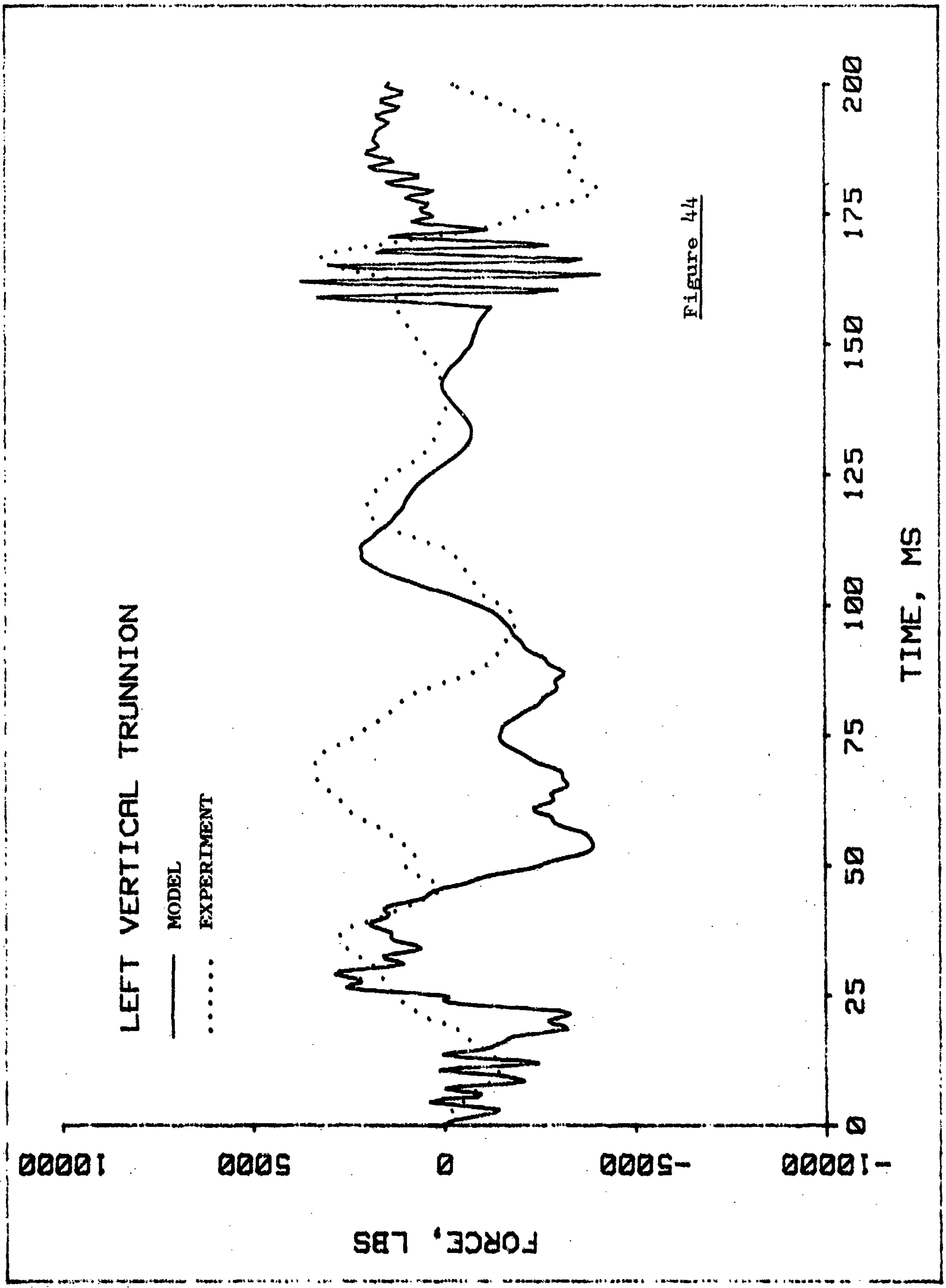

LEFT VERTICAL TRUNNION
— MODEL
······ EXPERIMENT
FORCE, LBS
10000
5000
0
-5000
-10000
TIME, MS
0
25
50
75
100
125
150
175
200

Figure 44

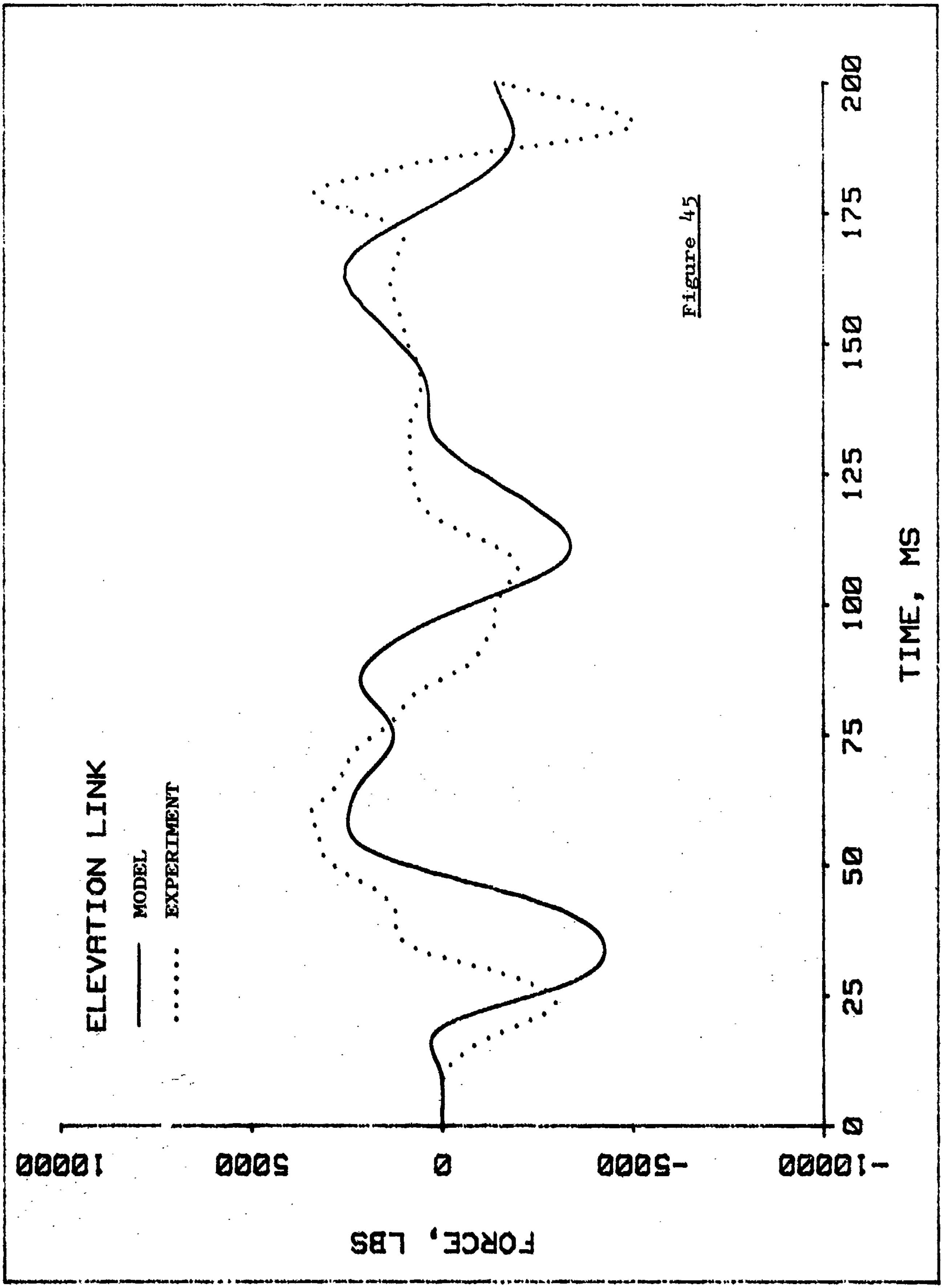

ELEVATION LINK
MODEL
EXPERIMENT
FORCE, LBS
10000
5000
0
-5000
-10000
TIME, MS
0
25
50
75
100
125
150
175
200

Figure 45

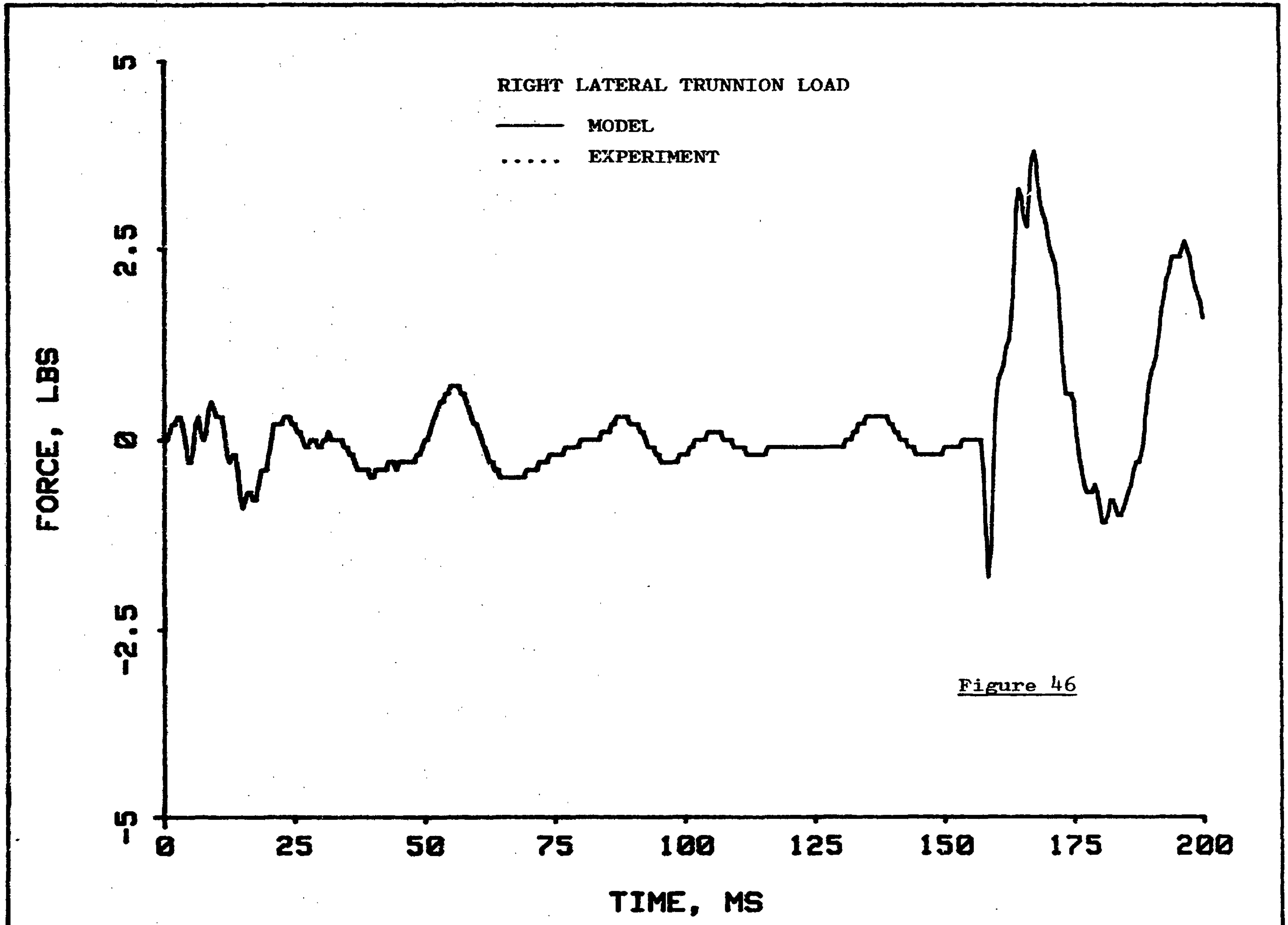
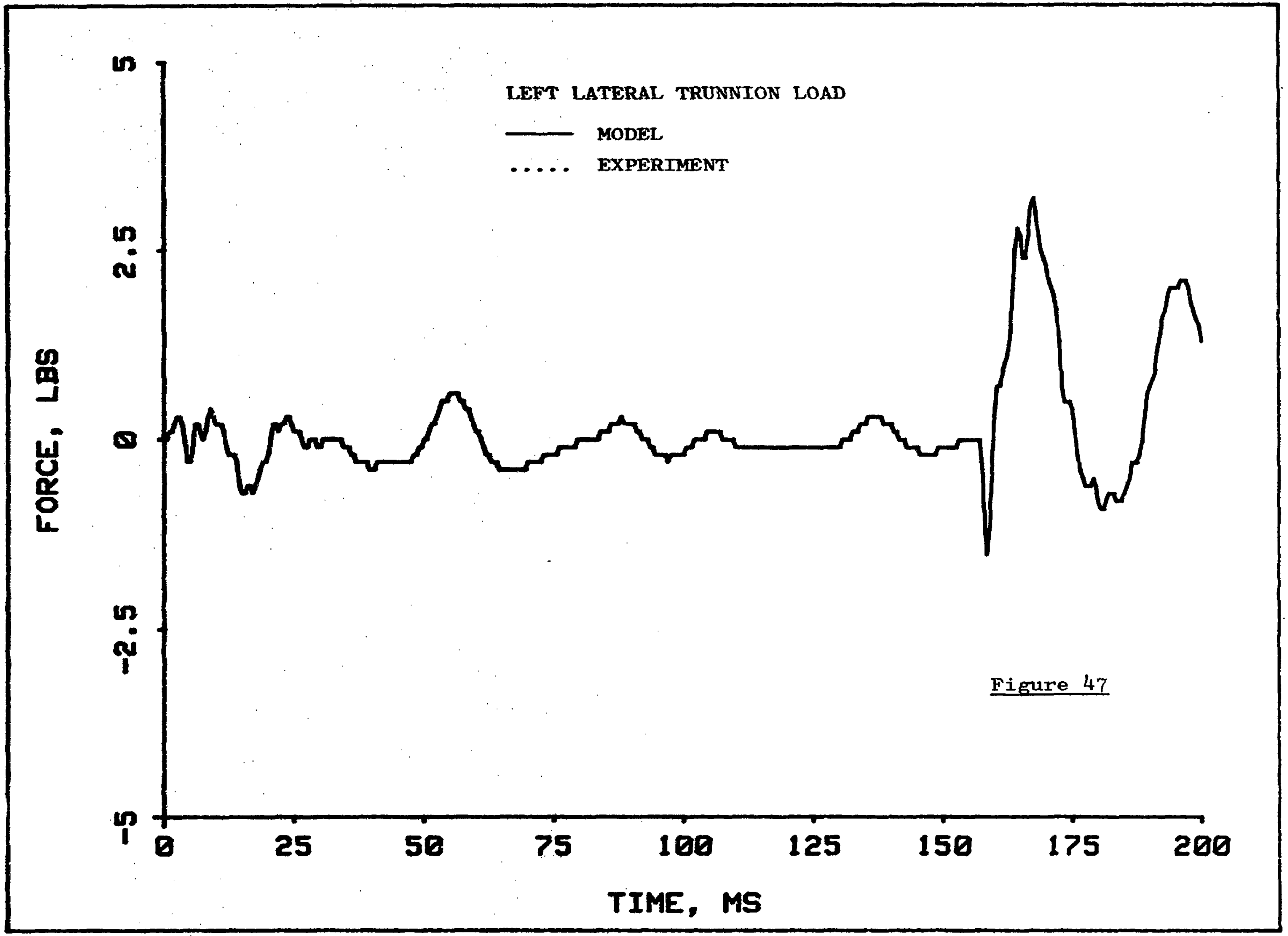

RIGHT LATERAL TRUNNION LOAD
MODEL
EXPERIMENT
FORCE, LBS
5
2.5
0
-2.5
-5
0
25
50
75
100
125
150
175
200
TIME, MS

Figure 46



LEFT LATERAL TRUNNION LOAD
MODEL
EXPERIMENT
FORCE, LBS
5
2.5
0
-2.5
-5
0
25
50
75
100
125
150
175
200
TIME, MS

Figure 47

6. CONCLUSIONS AND RECOMMENDATIONS

The following conclusions are based on the study herein conducted:

- A finite element (lumped parameter) analytical simulation model of the 75mm ADMAG gun system has been developed for the purpose of simulating the dynamic response characteristics of the physical gun system when subjected to single shot and burst mode firings.

- The simulation model has been correlated with experimental trunnion and elevation link load data obtained from a ten-round sample of 75mm APFSDS firings.

- For correlation purposes the stiffness characteristics of the M240 artillery mount have been introduced, however, the model is sufficiently general to accommodate introduction of any other gun mount configuration.

- The solution technique permits determination of the natural frequencies and normal mode shapes of the gun system, as well as the time-dependent response characteristics of each point along the gun system when subjected to a firing.

- Up to 114 natural frequencies and 114 corresponding normal mode shapes are available as output from the model. The normal mode shapes predict coupled modes of deformation. Within a given mode shape either axial deformation is coupled with in-plane bending, or torsional deformation is coupled with out-of-plane bending.

- Model output prescribes the lowest natural frequency of the system to be 11.77 cps (with a corresponding mode shape which predicts coupled axial deformation and in-plane bending). This is to be compared with an experimentally determined mean basic structural frequency of 10 cps prescribed by BRL, and 11.8 to 12.2 cps prescribed by ARES, Inc.

- Model output consisting of left and right horizontal, vertical and lateral trunnion loads and the elevation link load were compared with corresponding experimental data obtained from a ten-round sample of 75mm APFSDS firings conducted at Yuma Proving Ground during June 1979, as described in Section 5. As is evidenced from the comparisons presented in Section 5, excellent agreement has been achieved between model predictions and experimental data for the left and right horizontal and vertical trunnion loads and the elevation link load. Lateral experimental trunnion load data differs from theoretical predictions by a factor of 1000 (a discrepancy which has not as yet been resolved).

- Model output consisting of muzzle linear and angular displacements, velocities and accelerations remain uncorrelated with experimental data for the reasons cited in Section 5. However, noting that the model solution procedure requires first establishing a displacement and rotation field for each mass point of the system at each integration-time interval, and then uses the computed displacements and rotations at mass point m_2 to establish load data, lends, in view of the excellent theoretical/experimental load correlations achieved, credence to the model muzzle motion predictions presented in Appendix H.

In view of the potential of the simulation model herein developed to serve as an engineering tool for evaluating existing and proposed weapon system designs, as well as proposed design modifications and/or optimizations, the following recommendations are deemed appropriate:

- Implement the simulation model as presently configured for the 75mm ADMAG gun system for the purpose of isolating those design characteristics which have a dominant effect on gun tube muzzle motion, and in particular, the slope of the muzzle of the gun tube at shot ejection (which is a key parameter affecting weapon accuracy since it prescribes the initial conditions for the free-flight trajectory of the projectile). Recommended gun system design characteristics which should be examined include:

    - barrel guide location
    - support tube stiffness characteristics
    - elevation link offset
    - torsional coupling of support tube to gun tube
    - barrel spline design
    - barrel straightness

  In addition, examine the effect of the addition of muzzle weight and/or receiver ballast, as well as the effects of shot-to-shot variations in the interior ballistics and recoil mechanism cycles.

- Apply the simulation model to the 90mm gun system design for the purpose of predicting its dynamic response to a firing, and in particular, gun tube muzzle motion at shot ejection. Perform parametric studies relative to key gun system design parameters for the purpose of providing design guidance feedback.

- Modify the simulation model to include the effects of projectile spin, and apply the modified model to the 105mm gun system design (for which there exists an extremely broad and well documented experimental data base). Correlate model output with existing experimental data, and with output from other theoretical models.

ACKNOWLEDGEMENTS

The authors wish to express their sincere appreciation to Mr. Kenneth D. Rubin, ARRADCOM, Dover, N.J., and to Mr. James O. Pilcher, BRL, Aberdeen Proving Ground, MD, for their valuable technical input, critique and support during the course of this effort. The authors also wish to express their thanks to Mr. Pilcher for his assistance in generating static deflection test data of the gun system, and to Mr. Pilcher and his staff for providing the detailed firing test data needed to serve as a basis for model correlation. In addition, the authors would also like to express their thanks to Miss Caledonia L. Henry, BRL, for her assistance during the initial stages of computer program debugging and during the final stages of model correlation.

Last, but by no means least, the authors wish to express their appreciation to ARES, Inc., Port Clinton, Ohio, for their aid and cooperation in providing us with access to their facilities, 75mm ADMAG gun system and detailed gun system drawings and engineering data. Furthermore, additional appreciation is expressed to ARES, Inc. for their demonstrated interest in advancing the state-of-the-art in gun dynamics simulation modeling, and for providing support to extend the validation effort of the simulation model herein developed beyond the scope of this contractual effort.

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APPENDIX A

This appendix presents the elements of the 114 x 114 system stiffness matrix, relative to earth-fixed coordinates, in terms of the elements of the individual (sub-structure) stiffness matrices.

$$k'_{1\,1} = k'^{(1,2)}_{1\,1} + k'^{(1,8)}_{1\,1} + k'^{(1,9)}_{1\,1}$$

$$k'_{1\,2} = k'^{(1,2)}_{1\,2} + k'^{(1,8)}_{1\,2} + k'^{(1,9)}_{1\,2}$$

$$k'_{1\,j} = 0 \; ; \; j = 3, 4, 5$$

$$k'_{1\,6} = - k'^{(1,2)}_{1\,6} - k'^{(1,8)}_{1\,6} - k'^{(1,9)}_{1\,6}$$

$$k'_{1\,7} = - k'^{(1,2)}_{1\,1}$$

$$k'_{1\,8} = - k'^{(1,2)}_{1\,2}$$

$$k'_{1\,j} = 0 \; ; \; j = 9, 10, 11$$

$$k'_{1\,12} = - k'^{(1,2)}_{1\,6}$$

$$k'_{1\,j} = 0 \; ; \; j = 13, 14, 15, \ldots, 42$$

$$k'_{1\,43} = - k'^{(1,8)}_{1\,1}$$

$$k'_{1\,44} = - k'^{(1,8)}_{1\,2}$$

$$k'_{1\,j} = 0 \; ; \; j = 45, 46, 47$$

$$k'_{1\,48} = - k'^{(1,8)}_{1\,6}$$

$$k'_{1\,49} = - k'^{(1,9)}_{1\,1}$$

$$k'_{1\,50} = - k'^{(1,9)}_{1\,2}$$

$$k'_{1\,j} = 0 \; ; \; j = 51, 52, 53$$

$$k'_{1\,54} = - k'^{(1,9)}_{1\,6}$$

$$k'_{1\,j} = 0 \; ; \; j = 55, 56, 57, \ldots, 114$$

$$k'_{2\,2} = k'^{(1,2)}_{2\,2} + k'^{(1,8)}_{2\,2} + k'^{(1,9)}_{2\,2}$$

$$k'_{2\,j} = 0 \; ; \; j = 3,\ 4,\ 5$$

$$k'_{2\,6} = k'^{(1,2)}_{2\,6} + k'^{(1,8)}_{2\,6} + k'^{(1,9)}_{2\,6}$$

$$k'_{2\,7} = -\,k'^{(1,2)}_{1\,2}$$

$$k'_{2\,8} = -\,k'^{(1,2)}_{2\,2}$$

$$k'_{2\,j} = 0 \; ; \; j = 9,\ 10,\ 11$$

$$k'_{2\,12} = k'^{(1,2)}_{2\,6}$$

$$k'_{2\,j} = 0 \; ; \; j = 13,\ 14,\ 15,\ \ldots,\ 42$$

$$k'_{2\,43} = -\,k'^{(1,8)}_{1\,2}$$

$$k'_{2\,44} = -\,k'^{(1,8)}_{2\,2}$$

$$k'_{2\,j} = 0 \; ; \; j = 45,\ 46,\ 47$$

$$k'_{2\,48} = k'^{(1,8)}_{2\,6}$$

$$k'_{2\,49} = -\,k'^{(1,9)}_{1\,2}$$

$$k'_{2\,50} = -\,k'^{(1,9)}_{2\,2}$$

$$k'_{2\,j} = 0 \; ; \; j = 51,\ 52,\ 53$$

$$k'_{2\,54} = k'^{(1,9)}_{2\,6}$$

$$k'_{2\,j} = 0 \; ; \; j = 55,\ 56,\ 57,\ \ldots,\ 114$$

$$k'_{3\,3} = k'^{(1,2)}_{3\,3} + k'^{(1,8)}_{3\,3} + k'^{(1,9)}_{3\,3}$$

$$k'_{3\,4} = k'^{(1,2)}_{3\,4} + k'^{(1,8)}_{3\,4} + k'^{(1,9)}_{3\,4}$$

$$k'_{3\,5} = - k'^{(1,2)}_{3\,5} - k'^{(1,8)}_{3\,5} - k'^{(1,9)}_{3\,5}$$

$$k'_{3\,j} = 0 \; ; \; j = 6,\ 7,\ 8$$

$$k'_{3\,9} = - k'^{(1,2)}_{3\,3}$$

$$k'_{3\,10} = k'^{(1,2)}_{3\,4}$$

$$k'_{3\,11} = - k'^{(1,2)}_{3\,5}$$

$$k'_{3\,j} = 0 \; ; \; j = 12,\ 13,\ 14,\ \ldots,\ 44$$

$$k'_{3\,45} = - k'^{(1,8)}_{3\,3}$$

$$k'_{3\,46} = k'^{(1,8)}_{3\,4}$$

$$k'_{3\,47} = - k'^{(1,8)}_{3\,5}$$

$$k'_{3\,j} = 0 \; ; \; j = 48,\ 49,\ 50$$

$$k'_{3\,51} = - k'^{(1,9)}_{3\,3}$$

$$k'_{3\,52} = k'^{(1,9)}_{3\,4}$$

$$k'_{3\,53} = - k'^{(1,9)}_{3\,5}$$

$$k'_{3\,j} = 0 \; ; \; j = 54,\ 55,\ 56,\ \ldots,\ 114$$

$$k'_{4\,4} = k'^{(1,2)}_{4\,4} + k'^{(1,8)}_{4\,4} + k'^{(1,9)}_{4\,4}$$

$$k'_{4\,5} = k'^{(1,2)}_{4\,5} + k'^{(1,8)}_{4\,5} + k'^{(1,9)}_{4\,5}$$

$$k'_{4\,j} = 0 \; ; \; j = 6,\ 7,\ 8$$

$$k'_{4\,9} = -\,k'^{(1,2)}_{3\,4}$$

$$k'_{4\,10} = k'^{(1,2)}_{4\,10}$$

$$k'_{4\,11} = k'^{(1,2)}_{4\,11}$$

$$k'_{4\,j} = 0 \; ; \; j = 12,\ 13,\ 14,\ \ldots,\ 44$$

$$k'_{4\,45} = -\,k'^{(1,8)}_{3\,4}$$

$$k'_{4\,46} = k'^{(1,8)}_{4\,10}$$

$$k'_{4\,47} = k'^{(1,8)}_{4\,11}$$

$$k'_{4\,j} = 0 \; ; \; j = 48,\ 49,\ 50$$

$$k'_{4\,51} = -\,k'^{(1,9)}_{3\,4}$$

$$k'_{4\,52} = k'^{(1,9)}_{4\,10}$$

$$k'_{4\,53} = k'^{(1,9)}_{4\,11}$$

$$k'_{4\,j} = 0 \; ; \; j = 54,\ 55,\ 56,\ \ldots,\ 114$$

$$k'_{5\,5} = k'^{(1,2)}_{5\,5} + k'^{(1,8)}_{5\,5} + k'^{(1,9)}_{5\,5}$$

$$k'_{5\,j} = 0\ ;\ j = 6,\ 7,\ 8$$

$$k'_{5\,9} = k'^{(1,2)}_{3\,5}$$

$$k'_{5\,10} = k'^{(1,2)}_{4\,11}$$

$$k'_{5\,11} = k'^{(1,2)}_{5\,11}$$

$$k'_{5\,j} = 0\ ;\ j = 12,\ 13,\ 14,\ \ldots,\ 44$$

$$k'_{5\,45} = k'^{(1,8)}_{3\,5}$$

$$k'_{5\,46} = k'^{(1,8)}_{4\,11}$$

$$k'_{5\,47} = k'^{(1,8)}_{5\,11}$$

$$k'_{5\,j} = 0\ ;\ j = 48,\ 49,\ 50$$

$$k'_{5\,51} = k'^{(1,9)}_{3\,5}$$

$$k'_{5\,52} = k'^{(1,9)}_{4\,11}$$

$$k'_{5\,53} = k'^{(1,9)}_{5\,11}$$

$$k'_{5\,j} = 0\ ;\ j = 54,\ 55,\ 56,\ \ldots,\ 114$$

$$k'_{6\,6} = k'^{(1,2)}_{6\,6} + k'^{(1,8)}_{6\,6} + k'^{(1,9)}_{6\,6}$$

$$k'_{6\,7} = k'^{(1,2)}_{1\,6}$$

$$k'_{6\,8} = -\,k'^{(1,2)}_{2\,6}$$

$$k'_{6\,j} = 0 \;;\; j = 9,\ 10,\ 11$$

$$k'_{6\,12} = k'^{(1,2)}_{6\,12}$$

$$k'_{6\,j} = 0 \;;\; j = 13,\ 14,\ 15,\ \ldots,\ 42$$

$$k'_{6\,43} = k'^{(1,8)}_{1\,6}$$

$$k'_{6\,44} = -\,k'^{(1,8)}_{2\,6}$$

$$k'_{6\,j} = 0 \;;\; j = 45,\ 46,\ 47$$

$$k'_{6\,48} = k'^{(1,8)}_{6\,12}$$

$$k'_{6\,49} = k'^{(1,9)}_{1\,6}$$

$$k'_{6\,50} = -\,k'^{(1,9)}_{2\,6}$$

$$k'_{6\,j} = 0 \;;\; j = 51,\ 52,\ 53$$

$$k'_{6\,54} = k'^{(1,9)}_{6\,12}$$

$$k'_{6\,j} = 0 \;;\; j = 55,\ 56,\ 57,\ \ldots,\ 114$$

$$k'_{7\ 7} = k'^{(1,2)}_{1\ 1} + k'^{(2,3)}_{1\ 1} + k'^{(2,8)}_{1\ 1} + k'^{(2,9)}_{1\ 1} + k^{(2,g)}_{x'}$$

$$k'_{7\ 8} = k'^{(1,2)}_{1\ 2} + k'^{(2,3)}_{1\ 2} + k'^{(2,8)}_{1\ 2} + k'^{(2,9)}_{1\ 2}$$

$$k'_{7\ j} = 0\ ;\ j = 9,\ 10,\ 11$$

$$k'_{7\ 12} = k'^{(1,2)}_{1\ 6} - k'^{(2,3)}_{1\ 6} + k'^{(2,8)}_{1\ 6} + k'^{(2,9)}_{1\ 6}$$

$$k'_{7\ 13} = -\ k'^{(2,3)}_{1\ 1}$$

$$k'_{7\ 14} = -\ k'^{(2,3)}_{1\ 2}$$

$$k'_{7\ j} = 0\ ;\ j = 15,\ 16,\ 17$$

$$k'_{7\ 18} = -\ k'^{(2,3)}_{1\ 6}$$

$$k'_{7\ j} = 0\ ;\ j = 19,\ 20,\ 21,\ \ldots,\ 42$$

$$k'_{7\ 43} = -\ k'^{(2,8)}_{1\ 1}$$

$$k'_{7\ 44} = -\ k'^{(2,8)}_{1\ 2}$$

$$k'_{7\ j} = 0\ ;\ j = 45,\ 46,\ 47$$

$$k'_{7\ 48} = k'^{(2,8)}_{1\ 6}$$

$$k'_{7\ 49} = -\ k'^{(2,9)}_{1\ 1}$$

$$k'_{7\ 50} = -\ k'^{(2,9)}_{1\ 2}$$

$$k'_{7\ j} = 0\ ;\ j = 51,\ 52,\ 53$$

$$k'_{7\ 54} = k'^{(2,9)}_{1\ 6}$$

$$k'_{7\ j} = 0\ ;\ j = 55,\ 56,\ 57,\ \ldots,\ 114$$

$$k'_{8\,8} = k'^{(1,2)}_{2\,2} + k'^{(2,3)}_{2\,2} + k'^{(2,8)}_{2\,2} + k'^{(2,9)}_{2\,2} + k^{(2,g)}_{y'}$$

$$k'_{8\,j} = 0 \;;\; j = 9,\ 10,\ 11$$

$$k'_{8\,12} = -\,k'^{(1,2)}_{2\,6} + k'^{(2,3)}_{2\,6} - k'^{(2,8)}_{2\,6} - k'^{(2,9)}_{2\,6}$$

$$k'_{8\,13} = -\,k'^{(2,3)}_{1\,2}$$

$$k'_{8\,14} = -\,k'^{(2,3)}_{2\,2}$$

$$k'_{8\,j} = 0 \;;\; j = 15,\ 16,\ 17$$

$$k'_{8\,18} = k'^{(2,3)}_{2\,6}$$

$$k'_{8\,j} = 0 \;;\; j = 19,\ 20,\ 21,\ \ldots,\ 42$$

$$k'_{8\,43} = -\,k'^{(2,8)}_{1\,2}$$

$$k'_{8\,44} = -\,k'^{(2,8)}_{2\,2}$$

$$k'_{8\,j} = 0 \;;\; j = 45,\ 46,\ 47$$

$$k'_{8\,48} = -\,k'^{(2,8)}_{2\,6}$$

$$k'_{8\,49} = -\,k'^{(2,9)}_{1\,2}$$

$$k'_{8\,50} = -\,k'^{(2,9)}_{2\,2}$$

$$k'_{8\,j} = 0 \;;\; j = 51,\ 52,\ 53$$

$$k'_{8\,54} = -\,k'^{(2,9)}_{2\,6}$$

$$k'_{8\,j} = 0 \;;\; j = 55,\ 56,\ 57,\ \ldots,\ 114$$

$$k'_{9\,9} = k'^{(1,2)}_{3\,3} + k'^{(2,3)}_{3\,3} + k'^{(2,8)}_{3\,3} + k'^{(2,9)}_{3\,3} + k^{(2,g)}_{z'}$$

$$k'_{9\,10} = - k'^{(1,2)}_{3\,4} + k'^{(2,3)}_{3\,4} - k'^{(2,8)}_{3\,4} - k'^{(2,9)}_{3\,4}$$

$$k'_{9\,11} = k'^{(1,2)}_{3\,5} - k'^{(2,3)}_{3\,5} + k'^{(2,8)}_{3\,5} + k'^{(2,9)}_{3\,5}$$

$$k'_{9\,j} = 0 \; ; \; j = 12,\ 13,\ 14$$

$$k'_{9\,15} = - k'^{(2,3)}_{3\,3}$$

$$k'_{9\,16} = k'^{(2,3)}_{3\,4}$$

$$k'_{9\,17} = - k'^{(2,3)}_{3\,5}$$

$$k'_{9\,j} = 0 \; ; \; j = 18,\ 19,\ 20,\ \ldots,\ 44$$

$$k'_{9\,45} = - k'^{(2,8)}_{3\,3}$$

$$k'_{9\,46} = - k'^{(2,8)}_{3\,4}$$

$$k'_{9\,47} = k'^{(2,8)}_{3\,5}$$

$$k'_{9\,j} = 0 \; ; \; j = 48,\ 49,\ 50$$

$$k'_{9\,51} = - k'^{(2,9)}_{3\,3}$$

$$k'_{9\,52} = - k'^{(2,9)}_{3\,4}$$

$$k'_{9\,53} = k'^{(2,9)}_{3\,5}$$

$$k'_{9\,j} = 0 \; ; \; j = 54,\ 55,\ 56,\ \ldots,\ 114$$

$$k'_{10\,10} = k'^{(1,2)}_{4\,4} + k'^{(2,3)}_{4\,4} + k'^{(2,8)}_{4\,4} + k'^{(2,9)}_{4\,4} + k^{(2,g)}_{\theta_x}{}'$$

$$k'_{10\,11} = k'^{(1,2)}_{4\,5} + k'^{(2,3)}_{4\,5} + k'^{(2,8)}_{4\,5} + k'^{(2,9)}_{4\,5}$$

$$k'_{10\,j} = 0 \; ; \; j = 12,\ 13,\ 14$$

$$k'_{10\,15} = -\,k'^{(2,3)}_{3\,4}$$

$$k'_{10\,16} = k'^{(2,3)}_{4\,10}$$

$$k'_{10\,17} = k'^{(2,3)}_{4\,11}$$

$$k'_{10\,j} = 0 \; ; \; j = 18,\ 19,\ 20,\ \ldots,\ 44$$

$$k'_{10\,45} = k'^{(2,8)}_{3\,4}$$

$$k'_{10\,46} = k'^{(2,8)}_{4\,10}$$

$$k'_{10\,47} = k'^{(2,8)}_{4\,11}$$

$$k'_{10\,j} = 0 \; ; \; j = 48,\ 49,\ 50$$

$$k'_{10\,51} = k'^{(2,9)}_{3\,4}$$

$$k'_{10\,52} = k'^{(2,9)}_{4\,10}$$

$$k'_{10\,53} = k'^{(2,9)}_{4\,11}$$

$$k'_{10\,j} = 0 \; ; \; j = 54,\ 55,\ 56,\ \ldots,\ 114$$

$$k'_{11\ 11} = k'^{(1,2)}_{5\ 5} + k'^{(2,3)}_{5\ 5} + k'^{(2,8)}_{5\ 5} + k'^{(2,9)}_{5\ 5} + k^{(2,g)}_{\theta_y}{}'$$

$$k'_{11\ j} = 0\ ;\ j = 12,\ 13,\ 14$$

$$k'_{11\ 15} = k'^{(2,3)}_{3\ 5}$$

$$k'_{11\ 16} = k'^{(2,3)}_{4\ 11}$$

$$k'_{11\ 17} = k'^{(2,3)}_{5\ 11}$$

$$k'_{11\ j} = 0\ ;\ j = 18,\ 19,\ 20,\ \ldots,\ 44$$

$$k'_{11\ 45} = -\ k'^{(2,8)}_{3\ 5}$$

$$k'_{11\ 46} = k'^{(2,8)}_{4\ 11}$$

$$k'_{11\ 47} = k'^{(2,8)}_{5\ 11}$$

$$k'_{11\ j} = 0\ ;\ j = 48,\ 49,\ 50$$

$$k'_{11\ 51} = -\ k'^{(2,9)}_{3\ 5}$$

$$k'_{11\ 52} = k'^{(2,9)}_{4\ 11}$$

$$k'_{11\ 53} = k'^{(2,9)}_{5\ 11}$$

$$k'_{11\ j} = 0\ ;\ j = 54,\ 55,\ 56,\ \ldots,\ 114$$

$$k'_{12\ 12} = k'^{(1,2)}_{6\ 6} + k'^{(2,3)}_{6\ 6} + k'^{(2,8)}_{6\ 6} + k'^{(2,9)}_{6\ 6} + k^{(2,g)}_{\theta_z}{}'$$

$$k'_{12\ 13} = k'^{(2,3)}_{1\ 6}$$

$$k'_{12\ 14} = -\ k'^{(2,3)}_{2\ 6}$$

$$k'_{12\ j} = 0\ ;\ j = 15,\ 16,\ 17$$

$$k'_{12\ 18} = k'^{(2,3)}_{6\ 12}$$

$$k'_{12\ j} = 0\ ;\ j = 19,\ 20,\ 21,\ \ldots,\ 42$$

$$k'_{12\ 43} = -\ k'^{(2,8)}_{1\ 6}$$

$$k'_{12\ 44} = k'^{(2,8)}_{2\ 6}$$

$$k'_{12\ j} = 0\ ;\ j = 45,\ 46,\ 47$$

$$k'_{12\ 48} = k'^{(2,8)}_{6\ 12}$$

$$k'_{12\ 49} = -\ k'^{(2,9)}_{1\ 6}$$

$$k'_{12\ 50} = k'^{(2,9)}_{2\ 6}$$

$$k'_{12\ j} = 0\ ;\ j = 51,\ 52,\ 53$$

$$k'_{12\ 54} = k'^{(2,9)}_{6\ 12}$$

$$k'_{12\ j} = 0\ ;\ j = 55,\ 56,\ 57,\ \ldots,\ 114$$

$$k'_{13\ 13} = k'^{(2,3)}_{1\ 1} + k'^{(3,4)}_{1\ 1} + k'^{(3,8)}_{1\ 1} + k'^{(3,9)}_{1\ 1}$$

$$k'_{13\ 14} = k'^{(2,3)}_{1\ 2} + k'^{(3,4)}_{1\ 2} + k'^{(3,8)}_{1\ 2} + k'^{(3,9)}_{1\ 2}$$

$$k'_{13\ j} = 0 \ ; \ j = 15,\ 16,\ 17$$

$$k'_{13\ 18} = k'^{(2,3)}_{1\ 6} - k'^{(3,4)}_{1\ 6}$$

$$k'_{13\ 19} = -\ k'^{(3,4)}_{1\ 1}$$

$$k'_{13\ 20} = -\ k'^{(3,4)}_{1\ 2}$$

$$k'_{13\ j} = 0 \ ; \ j = 21,\ 22,\ 23$$

$$k'_{13\ 24} = -\ k'^{(3,4)}_{1\ 6}$$

$$k'_{13\ j} = 0 \ ; \ j = 25,\ 26,\ 27,\ \ldots,\ 42$$

$$k'_{13\ 43} = -\ k'^{(3,8)}_{1\ 1}$$

$$k'_{13\ 44} = -\ k'^{(3,8)}_{1\ 2}$$

$$k'_{13\ j} = 0 \ ; \ j = 45,\ 46,\ 47,\ 48$$

$$k'_{13\ 49} = -\ k'^{(3,9)}_{1\ 1}$$

$$k'_{13\ 50} = -\ k'^{(3,9)}_{1\ 2}$$

$$k'_{13\ j} = 0 \ ; \ j = 51,\ 52,\ 53,\ \ldots,\ 114$$

$$k'_{14\;14} = k'^{(2,3)}_{2\;2} + k'^{(3,4)}_{2\;2} + k'^{(3,8)}_{2\;2} + k'^{(3,9)}_{2\;2}$$

$$k'_{14\;j} = 0 \;;\; j = 15,\ 16,\ 17$$

$$k'_{14\;18} = -\,k'^{(2,3)}_{2\;6} + k'^{(3,4)}_{2\;6}$$

$$k'_{14\;19} = -\,k'^{(3,4)}_{1\;2}$$

$$k'_{14\;20} = -\,k'^{(3,4)}_{2\;2}$$

$$k'_{14\;j} = 0 \;;\; j = 21,\ 22,\ 23$$

$$k'_{14\;24} = k'^{(3,4)}_{2\;6}$$

$$k'_{14\;j} = 0 \;;\; j = 25,\ 26,\ 27,\ \ldots,\ 42$$

$$k'_{14\;43} = -\,k'^{(3,8)}_{1\;2}$$

$$k'_{14\;44} = -\,k'^{(3,8)}_{2\;2}$$

$$k'_{14\;j} = 0 \;;\; j = 45,\ 46,\ 47,\ 48$$

$$k'_{14\;49} = -\,k'^{(3,9)}_{1\;2}$$

$$k'_{14\;50} = -\,k'^{(3,9)}_{2\;2}$$

$$k'_{14\;j} = 0 \;;\; j = 51,\ 52,\ 53,\ \ldots,\ 114$$

$$k'_{15\,15} = k'^{(2,3)}_{3\,3} + k'^{(3,4)}_{3\,3}$$

$$k'_{15\,16} = -\,k'^{(2,3)}_{3\,4} + k'^{(3,4)}_{3\,4}$$

$$k'_{15\,17} = k'^{(2,3)}_{3\,5} - k'^{(3,4)}_{3\,5}$$

$$k'_{15\,j} = 0\ ;\ j = 18,\ 19,\ 20$$

$$k'_{15\,21} = -\,k'^{(3,4)}_{3\,3}$$

$$k'_{15\,22} = k'^{(3,4)}_{3\,4}$$

$$k'_{15\,23} = -\,k'^{(3,4)}_{3\,5}$$

$$k'_{15\,j} = 0\ ;\ j = 24,\ 25,\ 26,\ \ldots,\ 114$$

$$k'_{16\,16} = k'^{(2,3)}_{4\,4} + k'^{(3,4)}_{4\,4}$$

$$k'_{16\,17} = k'^{(2,3)}_{4\,5} + k'^{(3,4)}_{4\,5}$$

$$k'_{16\,j} = 0\ ;\ j = 18,\ 19,\ 20$$

$$k'_{16\,21} = -\,k'^{(3,4)}_{3\,4}$$

$$k'_{16\,22} = k'^{(3,4)}_{4\,10}$$

$$k'_{16\,23} = k'^{(3,4)}_{4\,11}$$

$$k'_{16\,j} = 0\ ;\ j = 24,\ 25,\ 26,\ \ldots,\ 114$$

$$k'_{17\ 17} = k'^{(2,3)}_{5\ 5} + k'^{(3,4)}_{5\ 5}$$

$$k'_{17\ j} = 0 \ ; \ j = 18,\ 19,\ 20$$

$$k'_{17\ 21} = k'^{(3,4)}_{3\ 5}$$

$$k'_{17\ 22} = k'^{(3,4)}_{4\ 11}$$

$$k'_{17\ 23} = k'^{(3,4)}_{5\ 11}$$

$$k'_{17\ j} = 0 \ ; \ j = 24,\ 25,\ 26,\ \ldots,\ 114$$

$$k'_{18\ 18} = k'^{(2,3)}_{6\ 6} + k'^{(3,4)}_{6\ 6}$$

$$k'_{18\ 19} = k'^{(3,4)}_{1\ 6}$$

$$k'_{18\ 20} = -\ k'^{(3,4)}_{2\ 6}$$

$$k'_{18\ j} = 0 \ ; \ j = 21,\ 22,\ 23$$

$$k'_{18\ 24} = k'^{(3,4)}_{6\ 12}$$

$$k'_{18\ j} = 0 \ ; \ j = 25,\ 26,\ 27,\ \ldots,\ 114$$

$$k'_{19\,19} = k'^{(3,4)}_{1\,1} + k'^{(4,5)}_{1\,1}$$

$$k'_{19\,20} = k'^{(3,4)}_{1\,2} + k'^{(4,5)}_{1\,2}$$

$$k_{19\,j} = 0 \;;\; j = 21,\ 22,\ 23$$

$$k'_{19\,24} = k'^{(3,4)}_{1\,6} - k'^{(4,5)}_{1\,6}$$

$$k'_{19\,25} = -\,k'^{(4,5)}_{1\,1}$$

$$k'_{19\,26} = -\,k'^{(4,5)}_{1\,2}$$

$$k'_{19\,j} = 0 \;;\; j = 27,\ 28,\ 29$$

$$k'_{19\,30} = -\,k'^{(4,5)}_{1\,6}$$

$$k'_{19\,j} = 0 \;;\; j = 31,\ 32,\ 33,\ \ldots,\ 114$$

$$k'_{20\,20} = k'^{(3,4)}_{2\,2} + k'^{(4,5)}_{2\,2}$$

$$k'_{20\,j} = 0 \;;\; j = 21,\ 22,\ 23$$

$$k'_{20\,24} = -\,k'^{(3,4)}_{2\,6} + k'^{(4,5)}_{2\,6}$$

$$k'_{20\,25} = -\,k'^{(4,5)}_{1\,2}$$

$$k'_{20\,26} = -\,k'^{(4,5)}_{2\,2}$$

$$k'_{20\,j} = 0 \;;\; j = 27,\ 28,\ 29$$

$$k'_{20\,30} = k'^{(4,5)}_{2\,6}$$

$$k'_{20\,j} = 0 \;;\; j = 31,\ 32,\ 33,\ \ldots,\ 114$$

$$k'_{21\ 21} = k'^{(3,4)}_{3\ 3} + k'^{(4,5)}_{3\ 3}$$

$$k'_{21\ 22} = - k'^{(3,4)}_{3\ 4} + k'^{(4,5)}_{3\ 4}$$

$$k'_{21\ 23} = k'^{(3,4)}_{3\ 5} - k'^{(4,5)}_{3\ 5}$$

$$k'_{21\ j} = 0 \ ; \ j = 24,\ 25,\ 26$$

$$k'_{21\ 27} = - k'^{(4,5)}_{3\ 3}$$

$$k'_{21\ 28} = k'^{(4,5)}_{3\ 4}$$

$$k'_{21\ 29} = - k'^{(4,5)}_{3\ 5}$$

$$k'_{21\ j} = 0 \ ; \ j = 30,\ 31,\ 32,\ \ldots,\ 114$$

$$k'_{22\ 22} = k'^{(3,4)}_{4\ 4} + k'^{(4,5)}_{4\ 4}$$

$$k'_{22\ 23} = k'^{(3,4)}_{4\ 5} + k'^{(4,5)}_{4\ 5}$$

$$k'_{22\ j} = 0 \ ; \ j = 24,\ 25,\ 26$$

$$k'_{22\ 27} = - k'^{(4,5)}_{3\ 4}$$

$$k'_{22\ 28} = k'^{(4,5)}_{4\ 10}$$

$$k'_{22\ 29} = k'^{(4,5)}_{4\ 11}$$

$$k'_{22\ j} = 0 \ ; \ j = 30,\ 31,\ 32,\ \ldots,\ 114$$

$$k'_{23\,23} = k'^{(3,4)}_{5\,5} + k'^{(4,5)}_{5\,5}$$

$$k'_{23\,j} = 0 \;;\; j = 24,\ 25,\ 26$$

$$k'_{23\,27} = k'^{(4,5)}_{3\,5}$$

$$k'_{23\,28} = k'^{(4,5)}_{4\,11}$$

$$k'_{23\,29} = k'^{(4,5)}_{5\,11}$$

$$k'_{23\,j} = 0 \;;\; j = 30,\ 31,\ 32,\ \ldots,\ 114$$

$$k'_{24\,24} = k'^{(3,4)}_{6\,6} + k'^{(4,5)}_{6\,6}$$

$$k'_{24\,25} = k'^{(4,5)}_{1\,6}$$

$$k'_{24\,26} = -\,k'^{(4,5)}_{2\,6}$$

$$k'_{24\,j} = 0 \;;\; j = 27,\ 28,\ 29$$

$$k'_{24\,30} = k'^{(4,5)}_{6\,12}$$

$$k'_{24\,j} = 0 \;;\; j = 31,\ 32,\ 33,\ \ldots,\ 114$$

$$k'_{25\ 25} = k'^{(4,5)}_{1\ 1} + k'^{(5,6)}_{1\ 1}$$

$$k'_{25\ 26} = k'^{(4,5)}_{1\ 2} + k'^{(5,6)}_{1\ 2}$$

$$k'_{25\ j} = 0 \ ; \ j = 27,\ 28,\ 29$$

$$k'_{25\ 30} = k'^{(4,5)}_{1\ 6} - k'^{(5,6)}_{1\ 6}$$

$$k'_{25\ 31} = -\, k'^{(5,6)}_{1\ 1}$$

$$k'_{25\ 32} = -\, k'^{(5,6)}_{1\ 2}$$

$$k'_{25\ j} = 0 \ ; \ j = 33,\ 34,\ 35$$

$$k'_{25\ 36} = -\, k'^{(5,6)}_{1\ 6}$$

$$k'_{25\ j} = 0 \ ; \ j = 37,\ 38,\ 39,\ \ldots,\ 114$$

$$k'_{26\ 26} = k'^{(4,5)}_{2\ 2} + k'^{(5,6)}_{2\ 2}$$

$$k'_{26\ j} = 0 \ ; \ j = 27,\ 28,\ 29$$

$$k'_{26\ 30} = -\, k'^{(4,5)}_{2\ 6} + k'^{(5,6)}_{2\ 6}$$

$$k'_{26\ 31} = -\, k'^{(5,6)}_{1\ 2}$$

$$k'_{26\ 32} = -\, k'^{(5,6)}_{2\ 2}$$

$$k'_{26\ j} = 0 \ ; \ j = 33,\ 34,\ 35$$

$$k'_{26\ 36} = k'^{(5,6)}_{2\ 6}$$

$$k'_{26\ j} = 0 \ ; \ j = 37,\ 38,\ 39,\ \ldots,\ 114$$

$$k'_{27\,27} = k'^{(4,5)}_{3\,3} + k'^{(5,6)}_{3\,3}$$

$$k'_{27\,28} = - k'^{(4,5)}_{3\,4} + k'^{(5,6)}_{3\,4}$$

$$k'_{27\,29} = k'^{(4,5)}_{3\,5} - k'^{(5,6)}_{3\,5}$$

$$k'_{27\,j} = 0 \; ; \; j = 30,\ 31,\ 32$$

$$k'_{27\,33} = - k'^{(5,6)}_{3\,3}$$

$$k'_{27\,34} = k'^{(5,6)}_{3\,4}$$

$$k'_{27\,35} = - k'^{(5,6)}_{3\,5}$$

$$k'_{27\,j} = 0 \; ; \; j = 36,\ 37,\ 38,\ \ldots,\ 114$$

$$k'_{28\,28} = k'^{(4,5)}_{4\,4} + k'^{(5,6)}_{4\,4}$$

$$k'_{28\,29} = k'^{(4,5)}_{4\,5} + k'^{(5,6)}_{4\,5}$$

$$k'_{28\,j} = 0 \; ; \; j = 30,\ 31,\ 32$$

$$k'_{28\,33} = - k'^{(5,6)}_{3\,4}$$

$$k'_{28\,34} = k'^{(5,6)}_{4\,10}$$

$$k'_{28\,35} = k'^{(5,6)}_{4\,11}$$

$$k'_{28\,j} = 0 \; ; \; j = 36,\ 37,\ 38,\ \ldots,\ 114$$

$$k'_{29\ 29} = k'^{(4,5)}_{5\ 5} + k'^{(5,6)}_{5\ 5}$$

$$k'_{29\ j} = 0\ ;\ j = 30,\ 31,\ 32$$

$$k'_{29\ 33} = k'^{(5,6)}_{3\ 5}$$

$$k'_{29\ 34} = k'^{(5,6)}_{4\ 11}$$

$$k'_{29\ 35} = k'^{(5,6)}_{5\ 11}$$

$$k'_{29\ j} = 0\ ;\ j = 36,\ 37,\ 38,\ \ldots,\ 114$$

$$k'_{30\ 30} = k'^{(4,5)}_{6\ 6} + k'^{(5,6)}_{6\ 6}$$

$$k'_{30\ 31} = k'^{(5,6)}_{1\ 6}$$

$$k'_{30\ 32} = -\ k'^{(5,6)}_{2\ 6}$$

$$k'_{30\ j} = 0\ ;\ j = 33,\ 34,\ 35$$

$$k'_{30\ 36} = k'^{(5,6)}_{6\ 12}$$

$$k'_{30\ j} = 0\ ;\ j = 37,\ 38,\ 39,\ \ldots,\ 114$$

$$k'_{31\ 31} = k'^{(5,6)}_{1\ 1} + k'^{(6,7)}_{1\ 1}$$

$$k'_{31\ 32} = k'^{(5,6)}_{1\ 2} + k'^{(6,7)}_{1\ 2}$$

$$k'_{31\ j} = 0 \ ; \ j = 33,\ 34,\ 35$$

$$k'_{31\ 36} = k'^{(5,6)}_{1\ 6} - k'^{(6,7)}_{1\ 6}$$

$$k'_{31\ 37} = - k'^{(6,7)}_{1\ 1}$$

$$k'_{31\ 38} = - k'^{(6,7)}_{1\ 2}$$

$$k'_{31\ j} = 0 \ ; \ j = 39,\ 40,\ 41$$

$$k'_{31\ 42} = - k'^{(6,7)}_{1\ 6}$$

$$k'_{31\ j} = 0 \ ; \ j = 43,\ 44,\ 45,\ \ldots,\ 114$$

$$k'_{32\ 32} = k'^{(5,6)}_{2\ 2} + k'^{(6,7)}_{2\ 2}$$

$$k'_{32\ j} = 0 \ ; \ j = 33,\ 34,\ 35$$

$$k'_{32\ 36} = - k'^{(5,6)}_{2\ 6} + k'^{(6,7)}_{2\ 6}$$

$$k'_{32\ 37} = - k'^{(6,7)}_{1\ 2}$$

$$k'_{32\ 38} = - k'^{(6,7)}_{2\ 2}$$

$$k'_{32\ j} = 0 \ ; \ j = 39,\ 40,\ 41$$

$$k'_{32\ 42} = k'^{(6,7)}_{2\ 6}$$

$$k'_{32\ j} = 0 \ ; \ j = 43,\ 44,\ 45,\ \ldots,\ 114$$

$$k'_{33\,33} = k'^{(5,6)}_{3\,3} + k'^{(6,7)}_{3\,3}$$

$$k'_{33\,34} = -\,k'^{(5,6)}_{3\,4} + k'^{(6,7)}_{3\,4}$$

$$k'_{33\,35} = k'^{(5,6)}_{3\,5} - k'^{(6,7)}_{3\,5}$$

$$k'_{33\,j} = 0 \; ; \; j = 36,\ 37,\ 38$$

$$k'_{33\,39} = -\,k'^{(6,7)}_{3\,3}$$

$$k'_{33\,40} = k'^{(6,7)}_{3\,4}$$

$$k'_{33\,41} = -\,k'^{(6,7)}_{3\,5}$$

$$k'_{33\,j} = 0 \; ; \; j = 42,\ 43,\ 44,\ \ldots,\ 114$$

$$k'_{34\,34} = k'^{(5,6)}_{4\,4} + k'^{(6,7)}_{4\,4}$$

$$k'_{34\,35} = k'^{(5,6)}_{4\,5} + k'^{(6,7)}_{4\,5}$$

$$k'_{34\,j} = 0 \; ; \; j = 36,\ 37,\ 38$$

$$k'_{34\,39} = -\,k'^{(6,7)}_{3\,4}$$

$$k'_{34\,40} = k'^{(6,7)}_{4\,10}$$

$$k'_{34\,41} = k'^{(6,7)}_{4\,11}$$

$$k'_{34\,j} = 0 \; ; \; j = 42,\ 43,\ 44,\ \ldots,\ 114$$

$$k'_{35\,35} = k'^{(5,6)}_{5\,5} + k'^{(6,7)}_{5\,5}$$

$$k'_{35\,j} = 0 \; ; \; j = 36,\ 37,\ 38$$

$$k'_{35\,39} = k'^{(6,7)}_{3\,5}$$

$$k'_{35\,40} = k'^{(6,7)}_{4\,11}$$

$$k'_{35\,41} = k'^{(6,7)}_{5\,11}$$

$$k'_{35\,j} = 0 \; ; \; j = 42,\ 43,\ 44,\ \ldots,\ 114$$

$$k'_{36\,36} = k'^{(5,6)}_{6\,6} + k'^{(6,7)}_{6\,6}$$

$$k'_{36\,37} = k'^{(6,7)}_{1\,6}$$

$$k'_{36\,38} = -\,k'^{(6,7)}_{2\,6}$$

$$k'_{36\,j} = 0 \; ; \; j = 39,\ 40,\ 41$$

$$k'_{36\,42} = k'^{(6,7)}_{6\,12}$$

$$k'_{36\,j} = 0 \; ; \; j = 43,\ 44,\ 45,\ \ldots,\ 114$$

$$k'_{37\ 37} = k'^{(6,7)}_{1\ 1} + k'^{(7,15)}_{1\ 1}$$

$$k'_{37\ 38} = k'^{(6,7)}_{1\ 2} + k'^{(7,15)}_{1\ 2}$$

$$k'_{37\ j} = 0\ ;\ j = 39,\ 40,\ 41$$

$$k'_{37\ 42} = k'^{(6,7)}_{1\ 6}$$

$$k'_{37\ j} = 0\ ;\ j = 43,\ 44,\ 45,\ \ldots,\ 84$$

$$k'_{37\ 85} = -\ k'^{(7,15)}_{1\ 1}$$

$$k'_{37\ 86} = -\ k'^{(7,15)}_{1\ 2}$$

$$k'_{37\ j} = 0\ ;\ j = 87,\ 88,\ 89,\ \ldots,\ 114$$

$$k'_{38\ 38} = k'^{(6,7)}_{2\ 2} + k'^{(7,15)}_{2\ 2}$$

$$k'_{38\ j} = 0\ ;\ j = 39,\ 40,\ 41$$

$$k'_{38\ 42} = -\ k'^{(6,7)}_{2\ 6}$$

$$k'_{38\ j} = 0\ ;\ j = 43,\ 44,\ 45,\ \ldots,\ 84$$

$$k'_{38\ 85} = -\ k'^{(7,15)}_{1\ 2}$$

$$k'_{38\ 86} = -\ k'^{(7,15)}_{2\ 2}$$

$$k'_{38\ j} = 0\ ;\ j = 87,\ 88,\ 89,\ \ldots,\ 114$$

$$k'_{39\ 39} = k'^{(6,7)}_{3\ 3} + k'^{(7,15)}_{3\ 3}$$

$$k'_{39\ 40} = -\ k'^{(6,7)}_{3\ 4}$$

$$k'_{39\ 41} = k'^{(6,7)}_{3\ 5}$$

$$k'_{39\ j} = 0\ ;\ j = 42,\ 43,\ 44,\ \ldots,\ 86$$

$$k'_{39\ 87} = -\ k'^{(7,15)}_{3\ 3}$$

$$k'_{39\ j} = 0\ ;\ j = 88,\ 89,\ 90,\ \ldots,\ 114$$

$$k'_{40\ 40} = k'^{(6,7)}_{4\ 4} + k'^{(7,15)}_{4\ 4}$$

$$k'_{40\ 41} = k'^{(6,7)}_{4\ 5} + k'^{(7,15)}_{4\ 5}$$

$$k'_{40\ j} = 0\ ;\ j = 42,\ 43,\ 44,\ \ldots,\ 87$$

$$k'_{40\ 88} = -\ k'^{(7,15)}_{4\ 4}$$

$$k'_{40\ 89} = -\ k'^{(7,15)}_{4\ 5}$$

$$k'_{40\ j} = 0\ ;\ j = 90,\ 91,\ 92,\ \ldots,\ 114$$

$$k'_{41\ 41} = k'^{(6,7)}_{5\ 5} + k'^{(7,15)}_{5\ 5}$$

$$k'_{41\ j} = 0 \ ; \ j = 42, 43, 44, \ldots, 87$$

$$k'_{41\ 88} = - k'^{(7,15)}_{4\ 5}$$

$$k'_{41\ 89} = - k'^{(7,15)}_{5\ 5}$$

$$k'_{41\ j} = 0 \ ; \ j = 90, 91, 92, \ldots, 114$$

$$k'_{42\ 42} = k'^{(6,7)}_{6\ 6} + k'^{(7,15)}_{6\ 6}$$

$$k'_{42\ j} = 0 \ ; \ j = 43, 44, 45, \ldots, 89$$

$$k'_{42\ 90} = - k'^{(7,15)}_{6\ 6}$$

$$k'_{42\ j} = 0 \ ; \ j = 91, 92, 93, \ldots, 114$$

$$k'_{43\,43} = k'^{(1,8)}_{1\,1} + k'^{(2,8)}_{1\,1} + k'^{(3,8)}_{1\,1} + k'^{(8,9)}_{1\,1} + k'^{(8,10)}_{1\,1} + k'^{(8,11)}_{1\,1}$$

$$k'_{43\,44} = k'^{(1,8)}_{1\,2} + k'^{(2,8)}_{1\,2} + k'^{(3,8)}_{1\,2} + k'^{(8,9)}_{1\,2} + k'^{(8,10)}_{1\,2} + k'^{(8,11)}_{1\,2}$$

$$k'_{43\,45} = 0$$

$$k'_{43\,46} = k'^{(8,9)}_{1\,4}$$

$$k'_{43\,47} = k'^{(8,9)}_{1\,5}$$

$$k'_{43\,48} = k'^{(1,8)}_{1\,6} - k'^{(2,8)}_{1\,6} - k'^{(8,11)}_{1\,6}$$

$$k'_{43\,49} = - k'^{(8,9)}_{1\,1}$$

$$k'_{43\,50} = - k'^{(8,9)}_{1\,2}$$

$$k'_{43\,51} = 0$$

$$k'_{43\,52} = k'^{(8,9)}_{1\,4}$$

$$k'_{43\,53} = k'^{(8,9)}_{1\,5}$$

$$k'_{43\,54} = 0$$

$$k'_{43\,55} = - k'^{(8,10)}_{1\,1}$$

$$k'_{43\,56} = - k'^{(8,10)}_{1\,2}$$

$$k'_{43\,j} = 0 \;;\; j = 57,\, 58,\, 59,\, 60$$

$$k'_{43\,61} = - k'^{(8,11)}_{1\,1}$$

$$k'_{43\,62} = - k'^{(8,11)}_{1\,2}$$

$$k'_{43\,j} = 0 \;;\; j = 63,\, 64,\, 65$$

$$k'_{43\,66} = - k'^{(8,11)}_{1\,6}$$

$$k'_{43\,j} = 0 \;;\; j = 67,\, 68,\, 69,\, \ldots,\, 114$$

$$k'_{44\ 44} = k'^{(1,8)}_{2\ 2} + k'^{(2,8)}_{2\ 2} + k'^{(3,8)}_{2\ 2} + k'^{(8,9)}_{2\ 2} + k'^{(8,10)}_{2\ 2} + k'^{(8,11}_{2\ 2}$$

$$k'_{44\ 45} = 0$$

$$k'_{44\ 46} = k'^{(8,9)}_{2\ 4}$$

$$k'_{44\ 47} = -\ k'^{(8,9)}_{1\ 4}$$

$$k'_{44\ 48} = -\ k'^{(1,8)}_{2\ 6} + k'^{(2,8)}_{2\ 6} + k'^{(8,11)}_{2\ 6}$$

$$k'_{44\ 49} = -\ k'^{(8,9)}_{1\ 2}$$

$$k'_{44\ 50} = -\ k'^{(8,9)}_{2\ 2}$$

$$k'_{44\ 51} = 0$$

$$k'_{44\ 52} = k'^{(8,9)}_{2\ 4}$$

$$k'_{44\ 53} = -\ k'^{(8,9)}_{1\ 4}$$

$$k'_{44\ 54} = 0$$

$$k'_{44\ 55} = -\ k'^{(8,10)}_{1\ 2}$$

$$k'_{44\ 56} = -\ k'^{(8,10)}_{2\ 2}$$

$$k'_{44\ j} = 0\ ;\ j = 57,\ 58,\ 59,\ 60$$

$$k'_{44\ 61} = -\ k'^{(8,11)}_{1\ 2}$$

$$k'_{44\ 62} = -\ k'^{(8,11)}_{2\ 2}$$

$$k'_{44\ j} = 0\ ;\ j = 63,\ 64,\ 65$$

$$k'_{44\ 66} = k'^{(8,11)}_{2\ 6}$$

$$k'_{44\ j} = 0\ ;\ j = 67,\ 68,\ 69,\ \ldots,\ 114$$

$$k'_{45\;45} = k'^{(1,8)}_{3\;3} + k'^{(2,8)}_{3\;3} + k'^{(8,9)}_{3\;3} + k'^{(8,10)}_{3\;3} + k'^{(8,11)}_{3\;3}$$

$$k'_{45\;46} = - k'^{(1,8)}_{3\;4} + k'^{(2,8)}_{3\;4} + k'^{(8,11)}_{3\;4}$$

$$k'_{45\;47} = k'^{(1,8)}_{3\;5} - k'^{(2,8)}_{3\;5} - k'^{(8,11)}_{3\;5}$$

$$k'_{45\;j} = 0 \;;\; j = 48,\ 49,\ 50$$

$$k'_{45\;51} = - k'^{(8,9)}_{3\;3}$$

$$k'_{45\;j} = 0 \;;\; j = 52,\ 53,\ 54,\ 55,\ 56$$

$$k'_{45\;57} = - k'^{(8,10)}_{3\;3}$$

$$k'_{45\;j} = 0 \;;\; j = 58,\ 59,\ 60,\ 61,\ 62$$

$$k'_{45\;63} = - k'^{(8,11)}_{3\;3}$$

$$k'_{45\;64} = k'^{(8,11)}_{3\;4}$$

$$k'_{45\;65} = - k'^{(8,11)}_{3\;5}$$

$$k'_{45\;j} = 0 \;;\; j = 66,\ 67,\ 68,\ \ldots,\ 114$$

$$k'_{46\,46} = k'^{(1,8)}_{4\,4} + k'^{(2,8)}_{4\,4} + k'^{(8,9)}_{4\,4} + k'^{(8,10)}_{4\,4} + k'^{(8,11)}_{4\,4}$$

$$k'_{46\,47} = k'^{(1,8)}_{4\,5} + k'^{(2,8)}_{4\,5} + k'^{(8,9)}_{4\,5} + k'^{(8,10)}_{4\,5} + k'^{(8,11)}_{4\,5}$$

$$k'_{46\,48} = 0$$

$$k'_{46\,49} = - k'^{(8,9)}_{1\,4}$$

$$k'_{46\,50} = - k'^{(8,9)}_{2\,4}$$

$$k'_{46\,51} = 0$$

$$k'_{46\,52} = k'^{(8,9)}_{4\,10}$$

$$k'_{46\,53} = k'^{(8,9)}_{4\,11}$$

$$k'_{46\,j} = 0 \;;\; j = 54,\ 55,\ 56,\ 57$$

$$k'_{46\,58} = - k'^{(8,10)}_{4\,4}$$

$$k'_{46\,59} = - k'^{(8,10)}_{4\,5}$$

$$k'_{46\,j} = 0 \;;\; j = 60,\ 61,\ 62$$

$$k'_{46\,63} = - k'^{(8,11)}_{3\,4}$$

$$k'_{46\,64} = k'^{(8,11)}_{4\,10}$$

$$k'_{46\,65} = k'^{(8,11)}_{4\,11}$$

$$k'_{46\,j} = 0 \;;\; j = 66,\ 67,\ 68,\ \ldots,\ 114$$

$$k'_{47\,47} = k'^{(1,8)}_{5\,5} + k'^{(2,8)}_{5\,5} + k'^{(8,9)}_{5\,5} + k'^{(8,10)}_{5\,5} + k'^{(8,11)}_{5\,5}$$

$$k'_{47\,48} = 0$$

$$k'_{47\,49} = -\,k'^{(8,9)}_{1\,5}$$

$$k'_{47\,50} = k'^{(8,9)}_{1\,4}$$

$$k'_{47\,51} = 0$$

$$k'_{47\,52} = k'^{(8,9)}_{4\,11}$$

$$k'_{47\,53} = k'^{(8,9)}_{5\,11}$$

$$k'_{47\,j} = 0 \;;\; j = 54,\ 55,\ 56,\ 57$$

$$k'_{47\,58} = -\,k'^{(8,10)}_{4\,5}$$

$$k'_{47\,59} = -\,k'^{(8,10)}_{5\,5}$$

$$k'_{47\,j} = 0 \;;\; j = 60,\ 61,\ 62$$

$$k'_{47\,63} = k'^{(8,11)}_{3\,5}$$

$$k'_{47\,64} = k'^{(8,11)}_{4\,11}$$

$$k'_{47\,65} = k'^{(8,11)}_{5\,11}$$

$$k'_{47\,j} = 0 \;;\; j = 66,\ 67,\ 68,\ \ldots,\ 114$$

$$k'_{48\ 48} = k'^{(1,8)}_{6\ 6} + k'^{(2,8)}_{6\ 6} + k'^{(8,9)}_{6\ 6} + k'^{(8,10)}_{6\ 6} + k'^{(8,11)}_{6\ 6}$$

$$k'_{48\ j} = 0 \ ; \ j = 49, 50, 51, 52, 53$$

$$k'_{48\ 54} = - k'^{(8,9)}_{6\ 6}$$

$$k'_{48\ j} = 0 \ ; \ j = 55, 56, 57, 58, 59$$

$$k'_{48\ 60} = - k'^{(8,10)}_{6\ 6}$$

$$k'_{48\ 61} = k'^{(8,11)}_{1\ 6}$$

$$k'_{48\ 62} = - k'^{(8,11)}_{2\ 6}$$

$$k'_{48\ j} = 0 \ ; \ j = 63, 64, 65$$

$$k'_{48\ 66} = k'^{(8,11)}_{6\ 12}$$

$$k'_{48\ j} = 0 \ ; \ j = 67, 68, 69, \ldots, 114$$

$$k'_{49\,49} = k'^{(1,9)}_{1\,1} + k'^{(2,9)}_{1\,1} + k'^{(3,9)}_{1\,1} + k'^{(8,9)}_{1\,1} + k'^{(9,10)}_{1\,1} + k'^{(9,11)}_{1\,1}$$

$$k'_{49\,50} = k'^{(1,9)}_{1\,2} + k'^{(2,9)}_{1\,2} + k'^{(3,9)}_{1\,2} + k'^{(8,9)}_{1\,2} + k'^{(9,10)}_{1\,2} + k'^{(9,11)}_{1\,2}$$

$$k'_{49\,51} = 0$$

$$k'_{49\,52} = - k'^{(8,9)}_{1\,4}$$

$$k'_{49\,53} = - k'^{(8,9)}_{1\,5}$$

$$k'_{49\,54} = k'^{(1,9)}_{1\,6} - k'^{(2,9)}_{1\,6} - k'^{(9,11)}_{1\,6}$$

$$k'_{49\,55} = - k'^{(9,10)}_{1\,1}$$

$$k'_{49\,56} = - k'^{(9,10)}_{1\,2}$$

$$k'_{49\,j} = 0 \; ; \; j = 57,\ 58,\ 59,\ 60$$

$$k'_{49\,61} = - k'^{(9,11)}_{1\,1}$$

$$k'_{49\,62} = - k'^{(9,11)}_{1\,2}$$

$$k'_{49\,j} = 0 \; ; \; j = 63,\ 64,\ 65$$

$$k'_{49\,66} = - k'^{(9,11)}_{1\,6}$$

$$k'_{49\,j} = 0 \; ; \; j = 67,\ 68,\ 69,\ \ldots,\ 114$$

$$k'_{50\,50} = k'^{(1,9)}_{2\,2} + k'^{(2,9)}_{2\,2} + k'^{(3,9)}_{2\,2} + k'^{(8,9)}_{2\,2} + k'^{(9,10)}_{2\,2} + k'^{(9,11)}_{2\,2}$$

$$k'_{50\,51} = 0$$

$$k'_{50\,52} = -\,k'^{(8,9)}_{2\,4}$$

$$k'_{50\,53} = k'^{(8,9)}_{1\,4}$$

$$k'_{50\,54} = -\,k'^{(1,9)}_{2\,6} + k'^{(2,9)}_{2\,6} + k'^{(9,11)}_{2\,6}$$

$$k'_{50\,55} = -\,k'^{(9,10)}_{1\,2}$$

$$k'_{50\,56} = -\,k'^{(9,10)}_{2\,2}$$

$$k'_{50\,j} = 0\ ;\ j = 57,\ 58,\ 59,\ 60$$

$$k'_{50\,61} = -\,k'^{(9,11)}_{1\,2}$$

$$k'_{50\,62} = -\,k'^{(9,11)}_{2\,2}$$

$$k'_{50\,j} = 0\ ;\ j = 63,\ 64,\ 65$$

$$k'_{50\,66} = k'^{(9,11)}_{2\,6}$$

$$k'_{50\,j} = 0\ ;\ j = 67,\ 68,\ 69,\ \ldots,\ 114$$

$$k'_{51\,51} = k'^{(1,9)}_{3\,3} + k'^{(2,9)}_{3\,3} + k'^{(8,9)}_{3\,3} + k'^{(9,10)}_{3\,3} + k'^{(9,11)}_{3\,3}$$

$$k'_{51\,52} = -\,k'^{(1,9)}_{3\,4} + k'^{(2,9)}_{3\,4} + k'^{(9,11)}_{3\,4}$$

$$k'_{51\,53} = k'^{(1,9)}_{3\,5} - k'^{(2,9)}_{3\,5} - k'^{(9,11)}_{3\,5}$$

$$k'_{51\,j} = 0\ ;\ j = 54,\ 55,\ 56$$

$$k'_{51\,57} = -\,k'^{(9,10)}_{3\,3}$$

$$k'_{51\,j} = 0\ ;\ j = 58,\ 59,\ 60,\ 61,\ 62$$

$$k'_{51\,63} = -\,k'^{(9,11)}_{3\,3}$$

$$k'_{51\,64} = k'^{(9,11)}_{3\,4}$$

$$k'_{51\,65} = -\,k'^{(9,11)}_{3\,5}$$

$$k'_{51\,j} = 0\ ;\ j = 66,\ 67,\ 68,\ \ldots,\ 114$$

$$k'_{52\,52} = k'^{(1,9)}_{4\,4} + k'^{(2,9)}_{4\,4} + k'^{(8,9)}_{4\,4} + k'^{(9,10)}_{4\,4} + k'^{(9,11)}_{4\,4}$$

$$k'_{52\,53} = k'^{(1,9)}_{4\,5} + k'^{(2,9)}_{4\,5} + k'^{(8,9)}_{4\,5} + k'^{(9,10)}_{4\,5} + k'^{(9,11)}_{4\,5}$$

$$k'_{52\,j} = 0 \;;\; j = 54,\ 55,\ 56,\ 57$$

$$k'_{52\,58} = -\,k'^{(9,10)}_{4\,4}$$

$$k'_{52\,59} = -\,k'^{(9,10)}_{4\,5}$$

$$k'_{52\,j} = 0 \;;\; j = 60,\ 61,\ 62$$

$$k'_{52\,63} = -\,k'^{(9,11)}_{3\,4}$$

$$k'_{52\,64} = k'^{(9,11)}_{4\,10}$$

$$k'_{52\,65} = k'^{(9,11)}_{4\,11}$$

$$k'_{52\,j} = 0 \;;\; j = 66,\ 67,\ 68,\ \ldots,\ 114$$

$$k'_{53\,53} = k'^{(1,9)}_{5\,5} + k'^{(2,9)}_{5\,5} + k'^{(8,9)}_{5\,5} + k'^{(9,10)}_{5\,5} + k'^{(9,11)}_{5\,5}$$

$$k'_{53\,j} = 0 \; ; \; j = 54,\ 55,\ 56,\ 57$$

$$k'_{53\,58} = -\,k'^{(9,10)}_{4\,5}$$

$$k'_{53\,59} = -\,k'^{(9,10)}_{5\,5}$$

$$k'_{53\,j} = 0 \; ; \; j = 60,\ 61,\ 62$$

$$k'_{53\,63} = k'^{(9,11)}_{3\,5}$$

$$k'_{53\,64} = k'^{(9,11)}_{4\,11}$$

$$k'_{53\,65} = k'^{(9,11)}_{5\,11}$$

$$k'_{53\,j} = 0 \; ; \; j = 66,\ 67,\ 68,\ \ldots,\ 114$$

$$k'_{54\,54} = k'^{(1,9)}_{6\,6} + k'^{(2,9)}_{6\,6} + k'^{(8,9)}_{6\,6} + k'^{(9,10)}_{6\,6} + k'^{(9,11)}_{6\,6}$$

$$k'_{54\,j} = 0 \; ; \; j = 55,\ 56,\ 57,\ 58,\ 59$$

$$k'_{54\,60} = -\,k'^{(9,10)}_{6\,6}$$

$$k'_{54\,61} = k'^{(9,11)}_{1\,6}$$

$$k'_{54\,62} = -\,k'^{(9,11)}_{2\,6}$$

$$k'_{54\,j} = 0 \; ; \; j = 63,\ 64,\ 65$$

$$k'_{54\,66} = k'^{(9,11)}_{6\,12}$$

$$k'_{54\,j} = 0 \; ; \; j = 67,\ 68,\ 69,\ \ldots,\ 114$$

$$k'_{55\ 55} = k'^{(8,10)}_{1\ 1} + k'^{(9,10)}_{1\ 1}$$

$$k'_{55\ 56} = k'^{(8,10)}_{1\ 2} + k'^{(9,10)}_{1\ 2}$$

$$k'_{55\ j} = 0\ ;\ j = 57,\ 58,\ 59,\ \ldots,\ 114$$

$$k'_{56\ 56} = k'^{(8,10)}_{2\ 2} + k'^{(9,10)}_{2\ 2}$$

$$k'_{56\ j} = 0\ ;\ j = 57,\ 58,\ 59,\ \ldots,\ 114$$

$$k'_{57\ 57} = k'^{(8,10)}_{3\ 3} + k'^{(9,10)}_{3\ 3}$$

$$k'_{57\ j} = 0\ ;\ j = 58,\ 59,\ 60,\ \ldots,\ 114$$

$$k'_{58\ 58} = k'^{(8,10)}_{4\ 4} + k'^{(9,10)}_{4\ 4}$$

$$k'_{58\ 59} = k'^{(8,10)}_{4\ 5} + k'^{(9,10)}_{4\ 5}$$

$$k'_{58\ j} = 0\ ;\ j = 60,\ 61,\ 62,\ \ldots,\ 114$$

$$k'_{59\ 59} = k'^{(8,10)}_{5\ 5} + k'^{(9,10)}_{5\ 5}$$

$$k'_{59\ j} = 0\ ;\ j = 60,\ 61,\ 62,\ \ldots,\ 114$$

$$k'_{60\ 60} = k'^{(8,10)}_{6\ 6} + k'^{(9,10)}_{6\ 6}$$

$$k'_{60\ j} = 0\ ;\ j = 61,\ 62,\ 63,\ \ldots,\ 114$$

$$k'_{61\,61} = k'^{(8,11)}_{1\,1} + k'^{(9,11)}_{1\,1} + k'^{(11,12)}_{1\,1}$$

$$k'_{61\,62} = k'^{(8,11)}_{1\,2} + k'^{(9,11)}_{1\,2} + k'^{(11,12)}_{1\,2}$$

$$k'_{61\,j} = 0 \; ; \; j = 63,\ 64,\ 65$$

$$k'_{61\,66} = k'^{(8,11)}_{1\,6} + k'^{(9,11)}_{1\,6} - k'^{(11,12)}_{1\,6}$$

$$k'_{61\,67} = -\,k'^{(11,12)}_{1\,1}$$

$$k'_{61\,68} = -\,k'^{(11,12)}_{1\,2}$$

$$k'_{61\,j} = 0 \; ; \; j = 69,\ 70,\ 71$$

$$k'_{61\,72} = -\,k'^{(11,12)}_{1\,6}$$

$$k'_{61\,j} = 0 \; ; \; j = 73,\ 74,\ 75,\ \ldots,\ 114$$

$$k'_{62\,62} = k'^{(8,11)}_{2\,2} + k'^{(9,11)}_{2\,2} + k'^{(11,12)}_{2\,2}$$

$$k'_{62\,j} = 0 \; ; \; j = 63,\ 64,\ 65$$

$$k'_{62\,66} = -\,k'^{(8,11)}_{2\,6} - k'^{(9,11)}_{2\,6} + k'^{(11,12)}_{2\,6}$$

$$k'_{62\,67} = -\,k'^{(11,12)}_{1\,2}$$

$$k'_{62\,68} = -\,k'^{(11,12)}_{2\,2}$$

$$k'_{62\,j} = 0 \; ; \; j = 69,\ 70,\ 71$$

$$k'_{62\,72} = k'^{(11,12)}_{2\,6}$$

$$k'_{62\,j} = 0 \; ; \; j = 73,\ 74,\ 75,\ \ldots,\ 114$$

$$k'_{63\,63} = k'^{(8,11)}_{3\,3} + k'^{(9,11)}_{3\,3} + k'^{(11,12)}_{3\,3}$$

$$k'_{63\,64} = - k'^{(8,11)}_{3\,4} - k'^{(9,11)}_{3\,4} + k'^{(11,12)}_{3\,4}$$

$$k'_{63\,65} = k'^{(8,11)}_{3\,5} + k'^{(9,11)}_{3\,5} - k'^{(11,12)}_{3\,5}$$

$$k'_{63\,j} = 0 \; ; \; j = 66,\ 67,\ 68$$

$$k'_{63\,69} = - k'^{(11,12)}_{3\,3}$$

$$k'_{63\,70} = k'^{(11,12)}_{3\,4}$$

$$k'_{63\,71} = - k'^{(11,12)}_{3\,5}$$

$$k'_{63\,j} = 0 \; ; \; j = 72,\ 73,\ 74,\ \ldots,\ 114$$

$$k'_{64\,64} = k'^{(8,11)}_{4\,4} + k'^{(9,11)}_{4\,4} + k'^{(11,12)}_{4\,4}$$

$$k'_{64\,65} = k'^{(8,11)}_{4\,5} + k'^{(9,11)}_{4\,5} + k'^{(11,12)}_{4\,5}$$

$$k'_{64\,j} = 0 \; ; \; j = 66,\ 67,\ 68$$

$$k'_{64\,69} = - k'^{(11,12)}_{3\,4}$$

$$k'_{64\,70} = k'^{(11,12)}_{4\,10}$$

$$k'_{64\,71} = k'^{(11,12)}_{4\,11}$$

$$k'_{64\,j} = 0 \; ; \; j = 72,\ 73,\ 74,\ \ldots,\ 114$$

$$k'_{65\,65} = k'^{(8,11)}_{5\,5} + k'^{(9,11)}_{5\,5} + k'^{(11,12)}_{5\,5}$$

$$k'_{65\,j} = 0 \; ; \; j = 66,\ 67,\ 68$$

$$k'_{65\,69} = k'^{(11,12)}_{3\,5}$$

$$k'_{65\,70} = k'^{(11,12)}_{4\,11}$$

$$k'_{65\,71} = k'^{(11,12)}_{5\,11}$$

$$k'_{65\,j} = 0 \; ; \; j = 72,\ 73,\ 74,\ \ldots,\ 114$$

$$k'_{66\,66} = k'^{(8,11)}_{6\,6} + k'^{(9,11)}_{6\,6} + k'^{(11,12)}_{6\,6}$$

$$k'_{66\,67} = k'^{(11,12)}_{1\,6}$$

$$k'_{66\,68} = -\,k'^{(11,12)}_{2\,6}$$

$$k'_{66\,j} = 0 \; ; \; j = 69,\ 70,\ 71$$

$$k'_{66\,72} = k'^{(11,12)}_{6\,12}$$

$$k'_{66\,j} = 0 \; ; \; j = 73,\ 74,\ 75,\ \ldots,\ 114$$

$$k'_{67\,67} = k'^{(11,12)}_{1\,1} + k'^{(12,13)}_{1\,1}$$

$$k'_{67\,68} = k'^{(11,12)}_{1\,2} + k'^{(12,13)}_{1\,2}$$

$$k'_{67\,j} = 0 \;;\; j = 69,\ 70,\ 71$$

$$k'_{67\,72} = k'^{(11,12)}_{1\,6} - k'^{(12,13)}_{1\,6}$$

$$k'_{67\,73} = -\,k'^{(12,13)}_{1\,1}$$

$$k'_{67\,74} = -\,k'^{(12,13)}_{1\,2}$$

$$k'_{67\,j} = 0 \;;\; j = 75,\ 76,\ 77$$

$$k'_{67\,78} = -\,k'^{(12,13)}_{1\,6}$$

$$k'_{67\,j} = 0 \;;\; j = 79,\ 80,\ 81,\ \ldots,\ 114$$

$$k'_{68\,68} = k'^{(11,12)}_{2\,2} + k'^{(12,13)}_{2\,2}$$

$$k'_{68\,j} = 0 \;;\; j = 69,\ 70,\ 71$$

$$k'_{68\,72} = -\,k'^{(11,12)}_{2\,6} + k'^{(12,13)}_{2\,6}$$

$$k'_{68\,73} = -\,k'^{(12,13)}_{1\,2}$$

$$k'_{68\,74} = -\,k'^{(12,13)}_{2\,2}$$

$$k'_{68\,j} = 0 \;;\; j = 75,\ 76,\ 77$$

$$k'_{68\,78} = k'^{(12,13)}_{2\,6}$$

$$k'_{68\,j} = 0 \;;\; j = 79,\ 80,\ 81,\ \ldots,\ 114$$

$$k'_{69\,69} = k'^{(11,12)}_{3\,3} + k'^{(12,13)}_{3\,3}$$

$$k'_{69\,70} = -\,k'^{(11,12)}_{3\,4} + k'^{(12,13)}_{3\,4}$$

$$k'_{69\,71} = k'^{(11,12)}_{3\,5} - k'^{(12,13)}_{3\,5}$$

$$k'_{69\,j} = 0\ ;\ j = 72,\ 73,\ 74$$

$$k'_{69\,75} = -\,k'^{(12,13)}_{3\,3}$$

$$k'_{69\,76} = k'^{(12,13)}_{3\,4}$$

$$k'_{69\,77} = -\,k'^{(12,13)}_{3\,5}$$

$$k'_{69\,j} = 0\ ;\ j = 78,\ 79,\ 80,\ \ldots,\ 114$$

$$k'_{70\,70} = k'^{(11,12)}_{4\,4} + k'^{(12,13)}_{4\,4}$$

$$k'_{70\,71} = k'^{(11,12)}_{4\,5} + k'^{(12,13)}_{4\,5}$$

$$k'_{70\,j} = 0\ ;\ j = 72,\ 73,\ 74$$

$$k'_{70\,75} = -\,k'^{(12,13)}_{3\,4}$$

$$k'_{70\,76} = k'^{(12,13)}_{4\,10}$$

$$k'_{70\,77} = k'^{(12,13)}_{4\,11}$$

$$k'_{70\,j} = 0\ ;\ j = 78,\ 79,\ 80,\ \ldots,\ 114$$

$$k'_{71\,71} = k'^{(11,12)}_{5\,5} + k'^{(12,13)}_{5\,5}$$

$$k'_{71\,j} = 0 \;;\; j = 72,\ 73,\ 74$$

$$k'_{71\,75} = k'^{(12,13)}_{3\,5}$$

$$k'_{71\,76} = k'^{(12,13)}_{4\,11}$$

$$k'_{71\,77} = k'^{(12,13)}_{5\,11}$$

$$k'_{71\,j} = 0 \;;\; j = 78,\ 79,\ 80,\ \ldots,\ 114$$

$$k'_{72\,72} = k'^{(11,12)}_{6\,6} + k'^{(12,13)}_{6\,6}$$

$$k'_{72\,73} = k'^{(12,13)}_{1\,6}$$

$$k'_{72\,74} = -\,k'^{(12,13)}_{2\,6}$$

$$k'_{72\,j} = 0 \;;\; j = 75,\ 76,\ 77$$

$$k'_{72\,78} = k'^{(12,13)}_{6\,12}$$

$$k'_{72\,j} = 0 \;;\; j = 79,\ 80,\ 81,\ \ldots,\ 114$$

$$k'_{73\,73} = k'^{(12,13)}_{1\,1} + k'^{(13,14)}_{1\,1}$$

$$k'_{73\,74} = k'^{(12,13)}_{1\,2} + k'^{(13,14)}_{1\,2}$$

$$k'_{73\,j} = 0 \; ; \; j = 75, 76, 77$$

$$k'_{73\,78} = k'^{(12,13)}_{1\,6} - k'^{(13,14)}_{1\,6}$$

$$k'_{73\,79} = - k'^{(13,14)}_{1\,1}$$

$$k'_{73\,80} = - k'^{(13,14)}_{1\,2}$$

$$k'_{73\,j} = 0 \; ; \; j = 81, 82, 83$$

$$k'_{73\,84} = - k'^{(13,14)}_{1\,6}$$

$$k'_{73\,j} = 0 \; ; \; j = 85, 86, 87, \ldots, 114$$

$$k'_{74\,74} = k'^{(12,13)}_{2\,2} + k'^{(13,14)}_{2\,2}$$

$$k'_{74\,j} = 0 \; ; \; j = 75, 76, 77$$

$$k'_{74\,78} = - k'^{(12,13)}_{2\,6} + k'^{(13,14)}_{2\,6}$$

$$k'_{74\,79} = - k'^{(13,14)}_{1\,2}$$

$$k'_{74\,80} = - k'^{(13,14)}_{2\,2}$$

$$k'_{74\,j} = 0 \; ; \; j = 81, 82, 83$$

$$k'_{74\,84} = k'^{(13,14)}_{2\,6}$$

$$k'_{74\,j} = 0 \; ; \; j = 85, 86, 87, \ldots, 114$$

$$k'_{75\ 75} = k'^{(12,13)}_{3\ 3} + k'^{(13,14)}_{3\ 3}$$

$$k'_{75\ 76} = - k'^{(12,13)}_{3\ 4} + k'^{(13,14)}_{3\ 4}$$

$$k'_{75\ 77} = k'^{(12,13)}_{3\ 5} - k'^{(13,14)}_{3\ 5}$$

$$k'_{75\ j} = 0 \ ; \ j = 78,\ 79,\ 80$$

$$k'_{75\ 81} = - k'^{(13,14)}_{3\ 3}$$

$$k'_{75\ 82} = k'^{(13,14)}_{3\ 4}$$

$$k'_{75\ 83} = - k'^{(13,14)}_{3\ 5}$$

$$k'_{75\ j} = 0 \ ; \ j = 84, 85,\ 86,\ \ldots,\ 114$$

$$k'_{76\ 76} = k'^{(12,13)}_{4\ 4} + k'^{(13,14)}_{4\ 4}$$

$$k'_{76\ 77} = k'^{(12,13)}_{4\ 5} + k'^{(13,14)}_{4\ 5}$$

$$k'_{76\ j} = 0 \ ; \ j = 78,\ 79,\ 80$$

$$k'_{76\ 81} = - k'^{(13,14)}_{3\ 4}$$

$$k'_{76\ 82} = k'^{(13,14)}_{4\ 10}$$

$$k'_{76\ 83} = k'^{(13,14)}_{4\ 11}$$

$$k'_{76\ j} = 0 \ ; \ j = 85,\ 86,\ 87,\ \ldots,\ 114$$

$$k'_{77\ 77} = k'^{(12,13)}_{5\ 5} + k'^{(13,14)}_{5\ 5}$$

$$k'_{77\ j} = 0 \ ; \ j = 78,\ 79,\ 80$$

$$k'_{77\ 81} = k'^{(13,14)}_{3\ 5}$$

$$k'_{77\ 82} = k'^{(13,14)}_{4\ 11}$$

$$k'_{77\ 83} = k'^{(13,14)}_{5\ 11}$$

$$k'_{77\ j} = 0 \ ; \ j = 84,\ 85,\ 86,\ \ldots,\ 114$$

$$k'_{78\ 78} = k'^{(12,13)}_{6\ 6} + k'^{(13,14)}_{6\ 6}$$

$$k'_{78\ 79} = k'^{(13,14)}_{1\ 6}$$

$$k'_{78\ 80} = -\ k'^{(13,14)}_{2\ 6}$$

$$k'_{78\ j} = 0 \ ; \ j = 81,\ 82,\ 83$$

$$k'_{78\ 84} = k'^{(13\ 14)}_{6\ 12}$$

$$k'_{78\ j} = 0 \ ; \ j = 85,\ 86,\ 87,\ \ldots,\ 114$$

$$k'_{79\,79} = k'^{(13,14)}_{1\,1} + k'^{(14,15)}_{1\,1}$$

$$k'_{79\,80} = k'^{(13,14)}_{1\,2} + k'^{(14,15)}_{1\,2}$$

$$k'_{79\,j} = 0 \;;\; j = 81,\ 82,\ 83$$

$$k'_{79\,84} = k'^{(13,14)}_{1\,6} - k'^{(14,15)}_{1\,6}$$

$$k'_{79\,85} = - k'^{(14,15)}_{1\,1}$$

$$k'_{79\,86} = - k'^{(14,15)}_{1\,2}$$

$$k'_{79\,j} = 0 \;;\; j = 87,\ 88,\ 89$$

$$k'_{79\,90} = - k'^{(14,15)}_{1\,6}$$

$$k'_{79\,j} = 0 \;;\; j = 91,\ 91,\ 93,\ \ldots,\ 114$$

$$k'_{80\,80} = k'^{(13,14)}_{2\,2} + k'^{(14,15)}_{2\,2}$$

$$k'_{80\,j} = 0 \;;\; j = 81,\ 82,\ 83$$

$$k'_{80\,84} = - k'^{(13,14)}_{2\,6} + k'^{(14,15)}_{2\,6}$$

$$k'_{80\,85} = - k'^{(14,15)}_{1\,2}$$

$$k'_{80\,86} = - k'^{(14,15)}_{2\,2}$$

$$k'_{80\,j} = 0 \;;\; j = 87,\ 88,\ 89$$

$$k'_{80\,90} = k'^{(14,15)}_{2\,6}$$

$$k'_{80\,j} = 0 \;;\; j = 91,\ 92,\ 93,\ \ldots,\ 114$$

$$k'_{81\ 81} = k'^{(13,14)}_{3\ 3} + k'^{(14,15)}_{3\ 3}$$

$$k'_{81\ 82} = -\,k'^{(13,14)}_{3\ 4} + k'^{(14,15)}_{3\ 4}$$

$$k'_{81\ 83} = k'^{(13,14)}_{3\ 5} - k'^{(14,15)}_{3\ 5}$$

$$k'_{81\ j} = 0\ ;\ j = 84,\ 85,\ 86$$

$$k'_{81\ 87} = -\,k'^{(14,15)}_{3\ 3}$$

$$k'_{81\ 88} = k'^{(14,15)}_{3\ 4}$$

$$k'_{81\ 89} = -\,k'^{(14,15)}_{3\ 5}$$

$$k'_{81\ j} = 0\ ;\ j = 90,\ 91,\ 92,\ \ldots,\ 114$$

$$k'_{82\ 82} = k'^{(13,14)}_{4\ 4} + k'^{(14,15)}_{4\ 4}$$

$$k'_{82\ 83} = k'^{(13,14)}_{4\ 5} + k'^{(14,15)}_{4\ 5}$$

$$k'_{82\ j} = 0\ ;\ j = 84,\ 85,\ 86$$

$$k'_{82\ 87} = -\,k'^{(14,15)}_{3\ 4}$$

$$k'_{82\ 88} = k'^{(14,15)}_{4\ 10}$$

$$k'_{82\ 89} = k'^{(14,15)}_{4\ 11}$$

$$k'_{82\ j} = 0\ ;\ j = 90,\ 91,\ 92,\ \ldots,\ 114$$

$$k'_{83\;83} = k'^{(13,14)}_{5\;5} + k'^{(14,15)}_{5\;5}$$

$$k'_{83\;j} = 0 \;;\; j = 84,\ 85,\ 86$$

$$k'_{83\;87} = k'^{(14,15)}_{3\;5}$$

$$k'_{83\;88} = k'^{(14,15)}_{4\;11}$$

$$k'_{83\;89} = k'^{(14,15)}_{5\;11}$$

$$k'_{83\;j} = 0 \;;\; j = 90,\ 91,\ 92,\ \ldots,\ 114$$

$$k'_{84\;84} = k'^{(13,14)}_{6\;6} + k'^{(14,15)}_{6\;6}$$

$$k'_{84\;85} = k'^{(14,15)}_{1\;6}$$

$$k'_{84\;86} = -\,k'^{(14,15)}_{2\;6}$$

$$k'_{84\;j} = 0 \;;\; j = 87,\ 88,\ 89$$

$$k'_{84\;90} = k'^{(14,15)}_{6\;12}$$

$$k'_{84\;j} = 0 \;;\; j = 91,\ 92,\ 93,\ \ldots,\ 114$$

$$k'_{85\,85} = k'^{(7,15)}_{1\,1} + k'^{(14,15)}_{1\,1} + k'^{(15,16)}_{1\,1}$$

$$k'_{85\,86} = k'^{(7,15)}_{1\,2} + k'^{(14,15)}_{1\,2} + k'^{(15,16)}_{1\,2}$$

$$k'_{85\,j} = 0 \;;\; j = 87,\ 88,\ 89$$

$$k'_{85\,90} = k'^{(14,15)}_{1\,6} - k'^{(15,16)}_{1\,6}$$

$$k'_{85\,91} = -\,k'^{(15,16)}_{1\,1}$$

$$k'_{85\,92} = -\,k'^{(15,16)}_{1\,2}$$

$$k'_{85\,j} = 0 \;;\; j = 93,\ 94,\ 95$$

$$k'_{85\,96} = -\,k'^{(15,16)}_{1\,6}$$

$$k'_{85\,j} = 0 \;;\; j = 97,\ 98,\ 99,\ \ldots,\ 114$$

$$k'_{86\,86} = k'^{(7,15)}_{2\,2} + k'^{(14,15)}_{2\,2} + k'^{(15,16)}_{2\,2}$$

$$k'_{86\,j} = 0 \;;\; j = 87,\ 88,\ 89$$

$$k'_{86\,90} = -\,k'^{(14,15)}_{2\,6} + k'^{(15,16)}_{2\,6}$$

$$k'_{86\,91} = -\,k'^{(15,16)}_{1\,2}$$

$$k'_{86\,92} = -\,k'^{(15,16)}_{2\,2}$$

$$k'_{86\,j} = 0 \;;\; j = 93,\ 94,\ 95$$

$$k'_{86\,96} = k'^{(15,16)}_{2\,6}$$

$$k'_{86\,j} = 0 \;;\; j = 97,\ 98,\ 99,\ \ldots,\ 114$$

$$k'_{87\ 87} = k'^{(7,15)}_{3\ 3} + k'^{(14,15)}_{3\ 3} + k'^{(15,16)}_{3\ 3}$$

$$k'_{87\ 88} = - k'^{(14,15)}_{3\ 4} + k'^{(15,16)}_{3\ 4}$$

$$k'_{87\ 89} = k'^{(14,15)}_{3\ 5} - k'^{(15,16)}_{3\ 5}$$

$$k'_{87\ j} = 0 \ ; \ j = 90,\ 91,\ 92$$

$$k'_{87\ 93} = - k'^{(15,16)}_{3\ 3}$$

$$k'_{87\ 94} = k'^{(15,16)}_{3\ 4}$$

$$k'_{87\ 95} = - k'^{(15,16)}_{3\ 5}$$

$$k'_{87\ j} = 0 \ ; \ j = 96,\ 97,\ 98,\ \ldots,\ 114$$

$$k'_{88\ 88} = k'^{(7,15)}_{4\ 4} + k'^{(14,15)}_{4\ 4} + k'^{(15,16)}_{4\ 4}$$

$$k'_{88\ 89} = k'^{(7,15)}_{4\ 5} + k'^{(14,15)}_{4\ 5} + k'^{(15,16)}_{4\ 5}$$

$$k'_{88\ j} = 0 \ ; \ j = 90,\ 91,\ 92$$

$$k'_{88\ 93} = - k'^{(15,16)}_{3\ 4}$$

$$k'_{88\ 94} = k'^{(15,16)}_{4\ 10}$$

$$k'_{88\ 95} = k'^{(15,16)}_{4\ 11}$$

$$k'_{88\ j} = 0 \ ; \ j = 96,\ 97,\ 98,\ \ldots,\ 114$$

$$k'_{89\,89} = k'^{(7,15)}_{5\,5} + k'^{(14,15)}_{5\,5} + k'^{(15,16)}_{5\,5}$$

$$k'_{89\,j} = 0 \;;\; j = 90,\ 91,\ 92$$

$$k'_{89\,93} = k'^{(15,16)}_{3\,5}$$

$$k'_{89\,94} = k'^{(15,16)}_{4\,11}$$

$$k'_{89\,95} = k'^{(15,16)}_{5\,11}$$

$$k'_{89\,j} = 0 \;;\; j = 96,\ 97,\ 98,\ \ldots,\ 114$$

$$k'_{90\,90} = k'^{(7,15)}_{6\,6} + k'^{(14,15)}_{6\,6} + k'^{(15,16)}_{6\,6}$$

$$k'_{90\,91} = k'^{(15,16)}_{1\,6}$$

$$k'_{90\,92} = -\,k'^{(15,16)}_{2\,6}$$

$$k'_{90\,j} = 0 \;;\; j = 93,\ 94,\ 95$$

$$k'_{90\,96} = k'^{(15,16)}_{6\,12}$$

$$k'_{90\,j} = 0 \;;\; j = 97,\ 98,\ 99,\ \ldots,\ 114$$

$$k'_{91\,91} = k'^{(15,16)}_{1\,1} + k'^{(16,17)}_{1\,1}$$

$$k'_{91\,92} = k'^{(15,16)}_{1\,2} + k'^{(16,17)}_{1\,2}$$

$$k'_{91\,j} = 0 \; ; \; j = 93,\ 94,\ 95$$

$$k'_{91\,96} = k'^{(15,16)}_{1\,6} - k'^{(16,17)}_{1\,6}$$

$$k'_{91\,97} = - k'^{(16,17)}_{1\,1}$$

$$k'_{91\,98} = - k'^{(16,17)}_{1\,2}$$

$$k'_{91\,j} = 0 \; ; \; j = 99,\ 100,\ 101$$

$$k'_{91\,102} = - k'^{(16,17)}_{1\,6}$$

$$k'_{91\,j} = 0 \; ; \; j = 103,\ 104,\ 105,\ \ldots,\ 114$$

$$k'_{92\,92} = k'^{(15,16)}_{2\,2} + k'^{(16,17)}_{2\,2}$$

$$k'_{92\,j} = 0 \; ; \; j = 93,\ 94,\ 95$$

$$k'_{92\,96} = - k'^{(15,16)}_{2\,6} + k'^{(16,17)}_{2\,6}$$

$$k'_{92\,97} = - k'^{(16,17)}_{1\,2}$$

$$k'_{92\,98} = - k'^{(16,17)}_{2\,2}$$

$$k'_{92\,j} = 0 \; ; \; j = 99,\ 100,\ 101$$

$$k'_{92\,102} = k'^{(16,17)}_{2\,6}$$

$$k'_{92\,j} = 0 \; ; \; j = 103,\ 104,\ 105,\ \ldots,\ 114$$

$$k'_{93\ 93} = k'^{(15,16)}_{3\ 3} + k'^{(16,17)}_{3\ 3}$$

$$k'_{93\ 94} = - k'^{(15,16)}_{3\ 4} + k'^{(16,17)}_{3\ 4}$$

$$k'_{93\ 95} = k'^{(15,16)}_{3\ 5} - k'^{(16,17)}_{3\ 5}$$

$$k'_{93\ j} = 0 \ ; \ j = 96,\ 97,\ 98$$

$$k'_{93\ 99} = - k'^{(16,17)}_{3\ 3}$$

$$k'_{93\ 100} = k'^{(16,17)}_{3\ 4}$$

$$k'_{93\ 101} = - k'^{(16,17)}_{3\ 5}$$

$$k'_{93\ j} = 0 \ ; \ j = 102,\ 103,\ 104,\ \ldots,\ 114$$

$$k'_{94\ 94} = k'^{(15,16)}_{4\ 4} + k'^{(16,17)}_{4\ 4}$$

$$k'_{94\ 95} = k'^{(15,16)}_{4\ 5} + k'^{(16,17)}_{4\ 5}$$

$$k'_{94\ j} = 0 \ ; \ j = 96,\ 97,\ 98$$

$$k'_{94\ 99} = - k'^{(16,17)}_{3\ 4}$$

$$k'_{94\ 100} = k'^{(16,17)}_{4\ 10}$$

$$k'_{94\ 101} = k'^{(16,17)}_{4\ 11}$$

$$k'_{94\ j} = 0 \ ; \ j = 102,\ 103,\ 104,\ \ldots,\ 114$$

$$k'_{95\,95} = k'^{(15,16)}_{5\,5} + k'^{(16,17)}_{5\,5}$$

$$k'_{95\,j} = 0\ ;\ j = 96,\ 97,\ 98$$

$$k'_{95\,99} = k'^{(16,17)}_{3\,5}$$

$$k'_{95\,100} = k'^{(16,17)}_{4\,11}$$

$$k'_{95\,101} = k'^{(16,17)}_{5\,11}$$

$$k'_{95\,j} = 0\ ;\ j = 102,\ 103,\ 104,\ \ldots,\ 114$$

$$k'_{96\,96} = k'^{(15,16)}_{6\,6} + k'^{(16,17)}_{6\,6}$$

$$k'_{96\,97} = k'^{(16,17)}_{1\,6}$$

$$k'_{96\,98} = -\,k'^{(16,17)}_{2\,6}$$

$$k'_{96\,j} = 0\ ;\ j = 99,\ 100,\ 101$$

$$k'_{96\,102} = k'^{(16,17)}_{6\,12}$$

$$k'_{96\,j} = 0\ ;\ j = 103,\ 104,\ 105,\ \ldots,\ 114$$

$$k'_{97\,97} = k'^{(16,17)}_{1\,1} + k'^{(17,18)}_{1\,1}$$

$$k'_{97\,98} = k'^{(16,17)}_{1\,2} + k'^{(17,18)}_{1\,2}$$

$$k'_{97\,j} = 0 \;;\; j = 99,\ 100,\ 101$$

$$k'_{97\,102} = k'^{(16,17)}_{1\,6} - k'^{(17,18)}_{1\,6}$$

$$k'_{97\,103} = -\,k'^{(17,18)}_{1\,1}$$

$$k'_{97\,104} = -\,k'^{(17,18)}_{1\,2}$$

$$k'_{97\,j} = 0 \;;\; j = 105,\ 106,\ 107$$

$$k'_{97\,108} = -\,k'^{(17,18)}_{1\,6}$$

$$k'_{97\,j} = 0 \;;\; j = 109,\ 110,\ 111,\ \ldots,\ 114$$

$$k'_{98\,98} = k'^{(16,17)}_{2\,2} + k'^{(17,18)}_{2\,2}$$

$$k'_{98\,j} = 0 \;;\; j = 99,\ 100,\ 101$$

$$k'_{98\,102} = -\,k'^{(16,17)}_{2\,6} + k'^{(17,18)}_{2\,6}$$

$$k'_{98\,103} = -\,k'^{(17,18)}_{1\,2}$$

$$k'_{98\,104} = -\,k'^{(17,18)}_{2\,2}$$

$$k'_{98\,j} = 0 \;;\; j = 105,\ 106,\ 107$$

$$k'_{98\,108} = k'^{(17,18)}_{2\,6}$$

$$k'_{98\,j} = 0 \;;\; j = 109,\ 110,\ 111,\ \ldots,\ 114$$

$$k'_{99\ 99} = k'^{(16,17)}_{3\ 3} + k'^{(17,18)}_{3\ 3}$$

$$k'_{99\ 100} = - k'^{(16,17)}_{3\ 4} + k'^{(17,18)}_{3\ 4}$$

$$k'_{99\ 101} = k'^{(16,17)}_{3\ 5} - k'^{(17,18)}_{3\ 5}$$

$$k'_{99\ j} = 0 \ ; \ j = 102,\ 103,\ 104$$

$$k'_{99\ 105} = - k'^{(17,18)}_{3\ 3}$$

$$k'_{99\ 106} = k'^{(17,18)}_{3\ 4}$$

$$k'_{99\ 107} = - k'^{(17,18)}_{3\ 5}$$

$$k'_{99\ j} = 0 \ ; \ j = 108,\ 109,\ 110,\ \ldots,\ 114$$

$$k'_{100\ 100} = k'^{(16,17)}_{4\ 4} + k'^{(17,18)}_{4\ 4}$$

$$k'_{100\ 101} = k'^{(16,17)}_{4\ 5} + k'^{(17,18)}_{4\ 5}$$

$$k'_{100\ j} = 0 \ ; \ j = 102,\ 103,\ 104$$

$$k'_{100\ 105} = - k'^{(17,18)}_{3\ 4}$$

$$k'_{100\ 106} = k'^{(17,18)}_{4\ 10}$$

$$k'_{100\ 107} = k'^{(17,18)}_{4\ 11}$$

$$k'_{100\ j} = 0 \ ; \ j = 108,\ 109,\ 110,\ \ldots,\ 114$$

$$k'_{101\ 101} = k'^{(16,17)}_{5\ 5} + k'^{(17,18)}_{5\ 5}$$

$$k'_{101\ j} = 0 \ ; \ j = 102,\ 103,\ 104$$

$$k'_{101\ 105} = k'^{(17,18)}_{3\ 5}$$

$$k'_{101\ 106} = k'^{(17,18)}_{4\ 11}$$

$$k'_{101\ 107} = k'^{(17,18)}_{5\ 11}$$

$$k'_{101\ j} = 0 \ ; \ j = 108,\ 109,\ 110,\ \ldots,\ 114$$

$$k'_{102\ 102} = k'^{(16,17)}_{6\ 6} + k'^{(17,18)}_{6\ 6}$$

$$k'_{102\ 103} = k'^{(17,18)}_{1\ 6}$$

$$k'_{102\ 104} = -\ k'^{(17,18)}_{2\ 6}$$

$$k'_{102\ j} = 0 \ ; \ j = 105,\ 106,\ 107$$

$$k'_{102\ 108} = k'^{(17,18)}_{6\ 12}$$

$$k'_{102\ j} = 0 \ ; \ j = 109,\ 110,\ 111,\ \ldots,\ 114$$

$$k'_{103\ 103} = k'^{(17,18)}_{1\ 1} + k'^{(18,19)}_{1\ 1}$$

$$k'_{103\ 104} = k'^{(17,18)}_{1\ 2} + k'^{(18,19)}_{1\ 2}$$

$$k'_{103\ j} = 0 \ ; \ j = 105,\ 106,\ 107$$

$$k'_{103\ 108} = k'^{(17,18)}_{1\ 6} - k'^{(18,19)}_{1\ 6}$$

$$k'_{103\ 109} = - k'^{(18,19)}_{1\ 1}$$

$$k'_{103\ 110} = - k'^{(18,19)}_{1\ 2}$$

$$k'_{103\ j} = 0 \ ; \ j = 111,\ 112,\ 113$$

$$k'_{103\ 114} = - k'^{(18,19)}_{1\ 6}$$

$$k'_{104\ 104} = k'^{(17,18)}_{2\ 2} + k'^{(18,19)}_{2\ 2}$$

$$k'_{104\ j} = 0 \ ; \ j = 105,\ 106,\ 107$$

$$k'_{104\ 108} = - k'^{(17,18)}_{2\ 6} + k'^{(18,19)}_{2\ 6}$$

$$k'_{104\ 109} = - k'^{(18,19)}_{1\ 2}$$

$$k'_{104\ 110} = - k'^{(18,19)}_{2\ 2}$$

$$k'_{104\ j} = 0 \ ; \ j = 111,\ 112,\ 113$$

$$k'_{104\ 114} = k'^{(18,19)}_{2\ 6}$$

$$k'_{105\ 105} = k'^{(17,18)}_{3\ 3} + k'^{(18,19)}_{3\ 3}$$

$$k'_{105\ 106} = -\ k'^{(17,18)}_{3\ 4} + k'^{(18,19)}_{3\ 4}$$

$$k'_{105\ 107} = k'^{(17,18)}_{3\ 5} - k'^{(18,19)}_{3\ 5}$$

$$k'_{105\ j} = 0\ ;\ j = 108,\ 109,\ 110$$

$$k'_{105\ 111} = -\ k'^{(18,19)}_{3\ 3}$$

$$k'_{105\ 112} = k'^{(18,19)}_{3\ 4}$$

$$k'_{105\ 113} = -\ k'^{(18,19)}_{3\ 5}$$

$$k'_{105\ 114} = 0$$

$$k'_{106\ 106} = k'^{(17,18)}_{4\ 4} + k'^{(18,19)}_{4\ 4}$$

$$k'_{106\ 107} = k'^{(17\ 18)}_{4\ 5} + k'^{(18,19)}_{4\ 5}$$

$$k'_{106\ j} = 0\ ;\ j = 108,\ 109,\ 110$$

$$k'_{106\ 111} = -\ k'^{(18,19)}_{3\ 4}$$

$$k'_{106\ 112} = k'^{(18,19)}_{4\ 10}$$

$$k'_{106\ 113} = k'^{(18,19)}_{4\ 11}$$

$$k'_{106\ 114} = 0$$

$$k'_{107\ 107} = k'^{(17,18)}_{5\ 5} + k'^{(18,19)}_{5\ 5}$$

$$k'_{107\ j} = 0\ ;\ j = 108,\ 109\ 110$$

$$k'_{107\ 111} = k'^{(18,19)}_{3\ 5}$$

$$k'_{107\ 112} = k'^{(18,19)}_{4\ 11}$$

$$k'_{107\ 113} = k'^{(18,19)}_{5\ 11}$$

$$k'_{107\ 114} = 0$$

$$k'_{108\ 108} = k'^{(17,18)}_{6\ 6} + k'^{(18,19)}_{6\ 6}$$

$$k'_{108\ 109} = k'^{(18,19)}_{1\ 6}$$

$$k'_{108\ 110} = -\ k'^{(18,19)}_{2\ 6}$$

$$k'_{108\ j} = 0\ ;\ j = 111,\ 112,\ 113$$

$$k'_{108\ 114} = k'^{(18,19)}_{6\ 12}$$

$$k'_{109\ 109} = k'^{(18,19)}_{1\ 1}$$

$$k'_{109\ 110} = k'^{(18,19)}_{1\ 2}$$

$$k'_{109\ j} = 0\ ;\ j = 111,\ 112,\ 113$$

$$k'_{109\ 114} = k'^{(18,19)}_{1\ 6}$$

$$k'_{110\ 110} = k'^{(18,19)}_{2\ 2}$$

$$k'_{110\ j} = 0 \ ; \ j = 111,\ 112,\ 113$$

$$k'_{110\ 114} = -\ k'^{(18,19)}_{2\ 6}$$

$$k'_{111\ 111} = k'^{(18,19)}_{3\ 3}$$

$$k'_{111\ 112} = -\ k'^{(18,19)}_{3\ 4}$$

$$k'_{111\ 113} = k'^{(18,19)}_{3\ 5}$$

$$k'_{111\ 114} = 0$$

$$k'_{112\ 112} = k'^{(18,19)}_{4\ 4}$$

$$k'_{112\ 113} = k'^{(18,19)}_{4\ 5}$$

$$k'_{112\ 114} = 0$$

$$k'_{113\ 113} = k'^{(18,19)}_{5\ 5}$$

$$k'_{113\ 114} = 0$$

$$k'_{114\ 114} = k'^{(18,19)}_{6\ 6}$$

APPENDIX B

This appendix presents the elements of the 114 x 114 system inertia matrix, relative to earth-fixed coordinates, in terms of the elements of the individual (sub-structure) inertia matrices.

$m'_{1\,1} = m_1$

$m'_{1\,j} = 0 \; ; \; j = 2, 3, 4, \ldots, 114$

$m'_{2\,2} = m_1$

$m'_{2\,j} = 0 \; ; \; j = 3, 4, 5, \ldots, 114$

$m'_{3\,3} = m_1$

$m'_{3\,j} = 0 \; ; \; j = 4, 5, 6, \ldots, 114$

$m'_{4\,4} = I_{x_1} \, Cos^2 \beta_o + I_{y_1} \, Sin^2 \beta_o$

$m'_{4\,5} = (I_{x_1} - I_{y_1}) \, Cos \, \beta_o \, Sin \, \beta_o$

$m'_{4\,j} = 0 \; ; \; j = 6, 7, 8, \ldots, 114$

$m'_{5\,5} = I_{x_1} \, Sin^2 \beta_o + I_{y_1} \, Cos^2 \beta_o$

$m'_{5\,j} = 0 \; ; \; j = 6, 7, 8, \ldots, 114$

$m'_{6\,6} = I_{z_1}$

$m'_{6\,j} = 0 \; ; \; j = 7, 8, 9, \ldots, 114$

$$m'_{7\,7} = m_2$$

$$m'_{7\,j} = 0 \; ; \; j = 8, 9, 10, \ldots, 114$$

$$m'_{8\,8} = m_2$$

$$m'_{8\,j} = 0 \; ; \; j = 9, 10, 11, \ldots, 114$$

$$m'_{9\,9} = m_2$$

$$m'_{9\,j} = 0 \; ; \; j = 10, 11, 12, \ldots, 114$$

$$m'_{10\,10} = I_{x_2} \cos^2\beta_o + I_{y_2} \sin^2\beta_o$$

$$m'_{10\,11} = (I_{x_2} - I_{y_2}) \sin\beta_o \cos\beta_o$$

$$m'_{10\,j} = 0 \; ; \; j = 12, 13, 14, \ldots, 114$$

$$m'_{11\,11} = I_{x_2} \sin^2\beta_o + I_{y_2} \cos^2\beta_o$$

$$m'_{11\,j} = 0 \; ; \; j = 12, 13, 14, \ldots, 114$$

$$m'_{12\,12} = I_{z_2}$$

$$m'_{12\,j} = 0 \; ; \; j = 13, 14, 15, \ldots, 114$$

$m'_{13\ 13} = m_3$

$m'_{13\ j} = 0\ ;\ j = 14, 15, 16, \ldots, 114$

$m'_{14\ 14} = m_3$

$m'_{14\ j} = 0\ ;\ j = 15, 16, 17, \ldots, 114$

$m'_{15\ 15} = m_3$

$m'_{15\ j} = 0\ ;\ j = 16, 17, 18, \ldots, 114$

$m'_{16\ 16} = I_{x_3}\ \mathrm{Cos}^2\beta_o + I_{y_3}\ \mathrm{Sin}^2\beta_o$

$m'_{16\ 17} = (I_{x_3} - I_{y_3})\ \mathrm{Sin}\,\beta_o\ \mathrm{Cos}\,\beta_o$

$m'_{16\ j} = 0\ ;\ j = 18, 19, 20, \ldots, 114$

$m'_{17\ 17} = I_{x_3}\ \mathrm{Sin}^2\beta_o + I_{y_3}\ \mathrm{Cos}\,\beta_o$

$m'_{17\ j} = 0\ ;\ j = 18, 19, 20, \ldots, 114$

$m'_{18\ 18} = I_{z_3}$

$m'_{18\ j} = 0\ ;\ j = 19, 20, 21, \ldots, 114$

$m'_{25\ 25} = m_5$

$m'_{25\ j} = 0\ ;\ j = 26, 27, 28, \ldots, 114$

$m'_{26\ 26} = m_5$

$m'_{26\ j} = 0\ ;\ j = 27, 28, 29, \ldots, 114$

$m'_{27\ 27} = m_5$

$m'_{27\ j} = 0\ ;\ j = 28, 29, 30, \ldots, 114$

$m'_{28\ 28} = I_{x_5} \cos^2 \beta_5 + I_{y_5} \sin^2 \beta_5$

$m'_{28\ 29} = (I_{x_5} - I_{y_5}) \sin \beta_5 \cos \beta_5$

$m'_{28\ j} = 0\ ;\ j = 30, 31, 32, \ldots, 114$

$m'_{29\ 29} = I_{x_5} \sin^2 \beta_5 + I_{y_5} \cos^2 \beta_5$

$m'_{29\ j} = 0\ ;\ j = 30, 31, 32, \ldots, 114$

$m'_{30\ 30} = I_{z_5}$

$m'_{30\ j} = 0\ ;\ j = 31, 32, 33, \ldots, 114$

$$m'_{19\ 19} = m_4$$

$$m'_{19\ j} = 0 \ ; \ j = 20, 21, 22, \ldots, 114$$

$$m'_{20\ 20} = m_4$$

$$m'_{20\ j} = 0 \ ; \ j = 21, 22, 23, \ldots, 114$$

$$m'_{21\ 21} = m_4$$

$$m'_{21\ j} = 0 \ ; \ j = 22, 23, 24, \ldots, 114$$

$$m'_{22\ 22} = I_{x_4} \cos^2\beta_4 + I_{y_4} \sin^2\beta_4$$

$$m'_{22\ 23} = (I_{x_4} - I_{y_4}) \sin\beta_4 \cos\beta_4$$

$$m'_{22\ j} = 0 \ ; \ j = 24, 25, 26, \ldots, 114$$

$$m'_{23\ 23} = I_{x_4} \sin^2\beta_4 + I_{y_4} \cos^2\beta_4$$

$$m'_{23\ j} = 0 \ ; \ j = 24, 25, 26, \ldots, 114$$

$$m'_{24\ 24} = I_{z_4}$$

$$m'_{24\ j} = 0 \ ; \ j = 25, 26, 27, \ldots, 114$$

$$m'_{31\,31} = m_6$$

$$m'_{31\,j} = 0 \;;\; j = 32, 33, 34, \ldots, 114$$

$$m'_{32\,32} = m_6$$

$$m'_{32\,j} = 0 \;;\; j = 33, 34, 35, \ldots, 114$$

$$m'_{33\,33} = m_6$$

$$m'_{33\,j} = 0 \;;\; j = 34, 35, 36, \ldots, 114$$

$$m'_{34\,34} = I_{x_6} \cos^2 \beta_6 + I_{y_6} \sin^2 \beta_6$$

$$m'_{34\,35} = (I_{x_6} - I_{y_6}) \sin \beta_6 \cos \beta_6$$

$$m'_{34\,j} = 0 \;;\; j = 36, 37, 38, \ldots, 114$$

$$m'_{35\,35} = I_{x_6} \sin^2 \beta_6 + I_{y_6} \cos^2 \beta_6$$

$$m'_{35\,j} = 0 \;;\; j = 36, 37, 38, \ldots, 114$$

$$m'_{36\,36} = I_{z_6}$$

$$m'_{36\,j} = 0 \;;\; j = 37, 38, 39, \ldots, 114$$

$$m'_{37\ 37} = m_7$$

$$m'_{37\ j} = 0 \ ; \ j = 38, 39, 40, \ldots, 114$$

$$m'_{38\ 38} = m_7$$

$$m'_{38\ j} = 0 \ ; \ j = 39, 40, 41, \ldots, 114$$

$$m'_{39\ 39} = m_7$$

$$m'_{39\ j} = 0 \ ; \ j = 40, 41, 42, \ldots, 114$$

$$m'_{40\ 40} = I_{x_7} \text{Cos}^2 \beta_7 + I_{y_7} \text{Sin}^2 \beta_7$$

$$m'_{40\ 41} = (I_{x_7} - I_{y_7}) \text{Sin}\, \beta_7 \text{Cos}\, \beta_7$$

$$m'_{40\ j} = 0 \ ; \ j = 42, 43, 44, \ldots, 114$$

$$m'_{41\ 41} = I_{x_7} \text{Sin}^2 \beta_7 + I_{y_7} \text{Cos}^2 \beta_7$$

$$m'_{41\ j} = 0 \ ; \ j = 42, 43, 44, \ldots, 114$$

$$m'_{42\ 42} = I_{z_7}$$

$$m'_{42\ j} = 0 \ ; \ j = 43, 44, 45, \ldots, 114$$

$$m'_{43\ 43} = m_8$$

$$m'_{43\ j} = 0 \ ; \ j = 44, 45, 46, \ldots, 114$$

$$m'_{44\ 44} = m_8$$

$$m'_{44\ j} = 0 \ ; \ j = 45, 46, 47, \ldots, 114$$

$$m'_{45\ 45} = m_8$$

$$m'_{45\ j} = 0 \ ; \ j = 46, 47, 48, \ldots, 114$$

$$m'_{46\ 46} = I_{x_8} \operatorname{Cos}^2 \beta_o + I_{y_8} \operatorname{Sin}^2 \beta_o$$

$$m'_{46\ 47} = (I_{x_8} - I_{y_8}) \operatorname{Sin} \beta_o \operatorname{Cos} \beta_o$$

$$m'_{46\ j} = 0 \ ; \ j = 48, 49, 50, \ldots, 114$$

$$m'_{47\ 47} = I_{x_8} \operatorname{Sin}^2 \beta_o + I_{y_8} \operatorname{Cos}^2 \beta_o$$

$$m'_{47\ j} = 0 \ ; \ j = 48, 49, 50, \ldots, 114$$

$$m'_{48\ 48} = I_{z_8}$$

$$m'_{48\ j} = 0 \ ; \ j = 49, 50, 51, \ldots, 114$$

$m'_{49\ 49} = m_9$

$m'_{49\ j} = 0 \ ; \ j = 50, 51, 52, \ldots, 114$

$m'_{50\ 50} = m_9$

$m'_{50\ j} = 0 \ ; \ j = 51, 52, 53, \ldots, 114$

$m'_{51\ 51} = m_9$

$m'_{51\ j} = 0 \ ; \ j = 52, 53, 54, \ldots, 114$

$m'_{52\ 52} = I_{x_9} \, \mathrm{Cos}^2 \beta_o + I_{y_9} \, \mathrm{Sin}^2 \beta_o$

$m'_{52\ 53} = (I_{x_9} - I_{y_9}) \, \mathrm{Sin}\, \beta_o \, \mathrm{Cos}\, \beta_o$

$m'_{52\ j} = 0 \ ; \ j = 54, 55, 56, \ldots, 114$

$m'_{53\ 53} = I_{x_9} \, \mathrm{Sin}^2 \beta_o + I_{y_9} \, \mathrm{Cos}^2 \beta_o$

$m'_{53\ j} = 0 \ ; \ j = 54, 55, 56, \ldots, 114$

$m'_{54\ 54} = I_{z_9}$

$m'_{54\ j} = 0 \ ; \ j = 55, 56, 57, \ldots, 114$

$$m'_{55\ 55} = m_{10}$$

$$m'_{55\ j} = 0\ ;\ j = 56, 57, 58, \ldots, 114$$

$$m'_{56\ 56} = m_{10}$$

$$m'_{56\ j} = 0\ ;\ j = 57, 58, 59, \ldots, 114$$

$$m'_{57\ 57} = m_{10}$$

$$m'_{57\ j} = 0\ ;\ j = 58, 59, 60, \ldots, 114$$

$$m'_{58\ 58} = I_{x_{10}} \cos^2\beta_o + I_{y_{10}} \sin^2\beta_o$$

$$m'_{58\ 59} = (I_{x_{10}} - I_{y_{10}}) \sin\beta_o \cos\beta_o$$

$$m'_{58\ j} = 0\ ;\ j = 60, 61, 62, \ldots, 114$$

$$m'_{59\ 59} = I_{x_{10}} \sin^2\beta_o + I_{y_{10}} \cos^2\beta_o$$

$$m'_{59\ j} = 0\ ;\ j = 60, 61, 62, \ldots, 114$$

$$m'_{60\ 60} = I_{z_{10}}$$

$$m'_{60\ j} = 0\ ;\ j = 61, 62, 63, \ldots, 114$$

$$m'_{61\,61} = m_{11}$$

$$m'_{61\,j} = 0 \; ; \; j = 62, 63, 64, \ldots, 114$$

$$m'_{62\,62} = m_{11}$$

$$m'_{62\,j} = 0 \; ; \; j = 63, 64, 65, \ldots, 114$$

$$m'_{63\,63} = m_{11}$$

$$m'_{63\,j} = 0 \; ; \; j = 64, 65, 66, \ldots, 114$$

$$m'_{64\,64} = I_{x_{11}} \cos^2\beta_{11} + I_{y_{11}} \sin^2\beta_{11}$$

$$m'_{64\,65} = (I_{x_{11}} - I_{y_{11}}) \sin\beta_{11} \cos\beta_{11}$$

$$m'_{64\,j} = 0 \; ; \; j = 66, 67, 68, \ldots, 114$$

$$m'_{65\,65} = I_{x_{11}} \sin^2\beta_{11} + I_{y_{11}} \cos^2\beta_{11}$$

$$m'_{65\,j} = 0 \; ; \; j = 66, 67, 68, \ldots, 114$$

$$m'_{66\,66} = I_{z_{11}}$$

$$m'_{66\,j} = 0 \; ; \; j = 67, 68, 69, \ldots, 114$$

$$m'_{67\,67} = m_{12}$$

$$m'_{67\,j} = 0 \; ; \; j = 68, 69, 70, \ldots, 114$$

$$m'_{68\,68} = m_{12}$$

$$m'_{68\,j} = 0 \; ; \; j = 69, 70, 71, \ldots, 114$$

$$m'_{69\,69} = m_{12}$$

$$m'_{69\,j} = 0 \; ; \; j = 70, 71, 72, \ldots, 114$$

$$m'_{70\,70} = I_{x_{12}} \cos^2 \beta_{12} + I_{y_{12}} \sin^2 \beta_{12}$$

$$m'_{70\,71} = (I_{x_{12}} - I_{y_{12}}) \sin \beta_{12} \cos \beta_{12}$$

$$m'_{70\,j} = 0 \; ; \; j = 72, 73, 74, \ldots, 114$$

$$m'_{71\,71} = I_{x_{12}} \sin^2 \beta_{12} + I_{y_{12}} \cos^2 \beta_{12}$$

$$m'_{71\,j} = 0 \; ; \; j = 72, 73, 74, \ldots, 114$$

$$m'_{72\,72} = I_{z_{12}}$$

$$m'_{72\,j} = 0 \; ; \; j = 73, 74, 75, \ldots, 114$$

$$m'_{73\,73} = m_{13}$$

$$m'_{73\,j} = 0 \;;\; j = 74, 75, 76, \ldots, 114$$

$$m'_{74\,74} = m_{13}$$

$$m'_{74\,j} = 0 \;;\; j = 75, 76, 77, \ldots, 114$$

$$m'_{75\,75} = m_{13}$$

$$m'_{75\,j} = 0 \;;\; j = 76, 77, 78, \ldots, 114$$

$$m'_{76\,76} = I_{x_{13}} \operatorname{Cos}^2 \beta_{13} + I_{y_{13}} \operatorname{Sin}^2 \beta_{13}$$

$$m'_{76\,77} = (I_{x_{13}} - I_{y_{13}}) \operatorname{Sin} \beta_{13} \operatorname{Cos} \beta_{13}$$

$$m'_{76\,j} = 0 \;;\; j = 78, 79, 80, \ldots, 114$$

$$m'_{77\,77} = I_{x_{13}} \operatorname{Sin}^2 \beta_{13} + I_{y_{13}} \operatorname{Cos}^2 \beta_{13}$$

$$m'_{77\,j} = 0 \;;\; j = 78, 79, 80, \ldots, 114$$

$$m'_{78\,78} = I_{z_{13}}$$

$$m'_{78\,j} = 0 \;;\; j = 79, 80, 81, \ldots, 114$$

$$m'_{79\ 79} = m_{14}$$

$$m'_{79\ j} = 0 \ ; \ j = 80, 81, 82, \ldots, 114$$

$$m'_{80\ 80} = m_{14}$$

$$m'_{80\ j} = 0 \ ; \ j = 81, 82, 83, \ldots, 114$$

$$m'_{81\ 81} = m_{14}$$

$$m'_{81\ j} = 0 \ ; \ j = 82, 83, 84, \ldots, 114$$

$$m'_{82\ 82} = I_{x_{14}} \cos^2 \beta_{14} + I_{y_{14}} \sin^2 \beta_{14}$$

$$m'_{82\ 83} = (I_{x_{14}} - I_{y_{14}}) \sin \beta_{14} \cos \beta_{14}$$

$$m'_{82\ j} = 0 \ ; \ j = 84, 85, 86, \ldots, 114$$

$$m'_{83\ 83} = I_{x_{14}} \sin^2 \beta_{14} + I_{y_{14}} \cos^2 \beta_{14}$$

$$m'_{83\ j} = 0 \ ; \ j = 84, 85, 86, \ldots, 114$$

$$m'_{84\ 84} = I_{z_{14}}$$

$$m'_{84\ j} = 0 \ ; \ j = 85, 86, 87, \ldots, 114$$

$m'_{85\ 85} = m_{15}$

$m'_{85\ j} = 0$; $j = 86, 87, 88, \ldots, 114$

$m'_{86\ 86} = m_{15}$

$m'_{86\ j} = 0$; $j = 87, 88, 89, \ldots, 114$

$m'_{87\ 87} = m_{15}$

$m'_{87\ j} = 0$; $j = 88, 89, 90, \ldots, 114$

$m'_{88\ 88} = I_{x_{15}} \operatorname{Cos}^2 \beta_{15} + I_{y_{15}} \operatorname{Sin}^2 \beta_{15}$

$m'_{88\ 89} = (I_{x_{15}} - I_{y_{15}}) \operatorname{Sin} \beta_{15} \operatorname{Cos} \beta_{15}$

$m'_{88\ j} = 0$; $j = 90, 91, 92, \ldots, 114$

$m'_{89\ 89} = I_{x_{15}} \operatorname{Sin}^2 \beta_{15} + I_{y_{15}} \operatorname{Cos}^2 \beta_{15}$

$m'_{89\ j} = 0$; $j = 90, 91, 92, \ldots, 114$

$m'_{90\ 90} = I_{z_{15}}$

$m'_{90\ j} = 0$; $j = 91, 92, 93, \ldots, 114$

$$m'_{91\,91} = m_{16}$$

$$m'_{91\,j} = 0 \; ; \; j = 92, 93, 94, \ldots, 114$$

$$m'_{92\,92} = m_{16}$$

$$m'_{92\,j} = 0 \; ; \; j = 93, 94, 95, \ldots, 114$$

$$m'_{93\,93} = m_{16}$$

$$m'_{93\,j} = 0 \; ; \; j = 94, 95, 96, \ldots, 114$$

$$m'_{94\,94} = I_{x_{16}} \operatorname{Cos}^2 \beta_{16} + I_{y_{16}} \operatorname{Sin}^2 \beta_{16}$$

$$m'_{94\,95} = (I_{x_{16}} - I_{y_{16}}) \operatorname{Sin} \beta_{16} \operatorname{Cos} \beta_{16}$$

$$m'_{94\,j} = 0 \; ; \; j = 96, 97, 98, \ldots, 114$$

$$m'_{95\,95} = I_{x_{16}} \operatorname{Sin}^2 \beta_{16} + I_{y_{16}} \operatorname{Cos}^2 \beta_{16}$$

$$m'_{95\,j} = 0 \; ; \; j = 96, 97, 98, \ldots, 114$$

$$m'_{96\,96} = I_{z_{16}}$$

$$m'_{96\,j} = 0 \; ; \; j = 97, 98, 99, \ldots, 114$$

$$m'_{97\ 97} = m_{17}$$

$$m'_{97\ j} = 0\ ;\ j = 98,\ 99,\ 100,\ \ldots,\ 114$$

$$m'_{98\ 98} = m_{17}$$

$$m'_{98\ j} = 0\ ;\ j = 99,\ 100,\ 101,\ \ldots,\ 114$$

$$m'_{99\ 99} = m_{17}$$

$$m'_{99\ j} = 0\ ;\ j = 100,\ 101,\ 102,\ \ldots,\ 114$$

$$m'_{100\ 100} = I_{x_{17}} \mathrm{Cos}^2 \beta_{17} + I_{y_{17}} \mathrm{Sin}^2 \beta_{17}$$

$$m'_{100\ 101} = (I_{x_{17}} - I_{y_{17}}) \mathrm{Sin}\, \beta_{17} \mathrm{Cos}\, \beta_{17}$$

$$m'_{100\ j} = 0\ ;\ j = 102,\ 103,\ 104,\ \ldots,\ 114$$

$$m'_{101\ 101} = I_{x_{17}} \mathrm{Sin}^2 \beta_{17} + I_{y_{17}} \mathrm{Cos}^2 \beta_{17}$$

$$m'_{101\ j} = 0\ ;\ j = 102,\ 103,\ 104,\ \ldots,\ 114$$

$$m'_{102\ 102} = I_{z_{17}}$$

$$m'_{102\ j} = 0\ ;\ j = 103,\ 104,\ 105,\ \ldots,\ 114$$

$$m'_{103\ 103} = m_{18}$$

$$m'_{103\ j} = 0\ ;\ j = 104,\ 105,\ 106,\ \ldots,\ 114$$

$$m'_{104\ 104} = m_{18}$$

$$m'_{104\ j} = 0\ ;\ j = 105,\ 106,\ 107,\ \ldots,\ 114$$

$$m'_{105\ 105} = m_{18}$$

$$m'_{105\ j} = 0\ ;\ j = 106,\ 107,\ 108,\ \ldots,\ 114$$

$$m'_{106\ 106} = I_{x_{18}} \operatorname{Cos}^2 \beta_{18} + I_{y_{18}} \operatorname{Sin}^2 \beta_{18}$$

$$m'_{106\ 107} = (I_{x_{18}} - I_{y_{18}}) \operatorname{Sin}\ {}_{18} \operatorname{Cos}\ {}_{18}$$

$$m'_{106\ j} = 0\ ;\ j = 108,\ 109,\ 110,\ \ldots,\ 114$$

$$m'_{107\ 107} = I_{x_{18}} \operatorname{Sin}^2 \beta_{18} + I_{y_{18}} \operatorname{Cos}^2 \beta_{18}$$

$$m'_{107\ j} = 0\ ;\ j = 108,\ 109,\ 110,\ \ldots,\ 114$$

$$m'_{108\ 108} = I_{z_{18}}$$

$$m'_{108\ j} = 0\ ;\ j = 109,\ 110,\ 111,\ \ldots,\ 114$$

$$m'_{109\ 109} = m_{19}$$

$$m'_{109\ j} = 0 \ ; \ j = 110, 111, 112, \ldots, 114$$

$$m'_{110\ 110} = m_{19}$$

$$m'_{110\ j} = 0 \ ; \ j = 111, 112, 113, 114$$

$$m'_{111\ 111} = m_{19}$$

$$m'_{111\ j} = 0 \ ; \ j = 112, 113, 114$$

$$m'_{112\ 112} = I_{x_{19}} \operatorname{Cos}^2 \beta_{19} + I_{y_{19}} \operatorname{Sin}^2 \beta_{19}$$

$$m'_{112\ 113} = (I_{x_{19}} - I_{y_{19}}) \operatorname{Sin} \beta_{19} \operatorname{Cos} \beta_{19}$$

$$m'_{112\ 114} = 0$$

$$m'_{113\ 113} = I_{x_{19}} \operatorname{Sin}^2 \beta_{19} + I_{y_{19}} \operatorname{Cos}^2 \beta_{19}$$

$$m'_{113\ 114} = 0$$

$$m'_{114\ 114} = I_{z_{19}}$$

APPENDIX C

This appendix presents the numerical values of the elements of the 114 x 114 system stiffness matrix.

$k'_{1\,1} = 85.48 \times 10^6$ lb/in

$k'_{1\,2} = 0.3886 \times 10^6$ lb/in

$k'_{1\,j} = 0$; $j = 3, 4, 5$

$k'_{1\,6} = -4.293 \times 10^6$ lb

$k'_{1\,7} = -85.48 \times 10^6$ lb/in

$k'_{1\,8} = -0.5573 \times 10^6$ lb/in

$k'_{1\,j} = 0$; $j = 9, 10, 11$

$k'_{1\,12} = - -2.449 \times 10^6$ lb

$k'_{1\,j} = 0$; $j = 13, 14, 15, \ldots, 42$

$k'_{1\,43} = -0.0007156 \times 10^6$ lb/in

$k'_{1\,44} = 0.08436 \times 10^6$ lb/in

$k'_{1\,j} = 0$; $j = 45, 46, 47$

$k'_{1\,48} = -0.9221 \times 10^6$ lb

$k'_{1\,49} = -0.0007156 \times 10^6$ lb/in

$k'_{1\,50} = 0.08436 \times 10^6$ lb/in

$k'_{1\,j} = 0$; $j = 51, 52, 53$

$k'_{1\,54} = -0.9221 \times 10^6$ lb/in

$k'_{1\,j} = 0$; $j = 55, 56, 57, \ldots, 114$

$k'_{2\,2} = 39.68 \times 10^6$ lb/in

$k'_{2\,j} = 0$; j = 3, 4, 5

$k'_{2\,6} = 506.1 \times 10^6$ lb

$k'_{2\,7} = -\,0.5573 \times 10^6$ lb/in

$k'_{2\,8} = -\,19.79 \times 10^6$ lb/in

$k'_{2\,j} = 0$; j = 9, 10, 11

$k'_{2\,12} = 288.7 \times 10^6$ lb

$k'_{2\,j} = 0$; j = 13, 14, 15, ..., 42

$k'_{2\,43} = 0.08436 \times 10^6$ lb/in

$k'_{2\,44} = -\,9.945 \times 10^6$ lb/in

$k'_{2\,j} = 0$; j = 45, 46, 47

$k'_{2\,48} = 108.7 \times 10^6$ lb

$k'_{2\,49} = 0.08436 \times 10^6$ lb/in

$k'_{2\,50} = -\,9.945 \times 10^6$ lb/in

$k'_{2\,j} = 0$; j = 51, 52, 53

$k'_{2\,54} = 108.7 \times 10^6$ lb

$k'_{2\,j} = 0$; j = 55, 56, 57, ..., 114

$k'_{3\,3} = 47.62 \times 10^6$ lb/in

$k'_{3\,4} = 5.006 \times 10^6$ lb

$k'_{3\,5} = -\ 590.2 \times 10^6$ lb

$k'_{3\,j} = 0\ ;\ j = 6, 7, 8$

$k'_{3\,9} = -\ 19.04 \times 10^6$ lb/in

$k'_{3\,10} = 2.356 \times 10^6$ lb

$k'_{3\,11} = -\ 277.8 \times 10^6$ lb

$k'_{3\,j} = 0\ ;\ j = 12, 13, 14, \ldots, 44$

$k'_{3\,45} = -\ 14.29 \times 10^6$ lb/in

$k'_{3\,46} = 1.325 \times 10^6$ lb

$k'_{3\,47} = -\ 156.2 \times 10^6$ lb

$j'_{3\,j} = 0\ ;\ j = 48, 49, 50$

$k'_{3\,51} = -\ 14.29 \times 10^6$ lb/in

$k'_{3\,52} = 1.325 \times 10^6$ lb

$k'_{3\,53} = -\ 156.2 \times 10^6$ lb

$k'_{3\,j} = 0\ ;\ j = 54, 55, 56, \ldots, 114$

$k'_{4\ 4} = 4421 \times 10^6$ in-lb

$k'_{4\ 5} = -142.9 \times 10^6$ in-lb

$k'_{4\ j} = 0$; $j = 6, 7, 8$

$k'_{4\ 9} = -2.356 \times 10^6$ lb

$k'_{4\ 10} = -3324 \times 10^6$ in-lb

$k'_{4\ 11} = 6.945 \times 10^6$ in-lb

$k'_{4\ j} = 0$; $j = 12, 13, 14, \ldots, 44$

$k'_{4\ 45} = -1.325 \times 10^6$ lb

$k'_{4\ 46} = -548.1 \times 10^6$ in-lb

$k'_{4\ 47} = 4.651 \times 10^6$ in-lb

$k'_{4\ j} = 0$; $j = 48, 49, 50$

$k'_{4\ 51} = -1.325 \times 10^6$ lb

$k'_{4\ 52} = -548.1 \times 10^6$ in-lb

$k'_{4\ 53} = 4.651 \times 10^6$ in-lb

$k'_{4\ j} = 0$; $j = 54, 55, 56, \ldots, 114$

$k'_{5\ 5} = 21270 \times 10^6$ in-lb

$k'_{5\ j} = 0\ ;\ j = 6,\ 7,\ 8$

$k'_{5\ 9} = 277.8 \times 10^6$ lb

$k'_{5\ 10} = 6.945 \times 10^6$ in-lb

$k'_{5\ 11} = -\ 4142 \times 10^6$ in-lb

$k'_{5\ j} = 0\ ;\ j = 12,\ 13,\ 14,\ \ldots,\ 44$

$k'_{5\ 45} = 156.2 \times 10^6$ lb

$k'_{5\ 46} = 4.651 \times 10^6$ in-lb

$k'_{5\ 47} = -\ 1096 \times 10^6$ in-lb

$k'_{5\ j} = 0\ ;\ j = 48,\ 49,\ 50$

$k'_{5\ 51} = 156.2 \times 10^6$ lb

$k'_{5\ 52} = 4.651 \times 10^6$ in-lb

$k'_{5\ 53} = -\ 1096 \times 10^6$ in-lb

$k'_{5\ j} = 0\ ;\ j = 54,\ 55,\ 56,\ \ldots,\ 114$

$k'_{6\,6} = 19000 \times 10^6$ in-lb

$k'_{6\,7} = 2.449 \times 10^6$ lb

$k'_{6\,8} = -288.7 \times 10^6$ lb

$k'_{6\,j} = 0 \; ; = 9, 10, 11$

$k'_{6\,12} = -6417 \times 10^6$ in-lb

$k'_{6\,j} = 0 \; ; \; j = 13, 14, 15, \ldots, 42$

$k'_{6\,43} = 0.9221 \times 10^6$ lb

$k'_{6\,44} = -108.7 \times 10^6$ lb

$k'_{6\,j} = 0 \; ; \; j = 45, 46, 47$

$k'_{6\,48} = 298.9 \times 10^6$ in-lb

$k'_{6\,49} = 0.9221 \times 10^6$ lb

$k'_{6\,50} = -108.7 \times 10^6$ lb

$k'_{6\,j} = 0 \; ; \; j = 51, 52, 53$

$k'_{6\,54} = 298.9 \times 10^6$ in-lb

$k'_{6\,j} = 0 \; ; \; j = 55, 56, 57, \ldots, 114$

$k'_{7\ 7} = 493.4 \times 10^6$ lb/in

$k'_{7\ 8} = 1.963 \times 10^6$ lb/in

$k'_{7\ j} = 0$; $j = 9, 10, 11$

$k'_{7\ 12} = 0.3764 \times 10^6$ lb

$k'_{7\ 13} = -356.9 \times 10^6$ lb/in

$k'_{7\ 14} = -2.239 \times 10^6$ lb/in

$k'_{7\ j} = 0$; $j = 15, 16, 17$

$k'_{7\ 18} = -5.123 \times 10^6$ lb

$k'_{7\ j} = 0$; $j = 19, 20, 21, \ldots, 42$

$k'_{7\ 43} = -0.003535 \times 10^6$ lb/in

$k'_{7\ 44} = 0.4167 \times 10^6$ lb/in

$k'_{7\ j} = 0$; $j = 45, 46, 47$

$k'_{7\ 48} = 1.525 \times 10^6$ lb

$k'_{7\ 49} = -0.003535 \times 10^6$ lb/in

$k'_{7\ 50} = 0.4167 \times 10^6$ lb/in

$k'_{7\ j} = 0$; $j = 51, 52, 53$

$k'_{7\ 54} = 1.525 \times 10^6$ lb

$k'_{7\ j} = 0$; $j = 55, 56, 57, \ldots, 114$

$k'_{8\ 8} = 263.3 \times 10^6$ lb/in

$k'_{8\ j} = 0$; $j = 9, 10, 11$

$k'_{8\ 12} = -44.20 \times 10^6$ lb

$k'_{8\ 13} = -2.239 \times 10^6$ lb/in

$k'_{8\ 14} = -92.96 \times 10^6$ lb/in

$k'_{8\ j} = 0$; $j = 15, 16, 17$

$k'_{8\ 18} = 604.1 \times 10^6$ lb

$k'_{8\ j} = 0$; $j = 19, 20, 21, \ldots, 42$

$k'_{8\ 43} = 0.4167 \times 10^6$ lb/in

$k'_{8\ 44} = -49.13 \times 10^6$ lb/in

$k'_{8\ j} = 0$; $j = 45, 46, 47$

$k'_{8\ 48} = -179.8 \times 10^6$ lb

$k'_{8\ 49} = 0.4167 \times 10^6$ lb/in

$k'_{8\ 50} = -49.13 \times 10^6$ lb/in

$k'_{8\ j} = 0$; $j = 51, 52, 53$

$k'_{8\ 54} = -179.8 \times 10^6$ lb

$k'_{8\ j} = 0$; $j = 55, 56, 57, \ldots, 114$

$k'_{9\,9} = 216.0 \times 10^6$ lb/in

$k'_{9\,10} = -0.5093 \times 10^6$ lb

$k'_{9\,11} = 60.10 \times 10^6$ lb

$k'_{9\,j} = 0 \; ; \; j = 12, 13, 14$

$k'_{9\,15} = -92.13 \times 10^6$ lb/in

$k'_{9\,16} = 5.079 \times 10^6$ lb

$k'_{9\,17} = -598.8 \times 10^6$ lb

$k'_{9\,j} = 0 \; ; \; j = 18, 19, 20, \ldots, 44$

$k'_{9\,45} = -52.07 \times 10^6$ lb/in

$k'_{9\,46} = -1.616 \times 10^6$ lb

$k'_{9\,47} = 190.6 \times 10^6$ lb

$k'_{9\,j} = 0 \; ; \; j = 48, 49, 50$

$k'_{9\,51} = -52.07 \times 10^6$ lb/in

$k'_{9\,52} = -1.616 \times 10^6$ lb

$k'_{9\,53} = 190.6 \times 10^6$ lb

$k'_{9\,j} = 0 \; ; \; j = 54, 55, 56, \ldots, 114$

$k'_{10\ 10} = 34990 \times 10^6$ in-lb

$k'_{10\ 11} = -439.4 \times 10^6$ in-lb

$k'_{10\ j} = 0$; $j = 12, 13, 14$

$k'_{10\ 15} = -5.079 \times 10^6$ lb

$k'_{10\ 16} = -16223 \times 10^6$ in-lb

$k'_{10\ 17} = 171.6 \times 10^6$ in-lb

$k'_{10\ j} = 0$; $j = 18, 19, 20, \ldots, 44$

$k'_{10\ 45} = 1.616 \times 10^6$ lb

$k'_{10\ 46} = -1637 \times 10^6$ in-lb

$k'_{10\ 47} = 51.23 \times 10^6$ in-lb

$k'_{10\ j} = 0$; $j = 48, 49, 50$

$k'_{10\ 51} = 1.616 \times 10^6$ lb

$k'_{10\ 52} = -1637 \times 10^6$ in-lb

$k'_{10\ 53} = 51.23 \times 10^6$ in-lb

$k'_{10\ j} = 0$; $= 54, 55, 56, \ldots, 114$

$k'_{11\ 11} = 86490 \times 10^6$ in-lb

$k'_{11\ j} = 0$; $j = 12, 13, 14$

$k'_{11\ 15} = 598.8 \times 10^6$ lb

$k'_{11\ 16} = 171.6 \times 10^6$ in-lb

$k'_{11\ 17} = -36450 \times 10^6$ in-lb

$k'_{11\ j} = 0$; $j = 18, 19, 20, \ldots, 44$

$k'_{11\ 45} = -190.6 \times 10^6$ lb

$k'_{11\ 46} = 51.23 \times 10^6$ in-lb

$k'_{11\ 47} = -7675 \times 10^6$ in-lb

$k'_{11\ j} = 0$; $j = 48, 49, 50$

$k'_{11\ 51} = -190.6 \times 10^6$ lb

$k'_{11\ 52} = 51.23 \times 10^6$ in-lb

$k'_{11\ 53} = -7675 \times 10^6$ in-lb

$k'_{11\ j} = 0$; $j = 54, 55, 56, \ldots, 114$

$k'_{12\ 12} = 81000 \times 10^6$ in-lb

$k'_{12\ 13} = 5.123 \times 10^6$ lb

$k'_{12\ 14} = -\ 604.1 \times 10^6$ lb

$k'_{12\ j} = 0\ ;\ j = 15,\ 16,\ 17$

$k'_{12\ 18} = -\ 51570 \times 10^6$ in-lb

$k'_{12\ j} = 0\ ;\ j = 19,\ 20,\ 21,\ \ldots,\ 42$

$k'_{12\ 43} = -\ 1.525 \times 10^6$ lb

$k'_{12\ 44} = 179.8 \times 10^6$ lb

$k'_{12\ j} = 0\ ;\ j = 45,\ 46,\ 47$

$k'_{12\ 48} = -\ 1998 \times 10^6$ in-lb

$k'_{12\ 49} = -\ 1.525 \times 10^6$ lb

$k'_{12\ 50} = 179.8 \times 10^6$ lb

$k'_{12\ j} = 0\ ;\ j = 51,\ 52,\ 53$

$k'_{12\ 54} = -\ 1998 \times 10^6$ in-lb

$k'_{12\ j} = 0\ ;\ j = 55,\ 56,\ 57,\ \ldots,\ 114$

$k'_{13\ 13} = 370.0 \times 10^6$ lb/in

$k'_{13\ 14} = 2.334 \times 10^6$ lb/in

$k'_{13\ j} = 0$; $j = 15, 16, 17$

$k'_{13\ 18} = 4.932 \times 10^6$ lb

$k'_{13\ 19} = -\ 11.48 \times 10^6$ lb/in

$k'_{13\ 20} = -\ 0.08129 \times 10^6$ lb/in

$k'_{13\ j} = 0$; $j = 21, 22, 23$

$k'_{13\ 24} = -\ 0.1910 \times 10^6$ lb

$k'_{13\ j} = 0$; $j = 25, 26, 27, \ldots, 42$

$k'_{13\ 43} = -\ 0.8000 \times 10^6$ lb/in

$k'_{13\ 44} = -\ 0.006794 \times 10^6$ lb/in

$k'_{13\ j} = 0$; $j = 45, 46, 47, 48$

$k'_{13\ 49} = -\ 0.8000 \times 10^6$ lb/in

$k'_{13\ 50} = -\ 0.006794 \times 10^6$ lb/in

$k'_{13\ j} = 0$; $j = 51, 52, 53, \ldots, 114$

$k'_{14\ 14} = 94.86 \times 10^6$ lb/in

$k'_{14\ j} = 0$; $j = 15, 16, 17$

$k'_{14\ 18} = -581.6 \times 10^6$ lb

$k'_{14\ 19} = -0.08129 \times 10^6$ lb/in

$k'_{14\ 20} = -1.897 \times 10^6$ lb/in

$k'_{14\ j} = 0$; $j = 21, 22, 23$

$k'_{14\ 24} = 22.52 \times 10^6$ lb

$k'_{14\ j} = 0$; $j = 25, 26, 27, \ldots, 42$

$k'_{14\ 43} = -0.006794 \times 10^6$ lb/in

$k'_{14\ 44} = 0$

$k'_{14\ j} = 0$; $j = 45, 46, 47, 48$

$k'_{14\ 49} = -0.006794 \times 10^6$ lb/in

$k'_{14\ 50} = 0$

$k'_{14\ j} = 0$; $j = 51, 52, 53, \ldots, 114$

$k'_{15\ 15} = 94.16 \times 10^6$ lb/in

$k'_{15\ 16} = -\ 4.875 \times 10^6$ lb

$k'_{15\ 17} = 574.7 \times 10^6$ lb

$k'_{15\ j} = 0\ ;\ j = 18,\ 19,\ 20$

$k'_{15\ 21} = -\ 2.029 \times 10^6$ lb/in

$k'_{15\ 22} = 0.2043 \times 10^6$ lb

$k'_{15\ 23} = -\ 24.09 \times 10^6$ lb

$k'_{15\ j} = 0\ ;\ j = 24,\ 25,\ 26,\ \ldots,\ 114$

$k'_{16\ 16} = 16230 \times 10^6$ in-lb

$k'_{16\ 17} = -\ 242.4 \times 10^6$ in-lb

$k'_{16\ j} = 0\ ;\ j = 18,\ 19,\ 20$

$k'_{16\ 21} = -\ 0.2043 \times 10^6$

$k'_{16\ 22} = -\ 0.08484 \times 10^6$ in-lb

$k'_{16\ 23} = -\ 0.03040 \times 10^6$ in-lb

$k'_{16\ j} = 0\ ;\ j = 24,\ 25,\ 26,\ \ldots,\ 114$

$k'_{17\,17} = 44810 \times 10^6$ in-lb

$k'_{17\,j} = 0$; $j = 18, 19, 20$

$k'_{17\,21} = 24.09 \times 10^6$ lb

$k'_{17\,22} = -0.03040 \times 10^6$ in-lb

$k'_{17\,23} = 3.504 \times 10^6$ in-lb

$k'_{17\,j} = 0$; $j = 24, 25, 26, \ldots, 114$

$k'_{18\,18} = 59930 \times 10^6$ in-lb

$k'_{18\,19} = 0.1910 \times 10^6$ lb

$k'_{18\,20} = -22.52 \times 10^6$ lb

$k'_{18\,j} = 0$; $j = 21, 22, 23$

$k'_{18\,24} = 33.23 \times 10^6$ in-lb

$k'_{18\,j} = 0$; $j = 25, 26, 27, \ldots, 114$

$k'_{19\ 19} = 21.14 \times 10^6$ lb/in

$k'_{19\ 20} = 0.1627 \times 10^6$ lb/in

$k'_{19\ j} = 0$; $j = 21, 22, 23$

$k'_{19\ 24} = 0.0263 \times 10^6$ lb

$k'_{19\ 25} = -\ 9.662 \times 10^6$ lb/in

$k'_{19\ 26} = -\ 0.08141 \times 10^6$ lb/in

$k'_{19\ j} = 0$; $j = 27, 28, 29$

$k'_{19\ 30} = -\ 0.1647 \times 10^6$ lb

$k'_{19\ j} = 0$; $j = 31, 32, 33, \ldots, 114$

$k'_{20\ 20} = 3.304 \times 10^6$ lb/in

$k'_{20\ j} = 0$; $j = 21, 22, 23$

$k'_{20\ 24} = -\ 5.82 \times 10^6$ lb/in

$k'_{20\ 25} = -\ 0.08141 \times 10^6$ lb/in

$k'_{20\ 26} = -\ 1.407 \times 10^6$ lb/in

$k'_{20\ j} = 0$; $j = 27, 28, 29$

$k'_{20\ 30} = 16.70 \times 10^6$ lb

$k'_{20\ j} = 0$; $j = 31, 32, 33, \ldots, 114$

$$k'_{21\ 21} = 3.648 \times 10^6 \text{ lb/in}$$

$$k'_{21\ 22} = -\ 0.0147 \times 10^6 \text{ lb}$$

$$k'_{21\ 23} = 4.86 \times 10^6 \text{ lb}$$

$$k'_{21\ j} = 0\ ;\ j = 24,\ 25,\ 26$$

$$k'_{21\ 27} = -\ 1.619 \times 10^6 \text{ lb/in}$$

$$k'_{21\ 28} = 0.1896 \times 10^6 \text{ lb}$$

$$k'_{21\ 29} = -\ 19.23 \times 10^6 \text{ lb}$$

$$k'_{21\ j} = 0\ ;\ j = 30,\ 31,\ 32,\ \ldots,\ 114$$

$$k'_{22\ 22} = 0.2338 \times 10^6 \text{ in-lb}$$

$$k'_{22\ 23} = -\ 9.092 \times 10^6 \text{ in-lb}$$

$$k'_{22\ j} = 0\ ;\ j = 24,\ 25,\ 26$$

$$k'_{22\ 27} = -\ 0.1896 \times 10^6 \text{ lb}$$

$$k'_{22\ 28} = -\ 0.06325 \times 10^6 \text{ in-lb}$$

$$k'_{22\ 29} = -\ 0.2322 \times 10^6 \text{ in-lb}$$

$$k'_{22\ j} = 0\ ;\ j = 30,\ 31,\ 32,\ \ldots,\ 114$$

$k'_{23\ 23} = 1002 \times 10^6$ in-lb

$k'_{23\ j} = 0 \; ; \; j = 24, 25, 26$

$k'_{23\ 27} = 19.23 \times 10^6$ lb

$k'_{23\ 28} = -\ 0.2322 \times 10^6$ in-lb

$k'_{23\ 29} = 23.48 \times 10^6$ in-lb

$k'_{23\ j} = 0 \; ; \; j = 30, 31, 32, \ldots, 114$

$k'_{24\ 24} = 845.4 \times 10^6$ in-lb

$k'_{24\ 25} = 0.1647 \times 10^6$ lb

$k'_{24\ 26} = -\ 16.70 \times 10^6$ lb

$k'_{24\ j} = 0 \; ; \; j = 27, 28, 29$

$k'_{24\ 30} = 52.79 \times 10^6$ in-lb

$k'_{24\ j} = 0 \; ; \; j = 31, 32, 33, \ldots, 114$

$k'_{25\ 25} = 18.30 \times 10^6$ lb/in

$k'_{25\ 26} = 0.1670 \times 10^6$ lb/in

$k'_{25\ j} = 0$; $j = 27, 28, 29$

$k'_{25\ 30} = 0.0282 \times 10^6$ lb

$k'_{25\ 31} = -\ 8.639 \times 10^6$ lb/in

$k'_{25\ 32} = -\ 0.08561 \times 10^6$ lb/in

$k'_{25\ j} = 0$; $j = 33, 34, 35$

$k'_{25\ 36} = -\ 0.1365 \times 10^6$ lb

$k'_{25\ j} = 0$; $j = 37, 38, 39, \ldots, 114$

$k'_{26\ 26} = 2.431 \times 10^6$ lb/in

$k'_{26\ j} = 0$; $j = 27, 28, 29,$

$k'_{26\ 30} = -\ 4.56 \times 10^6$ lb

$k'_{26\ 31} = -\ 0.08561 \times 10^6$ lb/in

$k'_{26\ 32} = -\ 1.024 \times 10^6$ lb/in

$k'_{26\ j} = 0$; $j = 33, 34, 35$

$k'_{26\ 36} = 12.14 \times 10^6$ lb

$k'_{26\ j} = 0$; $j = 37, 38, 39, \ldots, 114$

$k'_{27\ 27} = 2.978 \times 10^6$ lb/in

$k'_{27\ 28} = -0.0082 \times 10^6$ lb

$k'_{27\ 29} = 3.09 \times 10^6$ lb

$k'_{27\ j} = 0$; j = 30, 31, 32

$k'_{27\ 33} = -1.359 \times 10^6$ lb/in

$k'_{27\ 34} = 0.1814 \times 10^6$ lb

$k'_{27\ 35} = -16.14 \times 10^6$ lb

$k'_{27\ j} = 0$; j = 36, 37, 38, ..., 114

$k'_{28\ 28} = 0.2173 \times 10^6$ in-lb

$k'_{28\ 29} = -8.176 \times 10^6$ in-lb

$k'_{28\ j} = 0$; j = 30, 31, 32

$k'_{28\ 33} = -0.1814 \times 10^6$ lb

$k'_{28\ 34} = -0.05853 \times 10^6$ in-lb

$k'_{28\ 35} = -0.4033 \times 10^6$ in-lb

$k'_{28\ j} = 0$; j = 36, 37, 38, ..., 114

$k'_{29\ 29} = 780.7 \times 10^6$ in-lb

$k'_{29\ j} = 0$; $j = 30, 31, 32$

$k'_{29\ 33} = 16.14 \times 10^6$ lb

$k'_{29\ 34} = -0.4033 \times 10^6$ in-lb

$k'_{29\ 35} = 35.82 \times 10^6$ in-lb

$k'_{29\ j} = 0$; $j = 36, 37, 38, \ldots, 114$

$k'_{30\ 30} = 574.4 \times 10^6$ in-lb

$k'_{30\ 31} = 0.1365 \times 10^6$ lb

$k'_{30\ 32} = -12.14 \times 10^6$ lb

$k'_{30\ j} = 0$; $j = 33, 34, 35$

$k'_{30\ 36} = 57.81 \times 10^6$ in-lb

$k'_{30\ j} = 0$; $j = 37, 38, 39, \ldots, 114$

$k'_{31\ 31} = 16.12 \times 10^6$ lb/in

$k'_{31\ 32} = 0.2235 \times 10^6$ lb/in

$k'_{31\ j} = 0$; $j = 33, 34, 35$

$k'_{31\ 36} = -\ 0.0171 \times 10^6$ lb

$k'_{31\ 37} = -\ 7.484 \times 10^6$ lb/in

$k'_{31\ 38} = -\ 0.1379 \times 10^6$ lb/in

$k'_{31\ j} = 0$; $j = 39, 40, 41$

$k'_{31\ 42} = -\ 0.1536 \times 10^6$ lb

$k'_{31\ j} = 0$; $j = 43, 44, 45, \ldots, 114$

$k'_{32\ 32} = 1.681 \times 10^6$ lb/in

$k'_{32\ j} = 0$; $j = 33, 34, 35$

$k'_{32\ 36} = -\ 4.536 \times 10^6$ lb

$k'_{32\ 37} = -\ 0.1379 \times 10^6$ lb/in

$k'_{32\ 38} = -\ 0.6571 \times 10^6$ lb/in

$k'_{32\ j} = 0$; $j = 39, 40, 41$

$k'_{32\ 42} = 7.604 \times 10^6$ lb

$k'_{32\ j} = 0$; $j = 43, 44, 45, \ldots, 114$

$k'_{33\ 33} = 2.488 \times 10^6$ lb/in

$k'_{33\ 34} = 0.0837 \times 10^6$ lb

$k'_{33\ 35} = 3.01 \times 10^6$ lb

$k'_{33\ j} = 0 \; ; \; j = 36, 37, 38$

$k'_{33\ 39} = -\ 1.129 \times 10^6$ lb/in

$k'_{33\ 40} = 0.2651 \times 10^6$ lb

$k'_{33\ 41} = -\ 13.13 \times 10^6$ lb

$k'_{33\ j} = 0 \; ; \; j = 42, 43, 44, \ldots, 114$

$k'_{34\ 34} = 0.2767 \times 10^6$ in-lb

$k'_{34\ 35} = -\ 9.351 \times 10^6$ in-lb

$k'_{34\ j} = 0 \; ; \; j = 36, 37, 38$

$k'_{34\ 39} = -\ 0.2651 \times 10^6$ lb

$k'_{34\ 40} = -\ 0.04311 \times 10^6$ in-lb

$k'_{34\ 41} = -\ 0.7163 \times 10^6$ in-lb

$k'_{34\ j} = 0 \; ; \; j = 42, 43, 44, \ldots, 114$

$k'_{35\,35} = 617.2 \times 10^6$ in-lb

$k'_{35\,j} = 0$; $j = 36, 37, 38$

$k'_{35\,39} = 13.13 \times 10^6$ lb

$k'_{35\,40} = -0.7163 \times 10^6$ in-lb

$k'_{35\,41} = 35.41 \times 10^6$ in-lb

$k'_{35\,j} = 0$; $j = 42, 43, 44, \ldots, 114$

$k'_{36\,36} = 362.9 \times 10^6$ in-lb

$k'_{36\,37} = 0.1536 \times 10^6$ lb

$k'_{36\,38} = -7.604 \times 10^6$ lb

$k'_{36\,j} = 0$; $j = 39, 40, 41$

$k'_{36\,42} = 44.58 \times 10^6$ in-lb

$k'_{36\,j} = 0$; $j = 43, 44, 45, \ldots, 114$

$k'_{37\ 37} = 7.488 \times 10^6$ lb/in

$k'_{37\ 38} = -0.3405 \times 10^6$ lb/in

$k'_{37\ j} = 0\ ;\ j = 39, 40, 41$

$k'_{37\ 42} = 0.1536 \times 10^6$ lb

$k'_{37\ j} = 0\ ;\ j = 43, 44, 45, \ldots, 84$

$k'_{37\ 85} = -0.004058 \times 10^6$ lb/in

$k'_{37\ 86} = 0.4784 \times 10^6$ lb/in

$k'_{37\ j} = 0\ ;\ j = 87, 88, 89, \ldots, 114$

$k'_{38\ 38} = 57.06 \times 10^6$ lb/in

$k'_{38\ j} = 0\ ;\ j = 39, 40, 41$

$k'_{38\ 42} = -7.604 \times 10^6$ lb

$k'_{38\ j} = 0\ ;\ j = 43, 44, 45, \ldots, 84$

$k'_{38\ 85} = 0.4784 \times 10^6$ lb/in

$k'_{38\ 86} = -56.40 \times 10^6$ lb/in

$k'_{38\ j} = 0\ ;\ j = 87, 88, 89, \ldots, 114$

$k'_{39\ 39} = 57.53 \times 10^6$ lb/in

$k'_{39\ 40} = -\ 0.2651 \times 10^6$ lb

$k'_{39\ 41} = 13.13 \times 10^6$ lb

$k'_{39\ j} = 0\ ;\ j = 42, 43, 44, \ldots, 86$

$k'_{39\ 87} = -\ 56.40 \times 10^6$ lb/in

$k'_{39\ j} = 0\ ;\ j = 88, 89, 90, \ldots, 114$

$k'_{40\ 40} = 649.9 \times 10^6$ in-lb

$k'_{40\ 41} = -\ 1.849 \times 10^6$ in-lb

$k'_{40\ j} = 0\ ;\ j = 42, 43, 44, \ldots, 87$

$k'_{40\ 88} = -\ 649.7 \times 10^6$ in-lb

$k'_{40\ 89} = -\ 3.597 \times 10^6$ in-lb

$k'_{40\ j} = 0\ ;\ j = 90, 91, 92, \ldots, 114$

$k'_{41\ 41} = 495.3 \times 10^6$ in-lb

$k'_{41\ j} = 0$; $j = 42, 43, 44, \ldots, 87$

$k'_{41\ 88} = -\ 3.597 \times 10^6$ in-lb

$k'_{41\ 89} = -\ 225.6 \times 10^6$ in-lb

$k'_{41\ j} = 0$; $j = 90, 91, 92, \ldots, 114$

$k'_{42\ 42} = 357.9 \times 10^6$ in-lb

$k'_{42\ j} = 0$; $j = 43, 44, 45, \ldots, 89$

$k'_{42\ 90} = -\ 225.6 \times 10^6$ in-lb

$k'_{42\ j} = 0$; $j = 91, 92, 93, \ldots, 114$

$k'_{43\ 43} = 212.0 \times 10^6$ lb/in

$k'_{43\ 44} = 0.4815 \times 10^6$ lb/in

$k'_{43\ 45} = 0$

$k'_{43\ 46} = 0.3908 \times 10^6$ lb

$k'_{43\ 47} = -309.0 \times 10^6$ lb

$k'_{43\ 48} = -1.536 \times 10^6$ lb

$k'_{43\ 49} = -76.29 \times 10^6$ lb/in

$k'_{43\ 50} = -0.09649 \times 10^6$ lb/in

$k'_{43\ 51} = 0$

$k'_{43\ 52} = 0.3908 \times 10^6$ lb

$k'_{43\ 53} = -309.0 \times 10^6$ lb

$k'_{43\ 54} = 0$

$k'_{43\ 55} = -30.60 \times 10^6$ lb/in

$k'_{43\ 56} = -0.1342 \times 10^6$ lb/in

$k'_{43\ j} = 0$; $j = 57, 58, 59, 60$

$k'_{43\ 61} = -104.4 \times 10^6$ lb/in

$k'_{43\ 62} = -0.7450 \times 10^6$ lb/in

$k'_{43\ j} = 0$; $j = 63, 64, 65$

$k'_{43\ 66} = -0.9329 \times 10^6$ lb/in

$k'_{43\ j} = 0$; $j = 67, 68, 69, \ldots, 114$

$k'_{44\ 44} = 155.3 \times 10^6$ lb/in

$k'_{44\ 45} = 0$

$k'_{44\ 46} = 262.9 \times 10^6$ lb

$k'_{44\ 47} = -0.3908 \times 10^6$ lb

$k'_{44\ 48} = 181.1 \times 10^6$ lb

$k'_{44\ 49} = -0.09649 \times 10^6$ lb/in

$k'_{44\ 50} = -64.92 \times 10^6$ lb/in

$k'_{44\ 51} = 0$

$k'_{44\ 52} = 262.9 \times 10^6$ lb

$k'_{44\ 53} = -0.3908 \times 10^6$ lb

$k'_{44\ 54} = 0$

$k'_{44\ 55} = -0.1342 \times 10^6$ lb/in

$k'_{44\ 56} = -14.78 \times 10^6$ lb/in

$k'_{44\ j} = 0$; $j = 57, 58, 59, 60$

$k'_{44\ 61} = -0.7450 \times 10^6$ lb/in

$k'_{44\ 62} = -16.53 \times 10^6$ lb/in

$k'_{44\ j} = 0$; $j = 63, 64, 65$

$k'_{44\ 66} = 110.0 \times 10^6$ lb/in

$k'_{44\ j} = 0$; $j = 67, 68, 69, \ldots, 114$

$k'_{45\ 45} = 943.4 \times 10^6$ lb/in

$k'_{45\ 46} = 1.577 \times 10^6$ lb

$k'_{45\ 47} = -186.0 \times 10^6$ lb

$k'_{45\ j} = 0$; $j = 48, 49, 50$

$k'_{45\ 51} = -288.7 \times 10^6$ lb/in

$k'_{45\ j} = 0$; $j = 52, 53, 54, 55, 56$

$k'_{45\ 57} = -565.5 \times 10^6$ lb/in

$k'_{45\ j} = 0$; $j = 58, 59, 60, 61, 62$

$k'_{45\ 63} = -22.78 \times 10^6$ lb

$k'_{45\ 64} = 1.286 \times 10^6$ lb

$k'_{45\ 65} = -151.6 \times 10^6$ lb

$k'_{45\ j} = 0$; $j = 66, 67, 68, \ldots, 114$

$k'_{46\,46} = 6618 \times 10^6$ in-lb

$k'_{46\,47} = -487.4 \times 10^6$ in-lb

$k'_{46\,48} = 0$

$k'_{46\,49} = -0.3908 \times 10^6$ lb

$k'_{46\,50} = -262.9 \times 10^6$ lb

$k'_{46\,51} = 0$

$k'_{46\,52} = -1202 \times 10^6$ in-lb

$k'_{46\,53} = 368.1 \times 10^6$ in-lb

$k'_{46\,j} = 0$; $j = 54, 55, 56, 57$

$k'_{46\,58} = -259.2 \times 10^6$ in-lb

$k'_{46\,59} = 2.354 \times 10^6$ in-lb

$k'_{46\,j} = 0$; $j = 60, 61, 62$

$k'_{46\,63} = -1.286 \times 10^6$ lb

$k'_{46\,64} = -842.2 \times 10^6$ in-lb

$k'_{46\,65} = 0.02205 \times 10^6$ in-lb

$k'_{46\,j} = 0$; $j = 66, 67, 68, \ldots, 114$

$k'_{47\ 47} = 64070 \times 10^6$ in-lb

$k'_{47\ 48} = 0$

$k'_{47\ 49} = 309.0 \times 10^6$ lb

$k'_{47\ 50} = 0.3908 \times 10^6$ lb

$k'_{47\ 51} = 0$

$k'_{47\ 52} = 368.1 \times 10^6$ in-lb

$k'_{47\ 53} = -44590 \times 10^6$ in-lb

$k'_{47\ j} = 0$; j = 54, 55, 56, 57

$k'_{47\ 58} = 2.354 \times 10^6$ in-lb

$k'_{47\ 59} = -536.7 \times 10^6$ in-lb

$k'_{47\ j} = 0$; j = 60, 61, 62

$k'_{47\ 63} = 151.6 \times 10^6$ lb

$k'_{47\ 64} = 0.02205 \times 10^6$ in-lb

$k'_{47\ 65} = -844.8 \times 10^6$ in-lb

$k'_{47\ j} = 0$; j = 66, 67, 68, ..., 114

$k'_{48\,48} = 7217 \times 10^6$ in-lb

$k'_{48\,j} = 0$; $j = 49, 50, 51, 52, 53$

$k'_{48\,54} = -\ 491.9 \times 10^6$ in-lb

$k'_{48\,j} = 0$; $j = 55, 56, 57, 58, 59$

$k'_{48\,60} = -\ 0.6834 \times 10^6$ in-lb

$k'_{48\,61} = 0.9329 \times 10^6$ lb

$k'_{48\,62} = -\ 110.0 \times 10^6$ lb

$k'_{48\,j} \doteq 0$; $j = 63, 64, 65$

$k'_{48\,66} = 131.3 \times 10^6$ in-lb

$k'_{48\,j} = 0$; $j = 67, 68, 69, \ldots, 114$

$k'_{49\ 49} = 212.0 \times 10^6$ lb/in

$k'_{49\ 50} = 0.4815 \times 10^6$ lb/in

$k'_{49\ 51} = 0$

$k'_{49\ 52} = -0.3908 \times 10^6$ lb

$k'_{49\ 53} = 309.0 \times 10^6$ lb

$k'_{49\ 54} = -1.536 \times 10^6$ lb

$k'_{49\ 55} = -30.60 \times 10^6$ lb/in

$k'_{49\ 56} = -0.1342 \times 10^6$ lb/in

$k'_{49\ j} = 0$; $j = 57, 58, 59, 60$

$k'_{49\ 61} = -104.4 \times 10^6$ lb/in

$k'_{49\ 62} = -0.7450 \times 10^6$ lb/in

$k'_{49\ j} = 0$; $j = 63, 64, 65$

$k'_{49\ 66} = -0.9329 \times 10^6$ lb/in

$k'_{49\ j} = 0$; $j = 67, 68, 69, \ldots, 114$

$k'_{50\ 50} = 155.3 \times 10^6$ lb/in

$k'_{50\ 51} = 0$

$k'_{50\ 52} = -262.9 \times 10^6$ lb

$k'_{50\ 53} = 0.3908 \times 10^6$ lb

$k'_{50\ 54} = 181.1 \times 10^6$ lb

$k'_{50\ 55} = -0.1342 \times 10^6$ lb/in

$k'_{50\ 56} = -14.78 \times 10^6$ lb/in

$k'_{50\ j} = 0$; $j = 57, 58, 59, 60$

$k'_{50\ 61} = -0.7450 \times 10^6$ lb/in

$k'_{50\ 62} = -16.53 \times 10^6$ lb/in

$k'_{50\ j} = 0$; $j = 63, 64, 65$

$k'_{50\ 66} = 110.0 \times 10^6$ lb

$k'_{50\ j} = 0$; $j = 67, 68, 69, \ldots, 114$

$k'_{51\,51} = 943.4 \times 10^6$ lb/in

$k'_{51\,52} = 1.577 \times 10^6$ lb

$k'_{51\,53} = -186.0 \times 10^6$ lb

$k'_{51\,j} = 0\,;\ j = 54, 55, 56$

$k'_{51\,57} = -565.5 \times 10^6$ lb/in

$k'_{51\,j} = 0\,;\ j = 58, 59, 60, 61, 62$

$k'_{51\,63} = -22.78 \times 10^6$ lb/in

$k'_{51\,64} = 1.286 \times 10^6$ lb

$k'_{51\,65} = -151.6 \times 10^6$ lb

$k'_{51\,j} = 0\,;\ j = 66, 67, 68, \ldots, 114$

$k'_{52\ 52} = 6618 \times 10^6$ in-lb

$k'_{52\ 53} = -\ 487.4 \times 10^6$ in-lb

$k'_{52\ j} = 0$; $j = 54, 55, 56, 57$

$k'_{52\ 58} = -\ 259.2 \times 10^6$ in-lb

$k'_{52\ 59} = 2.354 \times 10^6$ in-lb

$k'_{52\ j} = 0$; $j = 60, 61, 62$

$k'_{52\ 63} = -\ 1.286 \times 10^6$ lb

$k'_{52\ 64} = -\ 842.2 \times 10^6$ in-lb

$k'_{52\ 65} = 0.02205 \times 10^6$ in-lb

$k'_{52\ j} = 0$; $j = 66, 67, 68, \ldots, 114$

$k'_{53\ 53} = 64070 \times 10^6$ in-lb

$k'_{53\ j} = 0$; $j = 54, 55, 56, 57$

$k'_{53\ 58} = 2.354\ 10^6$ in-lb

$k'_{53\ 59} = -536.7 \times 10^6$ in-lb

$k'_{53\ j} = 0$; $j = 60, 61, 62$

$k'_{53\ 63} = 151.6 \times 10^6$ lb

$k'_{53\ 64} = 0.02205 \times 10^6$ in-lb

$k'_{53\ 65} = -844.8 \times 10^6$ in-lb

$k'_{53\ j} = 0$; $j = 66, 67, 68, \ldots, 114$

$k'_{54\ 54} = 7217 \times 10^6$ in-lb

$k'_{54\ j} = 0$; $= 55, 56, 57, 58, 59$

$k'_{54\ 60} = -0.6834 \times 10^6$ in-lb

$k'_{54\ 61} = 0.9329 \times 10^6$ lb

$k'_{54\ 62} = -110.0 \times 10^6$ lb

$k'_{54\ j} = 0$; $j = 63, 64, 65$

$k'_{54\ 66} = 131.3 \times 10^6$ in-lb

$k'_{54\ j} = 0$; $j = 67, 68, 69, \ldots, 114$

$k'_{55\,55} = 61.20 \times 10^6$ lb/in

$k'_{55\,56} = 0.2685 \times 10^6$ lb/in

$k'_{55\,j} = 0$; $j = 57, 58, 59, \ldots, 114$

$k'_{56\,56} = 29.55 \times 10^6$ lb/in

$k'_{56\,j} = 0$; $j = 57, 58, 59, \ldots, 114$

$k'_{57\,57} = 1131 \times 10^6$ lb/in

$k'_{57\,j} = 0$; $j = 58, 59, 60, \ldots, 114$

$k'_{58\,58} = 518.3 \times 10^6$ in-lb

$k'_{58\,59} = -4.708 \times 10^6$ in-lb

$k'_{58\,j} = 0$; $j = 60, 61, 62, \ldots, 114$

$k'_{59\,59} = 1073 \times 10^6$ in-lb

$k'_{59\,j} = 0$; $j = 60, 61, 62, \ldots, 114$

$k'_{60\,60} = 1.367 \times 10^6$ in-lb

$k'_{60\,j} = 0$; $j = 61, 62, 63, \ldots, 114$

$k'_{61\ 61} = 231.7 \times 10^6$ lb/in

$k'_{61\ 62} = 1.708 \times 10^6$ lb/in

$k'_{61\ j} = 0$; $j = 63, 64, 65$

$k'_{61\ 66} = 1.747 \times 10^6$ lb

$k'_{61\ 67} = -23.00 \times 10^6$ lb/in

$k'_{61\ 68} = -0.2176 \times 10^6$ lb/in

$k'_{61\ j} = 0$; $j = 69, 70, 71$

$k'_{61\ 72} = -0.1187 \times 10^6$ lb

$k'_{61\ j} = 0$; $j = 73, 74, 75, \ldots, 114$

$k'_{62\ 62} = 33.93 \times 10^6$ lb/in

$k'_{62\ j} = 0$; $j = 63, 64, 65$

$k'_{62\ 66} = -207.9 \times 10^6$ lb

$k'_{62\ 67} = -0.2176 \times 10^6$ lb/in

$k'_{62\ 68} = -0.8662 \times 10^6$ lb/in

$k'_{62\ j} = 0$; $j = 69, 70, 71$

$k'_{62\ 72} = 12.07 \times 10^6$ lb

$k'_{62\ j} = 0$; $j = 73, 74, 75, \ldots, 114$

$k'_{63\ 63} = 46.42 \times 10^6$ lb/in

$k'_{63\ 64} = -\ 2.453 \times 10^6$ lb

$k'_{63\ 65} = 291.1 \times 10^6$ lb

$k'_{63\ j} = 0$; $j = 66, 67, 68$

$k'_{63\ 69} = -0.8640 \times 10^6$ lb/in

$k'_{63\ 70} = 0.1187 \times 10^6$ lb

$k'_{63\ 71} = -\ 12.07 \times 10^6$ lb

$k'_{63\ j} = 0$; $j = 72, 73, 74, \ldots, 114$

$k'_{64\ 64} = 1736 \times 10^6$ in-lb

$k'_{64\ 65} = -\ 36.07 \times 10^6$ in-lb

$k'_{64\ j} = 0$; $j = 66, 67, 68$

$k'_{64\ 69} = -\ 0.1187 \times 10^6$ lb

$k'_{64\ 70} = -\ 51.42 \times 10^6$ in-lb

$k'_{64\ 71} = -\ 1.532 \times 10^6$ in-lb

$k'_{64\ j} = 0$; $j = 72, 73, 74, \ldots, 114$

$k'_{65\ 65} = 5959 \times 10^6$ in-lb

$k'_{65\ j} = 0\ ;\ j = 66, 67, 68$

$k'_{65\ 69} = 12.07 \times 10^6$ lb

$k'_{65\ 70} = -1.532 \times 10^6$ in-lb

$k'_{65\ 71} = 104.4 \times 10^6$ in-lb

$k'_{65\ j} = 0\ ;\ j = 72, 73, 74, \ldots, 114$

$k'_{66\ 66} = 2898 \times 10^6$ in-lb

$k'_{66\ 67} = 0.1187 \times 10^6$ lb

$k'_{66\ 68} = -12.07 \times 10^6$ lb

$k'_{66\ j} = 0\ ;\ j = 69, 70, 71$

$k'_{66\ 72} = 104.5\ 10^6$ in-lb

$k'_{66\ j} = 0\ ;\ j = 73, 75, 75, \ldots, 114$

$k'_{67\ 67} = 43.25 \times 10^6$ lb/in

$k'_{67\ 68} = 0.4277 \times 10^6$ lb/in

$k'_{67\ j} = 0$; $j = 69, 70, 71$

$k'_{67\ 72} = 0.0118 \times 10^6$ lb

$k'_{67\ 73} = -\ 20.25 \times 10^6$ lb/in

$k'_{67\ 74} = -\ 0.2101 \times 10^6$ lb/in

$k'_{67\ j} = 0$; $j = 75, 76, 77$

$k'_{67\ 78} = -\ 0.1069 \times 10^6$ lb

$k'_{67\ j} = 0$; $j = 79, 80, 81, \ldots, 114$

$k'_{68\ 68} = 1.578 \times 10^6$ lb/in

$k'_{68\ j} = 0$; $j = 69, 70, 71$

$k'_{68\ 72} = -\ 2.124 \times 10^6$ lb

$k'_{68\ 73} = -\ 0.2101 \times 10^6$ lb/in

$k'_{68\ 74} = -\ 0.7117 \times 10^6$ lb/in

$k'_{68\ j} = 0$; $j = 75, 76, 77$

$k'_{68\ 78} = 9.946 \times 10^6$ lb

$k'_{68\ j} = 0$; $j = 79, 80, 81, \ldots, 114$

$k'_{69\ 69} = 1.576 \times 10^6$ lb/in

$k'_{69\ 70} = -\ 0.0118 \times 10^6$ lb

$k'_{69\ 71} = 2.124 \times 10^6$ lb

$k'_{69\ j} = 0\ ;\ j = 72,\ 73,\ 74$

$k'_{69\ 75} = -\ 0.7118 \times 10^6$ lb/in

$k'_{69\ 76} = 0.1069 \times 10^6$ lb

$k'_{69\ 77} = -\ 9.946 \times 10^6$ lb

$k'_{69\ j} = 0\ ;\ j = 78,\ 79,\ 80,\ \ldots,\ 114$

$k'_{70\ 70} = 93.46 \times 10^6$ in-lb

$k'_{70\ 71} = -\ 3.975 \times 10^6$ in-lb

$k'_{70\ j} = 0\ ;\ j = 72,\ 73,\ 74$

$k'_{70\ 75} = -\ 0.1069 \times 10^6$ lb

$k'_{70\ 76} = -\ 41.08 \times 10^6$ in-lb

$k'_{70\ 77} = -\ 1.381 \times 10^6$ in-lb

$k'_{70\ j} = 0\ ;\ j = 78,\ 79,\ 80,\ \ldots,\ 114$

$$k'_{71\ 71} = 478.7 \times 10^6 \text{ in-lb}$$

$$k'_{71\ j} = 0 \;;\; j = 72, 73, 74$$

$$k'_{71\ 75} = 9.946 \times 10^6 \text{ lb}$$

$$k'_{71\ 76} = -1.381 \times 10^6 \text{ in-lb}$$

$$k'_{71\ 77} = 86.51 \times 10^6 \text{ in-lb}$$

$$k'_{71\ j} = 0 \;;\; j = 78, 79, 80, \ldots, 114$$

$$k'_{72\ 72} = 478.7 \times 10^6 \text{ in-lb}$$

$$k'_{72\ 73} = 0.1069 \times 10^6 \text{ lb}$$

$$k'_{72\ 74} = -9.946 \times 10^6 \text{ lb}$$

$$k'_{72\ j} = 0 \;;\; j = 75, 76, 77$$

$$k'_{72\ 78} = 86.53 \times 10^6 \text{ in-lb}$$

$$k'_{72\ j} = 0 \;;\; j = 79, 80, 81, \ldots, 114$$

$k'_{73\ 73} = 34.22 \times 10^6$ lb/in

$k'_{73\ 74} = 0.2937 \times 10^6$ lb/in

$k'_{73\ j} = 0$; j = 75, 76, 77

$k'_{73\ 78} = 0.07147 \times 10^6$ lb

$k'_{73\ 79} = -13.97 \times 10^6$ lb/in

$k'_{73\ 80} = -0.08355 \times 10^6$ lb/in

$k'_{73\ j} = 0$; j = 81, 82, 83

$k'_{73\ 84} = -0.03543 \times 10^6$ lb

$k'_{73\ j} = 0$; j = 85, 86, 87, ..., 114

$k'_{74\ 74} = 1.124 \times 10^6$ lb/in

$k'_{74\ j} = 0$; j = 75, 76, 77

$k'_{74\ 78} = -4.195 \times 10^6$ lb

$k'_{74\ 79} = -0.08355 \times 10^6$ lb/in

$k'_{74\ 80} = -0.4120 \times 10^6$ lb/in

$k'_{74\ j} = 0$; j = 81, 82, 83

$k'_{74\ 84} = 5.751 \times 10^6$ lb

$k'_{74\ j} = 0$; j = 85, 86, 87, ..., 114

$$k'_{75\ 75} = 1.123 \times 10^6 \text{ lb/in}$$

$$k'_{75\ 76} = -\ 0.07147 \times 10^6 \text{ lb}$$

$$k'_{75\ 77} = 4.195 \times 10^6 \text{ lb}$$

$$75_{\ j} = 0 \ ; \ j = 78, 79, 80$$

$$k'_{75\ 81} = -\ 0.4115 \times 10^6 \text{ lb/in}$$

$$k'_{75\ 82} = 0.03543 \times 10^6 \text{ lb}$$

$$k'_{75\ 83} = -\ 5.751 \times 10^6 \text{ lb}$$

$$k'_{75\ j} = 0 \ ; \ j = 84, 85, 86, \ldots, 114$$

$$k'_{76\ 76} = 65.78 \times 10^6 \text{ in-lb}$$

$$k'_{76\ 77} = -\ 2.722 \times 10^6 \text{ in-lb}$$

$$k'_{76\ j} = 0 \ ; \ j = 78, 79, 80$$

$$k'_{76\ 81} = -\ 0.03543 \times 10^6 \text{ lb}$$

$$k'_{76\ 82} = -\ 23.77 \times 10^6 \text{ in-lb}$$

$$k'_{76\ 83} = -\ 0.4586 \times 10^6 \text{ in-lb}$$

$$k'_{76\ j} = 0 \ ; \ j = 84, 85, 86, \ldots, 114$$

$k'_{77\ 77} = 355.8 \times 10^6$ in-lb

$k'_{77\ j} = 0$; $j = 78, 79, 80$

$k'_{77\ 81} = 5.751 \times 10^6$ lb

$k'_{77\ 82} = -\ 0.4586 \times 10^6$ in-lb

$k'_{77\ 83} = 50.67 \times 10^6$ in-lb

$k'_{77\ j} = 0$; $j = 84, 85, 86, \ldots, 114$

$k'_{78\ 78} = 355.8 \times 10^6$ in-lb

$k'_{78\ 79} = 0.03543 \times 10^6$ lb

$k'_{78\ 80} = -\ 5.751 \times 10^6$ lb

$k'_{78\ j} = 0$; $j = 81, 82, 83$

$k'_{78\ 84} = 50.67 \times 10^6$ in-lb

$k'_{78\ j} = 0$; $j = 85, 86, 87, \ldots, 114$

$k'_{79\ 79} = 23.56 \times 10^6$ lb/in

$k'_{79\ 80} = 0.1807 \times 10^6$ lb/in

$k'_{79\ j} = 0$; $j = 81, 82, 83$

$k'_{79\ 84} = 0.00015 \times 10^6$ lb

$k'_{79\ 85} = -\ 9.588 \times 10^6$ lb/in

$k'_{79\ 86} = -\ 0.09711 \times 10^6$ lb/in

$k'_{79\ j} = 0$; $j = 87, 88, 89$

$k'_{79\ 90} = -\ 0.03528 \times 10^6$ lb

$k'_{79\ j} = 0$; $j = 91, 92, 93, \ldots, 114$

$k'_{80\ 80} = 0.6562 \times 10^6$ lb/in

$k'_{80\ j} = 0$; $j = 81, 82, 83$

$k'_{80\ 84} = -\ 2.352 \times 10^6$ lb

$k'_{80\ 85} = -\ 0.09711 \times 10^6$ lb/in

$k'_{80\ 86} = -\ 0.2442 \times 10^6$ lb/in

$k'_{80\ j} = 0$; $j = 87, 88, 89$

$k'_{80\ 90} = 3.399 \times 10^6$ lb/in

$k'_{80\ j} = 0$; $j = 91, 92, 93, \ldots, 114$

$k'_{81\,81} = 0.6547 \times 10^6$ lb/in

$k'_{81\,82} = -\,0.00015 \times 10^6$ lb/in

$k'_{81\,83} = 2.352 \times 10^6$ lb

$k'_{81\,j} = 0$; $j = 84, 85, 86$

$k'_{81\,87} = -\,0.2432 \times 10^6$ lb/in

$k'_{81\,88} = 0.03528 \times 10^6$ lb

$k'_{81\,89} = -\,3.399 \times 10^6$ lb

$k'_{81\,j} = 0$; $j = 90, 91, 92, \ldots, 114$

$k'_{82\,82} = 37.61 \times 10^6$ in-lb

$k'_{82\,83} = -\,1.061 \times 10^6$ in-lb

$k'_{82\,j} = 0$; $j = 84, 85, 86$

$k'_{82\,87} = -\,0.03528 \times 10^6$ lb

$k'_{82\,88} = -\,13.83 \times 10^6$ in-lb

$k'_{82\,89} = -\,0.4571 \times 10^6$ in-lb

$k'_{82\,j} = 0$; $j = 90, 91, 92, \ldots, 114$

$k'_{83\ 83} = 174.9 \times 10^6$ in-lb

$k'_{83\ j} = 0$; $j = 84, 85, 86$

$k'_{83\ 87} = 3.399 \times 10^6$ lb

$k'_{83\ 88} = -0.4571 \times 10^6$ in-lb

$k'_{83\ 89} = 30.20 \times 10^6$ in-lb

$k'_{83\ j} = 0$; $j = 90, 91, 92, \ldots, 114$

$k'_{84\ 84} = 174.9 \times 10^6$ in-lb

$k'_{84\ 85} = 0.03528 \times 10^6$ lb

$k'_{84\ 86} = -3.399 \times 10^6$ lb

$k'_{84\ j} = 0$; $j = 87, 88, 89$

$k'_{84\ 90} = 30.21 \times 10^6$ in-lb

$k'_{84\ j} = 0$; $j = 91, 92, 93, \ldots, 114$

$k'_{85\,85} = 22.85 \times 10^6$ lb/in

$k'_{85\,86} = -\ 0.2760 \times 10^6$ lb/in

$k'_{85\,j} = 0\ ;\ j = 87,\ 88,\ 89$

$k'_{85\,90} = -\ 0.02368 \times 10^6$ lb

$k'_{85\,91} = -\ 13.26 \times 10^6$ lb/in

$k'_{85\,92} = -\ 0.1053 \times 10^6$ lb/in

$k'_{85\,j} = 0\ ;\ j = 93,\ 94,\ 95$

$k'_{85\,96} = -\ 0.05896 \times 10^6$ lb

$k'_{85\,j} = 0\ ;\ j = 97,\ 98,\ 99,\ \ldots,\ 114$

$k'_{86\,86} = 57.35 \times 10^6$ lb/in

$k'_{86\,j} = 0\ ;\ j = 87,\ 88,\ 89$

$k'_{86\,90} = 3.628 \times 10^6$ lb

$k'_{86\,91} = -\ 0.1053 \times 10^6$ lb/in

$k'_{86\,92} = -\ 0.7062 \times 10^6$ lb/in

$k'_{86\,j} = 0\ ;\ j = 93,\ 94,\ 95$

$k'_{86\,96} = 7.027 \times 10^6$ lb

$k'_{86\,j} = 0\ ;\ j = 97,\ 98,\ 99,\ \ldots,\ 114$

$k'_{87\ 87} = 57.35 \times 10^6$ lb/in

$k'_{87\ 88} = 0.02368 \times 10^6$ lb

$k'_{87\ 89} = -\ 3.628 \times 10^6$ lb

$k'_{87\ j} = 0\ ;\ j = 90,\ 91,\ 92$

$k'_{87\ 93} = -\ 0.7062 \times 10^6$ lb/in

$k'_{87\ 94} = 0.05896 \times 10^6$ lb

$k'_{87\ 95} = -\ 7.027 \times 10^6$ lb

$k'_{87\ j} = 0\ ;\ j = 96,\ 97,\ 98,\ \ldots,\ 114$

$k'_{88\ 88} = 682.5 \times 10^6$ in-lb

$k'_{88\ 89} = 2.442 \times 10^6$ in-lb

$k'_{88\ j} = 0\ ;\ j = 90,\ 91,\ 92$

$k'_{88\ 93} = -\ 0.05896 \times 10^6$ lb

$k'_{88\ 94} = -\ 18.98 \times 10^6$ in-lb

$k'_{88\ 95} = -\ 0.5472 \times 10^6$ in-lb

$k'_{88\ j} = 0\ ;\ j = 96,\ 97,\ 98,\ \ldots,\ 114$

$k'_{89\ 89} = 384.0 \times 10^6$ in-lb

$k'_{89\ j} = 0$; $j = 90, 91, 92$

$k'_{89\ 93} = 7.027 \times 10^6$ lb

$k'_{89\ 94} = - 0.5472 \times 10^6$ in-lb

$k'_{89\ 95} = 46.23 \times 10^6$ in-lb

$k'_{89\ j} = 0$; $j = 96, 97, 98, \ldots, 114$

$k'_{90\ 90} = 384.0 \times 10^6$ in-lb

$k'_{90\ 91} = 0.05896 \times 10^6$ lb

$k'_{90\ 92} = - 7.027 \times 10^6$ lb

$k'_{90\ j} = 0$; $j = 93, 94, 95$

$k'_{90\ 96} = 46.24\ 10^6$ in-lb

$k'_{90\ j} = 0$; $j = 97, 98, 99, \ldots, 114$

$k'_{91\ 91} = 35.62 \times 10^6$ lb/in

$k'_{91\ 92} = 0.2615 \times 10^6$ lb/in

$k'_{91\ j} = 0$; $j = 93, 94, 95$

$k'_{91\ 96} = -\ 0.04564 \times 10^6$ lb

$k'_{91\ 97} = -\ 22.36 \times 10^6$ lb/in

$k'_{91\ 98} = -\ 0.1562 \times 10^6$ lb/in

$k'_{91\ j} = 0$; $j = 99, 100, 101$

$k'_{91\ 102} = -\ 0.1046 \times 10^6$ lb

$k'_{91\ j} = 0$; $j = 103, 104, 105, \ldots, 114$

$k'_{92\ 92} = 2.985 \times 10^6$ lb/in

$k'_{92\ j} = 0$; $j = 93, 94, 95$

$k'_{92\ 96} = 6.413 \times 10^6$ lb

$k'_{92\ 97} = -\ 0.1562 \times 10^6$ lb/in

$k'_{92\ 98} = -\ 2.279 \times 10^6$ lb/in

$k'_{92\ j} = 0$; $j = 99, 100, 101$

$k'_{92\ 102} = 13.44 \times 10^6$ lb

$k'_{92\ j} = 0$; $j = 103, 104, 105, \ldots, 114$

$k'_{93\ 93} = 2.984 \times 10^6$ lb/in

$k'_{93\ 94} = 0.04564 \times 10^6$ lb

$k'_{93\ 95} = -\ 6.413 \times 10^6$ lb

$k'_{93\ j} = 0\ ;\ j = 96,\ 97,\ 98$

$k'_{93\ 99} = -\ 2.278 \times 10^6$ lb/in

$k'_{93\ 100} = 0.1046 \times 10^6$ lb

$k'_{93\ 101} = -\ 13.44 \times 10^6$ lb

$k'_{93\ j} = 0\ ;\ j = 102,\ 103,\ 104,\ \ldots,\ 114$

$k'_{94\ 94} = 51.01 \times 10^6$ in-lb

$k'_{94\ 95} = -\ 1.306 \times 10^6$ in-lb

$k'_{94\ j} = 0\ ;\ j = 96,\ 97,\ 98$

$k'_{94\ 99} = -\ 0.1046 \times 10^6$ lb

$k'_{94\ 100} = -\ 32.01 \times 10^6$ in-lb

$k'_{94\ 101} = -\ 0.5565 \times 10^6$ in-lb

$k'_{94\ j} = 0\ ;\ j = 102,\ 103,\ 104,\ \ldots,\ 114$

$k'_{95\ 95} = 212.9 \times 10^6$ in-lb

$k'_{95\ j} = 0$; $j = 96, 97, 98$

$k'_{95\ 99} = 13.44 \times 10^6$ lb

$k'_{95\ 100} = -\ 0.5565 \times 10^6$ in-lb

$k'_{95\ 101} = 39.28 \times 10^6$ in-lb

$k'_{95\ j} = 0$; $j = 102, 103, 104, \ldots, 114$

$k'_{96\ 96} = 212.9 \times 10^6$ in-lb

$k'_{96\ 97} = 0.1046 \times 10^6$ lb

$k'_{96\ 98} = -\ 13.44 \times 10^6$ lb

$k'_{96\ j} = 0$; $j = 99, 100, 101$

$k'_{96\ 102} = 39.28 \times 10^6$ in-lb

$k'_{96\ j} = 0$; $j = 103, 104, 105, \ldots, 114$

$k'_{97\ 97} = 44.72 \times 10^6$ lb/in

$k'_{97\ 98} = 0.3175 \times 10^6$ lb/in

$k'_{97\ j} = 0$; $j = 99, 100, 101$

$k'_{97\ 102} = -0.0033 \times 10^6$ lb

$k'_{97\ 103} = -22.36 \times 10^6$ lb/in

$k'_{97\ 104} = -0.1613 \times 10^6$ lb/in

$k'_{97\ j} = 0$; $j = 105, 106, 107$

$k'_{97\ 108} = -0.1079 \times 10^6$ lb

$k'_{97\ j} = 0$; $j = 109, 110, 111, \ldots, 114$

$k'_{98\ 98} = 4.558 \times 10^6$ lb/in

$k'_{98\ j} = 0$; $j = 99, 100, 101$

$k'_{98\ 102} = 0$

$k'_{98\ 103} = -0.1613 \times 10^6$ lb/in

$k'_{98\ 104} = -2.279 \times 10^6$ lb/in

$k'_{98\ j} = 0$; $j = 105, 106, 107$

$k'_{98\ 108} = 13.44 \times 10^6$ lb

$k'_{98\ j} = 0$; $j = 109, 110, 111, \ldots, 114$

$k'_{99\ 99} = 4.556 \times 10^6$ lb/in

$k'_{99\ 100} = 0.0033 \times 10^6$ lb

$k'_{99\ 101} = 0$

$k'_{99\ j} = 0$; $j = 102, 103, 104$

$k'_{99\ 105} = -2.278 \times 10^6$ lb/in

$k'_{99\ 106} = 0.1079 \times 10^6$ lb

$k'_{99\ 107} = -13.44 \times 10^6$ lb

$k'_{99\ j} = 0$; $j = 108, 109, 110, \ldots, 114$

$k'_{100\ 100} = 64.04 \times 10^6$ in-lb

$k'_{100\ 101} = -1.380 \times 10^6$ in-lb

$k'_{100\ j} = 0$; $j = 102, 103, 104$

$k'_{100\ 105} = -0.1079 \times 10^6$ lb

$k'_{100\ 106} = -32.01 \times 10^6$ in-lb

$k'_{100\ 107} = -0.5725 \times 10^6$ in-lb

$k'_{100\ j} = 0$; $j = 108, 109, 110, \ldots, 114$

$k'_{101\ 101} = 238.6 \times 10^6$ in-lb

$k'_{101\ j} = 0$; $j = 102, 103, 104$

$k'_{101\ 105} = 13.44 \times 10^6$ lb

$k'_{101\ 106} = -\ 0.5725 \times 10^6$ in-lb

$k'_{101\ 107} = 39.28 \times 10^6$ in-lb

$k'_{101\ j} = 0$; $j = 108, 109, 110, \ldots, 114$

$k'_{102\ 102} = 238.6 \times 10^6$ in-lb

$k'_{102\ 103} = 0.1079 \times 10^6$ lb

$k'_{102\ 104} = -\ 13.44 \times 10^6$ lb

$k'_{102\ j} = 0$; $j = 105, 106, 107$

$k'_{102\ 108} = 39.28 \times 10^6$ in-lb

$k'_{102\ j} = 0$; $j = 109, 110, 111, \ldots, 114$

$k'_{103\ 103} = 41.69 \times 10^6$ lb/in

$k'_{103\ 104} = 0.3794 \times 10^6$ lb/in

$k'_{103\ j} = 0$; $j = 105, 106, 107$

$k'_{103\ 108} = -0.0323 \times 10^6$ lb

$k'_{103\ 109} = -19.33 \times 10^6$ lb/in

$k'_{103\ 110} = -0.2181 \times 10^6$ lb/in

$k'_{103\ j} = 0$; $j = 11, 112, 113$

$k'_{103\ 114} = -0.1402 \times 10^6$ lb

$k'_{104\ 104} = 4.181 \times 10^6$ lb/in

$k'_{104\ j} = 0$; $j = 105, 106, 107$

$k'_{104\ 108} = -2.23 \times 10^6$ lb

$k'_{104\ 109} = -0.2181 \times 10^6$ lb/in

$k'_{104\ 110} = -1.902 \times 10^6$ lb/in

$k'_{104\ j} = 0$; $j = 111, 112, 113$

$k'_{104\ 114} = 11.21 \times 10^6$ lb

$k'_{105\ 105} = 4.178 \times 10^6$ lb/in

$k'_{105\ 106} = 0.0323 \times 10^6$ lb

$k'_{105\ 107} = 2.230 \times 10^6$ lb

$k'_{105\ j} = 0$; $j = 108, 109, 110$

$k'_{105\ 111} = -\ 1.900 \times 10^6$ lb/in

$k'_{105\ 112} = 0.1402 \times 10^6$ lb

$k'_{105\ 113} = -\ 11.21 \times 10^6$ lb

$k'_{105\ 114} = 0$

$k'_{106\ 106} = 58.25 \times 10^6$ lb/in

$k'_{106\ 107} = -\ 1.610 \times 10^6$ in-lb

$k'_{106\ j} = 0$; $j = 108, 109, 110$

$k'_{106\ 111} = -\ 0.1402 \times 10^6$ lb

$k'_{106\ 112} = -\ 26.21 \times 10^6$ in-lb

$k'_{106\ 113} = -\ 0.7452 \times 10^6$ in-lb

$k'_{106\ 114} = 0$

$k'_{107\ 107} = 218.2 \times 10^6$ in-lb

$k'_{107\ j} = 0$; j = 108, 109, 110

$k'_{107\ 111} = 11.21 \times 10^6$ lb

$k'_{107\ 112} = -0.7452 \times 10^6$ in-lb

$k'_{107\ 113} = 33.35 \times 10^6$ in-lb

$k'_{107\ 114} = 0$

$k'_{108\ 108} = 218.2 \times 10^6$ in-lb

$k'_{108\ 109} = 0.1402 \times 10^6$ lb

$k'_{108\ 110} = -11.21 \times 10^6$ lb

$k'_{108\ j} = 0$; j = 11, 112, 113

$k'_{108\ 114} = 33.35 \times 10^6$ in-lb

$k'_{109\ 109} = 19.33 \times 10^6$ lb/in

$k'_{109\ 110} = 0.2181 \times 10^6$ lb/in

$k'_{109\ j} = 0$; j = 11, 112, 113

$k'_{109\ 114} = 0.1402 \times 10^6$ lb

$k'_{110\ 110} = 1.902 \times 10^6$ lb/in

$k'_{110\ j} = 0$; $j = 111, 112, 113$

$k'_{110\ 114} = -11.21 \times 10^6$ lb

$k'_{111\ 111} = 1.900 \times 10^6$ lb/in

$k'_{111\ 112} = -0.1402 \times 10^6$ lb

$k'_{111\ 113} = 11.21 \times 10^6$ lb

$k'_{111\ 114} = 0$

$k'_{112\ 112} = 26.23 \times 10^6$ in-lb

$k'_{112\ 113} = -0.9091 \times 10^6$ in-lb

$k'_{112\ 114} = 0$

$k'_{113\ 113} = 98.88 \times 10^6$ in-lb

$k'_{113\ 114} = 0$

$k'_{114\ 114} = 98.89 \times 10^6$ in-lb

APPENDIX D

This appendix presents the numerical values of the elements of the 114 x 114 system inertia matrix.

$m'_{1\,1} = 1.028$ lb-sec^2/in

$m'_{1\,j} = 0$; $j = 2, 3, 4, \ldots, 114$

$m'_{2\,2} = 1.028$ lb-sec^2/in

$m'_{2\,j} = 0$; $j = 3, 4, 5, \ldots, 114$

$m'_{3\,3} = 1.028$ lb-sec^2/in

$m'_{3\,j} = 0$; $j = 4, 5, 6, \ldots, 114$

$m'_{4\,4} = 71.97$ in-lb-sec^2

$m'_{4\,5} = -0.4667$ in-lb-sec^2

$m'_{4\,j} = 0$; $j = 6, 7, 8, \ldots, 114$

$m'_{5\,5} = 127.0$ in-lb-sec^2

$m'_{5\,j} = 0$; $j = 6, 7, 8, \ldots, 114$

$m'_{6\,6} = 117.9$ in-lb-sec^2

$m'_{6\,j} = 0$; $j = 7, 8, 9, \ldots, 114$

$m'_{7\,7} = 8.607$ lb-sec^2/in

$m'_{7\,j} = 0$; $j = 8, 9, 10, \ldots, 114$

$m'_{8\,8} = 8.607$ lb-sec^2/in

$m'_{8\,j} = 0$; $j = 9, 10, 11, \ldots, 114$

$m'_{9\,9} = 8.607$ lb-sec^2/in

$m'_{9\,j} = 0$; $j = 10, 11, 12, \ldots, 114$

$m'_{10\,10} = 2841$ in-lb-sec^2

$m'_{10\,11} = -9.086$ in-lb-sec^2

$m'_{10\,j} = 0$; $j = 12, 13, 14, \ldots, 114$

$m'_{11\,11} = 3912$ in-lb-sec^2

$m'_{11\,j} = 0$; $j = 12, 13, 14, \ldots, 114$

$m'_{12\,12} = 5448$ in-lb-sec^2

$m'_{12\,j} = 0$; $j = 13, 14, 15, \ldots, 114$

$m'_{13\,13} = 0.3281$ lb-sec^2/in

$m'_{13\,j} = 0 \; ; \; j = 14, 15, 16, \ldots, 114$

$m'_{14\,14} = 0.3281$ lb-sec^2/in

$m'_{14\,j} = 0 \; ; \; j = 15, 16, 17, \ldots, 114$

$m'_{15\,15} = 0.3281$ lb-sec^2/in

$m'_{15\,j} = 0 \; ; \; j = 16, 17, 18, \ldots, 114$

$m'_{16\,16} = 17.51$ in-lb-sec^2

$m'_{16\,17} = -\,0.07955$ in-lb-sec^2

$m'_{16\,j} = 0 \; ; \; j = 18, 19, 20, \ldots, 114$

$m'_{17\,17} = 26.89$ in-lb-sec^2

$m'_{17\,j} = 0 \; ; \; j = 18, 19, 20, \ldots, 114$

$m'_{18\,18} = 20.40$ in-lb-sec^2

$m'_{18\,j} = 0 \; ; \; j = 19, 20, 21, \ldots, 114$

$m'_{19\ 19} = 0.1446$ lb-sec^2/in

$m'_{19\ j} = 0$; j = 20, 21, 22, ..., 114

$m'_{20\ 20} = 0.1446$ lb-sec^2/in

$m'_{20\ j} = 0$; j = 21, 22, 23, ..., 114

$m'_{21\ 21} = 0.1446$ lb-sec^2/in

$m'_{21\ j} = 0$; j = 22, 23, 24, ..., 114

$m'_{22\ 22} = 5.865$ in-lb-sec^2

$m'_{22\ 23} = -\ 0.03888$ in-lb-sec^2

$m'_{22\ j} = 0$; j = 24, 25, 26, ..., 114

$m'_{23\ 23} = 10.10$ in-lb-sec^2

$m'_{23\ j} = 0$; j = 24, 25, 26, ..., 114

$m'_{24\ 24} = 9.351$ in-lb-sec^2

$m'_{24\ j} = 0$; j = 25, 26, 27, ..., 114

$m'_{25\ 25} = 0.1235$ lb-sec^2/in

$m'_{25\ j} = 0$; $j = 26, 27, 28, \ldots, 114$

$m'_{26\ 26} = 0.1235$ lb-sec^2/in

$m'_{26\ j} = 0$; $j = 27, 28, 29, \ldots, 114$

$m'_{27\ 27} = 0.1235$ lb-sec^2/in

$m'_{27\ j} = 0$; $j = 28, 29, 30, \ldots, 114$

$m'_{28\ 28} = 3.962$ in-lb-sec^2

$m'_{28\ 29} = -\ 0.0450$ in-lb-sec^2

$m'_{28\ j} = 0$; $j = 30, 31, 32, \ldots, 114$

$m'_{29\ 29} = 8.220$ in-lb-sec^2

$m'_{29\ j} = 0$; $j = 30, 31, 32, \ldots, 114$

$m'_{30\ 30} = 7.352$ in-lb-sec^2

$m'_{30\ j} = 0$; $j = 31, 32, 33, \ldots, 114$

$$m'_{31\ 31} = 0.1121 \text{ lb-sec}^2/\text{in}$$

$$m'_{31\ j} = 0 \ ; \ j = 32, 33, 34, \ldots, 114$$

$$m'_{32\ 32} = 0.1121 \text{ lb-sec}^2/\text{in}$$

$$m'_{32\ j} = 0 \ ; \ j = 33, 34, 35, \ldots, 114$$

$$m'_{33\ 33} = 0.1121 \text{ lb-sec}^2/\text{in}$$

$$m'_{33\ j} = 0 \ ; \ j = 34, 35, 36, \ldots, 114$$

$$m'_{34\ 34} = 2.750 \text{ in-lb-sec}^2$$

$$m'_{34\ 35} = -\ 0.06931 \text{ in-lb-sec}^2$$

$$m'_{34\ j} = 0 \ ; \ j = 36, 37, 38, \ldots, 114$$

$$m'_{35\ 35} = 7.156 \text{ in-lb-sec}^2$$

$$m'_{35\ j} = 0 \ ; \ j = 36, 37, 38, \ldots, 114$$

$$m'_{36\ 36} = 5.841 \text{ in-lb-sec}^2$$

$$m'_{36\ j} = 0 \ ; \ j = 37, 38, 39, \ldots, 114$$

$m'_{37\ 37} = 0.1421$ lb-sec^2/in

$m'_{37\ j} = 0$; $j = 38, 39, 40, \ldots, 114$

$m'_{38\ 38} = 0.1421$ lb-sec^2/in

$m'_{38\ j} = 0$; $j = 39, 40, 41, \ldots, 114$

$m'_{39\ 39} = 0.1421$ lb-sec^2/in

$m'_{39\ j} = 0$; $j = 40, 41, 42, \ldots, 114$

$m'_{40\ 40} = 2.349$ in-lb-sec^2

$m'_{40\ 41} = -\ 0.03513$ in-lb-sec^2

$m'_{40\ j} = 0$; $j = 42, 43, 44, \ldots, 114$

$m'_{41\ 41} = 4.087$ in-lb-sec^2

$m'_{41\ j} = 0$; $j = 42, 43, 44, \ldots, 114$

$m'_{42\ 42} = 3.226$ in-lb-sec^2

$m'_{42\ j} = 0$; $j = 43, 44, 45, \ldots, 114$

$m'_{43\ 43} = 0.8085$ lb-sec^2/in

$m'_{43\ j} = 0$; $j = 44, 45, 46, \ldots, 114$

$m'_{44\ 44} = 0.8085$ lb-sec^2/in

$m'_{44\ j} = 0$; $j = 45, 46, 47, \ldots, 114$

$m'_{45\ 45} = 0.8085$ lb-sec^2/in

$m'_{45\ j} = 0$; $j = 46, 47, 48, \ldots, 114$

$m'_{46\ 46} = 10.03$ in-lb-sec^2

$m'_{46\ 47} = -0.9073$ in-lb-sec^2

$m'_{46\ j} = 0$; $j = 48, 49, 50, \ldots, 114$

$m'_{47\ 47} = 117.0$ in-lb-sec^2

$m'_{47\ j} = 0$; $j = 48, 49, 50, \ldots, 114$

$m'_{48\ 48} = 116.5$ in-lb-sec^2

$m'_{48\ j} = 0$; $j = 49, 50, 51, \ldots, 114$

$m'_{49\ 49} = 0.8085$ lb-sec^2/in

$m'_{49\ j} = 0$; $j = 50, 51, 52, \ldots, 114$

$m'_{50\ 50} = 0.8085$ lb-sec^2/in

$m'_{50\ j} = 0$; $j = 51, 52, 53, \ldots, 114$

$m'_{51\ 51} = 0.8085$ lb-sec^2/in

$m'_{51\ j} = 0$; $j = 52, 53, 54, \ldots, 114$

$m'_{52\ 52} = 10.03$ in-lb-sec^2

$m'_{52\ 53} = -0.9073$ in-lb-sec^2

$m'_{52\ j} = 0$; $j = 54, 55, 56, \ldots, 114$

$m'_{53\ 53} = 117.0$ in-lb-sec^2

$m'_{53\ j} = 0$; $j = 54, 55, 56, \ldots, 114$

$m'_{54\ 54} = 116.5$ in-lb-sec^2

$m'_{54\ j} = 0$; $j = 55, 56, 57, \ldots, 114$

$m'_{55\ 55} = 0.5384$ lb-sec^2/in

$m'_{55\ j} = 0$; $j = 56, 57, 58, \ldots, 114$

$m'_{56\ 56} = 0.5384$ lb-sec^2/in

$m'_{56\ j} = 0$; $j = 57, 58, 59, \ldots, 114$

$m'_{57\ 57} = 0.5384$ lb-sec^2/in

$m'_{57\ j} = 0$; $j = 58, 59, 60, \ldots, 114$

$m'_{58\ 58} = 6.432$ in-lb-sec^2

$m'_{58\ 59} = -\ 0.03859$ in-lb-sec^2

$m'_{58\ j} = 0$; $j = 60, 61, 62, \ldots, 114$

$m'_{59\ 59} = 10.98$ in-lb-sec^2

$m'_{59\ j} = 0$; $j = 60, 61, 62, \ldots, 114$

$m'_{60\ 60} = 10.98$ in-lb-sec^2

$m'_{60\ j} = 0$; $j = 61, 62, 63, \ldots, 114$

$m'_{61\ 61} = 0.4374$ lb-sec^2/in

$m'_{61\ j} = 0$; $j = 62, 63, 64, \ldots, 114$

$m'_{62\ 62} = 0.4374$ lb-sec^2/in

$m'_{62\ j} = 0$; $j = 63, 64, 65, \ldots, 114$

$m'_{63\ 63} = 0.4374$ lb-sec^2/in

$m'_{63\ j} = 0$; $j = 64, 65, 66, \ldots, 114$

$m'_{64\ 64} = 2.523$ in-lb-sec^2

$m'_{64\ 65} = -\ 0.2462$ in-lb-sec^2

$m'_{64\ j} = 0$; $j = 66, 67, 68, \ldots, 114$

$m'_{65\ 65} = 29.39$ in-lb-sec^2

$m'_{65\ j} = 0$; $j = 66, 67, 68, \ldots, 114$

$m'_{66\ 66} = 29.45$ in-lb-sec^2

$m'_{66\ j} = 0$; $j = 67, 68, 69, \ldots, 114$

$m'_{67\ 67} = 0.4281$ lb-sec^2/in

$m'_{67\ j} = 0$; j = 68, 69, 70, ..., 114

$m'_{68\ 68} = 0.4281$ lb-sec^2/in

$m'_{68\ j} = 0$; j = 69, 70, 71, ..., 114

$m'_{69\ 69} = 0.4281$ lb-sec^2/in

$m'_{69\ j} = 0$; j = 70, 71, 72, ..., 114

$m'_{70\ 70} = 2.363$ in-lb-sec^2

$m'_{70\ 71} = -0.2733$ in-lb-sec^2

$m'_{70\ j} = 0$; j = 72, 73, 74, ..., 114

$m'_{71\ 71} = 28.90$ in-lb-sec^2

$m'_{71\ j} = 0$; j = 72, 73, 74, ..., 114

$m'_{72\ 72} = 28.90$ in-lb-sec^2

$m'_{72\ j} = 0$; j = 73, 74, 75, ..., 114

$m'_{73\,73} = 0.3251$ lb-sec^2/in

$m'_{73\,j} = 0$; j = 74, 75, 76, ..., 114

$m'_{74\,74} = 0.3251$ lb-sec^2/in

$m'_{74\,j} = 0$; j = 75, 76, 77, ..., 114

$m'_{75\,75} = 0.3251$ lb-sec^2/in

$m'_{75\,j} = 0$; j = 76, 77, 78, ..., 114

$m'_{76\,76} = 1.531$ in-lb-sec^2

$m'_{76\,77} = -0.1726$ in-lb-sec^2

$m'_{76\,j} = 0$; j = 78, 79, 80, ..., 114

$m'_{77\,77} = 21.92$ in-lb-sec^2

$m'_{77\,j} = 0$; j = 78, 79, 80, ..., 114

$m'_{78\,78} = 21.92$ in-lb-sec^2

$m'_{78\,j} = 0$; j = 79, 80, 81, ..., 114

$m'_{79\ 79} = 0.2125$ lb-sec^2/in

$m'_{79\ j} = 0$; $j = 80, 81, 82, \ldots, 114$

$m'_{80\ 80} = 0.2125$ lb-sec^2/in

$m'_{80\ j} = 0$; $j = 81, 82, 83, \ldots, 114$

$m'_{81\ 81} = 0.2125$ lb-sec^2/in

$m'_{81\ j} = 0$; $j = 82, 83, 84, \ldots, 114$

$m'_{82\ 82} = 0.8146$ in-lb-sec^2

$m'_{82\ 83} = -0.1110$ in-lb-sec^2

$m'_{82\ j} = 0$; $j = 84, 85, 86, \ldots, 114$

$m'_{83\ 83} = 14.23$ in-lb-sec^2

$m'_{83\ j} = 0$; $j = 84, 85, 86, \ldots, 114$

$m'_{84\ 84} = 14.23$ in-lb-sec^2

$m'_{84\ j} = 0$; $j = 85, 86, 87, \ldots, 114$

$m'_{85\,85} = 0.1801$ lb-sec^2/in

$m'_{85\,j} = 0$; $j = 86, 87, 88, \ldots, 114$

$m'_{86\,86} = 0.1801$ lb-sec^2/in

$m'_{86\,j} = 0$; $j = 87, 88, 89, \ldots, 114$

$m'_{87\,87} = 0.1801$ lb-sec^2/in

$m'_{87\,j} = 0$; $j = 88, 89, 90, \ldots, 114$

$m'_{88\,88} = 0.6635$ in-lb-sec^2

$m'_{88\,89} = -\ 0.1069$ in-lb-sec^2

$m'_{88\,j} = 0$; $j = 90, 91, 92, \ldots, 114$

$m'_{89\,89} = 12.05$ in-lb-sec^2

$m'_{89\,j} = 0$; $j = 90, 91, 92, \ldots, 114$

$m'_{90\,90} = 12.05$ in-lb-sec^2

$m'_{90\,j} = 0$; $j = 91, 92, 93, \ldots, 114$

$m'_{91\ 91} = 0.07609$ lb-sec^2/in

$m'_{91\ j} = 0$; $j = 92, 93, 94, \ldots, 114$

$m'_{92\ 92} = 0.07609$ lb-sec^2/in

$m'_{92\ j} = 0$; $j = 93, 94, 95, \ldots, 114$

$m'_{93\ 93} = 0.07609$ lb-sec^2/in

$m'_{93\ j} = 0$; $j = 94, 95, 96, \ldots, 114$

$m'_{94\ 94} = 0.2798$ in-lb-sec^2

$m'_{94\ 95} = -\ 0.00601$ in-lb-sec^2

$m'_{94\ j} = 0$; $j = 96, 97, 98, \ldots, 114$

$m'_{95\ 95} = 1.022$ in-lb-sec^2

$m'_{95\ j} = 0$; $j = 96, 97, 98, \ldots, 114$

$m'_{96\ 96} = 1.022$ in-lb-sec^2

$m'_{96\ j} = 0$; $j = 97, 98, 99, \ldots, 114$

$m'_{97\ 97} = 0.07609$ lb-sec^2/in

$m'_{97\ j} = 0$; j = 98, 99, 100, ..., 114

$m'_{98\ 98} = 0.07609$ lb-sec^2/in

$m'_{98\ j} = 0$; j = 99, 100, 101, ..., 114

$m'_{99\ 99} = 0.07609$ lb-sec^2/in

$m'_{99\ j} = 0$; j = 100, 101, 102, ..., 114

$m'_{100\ 100} = 0.2798$ in-lb-sec^2

$m'_{100\ 101} = -\ 0.00587$ in-lb-sec^2

$m'_{100\ j} = 0$; j = 102, 103, 104, ..., 114

$m'_{101\ 101} = 1.022$ in-lb-sec^2

$m'_{101\ j} = 0$; j = 102, 103, 104, ..., 114

$m'_{102\ 102} = 1.022$ in-lb-sec^2

$m'_{102\ j} = 0$; j = 103, 104, 105, ..., 114

$m'_{103\ 103} = 0.07220$ lb-sec^2/in

$m'_{103\ j} = 0$; j = 104, 105, 106, ..., 114

$m'_{104\ 104} = 0.07220$ lb-sec^2/in

$m'_{104\ j} = 0$; j = 105, 106, 107, ..., 114

$m'_{105\ 105} = 0.07220$ lb-sec^2/in

$m'_{105\ j} = 0$; j = 106, 107, 108, ..., 114

$m'_{106\ 106} = 0.2586$ in-lb-sec^2

$m'_{106\ 107} = -0.00689$ in-lb-sec^2

$m'_{106\ j} = 0$; j = 108, 109, 110, ..., 114

$m'_{107\ 107} = 0.9282$ in-lb-sec^2

$m'_{107\ j} = 0$; j = 108, 109, 110, ..., 114

$m'_{108\ 108} = 0.9283$ in-lb-sec^2

$m'_{108\ j} = 0$; j = 109, 110, 111, ..., 114

$m'_{109\ 109} = 0.04790$ lb-sec^2/in

$m'_{109\ j} = 0$; j = 110, 111, 112, ..., 114

$m'_{110\ 110} = 0.04790$ lb-sec^2/in

$m'_{110\ j} = 0$; j = 111, 112, 113, 114

$m'_{111\ 111} = 0.04790$ lb-sec^2/in

$m'_{111\ j} = 0$; j = 112, 113, 114

$m'_{112\ 112} = 0.1936$ in-lb-sec^2

$m'_{112\ 113} = -0.00447$ in-lb-sec^2

$m'_{112\ 114} = 0$

$m'_{113\ 113} = 0.5509$ in-lb-sec^2

$m'_{113\ 114} = 0$

$m'_{114\ 114} = 0.5510$ in-lb-sec^2

APPENDIX E

This appendix presents output from the computer program "ADMAGEG" for the first ten normal modes of the 75mm ADMAG gun system mounted on the M240 artillery mount. The gun is prescribed to be in an in-battery configuration, and mount sidewall flexure in the neighborhood of the elevation link has been taken into account.

FIRST 10 NATURAL FREQUENCIES AND NORMAL MODE SHAPES

MODE NO. 1 FREQ. = 11.77 HZ GEN. MASS = .5164E+00 LB-SEC**2/IN

MASS PT.	X (IN/IN)	Y (IN/IN)	Z (IN/IN)	THETAX (RAD/IN)	THETAY (RAD/IN)	THETAZ (RAD/IN)
1	.00115022	-.13564988	.00000000	-.00000000	.00000000	.00464852
2	-.00000066	.00001234	.00000000	-.00000000	.00000000	.00464849
3	-.00051322	.06043543	.00000000	-.00000000	.00000000	.00464817
4	-.00145367	.17107826	-.00000000	.00000000	.00000000	.00468441
5	-.00256352	.28341220	-.00000001	.00000000	.00000000	.00480394
6	-.00388975	.40113811	-.00000001	.00000000	.00000000	.00517308
7	-.00657348	.53387348	-.00000002	-.00000000	.00000000	.00637604
8	.00028657	-.03400016	.00000000	-.00000000	.00000000	.00464891
9	.00028657	-.03400016	.00000000	-.00000000	.00000000	.00464891
10	.00028671	-.03401506	.00000000	-.00000000	.00000000	.00486181
11	-.00023788	.02782418	.00000000	-.00000000	.00000000	.00462521
12	-.00133754	.13959965	-.00000000	-.00000000	.00000000	.00312773
13	-.00229761	.22881203	-.00000000	-.00000000	.00000000	.00296527
14	-.00307262	.35439091	-.00000001	-.00000000	.00000000	.00561103
15	-.00493781	.53389027	-.00000002	-.00000000	.00000000	.00666115
16	-.00620786	.68531697	-.00000001	-.00000000	-.00000000	.00834377
17	-.00699876	.78698793	.00000000	-.00000000	-.00000000	.00881714
18	-.00784739	.89264682	.00000002	-.00000000	-.00000000	.00904337
19	-.00919053	1.00000000	.00000004	-.00000000	-.00000000	.00912174

MODE NO. 2 FREQ. = 16.11 HZ GEN. MASS = .4729E+01 LB-SEC**2/IN

MASS PT.	X (IN/IN)	Y (IN/IN)	Z (IN/IN)	THETAX (RAD/IN)	THETAY (RAD/IN)	THETAZ (RAD/IN)
1	-.00000000	.00000001	.00453586	.00000555	-.00002050	-.00000000
2	.00000000	-.00000000	.00513183	.00000487	-.00001984	-.00000000
3	.00000000	-.00000000	.00545321	.00000773	-.00002588	-.00000000
4	.00000000	-.00000001	.01575370	.53495233	.00378444	-.00000000
5	.00000000	-.00000001	.06135943	.73660689	.00502365	-.00000000
6	.00000000	-.00000002	.14808528	.50059917	.00170613	-.00000000
7	.00000000	-.00000002	.35062637	.00421218	-.00874638	-.00000000
8	-.00004601	-.00001841	.00499021	.00000759	-.00002147	.00000042
9	.00004601	.00001842	.00499021	.00000759	-.00002147	-.00000042
10	.00000000	.00000000	.00499023	.00000759	-.00002148	-.00000000
11	.00000000	-.00000000	.00545292	.00002180	-.00004259	-.00000000
12	.00000000	-.00000001	.01864667	.00048662	-.00076952	-.00000000
13	.00000000	-.00000001	.05488851	.00107892	-.00166068	-.00000000
14	.00000000	-.00000002	.15131761	.00216855	-.00500009	-.00000000
15	.00000000	-.00000002	.35065686	.00416999	-.00892337	-.00000000
16	.00000000	-.00000002	.55840126	.00419200	-.01156951	-.00000000
17	.00000000	-.00000002	.70052826	.00419914	-.01233095	-.00000000
18	.00000000	-.00000002	.84893747	.00420320	-.01267280	-.00000000
19	.00000000	-.00000002	1.00000000	.00420472	-.01278207	-.00000000

MODE NO. 3 FREQ. = 21.24 HZ GEN. MASS = .2251E+00 LB-SEC**2/IN

MASS PT.	X (IN/IN)	Y (IN/IN)	Z (IN/IN)	THETAX (RAD/IN)	THETAY (RAD/IN)	THETAZ (RAD/IN)
1	-.00000000	.00000000	.01035616	.00000006	-.00004166	-.00000000
2	-.00000000	-.00000000	.01156414	.00000011	-.00004128	-.00000000
3	-.00000000	-.00000000	.01224108	.00000005	-.00005518	-.00000000
4	-.00000000	-.00000000	.03586147	-.03492662	-.00204183	-.00000000
5	.00000000	-.00000000	.10042252	-.03365367	-.00383832	.00000000
6	.00000000	-.00000000	.20602212	-.00298159	-.00520285	.00000000
7	.00000000	.00000000	.34581045	-.00020260	-.00659041	.00000000
8	-.00009134	.00000032	.01127212	-.00000009	-.00004261	-.00000003
9	.00009134	-.00000032	.01127212	-.00000009	-.00004261	.00000003
10	-.00000000	.00000000	.01127222	-.00000009	-.00004263	-.00000000
11	-.00000000	-.00000000	.01210891	-.00000079	-.00007396	-.00000000
12	-.00000000	-.00000000	.03053512	-.00002576	-.00102626	-.00000000
13	-.00000000	-.00000000	.07478428	-.00006021	-.00189840	-.00000000
14	.00000000	-.00000000	.17345520	-.00009832	-.00482164	.00000000
15	.00000000	.00000000	.34586879	-.00019551	-.00713363	.00000000
16	.00000000	.00000000	.53748455	-.00016069	-.01160068	.00000000
17	.00000000	.00000000	.68323865	-.00014998	-.01289352	.00000000
18	.00000000	.00000000	.83960324	-.00014433	-.01346792	.00000000
19	.00000000	.00000000	1.00000000	-.00014244	-.01364546	.00000000

MODE NO. 4 FREQ. = 26.91 HZ GEN. MASS = .2643E+00 LB-SEC**2/IN

MASS PT.	X (IN/IN)	Y (IN/IN)	Z (IN/IN)	THETAX (RAD/IN)	THETAY (RAD/IN)	THETAZ (RAD/IN)
1	-.00102113	.12032308	.00000000	-.00000000	.00000000	-.00411767
2	-.00000149	.00013093	.00000000	-.00000000	.00000000	-.00411374
3	.00045115	-.05324090	.00000000	-.00000000	.00000000	-.00410314
4	.00104251	-.12306464	-.00000000	-.00000000	.00000060	-.00194603
5	.00113793	-.13280722	-.00000000	.00000000	.00000000	.00086647
6	.00038895	-.06615323	-.00000001	.00000000	.00000000	.00435527
7	-.00280151	.09217143	-.00000001	-.00000000	.00000000	.00860881
8	-.00026375	.03025851	.00000000	-.00000000	.00000000	-.00411416
9	-.00026375	.03025851	.00000000	-.00000000	.00000000	-.00411416
10	-.00026398	.03028453	.00000000	-.00000000	.00000000	-.00534006
11	.00019778	-.02415307	.00000000	-.00000000	.00000000	-.00404751
12	.00103624	-.10940408	-.00000000	-.00000000	.00000000	-.00184804
13	.00134110	-.13771743	-.00000000	-.00000000	.00000000	-.00005344
14	.00103724	-.08837769	-.00000001	-.00000000	.00000000	.00357249
15	-.00083997	.09227511	-.00000001	-.00000000	.00000000	.00917582
16	-.00300109	.34993409	-.00000001	-.00000000	-.00000000	.01597469
17	-.00457727	.55254762	.00000000	-.00000000	-.00000000	.01805380
18	-.00634549	.77269561	.00000001	-.00000000	-.00000000	.01903453
19	-.00918924	1.00000000	.00000003	-.00000000	-.00000000	.01935468

MODE NO. 5 FREQ. = 28.64 HZ GEN. INERTIA = .9137E+01 IN-LB-SEC**2

MASS PT.	X (IN/RAD)	Y (IN/RAD)	Z (IN/RAD)	THETAX (RAD/RAD)	THETAY (RAD/RAD)	THETAZ (RAD/RAD)
1	.00000000	-.00000001	-.02099925	-.00000127	.00003750	.00000000
2	-.00000000	-.00000000	-.02207152	-.00000369	.00003598	.00000000
3	-.00000000	.00000000	-.02267684	-.00000868	.00004963	.00000000
4	-.00000000	.00000001	-.04584582	-.92939913	-.00615570	.00000000
5	-.00000000	.00000001	-.14142493	.56252063	.01037670	-.00000000
6	-.00000000	.00000000	-.26179048	1.00000000	.01710600	-.00000000
7	.00000000	-.00000001	-.21317550	.00886096	-.00110710	-.00000000
8	.00008814	-.00001493	-.02183389	.00000609	.00004110	.00000037
9	-.00008814	.00001493	-.02183389	.00000609	.00004110	-.00000037
10	-.00000000	-.00000000	-.02183422	.00000609	.00004113	.00000000
11	-.00000000	.00000000	-.02283816	.00003603	.00009227	.00000000
12	-.00000000	.00000001	-.04931887	.00102168	.00139843	.00000000
13	-.00000000	.00000001	-.10131315	.00228615	.00192197	-.00000000
14	-.00000000	.00000000	-.17320172	.00454413	.00270025	-.00000000
15	-.00000000	-.00000001	-.21320617	.00876243	-.00030299	-.00000000
16	-.00000000	-.00000003	-.24053258	.00875223	.00281699	-.00000000
17	.00000000	-.00000004	-.27906008	.00875137	.00369882	-.00000000
18	.00000000	-.00000006	-.32471209	.00875189	.00407999	-.00000000
19	.00000000	-.00000007	-.37253103	.00875291	.00419231	-.00000000

MODE NO. 6 FREQ. = 38.21 HZ GEN. MASS = .2434E+01 LB-SEC**2/IN

MASS PT.	X (IN/IN)	Y (IN/IN)	Z (IN/IN)	THETAX (RAD/IN)	THETAY (RAD/IN)	THETAZ (RAD/IN)
1	-.00000000	-.00000000	-.39835853	.00000053	-.00030075	.00000000
2	-.00000000	-.00000000	-.38922749	.00000020	-.00029860	.00000000
3	-.00000000	.00000000	-.38556304	.00000040	-.00029287	.00000000
4	-.00000000	.00000000	-.38381245	.04801829	.00015158	-.00000000
5	-.00000000	-.00000000	-.36968328	-.13931203	-.00257736	-.00000000
6	.00000000	-.00000000	-.32544788	.15886932	-.00116491	-.00000000
7	.00000000	-.00000000	-.18906357	.00076694	-.00747087	-.00000000
8	-.00054178	-.00000198	-.39174037	.00000107	-.00025269	-.00000006
9	.00054178	.00000198	-.39174037	.00000107	-.00025269	.00000006
10	-.00000000	-.00000000	-.39175112	.00000107	-.00025294	-.00000000
11	-.00000000	.00000000	-.39014275	.00000165	-.00008764	.00000000
12	-.00000000	-.00000000	-.44356606	.00005335	.00233731	-.00000000
13	-.00000000	-.00000000	-.49353712	.00016737	.00042134	-.00000000
14	-.00000000	-.00000000	-.41995568	.00040701	-.00546718	-.00000000
15	-.00000000	-.00000000	-.18907014	.00077407	-.00916261	-.00000000
16	-.00000000	-.00000000	.11860100	.00086992	-.02062514	-.00000000
17	-.00000000	-.00000000	.38715933	.00090355	-.02433560	.00000000
18	-.00000000	-.00000000	.68722481	.00092236	-.02609088	.00000000
19	-.00000000	-.00000000	1.00000000	.00092911	-.02665244	.00000000

MODE NO. 7 FREQ. = 39.42 HZ GEN. INERTIA = .5258E+01 IN-LB-SEC**2

MASS PT.	X (IN/RAD)	Y (IN/RAD)	Z (IN/RAD)	THETAX (RAD/RAD)	THETAY (RAD/RAD)	THETAZ (RAD/RAD)
1	.00000000	.00000000	.15986667	.00000590	.00017019	-.00000000
2	.00000000	.00000000	.15476275	.00000401	.00016762	-.00000000
3	.00000000	-.00000000	.15255225	.00000507	.00017733	-.00000000
4	.00000000	-.00000000	.13113738	.21846157	.00347641	-.00000000
5	.00000000	-.00000000	.07618299	-.68798472	-.00354291	-.00000000
6	.00000000	-.00000000	-.05169357	1.00000000	.01770513	-.00000000
7	.00000000	-.00000000	-.02628949	.00922017	.00037659	-.00000000
8	.00032567	-.00002918	.15611266	.00001183	.00015188	.00000075
9	-.00032567	.00002918	.15611266	.00001183	.00015188	-.00000075
10	.00000000	.00000000	.15611722	.00001184	.00015203	.00000000
11	.00000000	-.00000000	.15445604	.00004395	.00012521	-.00000000
12	.00000000	-.00000000	.15476230	.00108669	.00016467	-.00000000
13	.00000000	-.00000000	.13529378	.00240150	.00122546	-.00000000
14	.00000000	-.00000000	.06131336	.00473794	.00356547	-.00000000
15	.00000000	-.00000000	-.02631161	.00911431	.00168801	-.00000000
16	.00000000	-.00000000	-.13153353	.00908975	.00833992	-.00000000
17	.00000000	-.00000000	-.24318477	.00908468	.01040770	-.00000000
18	.00000000	-.00000000	-.37213454	.00908317	.01136079	-.00000000
19	.00000000	-.00000000	-.50739001	.00908411	.01165807	-.00000000

MODE NO. 8 FREQ. = 56.15 HZ GEN. INERTIA = .1107E+02 IN-LB-SEC**2

MASS PT.	X (IN/RAD)	Y (IN/RAD)	Z (IN/RAD)	THETAX (RAD/RAD)	THETAY (RAD/RAD)	THETAZ (RAD/RAD)
1	.00004621	-.00505367	.00000000	.00000000	-.00000000	.00013864
2	.00000855	-.00061566	.00000000	.00000000	-.00000000	.00012214
3	-.00000245	.00073443	.00000000	.00000000	-.00000000	.00010733
4	.00028740	-.03306415	.00000000	.00000000	.00000000	-.00236823
5	.00115001	-.12034756	-.00000000	.00000000	.00000000	-.00422792
6	.00238205	-.22989186	-.00000000	-.00000000	.00000000	-.00411938
7	.00333039	-.27727861	-.00000000	.00000000	.00000000	.00102809
8	.00009634	-.00187747	.00000000	-.00000000	-.00000000	.00020653
9	.00009634	-.00187747	.00000000	.00000000	-.00000000	.00020653
10	.00009647	-.00188238	.00000000	-.00000000	-.00000000	1.00000000
11	.00010511	-.00286483	.00000000	-.00000000	-.00000000	-.00027667
12	.00086244	-.07971296	.00000000	.00000000	-.00000000	-.00381112
13	.00220417	-.20437693	.00000000	.00000000	-.00000000	-.00379190
14	.00277431	-.29646551	.00000000	.00000000	.00000000	-.00199497
15	.00257768	-.27734529	-.00000000	.00000000	.00000000	.00241300
16	.00112413	-.10398729	-.00000000	.00000000	-.00000000	.01411118
17	-.00039009	.09067888	.00000000	.00000000	-.00000000	.01834753
18	-.00225372	.32270806	.00000000	.00000000	-.00000000	.02050591
19	-.00535794	.57085505	.00000000	.00000000	-.00000000	.02123316

MODE NO. 9 FREQ. = 61.28 HZ GEN. MASS = .2680E+00 LB-SEC**2/IN

MASS PT.	X (IN/IN)	Y (IN/IN)	Z (IN/IN)	THETAX (RAD/IN)	THETAY (RAD/IN)	THETAZ (RAD/IN)
1	-.00000000	.00000000	.05334602	-.00000266	.00026772	-.00000000
2	-.00000000	.00000000	.04550531	-.00000217	.00025023	-.00000000
3	-.00000000	-.00000000	.04174173	-.00000249	.00028947	-.00000000
4	.00000000	-.00000000	-.02509617	.00138192	.00429163	.00000000
5	.00000000	.00000000	-.17238190	.00174627	.00683938	.00000000
6	.00000000	.00000000	-.34704702	-.00997028	.00642176	.00000000
7	.00000000	.00000000	-.46574917	-.00132538	.00233257	-.00000000
8	.00058550	.00000843	.04733888	-.00000374	.00027302	-.00000008
9	-.00058550	-.00000843	.04733888	-.00000374	.00027302	.00000008
10	-.00000000	.00000000	.04734222	-.00000374	.00027354	.00000000
11	-.00000000	-.00000000	.04159[illegible]04	-.00001054	.00051255	-.00000000
12	-.00000000	.00000000	-.08191832	-.00020296	.00614779	.00000000
13	-.00000000	.00000000	-.28554318	-.00037961	.00629537	.00000000
14	-.00000000	.00000000	-.44751998	-.00068788	.00402369	.00000000
15	-.00000000	.00000000	-.46586850	-.00128568	-.00072272	-.00000000
16	-.00000000	.00000000	-.20224858	-.00109719	-.02404646	-.00000000
17	-.00000000	-.00000000	.13799057	-.00102334	-.03255390	-.00000000
18	-.00000000	-.00000000	.55310728	-.00098015	-.03685316	-.00000000
19	-.00000000	-.00000000	1.00000000	-.00096466	-.03828224	-.00000000

MODE NO. 10 FREQ. = 61.83 HZ GEN. MASS = .3602E+00 LB-SEC**2/IN

MASS PT.	X (IN/IN)	Y (IN/IN)	Z (IN/IN)	THETAX (RAD/IN)	THETAY (RAD/IN)	THETAZ (RAD/IN)
1	.00083770	-.09776803	.00000000	.00000000	.00000000	.00330280
2	.00002158	-.00156827	.00000000	.00000000	.00000000	.00320353
3	-.00033722	.04086514	.00000000	.00000000	.00000000	.00326783
4	-.00022697	.02889902	-.00000000	.00000000	.00000000	-.00303542
5	.00133480	-.12883464	-.00000000	.00000000	.00000000	-.00853637
6	.00412350	-.37665348	-.00000000	-.00000000	.00000000	-.01019296
7	.00733915	-.53695228	-.00000000	-.00000000	.00000000	-.00100452
8	.00041324	-.02590032	.00000000	-.00000000	.00000000	.00318498
9	.00041324	-.02590032	.00000000	-.00000000	.00000000	.00318498
10	.00041418	-.02598054	.00000000	-.00000000	.00000000	-.01499814
11	.00011608	.00923337	.00000000	-.00000000	-.00000000	.00222850
12	.00100812	-.08111444	.00000000	-.00000000	-.00000000	-.00617788
13	.00344140	-.30718541	.00000000	-.00000000	.00000000	-.00741757
14	.00470747	-.51175991	-.00000000	-.00000000	.00000000	-.00544922
15	.00497561	-.53714581	-.00000000	-.00000000	.00000000	.00179954
16	.00259109	-.25272218	-.00000000	.00000000	-.00000000	.02514578
17	-.00016634	.10177467	.00000000	.00000000	-.00000000	.03387[illegible]84
18	-.00363741	.53393417	.00000000	.00000000	-.00000000	.03839227
19	-.00946761	1.00000000	.00000000	.00000000	-.00000000	.03992672

APPENDIX F

This appendix presents output from the computer program "FORCFCN" for the dynamic response of the 75mm ADMAG gun system mounted on the M24 artillery mount when subjected to a single shot firing. The first fifty lines of output obtained for the trunnion and elevation link loads (in BRL gauge coordinates) and muzzle linear and angular displacements, velocities and accelerations are presented.

TRUNNION LOADS(LBS, TENSION + FOR ELEV. LINK)

TIME (MS)	RIGHT HORIZ.	RIGHT VERTICAL	RIGHT LATERAL	LEFT HORIZ.	LEFT VERTICAL	LEFT LATERAL	ELEV. LINK
.5	-128.8	-15.3	.0	-133.3	-14.1	.0	.3
1.0	-1044.5	-99.6	.0	-1072.5	-93.8	.0	2.0
1.5	-2888.6	-282.3	.1	-2952.4	-270.6	.1	5.2
2.0	-5180.0	-614.9	.2	-5274.2	-598.2	.1	9.0
2.5	-7199.5	-1014.8	.2	-7296.3	-996.7	.2	12.1
3.0	-7811.5	-1341.1	.3	-7874.4	-1318.6	.3	12.3
3.5	-5747.5	-1468.2	.3	-5745.1	-1432.5	.3	7.2
4.0	-1870.7	-954.5	.2	-1826.2	-901.1	.2	-1.1
4.5	165.3	-222.8	-.0	143.4	-162.9	-.0	-5.5
5.0	-1947.0	321.5	-.3	-2135.6	375.2	-.3	-3.7
5.5	-5469.8	-12.2	-.3	-5781.2	40.7	-.3	-.2
6.0	-5669.2	-917.1	-.1	-5917.0	-847.7	-.1	-3.2
6.5	-2764.6	-1037.3	.2	-2827.2	-950.8	.2	-9.9
7.0	-1581.2	-665.1	.3	-1550.0	-585.3	.2	-10.5
7.5	-4815.3	-74.7	.1	-4884.7	-8.1	.1	-4.9
8.0	-9997.8	-475.5	-.0	-10253.1	-398.9	-.0	3.3
8.5	-12301.5	-1755.9	.1	-12628.4	-1645.5	.1	6.1
9.0	-11103.8	-2209.3	.4	-11345.0	-2079.9	.3	5.2
9.5	-9696.9	-1933.1	.5	-9819.6	-1837.0	.4	7.8
10.0	-10739.4	-1594.0	.4	-10845.7	-1548.5	.3	14.1
10.5	-14366.9	-799.3	.3	-14570.6	-765.6	.2	30.7
11.0	-16639.9	47.8	.3	-16925.0	116.6	.2	52.9
11.5	-14719.1	-479.8	.3	-14970.9	-387.1	.2	71.2
12.0	-11586.5	-1841.0	.1	-11760.2	-1785.2	.1	90.2
12.5	-10951.2	-2480.5	-.2	-11115.1	-2484.0	-.1	115.4
13.0	-12578.4	-1883.2	-.3	-12799.8	-1903.9	-.2	148.6
13.5	-13094.6	-659.1	-.2	-13352.1	-647.4	-.2	181.0
14.0	-11005.4	12.2	-.2	-11236.4	40.0	-.2	204.9
14.5	-8598.2	-292.0	-.4	-8805.4	-310.2	-.3	226.7
15.0	-7410.1	-891.0	-.7	-7640.3	-967.6	-.6	248.1
15.5	-7141.3	-1155.0	-.9	-7419.3	-1232.9	-.7	267.1
16.0	-7081.0	-1341.3	-.8	-7389.1	-1372.0	-.7	281.9
16.5	-6377.1	-1569.3	-.7	-6661.0	-1571.8	-.6	285.2
17.0	-5275.2	-1619.0	-.7	-5501.1	-1646.7	-.6	275.1
17.5	-4624.6	-1723.9	-.8	-4810.8	-1789.1	-.7	253.0
18.0	-4668.4	-2154.1	-.8	-4859.2	-2203.1	-.6	214.3
18.5	-5587.8	-2826.7	-.6	-5820.4	-2804.7	-.5	157.1
19.0	-7019.7	-3315.6	-.4	-7287.3	-3227.4	-.4	81.9
19.5	-7919.1	-3264.1	-.4	-8163.9	-3148.4	-.3	-13.9
20.0	-8077.2	-2949.2	-.4	-8239.2	-2815.3	-.3	-131.0
20.5	-8328.9	-2914.9	-.2	-8399.1	-2720.4	-.2	-268.5
21.0	-9435.1	-3216.8	.0	-9451.3	-2913.2	.0	-424.5
21.5	-11078.0	-3601.5	.2	-11075.6	-3184.1	.2	-597.8
22.0	-12149.2	-3810.7	.2	-12122.7	-3308.2	.2	-788.9
22.5	-12174.7	-3567.1	.2	-12090.4	-2992.1	.1	-996.0
23.0	-11740.8	-2801.6	.2	-11577.4	-2128.0	.2	-1215.0
23.5	-11763.1	-1783.4	.3	-11528.4	-978.6	.2	-1441.3
24.0	-12371.0	-1069.6	.3	-12080.3	-131.7	.3	-1671.6
24.5	-12566.3	-1015.4	.3	-12209.4	34.3	.3	-1905.4
25.0	-11696.2	-1273.2	.2	-11247.4	-122.4	.2	-2140.8

DISPLACEMENTS(IN) AND ROTATIONS(RAD) AT MASS POINT 19

TIME (MS)	X	Y	Z	THETAX	THETAY	THETAZ
.2	-.48054E-03	-.63651E-05	.15593E-10	-.46666E-09	-.16430E-10	-.32395E-07
.4	-.27698E-02	-.36362E-04	.86503E-10	-.26887E-08	-.94418E-10	-.87188E-07
.6	-.66200E-02	-.87486E-04	.17276E-09	-.53895E-08	-.19481E-09	-.95140E-07
.8	-.11891E-01	-.15871E-03	.21307E-09	-.65218E-08	-.25229E-09	-.10875E-06
1.0	-.18635E-01	-.25137E-03	.21856E-09	-.66918E-08	-.28260E-09	-.32534E-06
1.2	-.26981E-01	-.36737E-03	.24470E-09	-.78146E-08	-.34722E-09	-.91957E-06
1.4	-.36933E-01	-.50684E-03	.31238E-09	-.85094E-08	-.44226E-09	-.18758E-05
1.6	-.48348E-01	-.66774E-03	[illegible]	-.31446E-08	-.51801E-09	-.30129E-05
1.8	-.61110E-01	-.84820E-03	.15560E-09	.13207E-07	-.61569E-09	-.41757E-05
2.0	-.75356E-01	-.10490E-02	-.14856E-08	.39629E-07	-.82657E-09	-.53813E-05
2.2	-.91514E-01	-.12739E-02	-.72741E-08	.69978E-0[illegible]	-.91251E-09	-.67330E-05
2.4	-.11015E+00	-.15271E-02	-.21263E-07	.97932E-07	.66325E-10	-.81535E-05
2.6	-.13178E+00	-.18110E-02	-.46466E-07	.12062E-06	.35088E-08	-.92608E-05
2.8	-.15669E+00	-.21256E-02	-.80884E-07	.13829E-06	.10254E-07	-.95048E-05
3.0	-.18490E+00	-.24690E-02	-.11436E-06	.15271E-06	.19439E-07	-.84154E-05
3.2	-.21610E+00	-.28374E-02	-.12909E-06	.16652E-06	.28233E-07	-.57626E-05
3.4	-.24961E+00	-.32255E-02	-.10439E-06	.18260E-06	.32853E-07	-.15749E-05
3.6	-.28452E+00	-.36279E-02	-.23508E-07	.20159E-06	.30208E-07	.38716E-05
3.8	-.31994E+00	-.40419E-02	.12155E-06	.21972E-06	.18961E-07	.99913E-05
4.0	-.35532E+00	-.44702E-02	.32913E-06	.23028E-06	-.67246E-09	.15809E-04
4.2	-.39064E+00	-.49214E-02	.58971E-06	.22863E-06	-.27956E-07	.20021E-04
4.4	-.42633E+00	-.54085E-02	.88699E-06	.21602E-06	-.62353E-07	.21255E-04
4.6	-.46315E+00	-.59441E-02	.11960E-05	.19877E-06	-.10311E-06	.18403E-04
4.8	-.50187E+00	-.65369E-02	.14793E-05	.18398E-06	-.14792E-06	.10948E-04
5.0	-.54310E+00	-.71877E-02	.16846E-05	.17616E-06	-.19164E-06	-.69476E-06
5.2	-.58708E+00	-.78867E-02	.17484E-05	.17657E-06	-.22613E-06	-.14495E-04
5.4	-.63361E+00	-.86204E-02	.16045E-05	.18415E-06	-.24148E-06	-.30766E-0[illegible]
5.6	-.68214E+00	-.93636E-02	.11973E-05	.19644E-06	-.22854E-06	-.48230E-04
5.8	-.73193E+00	-.10061E-01	.49507E-06	.21082E-06	-.17984E-06	-.41696E-04
6.0	-.78225E+00	-.10773E-01	-.49135E-06	.22654E-06	-.97256E-07	.12717E-04
6.2	-.83260E+00	-.12993E-01	-.17808E-05	.24596E-06	.17271E-07	.21017E-03
6.4	-.88279E+00	-.19349E-01	-.34460E-05	.27268E-06	.18324E-06	.24278E-05
6.6	-.93309E+00	-.27006E-01	-.51444E-05	.30868E-06	.37608E-06	-.41432E-03
6.8	-.98396E+00	-.35292E-01	-.66628E-05	.35118E-06	.57032E-06	-.86991E-03
7.0	-.10360E+01	-.43570E-01	-.77566E-05	.39343E-06	.73204E-06	-.12076E-02
7.2	-.10897E+01	-.51560E-01	-.81923E-05	.42756E-06	.82670E-06	-.13832E-02
7.4	-.11454E+01	-.59392E-01	-.77959E-05	.44744E-06	[illegible]	-.14750E-02
7.6	-.12033E+01	-.67364E-01	-.64910E-05	.44997E-06	.71625E-06	-.15936E-02
7.8	-.12629E+01	-.75636E-01	-.43233E-05	.43539E-06	.49353E-06	-.18060E-02
8.0	-.13237E+01	-.84080E-01	-.14714E-05	.40726E-06	.17245E-06	-.20762E-02
8.2	-.13851E+01	-.92397E-01	.17580E-05	.37179E-06	-.21616E-06	-.23152E-02
8.4	-.14466E+01	-.10035E+00	.49555E-05	.33626E-06	-.62504E-06	-.24555E-02
8.6	-.15079E+01	-.10793E+00	.76592E-05	.30729E-06	-.99502E-06	-.25007E-02
8.8	-.15689E+01	[illegible]	[illegible]	.29004E-06	-.12647E-05	-.25140E-02
9.0	-.16299E+01	-.12280E+00	.99223E-05	.28929E-06	-.13830E-05	-.25642E-02
9.2	-.16913E+01	-.13045E+00	.89793E-05	.30542E-06	-.13219E-05	-.26754E-02
9.4	-.17536E+01	-.13823E+00	.66511E-05	33386E-06	-.10829E-05	-.28203E-02
9.6	-.18170E+01	-.14603E+00	.32068E-05	.36964E-06	-.69707E-06	-.29538E-02
9.8	-.18818E+01	-.15376E+00	-.90818E-06	.40883E-06	-.22028E-06	-.30543E-02
10.0	-.19477E+01	-.16146E+00	-.51415E-05	.44518E-06	.27772E-06	-.31364E-02

LINEAR(FT/SEC) AND ANGULAR(RAD/SEC) VELOCITIES AT MASS POINT 19

TIME (MS)	X	Y	Z	THETAX	THETAY	THETAZ
.2	-.59366E+00	-.77486E-02	.17577E-07	-.64581E-05	-.22579E-06	-.28789E-03
.4	-.12952E+01	-.17054E-01	.38066E-07	-.14442E-04	-.51319E-06	-.18595E-03
.6	-.19028E+01	-.25470E-01	.28711E-07	-.10478E-04	-.42447E-06	.66037E-04
.8	-.24941E+01	-.33989E-01	.59600E-08	-.14858E-05	-.16746E-06	-.38378E-03
1.0	-.31371E+01	-.43375E-01	.31868E-08	-.23031E-05	-.19952E-06	-.19445E-02
1.2	-.38177E+01	-.53305E-01	.20037E-07	-.78892E-05	-.44179E-06	-.39825E-02
1.4	-.44639E+01	-.62754E-01	.34050E-07	.61364E-05	-.45119E-06	-.54097E-02
1.6	-.50384E+01	-.71179E-01	.47630E-08	.52135E-04	-.34112E-06	-.58194E-02
1.8	-.56062E+01	-.79260E-01	-.26169E-06	.11054E-03	-.74795E-06	-.58294E-02
2.0	-.62961E+01	-.88375E-01	-.12959E-05	.14805E-03	-.12023E-05	-.63392E-02
2.2	-.72094E+01	-.99334E-01	-.38261E-05	.14784E-03	.11435E-05	-.71249E-02
2.4	-.83592E+01	-.11180E+00	-.80708E-05	.12738E-03	.99044E-05	-.67483E-02
2.6	-.96826E+01	-.12477E+00	-.12818E-04	.99865E-04	.25327E-04	-.38429E-02
2.8	-.11077E+02	-.13730E+00	-.15157E-04	.78386E-04	.41432E-04	.18088E-02
3.0	-.12413E+02	-.14861E+00	-.11471E-04	.68004E-04	.47966E-04	.92762E-02
3.2	-.13539E+02	-.15799E+00	.68718E-06	.72728E-04	.36667E-04	.17225E-01
3.4	-.14320E+02	-.16502E+00	.21096E-04	.88840E-04	.68584E-05	.24428E-01
3.6	-.11703E+02	-.17012E+00	.46866E-04	.97837E-04	-.34398E-04	.29559E-01
3.8	-.14771E+02	-.17506E+00	.73884E-04	.77389E-04	-.77787E-04	.30823E-01
4.0	-.14713E+02	-.18243E+00	.98427E-04	.23960E-04	-.11787E-03	.26246E-01
4.2	-.14749E+02	-.19452E+00	.11761E-03	-.39376E-04	-.15456E-03	.14700E-01
4.4	-.15049E+02	-.21225E+00	.12843E-03	-.81003E-04	-.18891E-03	-.33075E-02
4.6	-.15685E+02	-.23472E+00	.12650E-03	-.85270E-04	-.21689E-03	-.25689E-01
4.8	-.16626E+02	-.25934E+00	.10590E-03	-.58845E-04	-.22695E-03	-.48404E-01
5.0	-.17749E+02	-.28243E+00	.60747E-04	-.18458E-04	-.20328E-03	-.67281E-01
5.2	-.18883E+02	-.29910E+00	-.12285E-04	.21608E-04	-.13293E-03	-.70568E-01
5.4	-.19852E+02	-.31175E+00	-.11149E-03	.52096E-04	-.13737E-04	-.10020E+00
5.6	-.20537E+02	-.29480E+00	-.23190E-03	.68426E-04	.16200E-03	.61155E-02
5.8	-.20899E+02	-.30145E+00	-.35288E-03	.74624E-04	.32548E-03	-.31228E-01
6.0	-.20998E+02	-.41640E+00	-.46762E-03	.84630E-04	.49624E-03	.82272E+00
6.2	-.20940E+02	-.21003E+01	-.64321E-03	.11227E-03	.71519E-03	-.12237E+00
6.4	-.20910E+02	-.29507E+01	-.71396E-03	.15698E-03	.91343E-03	-.16775E+01
6.6	-.21037E+02	-.33724E+01	-.68654E-03	.20061E-03	.99356E-03	-.23385E+01
6.8	-.21396E+02	-.34860E+01	-.55151E-03	.21852E-03	.91987E-03	-.20773E+01
7.0	-.21997E+02	-.33926E+01	-.33341E-03	.19710E-03	.66791E-03	-.12622E+01
7.2	-.22788E+02	-.32765E+01	-.17333E-04	.13899E-03	.25533E-03	-.56534E+00
7.4	-.23664E+02	-.32730E+01	.35314E-03	.57251E-04	-.26854E-03	-.43872E+00
7.6	-.24491E+02	-.33830E+01	.73122E-03	-.31741E-04	-.83778E-03	-.82169E+00
7.8	-.25135E+02	-.35002E+01	.10628E-02	-.11104E-03	-.13780E-02	-.12695E+01
8.0	-.25517E+02	-.35140E+01	.12923E-02	-.16497E-03	-.18071E-02	-.13510E+01
8.2	-.25638E+02	-.33991E+01	.13701E-02	-.18355E-03	-.20393E-02	-.97865E+00
8.4	-.25579E+02	-.32276E+01	.12619E-02	-.16613E-03	-.19994E-02	-.42871E+00
8.6	-.25460E+02	-.31039E+01	.95993E-03	-.11949E-03	-.16481E-02	-.80555E-01
8.8	-.25398E+02	-.30849E+01	.46976E-03	-.48117E-04	-.10048E-02	-.11459E+00
9.0	-.25477E+02	-.31476E+01	-.89250E-04	.40864E-04	-.15489E-03	-.40780E+00
9.2	-.25736E+02	-.32221E+01	-.69295E-03	.11614E-03	.76498E-03	-.67841E+00
9.4	-.26165E+02	-.32525E+01	-.1228[illegible]E-02	.16397E-03	.15997E-02	-.72937E+00
9.6	-.26703E+02	-.32360E+01	-.16106E-02	.19098E-03	.21022E-02	-.58639E+00
9.8	-.27251E+02	-.32112E+01	-.17796E-02	.19591E-03	.24985E-02	-.43462E+00
10.0	-.27694E+02	-.32149E+01	-.17078E-02	.15866E-03	.24196E-02	-.43861E+00

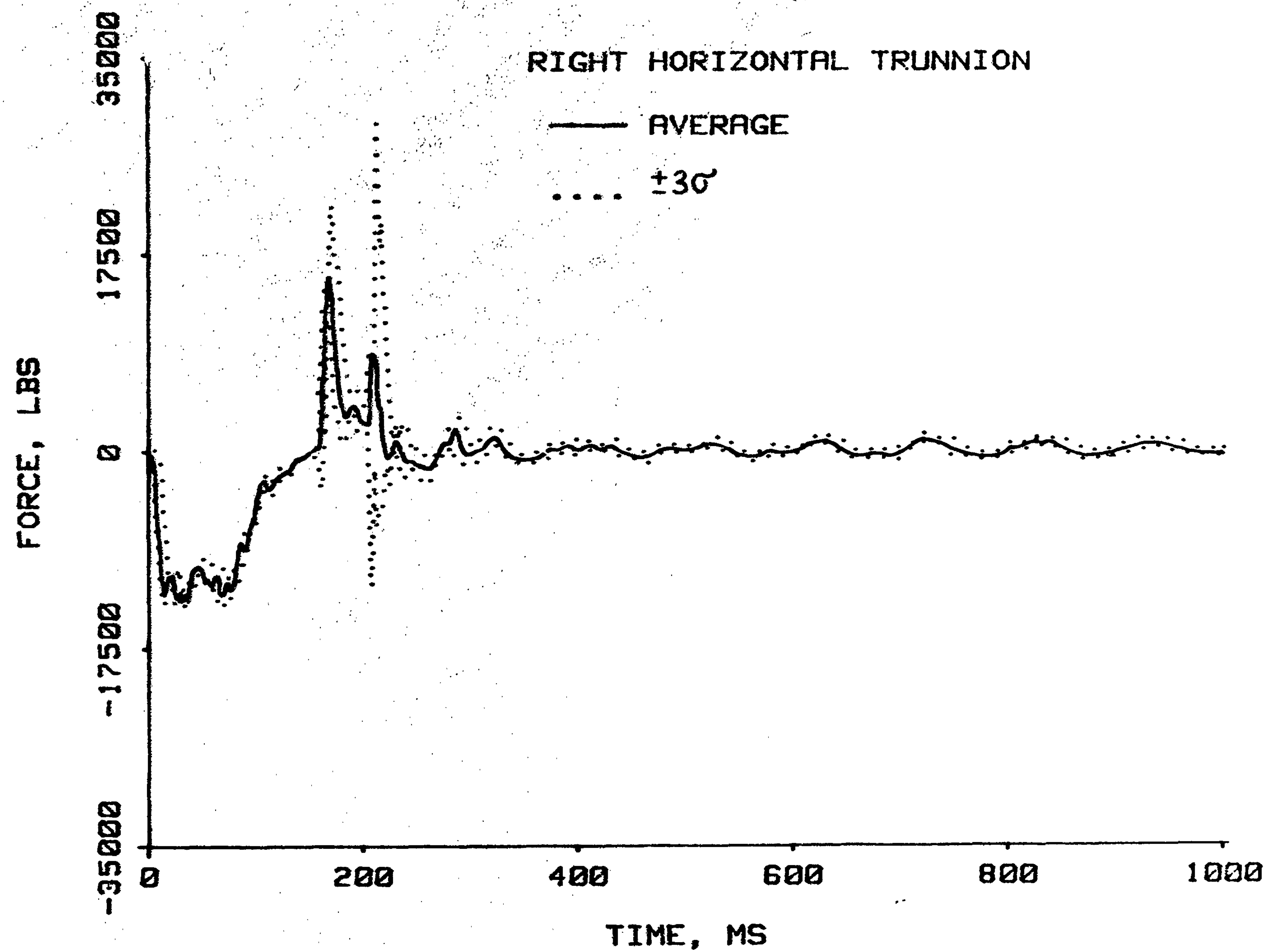
LINEAR(FT/SEC**2) AND ANGULAR(RAD/SEC**2) ACCELERATIONS AT MASS POINT 19

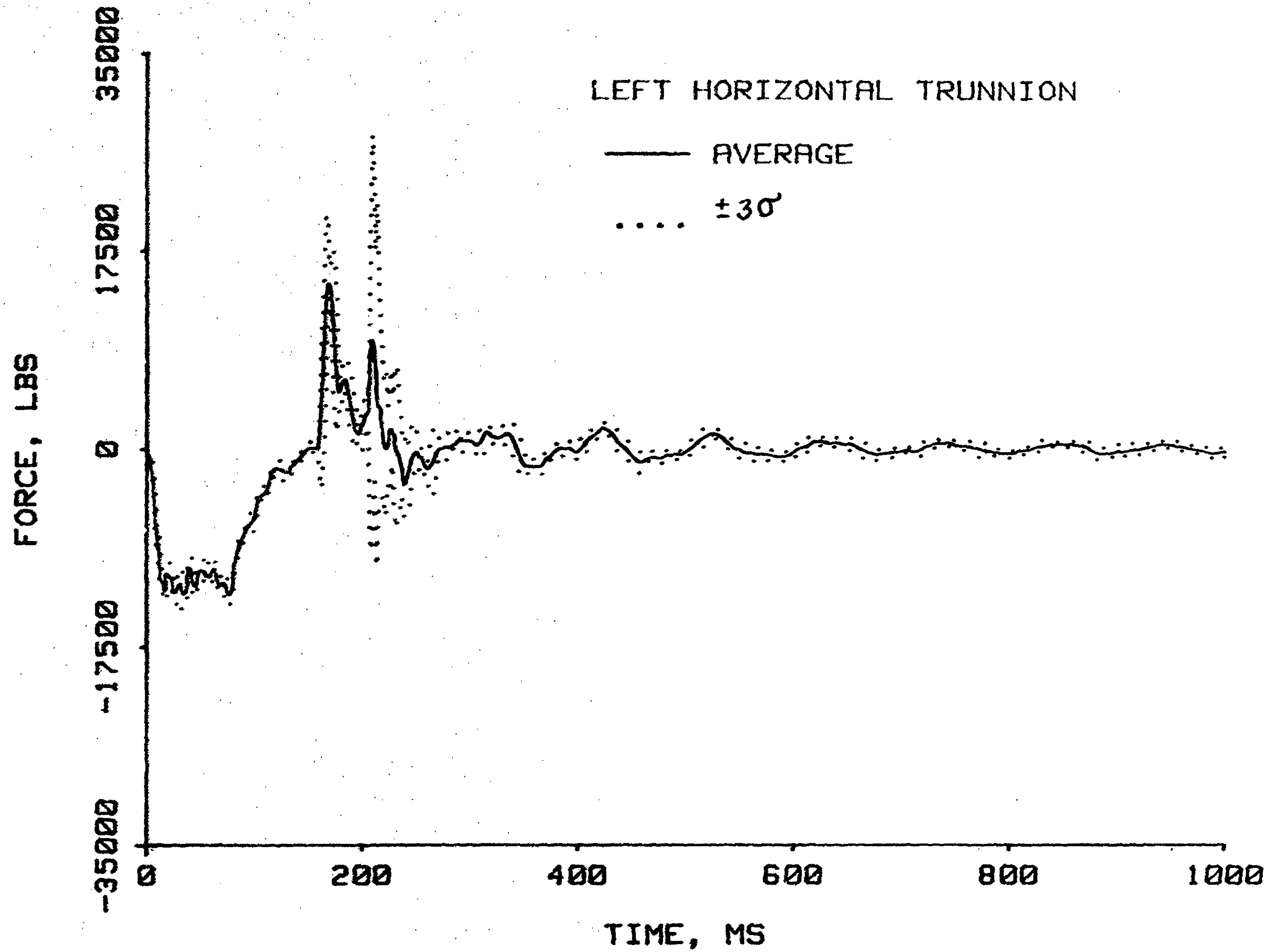
TIME (MS)	X	Y	Z	THETAX	THETAY	THETAZ
.2	-.93530E+02	-.31987E+01	.13898E-03	-.52234E-01	-.18260E-02	-.65562E+00
.4	.12972E+03	-.16645E+01	.37366E-04	-.14006E-01	-.63311E-03	.14784E+01
.6	-.14740E+02	-.50127E+01	-.11263E-03	.46544E-01	.12984E-02	.25701E+00
.8	-.76685E+03	-.15838E+02	-.82687E-04	.28041E-01	.81093E-03	-.51013E+01
1.0	-.18470E+04	-.30106E+02	.51035E-04	-.31992E-01	-.10393E-02	-.98823E+01
1.2	-.28316E+04	-.42446E+02	.95094E-04	-.29486E-03	-.90492E-03	-.94385E+01
1.4	-.36089E+04	-.51707E+02	.22513E-04	.15289E+00	.73083E-03	-.44394E+01
1.6	-.44905E+04	-.61912E+02	-.45843E-03	.28890E+00	-.31168E-03	-.26487E+00
1.8	-.58538E+04	-.77203E+02	-.26590E-02	.26418E+00	-.34904E-02	-.85955E+00
2.0	-.77322E+04	-.97266E+02	-.83562E-02	.97962E-01	.14742E-02	-.41050E+01
2.2	-.95866E+04	-.11435E+03	-.17218E-01	-.68477E-01	.25399E-01	-.24743E+01
2.4	-.10925E+05	-.12294E+03	-.24248E-01	-.13861E+00	.62697E-01	.74116E+01
2.6	-.11536E+05	-.12259E+03	-.20710E-01	-.12775E+00	.86400E-01	.21764E+02
2.8	-.11358E+05	-.11542E+03	.50791E-03	-.83473E-01	.65508E-01	.33870E+02
3.0	-.10345E+05	-.10311E+03	.38529E-01	-.16211E-01	-.75651E-02	.39632E+02
3.2	-.85119E+04	-.86891E+02	.82863E-01	.61307E-01	-.10600E+00	.38854E+02
3.4	-.61445E+04	-.70015E+02	.11867E+00	.83417E-01	-.18567E+00	.32109E+02
3.6	-.38547E+04	-.58800E+02	.13544E+00	-.14337E-01	-.21853E+00	.17683E+02
3.8	-.23262E+04	-.59644E+02	.13160E+00	-.19390E+00	-.21067E+00	-.67706E+01
4.0	-.19325E+04	-.74232E+02	.11152E+00	-.31935E+00	-.19046E+00	-.40012E+02
4.2	-.25266E+04	-.96935E+02	.77905E-01	-.28526E+00	-.17795E+00	-.74989E+02
4.4	-.37216E+04	-.11945E+03	.26649E-01	-.11752E+00	-.16248E+00	-.10325E+03
4.6	-.50424E+04	-.13344E+03	-.50993E-01	.67836E-01	-.10742E+00	-.11709E+03
4.8	-.59952E+04	-.13114E+03	-.16006E+00	.18112E+00	.20348E-01	-.10455E+03
5.0	-.62130E+04	-.11058E+03	-.29442E+00	.21061E+00	.22726E+00	-.82062E+02
5.2	-.55592E+04	-.70514E+02	-.43516E+00	.18243E+00	.48275E+00	-.21942E+02
5.4	-.41647E+04	-.42289E+02	-.54947E+00	.11823E+00	.69168E+00	-.18593E+03
5.6	-.23964E+04	.22957E+03	-.65912E+00	.47867E-01	.11734E+01	.14510E+04
5.8	-.69999E+03	-.21487E+03	-.59567E+00	.25932E-01	.85512E+00	-.12731E+04
6.0	.47268E+03	-.24057E+04	-.56166E+00	.89442E-01	.73578E+00	.64967E+04
6.2	.10551E+04	-.15167E+05	-.14286E+01	.17647E+00	.20854E+01	-.19336E+05
6.4	.48457E+03	-.27209E+04	-.75119E-01	.24269E+00	.69548E+00	-.54227E+04
6.6	-.53094E+03	-.13002E+04	.36851E+00	.17185E+00	.54031E-01	-.94048E+03
6.8	-.17623E+04	.96167E+02	.88672E+00	-.45421E-02	-.81194E+00	.32123E+04
7.0	-.28836E+04	.68755E+03	.13815E+01	-.20695E+00	-.16920E+01	.43293E+04
7.2	-.36119E+04	.36901E+03	.17509E+01	-.36286E+00	-.23914E+01	.22359E+04
7.4	-.37206E+04	-.31530E+03	.19144E+01	-.44099E+00	-.27918E+01	-.90624E+03
7.6	-.31215E+04	-.66971E+03	.18209E+01	-.43491E+00	-.28387E+01	-.25150E+04
7.8	-.19568E+04	-.38585E+03	.14480E+01	-.34467E+00	-.24946E+01	-.15729E+04
8.0	-.58064E+03	.28605E+03	.80492E+00	-.18592E+00	-.17240E+01	.83191E+03
8.2	.55035E+03	.81192E+03	-.56788E-01	.52540E-03	-.53278E+00	.26364E+04
8.4	.11484E+04	.82389E+03	-.10320E+01	.16773E+00	.96692E+00	.25185E+04
8.6	.11159E+04	.37767E+03	-.19666E+01	.29114E+00	.25305E+01	.80343E+03
8.8	.55362E+03	-.15128E+03	-.26850E+01	.42580E+00	.38302E+01	-.10271E+04
9.0	-.32810E+03	-.39912E+03	-.30336E+01	.43402E+00	.45523E+01	-.16469E+04
9.2	-.12860E+04	-.28303E+03	-.29242E+01	.30717E+00	.45141E+01	-.88166E+03
9.4	-.20676E+04	-.89135E+01	-.23567E+01	.17960E+00	.37148E+01	.34368E+03
9.6	-.24350E+04	.14724E+03	-.14161E+01	.90414E-01	.23077E+01	.91500E+03
9.8	-.22406E+04	.75710E+02	-.25028E+00	-.60981E-01	.53711E+00	.45203E+03
10.0	-.15171E+04	-.10486E+03	.95998E+00	-.32445E+00	-.13164E+01	-.49567E+03

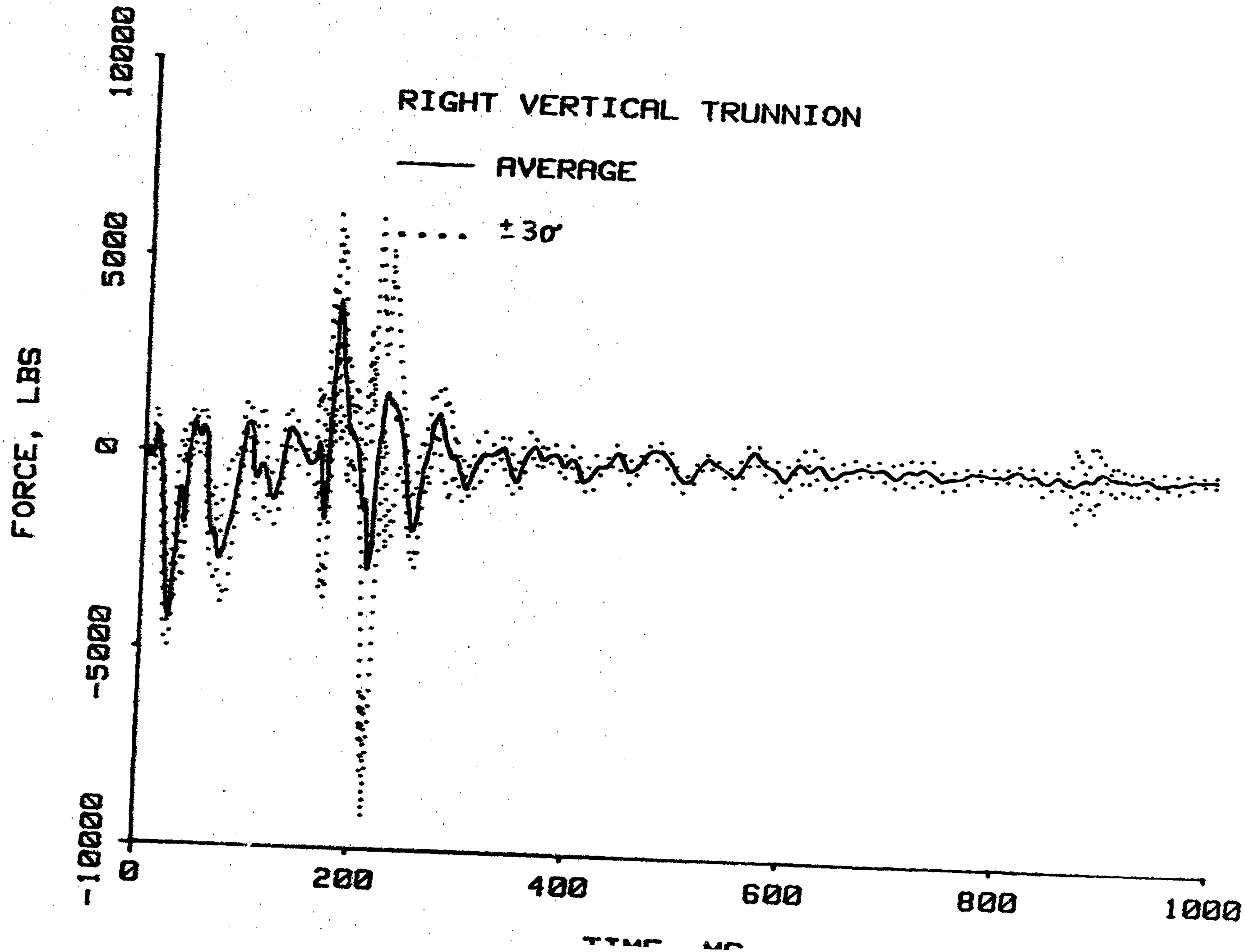
APPENDIX G

This appendix presents experimental data consisting of right and left horizontal, vertical and lateral trunnion gauge loads and the elevation link load obtained from the ten-round sample of 75mm APFSDS firings conducted at Yuma Proving Ground during June 1979. Set A presents a description of these data during the first 1000 ms subsequent to firing; Set B presents an expanded view of each parameter during the first 200 ms. The solid curve represents the mean of the ten-round sample; the dotted curves represent plus and minus three standard deviations from the mean.

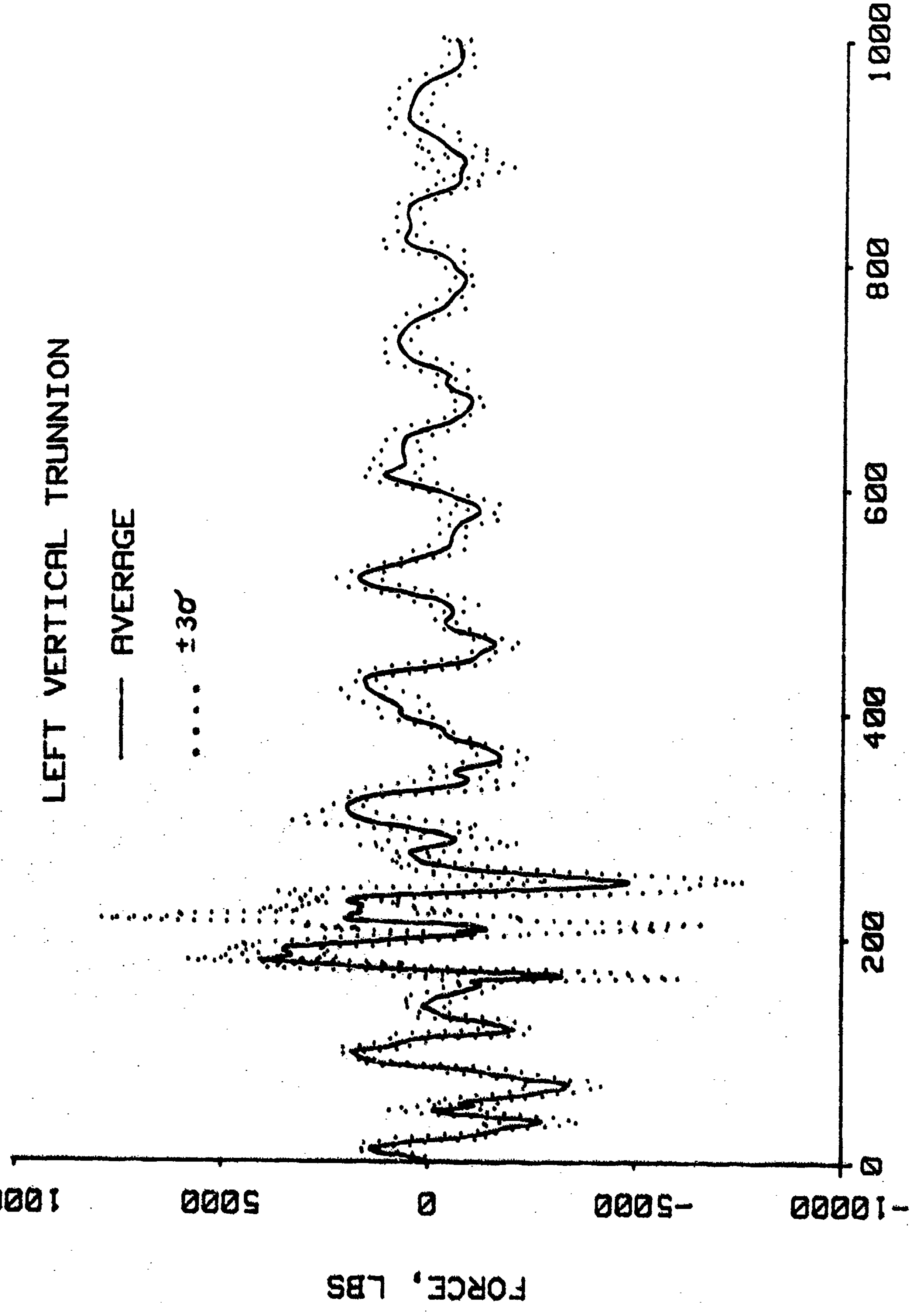
Set A

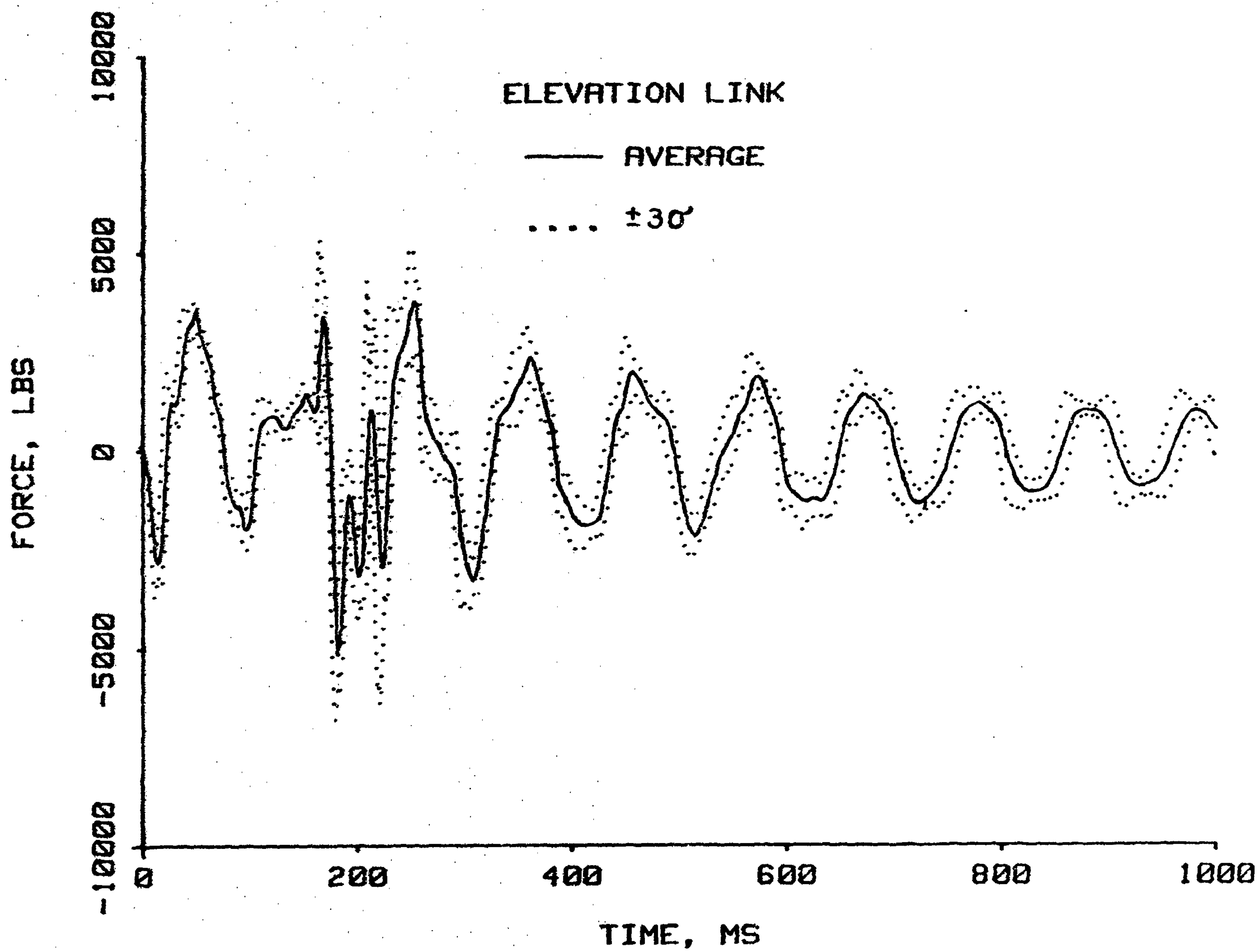

RIGHT HORIZONTAL TRUNNION
AVERAGE
±3σ
FORCE, LBS
35000
17500
0
-17500
-35000
0
200
400
600
800
1000
TIME, MS

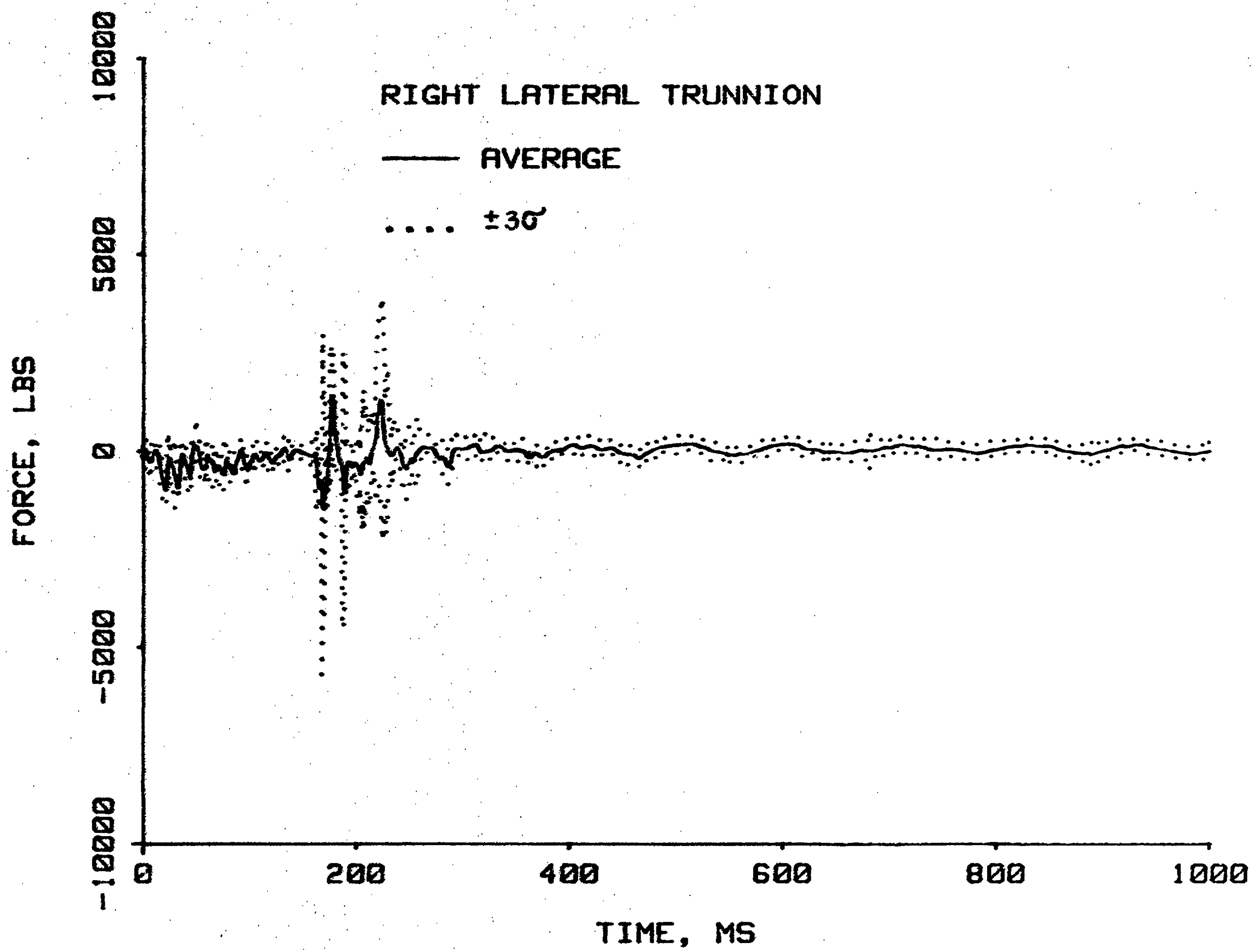

LEFT HORIZONTAL TRUNNION
AVERAGE
±3σ
35000
17500
0
-17500
-35000
FORCE, LBS
0
200
400
600
800
1000
TIME, MS

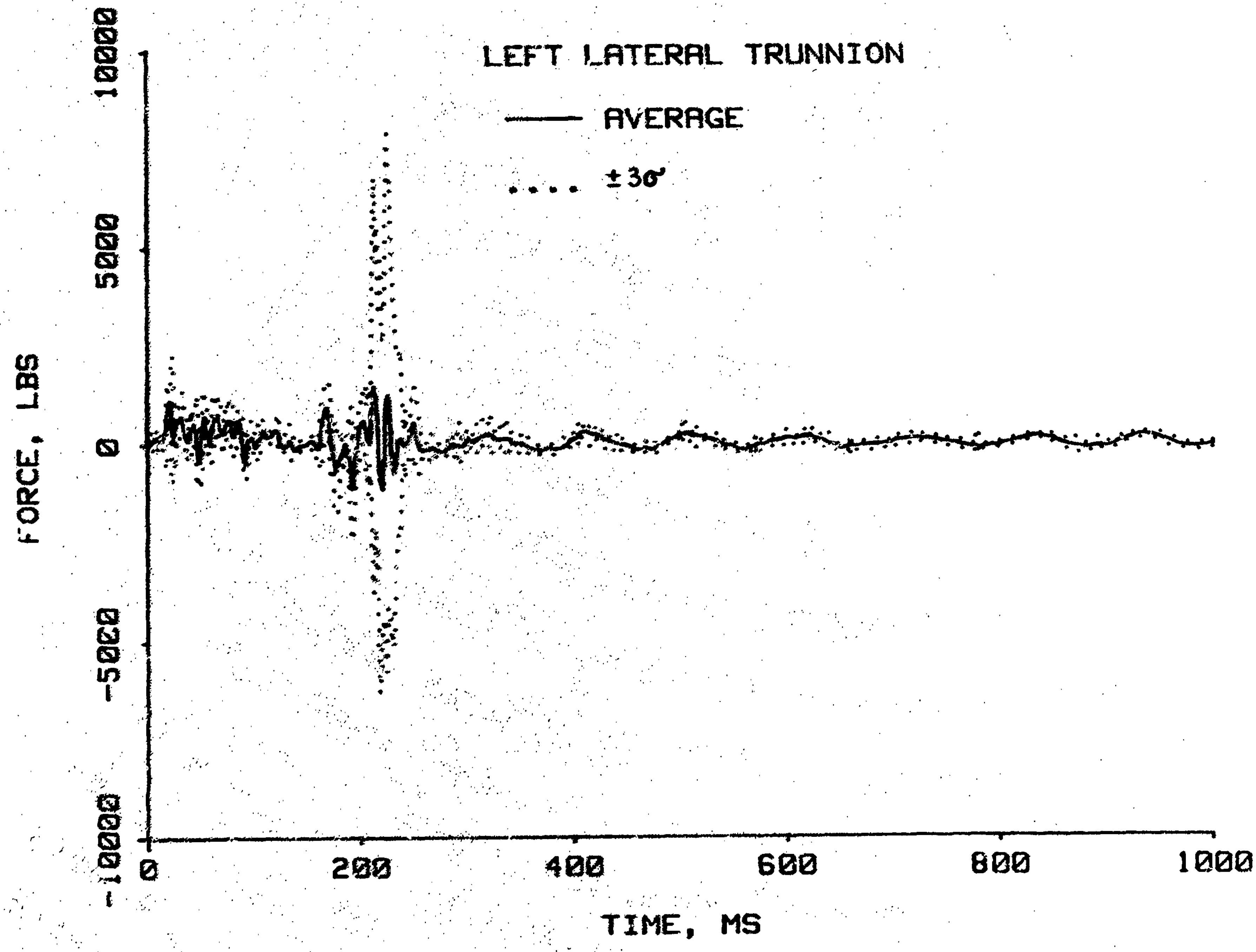


RIGHT VERTICAL TRUNNION
AVERAGE
±3σ
FORCE, LBS
10000
5000
0
-5000
-10000
0
200
400
600
800
1000

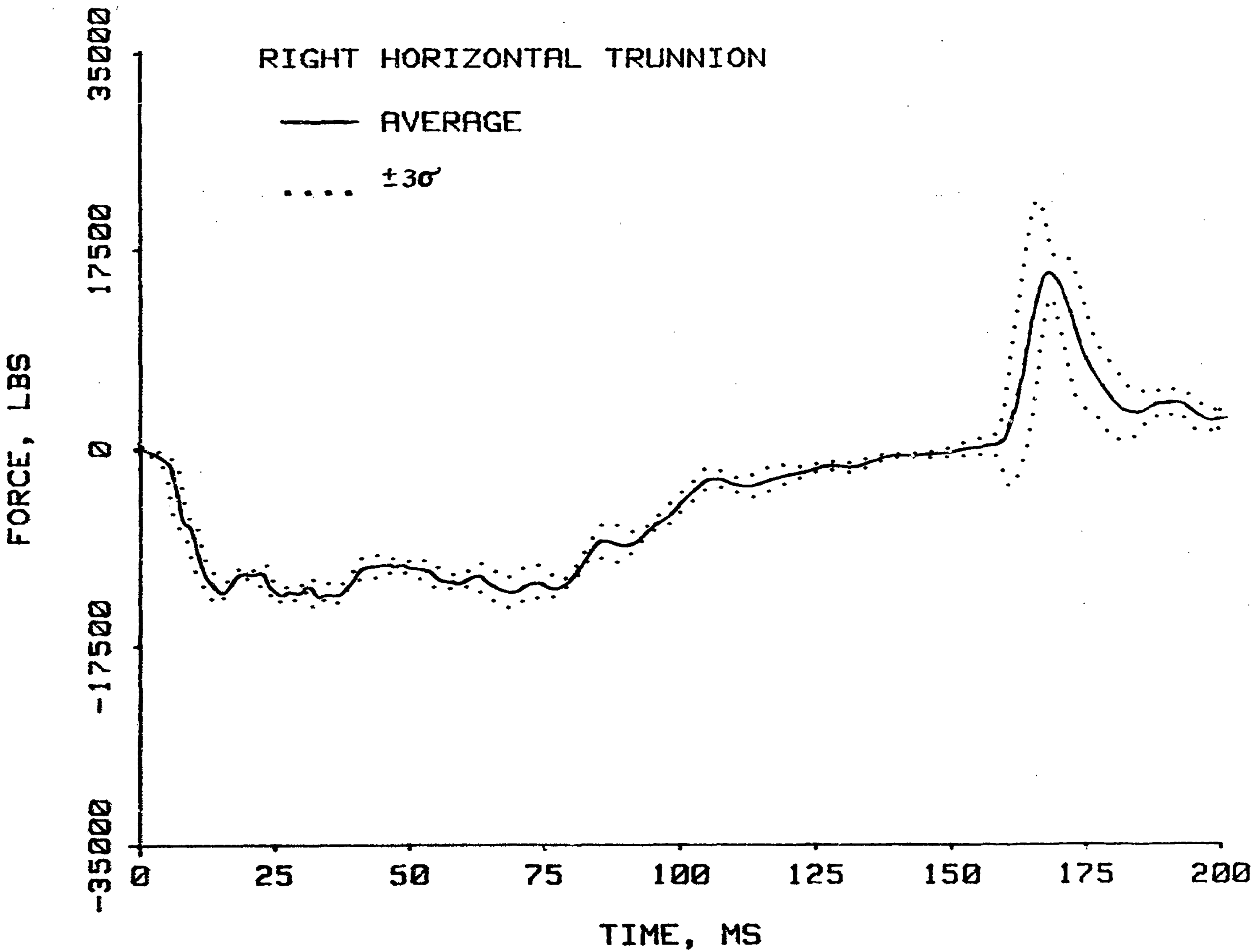

LEFT VERTICAL TRUNNION
— AVERAGE
.... ±3σ
FORCE, LBS
10000
5000
0
-5000
-10000
TIME, MS
0
200
400
600
800
1000

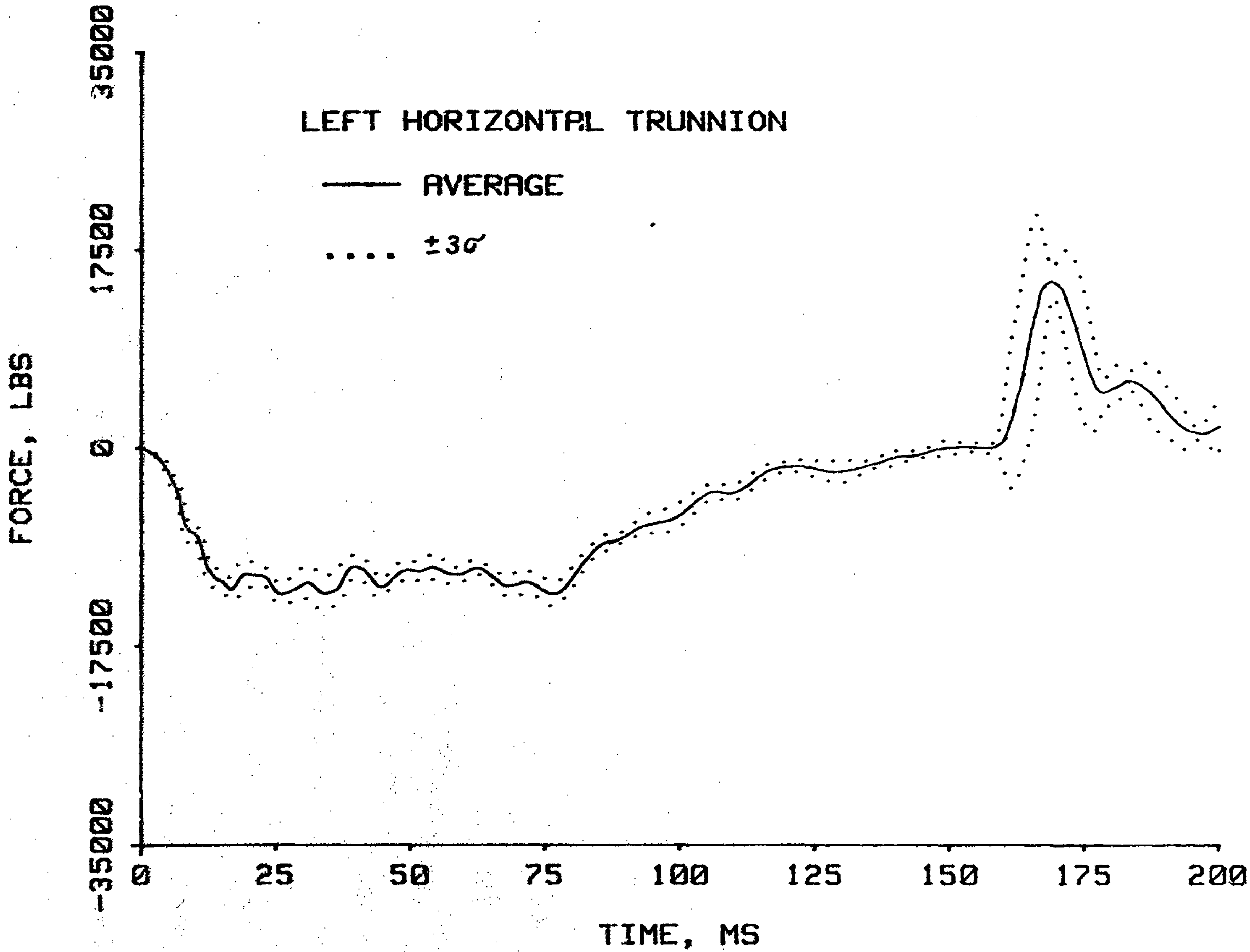

ELEVATION LINK
AVERAGE
±3σ
FORCE, LBS
10000
5000
0
-5000
-10000
TIME, MS
0
200
400
600
800
1000

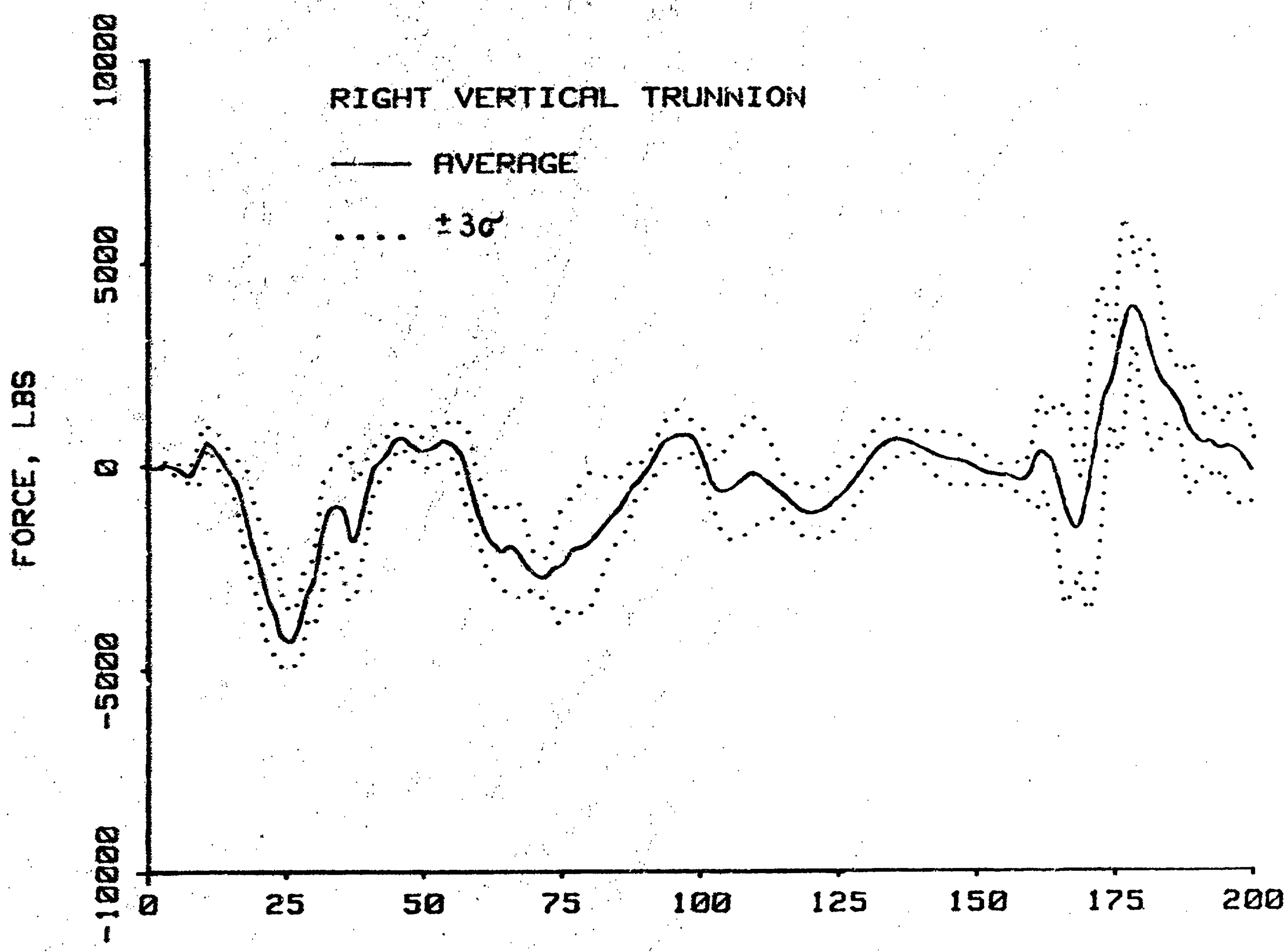

RIGHT LATERAL TRUNNION
AVERAGE
±3σ
FORCE, LBS
10000
5000
0
-5000
-10000
0
200
400
600
800
1000
TIME, MS

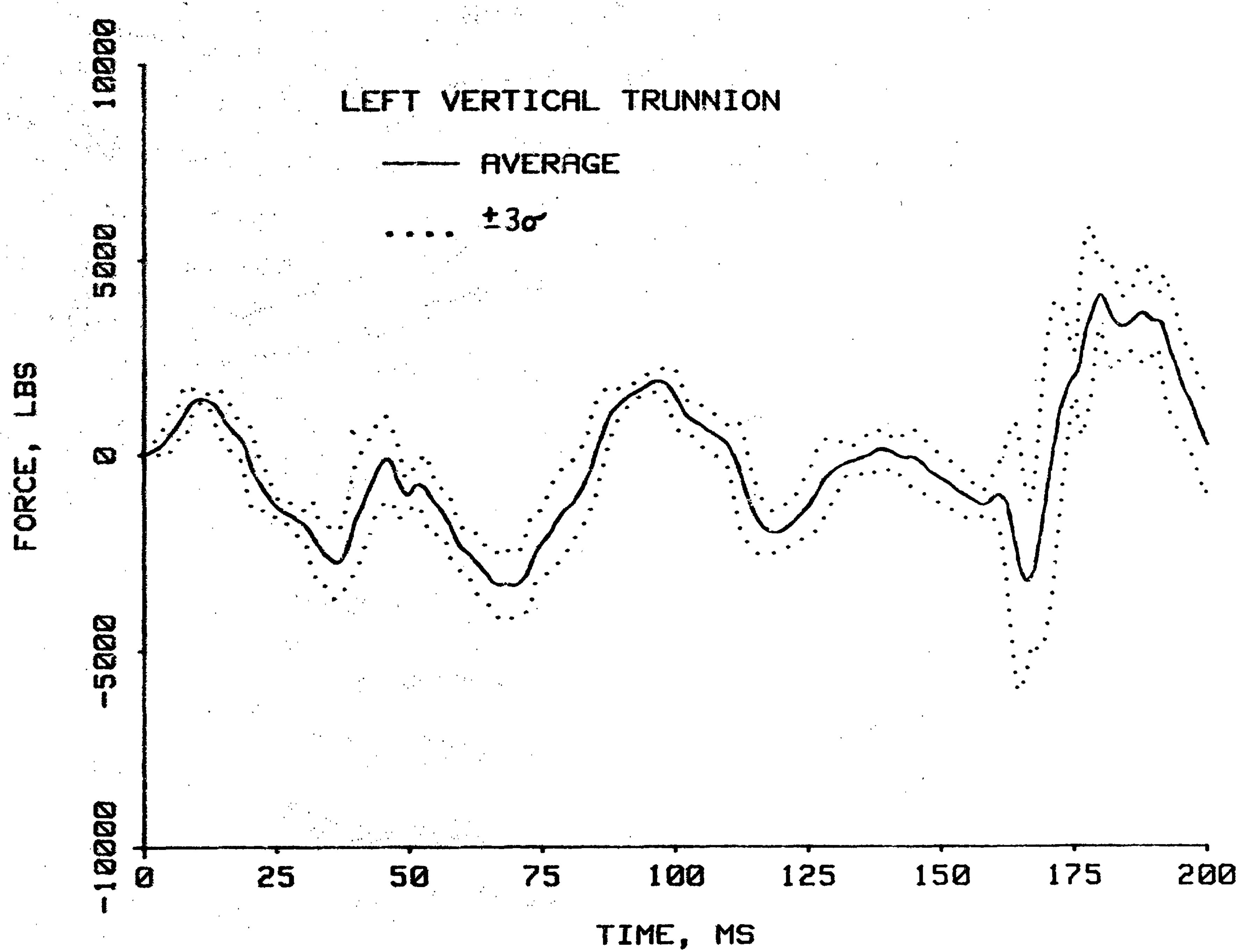

LEFT LATERAL TRUNNION
AVERAGE
±3σ
FORCE, LBS
10000
5000
0
-5000
-10000
0
200
400
600
800
1000
TIME, MS

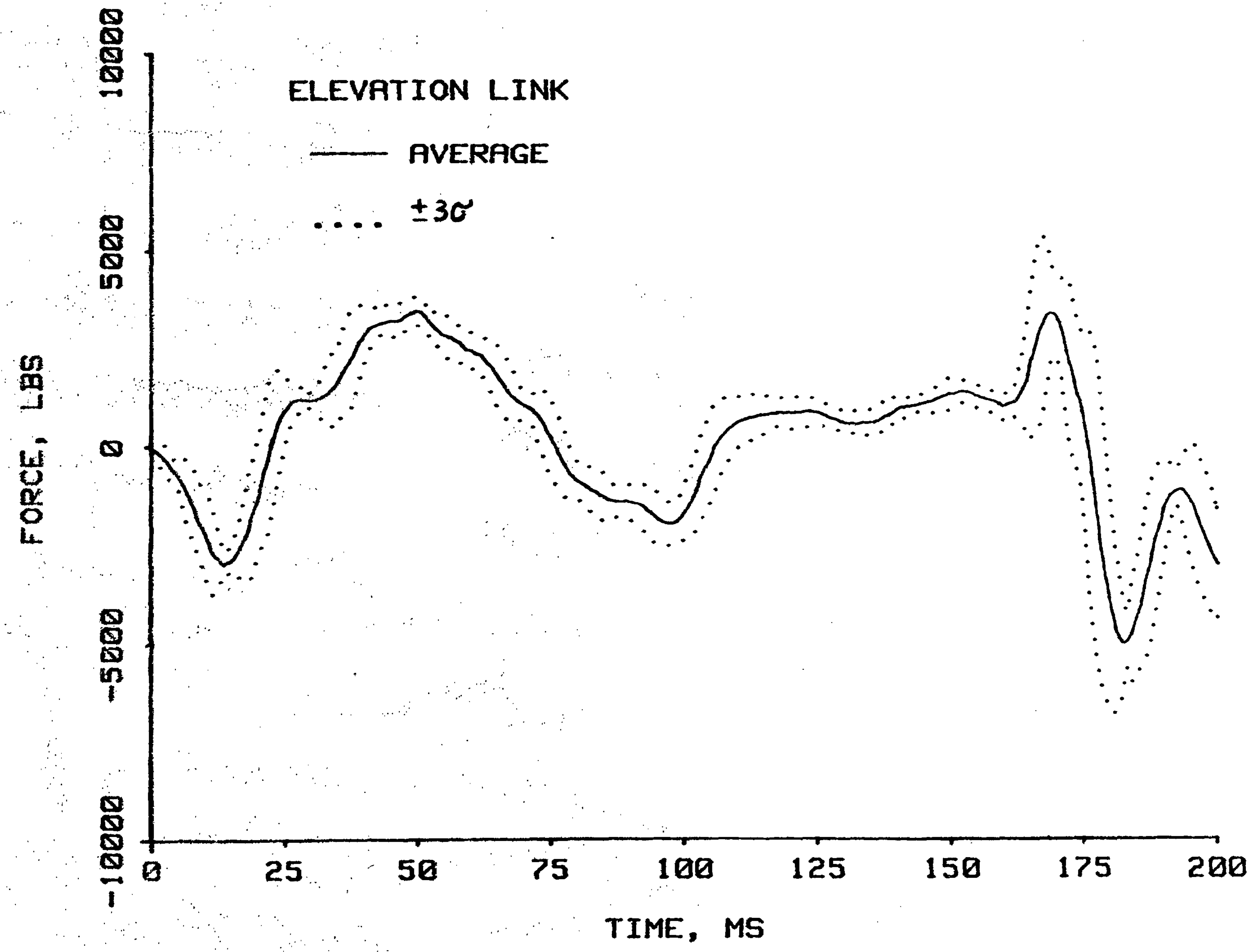
Set B


RIGHT HORIZONTAL TRUNNION
AVERAGE
±3σ
35000
17500
0
-17500
-35000
FORCE, LBS
0
25
50
75
100
125
150
175
200
TIME, MS

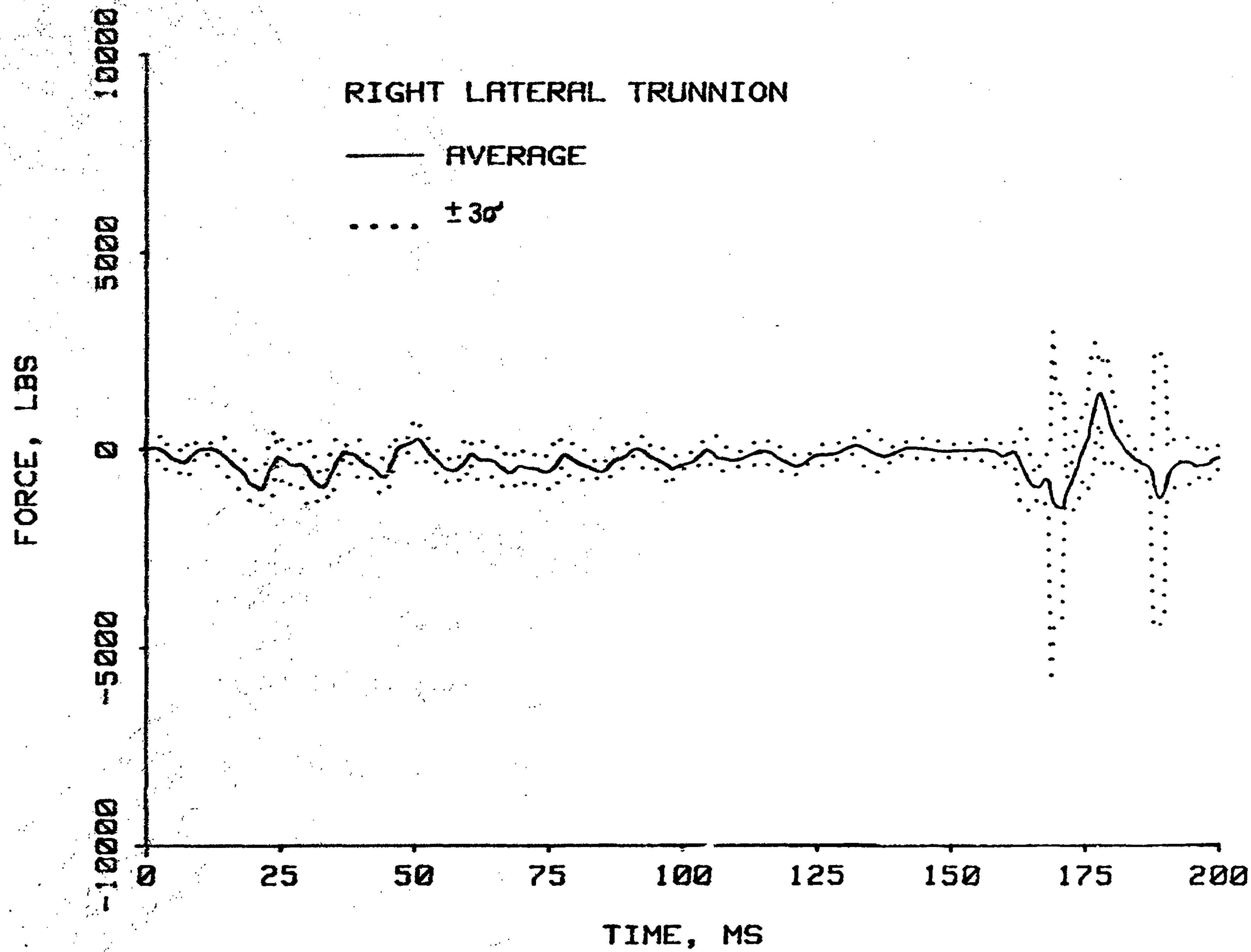

LEFT HORIZONTAL TRUNNION
AVERAGE
±3σ
FORCE, LBS
35000
17500
0
-17500
-35000
0
25
50
75
100
125
150
175
200
TIME, MS

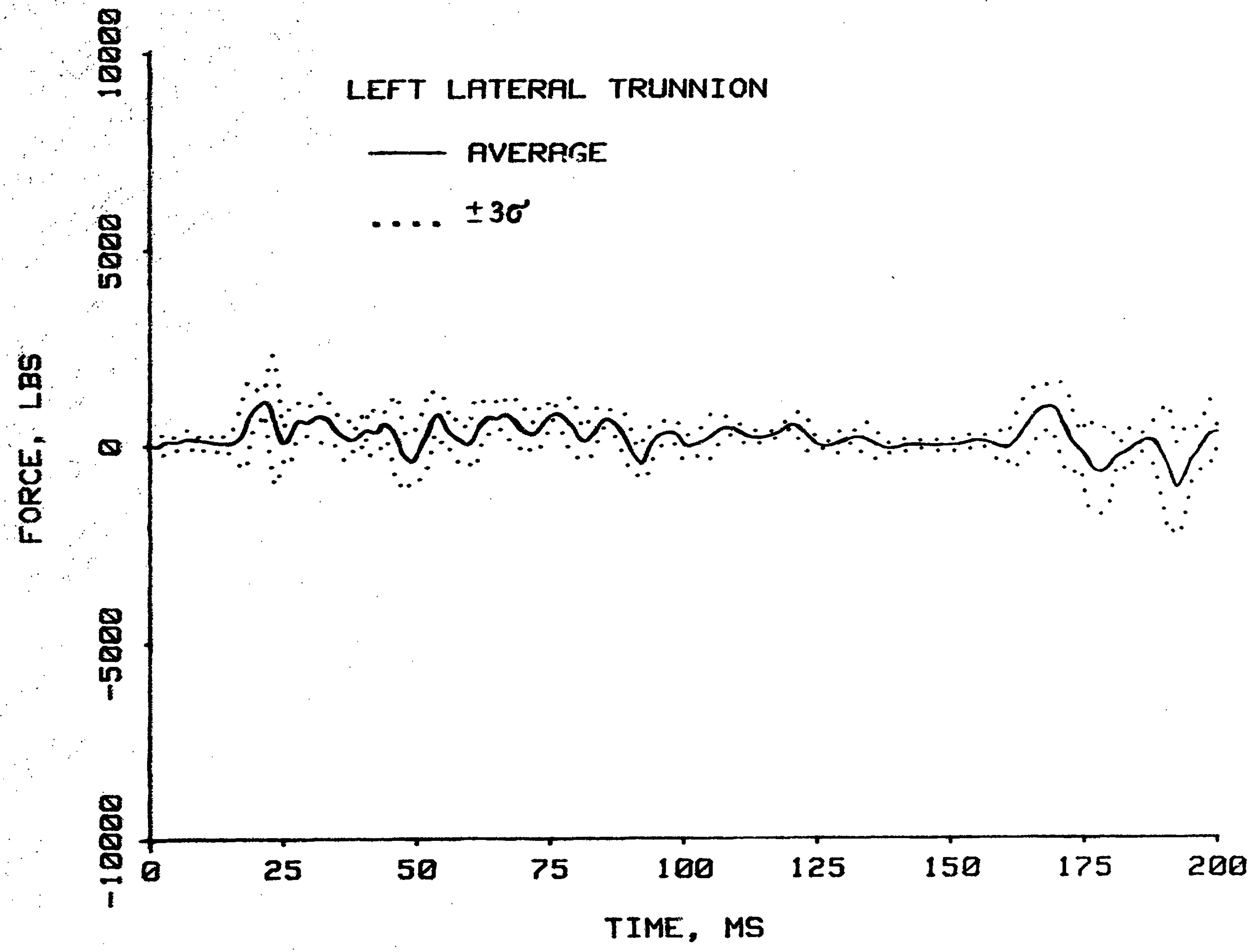

RIGHT VERTICAL TRUNNION
AVERAGE
$\pm 3\sigma$
FORCE, LBS
10000
5000
0
-5000
-10000
0
25
50
75
100
125
150
175
200


LEFT VERTICAL TRUNNION
AVERAGE
±3σ
FORCE, LBS
10000
5000
0
-5000
-10000
0
25
50
75
100
125
150
175
200
TIME, MS



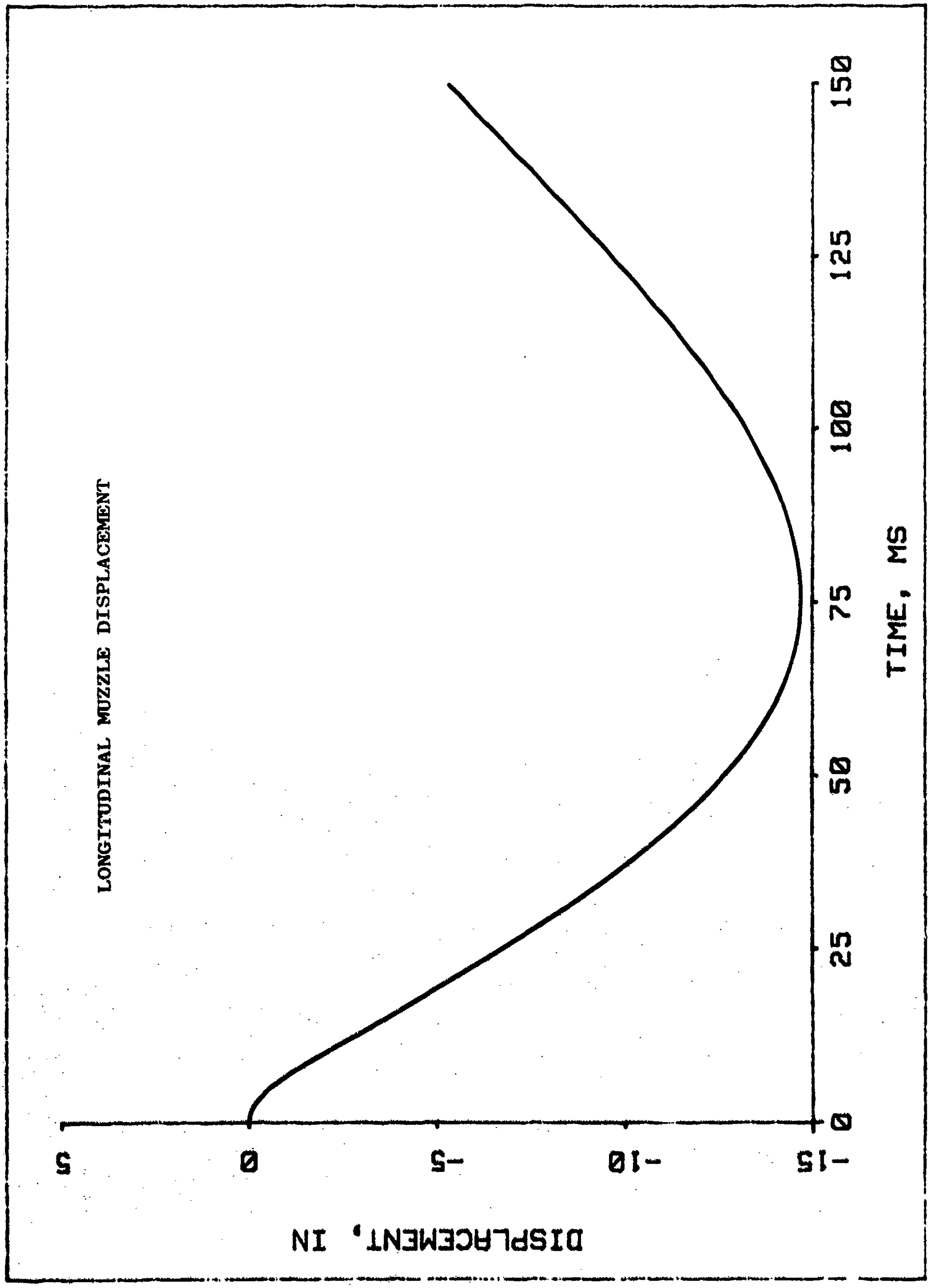
ELEVATION LINK
AVERAGE
±3σ
FORCE, LBS
10000
5000
0
-5000
-10000
0
25
50
75
100
125
150
175
200
TIME, MS

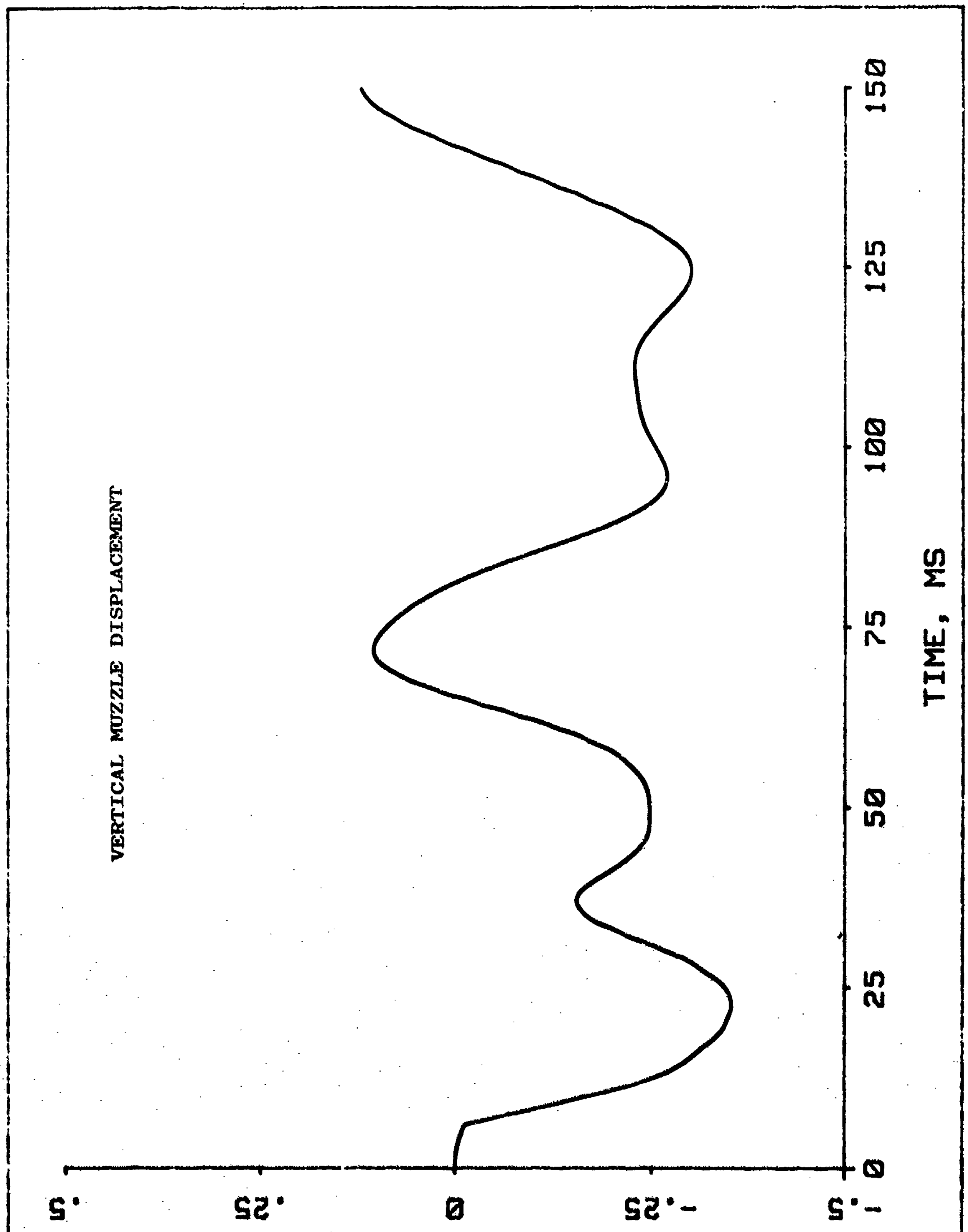

RIGHT LATERAL TRUNNION
AVERAGE
± 3σ
FORCE, LBS
10000
5000
0
-5000
-10000
0
25
50
75
100
125
150
175
200
TIME, MS


LEFT LATERAL TRUNNION
AVERAGE
±3σ
10000
5000
0
-5000
-10000
FORCE, LBS
0
25
50
75
100
125
150
175
200
TIME, MS

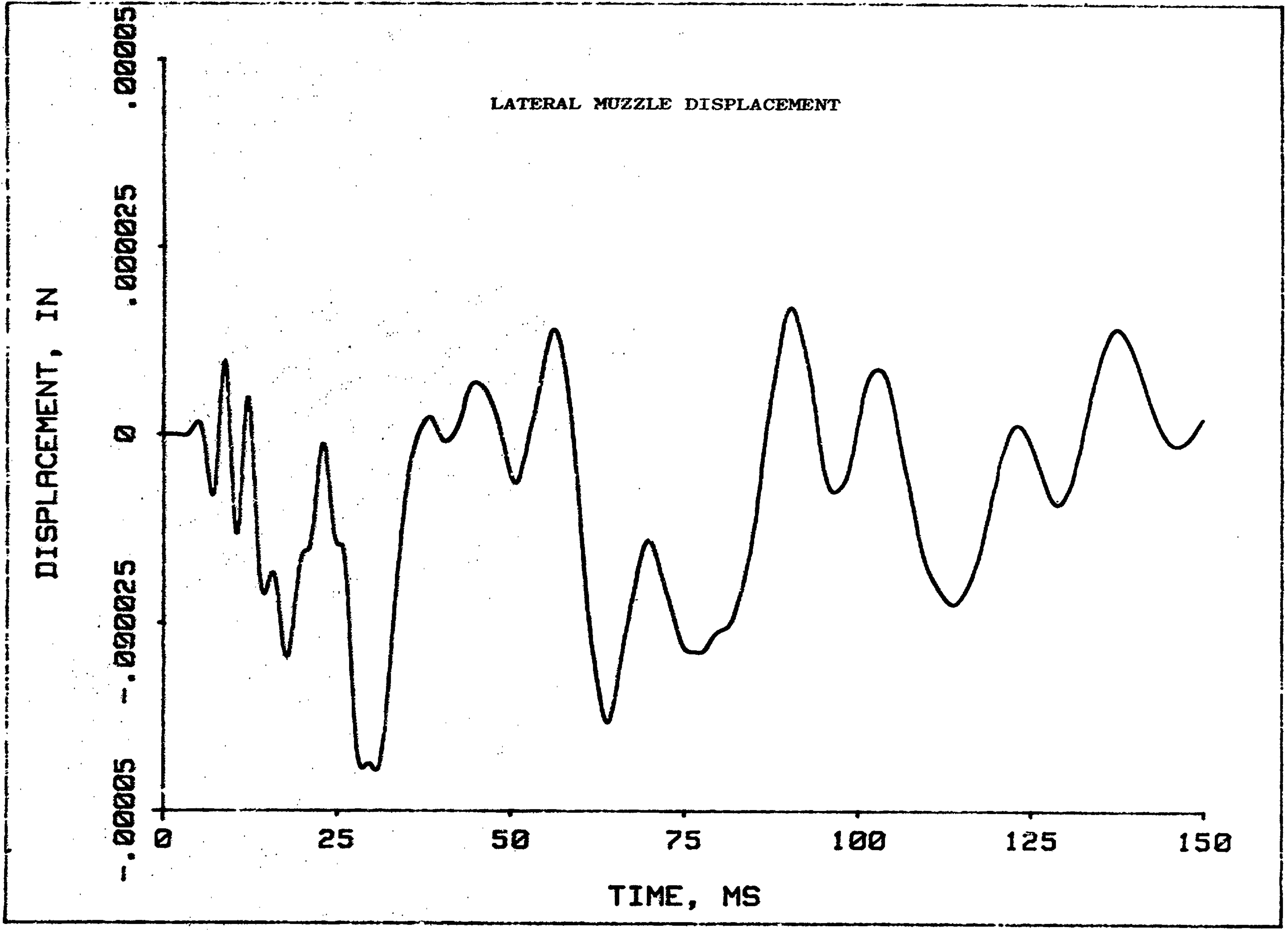
APPENDIX H

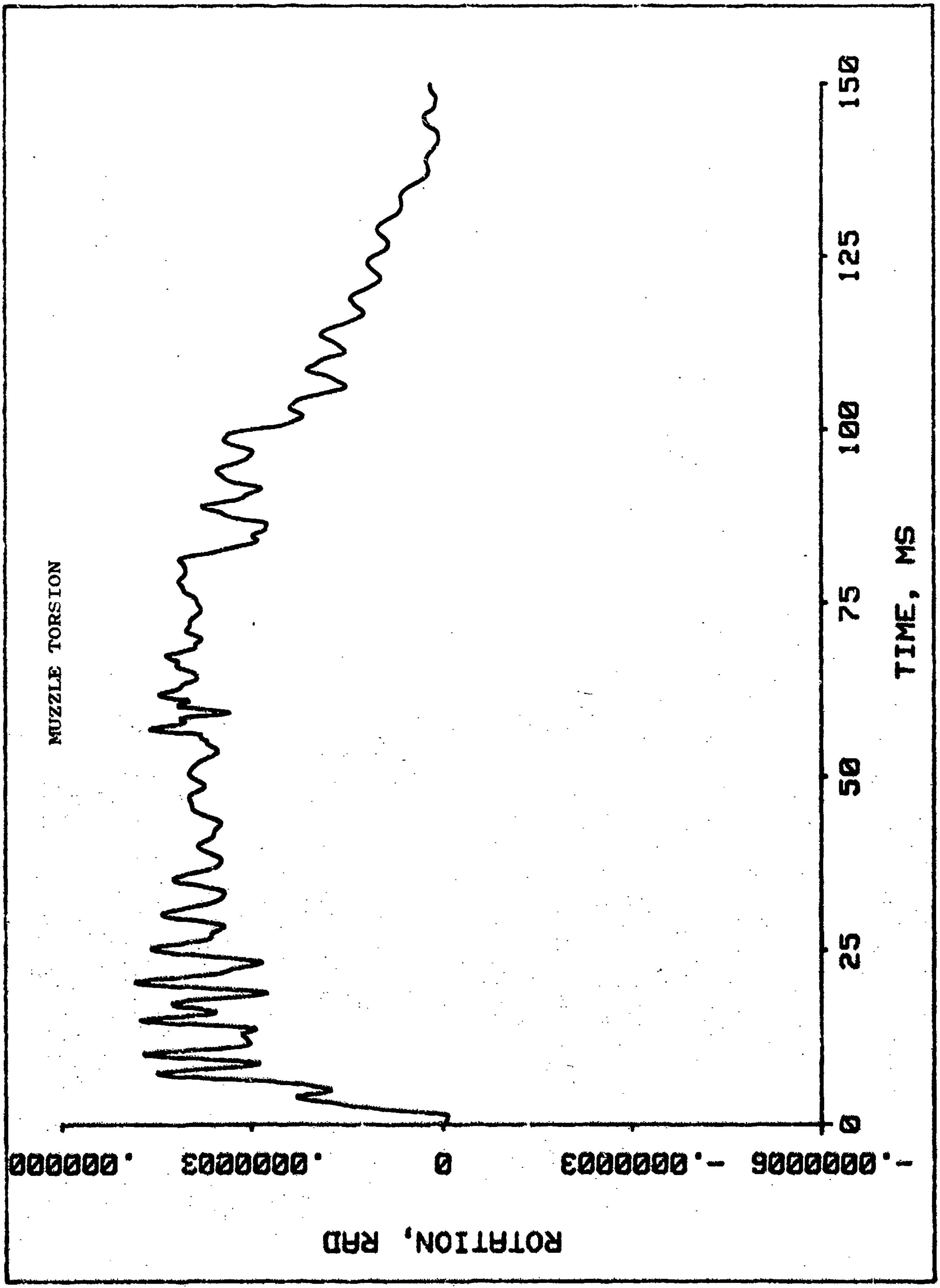
This appendix presents model output for muzzle linear and angular displacements, velocities and accelerations during the first 150 ms. subsequent to the firing of Round No. 67 of the ten-round sample.

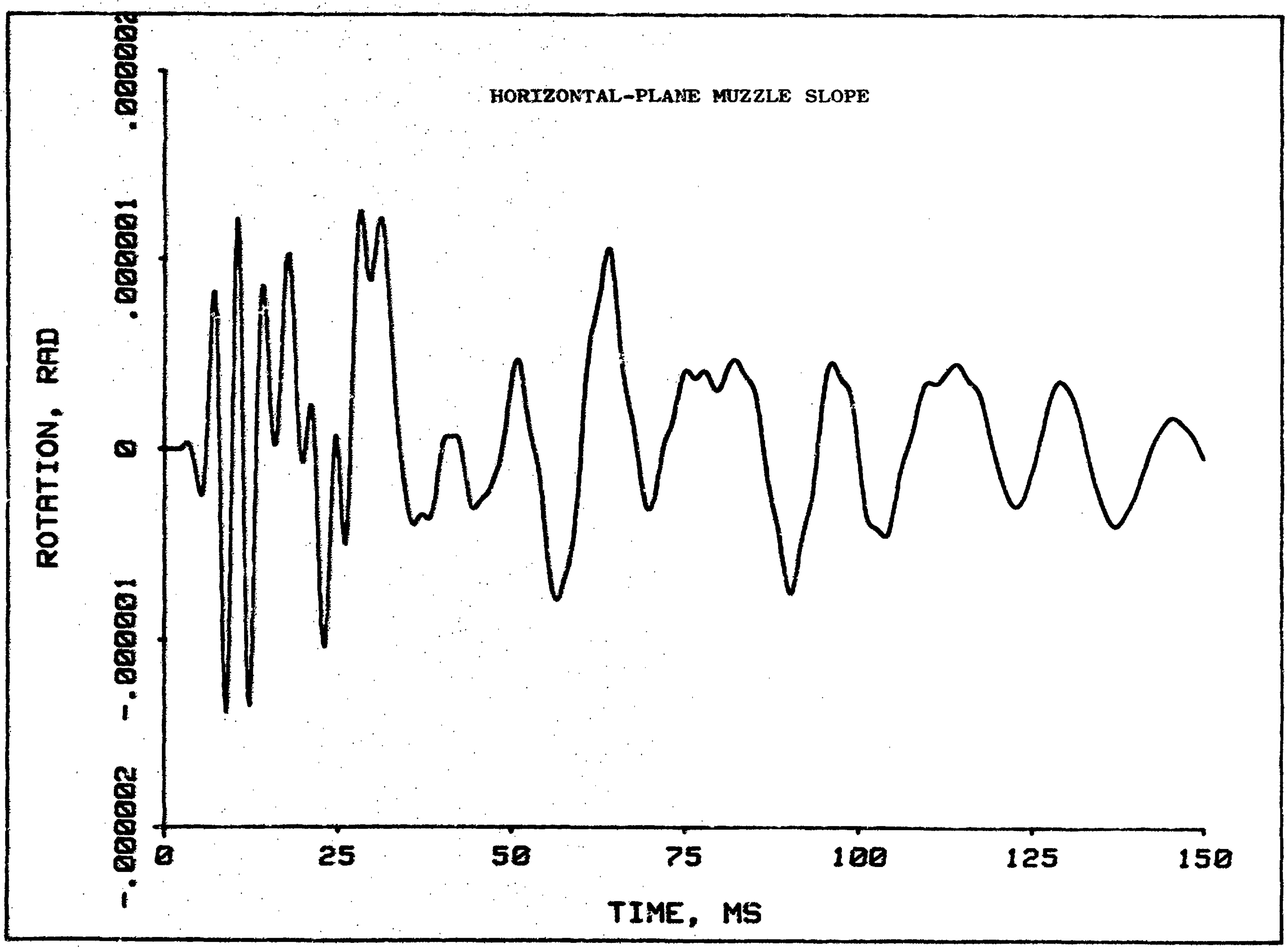

LONGITUDINAL MUZZLE DISPLACEMENT
DISPLACEMENT, IN
5
0
-5
-10
-15
TIME, MS
0
25
50
75
100
125
150

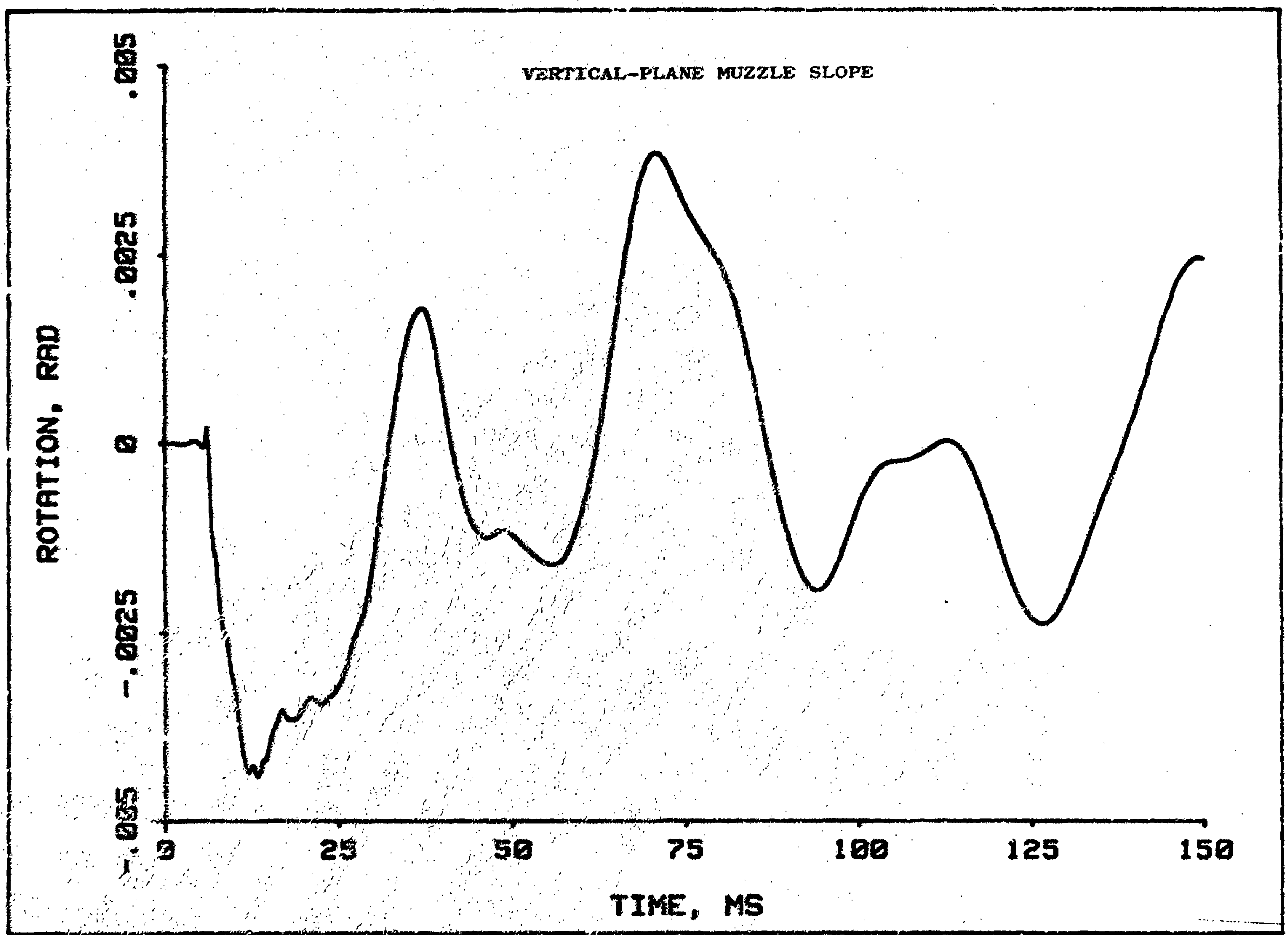


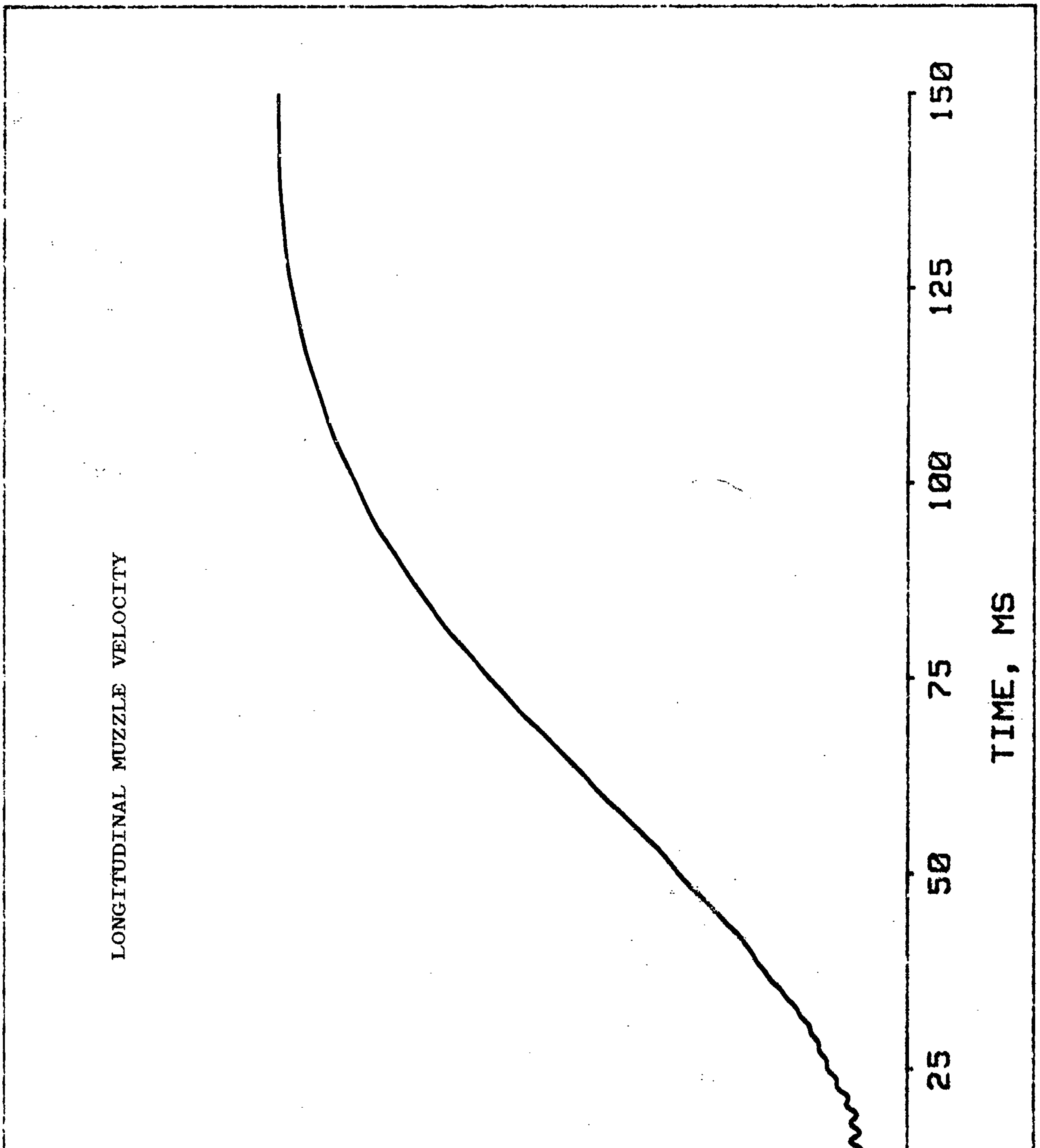
VERTICAL MUZZLE DISPLACEMENT
DISPLACEMENT, IN
.5
.25
0
-.25
-.5
TIME, MS
0
25
50
75
100
125
150

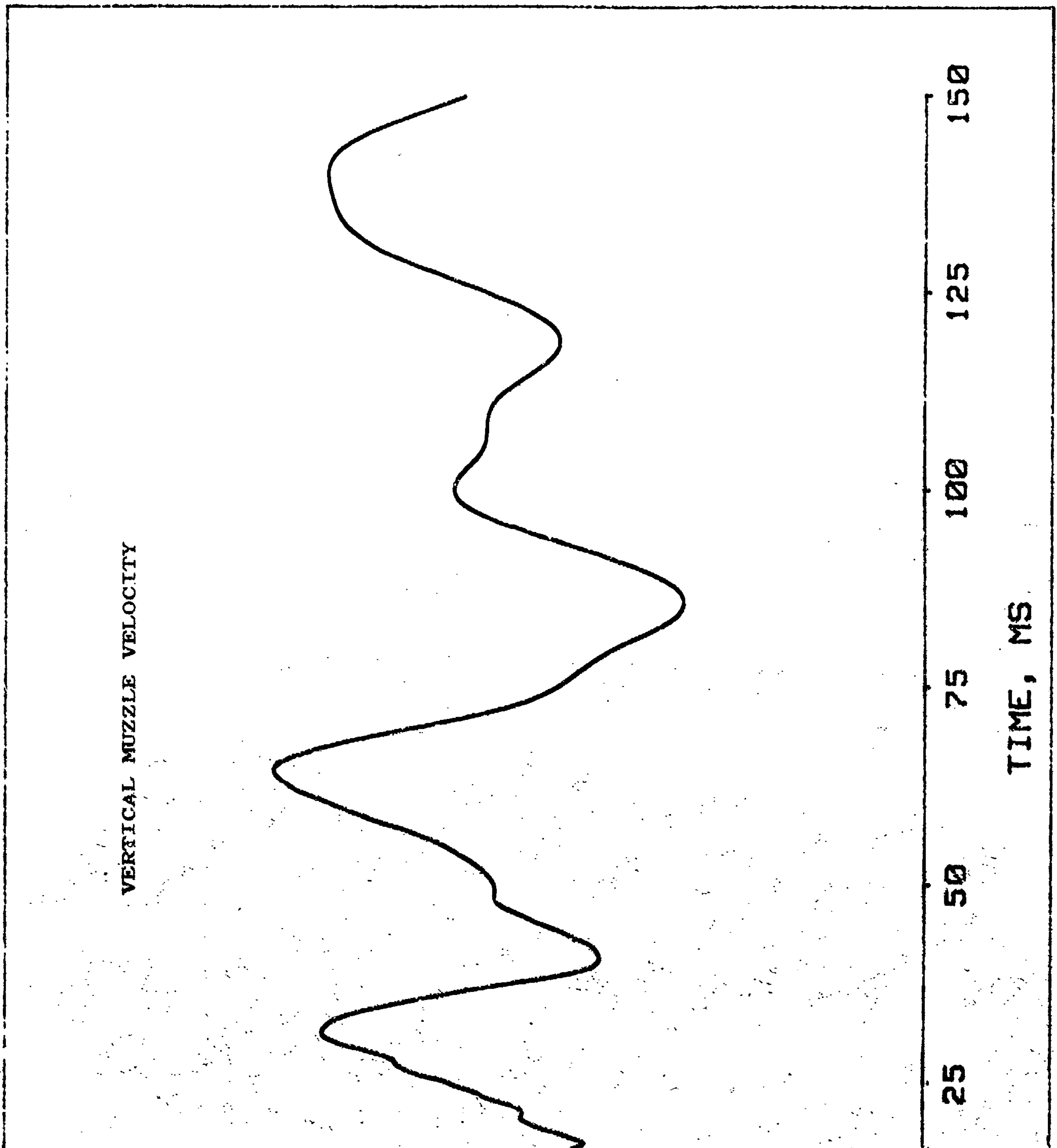

LATERAL MUZZLE DISPLACEMENT
DISPLACEMENT, IN
.00005
.000025
0
-.000025
-.00005
0
25
50
75
100
125
150
TIME, MS

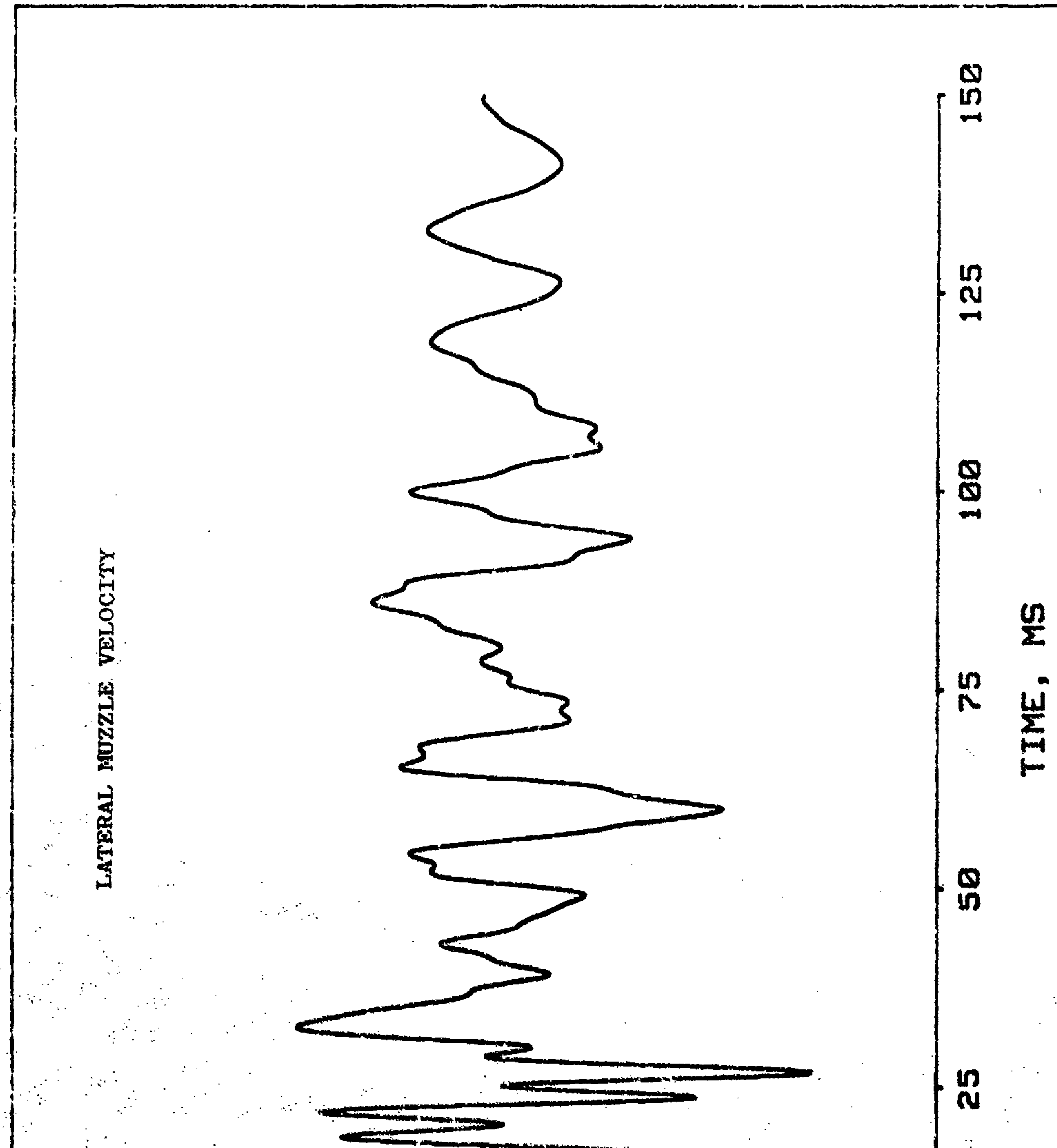

MUZZLE TORSION
TIME, MS
0
25
50
75
100
125
150
ROTATION, RAD
.0000006
.0000003
0
-.0000003
-.0000006

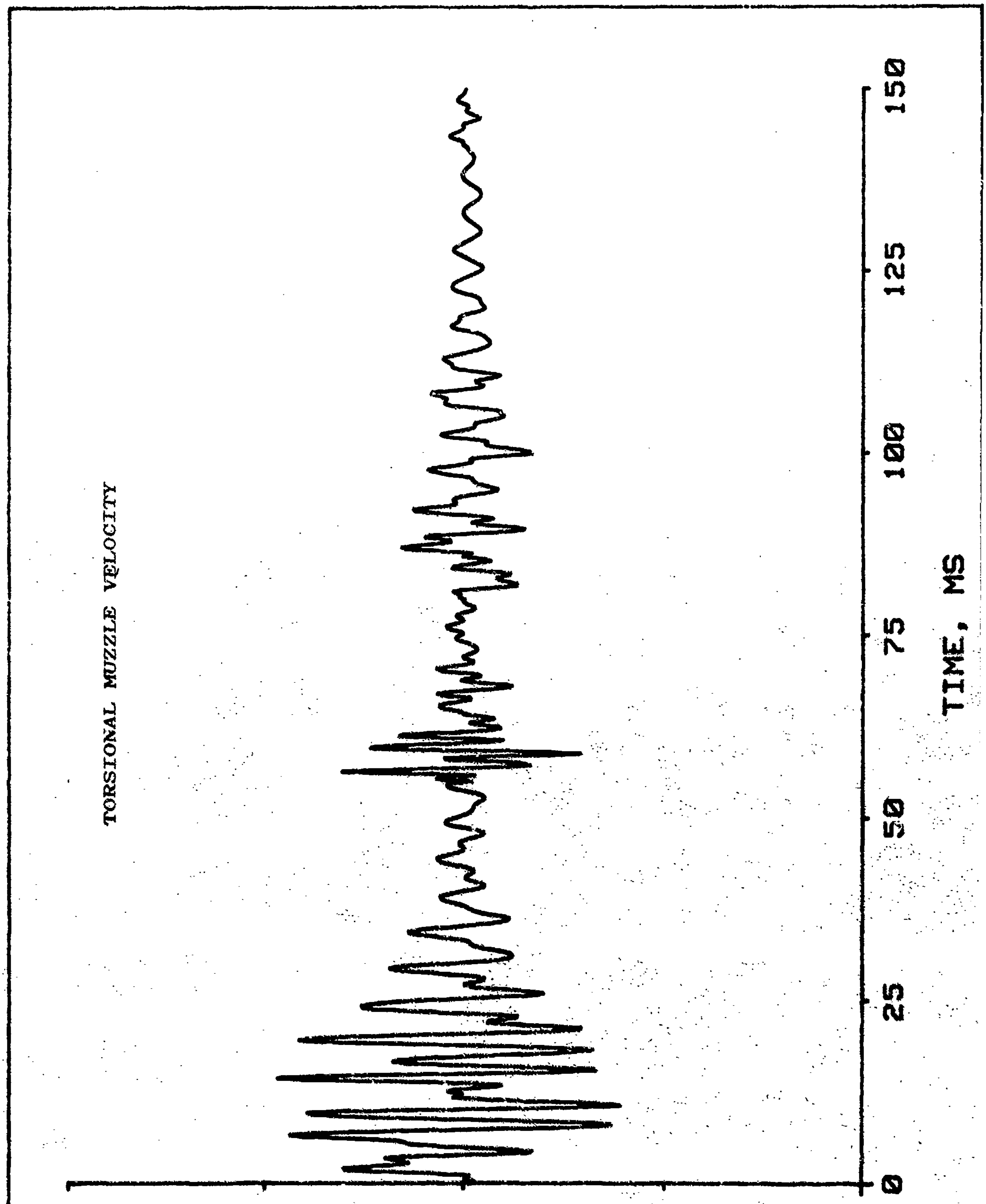

HORIZONTAL-PLANE MUZZLE SLOPE
ROTATION, RAD
.000002
.000001
0
-.000001
-.000002
0
25
50
75
100
125
150
TIME, MS

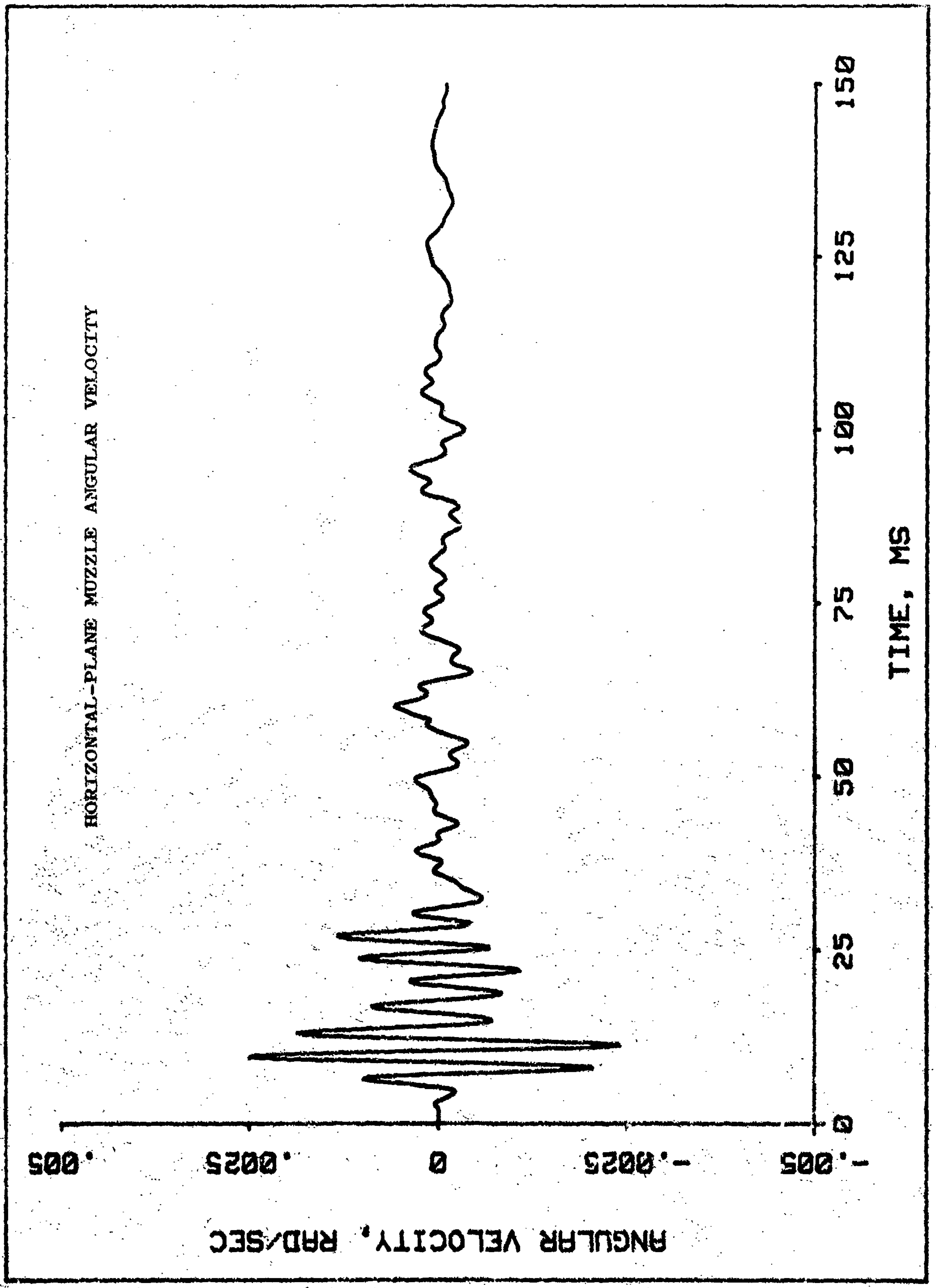

VERTICAL-PLANE MUZZLE SLOPE
ROTATION, RAD
.005
.0025
0
-.0025
-.005
0
25
50
75
100
125
150
TIME, MS

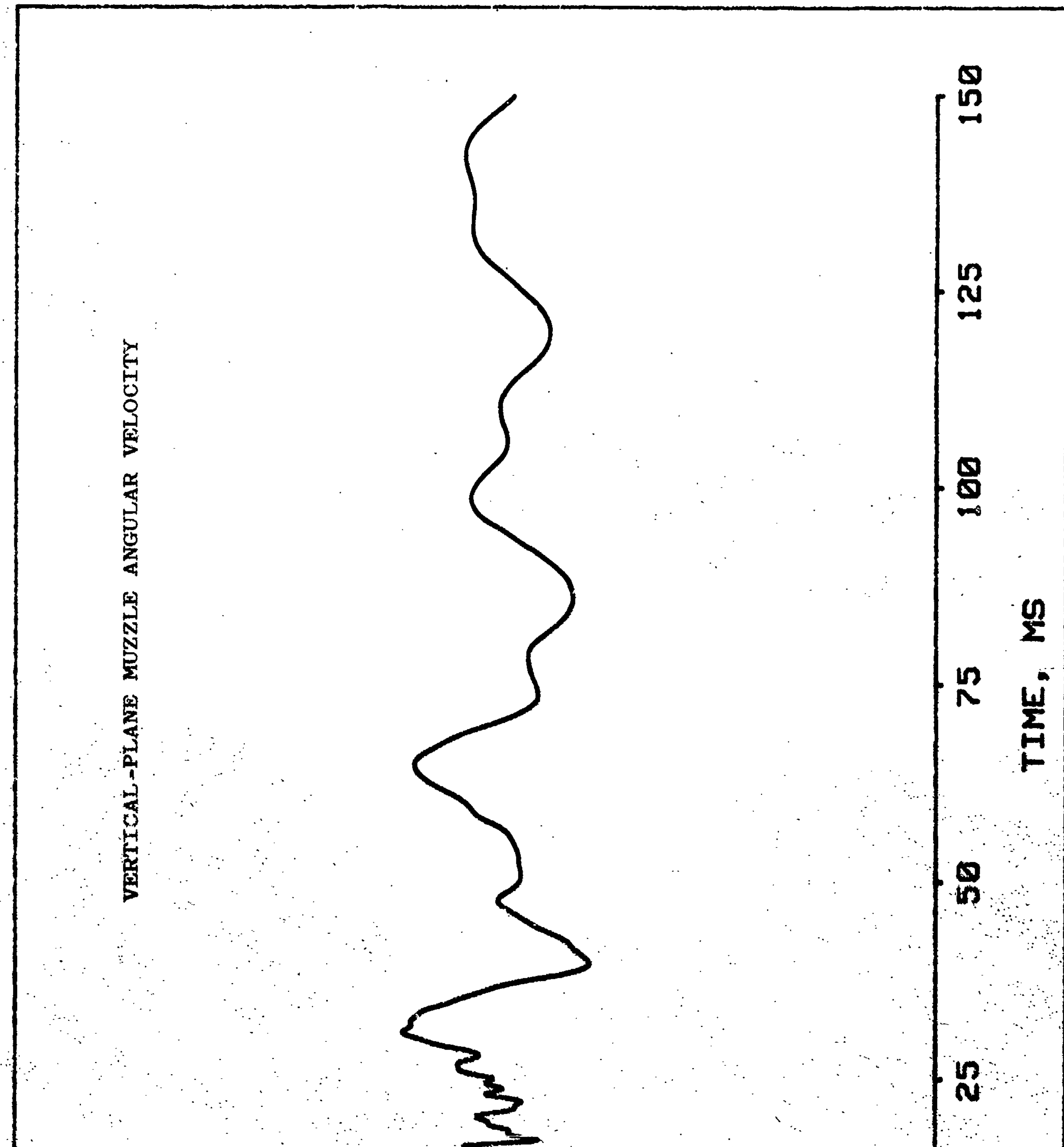

LONGITUDINAL MUZZLE VELOCITY
VELOCITY, FT/SEC
30
15
0
-15
-30
TIME, MS
0
25
50
75
100
125
150

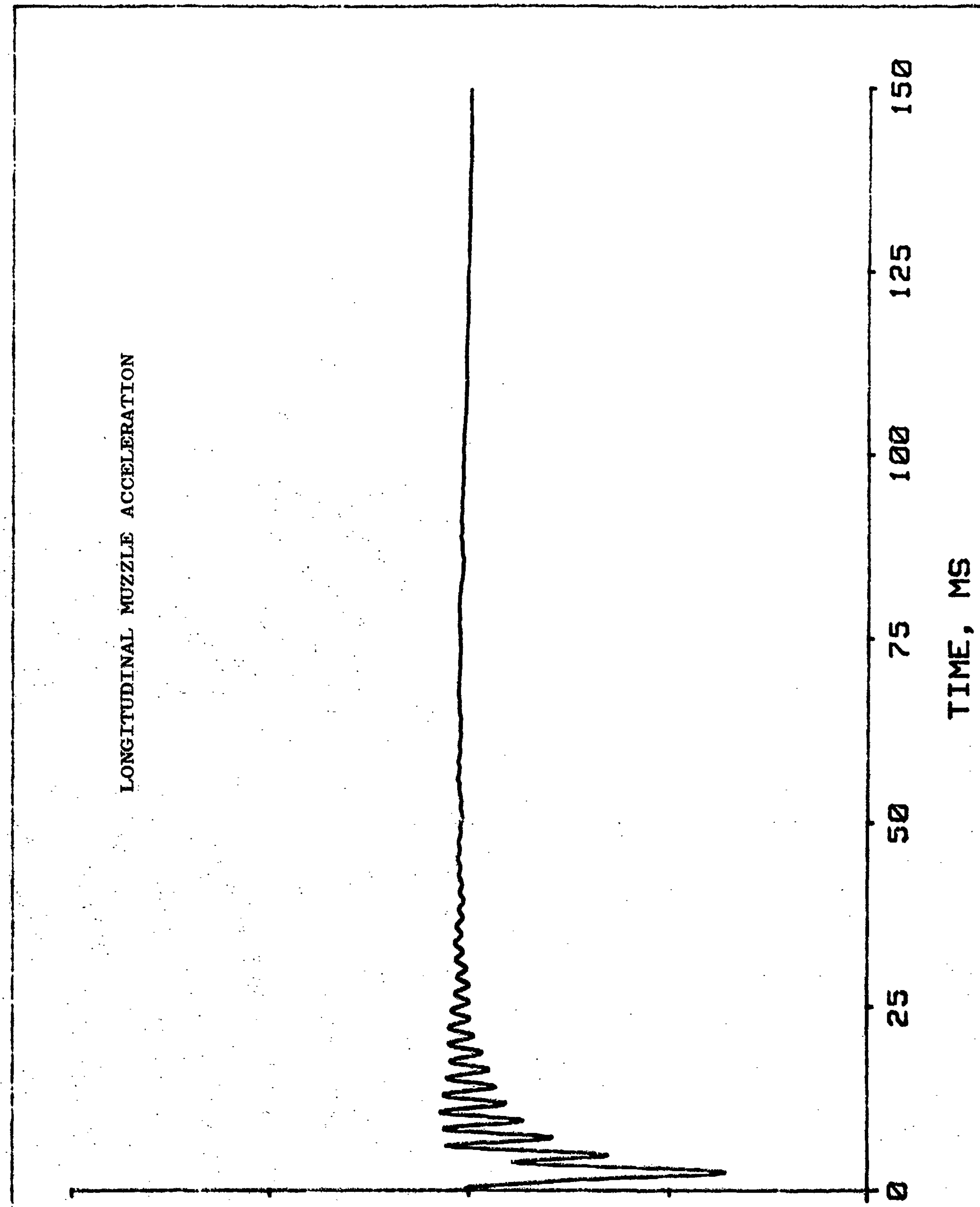

VERTICAL MUZZLE VELOCITY
VELOCITY, FT/SEC
TIME, MS
5
2.5
0
-2.5
-5
0
25
50
75
100
125
150

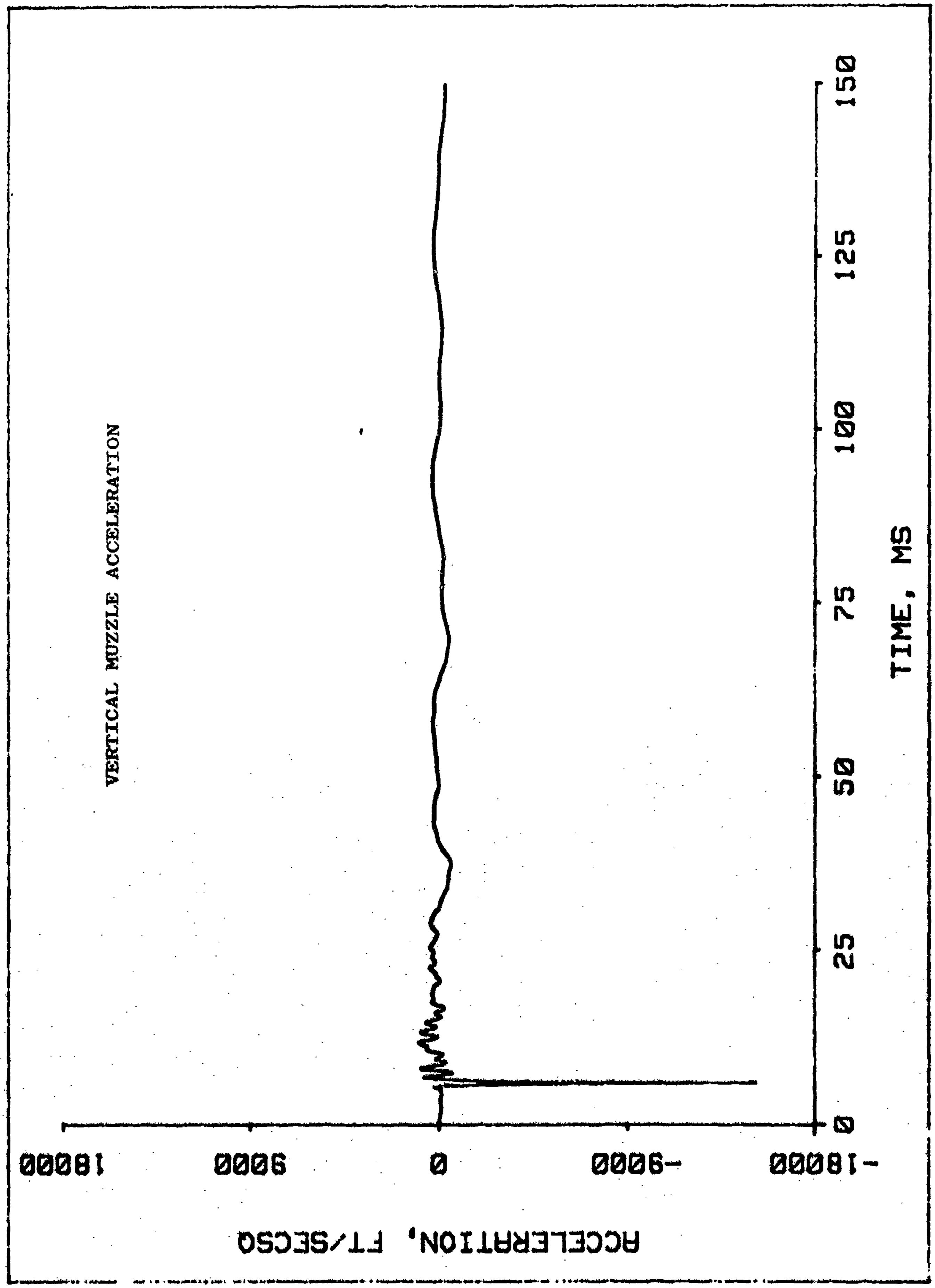

LATERAL MUZZLE VELOCITY
VELOCITY, FT/SEC
.002
.001
0
-.001
-.002
TIME, MS
0
25
50
75
100
125
150

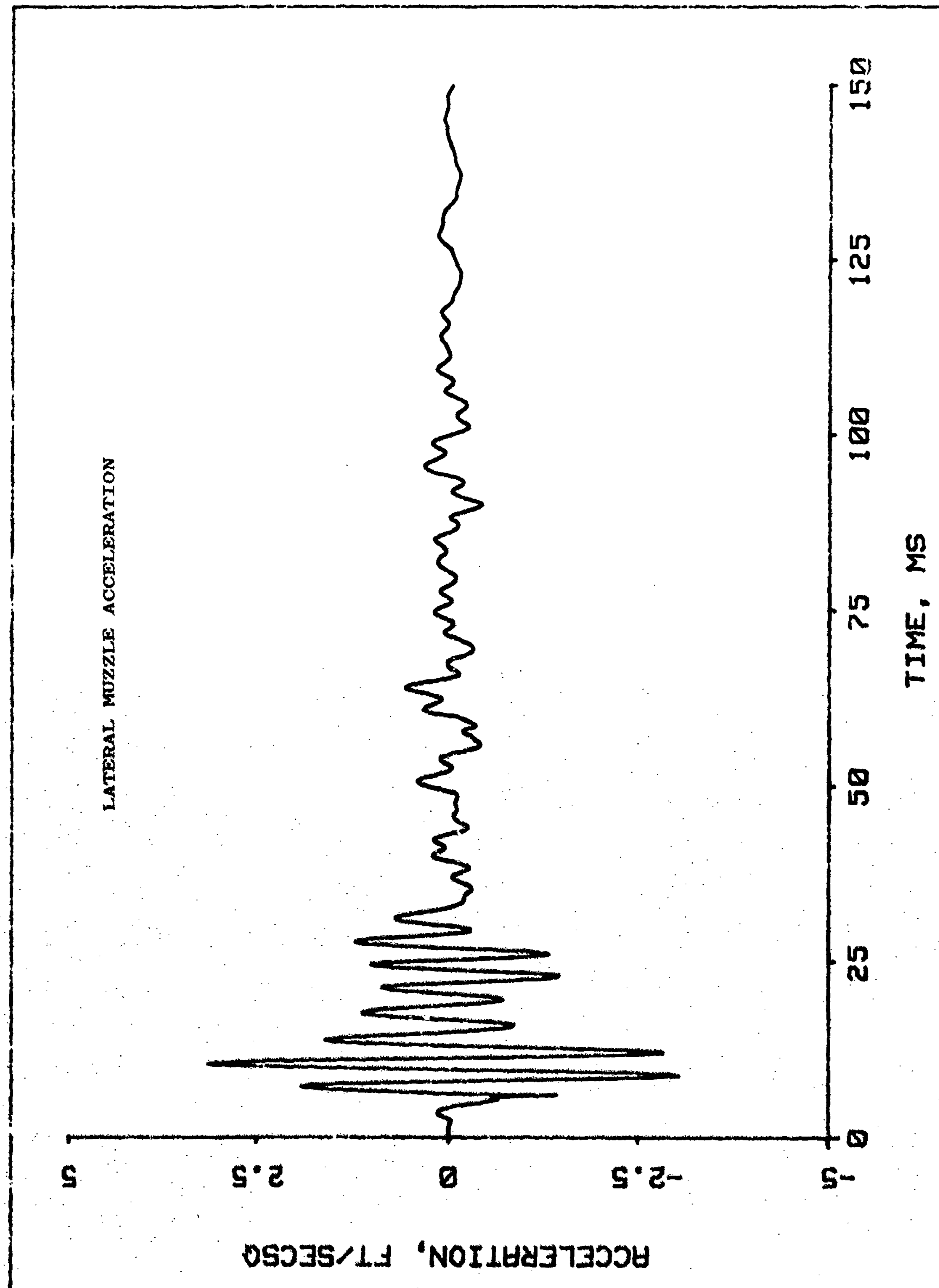

TORSIONAL MUZZLE VELOCITY
ANGULAR VELOCITY, RAD/SEC
.0005
.00025
0
-.00025
-.0005
0
25
50
75
100
125
150
TIME, MS

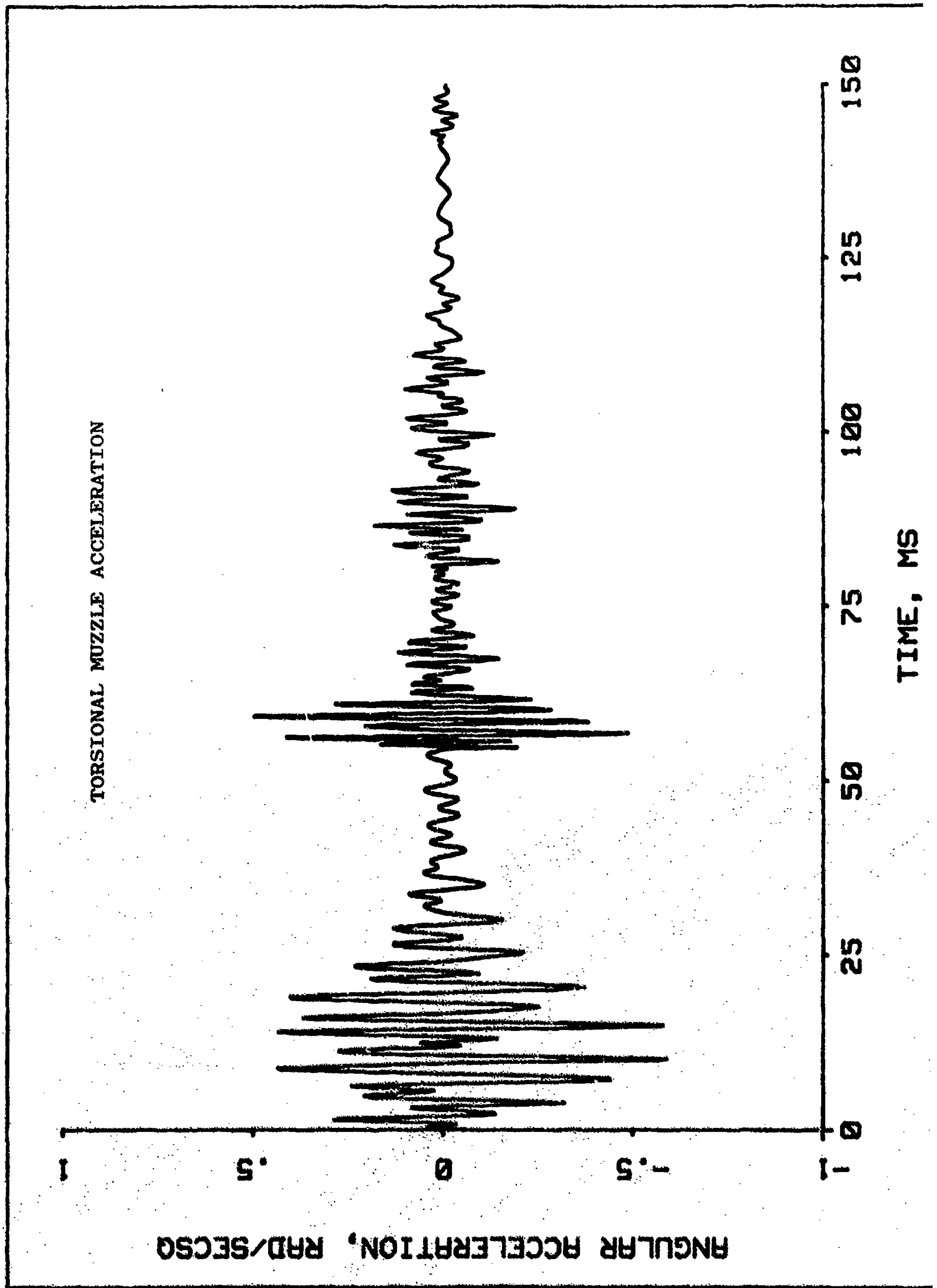

HORIZONTAL-PLANE MUZZLE ANGULAR VELOCITY
ANGULAR VELOCITY, RAD/SEC
.005
.0025
0
-.0025
-.005
TIME, MS
0
25
50
75
100
125
150

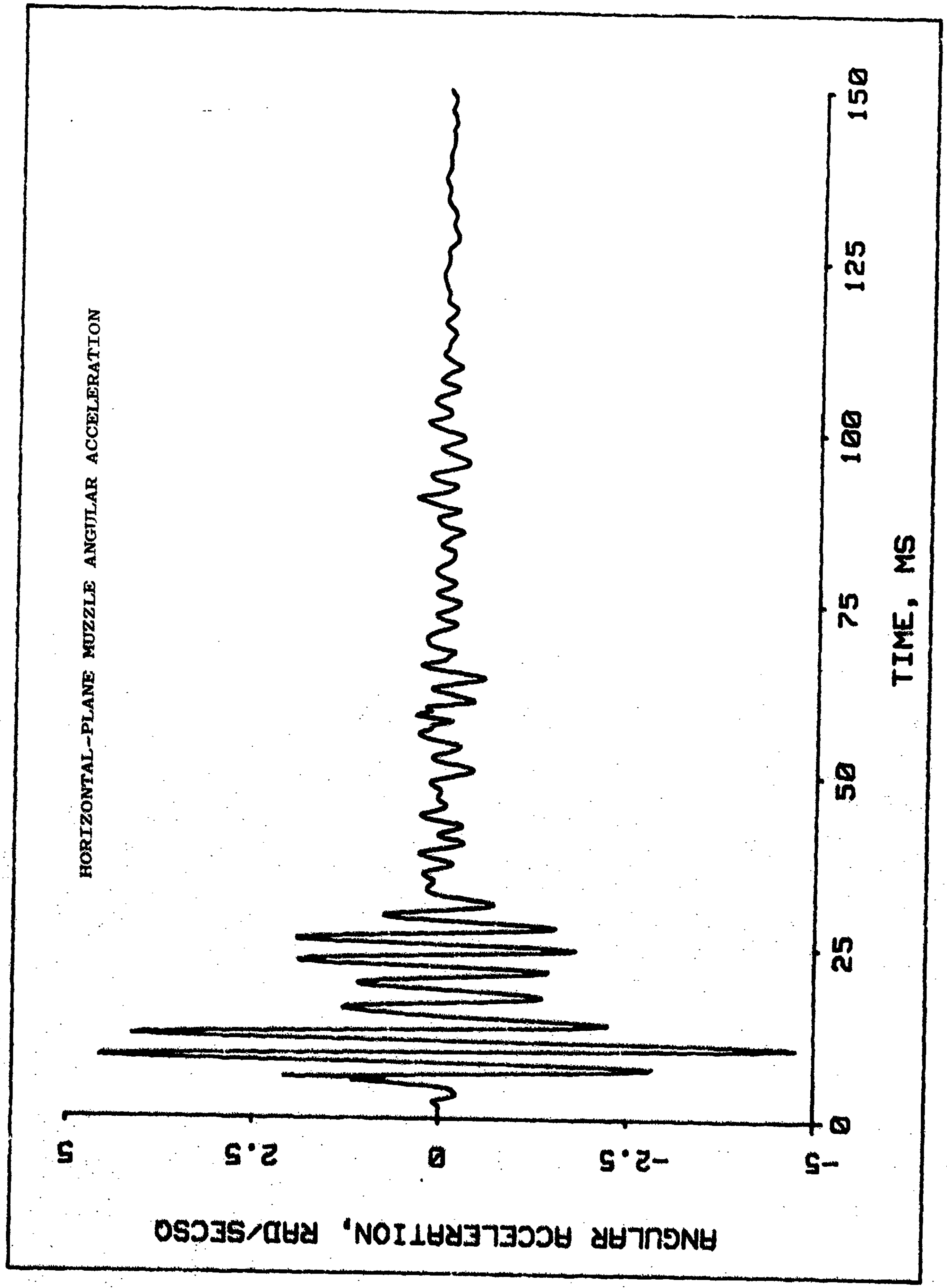

VERTICAL-PLANE MUZZLE ANGULAR VELOCITY
ANGULAR VELOCITY, RAD/SEC
3
1.5
0
-1.5
-3
TIME, MS
0
25
50
75
100
125
150

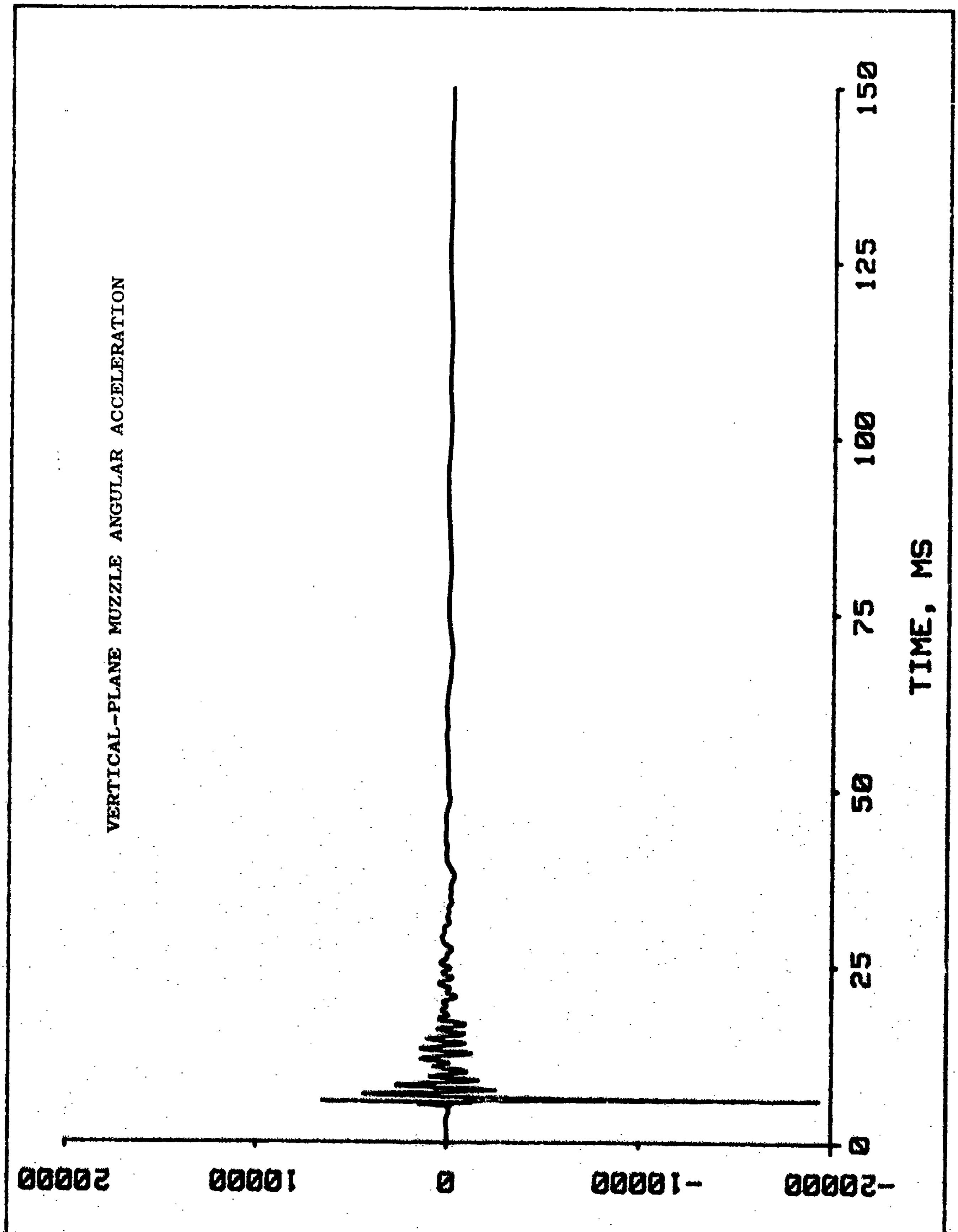

LONGITUDINAL MUZZLE ACCELERATION
0
25
50
75
100
125
150
TIME, MS
-18000
-9000
0
9000
18000
ACCELERATION, FT/SECSQ


VERTICAL MUZZLE ACCELERATION
ACCELERATION, FT/SECSQ
18000
9000
0
-9000
-18000
TIME, MS
0
25
50
75
100
125
150


LATERAL MUZZLE ACCELERATION
ACCELERATION, FT/SECSQ
5
2.5
0
-2.5
-5
TIME, MS
0
25
50
75
100
125
150


TORSIONAL MUZZLE ACCELERATION
ANGULAR ACCELERATION, RAD/SECSQ
1
.5
0
-.5
-1
TIME, MS
0
25
50
75
100
125
150


HORIZONTAL-PLANE MUZZLE ANGULAR ACCELERATION
ANGULAR ACCELERATION, RAD/SECSQ
5
2.5
0
-2.5
-5
0
25
50
75
100
125
150
TIME, MS


VERTICAL-PLANE MUZZLE ANGULAR ACCELERATION
ANGULAR ACCELERATION, RAD/SECSQ
20000
10000
0
-10000
-20000
TIME, MS
0
25
50
75
100
125
150

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