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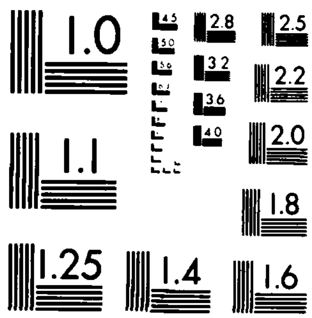

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THESIS

Implementation of a Reliability Shorthand
on the TI-59 Handheld Calculator

by

Hans-Eberhard Peters

October 1982

Thesis Advisor: J.D.Esary

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Assuming constant failure rates, basic structures are used to show how the shorthand can be applied. Several examples are worked out that show, how, with component failure rates as input, a handheld calculator can be

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Implementation of a Reliability Shorthand on the TI-59 Handheld Calculator

by

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Submitted in partial fulfillment of the requirements for the degree of

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from the

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It is shown how a reliability shorthand can be implemented on a handheld calculator.

Assuming constant failure rates, basic structures are used to show how the shorthand can be applied. Several examples are worked out that show, how, with component failure rates as input, a handheld calculator can be used to compute the reliability of a system.

Two TI-59 programs are provided as a computational aid.

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I. INTRODUCTION

Systems and components can be in either of two states: either they are functioning or they have failed. The ability, that a system stays functioning over a predetermined time interval is called its reliability. It is generally not realistic to assume that a system, say a lightbulb, will fail at a specified time, but rather that T, the time to failure, is a random variable which has a probability distribution that can be specified. The probability distribution for a time to failure is called its life distribution. In this paper we will solely be concerned with one specific type of life distribution which is especially important in reliability theory and practice, the exponential distribution. It has the property that the remaining life of a used component is independent of its age (the "memoryless" property), i.e. a functioning component is always as good as new, the failure rate is constant. The memoryless property is the basis for a reliability shorthand, one that can be implemented on a handheld calculator.

Depending on the size, structure and life distribution of a system, probability statements about its time to failure are in general not easily achieved. Forming the sum of independent life lengths (i.e. convolving the corresponding life distributions) requires knowledge of integral calculus and computations can become rather tedious.

In the case of the exponential distribution, though, computations can be simplified by translating the problem into a simple shorthand notation and using this shorthand as input for some computing device.

In this paper we will show how a reliability shorthand can be implemented on a handheld calculator. Basic structures are used to show how the shorthand can be applied. Two TI-59 programs are provided as a computational aid. Formulas for the convolution of up to four exponential random variables can be found in Appendix A. Appendix B contains a user guide to the TI-59 programs.

II. THE CONCEPT OF A RELIABILITY SHORTHAND

A. BASIC NOTATION

The survival function of a life length can be derived from the distribution function.

Let

T : life length

$F(t) = P(T \leq t)$ be the distribution function of

Then

$$\bar{F}(t) = P(T>t)$$
$$= 1-F(t)$$

is the survival function of T.

In the case of the exponential distribution, $\bar{F}(t) = e^{-\lambda t}$, where λ is the failure rate. Translated into shorthand, the life distribution is denoted

EXP(λ).

B. CONVOLUTION OF DISTRIBUTIONS

When independent random lives are summed up, the corresponding life distributions have to be convolved to determine the probability that the sum of the lives will exceed a specified time t. Let

T_1, T_2 : independent life lengths

$\bar{F}_1(t), \bar{F}_2(t)$: the corresponding survival functions

$f_1(t), f_2(t)$: the corresponding density functions

$T = T_1 + T_2$: the total life length

Then

$$\bar{F}(t) = P(T>t)$$
$$= P(T_1 + T_2 > t)$$
$$= \bar{F}_1(t) + \int_0^t \bar{F}_2(t-s) f_1(s)\, ds.$$

This means that T will exceed a specified time t when

-either T_1 exceeds t

-or T_1 is smaller than t, say equal to s, and T_2 exceeds t-s.

Integration with respect to s (i.e. summing over all possible values of s) is called the convolution of T_1 and T_2. When T_1 and T_2 are both exponentially distributed with failure rates λ_1 and λ_2, i.e.

$$\bar{F}_1(t) = e^{-\lambda_1 t}$$
$$\bar{F}_2(t) = e^{-\lambda_2 t},$$

then the survival function of T is

$$\bar{F}(t) = e^{-\lambda_1 t} + \int_0^t e^{-\lambda_2 (t-s)} \lambda_1 e^{-\lambda_1 s} \quad ds.$$

Translated into shorthand, the survival function is denoted

$$\mathrm{EXP}(\lambda_1) + \mathrm{EXP}(\lambda_2).$$

This shorthand notation is heuristically apparent. We can visualize a 1 component / 1 spare system with $EXp(\lambda_1)$ and $Exp(\lambda_2)$ lives respectively. From component 1 the system has an $EXP(\lambda_1)$ life to begin with. When component 1 fails, the system has an extra $EXP(\lambda_2)$ life.

C. MIXTURE OF DISTRIBUTIONS

1. MIX-Notation

In the previous chapter, we formed the sum of independent random lives, which each had weight one, i.e.

$$T = T_1 + T_2 .$$

Now consider

$$T = \begin{cases} T_1 & \text{with probability } p_1 \\ \\ T_2 & \text{with probability } p_2 \end{cases}$$

where $p_1 + p_2 = 1$.

Let D_1 and D_2 be the probability distributions of the random variables T_1 and T_2 respectively. The corresponding survival functions are $\overline{F}_1(t)$ and $\overline{F}_2(t)$.

Then

$$\overline{F}(t) = p_1 \overline{F}_1(t) + p_2 \overline{F}_2(t) .$$

In shorthand, the mixture of distributions D_1 and D_2 with respect to the mixing probabilities p_1 and p_2 is denoted

$$MIX[\ p_1 D_1\ ,\ p_2 D_2\ ].$$

2. Distributive Law

Now let

$$T = T_3 + T'$$

where

$$T' = \begin{cases} T_1 \text{ with probability } p \\ \\ T_2 \text{ with probability } 1-p. \end{cases}$$

Then

$$T = T_3 + \begin{cases} T_1 \text{ with probability } p \\ \\ T_2 \text{ with probability } 1-p. \end{cases}$$

$$T = \begin{cases} T_3 + T_1 \quad \text{with probability } p \\ \\ T_3 + T_2 \quad \text{with probability } 1-p. \end{cases}$$

The distributive law holds due to the fact that the sum of the mixing probabilities for T_1 and T_2 is equal to one.

The survival function of T can be found by convolution:

$$\bar{F}(t) = \bar{F}_3(t) + \int_0^t (p\bar{F}_1(t-s) + (1-p)\bar{F}_2(t-s))f_3(s)ds.$$

With D_1, D_2, D_3 being the probability distributions for T_1, T_2, T_3, the distributive law can be applied to the shorthand notation:

$$D_3 + \text{MIX}\,[pD_1, (1-p)D_2] = \text{MIX}\,[\,p((D_1 + D_3), \\ (1-p)(D_2 + D_3)\,].$$

Graphically this can be represented as follows:

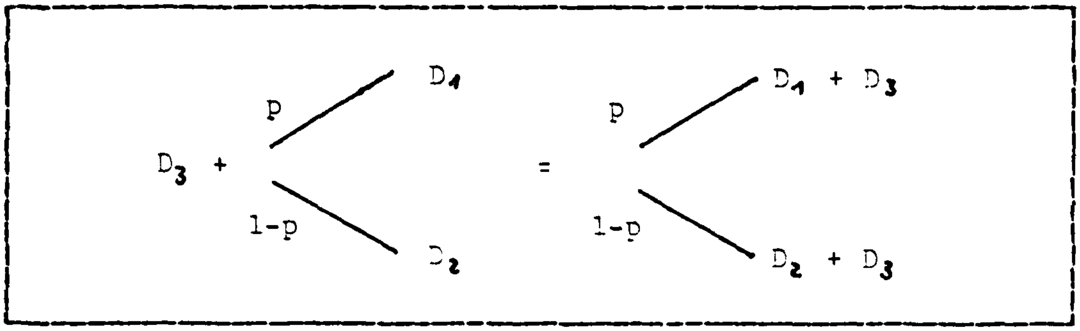


Figure 1: Distributive Property of the MIX-Notation

3. <u>Degeneracy at the Origin</u>

Let

$$P(T=0) = 1.$$

Then the distribution of T is degenerate at zero.

In shorthand notation, such a distribution is called the ZERO-distribution.

Now let $T = T_1 + T_0$

where T_1 and T_0 have probability distributions D_1 and ZERO and survival functions $\bar{F}_1(t)$ and $\bar{F}_0(t)$ respectively.

Then

$$\begin{aligned}\bar{F}(t) &= \bar{F}_1(t) + \int_0^t \bar{F}_0(t-s) f_1(s)\,ds \\ &= \bar{F}_1(t).\end{aligned}$$

The ZERO-distribution doesn't add anything to another distribution, so for instance

$$D_1 + ZERO = D_1$$

$$D_2 + MIX[pD_1, (1-p) ZERO] = MIX[p(D_1 + D_2), (1-p) D_2].$$

III. APPLYING A RELIABILITY SHORTHAND

After this brief survey over the concept of a reliability shorthand we will now show how the shorthand can be applied. To do so we will use basic structures. Part A of this chapter will give examples whose representation in shorthand requires only basic notation described in Chapter II, Parts A and B, whereas Part B of this chapter will give examples whose representation in shorthand makes use of the MIX-notation and the ZERO-distribution.

A. SUMS OF EXPONENTIALS WITH WEIGHT ONE

1. Simple Series System

A series system is a system which is functioning, when all its components are functioning. A two-component series system can be graphically represented as shown in Fig.2 .

Let

T : life of the system

T_1: life of component 1

T_2: life of component 2

$\bar{F}_1(t)$ = survival function of component 1

$= e^{-\lambda_1 t}$

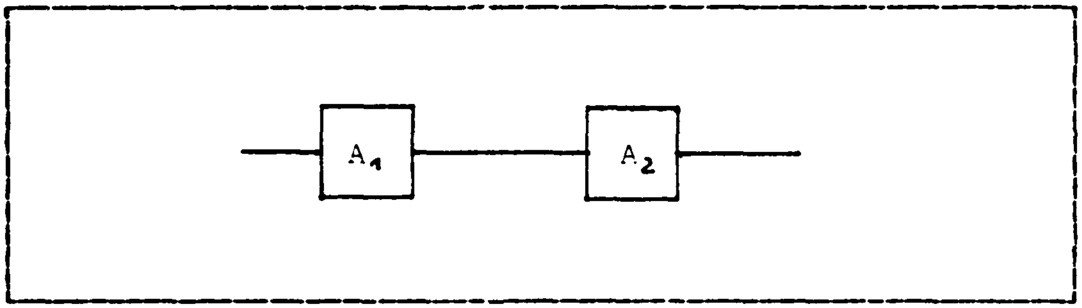


Figure 2: Two-Component Series System

$\bar{F}_2(t)$ = survival function of component 2

$= e^{-\lambda_2 t}$.

Then

$T = \min(T_1, T_2)$

$\bar{F}(t)$ = survival function of the system

$= P(\min(T_1, T_2) > t)$

$= P(T_1 > t, T_2 > t)$

Assuming independence of the two components

$\bar{F}(t) = P(T_1 > t)\ P(T_2 > t)$

$= \bar{F}_1(t)\ \bar{F}_2(t)$

$= e^{-\lambda_1 t}\ e^{-\lambda_2 t}$

$= e^{-(\lambda_1 + \lambda_2) t}$.

The shorthand notation for this system is

$\mathrm{EXP}\ (\lambda_1 + \lambda_2)$.

This is intuitively apparent, as the system has an exponential survival function with failure rate $\lambda_1 + \lambda_2$.

2. Simple Parallel System

A parallel system is a system which is functioning, when at least one of its components is functioning. A two-component parallel system can be graphically represented as follows:

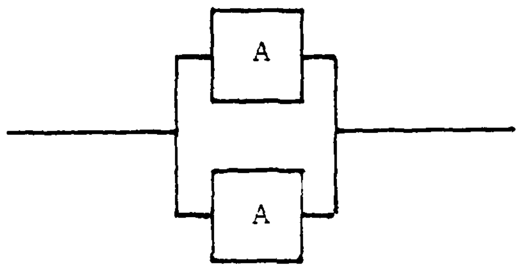


Figure 3: Two-Component Parallel System

Let

$$T_1 \sim EXP(\lambda),\ T_2 \sim EXP(\lambda).$$

Then

$$T = \max(T_1, T_2)$$

$$\begin{aligned}\bar{F}(t) &= P(\max(T_1, T_2) > t) \\ &= 1 - P(\max(T_1, T_2) \leqslant t) \\ &= 1 - P(T_1 \leqslant t, T_2 \leqslant t)\end{aligned}$$

Assuming independence of the two components,

$$\bar{F}(t) = 1 - P(T_1 \leq t)\ P(T_2 \leq t)$$

$$= 1 - F_1(t)\ F_2(t)$$

$$= 1 - (1-e^{-\lambda t})(1-e^{-\lambda t})$$

$$= 1 - (1-2e^{-\lambda t} + e^{-2\lambda t})$$

$$= 2e^{-\lambda t} - e^{-2\lambda t}.$$

The shorthand notation for the system is

$$\mathrm{EXP}(2\lambda) + \mathrm{EXP}(\lambda).$$

This follows intuition as the system has an EXP(2λ) life to begin with and when one component fails it has an extra EXP(λ) life due to the memoryless property of the exponential distribution.

3. Standby-System with Dissimilar Components

Suppose a system consists of two components, one active and one spare. The active component stays in service until it fails and then immediately is replaced by the spare.

Let the time to failure of the two components be $T_1 \sim \mathrm{EXP}(\lambda_1)$ and $T_2 \sim \mathrm{EXP}(\lambda_2)$ respectively.

Then the system time to failure is

$$T = T_1 + T_2$$

and the survival function of the system is

$$\bar{F}(t) = P(T > t)$$

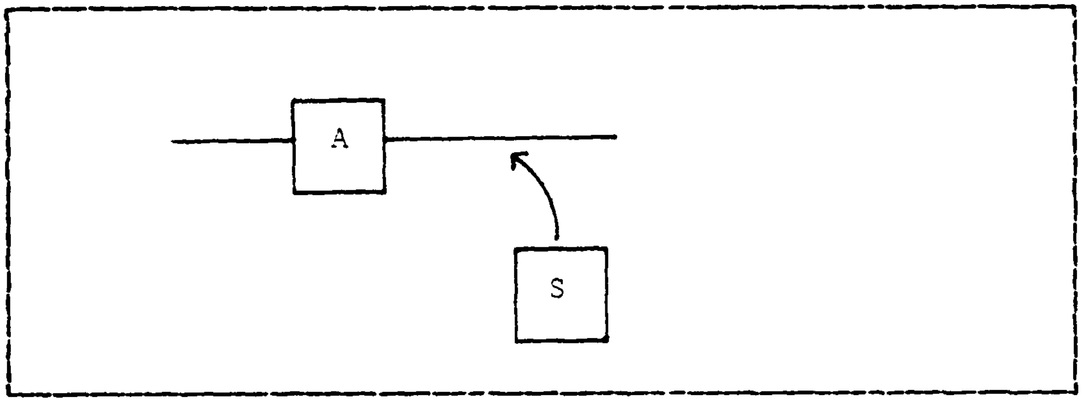


Figure 4: Standby System

$$= \bar{F}_1(t) + \int_0^t \bar{F}_2(t-s)\, f_1(s)\, ds$$

$$= e^{-\lambda_1 t} + \int_0^t e^{-\lambda_2 (t-s)}\, \lambda_1\, e^{-\lambda_1 s}\, ds$$

$$= \frac{\lambda_1}{\lambda_1 - \lambda_2} e^{-\lambda_2 t} - \frac{\lambda_2}{\lambda_1 - \lambda_2} e^{-\lambda_1 t}$$

The shorthand notation for the system's survival function should be obvious. The system has an EXP(λ_1) life from the active component and an additional EXP(λ_2) life from the spare. So the shorthand notation is

EXP(λ_1) + EXP(λ_2).

B. SUMS OF EXPONENTIALS WITH WEIGHT BETWEEN ZERO AND ONE

The examples given in the previous chapter only involved exponential lives with weight one. Now we will look at some structures, whose survival function has a shorthand notation which includes the MIX-notation and/or the ZERO-distribution.

1. Parallel System with Dissimilar Failure Rates

The notion of a parallel system has been introduced in Chapter III.A.2 . We now look at the case where

$$T_1 \sim EXP(\lambda_1) \text{ and } T_2 \sim EXP(\lambda_2).$$

Then

$$T = \max(T_1, T_2)$$

$$\begin{aligned}\bar{F}(t) &= P(\max(T_1, T_2) > t) \\ &= 1-P(\max(T_1, T_2) \leqslant t) \\ &= 1-P(T_1 \leqslant t, T_2 \leqslant t)\end{aligned}$$

Assuming independence of the two components

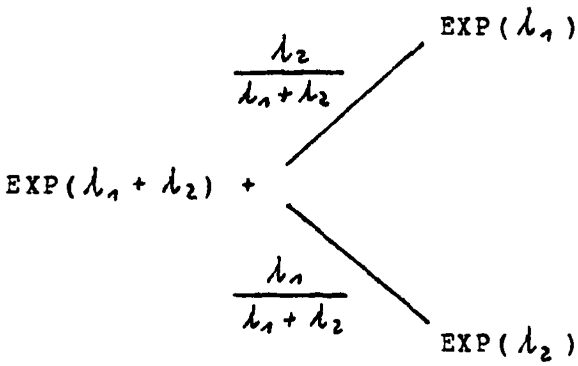
$$\begin{aligned}\bar{F}(t) &= 1-P(T_1 \leqslant t)\ P(T_2 \leqslant t) \\ &= 1-F_1(t)\ F_2(t) \\ &= 1-(1-e^{-\lambda_1 t})(1-e^{-\lambda_2 t}) \\ &= 1-(1-e^{-\lambda_1 t} - e^{-\lambda_2 t} + e^{-(\lambda_1+\lambda_2)t}) \\ &= e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1+\lambda_2)t}.\end{aligned}$$

To find the shorthand notation of the system consider all the ways which lead to the survival of the system:

- either both components survive
- or component 1 fails and component 2 survives
- or component 2 fails and component 1 survives.

If one component fails and one survives, in $\frac{\lambda_2}{\lambda_1+\lambda_2}$ fraction of the cases the survivor will be component 1 and in $\frac{\lambda_1}{\lambda_1+\lambda_2}$ fraction of the cases it will be component 2.

This can graphically be represented as



Making use of the MIX-notation the shorthand notation then is

$$\mathrm{EXP}(\lambda_1 + \lambda_2) + \mathrm{MIX}[\frac{\lambda_2}{\lambda_1+\lambda_2}\mathrm{EXP}(\lambda_1), \frac{\lambda_1}{\lambda_1+\lambda_2}\mathrm{EXP}(\lambda_2)]$$

and using the distributive property it becomes

$$\mathrm{MIX}[\frac{\lambda_2}{\lambda_1+\lambda_2}(\mathrm{EXP}(\lambda_1) + \mathrm{EXP}(\lambda_1+\lambda_2)), \frac{\lambda_1}{\lambda_1+\lambda_2}(\mathrm{EXP}(\lambda_2) + \mathrm{EXP}(\lambda_1+\lambda_2))].$$

As a check to see that this shorthand notation represents the survival function of the system, we derive the survival function from the shorthand notation:

$$\bar{F}(t) = \frac{\lambda_2}{\lambda_1+\lambda_2}\left(e^{-\lambda_1 t} + \int_0^t e^{-(\lambda_1+\lambda_2)(t-s)}\lambda_1 e^{-\lambda_1 s}\,ds\right)$$

$$+ \frac{\lambda_1}{\lambda_1+\lambda_2}\left(e^{-\lambda_2 t} + \int_0^t e^{-(\lambda_1+\lambda_2)(t-s)}\lambda_2 e^{-\lambda_2 s}\,ds\right)$$

$$= e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1+\lambda_2)t} .$$

This verifies that the shorthand notation indeed represents the system's survival function.

2. Series System with One Spare

Let us now look at a two-component series system, whose components have dissimilar failure rates with one component having a spare:

Component 1 has the constant failure rate λ_1 and component 2 and the spare have the constant failure rate λ_2. The spare can only replace component 2.

Let

$\bar{F}_1(t)$: the survival function of component 1

$\bar{F}_2(t)$: the survival function of the standby system component 2 with its spare.

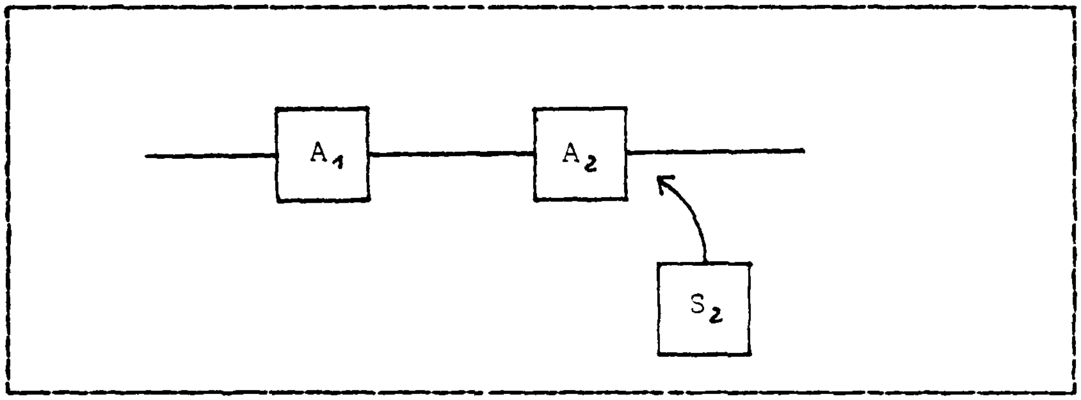


Figure 5: Series System with one Spare

The survival function for a standby system was derived in Chapter II.B. Therefore

$$\bar{F}_2(t) = e^{-\lambda_2 t} + \int_0^t e^{-\lambda_2(t-s)} \lambda_2 e^{-\lambda_2 s} ds$$
$$= e^{-\lambda_2 t} + \lambda_2 e^{-\lambda_2 t} \int_0^t ds$$
$$= (1+\lambda_2 t) e^{-\lambda_2 t} .$$

Now $\bar{F}_1(t) = e^{-\lambda_1 t}$

Then $\bar{F}(t) = \bar{F}_1(t)\,\bar{F}_2(t)$

$$= (1+\lambda_2 t) e^{-(\lambda_1+\lambda_2)t}.$$

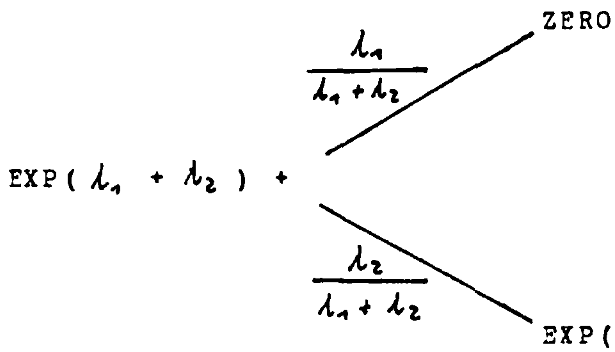
To translate the survival function into shorthand notation, let us consider the ways in which the system can survive:

-either both components survive

-or component 2 fails and its spare survives.

If one component fails, in $\frac{\lambda_1}{\lambda_1+\lambda_2}$ fraction of the time it will be component 1, which means that the system will not survive; in $\frac{\lambda_2}{\lambda_1+\lambda_2}$ fraction of the time the failing component will be component 2.

This can graphically be represented as



Using the MIX-notation the survival function then is

$$\text{EXP}(\lambda_1+\lambda_2) + \text{MIX}\left[\frac{\lambda_1}{\lambda_1+\lambda_2}\text{ZERO}, \frac{\lambda_2}{\lambda_1+\lambda_2}\text{EXP}(\lambda_1+\lambda_2)\right]$$

$$= \text{MIX}\left[\frac{\lambda_1}{\lambda_1+\lambda_2}(\text{ZERO} + \text{EXP}(\lambda_1+\lambda_2)), \right.$$

$$\left.\frac{\lambda_2}{\lambda_1+\lambda_2}(\text{EXP}(\lambda_1+\lambda_2) + \text{EXP}(\lambda_1+\lambda_2))\right]$$

$$= \text{MIX}\left[\frac{\lambda_1}{\lambda_1+\lambda_2}(\text{EXP}(\lambda_1+\lambda_2)), \right.$$

$$\left.\frac{\lambda_2}{\lambda_1+\lambda_2}(\text{EXP}(\lambda_1+\lambda_2) + \text{EXP}(\lambda_1+\lambda_2))\right].$$

To prove, that the shorthand notation does represent the survival function, we derive the latter from the shorthand:

$$\bar{F}(t) = \frac{\lambda_1}{\lambda_1+\lambda_2}\, e^{-(\lambda_1+\lambda_2)t} + \frac{\lambda_2}{\lambda_1+\lambda_2}\left(e^{-(\lambda_1+\lambda_2)t} + \right.$$

$$\int_0^t e^{-(\lambda_1+\lambda_2)(t-s)}(\lambda_1+\lambda_2)\, e^{-(\lambda_1+\lambda_2)s}\, ds\,)$$

$$= (\,1 + \lambda_2 t\,)\; e^{-(\lambda_1+\lambda_2)t}\,.$$

This is the previously found result and this verifies, that the shorthand notation does represent the system's survival function.

3. Two-out-of-Three System

As a last example in this chapter, we will look at a Two-out-of- Three system.

Consider a three component system, whose components have constant failure rates λ_1, λ_2 and λ_3 respectively. The system is functioning, as long as two out of three components are functioning (see Fig. 6).

In other words, the system is functioning as long as there is a path through the system .

Alternatively, the system can be visualized as a parallel-series system (compare Fig. 7).

The survival function of the system is

$$\begin{aligned}
\bar{F}(t) = {} & P(T_1 > t \wedge T_2 > t) + P(T_1 > t \wedge T_3 > t) \\
& + P(T_2 > t \wedge T_3 > t) \\
& - P((T_1 > t \wedge T_2 > t) \wedge (T_1 > t \wedge T_3 > t)) \\
& - P((T_1 > t \wedge T_2 > t) \wedge (T_2 > t \wedge T_3 > t)) \\
& - P((T_1 > t \wedge T_3 > t) \wedge (T_2 > t \wedge T_3 > t))
\end{aligned}$$

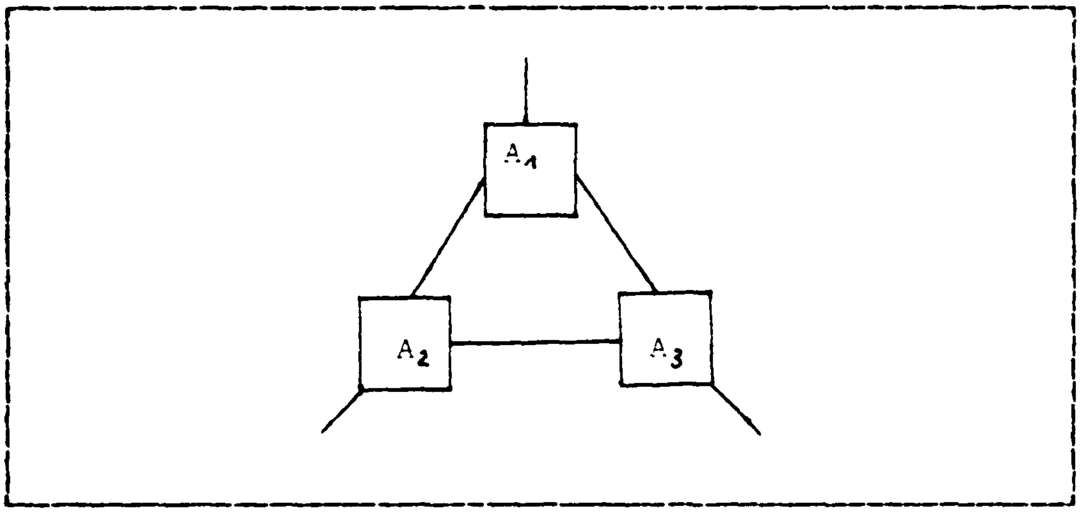


Figure 6: Two-out-of-Three System

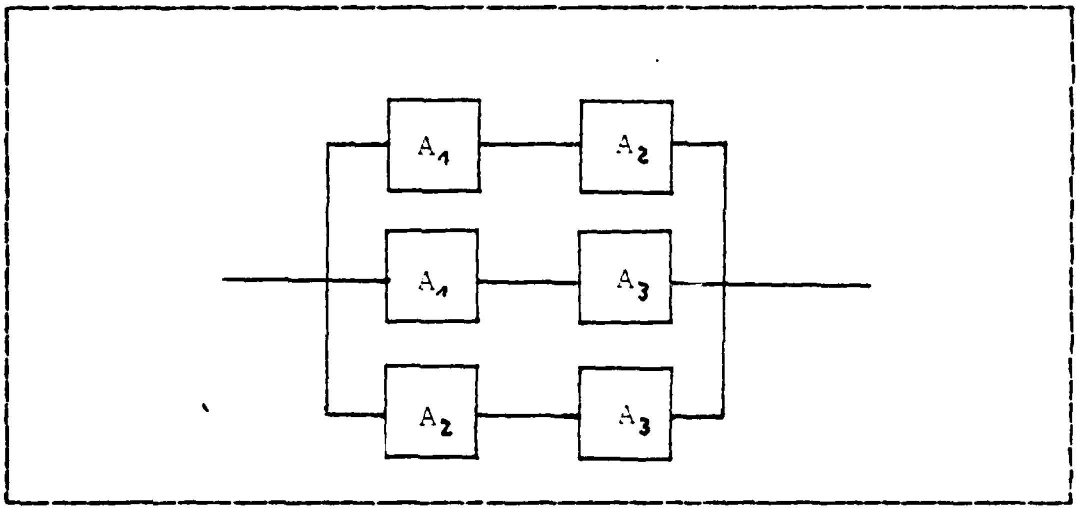


Figure 7: Two-out-of-Three System

$$+ P((T_1 > t \wedge T_2 > t) \wedge (T_1 > t \wedge T_3 > t)$$
$$\wedge P(T_2 > t \wedge T_3 > t)).$$

Thus

$$\bar{F}(t) = P(T_1 > t \wedge T_2 > t) + P(T_1 > t \wedge T_3 > t)$$
$$+ P(T_2 > t \wedge T_3 > t)$$
$$- 3P(T_1 > t \wedge T_2 > t \wedge T_3 > t)$$
$$+ P(T_1 > t \wedge T_2 > t \wedge T_3 > t)$$

Therefore, and assuming independence of the components,

$$\bar{F}(t) = P(T_1 > t)\ P(T_2 > t) + P(T_1 > T)\ P(T_3 > t)$$
$$+ P(T_2 > t)\ P(T_3 > t)$$
$$- 3P(T_1 > t)\ P(T_2 > t)\ P(T_3 > t)$$
$$+ P(T_1 > t)\ P(T_2 > t)\ P(T_3 > t)$$

$$= P(T_1 > t)\ P(T_2 > t) + P(T_1 > t)\ P(T_3 > t)$$
$$+ P(T_2 > t)\ P(T_3 > t)$$
$$- 2P(T_1 > t)\ P(T_2 > t)\ P(T_3 > t)$$
$$= e^{-(\lambda_1+\lambda_2)t} + e^{-(\lambda_1+\lambda_3)t} + e^{-(\lambda_2+\lambda_3)t}$$
$$- 2e^{-(\lambda_1+\lambda_2+\lambda_3)t} .$$

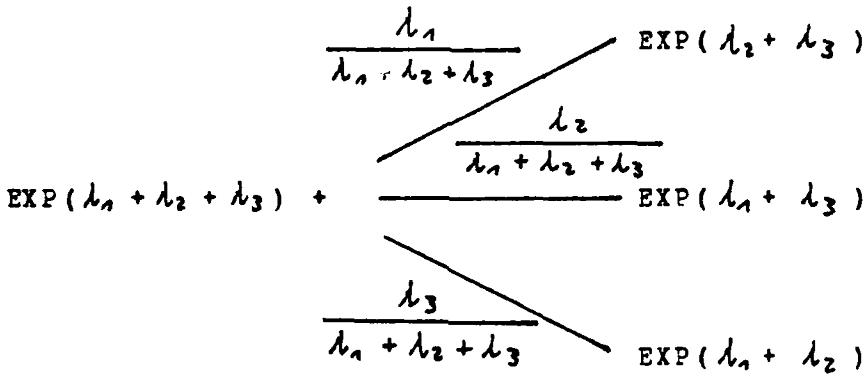
Now let us consider all the possible ways, in which the system can survive:

- either all components survive
- or component 1 fails and component 2 and 3 survive
- or component 2 fails and component 1 and 3 survive

- or component 3 fails and component 1 and 2 survive.

If a component fails and the other two survive, in $\frac{\lambda_i}{\lambda_1 + \lambda_2 + \lambda_3}$ fraction of the time it will be component i , i = 1,2,3 .

This can graphically be represented as



The shorthand notation then is

$$\text{EXP}(\lambda_1 + \lambda_2 + \lambda_3) + \text{MIX}\Big[\frac{\lambda_1}{\lambda_1+\lambda_2+\lambda_3}\text{EXP}(\lambda_2+\lambda_3),\; \frac{\lambda_2}{\lambda_1+\lambda_2+\lambda_3}\text{EXP}(\lambda_1+\lambda_3),\; \frac{\lambda_3}{\lambda_1+\lambda_2+\lambda_3}\text{EXP}(\lambda_1+\lambda_2)\Big],$$

$$= \text{MIX}\Big[\frac{\lambda_1}{\lambda_1+\lambda_2+\lambda_3}(\text{EXP}(\lambda_2+\lambda_3) + \text{EXP}(\lambda_1+\lambda_2+\lambda_3)),\; \frac{\lambda_2}{\lambda_1+\lambda_2+\lambda_3}(\text{EXP}(\lambda_1+\lambda_3) + \text{EXP}(\lambda_1+\lambda_2+\lambda_3)),\; \frac{\lambda_3}{\lambda_1+\lambda_2+\lambda_3}(\text{EXP}(\lambda_1+\lambda_2) + \text{EXP}(\lambda_1+\lambda_2+\lambda_3))\Big].$$

Again, as a check that the shorthand notation represents the survival function, let us derive the survival function from the shorthand notation:

$$
\begin{aligned}
F(t) = {} & \frac{\lambda_1}{\lambda_1+\lambda_2+\lambda_3}\Big[e^{-(\lambda_2+\lambda_3)t} \\
& \qquad + \int_0^t e^{-(\lambda_1+\lambda_2+\lambda_3)(t-s)} (\lambda_2+\lambda_3) e^{-(\lambda_2+\lambda_3)s}\, ds \Big] \\
& + \frac{\lambda_2}{\lambda_1+\lambda_2+\lambda_3}\Big[e^{-(\lambda_1+\lambda_3)t} \\
& \qquad + \int_0^t e^{-(\lambda_1+\lambda_2+\lambda_3)(t-s)} (\lambda_1+\lambda_3) e^{-(\lambda_1+\lambda_3)s}\, ds \Big] \\
& + \frac{\lambda_3}{\lambda_1+\lambda_2+\lambda_3}\Big[e^{-(\lambda_1+\lambda_2)t} \\
& \qquad + \int_0^t e^{-(\lambda_1+\lambda_2+\lambda_3)(t-s)} (\lambda_1+\lambda_2) e^{-(\lambda_1+\lambda_2)s}\, ds \Big] \\
= {} & e^{-(\lambda_2+\lambda_3)t} + e^{-(\lambda_1+\lambda_3)t} + e^{-(\lambda_1+\lambda_2)t} \\
& - 2e^{-(\lambda_1+\lambda_2+\lambda_3)t}
\end{aligned}
$$

The result again proves that the shorthand notation indeed represents the survival function of the system.

IV. IMPLEMENTING THE SHORTHAND ON THE TI-59

The concept of a reliability shorthand is introduced in the course "Reliability and Weapons System Effectiveness Measurements", OA 4302, at the Naval Postgraduate School, Monterey. Most students taking the course are in the Operations Research (OR) - Curriculum.

The choice of the TI-59 as the computing device, on which the shorthand was to be implemented, was based on the fact, that each student in the OR-Curriculum is issued a TI-59 for use in basic probability and statistics courses. Thus almost every student at the Naval Postgraduate School, who is introduced to the shorthand, is familiar with the TI-59 and has access to such a calculator.

A program, that uses the shorthand notation, times to failure and failure rates as input, should

- calculate the survival probability of basic structures / small systems and
- require moderate computation time.

To achieve these requirements it was decided to incorporate all solutions for the convolution of up to four exponential random variables in the program. The formulas that were used are given in Appendix A.

Two programs are provided in this paper.

Program 1 can be used when all rates are dissimilar or all are the same. It uses the formulas on pages 37 and 38 only.

Program 2 can be used for the general case. It makes use of all the formulas given in Appendix A. The program includes a sorting routine that determines the applicable formula from the entered failure rates.

A user guide to the two programs is provided in Appendix B.

V. SUMMARY

There is a reliability shorthand that denotes the survival function of a system, assuming that the failure rates of all components are constant.

This shorthand can be implemented on the TI-59 handheld calculator. With failure rates, time to failure and shorthand as input the TI-59 calculates the survival probability of the system.

Knowledge of calculus is not necessary to use this method, whereas the standard procedure, finding the survival probability by convolution, requires knowledge of integral calculus.

The choice of the TI-59 as the computing device for the implementation of the shorthand, though, implied limitations; the number of failure rates is limited due to the limited storage capacity of the TI-59, and computing times are comparatively long. The TI-59 can therefore only be used for smaller systems, preferably for the solution of classroom problems.

For the solution of larger problems, the shorthand should be implemented on a state-of-the-art personal computer using a general algorithm for the convolution of any number of exponential random variables.

APPENDIX A

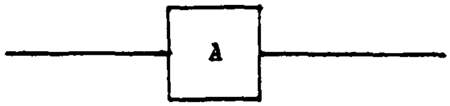
CONVOLUTION FORMULAS

Appendix A contains formulas for the convolution of up to four exponential random variables.

For the two special cases, when all random variables have the same failure rate and all have different failure rates, general formulas for the convolution of any number of exponential random variables are given.

These formulas are used in the two TI-59 programs provided in Appendix B.

System:



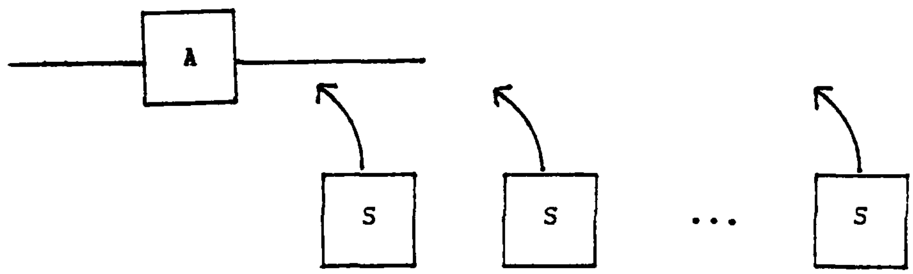
Shorthand: $EXP(\lambda)$

Survival Function: $\bar{F}(t) = e^{-\lambda t}$

Description:

A single active component with constant failure rate λ.

System:



Shorthand: $EXP(\lambda) + EXP(\lambda) + \ldots + EXP(\lambda)$

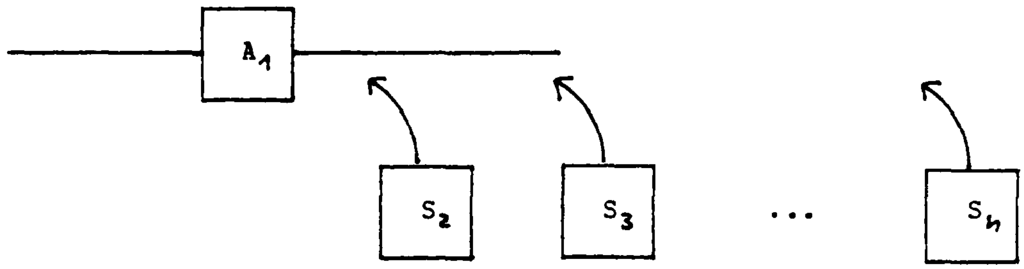
Survival Function: $$\bar{F}(t) = \left(\frac{(\lambda t)^0}{0!} + \frac{(\lambda t)^1}{1!} + \cdots + \frac{(\lambda t)^{n-1}}{(n-1)!} \right) e^{-\lambda t}$$

$$= \sum_{i=1}^{n} \frac{(\lambda t)^{i-1}}{(i-1)!} e^{-\lambda t}$$

Description:

A single active component with constant failure rate is supported by n-1 identical spares.

System:



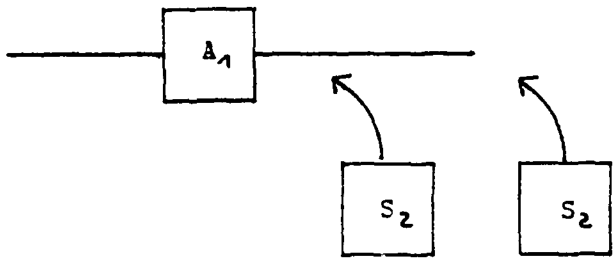
Shorthand: $EXP(\lambda_1) + EXP(\lambda_2) + \dots + EXP(\lambda_n)$

Survival Function: $\overline{F}(t) = \sum_{i=1}^{n} \left(\prod_{j \neq i} \frac{\lambda_j}{\lambda_j - \lambda_i} e^{-\lambda_i t} \right)$

Description:

A single active component with constant failure rate is supported by n-1 spares. The active component and the spares have all constant, but dissimilar failure rates.

System:



Shorthand: $EXP(\lambda_1) + EXP(\lambda_2) + EXP(\lambda_2)$

Survival Function: $\bar{F}(t) = Ae^{-\lambda_1 t} + (B + Ct)\, e^{-\lambda_2 t}$

where $A = \dfrac{\lambda_2^2}{(\lambda_2 - \lambda_1)^2}$

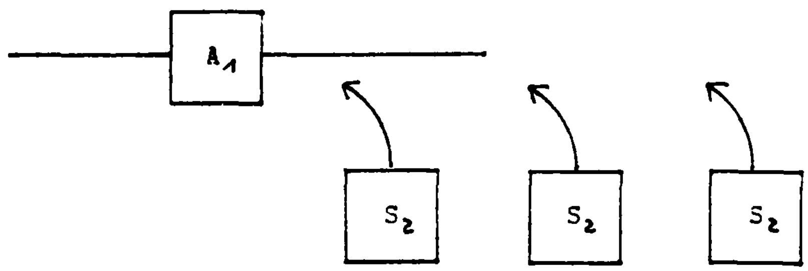
$B = 1 - A$

$C = \dfrac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2}$

Description:

A single active component with constant failure rate λ_1 is supported by two spares with identical constant failure rate λ_2.

System:



Shorthand: $EXP(\lambda_1) + EXP(\lambda_2) + EXP(\lambda_2) + EXP(\lambda_2)$

Survival Function: $\bar{F}(t) = Ae^{-\lambda_1 t} + (B + Ct + Dt^2)e^{-\lambda_2 t}$

where $A = \dfrac{\lambda_2^{\,3}}{(\lambda_2 - \lambda_1)^3}$

$B = 1 - A$

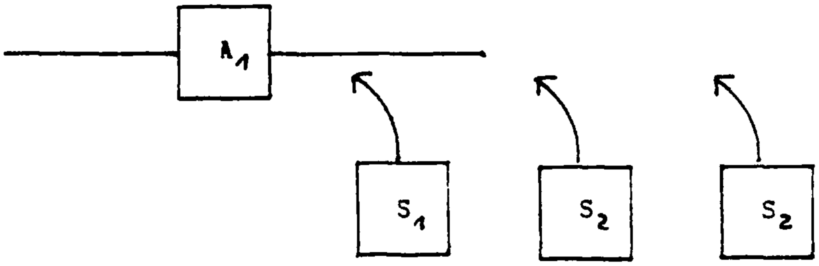
$C = \lambda_2 - \dfrac{\lambda_2^{\,3}}{(\lambda_1 - \lambda_2)^2}$

$D = \dfrac{\lambda_1 \lambda_2^{\,2}}{2(\lambda_1 - \lambda_2)}$

Description:

A single active component with constant failure rate λ_1 is supported by three spares with identical constant failure rate λ_2.

System:



Shorthand: $EXP(\lambda_1) + EXP(\lambda_1) + EXP(\lambda_2) + EXP(\lambda_2)$

Survival Function: $F(t) = (A + Bt)e^{-\lambda_1 t} + (C + Dt)e^{-\lambda_2 t}$

where $A = \dfrac{\lambda_2^3 - 3\lambda_2^2 \lambda_1}{(\lambda_2 - \lambda_1)^3}$

$B = \dfrac{\lambda_1 \lambda_2^2}{(\lambda_2 - \lambda_1)^2}$

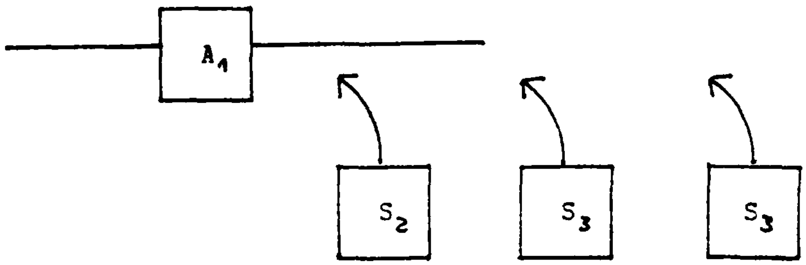
$C = 1 - A$

$D = \dfrac{\lambda_1^2 \lambda_2}{(\lambda_1 - \lambda_2)^2}$

Description:

A single active component with constant failure rate λ_1 is supported by one identical spare and two spares with dissimilar, constant failure rate λ_2.

System:



Shorthand: $EXP(\lambda_1) + EXP(\lambda_2) + EXP(\lambda_3) + EXP(\lambda_3)$

Survival Function: $\overline{F}(t) = Ae^{-\lambda_1 t} + Be^{-\lambda_2 t} + (C + Dt)e^{-\lambda_3 t}$

where
$$A = \frac{\lambda_2 \lambda_3^2}{(\lambda_2-\lambda_1)(\lambda_3-\lambda_1)^2}$$

$$B = \frac{\lambda_1 \lambda_3^2}{(\lambda_1-\lambda_2)(\lambda_3-\lambda_2)^2}$$

$$C = \frac{\lambda_1 \lambda_2}{(\lambda_1-\lambda_3)(\lambda_2-\lambda_3)} + \frac{\lambda_1 \lambda_2 \lambda_3}{(\lambda_1-\lambda_2)}\left(\frac{1}{(\lambda_1-\lambda_3)^2} - \frac{1}{(\lambda_2-\lambda_3)^2}\right)$$

$$D = \frac{\lambda_1 \lambda_2 \lambda_3}{(\lambda_1-\lambda_3)(\lambda_2-\lambda_3)}$$

Description:

A single active component with constant failure rate λ_1 has three spares. One spare has constant failure rate λ_2, two spares are identical with constant failure rate λ_3.

APPENDIX B

USER GUIDE TO TI-59 PROGRAMS

Appendix B contains a user guide to two TI-59 programs, which use reliability shorthand and failure rates as input to compute the survival probability of a system.

PROGRAM 1 is designed for the two special cases where the reliability shorthand is of the form

$$\mathrm{EXP}(\lambda) + \mathrm{EXP}(\lambda) + \ldots + \mathrm{EXP}(\lambda)$$

or

$$\mathrm{EXP}(\lambda_1) + \mathrm{EXP}(\lambda_2) + \ldots + \mathrm{EXP}(\lambda_n).$$

In the first case the number of terms is not limited, whereas in the second case the number of terms is limited to 40 due to limited storage capacity of the TI-59. In this case the number of terms can be increased to 70 by entering 9 in the display and pressing 2nd Op 1 7 .

PROGRAM 2 is designed to solve problems of the kind, that were introduced in Chapter III.B. . Due to limited memory of the TI-59 the number of exponential terms under one weight in shorthand notation is limited to four.

All results will be printed, if the TI-59 is connected to a TI PC-100A or TI PC-100C printer.

PROGRAM 1 : Procedure

1. Use any library module. Read in program 1 (side 1 of the magnetic card)
2. Press 2nd C' to initialize.
3. Enter n, the number of exponential terms to be convolved, in the display and press A.
4. Enter time t and press B.
5. Enter λ_i and press C . When all failure rates are the same, enter λ only once.
6. a) To find the survival probability of the system, when all failure rates are the same, press 2nd A'.

   b) To find the survival probability of the system, when all failure rates are dissimilar, press 2nd B'.

PROGRAM 1 : Sample Problems

1. Find the survival probability of a parallel system

   (compare Chapter III.A.2)

   a) λ = .3 , t = 7 , n = 2

   b) Shorthand notation:

      EXP(.6) + EXP(.3)

   c)

Enter	Comment	Press	Display
	Initialize	C'	0
2	n	A	0
7	t	B	7
.6	2λ	C	.3
.3	λ	C	.3
	$\bar{F}(t)$	B'	.2299172797

calculation takes 13 seconds

2. Find the survival probability of a standby-system with dissimilar components (compare Chapter III.A.3) .

a) $\lambda_1 = .4$, $\lambda_2 = .5$, $t = 6$, $n = 2$

b) Shorthand notation:

EXP(.4) + EXP(.5)

c)

Enter	Comment	Press	Display
	Initialize	C'	0
2	n	A	0
6	t	B	6
.4	λ_1	C	.4
.5	λ_2	C	.5
	$\overline{F}(t)$	B'	.254441493

calculation takes 13 seconds

3. Find the survival probability of a standby-system with one active component and four similar spares.

a) $\lambda = .3$, $t = 7$, $n = 5$

b) Shorthand notation:

EXP(.3) + EXP(.3) + EXP(.3) + EXP(.3) + EXP(.3)

c)

Enter	Comment	Press	Display
	Initialize	C'	0
5	n	A	0
7	t	B	7
.3	λ	C	.3
	$\overline{F}(t)$	A'	.9378738848

calculation takes 9 seconds

PROGRAM 2 : Procedure

CASE I : To find the convolution of up to four exponential random variables.

1. Use any library module.
   Re-Partition (enter 2 in the display, press 2nd Op 17).
   Read in all four sides of the magnetic card.
2. Press 2nd C' to initialize.
3. Enter n, the number of exponential terms to be convolved, in the display and press A.
4. Enter time t and press B.
5. Enter λ_i and press C (n entries) .
   REMARK: Failure rates, which appear only once in the expression, have to be entered before failure rates, that appear several times.
6. To find the survival probability of the system press E.

PROGRAM 2, CASE I : Sample Problems

(1) Shorthand notation

$EXP(\lambda_1) + EXP(\lambda_2) + EXP(\lambda_2)$

Sample values : $\lambda_1 = .3$, $\lambda_2 = .4$, $t = 7$

Procedure :

Enter	Comment	Press	Display
	Initialize	C'	0
3	n	A	0
7	t	B	7
.3	λ_1	C	.3
.4	λ_2	C	.4
.4	λ_2	C	.4
	$\overline{F}(t)$	E	.5363473866

calculation takes 14 seconds

(2) Shorthand notation

$EXP(\lambda_1) + EXP(\lambda_2) + EXP(\lambda_2) + EXP(\lambda_2)$

Sample values : $\lambda_1 = .2$, $\lambda_2 = .4$, $t = 3$

Procedure :

Enter	Comment	Press	Display
	Initialize	C'	0
4	n	A	0
3	t	B	3
.2	λ_1	C	.2
.4	λ_2	C	.4
.4	λ_2	C	.4
.4	λ_2	C	.4
	$\overline{F}(t)$	E	.9809746099

calculation takes 20 seconds

(3) Shorthand notation

$EXP(\lambda_1) + EXP(\lambda_1) + EXP(\lambda_2) + EXP(\lambda_2)$

Sample values : $\lambda_1 = .4$, $\lambda_2 = .3$, $t = 5$

Procedure :

Enter	Comment	Press	Display
	Initialize	C'	0
4	n	A	0
5	t	B	5
.4	λ_1	C	.4
.4	λ_1	C	.4
.3	λ_2	C	.3
.3	λ_2	C	.3
	$\bar{F}(t)$	E	.9029040721

calculation takes 20 seconds

(4) Shorthand notation

EXP(λ_1) + EXP(λ_2) + EXP(λ_3) + EXP(λ_3)

Sample values : λ_1 = .1 , λ_2 = .3 , λ_3 = .5 ,

t = 10

Procedure :

Enter	Comment	Press	Display
	Initialize	C'	0
4	n	A	0
10	t	B	10
.1	λ_1	C	.1
.3	λ_2	C	.3
.5	λ_3	C	.5
.5	λ_3	C	.5
	$\bar{F}(t)$	E	.7312684703

calculation takes 25 seconds

PROGRAM 2 : Procedure

CASE II : to solve problems of the kind, that were introduced in Chapter III.B. .

1. Derive the system's shorthand notation. Find either the
   - graphical representation or
   - the MIX-notation .
2. Use any library module.

   Re-Partition (enter 2 in the display, press 2nd Op 17).

   Read in all four sides of the magnetic card.
3. Press 2nd C' to initialize.
4. Enter time t and press B.
5. Repeat the following steps for each path of the graphical representation, i.e. for each convolution in the MIX-notation.

   a) Enter n, the number of exponential terms to be convolved, in the display and press A.

   b) Enter λ_i and press C.

   REMARK: Failure rates, which appear only once in the expression, have to be entered before failure rates, that appear several times.

   c) Enter p_i, the weight in the ith path, and press D.

   d) To find the part of the system's survival probability, that is contributed by the ith path, press E.

6. To find the survival probability of the system press 2nd E'.

PROGRAM 2, CASE II : Sample Problems

1. Find the survival probability of a parallel system with dissimilar failure rates (compare Chapter III.B.1).

   a) $\lambda_1 = .1$, $\lambda_2 = .2$, $t = 2$

   b) Shorthand notation

$$
\begin{tikzcd}
 & & \mathrm{EXP}(.1) \\
\mathrm{EXP}(.3) + \arrow[urr, dash, "(.2/.3)"] \arrow[drr, dash, "(.1/.3)"'] & & \\
 & & \mathrm{EXP}(.2)
\end{tikzcd}
$$

$$\overline{F}(t) = \mathrm{MIX}[\ (.2/.3)(\ \mathrm{EXP}(.1) + \mathrm{EXP}(.3)),$$
$$(.1/.3)(\ \mathrm{EXP}(.2) + \mathrm{EXP}(.3)\ ].$$

c)

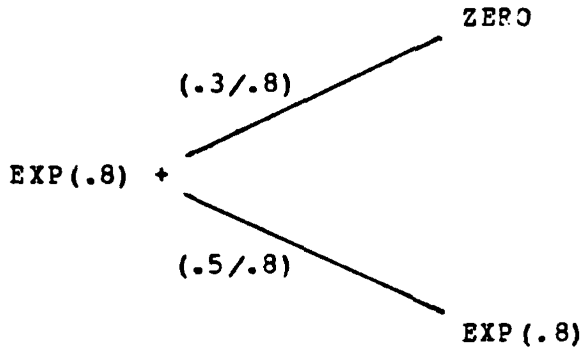
Procedure :

Enter	Comment	Press	Display
	Initialize	C'	0
2	t	B	2
2	n_1	A	0
.1	λ_1	C	.1
.3	$\lambda_1 + \lambda_2$	C	.3
(.2/.3)	P_1	D	.6666666667
		E	.635793541
2	n_2	A	0
.2	λ_2	C	.2
.3	$\lambda_1 + \lambda_2$	C	.3
(.1/.3)	P_2	D	.3333333333
		E	.304445622
	$\overline{F}(t)$	E'	.940239163

2. Find the survival probability of a series system with one spare as introduced in Chapter III.B.2 .

a) $\lambda_1 = .3$, $\lambda_2 = .5$, $t = 7$

b) Shorthand notation



$\overline{F}(t)$ = MIX[(.3/.8)(EXP(.8),

(.5/.8)(EXP(.8) + EXP(.8)].

c)

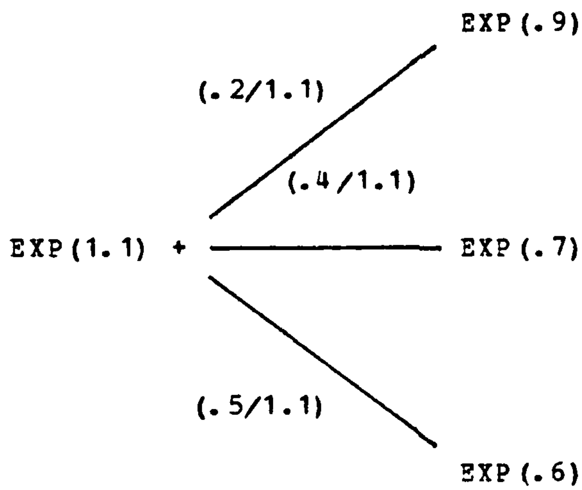
Procedure :

Enter	Comment	Press	Display
	Initialize	C'	0
7	t	B	7
1	n_1	A	0
.8	$\lambda_1 + \lambda_2$	C	.8
(.3/.8)	P_1	D	.375
		E	.0013866989
2	n_2	A	0
.8	$\lambda_1 + \lambda_2$	C	.8
.8	$\lambda_1 + \lambda_2$	C	.8
(.5/.8)	P_2	D	.625
		E	.0152536878
	$\overline{F}(t)$	E'	.0166403867

3. Find the survival probability of a Two-out-of-Three System as introduced in Chapter III.B.3 .

a) $\lambda_1 = .2$, $\lambda_2 = .4$, $\lambda_3 = .5$, $t = 9$

b) Shorthand notation



$$\bar{F}(t) = MIX[\ (.2/1.1)\,(\ EXP(.9)\ +\ EXP(1.1)),$$
$$(.4/1.1)\,(\ EXP(.7)\ +\ EXP(1.1)),$$
$$(.5/1.1)\,(\ EXP(.6)\ +\ EXP(1.1))\ ].$$

c)

Procedure :

Enter	Comment	Press	Display
	Initialize	C'	0
9	t	B	9
2	n_1	A	0
1.1	$\lambda_1 + \lambda_2 + \lambda_3$	C	1.1
.9	$\lambda_2 + \lambda_3$	C	.9
(.2/1.1)	P_1	D	.1818181818
		E	.0002624871
2	n_2	A	0
1.1	$\lambda_1 + \lambda_2 + \lambda_3$	C	1.1
.7	$\lambda_1 + \lambda_3$	C	.7
(.4/1.1)	P_2	D	.3636363636
		E	.0018043754
2	n_3	A	0
1.1	$\lambda_1 + \lambda_2 + \lambda_3$	C	1.1
.6	$\lambda_1 + \lambda_2$	C	.6
(.5/1.1)	P_3	D	.4545454545
		E	.0044892129
	$\overline{F}(t)$	E'	.0065560755

COMPUTER LISTINGS

PROGRAM 1

000	91	R/S
001	76	LBL
002	18	C'
003	29	CP
004	25	CLR
005	47	CMS
006	81	RST
007	76	LBL
008	11	A
009	42	STO
010	00	00
011	75	-
012	01	1
013	95	=
014	42	STO
015	09	09
016	29	CP
017	02	2
018	00	0
019	42	STO
020	08	08
021	01	1
022	42	STO
023	17	17
024	00	0
025	42	STO
026	18	18
027	91	R/S
028	76	LBL
029	12	B
030	42	STO
031	01	01
032	91	R/S
033	76	LBL
034	13	C
035	72	ST*
036	08	08
037	69	OP
038	28	28
039	91	R/S
040	76	LBL
041	16	A'
042	43	RCL
043	20	20
044	65	×
045	43	RCL
046	01	01
047	95	=
048	42	STO
049	02	02
050	00	0
051	01	1
052	42	STO
053	04	04
054	42	STO
055	05	05
056	42	STO
057	18	18
058	76	LBL
059	28	LOG
060	29	CP
061	43	RCL
062	09	09
063	67	EQ
064	29	CP
065	43	RCL
066	02	02
067	45	Y^X
068	43	RCL
069	05	05
070	55	÷
071	43	RCL
072	04	04
073	95	=
074	44	SUM
075	18	18
076	69	OP
077	39	39
078	69	OP
079	25	25
080	43	RCL
081	05	05
082	49	PRD
083	04	04
084	61	GTO
085	28	LOG
086	76	LBL
087	29	CP
088	43	RCL
089	02	02
090	94	+/-
091	22	INV
092	23	LNX
093	65	×
094	71	SBR
095	47	CMS
096	95	=
097	99	PRT
098	91	R/S
099	76	LBL
100	17	B'
101	02	2
102	00	0
103	42	STO
104	08	08
105	76	LBL
106	39	COS
107	02	2
108	00	0
109	42	STO
110	06	06
111	01	1
112	42	STO
113	16	16
114	76	LBL
115	38	SIN
116	29	CP
117	73	RC*
118	06	06
119	67	EQ

PROGRAM 1 continued

```
120  36 PGM
121  73 RC*
122  08  08
123  32 X:T
124  73 RC*
125  06  06
126  22 INV
127  67  EQ
128  30 TAN
129  69 DP
130  26  26
131  61 GTD
132  38 SIN
133  76 LBL
134  30 TAN
135  49 PRD
136  16  16
137  75  -
138  73 RC*
139  08  08
140  95  =
141  35 1/X
142  49 PRD
143  16  16
144  43 RCL
145  00  00
146  32 X:T
147  43 RCL
148  06  06
149  75  -
150  01  1
151  09  9
152  95  =
153  77  GE
154  36 PGM
155  69 DP
156  26  26
157  61 GTD
158  38 SIN
159  76 LBL
160  36 PGM
161  73 RC*
162  08  08
163  94 +/-
164  65  X
165  43 RCL
166  01  01
167  95  =
168  22 INV
169  23 LNX
170  65  X
171  43 RCL
172  16  16
173  95  =
174  44 SUM
175  18  18
176  43 RCL
177  00  00
178  32 X:T
179  43 RCL
180  08  08
181  75  -
182  01  1
183  09  9
184  95  =
185  67  EQ
186  37 P/R
187  69 DP
188  28  28
189  61 GTD
190  39 CDS
191  76 LBL
192  37 P/R
193  71 SBR
194  47 CMS
195  99 PRT
196  91 R/S
197  76 LBL
198  47 CMS
199  43 RCL
200  18  18
201  92 RTN
```

LABEL ADRESSES

```
002  18 C'
008  11  A
029  12  B
034  13  C
041  16 A'
059  28 LOG
087  29 CP
100  17 B'
106  39 CDS
115  38 SIN
134  30 TAN
160  36 PGM
192  37 P/R
198  47 CMS
```

PROGRAM 2

```
000  86  STF
001  00   00
002  61  GTO
003  15   E
004  76  LBL
005  18  C'
006  29  CP
007  25  CLR
008  47  CMS
009  91  R/S
010  76  LBL
011  11   A
012  42  STO
013  00   00
014  75   -
015  01   1
016  95   =
017  42  STO
018  09   09
019  29  CP
020  01   1
021  00   0
022  42  STO
023  08   08
024  01   1
025  42  STO
026  17   17
027  00   0
028  42  STO
029  18   18
030  91  R/S
031  76  LBL
032  12   B
033  42  STO
034  01   01
035  91  R/S
036  76  LBL
037  13   C
038  72  ST*
039  08   08
040  69  OP
041  28   28
042  91  R/S
043  76  LBL
044  14   D
045  49  PRD
046  17   17
047  91  R/S
048  76  LBL
049  16  A'
050  43  RCL
051  10   10
052  65   X
053  43  RCL
054  01   01
055  95   =
056  42  STO
057  02   02
058  00   0
059  01   1
060  42  STO
061  04   04
062  42  STO
063  05   05
064  42  STO
065  18   18
066  76  LBL
067  28  LOG
068  29  CP
069  43  RCL
070  09   09
071  67   EQ
072  29  CP
073  43  RCL
074  02   02
075  45  YX
076  43  RCL
077  05   05
078  55   ÷
079  43  RCL
080  04   04
081  95   =
082  44  SUM
083  18   18
084  69  OP
085  39   39
086  69  OP
087  25   25
088  43  RCL
089  05   05
090  49  PRD
091  04   04
092  61  GTO
093  28  LOG
094  76  LBL
095  29  CP
096  43  RCL
097  02   02
098  94  +/-
099  22  INV
100  23  LNX
101  65   X
102  71  SBR
103  47  CMS
104  95   =
105  91  R/S
106  76  LBL
107  17  B'
108  01   1
109  00   0
110  42  STO
111  08   08
112  76  LBL
113  39  COS
114  01   1
115  00   0
116  42  STO
117  06   06
118  01   1
119  42  STO
```

PROGRAM 2 continued

120	16	16	160	36	PGM	200	91	R/S
121	76	LBL	161	69	OP	201	76	LBL
122	38	SIN	162	26	26	202	47	CMS
123	29	CP	163	61	GTO	203	43	RCL
124	73	RC*	164	38	SIN	204	18	18
125	06	06	165	76	LBL	205	65	×
126	67	EQ	166	36	PGM	206	43	RCL
127	36	PGM	167	73	RC*	207	17	17
128	73	RC*	168	08	08	208	95	=
129	08	08	169	94	+/-	209	99	PRT
130	32	X:T	170	65	×	210	44	SUM
131	73	RC*	171	43	RCL	211	19	19
132	06	06	172	01	01	212	92	RTN
133	22	INV	173	95	=	213	76	LBL
134	67	EQ	174	22	INV	214	22	INV
135	30	TAN	175	23	LNX	215	43	RCL
136	69	OP	176	65	×	216	11	11
137	26	26	177	43	RCL	217	33	X²
138	61	GTO	178	16	16	218	55	÷
139	38	SIN	179	95	=	219	53	(
140	76	LBL	180	44	SUM	220	43	RCL
141	30	TAN	181	18	18	221	11	11
142	49	PRD	182	43	RCL	222	75	-
143	16	16	183	00	00	223	43	RCL
144	75	-	184	32	X:T	224	10	10
145	73	RC*	185	43	RCL	225	54	)
146	08	08	186	08	08	226	33	X²
147	95	=	187	75	-	227	95	=
148	35	1/X	188	09	9	228	42	STO
149	49	PRD	189	95	=	229	07	07
150	16	16	190	67	EQ	230	65	×
151	43	RCL	191	37	P/R	231	53	(
152	00	00	192	69	OP	232	43	RCL
153	32	X:T	193	28	28	233	10	10
154	43	RCL	194	61	GTO	234	94	+/-
155	06	06	195	39	COS	235	65	×
156	75	-	196	76	LBL	236	43	RCL
157	09	9	197	37	P/R	237	01	01
158	95	=	198	71	SBR	238	54	)
159	77	GE	199	47	CMS	239	22	INV

PROGRAM 2 continued

240	23	LNX
241	95	=
242	42	STO
243	18	18
244	43	RCL
245	07	07
246	94	+/-
247	85	+
248	01	1
249	85	+
250	43	RCL
251	10	10
252	65	×
253	43	RCL
254	11	11
255	55	÷
256	53	(
257	43	RCL
258	10	10
259	75	-
260	43	RCL
261	11	11
262	54	)
263	65	×
264	43	RCL
265	01	01
266	95	=
267	65	×
268	53	(
269	43	RCL
270	11	11
271	94	+/-
272	65	×
273	43	RCL
274	01	01
275	54	)
276	22	INV
277	23	LNX
278	95	=
279	44	SUM
280	18	18
281	71	SBR
282	47	CMS
283	91	R/S
284	76	LBL
285	23	LNX
286	43	RCL
287	11	11
288	45	YX
289	03	3
290	55	÷
291	53	(
292	43	RCL
293	11	11
294	75	-
295	43	RCL
296	10	10
297	54	)
298	45	YX
299	03	3
300	95	=
301	42	STO
302	07	07
303	65	×
304	53	(
305	43	RCL
306	10	10
307	94	+/-
308	65	×
309	43	RCL
310	01	01
311	54	)
312	22	INV
313	23	LNX
314	85	+
315	53	(
316	43	RCL
317	07	07
318	94	+/-
319	85	+
320	01	1
321	85	+
322	53	(
323	43	RCL
324	11	11
325	75	-
326	43	RCL
327	11	11
328	45	YX
329	03	3
330	55	÷
331	53	(
332	43	RCL
333	10	10
334	75	-
335	43	RCL
336	11	11
337	54	)
338	33	X²
339	54	)
340	65	×
341	43	RCL
342	01	01
343	85	+
344	43	RCL
345	10	10
346	65	×
347	43	RCL
348	11	11
349	33	X²
350	55	÷
351	53	(
352	02	2
353	65	×
354	53	(
355	43	RCL
356	10	10
357	75	-
358	43	RCL
359	11	11

PROGRAM 2 continued

360	54	)
361	54	)
362	65	×
363	43	RCL
364	01	01
365	33	X²
366	54	)
367	65	×
368	53	(
369	43	RCL
370	11	11
371	94	+/-
372	65	×
373	43	RCL
374	01	01
375	54	)
376	22	INV
377	23	LNX
378	95	=
379	44	SUM
380	18	18
381	71	SBR
382	47	CMS
383	91	R/S
384	76	LBL
385	24	CE
386	43	RCL
387	12	12
388	65	×
389	33	X²
390	75	-
391	03	3
392	65	×
393	43	RCL
394	12	12
395	33	X²
396	65	×
397	43	RCL
398	10	10
399	95	=
400	55	÷
401	53	(
402	53	(
403	43	RCL
404	12	12
405	75	-
406	43	RCL
407	10	10
408	54	)
409	65	×
410	33	X²
411	54	)
412	95	=
413	42	STO
414	07	07
415	85	+
416	43	RCL
417	10	10
418	65	×
419	43	RCL
420	12	12
421	33	X²
422	55	÷
423	53	(
424	43	RCL
425	12	12
426	75	-
427	43	RCL
428	10	10
429	54	)
430	33	X²
431	65	×
432	43	RCL
433	01	01
434	95	=
435	65	×
436	53	(
437	43	RCL
438	10	10
439	94	+/-
440	65	×
441	43	RCL
442	01	01
443	54	)
444	22	INV
445	23	LNX
446	95	=
447	42	STO
448	18	18
449	53	(
450	43	RCL
451	07	07
452	94	+/-
453	85	+
454	01	1
455	85	+
456	43	RCL
457	10	10
458	33	X²
459	65	×
460	43	RCL
461	12	12
462	55	÷
463	53	(
464	43	RCL
465	10	10
466	75	-
467	43	RCL
468	12	12
469	54	)
470	33	X²
471	65	×
472	43	RCL
473	01	01
474	54	)
475	65	×
476	53	(
477	43	RCL
478	12	12
479	94	+/-

PROGRAM 2 continued

480	65	×
481	43	RCL
482	01	01
483	54	)
484	22	INV
485	23	LNX
486	95	=
487	44	SUM
488	18	18
489	71	SBR
490	47	CMS
491	91	R/S
492	76	LBL
493	25	CLR
494	43	RCL
495	11	11
496	65	×
497	43	RCL
498	12	12
499	33	X²
500	95	=
501	55	÷
502	53	(
503	43	RCL
504	11	11
505	75	-
506	43	RCL
507	10	10
508	54	)
509	55	÷
510	53	(
511	43	RCL
512	12	12
513	75	-
514	43	RCL
515	10	10
516	54	)
517	33	X²
518	95	=
519	65	×
520	53	(
521	43	RCL
522	10	10
523	94	+/-
524	65	×
525	43	RCL
526	01	01
527	54	)
528	22	INV
529	23	LNX
530	95	=
531	42	STO
532	18	18
533	43	RCL
534	10	10
535	65	×
536	43	RCL
537	12	12
538	33	X²
539	55	÷
540	53	(
541	43	RCL
542	10	10
543	75	-
544	43	RCL
545	11	11
546	54	)
547	55	÷
548	53	(
549	43	RCL
550	12	12
551	75	-
552	43	RCL
553	11	11
554	54	)
555	33	X²
556	95	=
557	65	×
558	53	(
559	43	RCL
560	11	11
561	94	+/-
562	65	×
563	43	RCL
564	01	01
565	54	)
566	22	INV
567	23	LNX
568	95	=
569	44	SUM
570	18	18
571	43	RCL
572	10	10
573	65	×
574	43	RCL
575	11	11
576	55	÷
577	53	(
578	43	RCL
579	10	10
580	75	-
581	43	RCL
582	12	12
583	54	)
584	55	÷
585	53	(
586	43	RCL
587	11	11
588	75	-
589	43	RCL
590	12	12
591	54	)
592	95	=
593	42	STO
594	07	07
595	55	÷
596	43	RCL
597	10	10
598	65	×
599	43	RCL

PROGRAM 2 continued

```
600  11   11
601  65   ×
602  43  RCL
603  12   12
604  55   ÷
605  53   (
606  43  RCL
607  10   10
608  75   -
609  43  RCL
610  11   11
611  54   )
612  65   ×
613  53   (
614  53   (
615  43  RCL
616  10   10
617  75   -
618  43  RCL
619  12   12
620  54   )
621  33  X²
622  35  1/X
623  75   -
624  53   (
625  43  RCL
626  11   11
627  75   -
628  43  RCL
629  12   12
630  54   )
631  33  X²
632  35  1/X
633  54   )
634  95   =
635  85   +
636  43  RCL
637  07   07
638  65   ×
639  43  RCL
640  12   12
641  65   ×
642  43  RCL
643  01   01
644  95   =
645  65   ×
646  53   (
647  43  RCL
648  12   12
649  94  +/-
650  65   ×
651  43  RCL
652  01   01
653  54   )
654  22  INV
655  23  LNX
656  95   =
657  44  SUM
658  18   18
659  71  SBR
660  47  CMS
661  91  R/S
662  76  LBL
663  10  E'
664  98  ADV
665  43  RCL
666  19   19
667  99  PRT
668  91  R/S
669  76  LBL
670  15   E
671  87  IFF
672  00   00
673  60  DEG
674  81  RST
675  76  LBL
676  60  DEG
677  22  INV
678  86  STF
679  00   00
680  86  STF
681  40  IND
682  00   00
683  00   0
684  42  STO
685  07   07
686  43  RCL
687  11   11
688  32  X:T
689  87  IFF
690  01   01
691  16  A'
692  87  IFF
693  02   02
694  48  EXC
695  87  IFF
696  03   03
697  49  PRD
698  87  IFF
699  04   04
700  50  IXI
701  76  LBL
702  48  EXC
703  43  RCL
704  10   10
705  67   EQ
706  16  A'
707  61  GTO
708  17  B'
709  76  LBL
710  49  PRD
711  43  RCL
712  10   10
713  67   EQ
714  16  A'
715  43  RCL
716  12   12
717  67   EQ
718  22  INV
719  61  GTO
```

PROGRAM 2 continued

720	17	B'
721	76	LBL
722	50	I×I
723	43	RCL
724	10	10
725	22	INV
726	86	STF
727	40	IND
728	00	00
729	22	INV
730	67	EQ
731	57	ENG
732	01	1
733	42	STO
734	07	07
735	76	LBL
736	57	ENG
737	43	RCL
738	12	12
739	22	INV
740	67	EQ
741	58	FIX
742	02	2
743	44	SUM
744	07	07
745	86	STF
746	40	IND
747	07	07
748	87	IFF
749	03	03
750	59	INT
751	22	INV
752	86	STF
753	40	IND
754	07	07
755	76	LBL
756	58	FIX
757	43	RCL
758	12	12
759	32	X⇌T

760	43	RCL
761	13	13
762	22	INV
763	67	EQ
764	59	INT
765	04	4
766	44	SUM
767	07	07
768	76	LBL
769	59	INT
770	86	STF
771	40	IND
772	07	07
773	87	IFF
774	00	00
775	17	B'
776	87	IFF
777	03	03
778	16	A'
779	87	IFF
780	04	04
781	25	CLR
782	87	IFF
783	05	05
784	24	CE
785	87	IFF
786	06	06
787	23	LNX
788	91	R/S

LABEL ADRESSES

005	18	C'
011	11	A
032	12	B
037	13	C
044	14	D
049	16	A'
067	28	LOG
095	29	CP
107	17	B'
113	39	COS
122	38	SIN
141	30	TAN
166	36	PGM
197	37	P/R
202	47	CMS
214	22	INV
285	23	LNX
385	24	CE
494	25	CLR
664	10	E'
671	15	E
677	60	DEG
700	48	EXC
708	49	PRD
720	50	I×I
734	57	ENG
754	58	FIX
767	59	INT

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