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BISTATIC SURFACE CLUTTER RESOLUTION AREA AT SMALL GRAZING ANGLES

The MITRE Corporation

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Approximate expressions for the main beam bistatic surface clutter cell area in the resolution time-limited case are determined as a function of the bistatic scattering angle for a transmitter and receiver which subtend small grazing angles at the clutter cell. The bistatic angle is the angle subtended at the clutter cell by the lines joining it to the transmitter and receiver.

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The shape of the clutter cell is a function of the transmitter and receiver azimuthal beamwidths, the receiver resolution time, and the time-delay difference between the clutter return and the direct signal from the transmitter to receiver. For time-delay differences much greater than the receiver resolution time, the cell shape is a parallelogram. For small time-delay differences, the cell shape is approximately trapezoidal or triangular at small bistatic angles and rhomboidal or hexagonal at large bistatic angles.

Approximate expressions for the clutter cell area are derived for arbitrary time-delay differences. The clutter cell area increases with increasing bistatic angle and is proportional to the (secant)^2 of one-half the bistatic angle for all angles except those approaching 180°. The cell area is typically more than 30 dB larger in the forward-scattered direction than in the backscattered direction.

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SECTION I

INTRODUCTION

The radar return resulting from transmitter energy scattered by terrain or the sea to a radar receiver is known as surface clutter. The surface clutter area, which is within the main lobe of the transmitter and receiver antennas and which is further restricted by the receiver resolution time to be within a given range delay bin is defined as the clutter resolution area. If the transmitter and receiver are at separate sites, the radar is bistatic[1] as differentiated from monostatic for a common antenna or multistatic for two or more separate receiving sites.

The bistatic surface clutter geometry is shown in Figure 1. The transmitter, receiver, and clutter cell are located at points A, B, C, respectively, at separations $\overline{AB} = \rho_{AB}$, $\overline{AC} = \rho$, and $\overline{BC} = r$. The ground reference plane is the plane tangent to the Earth's surface at C . The main-lobe clutter return per pulse, at the receiver preamplifier and in the absence of range ambiguities, is given by

$$E_c = \int_{A_c} \underbrace{\left(\frac{P_t\ K\ \delta t\ L_t\ G_A}{4\pi\ \rho^2}\right)}_{\text{incident energy density}} \underbrace{\left(\frac{L_r\ G_B\ \lambda^2}{(4\pi)^2\ r^2}\right)}_{\text{fraction of reflected energy returned to receiver}} \underbrace{(\sigma_o\ dA)}_{\text{differential clutter RCS}} \qquad (1)$$

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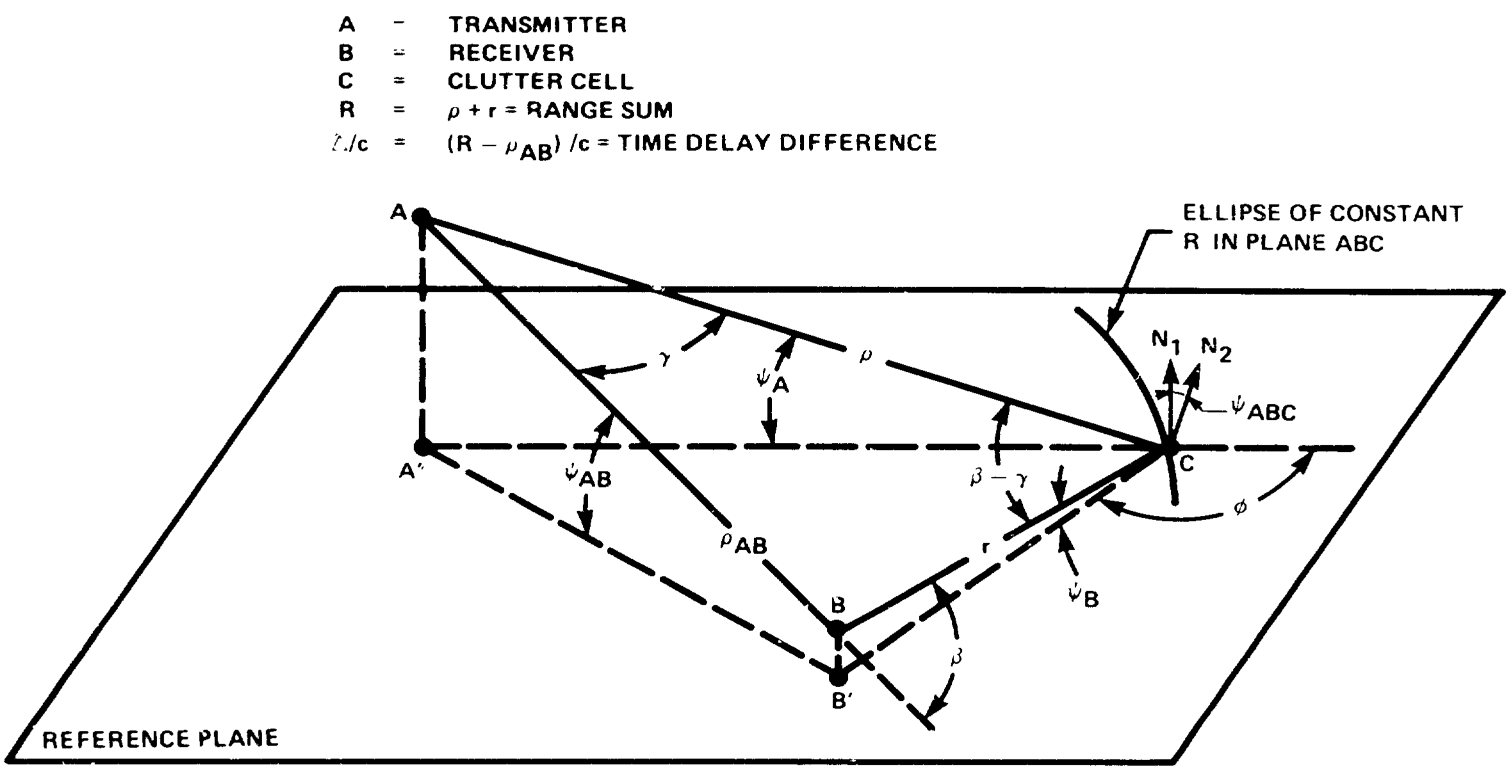

FIGURE 1. BISTATIC SURFACE CLUTTER GEOMETRY

where E_c is the clutter pulse energy, P_t is the transmitter peak power, G_A and G_B are the transmitter and receiver antenna power gains, respectively, in the direction of a differential clutter resolution area dA, K is the pulse compression ratio, δt is the receiver resolution time $\approx$ 1/receiver bandwidth, L_ρ and L_r are the one-way propagation losses (excess over free-space loss and including antenna losses) from the transmitter to the clutter cell and clutter cell to receiver, respectively, λ is the radar wavelength, σ_o is the clutter cross section per unit area (m^2/m^2), and A_c is the clutter resolution area corresponding to a specified range delay bin within the main lobes of the transmitter and receiving antennas.

For the ρr product and σ_o approximately constant over the resolution area A_c, Equation (1) reduces to

$$E_c \approx \left(\frac{P_t \; K \; \delta t \; L_\rho \; \overline{G_A \; G_B} \; L_r \; \lambda^2}{(4\pi)^3 \; \rho^2 \; r^2} \right) (\sigma_o \; A_c) \tag{2}$$

where $\overline{G_A \; G_B} \equiv \frac{1}{A_c} \int_{A_c} G_A \; G_B \; dA$ = mean power gain product within the main lobes of the transmitter and receiver antennas. The factor $\sigma_o \; A_c$ is the main beam clutter radar cross section.

Equation (2) is particularly useful in determining the Improvement Factor required of a Doppler processor to detect a weak return from a moving target in the presence of a much stronger quasi-stationary clutter return. For example, for a point target located at the clutter cell and at the peak gains G_{A_0}, G_{B_0} of the transmitter and receiver, respectively, the ratio of target return to main beam clutter return is given by $\left(G_{A_0} G_{B_0} / \overline{G_A G_B}\right)\left(\sigma_t/\sigma_0 A_c\right)$ where σ_t is the target radar cross section. The factor $\left(G_{A_0} G_{B_0} / \overline{G_A G_B}\right)$ is a function of the transmitter and receiver antenna pattern shapes. The target radar cross section σ_t and the clutter coefficient per unit area are found from published experimental data or appropriate scattering models. This paper is concerned with determining approximate, convenient expressions for the clutter resolution area A_c. The functional dependence of A_c upon the bistatic angle, which is the angle subtended at the clutter cell by the transmitter and receiver antenna [$\measuredangle ACB = \beta - \gamma$ in Figure 1], is of particular interest.

Approximate expressions for the monostatic clutter resolution area, corresponding to a bistatic angle $\beta - \gamma = 0$ and $\rho = r = R/2$, are well known for both the beamwidth-limited and resolution time-limited cases.[2] For the beamwidth-limited case, which occurs for sufficiently large grazing angles of incidence, the clutter resolution area is simply the illuminated area of the ground reference plane and is given by

$$A_c = \begin{cases} \frac{\pi}{4} D_\theta D_\xi \text{ , elliptical beam cross section} \\ D_\theta D_\xi \text{ , rectangular beam cross section} \end{cases} ;$$
$$\tan\psi > (2R/c\Delta t)\tan(\xi/2),\ \beta - \gamma = 0,\ \rho = r = \frac{R}{2} \tag{3}$$

where

$D_\theta = R \tan(\theta/2)$ = azimuthal width of clutter cell

$D_\xi = R \tan(\xi/2) \csc \psi$ = elevation width of clutter cell

$R = \rho + r$ ($= 2r = 2\rho$ for monostatic radar) = range sum

θ = main lobe azimuthal beamwidth of transmitter/receiver

ξ = main lobe elevation beamwidth of transmitter/receiver

ψ = beam grazing angle of incidence on the ground reference plane

c = velocity of wave propagation.

For the resolution time-limited case, which occurs for sufficiently small grazing angles of incidence, the clutter resolution area is approximately rectangular in shape and is given by

$$A_c = D_\theta\,(c\delta t/2) \sec \psi = [R \tan(\theta/2)]\,[(c\delta t/2) \sec \psi]\ ;$$

$$\tan \psi \leqslant (2R/c\delta t) \tan(\xi/2),\ c\delta t \ll R,\ \beta - \gamma = 0,\ \rho = r = \frac{R}{2} \qquad (4)$$

In Equations (3) and (4), the condition on the grazing angle of incidence follows from $(c\delta t/2) \sec \psi$ being greater or less, respectively, than D_ξ .

We have been unable to find in the literature convenient expressions for the bistatic clutter resolution area when the bistatic angle $\beta - \gamma \quad 0$. A convenient expression for bistatic volume clutter has been obtained for the case of Gaussian antenna functions when the time-delay difference, between the indirect signal from the volume clutter and the direct signal from the transmitter to the receiver, is large compared to the receiver resolution time.[3] The bistatic clutter resolution area has been determined exactly for specific antenna functions and for specific geometries by numerical integration techniques for both the beamwidth-limited case[4,5] and the resolution time-limited case.[6] In the following section, convenient expressions for the bistatic clutter resolution area are derived for the resolution time-limited case at small grazing angles. In Section III, we give some numerical results.

SECTION II

ANALYSIS

Small Grazing Angle Conditions

Consider the case where the angle ψ_{ABC} (see Figure 1), between the unit vector $\hat{N}_1$ normal to the reference plane and the unit vector $\hat{N}_2$ normal to the plane ABC, is small so that

$$\hat{N}_1 \cdot \hat{N}_2 = \cos \psi_{ABC} \approx 1 \qquad (5)$$

It follows from Equation (5) that

$$\cos \psi_A \approx \cos \psi_B \approx 1 \qquad (6a)$$

$$\measuredangle CA'B' \approx \gamma, \ \measuredangle A'CB' = \pi - \phi \approx \beta - \gamma, \ \measuredangle A'B'C \approx \pi - \beta \qquad (6b)$$

$$\theta_A \text{ and } \theta_B \text{ lie approximately in the plane ABC} \qquad (6c)$$

where

ψ_A, ψ_B = grazing angles subtended at the clutter cell by the transmitter and receiver, respectively

θ_A, θ_B = azimuthal beamwidths of the transmitter and receiver main beams, respectively

It should be noted that Equation (6b) follows from Equation (6a) but that Equation (6c) follows from the more restrictive Equation (5). In the following analysis, the clutter resolution area is derived subject to Equation (5) for pedagogical purposes but the results are also applicable to the less restrictive Equation (6a) when the azimuthal beamwidths do not necessarily lie in the plane ABC.

In addition to the low grazing angle conditions of Equation (6a), the grazing angles ψ_A and ψ_B must also be sufficiently small to satisfy the conditions for the resolution time-limited case of clutter cell area. Subject to the conditions of Equation (6a) and by analogy to the monostatic resolution time-limited conditions of Equation (4), the bistatic resolution time-limited conditions are given by

$$\begin{aligned} &\tan \psi_A \lesssim (2\rho/\delta\rho) \tan(\xi_A/2) \\ &\qquad\qquad\qquad\qquad\qquad ; \cos \psi_A \approx \cos \psi_B \approx 1 \qquad (7) \\ &\tan \psi_B \lesssim (2r/\delta r) \tan(\xi_B/2) \end{aligned}$$

where ξ_A and ξ_B are the main lobe elevation beamwidths of the transmitter and receiver, respectively, and $\delta\rho$ and δr are the range resolutions in plane ABC in the direction of the transmitter and receiver, respectively. The conditions of Equation (7) allow the ellipsoidal bistatic mapping, of the range resolutions $\delta\rho$ and δr in the plane ABC onto the reference plane, to be approximated in Equation (7) by the spherical monostatic mappings $\delta\rho \sec \psi_A$ and $\delta r \sec \psi_B$, respectively.

The clutter surface is assumed to be flat and contained within the reference plane through C . The effect of terrain shadowing, which can be significant at low grazing angles, is therefore neglected.

The time-delay difference Δ/c between the clutter return from the clutter cell at C and the direct signal from the transmitter to the receiver (see Figure 1) is given by

$$\Delta/c = (R - \rho_{AB})/c \tag{8}$$

where $R = \rho + r$ = range sum. The main lobe clutter resolution area A_c, centered at C and at a time-delay difference Δ/c within a resolution time δt, is the area of the Earth's surface which is within the field of view of the main beams of both the transmitter and receiver and which is bounded by two ellipsoids of revolution about the axis $\overline{AB}$ with foci at A and C, one ellipsoid with a range sum $R - (c\delta t/2)$ and the other ellipsoid with a range $R + (c\delta t/2)$. Subject to the grazing angle conditions of Eqs. (5) and (7) and a flat Earth approximation, the clutter resolution area A_c is closely approximated by the area A_c', in the plane ABC, which is within the field of view of the azimuthal main beamwidths of both the transmitter and receiver and which is bounded by the two ellipses of range sums $R + (c\delta t/2)$ and $R - (c\delta t/2)$ both with foci at A and B. Accordingly,

$$A_c \approx A_c' \text{ ; conditions of Equations (5) and (7), flat Earth approximation} \tag{9}$$

In the remainder of this section, the cell area A_c' is first determined for the two cases $\Delta/c \gg \delta t$ and $\Delta/c = 0$. The former case is applicable to all clutter cells except those in the vicinity of the forward-scattered direction or whose range resolution is large compared to the cell range. The latter case corresponds to the forward-scattered direction $\beta - \gamma = 180°$. This section concludes with an approximate determination (within 3 dB) of the area A_c' for arbitrary time delay difference Δ/c .

$\Delta/c >> \delta t$

The cell geometry in plane ABC is shown in Figures 2 and 3 for the case $\Delta/c >> \delta t$. In Figure 2, the cell area is limited by the receiver azimuthal beamwidth. The azimuthal angle subtended at the receiver by ellipse of range sum R (time-delay difference Δ/c) within the transmitter azimuthal beamwidth θ_A is designated η . The azimuthal angle subtended at the transmitter by the ellipse of range sum R within the receiver azimuthal beamwidth θ_B is designated χ . The transmitter beamwidth-limited case corresponds to the condition $\theta_A < \chi$ or equivalently $\theta_B \geq \eta$. The receiver beamwidth-limited case corresponds to the condition $\theta_B < \eta$ or equivalently $\theta_A \geq \chi$.

The condition $\Delta/c >> \delta t$ allows the following simplifications to be made concerning the computation of the cell area A_c' ; (1) The angle subtended by the cell at either the receiver or the transmitter is approximately the angle subtended by that portion of the cell on the ellipse of range sum R . (2) The cell range resolution in either the direction of the transmitter or the receiver is within

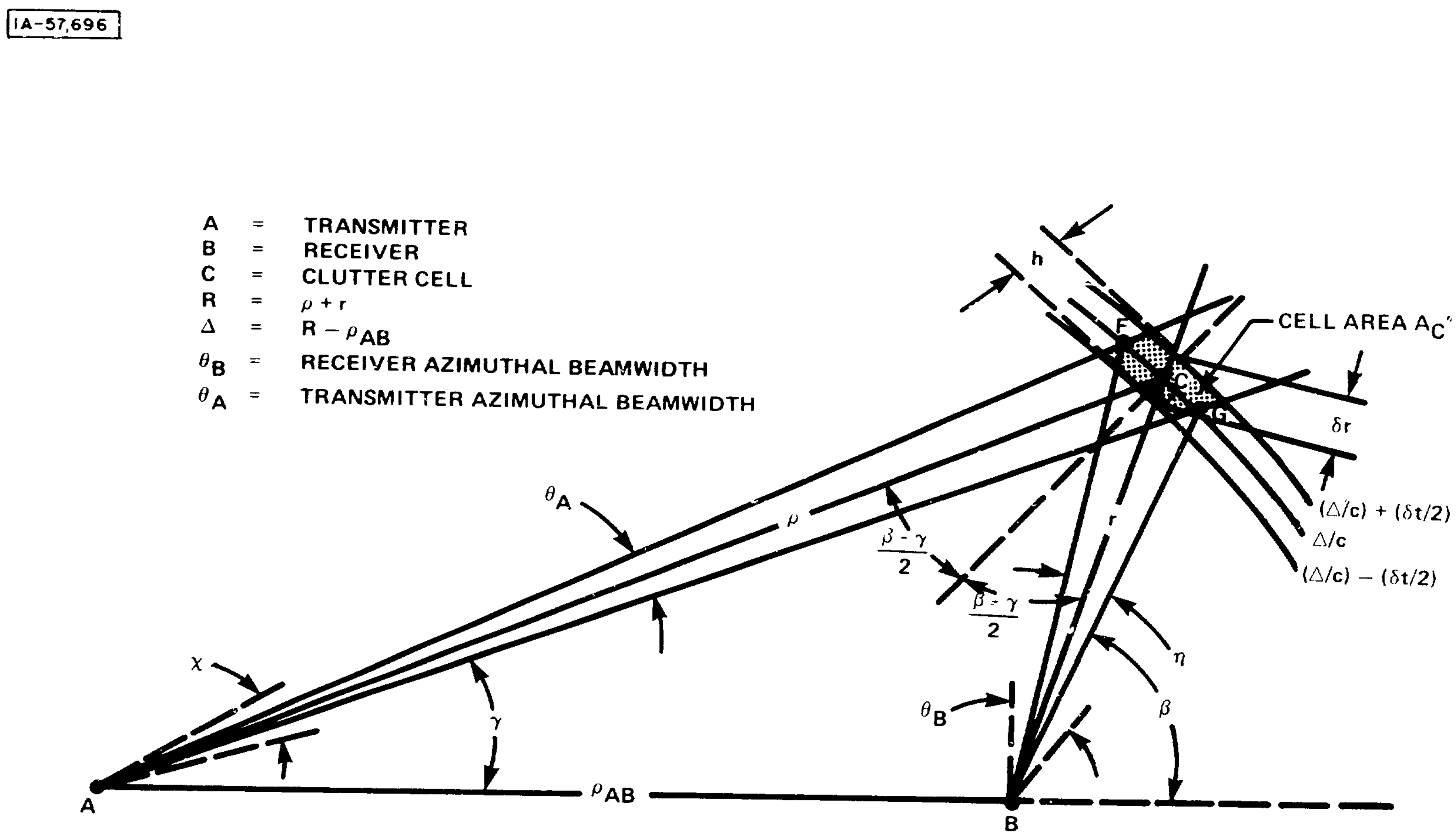

FIGURE 2. CELL AREA IN PLANE ABC, $\Delta/c \gg \delta t$

(TRANSMITTER BEAMWIDTH - LIMITED, $\theta_A < \chi$ OR $n \leqslant \theta_B$)

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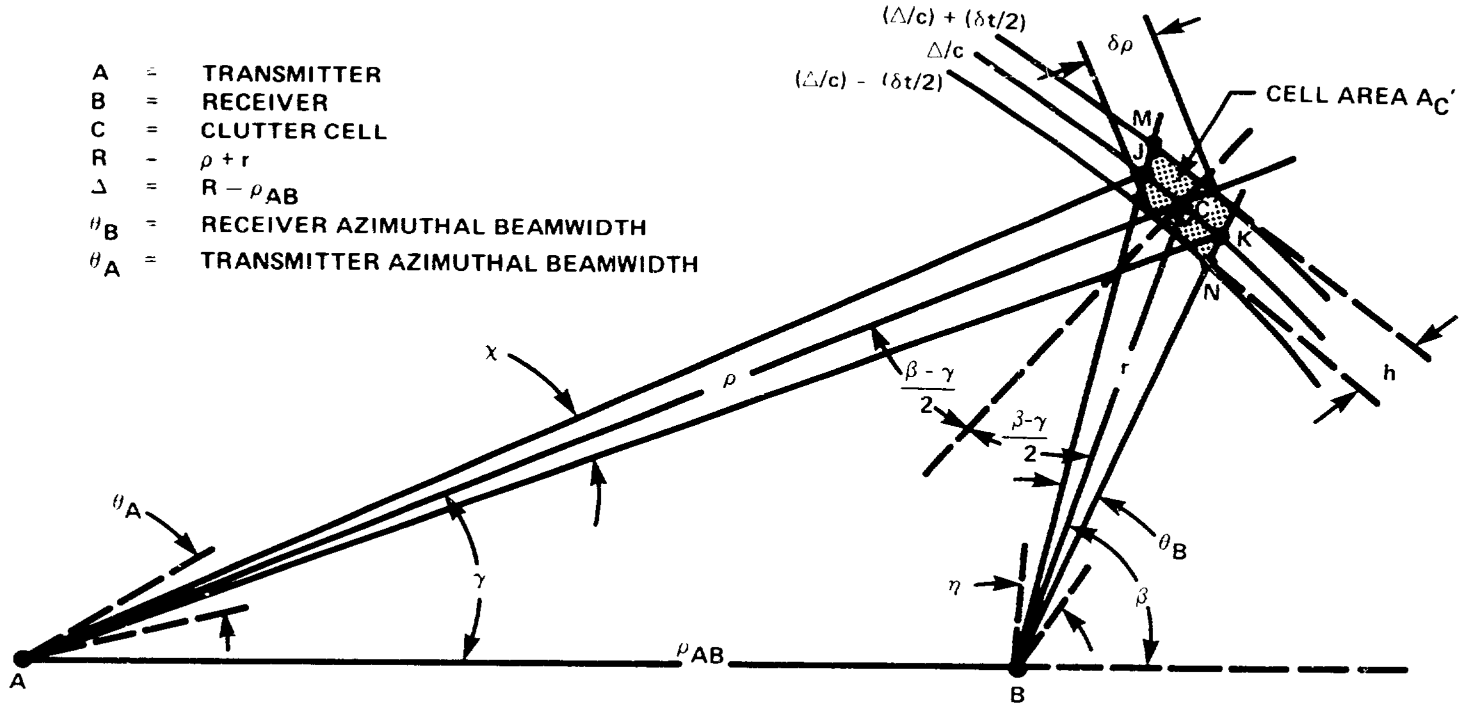

FIGURE 3. CELL AREA IN PLANE ABC, $\Delta/c \gg \delta t$

(RECEIVER BEAMWIDTH - LIMITED, $\theta_B < \eta$ OR $\chi \leqslant \theta_A$)

the fields of view of the transmitter and receiver. (3) The shape of the cell area A_c' is approximated by a parallelogram for moderate transmitter and receiver azimuthal beamwidths.

The parallelogram area A_c' is readily computed by utilizing the property of an ellipse that the bisector of the angle, subtended by the foci of the ellipse at an arbitrary point of the ellipse, is normal to the ellipse at that point. In Figures 2 and 3, it will be noted that the bisector of the bistatic angle $(\beta - \gamma)$ at point C is normal to the range-sum ellipse at C. In Figure 2, the area A_c' is given approximately by the parallelogram of altitude $h = \delta r \cos\left(\frac{\beta - \gamma}{2}\right)$ and base length $b = \overline{FG} = \overline{FC} + \overline{CG}$. In Figure 3, the area A_c' is given approximately by the parallelogram of altitude $h = \delta\rho \cos\left(\frac{\beta - \gamma}{2}\right)$ and base length $b = \overline{JK} = \overline{JC} + \overline{CK}$. Accordingly,

$$A_c' = bh = \begin{cases} [\overline{FG}] \cdot \left[\delta r \cos\left(\frac{\beta - \gamma}{2}\right)\right], & \eta \leq \theta_B \\ [\overline{JK}] \cdot \left[\delta\rho \cos\left(\frac{\beta - \gamma}{2}\right)\right], & \chi \leq \theta_A \end{cases}, \quad \Delta/c \gg \delta t \tag{10}$$

Applying the sine law to triangles AFC, ACG, BJC, and BCK, the lengths $\overline{FG}$ and $\overline{JK}$ are given by

$$\begin{aligned} \overline{FG} &= \frac{\rho \sin(\theta_A/2)}{\sin\left[\frac{\pi}{2} - \left(\frac{\theta_A}{2} - \frac{\beta - \gamma}{2}\right)\right]} + \frac{\rho \sin(\theta_A/2)}{\sin\left[\frac{\pi}{2} - \left(\frac{\theta_A}{2} + \frac{\beta - \gamma}{2}\right)\right]} \\ \overline{JK} &= \frac{r \sin(\theta_B/2)}{\sin\left[\frac{\pi}{2} - \left(\frac{\theta_B}{2} - \frac{\beta - \gamma}{2}\right)\right]} + \frac{r \sin(\theta_B/2)}{\sin\left[\frac{\pi}{2} - \left(\frac{\theta_B}{2} + \frac{\beta - \gamma}{2}\right)\right]} \end{aligned} \tag{11}$$

From Equation (A-30) of Appendix A, the range resolution δr and $\delta\rho$ for $\Delta/c >> \delta t$ are given by

$$\delta r = \left.\frac{\partial r}{\partial R}\right|_{\beta,\rho_{AB}} \cdot c\delta t = \left.\frac{\partial \rho}{\partial R}\right|_{\gamma,\rho_{AB}} \cdot c\delta t = \delta\rho$$

$$= \frac{c\delta t}{2}\sec^2\left(\frac{\beta-\gamma}{2}\right) \quad ; \Delta/c >> \delta t \tag{12}$$

From Equations (B-1) and (B-11) of Appendix B, the angle η is given by

$$\eta = 2\left\{\arctan\left[\left(1+\frac{2\rho_{AB}}{\Delta}\right)\tan\left(\frac{\gamma}{2}+\frac{\theta_A}{4}\right)\right] - \arctan\left[\left(1+\frac{2\rho_{AB}}{\Delta}\right)\tan\left(\frac{\gamma}{2}-\frac{\theta_A}{4}\right)\right]\right\}, \quad \eta \leq 180° \tag{13}$$

From Equations (B-23) and (B-26) of Appendix B, the angle χ is given by

$$\chi = 2\left\{\arctan\left[\left(1+\frac{2\rho_{AB}}{\Delta}\right)\tan\left(\frac{\pi-\beta}{2}+\frac{\theta_B}{4}\right)\right] - \arctan\left[\left(1+\frac{2\rho_{AB}}{\Delta}\right)\tan\left(\frac{\pi-\beta}{2}-\frac{\theta_B}{4}\right)\right]\right\}, \quad \chi \leq 180° \tag{14}$$

Substituting Equations (11)-(14) into Equation (10),

$$A_c' = \begin{cases} \dfrac{\rho\tan\left(\frac{\theta_A}{2}\right) c\delta t \sec^2\left(\frac{\beta-\gamma}{2}\right)}{1-\tan^2\left(\frac{\theta_A}{2}\right)\tan^2\left(\frac{\beta-\gamma}{2}\right)}, & \eta \leq \theta_B \\[2ex] \dfrac{r\tan\left(\frac{\theta_B}{2}\right) c\delta t \sec^2\left(\frac{\beta-\gamma}{2}\right)}{1-\tan^2\left(\frac{\theta_B}{2}\right)\tan^2\left(\frac{\beta-\gamma}{2}\right)}, & \chi \leq \theta_A \end{cases} \quad, \Delta/c >> \delta t \tag{15}$$

where η and χ are given by Equations (13) and (14).

Consider now the case when the transmitter and receiver azimuthal beamwidths are both very small angles. For $\theta_A << 1$ rad and $\theta_B << 1$ rad, the conditions of Equation (15) imply that η and χ are both small angles. From Equation (B-16) of Appendix B,

$$\eta \approx \theta_A \frac{\sin \beta}{\sin \gamma} = \theta_A \, \rho/r \quad ; \frac{\eta}{4} << 1 \text{ rad} \ , \frac{\theta_A}{4} << 1 \text{ rad} \qquad (16)$$

From Equation (B-29) of Appendix B,

$$\chi \approx \theta_B \frac{\sin \gamma}{\sin \beta} = \theta_B \, r/\rho \quad ; \frac{\chi}{4} << 1 \text{ rad} \ , \frac{\theta_B}{4} << 1 \text{ rad} \qquad (17)$$

For $\theta_A << 2$ rad and $\theta_B << 2$ rad, Equation (15) reduces to

$$A_c' = \begin{cases} \rho\theta_A \dfrac{c\delta t}{2} \sec^2\left(\dfrac{\beta - \gamma}{2}\right) , & \theta_A \leq \theta_B \dfrac{\sin \gamma}{\sin \beta} << 2 \text{ rad} \\ r\theta_B \dfrac{c\delta t}{2} \sec^2\left(\dfrac{\beta - \gamma}{2}\right) , & \theta_B \leq \theta_A \dfrac{\sin \beta}{\sin \gamma} << 2 \text{ rad} \end{cases} \quad ; \Delta/c >> \delta t \qquad (18)$$

In Equation (18), the condition $\tan^2\left(\frac{\theta_A}{2}\right) \tan^2\left(\frac{\beta - \gamma}{2}\right) << 1$ is implied by the condition $\Delta/c >> \delta t$ and $\theta_A << 2$ rad .

For $\Delta/c >> \delta t$, the clutter resolution area is given by either Equation (15) or Equation (18) provided that the additional conditions of Equation (9) are satisfied. It is clear from Equation (18) that for $\Delta/c >> \delta t$ the clutter resolution area increases with increasing bistatic angle and is proportional to the $(\text{secant})^2$ of one-half the bistatic angle.

$\underline{\Delta/c = 0}$

A clutter return at a time-delay difference $\Delta/c = 0$, corresponding to a bistatic angle $\beta - \gamma = 180°$ (see Figure 1), implies that the clutter cell location C is centered on the midpoint of the line $\overline{AB}$ and is distributed among all the points within the ellipsoid of range-sum $\rho_{AB} + c\delta t$ and with foci at A and B . Furthermore, the case $\Delta/c = 0$ implies that the transmitter and receiver are located in the reference plane because $\overline{AB}$ contains C which is in the reference plane. For $\Delta/c = 0$, the clutter cell is of large extent and, therefore, may not necessarily be locally flat on the Earth's spherical surface. The clutter resolution area A_c on the Earth's surface may be approximated by the cell area A_c' in the reference plane if $\overline{AB}$ is not appreciably larger than the chord $\overline{A''B''}$ between the points of intersection of the spherical Earth's surface with the ellipsoid of range-sum $\rho_{AB} + c\delta t$. Accordingly,

$$A_c \approx A_c' \; ; \qquad (\overline{AB} - \overline{A''B''})/\overline{AB} << 1, \; \Delta/c = 0 \tag{19}$$

The concept of clutter cell area requires discretion in its use for the case $\Delta/c = 0$. For this case, the cell dimension along $\overline{AB}$ is comparable to the range to the center of the cell and, therefore, violates the condition of Equation (2) that the ρr product is approximately constant within the cell. Nevertheless, the cell area for $\Delta/c = 0$, relative to that for other bistatic angles, may be estimated by the cell area concept of Equation (2).

The cell geometry in the reference plane is shown in Figure 4 for the case $\Delta/c = 0$. In Figure 4(a), the cell area is limited by the range-sum ellipse $\rho_{AB} + c\delta t$ and the transmitter and receiver azimuthal beamwidths. In Figure 4(b), the cell area is limited only by the azimuthal beamwidths. $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ are points of intersection of the range-sum ellipse $\rho_{AB} + c\delta t$ with the main beam extremal rays of the transmitter and receiver, respectively. The range-sum and beamwidth-limited cases correspond to $x_1 < x_2$ and $x \geq x_2$, respectively. $P_0(0, b)$ is the point of intersection of the range-sum ellipse $\rho_{AB} + c\delta t$ with the positive y-axis.

The range-sum ellipse $\rho_{AB} + c\delta t$, in x and y coordinates is given by

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1 \tag{20}$$

where

a = semi-axis along x axis = $(\rho_{AB} + c\delta t)/2$

b = semi-axis along y axis = $\overline{OP}_0 = [(\overline{AP}_0)^2 - (\overline{AO})^2]^{\frac{1}{2}}$

$$= \left[\left(\frac{c\delta t + \rho_{AB}}{2}\right)^2 - \left(\frac{\rho_{AB}}{2}\right)^2\right]^{\frac{1}{2}} = \frac{1}{2}[(c\delta t)^2 + 2c\delta t\ \rho_{AB}]^{\frac{1}{2}}$$

The points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ are specified by

$$y_1 = [x_1 + (\rho_{AB}/2)] \tan(\theta_A/2) \tag{21a}$$

$$y_2 = [(\rho_{AB}/2) - x_2] \tan(\theta_A/2) \tag{21b}$$

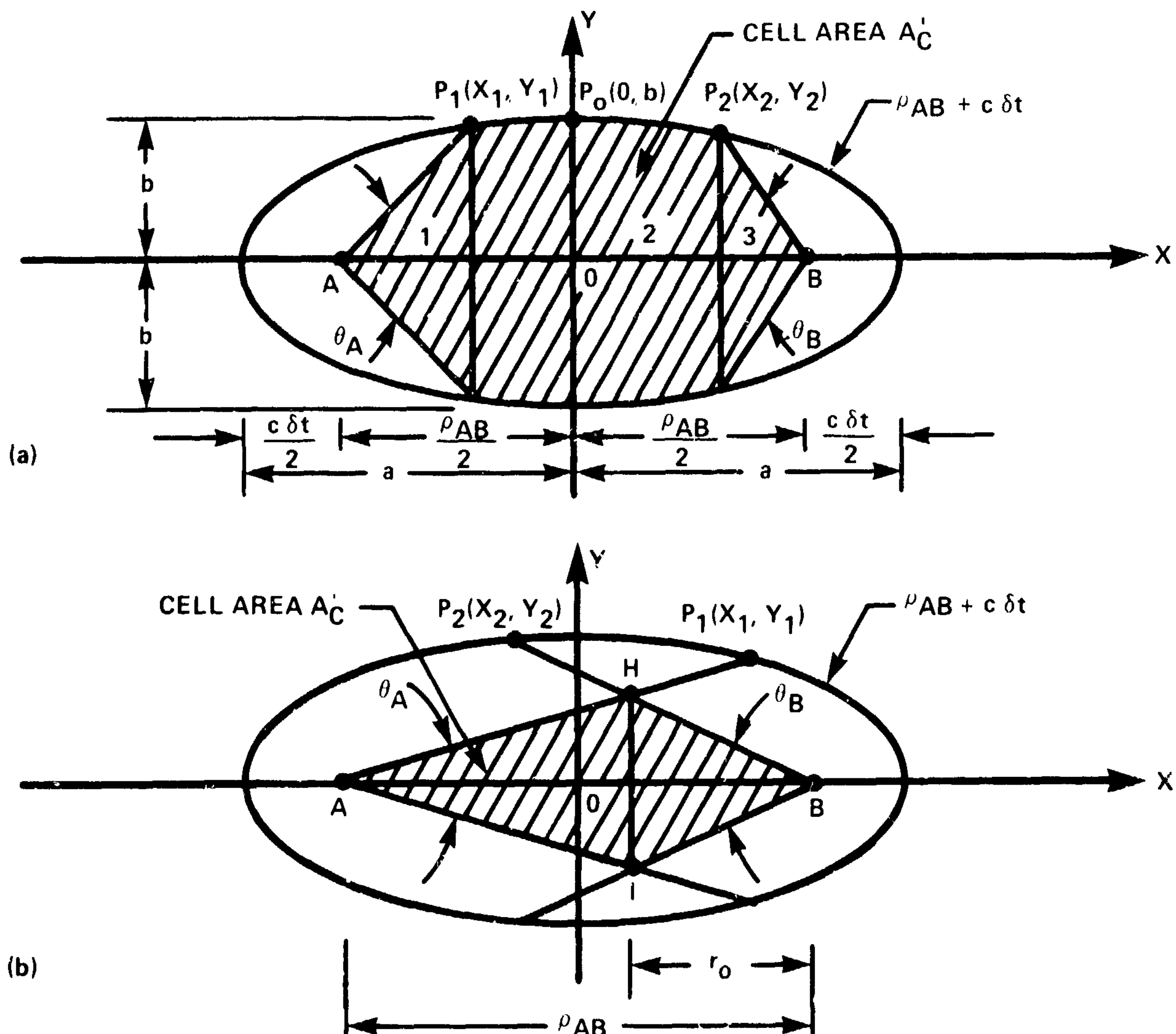

FIGURE 4. CELL AREA IN REFERENCE PLANE, $\Delta/c = 0$
(a) RESOLUTION TIME LIMITED ($X_1 < X_2$)
(b) BEAMWIDTH LIMITED ($X_1 > X_2$)

where x_1 and x_2 are found by substitution of Eqs. (21a) and (21b), respectively. Accordingly,

$$x_1 = -\frac{\rho_{AB}}{2K_A} \pm \frac{1}{2}\left[\left(\frac{\rho_{AB}}{K_A}\right)^2 - \frac{\rho_{AB}^2 - 4b^2\cot^2\left(\frac{\theta_A}{2}\right)}{K_A}\right]^{\frac{1}{2}} , \quad |x_1| \leq a$$

$$x_2 = \frac{\rho_{AB}}{2K_B} \pm \frac{1}{2}\left[\left(\frac{\rho_A}{K_B}\right)^2 - \frac{\rho_{AB}^2 - 4b^2\cot^2\left(\frac{\theta_B}{2}\right)}{K_B}\right]^{\frac{1}{2}} , \quad |x_2| \leq a \tag{22}$$

where

$$K_A = 1 + \left(\frac{b}{a}\right)^2 \cot^2\left(\frac{\theta_A}{2}\right)$$

$$K_B = 1 + \left(\frac{b}{a}\right)^2 \cot^2\left(\frac{\theta_B}{2}\right)$$

For $x_1 < x_2$, the clutter cell area A_c' is the sum of three areas whose composite sum is approximately hexagonal in shape with an area A_c' given by

$$A_c' = A_1 + A_2 + A_3 \ ; \quad \Delta/c = 0, \quad x_1 < x_2 \tag{23}$$

where

$$A_1 = \left(\frac{\rho_{AB}}{2} + x_1\right)^2 \tan(\theta_A/2)$$

$$A_2 = \int_{x_1}^{x_2} y\,dx = b\int_{x_1}^{x_2} [1 - (x/a)^2]\,dx$$

$$= \frac{bx_2}{2}\left[1 - \left(\frac{x_2}{a}\right)^2\right]^{\frac{1}{2}} - \frac{bx_1}{2}\left[1 - \left(\frac{x_1}{a}\right)^2\right]^{\frac{1}{2}} + \frac{ab}{2}\left[\sin^{-1}\left(\frac{x_2}{a}\right) - \sin^{-1}\left(\frac{x_1}{a}\right)\right]$$

$$A_3 = \left(\frac{\rho_{AB}}{2} - x_2\right)^2 \tan\left(\frac{\theta_B}{2}\right)$$

For $x_1 \geq x_2$, the clutter cell area is a rhomboid of length ρ_{AB} and width $\overline{HI}$ whose area A_c' is given by

$$A_c' = \frac{\rho_{AB}\ \overline{HI}}{2} = \frac{\rho_{AB}^2 \tan\left(\frac{\theta_A}{2}\right) \tan\left(\frac{\theta_B}{2}\right)}{\tan\left(\frac{\theta_B}{2}\right) + \tan\left(\frac{\theta_A}{2}\right)} \quad ;$$

$$\Delta/c = 0,\ x_1 \geq x_2 \qquad (24)$$

where

$$\overline{HI} = 2(\rho_{AB} - r_o) \tan\left(\frac{\theta_A}{2}\right) = 2r_o \tan\left(\frac{\theta_B}{2}\right)$$

$$r_o = \frac{\rho_{AB} \tan(\theta_A/2)}{\tan(\theta_A/2) + \tan(\theta_B/2)}$$

For $\Delta/c = 0$, corresponding to a bistatic angle $\beta - \gamma = 180°$, and for the conditions of Equation (19), the clutter resolution area A_c is given by Equations (23) and (24). The clutter resolution area is a maximum for $\beta - \gamma = 180°$ but is not infinite as might be inferred from extrapolation of Equation (15). For $\Delta/c = 0$, the cell area A_c is limited by the distance AB and the azimuthal beamwidths θ_A and θ_B.

<u>Arbitrary Δ/c</u>

For arbitrary Δ/c, the cell area in plane ABC may be determined, within an error of approximately 3 dB, by utilizing a quasi-parallelogram formula for bistatic angles $0 \leq \beta - \gamma < \pi - (\theta_A/2)$ rad and a quasi-rhomboidal formula for bistatic angles $\pi - (\theta_A/2) \leq \beta - \gamma < \pi$ rad.

The cell geometry for the case $0 \leq \beta - \gamma \leq \pi - (\theta_A/2)$ is shown in Figure 5. Figure 5 is the same as Figure 2 except that the segment $\overline{DE}$ and the angle η_{max} are defined.

The segment $\overline{DE}$ is the distance along $\overline{BC}$ which is illuminated by the transmitter and is determined by the intersection of the extremal rays of the transmitter main beam with the extended line from B through C. From Eq. (A-20) of Appendix A, $\overline{DE}$ is given by

$$DE = \begin{cases} \dfrac{\rho \sin \theta_A \sin(\beta - \gamma)}{\sin^2(\beta - \gamma) - \sin^2(\theta_A/2)} \; ; \; \dfrac{\theta_A}{2} < \beta - \gamma < \pi - \dfrac{\theta_A}{2} & \text{(a)} \\ \infty \; ; \quad 0 \leq \beta - \gamma \leq \dfrac{\theta_A}{2} \, , \; \pi - \dfrac{\theta_A}{2} \leq \beta - \gamma \leq \pi & \text{(b)} \end{cases} \tag{25}$$

For arbitrary Δ/c, the range resolution δr along r may be limited by either $\left.\frac{\partial r}{\partial R}\right|_{\beta,\rho_{AB}} c\delta t$ given by Equation (12), $\overline{DE}$, or by the separation ρ_{AB} between the transmitter or receiver. The range resolution δr is the least of these quantities. Accordingly,

$$\delta r = \begin{cases} \text{lesser of } \left.\frac{\partial r}{\partial R}\right|_{\beta,\rho_{AB}} c\delta t \text{ or } \overline{DE} \; ; \; 0 \leq \beta - \gamma < \pi - \dfrac{\theta_A}{2} & \text{(a)} \\ & ; \; \Delta/c \text{ arbitrary} \\ \rho_{AB} \; , \quad \pi - \dfrac{\theta_A}{2} \leq \beta - \gamma \leq \pi & \text{(b)} \end{cases} \tag{26}$$

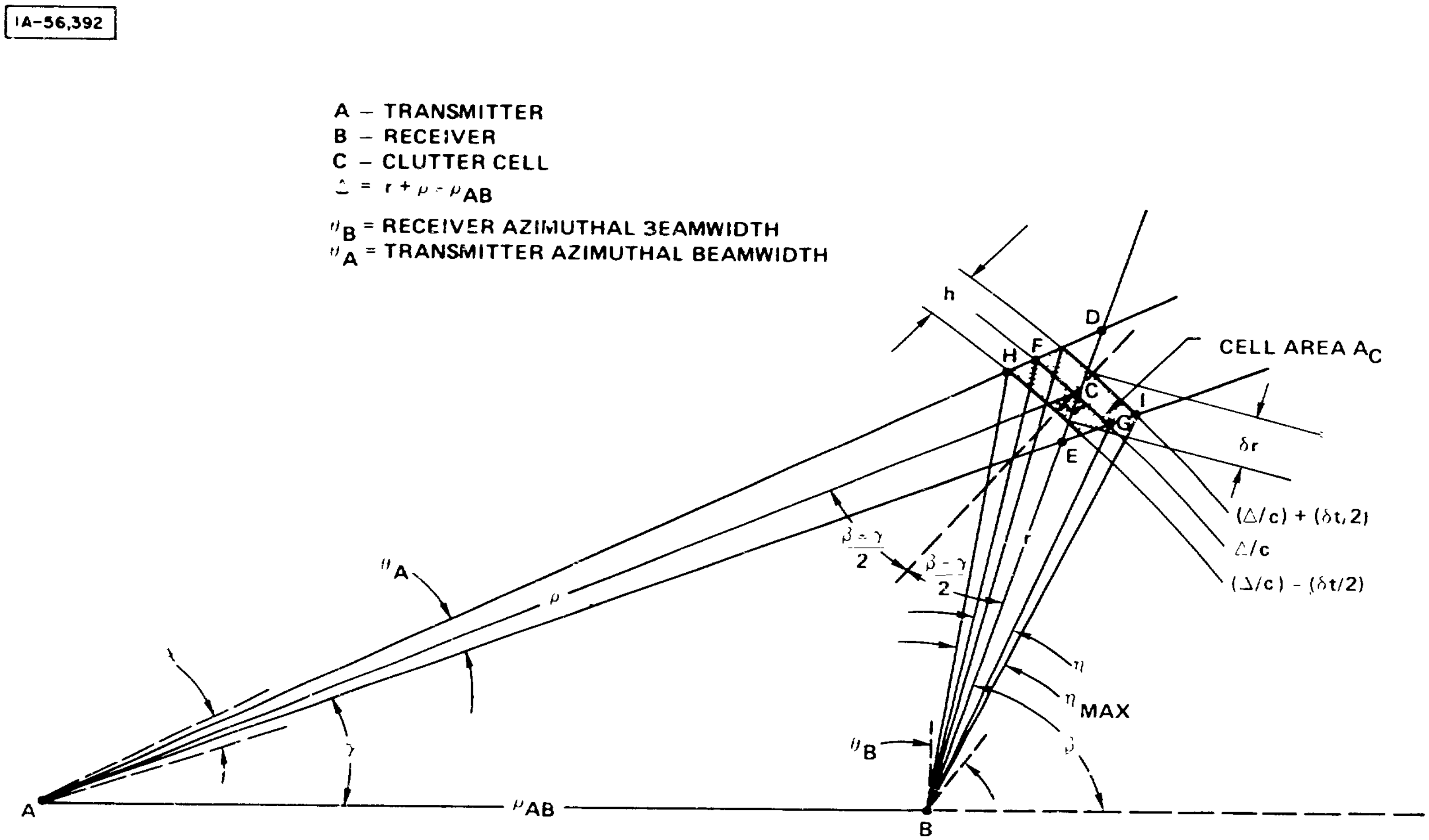

FIGURE 5. CELL AREA IN PLANE ABC, $0 \leqslant \beta - \gamma < \pi - (\theta_A/2)$ rad

The angle η_{max} is the angle subtended at the receiver by the entire clutter cell determined by the intersection H of the transmitter extremal ray $\overline{AD}$ with the range-sum ellipse $R - (c\delta t/2)$ and by the intersection I of the transmitter extremal ray $\overline{AE}$ with the range-sum ellipse $R + (\delta t/2)$. From Equation (B-19) of Appendix B, η_{max} is given by

$$\eta_{max} = 2\left\{\arctan\left[\left(1 + \frac{2\rho_{AB}}{|\Delta - (c\delta t/2)|}\right)\tan\left(\frac{\delta}{2} + \frac{\theta_A}{4}\right)\right] - \arctan\left[\left(1 + \frac{2\rho_{AB}}{\Delta + c\delta t/2}\right)\tan\left(\frac{\gamma}{2} - \frac{\theta_A}{4}\right)\right]\right\}, \quad \eta_{max} \leq 180° \tag{27}$$

For the case $\Delta/c >> \delta t$, $\eta_{max} \approx \eta$ where η is given by Eq. (13). However, for $\Delta/c \lesssim \delta t$ at large bistatic angles, $\eta << \eta_{max}$. For example, for $\Delta/c = 0$, $\eta = 0$ whereas $\eta_{max} = \tan^{-1}[y_1/(\rho_{AB} - x_1)]$ where y_1 and x_1 are given by Equations (21a) and (22a), respectively. Therefore, for arbitrary Δ/c, the cell area is more accurately characterized by the angle η_{max} than by η.

For $0 \leq \beta - \gamma \leq \pi - (\theta_A/2)$, we therefore approximate the cell area of Figure 5 by a quasi-parallelogram of altitude h and base $\overline{HI}$ whose area is given approximately by

$$A_c' \approx h\,\overline{HI} = \frac{2r\tan\left(\frac{\theta_W}{2}\right)}{1 + \tan\left(\frac{\theta_W}{2}\right)\tan\left(\frac{\beta - \gamma}{2}\right)} \times \Delta r\ , \quad 0 \leq \beta - \gamma \leq \pi - (\theta_A/2) \tag{28}$$

where

$$h = \delta r \cos\left(\frac{\beta - \gamma}{2}\right)$$

$$\overline{HI} = 2 \frac{\overline{CI}}{\sin\left(\frac{\theta_W}{2}\right)} = \frac{2r}{\sin\left[\frac{\pi}{2} - \left(\frac{\theta_W}{2} - \frac{\beta - \gamma}{2}\right)\right]}$$

$$= \frac{2r}{\cos\left(\frac{\theta_W}{2}\right) \cos\left(\frac{\beta - \gamma}{2}\right) + \sin\left(\frac{\theta_B}{2}\right) \sin\left(\frac{\beta - \gamma}{2}\right)}$$

$$\delta r = \text{lesser of } \left.\frac{\partial r}{\partial R}\right|_{\beta,\rho_{AB}} \cdot c\delta t = \frac{c\delta t}{2} \sec^2\left(\frac{\beta - \gamma}{2}\right) \text{ or } \overline{DE}$$

$$\theta_W = \text{lesser of } \theta_B \text{ or } \eta_{max}$$

For $\Delta/c \gg \delta t$, the cell is a parallelogram and Equation (28) reduces to the exact result given by Equation (15).

The cell geometry for the case $\pi - (\theta_A/2) \leq \beta - \gamma \leq \pi$ rad is shown in Figure 6. The clutter cell (shown shaded) may be approximated by the quasi-rhomboid AHBI of length ρ_{AB} and width $\overline{HI}$. In Figure 6, the center C of the clutter cell is not necessarily on the segment $\overline{AB}$ unless $\Delta/c = 0$, corresponding to $\beta - \gamma = 180°$, in which case AHBI is a rhomboid. The quasi-rhomboidal cell area A_c' is given by

$$A_c' = \frac{\rho_{AB}\,\overline{HI}}{2} = \frac{\rho_{AB}^2 \tan(\theta_A/2)\tan(\theta_W/2)}{\tan(\theta_A/2) + \tan(\theta_W/2)}, \quad \pi - \frac{\theta_A}{2} \leq \beta - \gamma \leq \pi \qquad (29)$$

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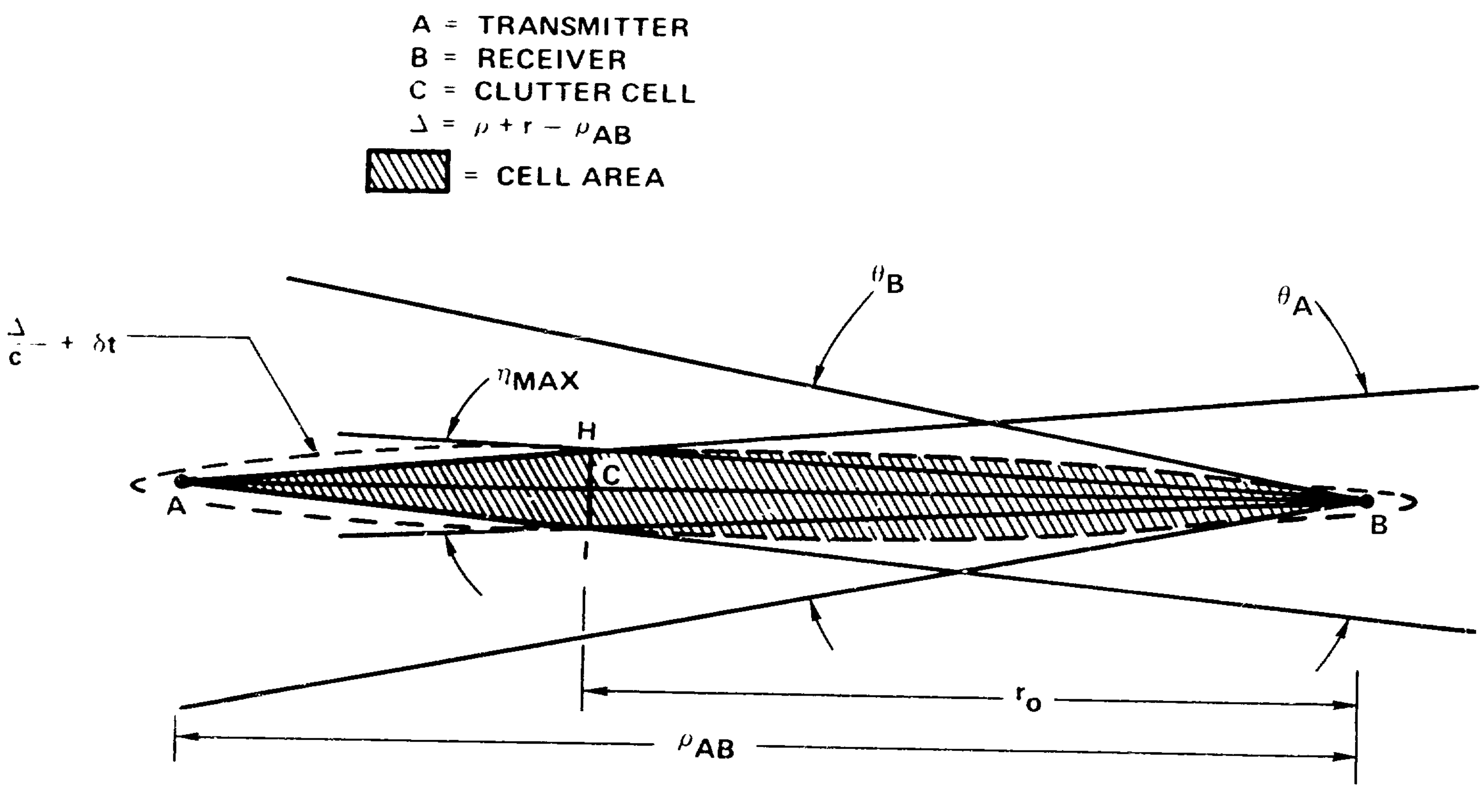

FIGURE 6. CELL AREA IN PLANE ABC, $\pi - (\theta_A/2) \leqslant \beta - \gamma \leqslant \pi$ rad

where

$$\overline{HI} = 2(\rho_{AB} - r_o)\tan(\theta_A/2) = 2r_o \tan(\theta_w/2)$$

$$r_o = \frac{\rho_{AB}\tan(\theta_A/2)}{\tan(\theta_w/2) + \tan(\theta_A/2)}$$

$$\theta_w = \text{lessor of } \theta_B \text{ or } \eta_{max} .$$

For $\Delta/c = 0$ and $\eta_{max} \leq \theta_B$, the cell is a rhomboid and Eq. (29) reduces to the exact result given by Equation (24). However, for $\Delta/c = 0$ and $\eta_{max} > \theta_B$, the cell is hexagonal-shaped so that Eq. (29) does not reduce to the exact result given by Equation (23).

The cell area A_c' for arbitrary Δ/c, is approximated by the quasi-parallelogram formula of Eq. (28) for $0 \leq \beta - \gamma < \pi - (\theta_A/2)$ rad and by the quasi-rhomboidal formula of Eq. (29) for $\pi - (\theta_A/2) \leq \beta - \gamma < \pi$ rad. The quasi-parallelogram formula is exact for $\Delta/c >> \delta t$ for which case the cell shape is a parallelogram. For small time-delay differences, the cell shape is trapezoidal or triangular at small bistatic angles and rhomboidal or hexagonal at large bistatic angles. The quadrilateral formulae of Eqs. (28) and (29) appear to have their poorest approximation when the cell shape is triangular (corresponding to $0 \leq \Delta/c = 2r/c \leq \delta t/2$ for $\beta - \gamma = 0$) in which case the quasi-parallelogram formula is too large by 3 dB.

SECTION III

NUMERICAL RESULTS

Although the cell area is a function of the path difference Δ and the bistatic angle $\beta - \gamma$, the parameters Δ and γ can be expressed as functions of r, β, and ρ_{AB}. Numerical results for the cell area may therefore be obtained as a function of the receiver range r and the azimuthal angle β for fixed values of the parameters θ_A, θ_B, and $c\delta t/\rho_{AB}$.

The normalized path difference, Δ/ρ_{AB}, between the indirect signal from the clutter cell and the direct signal from the transmitter to the receiver is given by Equation (A-11) of Appendix A. The parameter $\Delta/2\rho_{AB}$ (note the factor of one-half) is plotted in Figure 7 as a function of the normalized range r/ρ_{AB} and the azimuthal angle β. In the backscattered direction ($\beta = 0$), $\Delta/2\rho_{AB} = r/\rho_{AB}$ whereas in the forward-scattered direction ($\beta = 180°$), $\Delta/2\rho_{AB} = 0$.

The angle γ, subtended at the transmitter by the lines joining it to the clutter cell and receiver, is given by Equation (B-22) of Appendix B and is plotted in Figures 8 and 9. The angle γ is less than 10° for $r/\rho_{AB} \leq 0.1$ and less than 40° for $r/\rho_{AB} \leq 0.6$ at any azimuthal angle β.

The bistatic range resolutions, δr and $\delta\rho$, along the lines joining the clutter cell to the transmitter or receiver, respectively, IA-56,395

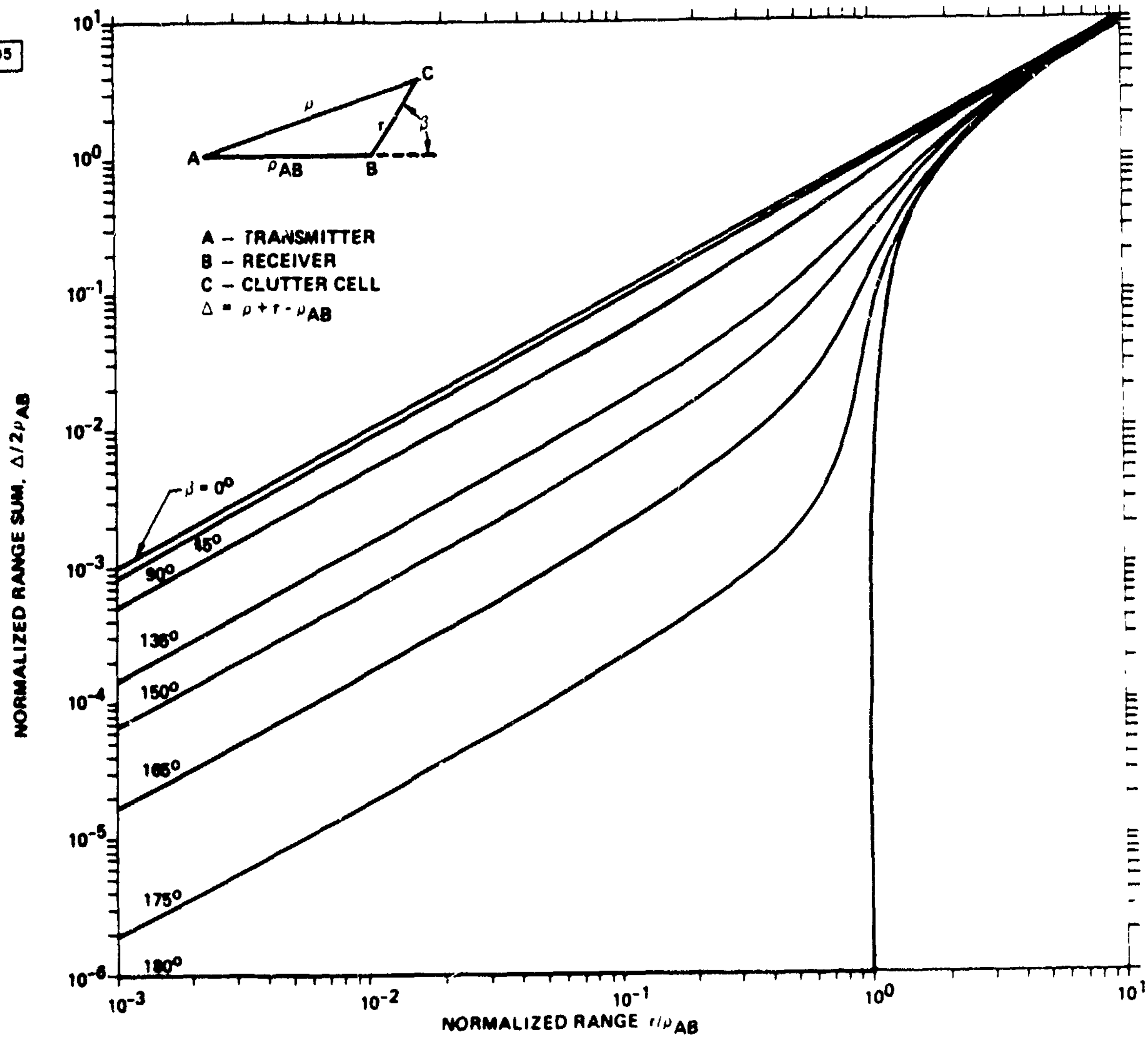

FIGURE 7. PATH DIFFERENCE BETWEEN INDIRECT AND DIRECT SIGNALS

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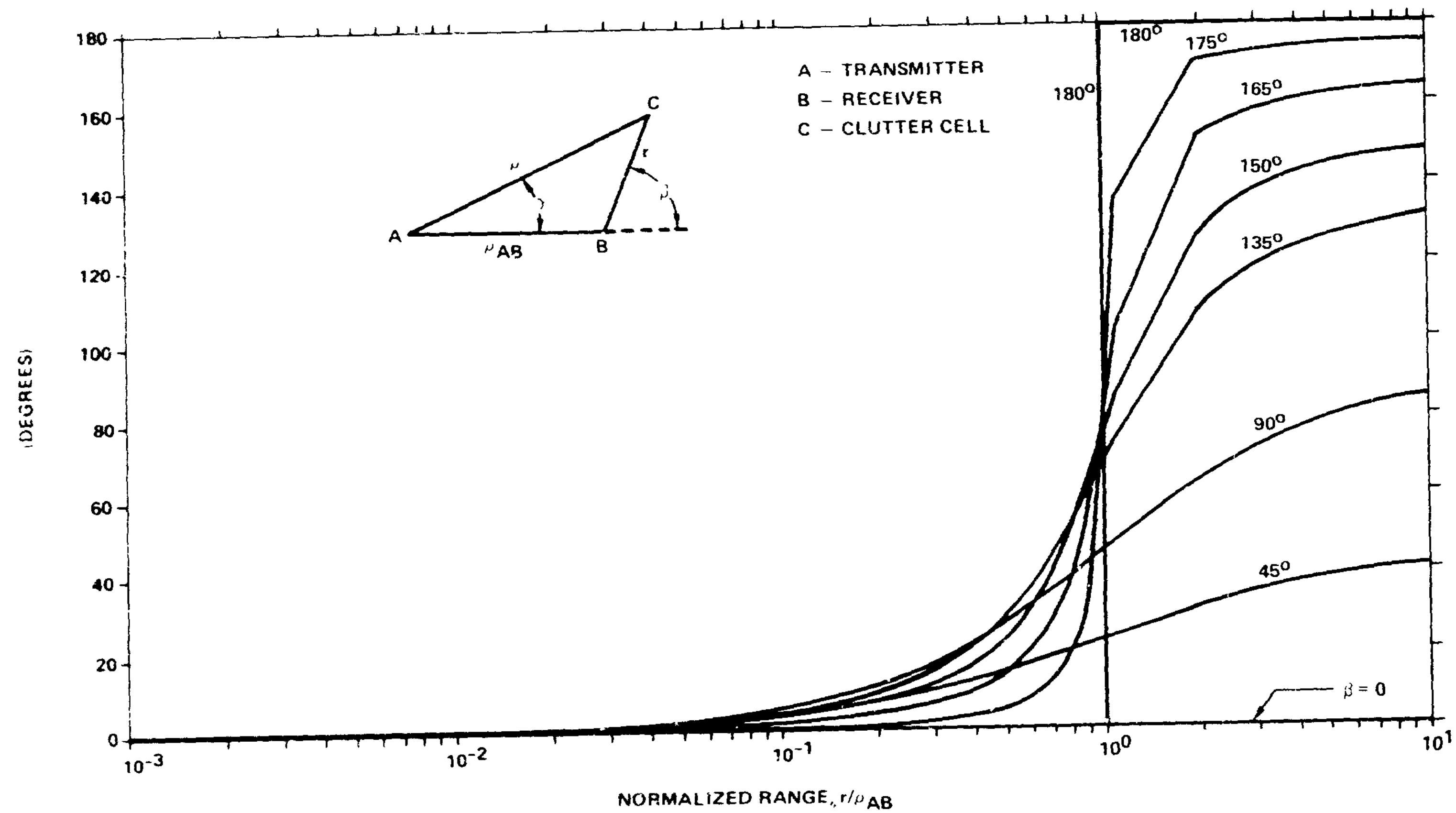

FIGURE 8. ANGLE γ SUBTENDED AT TRANSMITTER BY CLUTTER CELL AND RECEIVER

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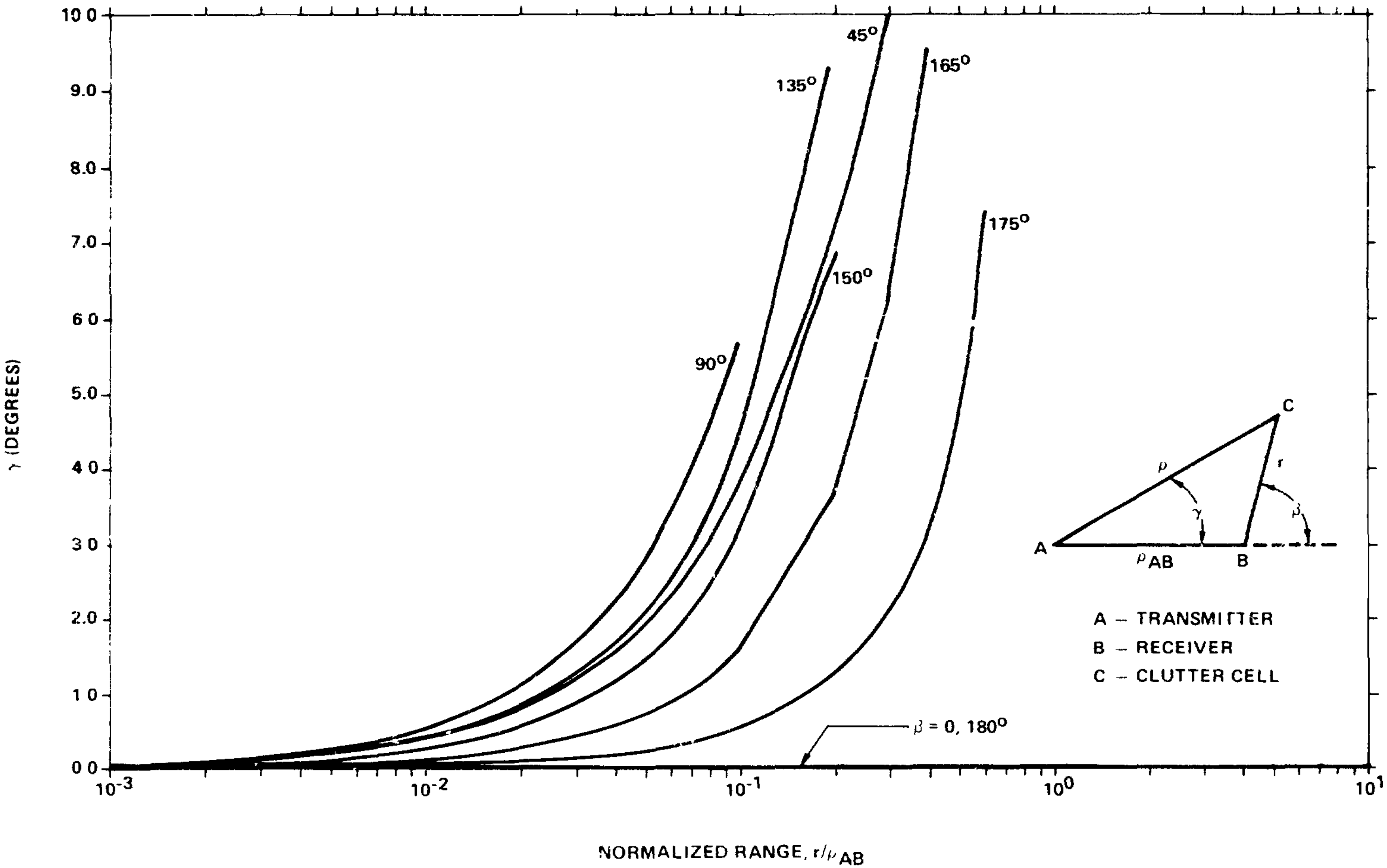

FIGURE 9. ANGLE γ ON AN EXPANDED SCALE, $\gamma \leqslant 10°$

and normalized with respect to the monostatic range resolution $c\delta t/2$, are given by Eq. (A-30) of Appendix A for the condition $\Delta/c >> \delta t$. The range resolutions δr and $\delta\rho$ are equal and are plotted in Figure 10. Although the range-sum resolution $\delta R = c\delta t$ for both monostatic and bistatic radars, the bistatic range resolutions $\delta r = \delta\rho$ can be quite different from that for monostatic radar. For monostatic radar, the range resolution is independent of the range and azimuth whereas for bistatic radar the range resolutions $\delta r = \delta\rho$ decrease with increasing range r and increase with increasing azimuthal angle β. The bistatic range resolutions δr and $\delta\rho$ are larger than the monostatic range resolution by a factor of approximately two for $0 \leq \beta \leq 90°$ and by much larger factors at the larger bistatic angles. It should be noted that the range resolution given by Eq. (A-30) is not valid for bistatic angles approaching 180° [see Equation (26)] because the condition $\Delta/c >> \delta t$ is not satisfied at very large bistatic angles.

Some examples, of numerical results for the normalized cell area A_c'/ρ_{AB}^2, are plotted in Figures 11, 12, and 13 as a function of the clutter cell location at a normalized receiver range r/ρ_{AB} and azimuthal angle β for a transmitter beamwidth $\theta_A = 1°$, receiver beam width $\theta_B = 3°$, and $c\delta t/\rho_{AB} = 1.6 \times 10^{-3}$, 1.6×10^{-4}, and and 1.6×10^{-5}, respectively. The quasi-parallelogram and quasi-rhomboidal formulae of Eqs. (28) and (29), respectively were utilized in plotting Figures 11-13. Two examples of the receiver resolution

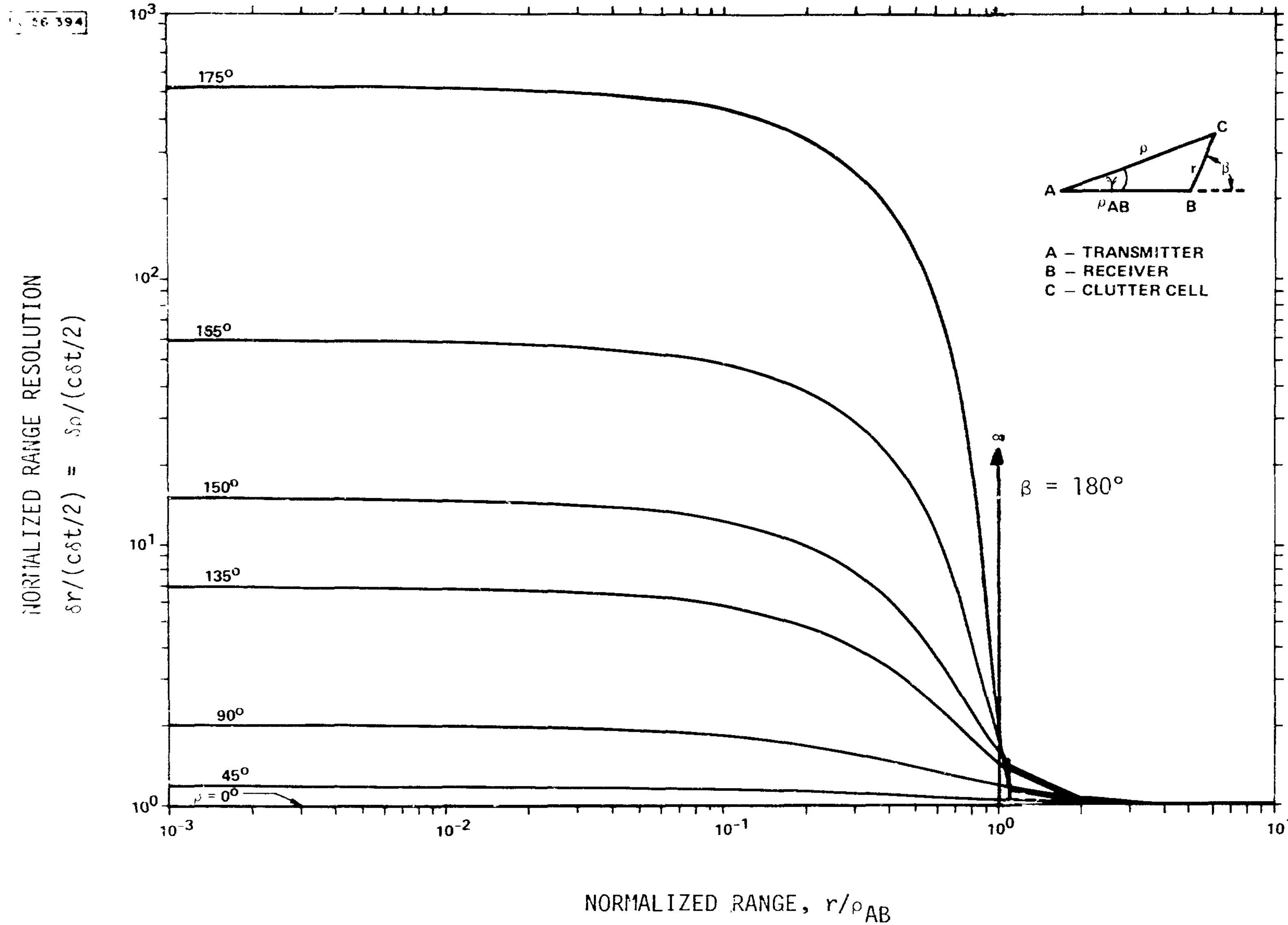

FIGURE 10. BISTATIC RANGE RESOLUTION ALONG $\overline{BC}$ OR $\overline{AC}$, $\Delta/c \gg \delta t$

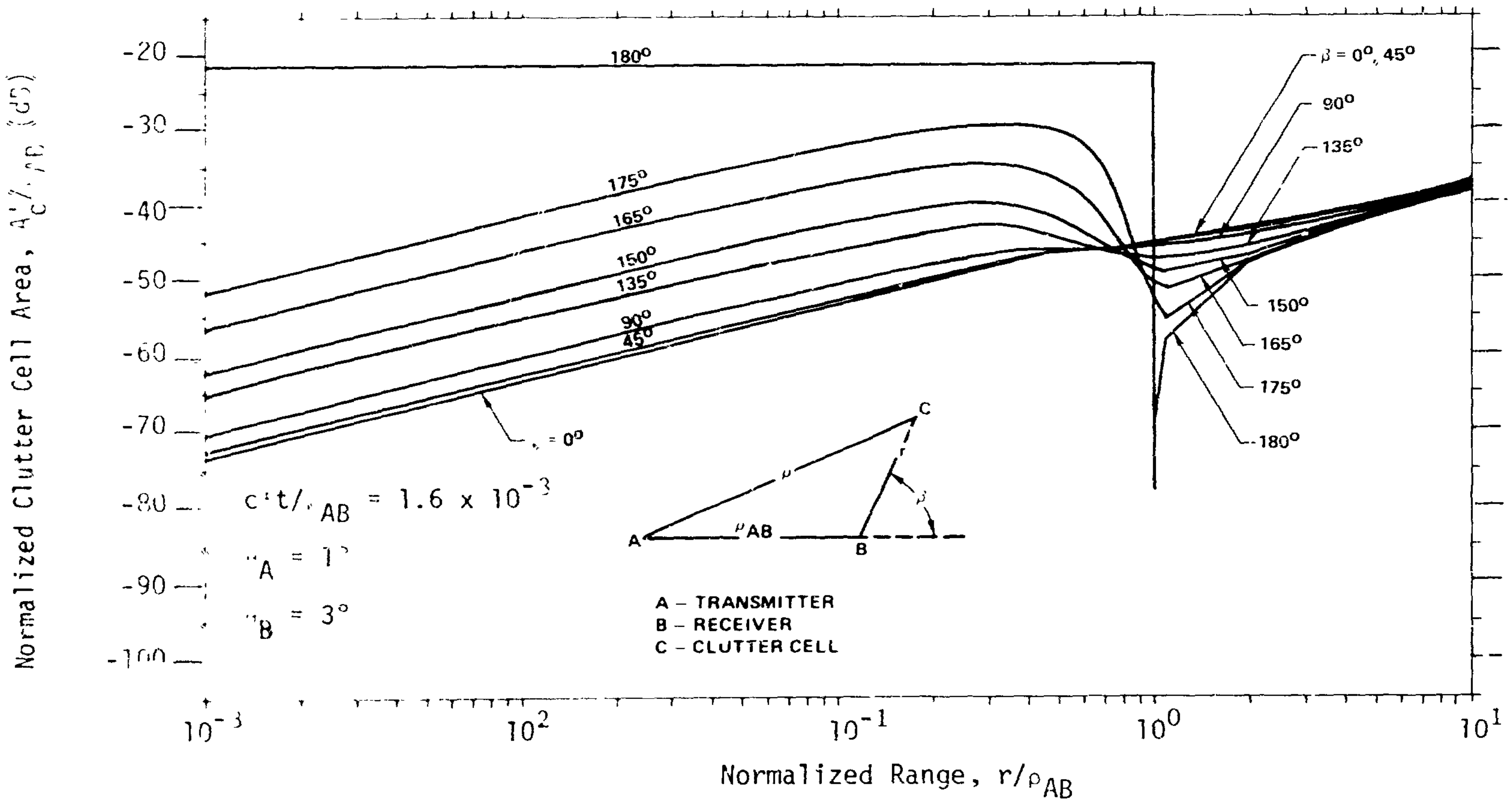

FIGURE 11. CLUTTER CELL AREA, NUMERICAL RESULTS FOR $c\delta t/\rho_{AB} = 1.6 \times 10^{-3}$

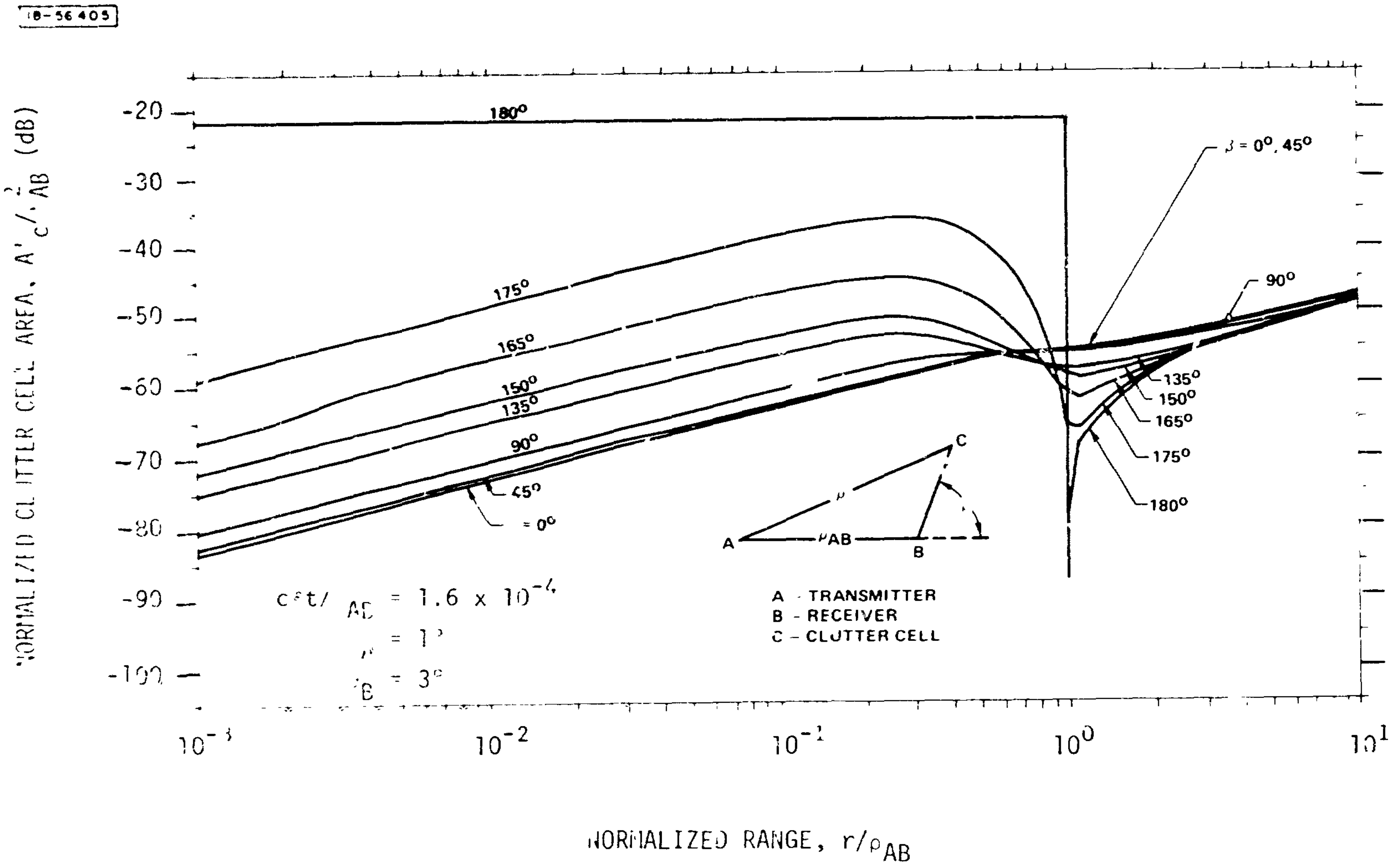

FIGURE 12. CLUTTER CELL AREA, NUMERICAL RESULTS FOR $c\delta t/\rho_{AB} = 1.6 \times 10^{-4}$

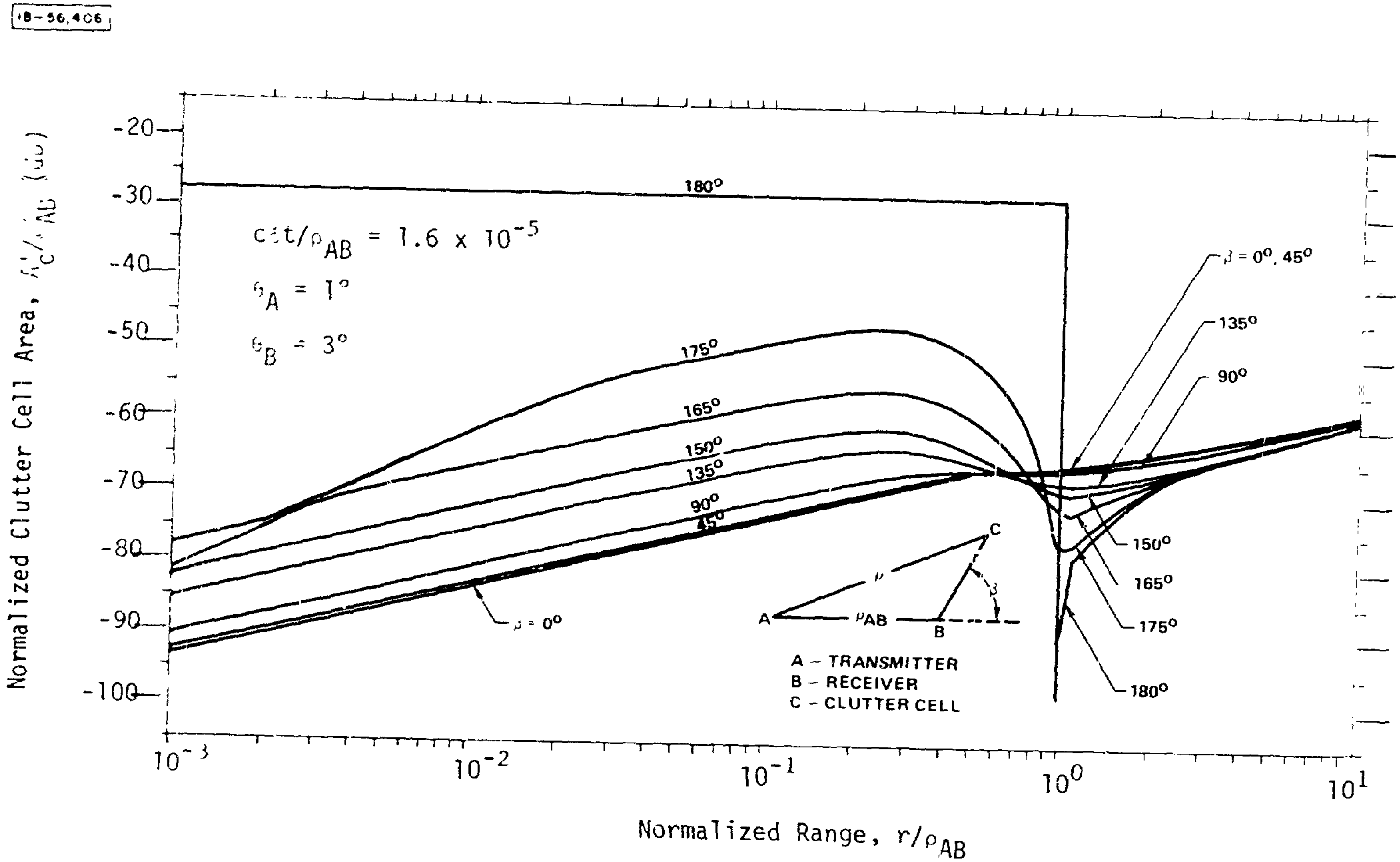

FIGURE 13. CLUTTER CELL AREA, NUMERICAL RESULTS FOR $c\delta t/\rho_{AB} = 1.6 \times 10^{-5}$

time δt and the baseline distance ρ_{AB} corresponding to the parameter $c\delta t/\rho_{AB} = 1.6 \times 10^{-4}$ are ($\delta t = 10^{-8}$s, $\rho_{AB} = 10$ nmi) and ($\delta t = 10^{-7}$, $\rho_{AB} = 100$ nmi). For $\rho_{AB} = 10$ nmi, a normalized cell area $A_c'/\rho_{AB}^2 = -22$ dB corresponds to a cell area $A_c' = +63$ dBm^2 .

In the backscattered or quasi-monostatic direction ($\beta = 0°$) and ($\beta = 180°$, $r/\rho_{AB} > 1$), the cell area increases monotonically with range r . In the forward-scattered direction ($\beta = 180°$, $r/\rho_{AB} < 1$), the cell area is constant for $0 < r < \rho_{AB}$ because the cell area is distributed over all points along $\overline{AB}$ at a time-delay difference $\Delta/c = 0$ for all points along $\overline{AB}$.

In Figures 11-13, the cell area increases with increasing azimuthal angle β for $r/\rho_{AB} \lesssim 0.5$. The cell area increases rather slowly for small azimuthal angles but increases rapidly at large azimuthal angles. For example, the increase in cell area over that for $\beta = 0°$ is only 3 dB and 8 dB for $\beta = 90°$ and 135°, respectively, but is 23 dB and 37 dB for $\beta = 175°$ amd 180°, respectively, at $r/\rho_{AB} = 0.3$ and $c\delta t/\rho_{AB} = 1.6 \times 10^{-4}$. It should also be noted that at small receiver ranges ($r/\rho_{AB} < 10^{-3}$), the cell area can be more than 50 dB larger in the forward-scattered direction than for $\beta = 0°$.

In Figures 11-13, the cell area is proportional to the receiver resolution time δt (for a fixed ρ_{AB}) for azimuthal angles as large as $\beta = 150°$. However, at the larger azimuthal angles the cell area is not proportional to the receiver resolution time δt because the range resolution is limited by the transmitter beam width θ_A and the baseline distance ρ_{AB} and because the angle subtended at the receiver by the clutter cell is a function of the resolution time δt. In the parallelogram formulae of Equations (15) and (18) for $\Delta/c >> \delta t$, the cell area is proportional to the receiver resolution time δt. For $c\delta t/\rho_{AB} = 1.6 \times 10^{-4}$, the parallelogram formulae of Equations (15) and (18) are within 0.8 dB of the results shown in Figure 12 for bistatic angles $\beta - \gamma$ as large as 165°.

The rapid increase in clutter cell area at large bistatic angles is not to be confused with the separate phnomenon of the rapid increase of the clutter cross section per unit area (scattering coefficient) which occurs at large bistatic angles for most clutter surfaces.

SECTION IV

CONCLUSIONS

Convenient, approximate expressions have been derived for the bistatic clutter cell area when limited by the receiver resolution time and when further restricted to the case in which the plane containing the transmitter, receiver, and clutter cell subtends a small angle with respect to the reference plane containing the clutter surface.

The cell shape is a parallelogram for large time-delay differences. For small time-delay differences, the cell shape is trapezoidal or triangular at small bistatic angles and is rhomboidal or hexagonal at large bistatic angles.

The cell area increases with increasing bistatic angle and is proportional to $(\text{secant})^2$ of one-half the bistatic angle excluding bistatic angles approaching 180°. The cell area is typically more than 30 dB larger in the forward-scattered direction than in the backscattered direction.

REFERENCES

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2. See, for example, F. E. Nathanson, "Radar Design Principles" (McGraw-Hill Book Co., N. Y., 1969), pp. 64-65. In this reference, the symbol R is used to denote the range from the transmitter/receiver to the clutter cell and should not be confused with the notation of the present paper in which $R = \rho + r$ = range sum = sum of the distances from the transmitter to clutter cell and from the clutter cell to the receiver. Please also note the typographical errors associated with Eqs. (2-35), (2-36), and Figure 2.5 of "Radar Design Principles."

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APPENDIX A

RANGE RESOLUTION OF CLUTTER CELL

With reference to Figure 5 the range resolution δr, when limited by the receiver resolution time δt and the transmitter azimuthal beam width θ_A at a delay time $t = \Delta/c$, is given for a constant direction β by

$$\delta r = \begin{cases} \left.\dfrac{\partial r}{\partial t}\right|_{\beta,\rho_{AB}} \delta t, & \left.\dfrac{\partial r}{\partial t}\right|_{\beta,\rho_{AB}} \delta t \leq \overline{DE} \\ \overline{DE}, & \left.\dfrac{\partial r}{\partial t}\right|_{\beta,\rho_{AB}} \delta t > \overline{DE} \end{cases} \tag{A-1}$$

$$\left.\frac{\partial r}{\partial t}\right|_{\beta,\rho_{AB}} = \left.\frac{\partial r}{\partial \Delta}\right|_{\beta,\rho_{AB}} \cdot \left.\frac{\partial \Delta}{\partial t}\right|_{\beta,\rho_{AB}} = \left.\frac{\partial r}{\partial \Delta}\right|_{\beta,\rho_{AB}} \cdot c \tag{A-2}$$

From the Cosine Law for the triangle ABC,

$$\rho^2 = r^2 + \rho_{AB}^2 - 2r\rho_{AB}\cos(\pi - \beta) \tag{A-3}$$

Substituting $\rho = \Delta + \rho_{AB} - r$ into Equation (A-3),

$$[\Delta + \rho_{AB} - r]^2 = r^2 + \rho_{AB}^2 - 2r\rho_{AB}\cos(\pi - \beta)$$

$$= \Delta^2 + 2\Delta\rho_{AB} + \rho_{AB}^2 - 2r(\Delta + \rho_{AB}) + r^2 \tag{A-4}$$

Solving Equation (A-4) for r and using the identity $1 - \cos(\pi - \beta) \equiv 2\sin^2(\frac{\pi - \beta}{2})$

$$r = \frac{(\Delta/2)(\Delta + 2\rho_{AB})}{\Delta + \rho_{AB}[1 - \cos(\pi-\beta)]} = \frac{(\Delta/2)[1 + (\Delta/2\rho_{AB})]}{(\Delta/2\rho_{AB}) + \sin^2(\frac{\pi-\beta}{2})} \qquad \text{(A-5)}$$

$$\left.\frac{\partial r}{\partial \Delta}\right|_{\beta,\rho_{AB}} = \frac{[(\Delta/2\rho_{AB}) + \sin^2(\frac{\pi-\beta}{2})][\frac{1}{2} + \frac{\Delta}{2\rho_{AB}}] - (\Delta/2)(1 + \frac{\Delta}{2\rho_{AB}})(1/2\rho_{AB})}{[(\Delta/2\rho_{AB}) + \sin^2(\frac{\pi-\beta}{2})]^2}$$

$$= \frac{(\frac{\Delta}{2\rho_{AB}})^2 + \frac{\Delta}{4\rho_{AB}} + \frac{\sin^2(\frac{\pi\ \beta}{2})}{2} + \frac{\Delta}{2\rho_{AB}}\sin^2(\frac{\pi-\beta}{2}) - \frac{\Delta}{4\rho_{AB}} - \frac{\Delta^2}{8\rho_{AB}^2}}{[(\Delta/2\rho_{AB}) + \sin^2(\frac{\pi-\beta}{2})]^2}$$

$$= \frac{\frac{1}{2}[\frac{\Delta}{2\rho_{AB}} + \sin^2(\frac{\pi-\beta}{2})]^2 + \frac{1}{2}\sin^2(\frac{\pi-\beta}{2})[1 - \sin^2(\frac{\pi-\beta}{2})]}{[(\Delta/2\rho_{AB}) + \sin^2(\frac{\pi-\beta}{2})]^2}$$

$$= \frac{1}{2}\left\{1 + \frac{(1/4)\sin^2(\pi-\beta)}{[\sin^2(\frac{\pi-\beta}{2}) + (\Delta/2\rho_{AB})]^2}\right\} \qquad \text{(A-6)}$$

Substituting Equations (A-2) and (A-6) into Equation (A-1),

$$\delta r = \begin{cases} (c\delta t/2)(1 + f), & (c\delta t/2)(1 + f) \le \overline{DE} \\ \overline{DE}, & (c\delta t/2)(1 + f) > \overline{DE} \end{cases} \qquad \text{(A-7)}$$

where $$f \equiv \frac{(1/4)\sin^2(\pi-\beta)}{[\sin^2(\frac{\pi-\beta}{2}) + (\Delta/2\rho_{AB})]^2} \tag{A-8}$$

Alternative expressions for $(1 + f)$ are given by Eqs. (A-29) and (A-30) at the end of this appendix.

The normalized range sum $\Delta/\rho_{AB} = (\rho/\rho_{AB}) + (r/\rho_{AB}) - 1$ may be expressed as functions of r/ρ_{AB} and β by solving Equation (A-5) for Δ/ρ_{AB}. Multiplying both sides of Equation (A-5) by $[(\Delta/2\rho_{AB}) + \sin^2(\frac{\pi-\beta}{2})$ and rearranging terms,

$$\frac{1}{4}\left(\frac{\Delta}{\rho_{AB}}\right)^2 + \frac{1}{2}\left(1 - \frac{r}{\rho_{AB}}\right)\frac{\Delta}{\rho_{AB}} - \frac{r}{\rho_{AB}}\sin^2\left(\frac{\pi-\beta}{2}\right) = 0 \tag{A-9}$$

Equation (A-8) is a quadratic equation of the dependent variable (Δ/ρ_{AB}). Solving Equation (A-8) for Δ/ρ_{AB},

$$\frac{\Delta}{\rho_{AB}} = -[1 - (r/\rho_{AB})] \pm \left[\left(1 - \frac{r}{\rho_{AB}}\right)^2 + \frac{4r}{\rho_{AB}}\sin^2\left(\frac{\pi-\beta}{2}\right)\right]^{\frac{1}{2}} \tag{A-10}$$

Choosing the root for which $(\Delta/\rho_{AB}) \geq 0$,

$$\frac{\Delta}{\rho_{AB}} = \begin{cases} -\left(1 - \frac{r}{\rho_{AB}}\right) + \left[\left(1 - \frac{r}{\rho_{AB}}\right)^2 + \frac{4r}{\rho_{AB}}\sin^2\left(\frac{\pi-\beta}{2}\right)\right]^{\frac{1}{2}}, & \beta < 180^\circ \\ 0, & \beta = 180^\circ,\ r/\rho_{AB} \leq 1 \\ 2\left(\frac{r}{\rho_{AB}} - 1\right), & \beta = 180^\circ,\ r/\rho_{AB} > 1\ . \end{cases} \tag{A-11}$$

The direction β, for which the function f is maximal for a given Δ/ρ_{AB}, is found by setting the derivative of f equal to zero where f is given by Equation (A-8). Accordingly,

$$\frac{df}{d\beta} = 0 = -[\sin^2(\frac{\pi-\beta}{2}) + (\Delta/2\rho_{AB})]^2 \frac{1}{2}\sin(\pi - \beta)\cos(\pi - \beta)$$

$$+ \frac{1}{2}\sin^2(\pi - \beta)[\sin^2(\frac{\pi-\beta}{2}) + (\Delta/2\rho_{AB})][\sin(\frac{\pi-\beta}{2})\cos(\frac{\pi-\beta}{2})]$$

$$= [\sin^2(\frac{\pi-\beta}{2}) + (\Delta/2\rho_{AB})]\cos(\pi - \beta) - \frac{\sin^2(\pi-\beta)}{2}$$

$$= [\frac{1 - \cos(\pi-\beta)}{2} + (\Delta/2\rho_{AB})]\cos(\pi - \beta) - \frac{1}{2} + \frac{\cos^2(\pi-\beta)}{2}$$

$$= \cos(\pi - \beta)[1 + (\Delta/\rho_{AB})] - 1 \qquad \text{(A-12)}$$

$$\cos(\pi - \beta) = [1 + (\Delta/\rho_{AB})]^{-1} = \cos(\pi - \beta_o) \qquad \text{(A-13)}$$

Substituting Equation (A-13) into Equation (A-8) by using the identities $\sin^2(\pi - \beta_o) \equiv 1 - \cos^2(\pi - \beta_o)$ and $\sin(\frac{\pi-\beta_o}{2}) \equiv \frac{1 - \cos(\pi\ \beta_o)}{2}$,

$$f_{max} = (\Delta/\rho_{AB})^{-1}[2 + (\Delta/\rho_{AB})]^{-1} \qquad \text{(A-14)}$$

The length $\overline{DE}$ in Equation (A-1) is given, with reference to Figure 5, by

$$\overline{DE} = \overline{DC} + \overline{CE} \qquad \text{(A-15)}$$

From the Sine Law for triangle ACE,

$$\overline{CE} = \frac{\rho \sin(\theta_A/2)}{\sin \measuredangle CEA} = \frac{\rho \sin(\theta_A/2)}{\sin[\pi - (\beta-\gamma) - \frac{\theta_A}{2}]}\ , \quad (\beta-\gamma) \leq \pi - \frac{\theta_A}{2} \tag{A-16}$$

where $\measuredangle CEA = [\pi - (\beta-\gamma) - \frac{\theta_A}{2}]$ for $(\beta - \gamma) \leq \pi - \frac{\theta_A}{2}$.

From the Law of Sines for triangle ACD,

$$\overline{DC} = \frac{\rho \sin(\theta_A/2)}{\sin \measuredangle CDA} = \frac{\rho \sin(\theta_A/2)}{\sin(\beta-\gamma - \frac{\theta_A}{2})}\ , \qquad \beta - \gamma \geq \frac{\theta_A}{2} \tag{A-17}$$

where $\measuredangle CDA = (\beta - \gamma) - \frac{\theta_A}{2}$ for $(\beta - \gamma) \geq \frac{\theta_A}{2}$

and $\rho = \Delta - r + \rho_{AB}$.

Substituting Equations (A-16) and (A-17) into Equation (A-15),

$$\overline{DE} = \rho \sin(\frac{\theta_A}{2})\left\{\frac{1}{\sin[\pi - (\beta-\gamma) - \frac{\theta_A}{2}]} + \frac{1}{\sin(\beta - \gamma - \frac{\theta_A}{2})}\right\}$$

$$= \rho \sin(\frac{\theta_A}{2})\left\{\frac{1}{\sin(\beta - \gamma + \frac{\theta_A}{2})} + \frac{1}{\sin(\beta - \gamma - \frac{\theta_A}{2})}\right\}$$

$$= \rho \sin(\frac{\theta_A}{2})\left\{\frac{2 \sin(\beta-\gamma) \cos(\theta_A/2)}{\sin^2(\beta-\gamma) - \sin^2(\theta_A/2)}\right\}$$

$$= \frac{\rho \sin\theta_A \sin(\beta-\gamma)}{\sin^2(\beta-\gamma) - \sin^2(\theta_A/2)} \; ; \; \frac{\theta_A}{2} < \beta - \gamma < \pi - \frac{\theta_A}{2} \tag{A-18}$$

When $(\beta - \gamma) = \frac{\theta_A}{2}$ or $\pi - \frac{\theta_A}{2}$, the point D or E occurs at infinity so that

$$\overline{DE} = \infty \; ; \; 0 \leq (\beta - \gamma) \leq \frac{\theta_A}{2} \quad , \qquad \pi - \frac{\theta_A}{2} < \beta - \gamma \leq \pi \tag{A-19}$$

Combining Equations (A-18) and (A-19),

$$\overline{DE} = \begin{cases} \dfrac{\rho \sin\theta_A \sin(\beta - \gamma)}{\sin^2(\beta - \gamma) - \sin^2(\theta_A/2)} \; ; & \dfrac{\theta_A}{2} < \beta - \gamma < \pi - \dfrac{\theta_A}{2} \\ \infty \; ; \quad 0 \leq \beta - \gamma \leq \dfrac{\theta_A}{2} & , \; \pi - \dfrac{\theta_A}{2} \leq \beta - \gamma \leq \pi \end{cases} \tag{A-20}$$

The range resolution $\delta r = (c\delta t/2)(1 + f)$, given by Eq. (A-7), is an explicit function of β and Δ/ρ_{AB}. The range resolution may also be expressed explicitly as a function of the bistantic angle $\beta - \gamma$. With reference to Figure 5, the ellipse through the point C, of time delay $(1/c)(\Delta + \rho_{AB})$ has a range sum R given by

$$R = \rho + r \tag{A-21}$$

The ellipse, of time delay $[(1/c)(\Delta + \rho_{AB}) + \delta t]$, has a corresponding range sum $R + dR$ where dR is given by

$$dR = c\delta t \tag{A-22}$$

Substituting Equation (A-22) into Equation (A-3),

$$(R - r)^2 = r^2 + \rho_{AB}^2 - 2r\,\rho_{AB}\cos(\pi - \beta) \qquad \text{(A-23)}$$

Expanding terms in Equation (A-23),

$$R^2 - 2Rr = \rho_{AB}^2 + 2r\,\rho_{AB}\cos\beta \qquad \text{(A-24)}$$

Differentiating Equation (A-24), with β and ρ_{AB} held constant,

$$2R\,dR - 2R\,\delta r - 2r\,dR = 2\rho_{AB}\cos\beta \qquad \text{(A-25)}$$

Substituting Equations (A-21) and (A-22) into Equation (A-25),

$$\delta r = \frac{(R - r)\,dR}{\rho_{AB}\cos\beta + R} = \frac{\rho\,c\delta t}{\rho_{AB}\cos\beta + \rho + r} = \frac{c\delta t}{\frac{\rho_{AB}}{\rho}\cos\beta + 1 + \frac{r}{\rho}} \qquad \text{(A-26)}$$

From the Sine Law for triangle ABC in Figure 5,

$$\frac{\rho_{AB}}{\rho} = \frac{\sin(\beta-\gamma)}{\sin(\pi-\beta)} = \frac{\sin(\beta-\gamma)}{\sin\beta} \qquad \text{(A-27)}$$

$$\frac{\rho_{AB}}{r} = \frac{\sin(\beta-\gamma)}{\sin\gamma} \qquad \text{(A-28)}$$

Substituting Equations (A-27) and (A-28) into Equation (A-26),

$$\delta r = \frac{c\delta t}{\frac{\sin(\beta-\gamma)}{\sin\beta} + 1 + \frac{\sin\gamma}{\sin\beta}} = \frac{c\delta t}{\frac{\sin(\beta-\gamma)\cos\beta + \sin\beta + \sin\gamma}{\sin\beta}}$$

$$= \frac{c\delta t \sin\beta}{[\sin\beta\cos\gamma - \sin\gamma\cos\beta]\cos\beta + \sin\beta + \sin\gamma}$$

$$= \frac{c\delta t}{1 + \cos\gamma\cos\beta - \sin\gamma\frac{\cos^2\beta}{\sin\beta} + \frac{\sin\gamma}{\sin\beta}}$$

$$= \frac{c\delta t}{1 + \cos\gamma\cos\beta + \frac{\sin\gamma}{\sin\beta}(1 - \cos^2\beta)}$$

$$= \frac{c\delta t}{1 + \cos\gamma\cos\beta + \sin\gamma\sin\beta}$$

$$= \frac{c\delta t}{1 + \cos(\beta-\gamma)} = \frac{c\delta t}{2\cos^2(\frac{\beta-\gamma}{2})} \tag{A-29}$$

The bistatic range resolution, for time-delay differences $\Delta/c >> \delta$ is therefore proportional to (secant)^2 of one-half the bistatic angle.

It can also be shown that the range resolution $\delta\rho$ in Figure 3 along a constant direction γ is given by $\delta\rho = \left.\frac{\partial\rho}{\partial t}\right|_{\gamma,\rho_{AB}} \delta t = \delta r$ because δr and $\delta\rho$ are symmetrical about the bisector of the bistatic angle (compare Figures 2 and 3). Since the geometries associated with the angles subtended by the clutter cell at A or B are identical for the transpositions of A with B, θ_A with θ_B, β with $\pi - \gamma$, γ with $\pi - \beta$, and ρ with r, it follows from Eqs. (A-7) and (A-29) that

$$\delta r = \delta\rho = \frac{c\delta t}{2} \sec^2\left(\frac{\beta - \gamma}{2}\right) = \frac{c\delta t}{2} \left\{ 1 + \frac{(1/4)\sin^2(\pi - \beta)}{\left[\sin^2\left(\frac{\pi - \beta}{2}\right) + (\Delta/2\rho_{AB})\right]^2} \right\}$$

$$= \frac{c\delta t}{2} \left\{ 1 + \frac{(1/4)\sin^2 \gamma}{\left[\sin^2\left(\frac{\gamma}{2}\right) + (\Delta/2\rho_{AB})\right]^2} \right\} \quad , \ \Delta/c >> \delta t \qquad \text{(A-30)}$$

APPENDIX B

ILLUMINATION ANGLES OF CLUTTER CELL

<u>Illumination Angles η and η_{max} Subtended by the Clutter Cell at the Receiver</u>

With reference to Figure 5,

$$\eta = \eta_1 + \eta_2 = \measuredangle FBG \,, \quad \eta_1 = \measuredangle FBC \,, \quad \eta_2 = \measuredangle CBG \tag{B-1}$$

$$\eta_{max} = \eta_3 + \eta_4 = \measuredangle HBI \quad \eta_3 = \measuredangle HBC \,, \quad \eta_4 = \measuredangle CBI \tag{B-2}$$

Consider first point C on the time-delay ellipse Δ/c and characterized by $\measuredangle CBL = \beta$ and $\measuredangle CAB = \gamma$. Applying the Law of Sines to triangle ABC ,

$$\frac{\rho_{AB}}{\sin(\beta-\gamma)} = \frac{r}{\sin\gamma} = \frac{\rho}{\sin(\pi-\beta)} = \frac{\rho}{\sin\beta} \tag{B-3}$$

Since $\Delta = \rho + r - \rho_{AB}$ where r and ρ are given by Eq. (B-3)

$$\frac{\rho}{\rho_{AB}} + \frac{r}{\rho_{AB}} = \frac{\Delta}{\rho_{AB}} + 1 = \frac{\sin\beta + \sin\gamma}{\sin(\beta-\gamma)} = \frac{2\sin(\frac{\beta+\gamma}{2})\cos(\frac{\beta-\gamma}{2})}{2\sin(\frac{\beta-\gamma}{2})\cos(\frac{\beta-\gamma}{2})} = \frac{\sin(\frac{\beta+\gamma}{2})}{\sin(\frac{\beta-\gamma}{2})} \tag{B-4}$$

Now consider the point F on the same ellipse of time delay Δ/c as point C and at $\measuredangle FBL = \beta \;\; \eta_1$ and $\measuredangle FAB = \gamma + \frac{\eta_A}{2}$. Applying the Law of Sines to triangle AFB .

$$\frac{\rho_{AB}}{\sin(\beta+\eta_1) - (\gamma+\frac{\theta_A}{2})} = \frac{\overline{FB}}{\sin(\gamma+\frac{\theta_A}{2})} = \frac{\overline{FA}}{\sin(\pi - \beta - \eta_1)} = \frac{\overline{FA}}{\sin(\beta + \eta_1)} \qquad \text{(B-5)}$$

Since $\Delta = \overline{FB} + \overline{FA} - \rho_{AB}$ where $\overline{FB}$ and $\overline{FA}$ are given by Eq. (B-5),

$$\frac{\overline{FB}}{\rho_{AB}} + \frac{\overline{FA}}{\rho_{AB}} = \frac{\Delta}{\rho_{AB}} + 1$$

$$= \frac{\sin[(\frac{\beta+\eta_1}{2}) + (\frac{\gamma}{2} + \frac{\theta_A}{4})]}{\sin[(\frac{\beta+\eta_1}{2}) - (\frac{\gamma}{2} - \frac{\theta_A}{4})]} = \frac{\sin[(\frac{\beta+\gamma}{2}) + (\frac{\eta_1}{2} + \frac{\theta_A}{4})]}{\sin[(\frac{\beta-\gamma}{2}) + (\frac{\eta_1}{2} + \frac{\theta_A}{4})]} \qquad \text{(B-6)}$$

Letting $x = \frac{\beta + \eta_1}{2}$, $y = \frac{\gamma}{2} + \frac{\theta_A}{4}$ and solving for x in Eq. (B-6),

$$\frac{\Delta}{\rho_{AB}} + 1 = \frac{\sin(x + y)}{\sin(x - y)} = \frac{\sin x \cos y + \cos x \sin y}{\sin x \cos y - \cos x \sin y} = \frac{\tan x \cos y + \sin y}{\tan x \cos y - \sin y} \qquad \text{(B-7)}$$

$$\tan x = (1 + \frac{2\rho_{AB}}{\Delta}) \tan y = \tan(\frac{\beta + \eta_1}{2}) = (1 + \frac{2\rho_{AB}}{\Delta}) \tan(\frac{\gamma}{2} + \frac{\theta_A}{4}) \qquad \text{(B-8)}$$

$$\eta_1 = 2 \arctan (\frac{\beta+\eta_1}{2}) - \beta$$

$$= 2 \arctan[(1 + \frac{2\rho_{AB}}{\Delta}) \tan(\frac{\gamma}{2} + \frac{\theta_A}{4})] - \beta \qquad \text{(B-9)}$$

Now consider the point G which occurs on the same ellipse of time delay Δ/c at $\measuredangle GBL = \beta - \eta_2$ and $\measuredangle GAL = \gamma - \frac{\theta_A}{2}$. Substituting $(\beta - \eta_2)$ for x and $(\gamma - \frac{\theta_A}{2})$ for y in Equation (B-7),

$$\eta_2 = \beta - 2 \arctan[(1 + \frac{2\rho_{AB}}{\Delta}) \tan(\frac{\gamma}{2} - \frac{\theta_A}{4})] \tag{B-10}$$

Adding Equations (B-9) and (B-10),

$$\eta_1 + \eta_2 = 2\left\{\arctan[(1 + \frac{2\rho_{AB}}{\Delta})\tan(\frac{\gamma}{2} + \frac{\theta_A}{4})] - \arctan[(1 + \frac{2\rho_{AB}}{\Delta}) \tan(\frac{\gamma}{2} - \frac{\theta_A}{4})]\right\} \tag{B-11}$$

$$\eta = \begin{cases} (\eta_1 + \eta_2) & , \eta_1 + \eta_2 \leq 180° \\ 360° - (\eta_1 + \eta_2), & \eta_1 + \eta_2 > 180° \end{cases} \tag{B-12}$$

where $\eta_1 + \eta_2$ is given by Equation (B-11).

For sufficiently small angles of η and θ_A , $\eta_1 \approx \eta_2$ and $\eta \approx 2\eta_1$. An expression for η , when η and θ_A are small angles, may be found by regrouping angles in Eq. (B-6) and by equating Eq. (B-4) to Eq. (B-6). Accordingly,

$$\frac{\Delta}{\rho_{AB}} + 1 = \frac{\sin(\frac{\beta+\gamma}{2})}{\sin(\frac{\beta-\gamma}{2})} = \frac{\sin[(\frac{\beta+\gamma}{2}) + (\frac{\eta_1}{2} + \frac{\theta_A}{4})]}{\sin[(\frac{\beta-\gamma}{2}) + (\frac{\eta_1}{2} + \frac{\theta_A}{4})]}$$

$$= \frac{\sin(\frac{\beta+\gamma}{2}) \cos(\frac{\eta_1}{2} + \frac{\theta_A}{4}) + \sin(\frac{\eta_1}{2} + \frac{\theta_A}{4}) \cos(\frac{\beta+\gamma}{2})}{\sin(\frac{\beta-\gamma}{2}) \cos(\frac{\eta_1}{2} + \frac{\theta_A}{4}) + \sin(\frac{\eta_1}{2} + \frac{\theta_A}{4}) \cos(\frac{\beta-\gamma}{2})} \tag{B-13}$$

For $\frac{\eta_1}{2} << 1$ rad and $\frac{\theta_A}{4} << 1$ rad, Equation (B-16) reduces to

$$\frac{\sin(\frac{\beta+\gamma}{2})}{\sin(\frac{\beta-\gamma}{2})} \approx \left[\frac{\sin(\frac{\beta+\gamma}{2})}{\sin(\frac{\beta-\gamma}{2})}\right]\left[\frac{1 + (\frac{\eta_1}{2} + \frac{\theta_A}{4}) \cot(\frac{\beta+\gamma}{2})}{1 + (\frac{\eta_1}{2} + \frac{\theta_A}{4}) \cot(\frac{\beta-\gamma}{2})}\right] \qquad \text{(B-14)}$$

Solving Equation (B-14) for η_1 ,

$$\eta_1 \approx \frac{\theta_A}{2} \frac{\sin \beta}{\sin \gamma} \; ; \; \frac{\eta_1}{2} << 1 \text{ rad}, \; \frac{\theta_A}{4} << 1 \text{ rad} \qquad \text{(B-15)}$$

Therefore,

$$\eta \approx 2\eta_1 \approx \theta_A \frac{\sin \beta}{\sin \gamma} = \theta_A \, \rho/r \quad ; \; \frac{\eta}{4} << 1 \text{ rad}, \; \frac{\theta_A}{4} << 1 \text{ rad} \qquad \text{(B-16)}$$

Consider now the point H on the ellipse of time delay $|(\Delta/c) - (\delta t/2)|$ where the absolute sign includes cases for which $\Delta/c < \delta t/2$. The point H occurs at angles $\measuredangle HBL = \beta + \eta_3$ and $\measuredangle HAB = \gamma + (\theta_A/2)$. Applying the Law of Sines to triangle AHB and using the identity $\Delta - (c\delta t/2) = \overline{HB} + \overline{HA} - \rho_{AB}$, one obtains an equation identical to Eq. (B-8) except that $x = \beta + \eta_3$ and $\Delta - (c\delta t/2)$ is substituted for Δ . Accordingly,

$$\eta_3 = 2 \arctan\left[(1 + \frac{2\rho_{AB}}{|\Delta - (c\delta t/2)|}) \tan(\frac{\gamma}{2} + \frac{\theta_A}{4})\right] - \beta \qquad \text{(B-17)}$$

Similarly, for point I on the ellipse of time delay $(\Delta/c) + (\delta t/2)$ at $\measuredangle IBL = \beta - \eta_4$ and $\measuredangle IAB = \gamma - \frac{\theta_A}{2}$,

$$\eta_4 = 2 \arctan\left\{\left[1 + \frac{2\rho_{AB}}{\Delta + (\delta t/2)}\right] \tan\left(\frac{\gamma}{2} - \frac{\theta_A}{4}\right)\right\} + \beta \tag{B-18}$$

Substituting Equations (B-17) and (B-18) into Equation (B-2),

$$\eta_3 + \eta_4 = 2\left\{\arctan\left[\left(1 + \frac{2\rho_{AB}}{|\Delta - (c\delta t/2)|}\right) \tan\left(\frac{\gamma}{2} + \frac{\theta_A}{4}\right)\right] - \arctan\left[\left(1 + \frac{2\rho_{AB}}{\Delta + (c\delta t/2)}\right) \tan\left(\frac{\gamma}{2} - \frac{\theta_A}{4}\right)\right]\right\} \tag{B-19}$$

$$\eta_{max} = \begin{cases} \eta_3 + \eta_4 & , \eta_3 + \eta_4 \leq 180° \\ 360° - (\eta_3 + \eta_4), & \eta_3 + \eta_4 > 180° \end{cases} \tag{B-20}$$

where $\eta_3 + \eta_4$ is given by Equation (B-19).

Illumination Angle γ

With reference to Figure 5, the angle $\gamma = \measuredangle CAB$ may be expressed as a function of β and r/ρ_{AB} by solving Eq. (B-3) for γ. Accordingly,

$$\frac{r}{\rho_{AB}} = \frac{\sin \gamma}{\sin(\beta - \gamma)} = \frac{\sin \gamma}{\sin \beta \cos \gamma - \cos \beta \sin \gamma} = \frac{1}{\sin \beta \cot \gamma - \cos \beta} \tag{B-21}$$

$$\gamma = \arctan\left[\left(\frac{r}{\rho_{AB}} \sin \beta\right) \Big/ \left(1 + \frac{r}{\rho_{AB}} \cos \beta\right)\right] \tag{B-22}$$

Illumination Angles χ and χ_{max} Subtended by the Clutter Cell At the Transmitter

With reference to Figure 3,

$$\chi = \chi_1 + \chi_2 = \measuredangle\, JAK \;,\; \chi_1 = \measuredangle\, JAC \;,\; \chi_2 = \measuredangle\, CAK$$

$$\chi_{max} = \chi_3 + \chi_4 = \measuredangle\, MAN \;,\; \chi_3 = \measuredangle\, MAC \;,\; \chi_4 = \measuredangle\, CAN \tag{B-23}$$

The geometries associated with the angles subtended by the clutter cell at A or B are identical for the transpositions of A with B, θ_A with θ_B, β with $\pi - \gamma$, γ with $\pi - \beta$, and ρ with r. The results given by Eqs. (B-3)-(B-20) for η and η_{max} are therefore identical for χ and χ_{max} provided that the above transpositions are made.

Accordingly, from Eqs. (B-9), (B-10), (B-11), and (B-12),

$$\chi_1 = 2 \arctan\left[\left(1 + \frac{2\rho_{AB}}{\Delta}\right) \tan\left(\frac{\pi - \beta}{2} + \frac{\theta_B}{4}\right)\right] - (\pi - \gamma) \tag{B-24}$$

$$\chi_2 = (\pi - \gamma) - 2 \arctan\left[\left(1 + \frac{2\rho_{AB}}{\Delta}\right) \tan\left(\frac{\pi - \beta}{2} - \frac{\theta_B}{4}\right)\right] \tag{B-25}$$

$$\begin{aligned}\chi_1 + \chi_2 = 2\Big\{&\arctan\left[\left(1 + \frac{2\rho_{AB}}{\Delta}\right) \tan\left(\frac{\pi - \beta}{2} + \frac{\theta_B}{4}\right)\right] \\ &- \arctan\left[\left(1 + \frac{2\rho_{AB}}{\Delta}\right) \tan\left(\frac{\pi - \beta}{2} - \frac{\theta_B}{4}\right)\right]\Big\}\end{aligned} \tag{B-26}$$

$$\chi = \begin{cases} (\chi_1 + \chi_2) & , \chi_1 + \chi_2 \le 180° \\ 360° - (\chi_1 + \chi_2), & \chi_1 + \chi_2 > 180° \end{cases} \tag{B-27}$$

where $\chi_3 + \chi_4$ is given by Equation (B-32).

For sufficiently small angles of χ and θ_B, $\chi_1 \approx \chi_2$ and $x \approx 2\chi$. From Equations (B-15) and (B-16),

$$\chi_1 \approx \frac{\theta_B}{2} \frac{\sin(\pi-\gamma)}{\sin(\pi-\beta)} = \frac{\theta_B}{2} \frac{\sin\gamma}{\sin\beta} \; ; \; \frac{\chi_1}{2} << 1 \text{ rad}, \; \frac{\theta_B}{4} << 1 \text{ rad} \qquad \text{(B-28)}$$

$$\chi \approx 2\chi_1 = \theta_B \frac{\sin\gamma}{\sin\beta} = \theta_B\, r/\rho \quad ; \; \frac{\chi}{4} << 1 \text{ rad}, \; \frac{\theta_B}{4} << 1 \text{ rad} \qquad \text{(B-29)}$$

From Equations (B-17), (B-18), (B-19), and (B-20),

$$\chi_3 = 2 \arctan\left\{\left[1 + \frac{2\rho_{AB}}{|\Delta - (c\delta t/2)|}\right] \tan\left(\frac{\pi - \beta}{2} + \frac{\theta_B}{4}\right)\right\} - (\pi - \gamma) \qquad \text{(B-30)}$$

$$\chi_4 = 2 \arctan\left\{\left[1 + \frac{2\rho_{AB}}{\Delta + (\delta t/2)}\right] \tan\left(\frac{\pi - \beta}{2} - \frac{\theta_B}{4}\right)\right\} + (\pi - \gamma) \qquad \text{(B-31)}$$

$$\chi_3 + \chi_4 = 2\left\{\arctan\left[\left(1 + \frac{2\rho_{AB}}{|\Delta - (c\delta t/2)|}\right) \tan\left(\frac{\pi - \beta}{2} + \frac{\theta_B}{4}\right)\right]\right. \qquad \text{(B-32)}$$

$$\left. - \arctan\left[\left(1 + \frac{2\rho_{AB}}{\Delta + (c\delta t/2)}\right) \tan\left(\frac{\pi - \beta}{2} - \frac{\theta_B}{4}\right)\right]\right\} \qquad \text{(B-32)}$$

$$\chi_{max} = \begin{cases} \chi_3 + \chi_4 & , \; \chi_3 + \chi_4 \leq 180° \\ 360° - (\chi_3 + \chi_4) & , \; \chi_3 + \chi_4 > 180° \end{cases} \qquad \text{(B-33)}$$

where $\chi_3 + \chi_4$ is given by Equation (B-32).

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