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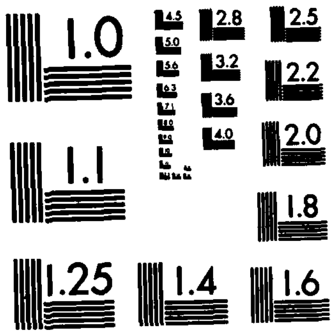

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A ROBUST RATIO-THRESHOLD TECHNIQUE TO MITIGATE TONE AND PARTIAL BAND JAMMING IN CODED FREQUENCY HOPPED COMMUNICATION LINKS

Final Report on

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1. INTRODUCTION

Frequency hopping has long been regarded as an effective technique to thwart intentional and unintentional interference. The latter includes also fading and multipath. An advantage of frequency hopping, as compared to direct sequence spread spectrum techniques, is that phase coherence is not required. This makes frequency hopped systems more robust, since they are dependent on one less parameter, which is particularly significant for fading channels wherein rapid phase variations make it difficult if not impossible to track. On the negative side, noncoherent systems performance is inferior to coherent systems, particularly when the number of bits per hop is small (or worse yet, fractional). More serious is the fact that tone or partial band jammers can cause severe degradations (of orders of magnitude) relative to full band Gaussian jammers. In contrast, theoretically direct sequence spread spectrum systems are no more vulnerable to partial band or tone jammers than they are to wideband (Gaussian) jammers. In Section 2, the extent of the degradation due to tone jamming of a frequency hopped multiple frequency shift keyed (MFSK) system is established: first for an uncoded system (Sec. 2.1) and then for a system which utilizes a powerful but practically implementable forward error-correcting code (Sec. 2.2).

In Section 3, a new robust mitigation technique for tone and partial band jamming is proposed. The resulting performance improvement for coded MFSK systems in unfaded channels is determined in Sec. 3.1 for the worst case tone jammer and in Sec. 3.2 for the worst case partial band jammer.

In Section 4, faded channels are considered and it is shown that, while performance is significantly degraded, the worst case jammer uses full-band Gaussian noise. Section 5 shows briefly how all results can be applied to a frequency hopped differential phase shift keyed (DPSK) modulation system.

Finally, since all coded results are based on theoretically achievable decoder performance, where the decoder is perfectly matched to the channel, Section 6 presents some simulation results for the same channels, but using practically implemented decoders which are not always matched to the channel. The conclusion of Section 6 is that practical decoders can be implemented which operate within about 1 dB of theoretically derived E_b/N_o values. Overall conclusions are that the mitigation technique improves performance by 3 to 6 dB for a coded system.

2. BACKGROUND - UNMITIGATED PERFORMANCE IN TONE AND PARTIAL BAND JAMMING

2.1 Uncoded System

Consider a tone jammer who has perfect knowledge of the MFSK communicator's parameters: signal power, timing and frequency slots. This last parameter gives the tone jammer a significant (unrealizable) advantage, which we shall reconsider later, for complete frequency uncertainty can reduce jammer power up to 4 dB.

Assume the communicator hops at rate R_H hops/second sending one of M possible contiguous frequency tones of power S. To keep the tones orthogonal, they must be spaced R_H Hz apart. Thus in the total bandwidth W, the frequency hopping communicator has available W/MR_H slots and if the jammer puts a tone of power slightly greater than S in the communicator's slot, he may affect the decision. Thus if he has total power J, he can jam almost the fraction

$$P_H = \frac{J/S}{W/MR_H} \tag{1}$$

of the total number of slots, each with power slightly above S. P_H is then the probability that during any given hop the communicator's slot is jammed. In such an event the conditional probability that any bit is in error is 1/2, so that the bit error rate

$$P_b = 1/2 \; P_H \tag{2}$$

<u>Without coding</u>, $\log_2 M$ bits are transmitted in each hop. Thus the energy/bit and energy/hop are related by

$$E_b = E_H/\log_2 M \tag{3}$$

Thus letting N_o=J/W be the effective noise density (corresponding to spreading the jammer power evenly over bandwidth W),

$$\frac{E_b}{N_o} = \frac{1}{\log_2 M}\frac{E_H}{N_o} = \frac{1}{\log_2 M}\frac{S/R_H}{J/W} = \left(\frac{M}{\log_2 M}\right)\frac{1}{P_H} \tag{4}$$

Combining (1) through (4), we obtain

$$P_b = \left(\frac{M}{2\log_2 M}\right)\frac{1}{E_b/N_o} \tag{5}$$

Note that for M = 2 and 4 the scale factor is 1 and it grows (gradually at first) for $M = 2^k$, $k \geq 3$.

For M = 2, it is simple to compare this with the performance of a partial band noise jammer. If the fraction ρ of the band is jammed with power N_o/ρ (so that the total noise density remains N_o = J/W), a particular hop is affected with probability ρ, and the conditional error probability is $1/2\exp(-\frac{E_b}{2N_o/\rho})$. Maximizing the overall bit error rate with respect to ρ, the jammer can produce a bit error rate

$$P_b = \underset{0<\rho\leq 1}{\mathrm{Max}} \quad \frac{\rho}{2}\exp - \left(\frac{E_b\rho}{2N_o}\right) = \frac{e^{-1}}{E_b/N_o}, \quad M=2 \tag{6}$$

[A union upper bound for M>2, produces the same result multipled by $(M-1)/\log_2 M$]

Thus it appears from (5) and (6) that the ideal tone jammer is about 4.3 dB more effective than the noise jammer - most of this advantage disappears if the jammer is unaware of the hopping frequencies (or if these are changed continuously - as is easily and commonly done).

2.2 Improvement Through Coding

This intolerable performance which requires E_b/N_o = 30 dB, for the modest bit error rate $P_b = 10^{-3}$ can be greatly improved by introducing redundancy and coding. This could be as trivial as repetition coding (time diversity), but since coder-decoder implementations have become inexpensive, being available at moderate data rates on a single or a few integrated circuits, we shall consider the limiting case of a long convolutional code and a sequential decoder operating at its cutoff rate. For a binary symmetric channel with symbol error rate P_s, this is given by

$$r_o = 1-\log_2[1+2\sqrt{P_s(1-P_s)}] \quad (7)$$

Then r_o is taken as the (maximum) code rate and its reciprocal, $1/r_o$, is the (minimum) redundancy. Actually, for the channel to be binary symmetric for M>2, we must interleave the $\log_2 M$ symbols per hop after coding and prior to modulation and deinterleave them after demodulation and before decoding. This interleaver needs only to ensure that the $\log_2 M$ symbols in a given hop correspond to code symbols which are far removed (a few constraint lengths) in the code, so the interleaving memory is quite small. For this case, equations (1) to (4) hold but with P_b replaced by P_s, and $\log_2 M$ symbols/hop and r_o bits/symbol so that there are now $r_o \log_2 M$ bits/hop.

Thus in this case

$$\frac{E_b}{N_o} = \frac{E_H/N_o}{r_o \log_2 M} = \frac{M}{r_o \log_2 M}\ \frac{1}{P_H}$$
$$= \left(\frac{M}{2\log_2 M}\right)\ \frac{1}{r_o P_s} \quad (8)$$

Thus solving (7) for P_s in terms of r_o, we have[1]

$$P_s = \left(1-\sqrt{1-\alpha^2}\right)/2 \quad \text{where} \quad \alpha = 2^{1-r_o-1} \tag{9}$$

Using (8) and (9), E_b/N_o is plotted as a function of $1/r_o$ in Figure 1 for M=2 or 4 (or for any M, but normalized by the leading factor).

This optimizes at a rate $r_o = 1/4$ with $E_b/N_o = 14.7$ dB, a tremendous improvement over uncoded performance, but still considerably above the required E_b/N_o for coded MFSK in white noise.

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[1]Note that, at least for M=2, the same argument used earlier for partial band noise jamming yields (8) normalized by e^{-1} (or 4.3 dB better)

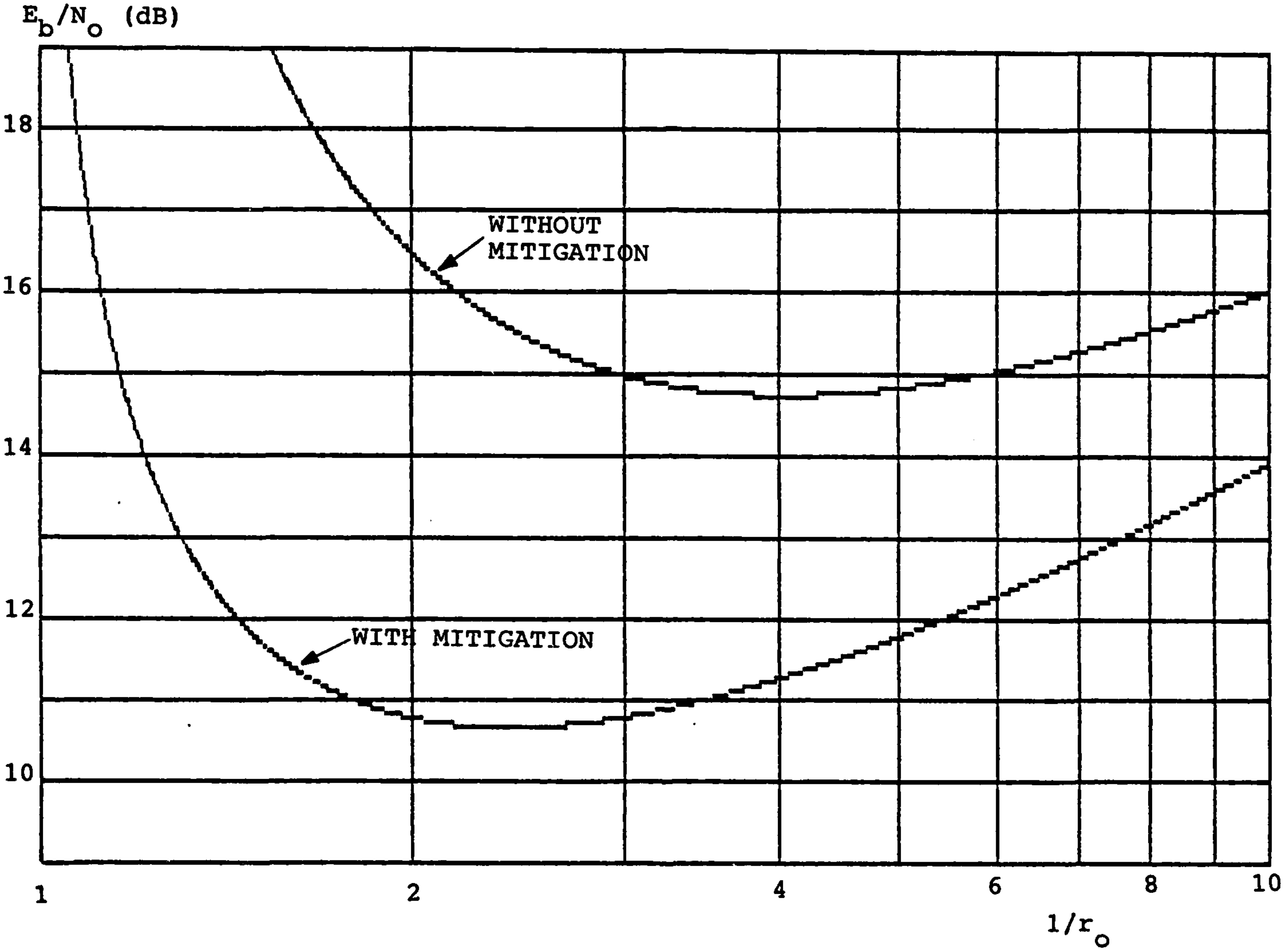


Figure 1. Minimum E_b/N_o for Coded MFSK and Tone Jaming - Without and With Mitigation Technique

3. THE RATIO-THRESHOLD MITIGATION TECHNIQUE

Let the filter outputs be ordered in ascending order so that

$$z_{min} = z_1 < z_2 < \ldots z_{M-1} < z_M = z_{max}$$

Then as always the $\log_2 M$ decision symbols are extracted from the index of z_M. In addition, we now derive a quality bit Q as follows:

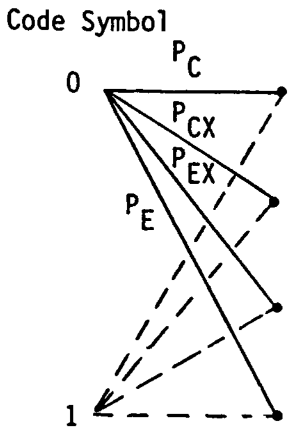
$$\text{if } z_M/z_{M-1} \geq \theta, \; Q=0 \text{ (good)}$$

$$\text{if } z_M/z_{M-1} < \theta, \; Q=1 \text{ (bad)}$$

where the ratio threshold $\theta \geq 1$ is a parameter chosen by t communicator. The use of this quality bit (along with t optimum choice of θ) constitutes the mitigation technique. Sec. 3.1 we consider a tone jammer in an unfaded channe. Initially we provide the jammer with the some unrealizable advantage of knowledge of the exact frequency transmitted within each slot, as in Sec. 2.

3.1 Tone Jamming for an Unfaded Channel

If the code symbols are interleaved prior to modulation and the decision symbols are correspondingly deinterleaved, as discussed above, but now with the quality bit Q associated with each of the $\log_2 M$ decision symbols upon deinterleaving, the result is a binary-input, quaternary-output channel as shown in Figure 2. We also indicate on the channel diagram the power level which the tone jammer must exceed in a communication slot to cause a hit (which results in a symbol error with probability 1/2). In Appendix A, we obtain the minimax solution (jammer strategy which maximizes degradation for communicator's choice of θ to minimize it) based on the convexity of r_o. Here we give a brief heuristic argument which leads to the correct answer as



Decision	Quality (0≡G, 1≡B)	Jammer Power Level to Produce Solid-line Transition
0	0	0
0	1	S/θ
1	1	S
1	0	$S\theta$

Figure 2. Channel with Ratio-Threshold Quality Measure

verified in the Appendix.

Suppose first that the jammer tries to overwhelm the communicator whenever he enters his slot, by jamming with power S so as to make resulting errors appear to have good quality. Then the fraction P_H of slots contain jammer power S and the remainder have none. The resulting channel is shown in Figure 3a and the average jammer power per slot is

$$\bar{J} = S\theta P_H \tag{10a}$$

Using (1) and (2) and proceeding as in (7) and (8), we find

$$\frac{E_b}{N_o} = \left(\frac{M}{2\log_2 M}\right)\frac{1}{r_o \theta P_s} \tag{11a}$$

On the other hand, suppose the jammer simply thwarts the quality measure by jamming the fraction P_H of the slots with power S and the fraction $1-P_H$ with power S/θ. Then the channel is that shown in Figure 3b and the resulting average jammer power per slot is

$$\bar{J} = P_H S + (1-P_H)S/\theta \tag{10b}$$

with resulting

$$\frac{E_b}{N_o} = \frac{M}{2\log_2 M} \; \frac{1}{r_o[P_s+(1-2P_s)/2\theta]}$$
$$= \frac{M}{2\log_2 M} \; \frac{2\theta}{r_o[2P_s(\theta-1)+1]} \tag{11b}$$

Note that (11a) is a decreasing fraction of θ while (11b) is an increasing function for $P_s < 1/2$.

This leads us to choose θ so as to equate[2] (11a) and (11b) which results in

$$\theta_o = [1+\sqrt{1-2(2-1/P_s)}]/2 \tag{12}$$

Then since the transition probabilities are the same, it matters not which strategy the jammer chooses. Proceeding as in (7), (8) and (9), we have from (11a) and (11b)

$$\frac{E_b}{N_o} = \left(\frac{M}{2\log_2 M}\right) \frac{1}{r_o \theta_o P_s} \tag{13}$$

where $P_s = (1-\sqrt{1-\alpha^2})/2, \;\; \alpha = 2^{1-r_o-1}$ (14)

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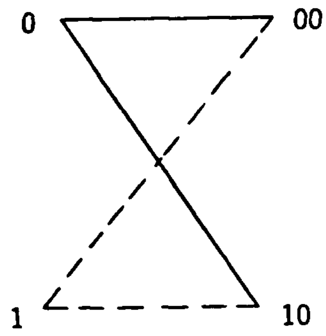
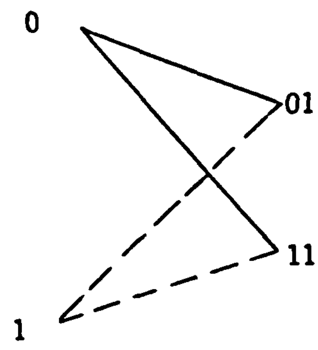
[2]The reason for choosing P_H to be the same for the two cases is justified in Appendix A.

and θ_o as given by (12) is the "improvement factor" over the unmitigated performance, as given by (8) and (9).

E_b/N_o is plotted as a function of $1/r_o$ in Figure 1. Comparison with the upper curve in the figure shows that the mitigation technique is quite effective against tone jamming.



Jammer Levels $S\theta$ and 0



Jammer Levels S and S/θ

Figure 3. Channels for Two Possible Tone Jammer Strategies

As a final observation, we note that the initial assumption that the jammer knows the frequency slot pattern exactly is completely unrealistic because in fact the communicator can vary the center frequency continuously for each successive hop. If the jammer's tone falls β Hz from any potential tone, his effective power is reduced by the factor

$$\left[\frac{\sin(\pi\beta/R_o)}{(\pi\beta/R_o)}\right]^2, \quad 0\leq\beta\leq R_o/2$$

which can be as small as $4/\pi^2$ (or -3.9 dB). A reasonable penalty on the jammer for not knowing frequency is 3 dB. This can be argued also on the basis that if he doubles the number of tones (but keeps the power constant as before), he will always come within $8/\pi^2$ (or -0.9 dB) from the desired levels established above. We shall use this 3 dB penalty in the comparison with partial band Gaussian jamming.

3.2 Partial Band Jamming for an Unfaded Channel

The effect of full band Gaussian noise on uncoded MFSK modulation is well known. It produces an error in one or more of the $L = \log_2 M$ bits conveyed by the tone with probability (Ref. 1)

$$P_M = \sum_{k=1}^{M-1} (-1)^{k-1} \binom{M-1}{k} \frac{e^{-\frac{kLE_b/N_o}{1+k}}}{1+k} \tag{15}$$

and the corresponding bit error probability is

$$P_E = \frac{M/2}{M-1} \; P_M \tag{16}$$

If the system is coded, E_b is replaced by E_s, the coded binary symbol energy.

Partial-band jamming usually refers to a jammer which places noise of density N_o/ρ in a fraction ρ of the band and leaves the remaining fraction $1-\rho$ noise-free. In this case, with the mitigation ratio-threshold set at $\theta = A$, the four transition probabilities (as in Fig. 2) are determined in Appendix B to be

$$P_E = \frac{M}{2(M-1)} \phi_0$$

$$P_{EX} = \frac{M}{2(M-1)} (\phi_1 - \phi_0)$$

$$P_{CX} = \frac{M-2}{2(M-1)} (\phi_1 - \phi_0) + (\phi_2 - \phi_1) \tag{17}$$

$$P_C = 1 - P_{CX} - P_{EX} - P_E$$

where $\phi_0 = f(\theta=1)$, $\phi_1 = f(\theta=A)$, $\phi_2 = f(\theta=1/A)$

and $$f(\theta) = \sum_{k=1}^{M-1} (-1)^{k-1} \binom{M-1}{k} \frac{\exp\left(\frac{-k\theta L \quad E_s/N_o}{1+k\theta}\right)}{1+k\theta}$$

$$E_s = rE_b$$

Then $r_o = 1 - \log(1+2\sqrt{P_E P_C} + 2\sqrt{P_{EX}P_{CX}})$ (18)

Clearly, setting $\theta = 1$ yields $\phi_0 = \phi_1 = \phi_2$ and thus $P_{EX} = P_{CX} = 0$, while P_E reduces to equations (16) and (15).

With even greater generality, we may allow for two levels of noise N_1 and N_2, over band fractions ρ and $1-\rho$, respectively. This would also cover the case where background (thermal) noise is present even when the jammer is not. To normalize to a common average noise density, we define

$$N_0 = \rho N_1 + (1-\rho) N_2 \tag{19}$$

so that the previous case ($\rho = 1$ or $N_2 = 0$) is also covered.

The expressions (17), derived for on-off noise, are easily modified for this two-level noise case to become

$$P_E = \frac{M}{2(M-1)} \left[\rho\phi_0{}^{(1)} + (1-\rho)\,\phi_0{}^{(2)}\right]$$

$$P_{EX} = \frac{M}{2(M-1)} \left\{\rho\left[\phi_1{}^{(1)} - \phi_0{}^{(1)}\right] + (1-\rho) \left[\phi_1{}^{(2)} - \phi_0{}^{(2)}\right]\right\}$$

$$P_{CX} = \frac{M-2}{2(M-1)}\left\{\rho\left[\phi_1{}^{(1)} - \phi_0{}^{(1)}\right] + (1-\rho) \left[\phi_1{}^{(2)} - \phi_0{}^{(2)}\right]\right\}$$

$$+ \rho\left[\phi_2{}^{(1)} - \phi_1{}^{(1)}\right] + (1-\rho)\left[\phi_2{}^{(2)} - \phi_1{}^{(2)}\right] \qquad (20)$$

$$P_C = 1 - P_{CX} - P_{EX} - P_E$$

where the superscripts (1) and (2) refer respectively to noise levels N_1 and N_2 rather than N_0.

For any given ρ, N_1/E_s and N_2/E_s, the various probabilities are obtained from (20). Substitution of (20) into (18) yields r_o. Also E_s/N_o is obtained from (19) and division by r_o yields E_b/N_o.

For several threshold values (A = 1,2,3,5,10) the worst case partial-band jammer (worst ρ, N_1 and N_2 for a given N_0) has been found and the resulting E_b/N_o is plotted as a function of $1/N_o$ in Figure 4 to 8 for M=2, 4, 8, 16 and 32. In most cases, the worst-case corresponds to N_2=0 (i.e., jammer on or off). However, for larger values of A, $N_2 \neq 0$ for the worst case. The advantage of mitigation (A>1) increases for lower redundancy ($1/r_o<2$). For small M (2 and 4), an optimum choice of θ=A produces performance almost equivalent to full-band Gaussian noise. Mitigation is not as effective for larger M but in all cases improvement is at least 2 dB at $r_o = 1/2$.

Figure 4. Worst Case Partial-band Jamming Unfaded 2-ary FSK

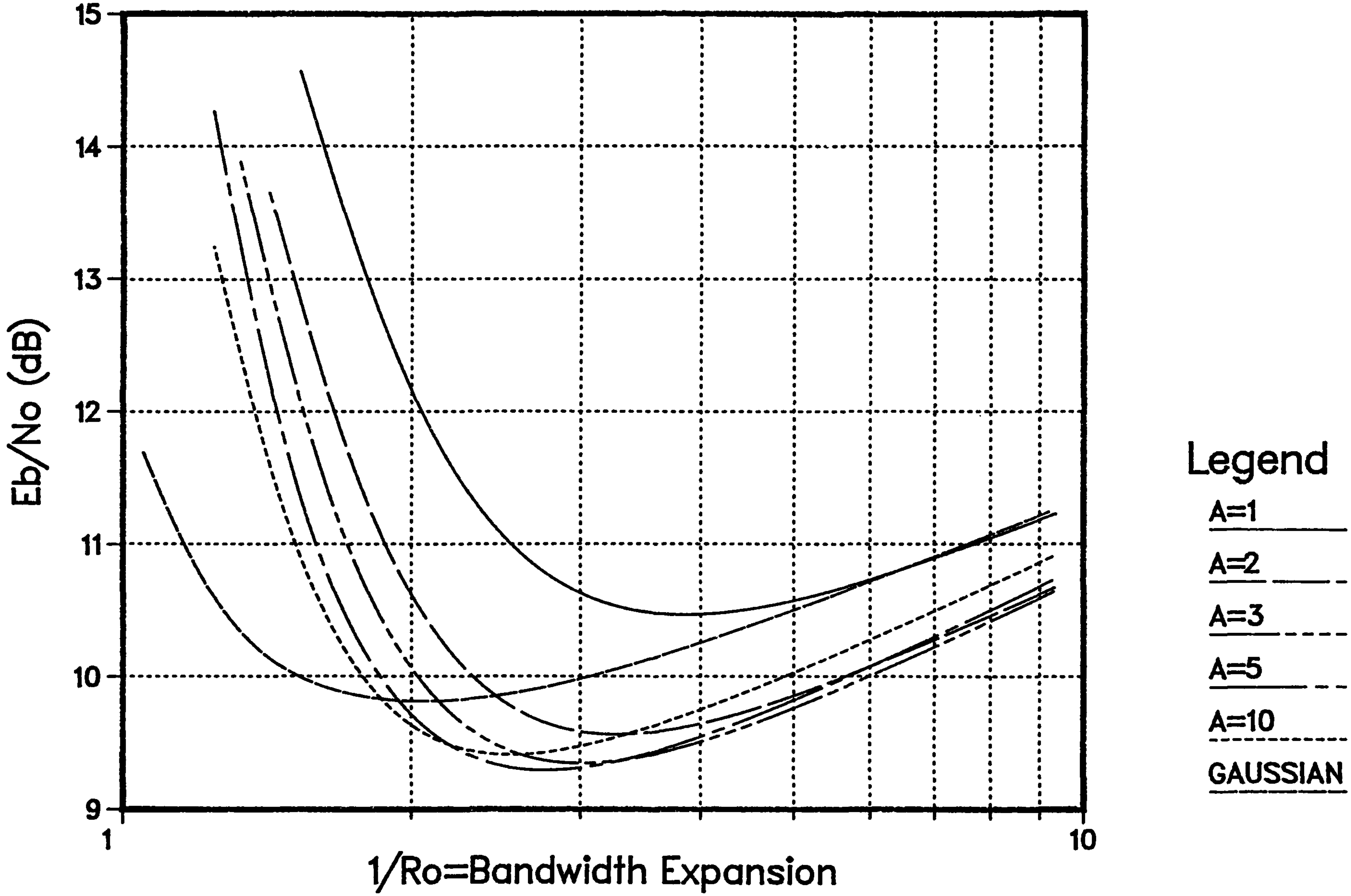


Figure 5. Worst Case Partial–band Jamming
Unfaded
4–ary FSK

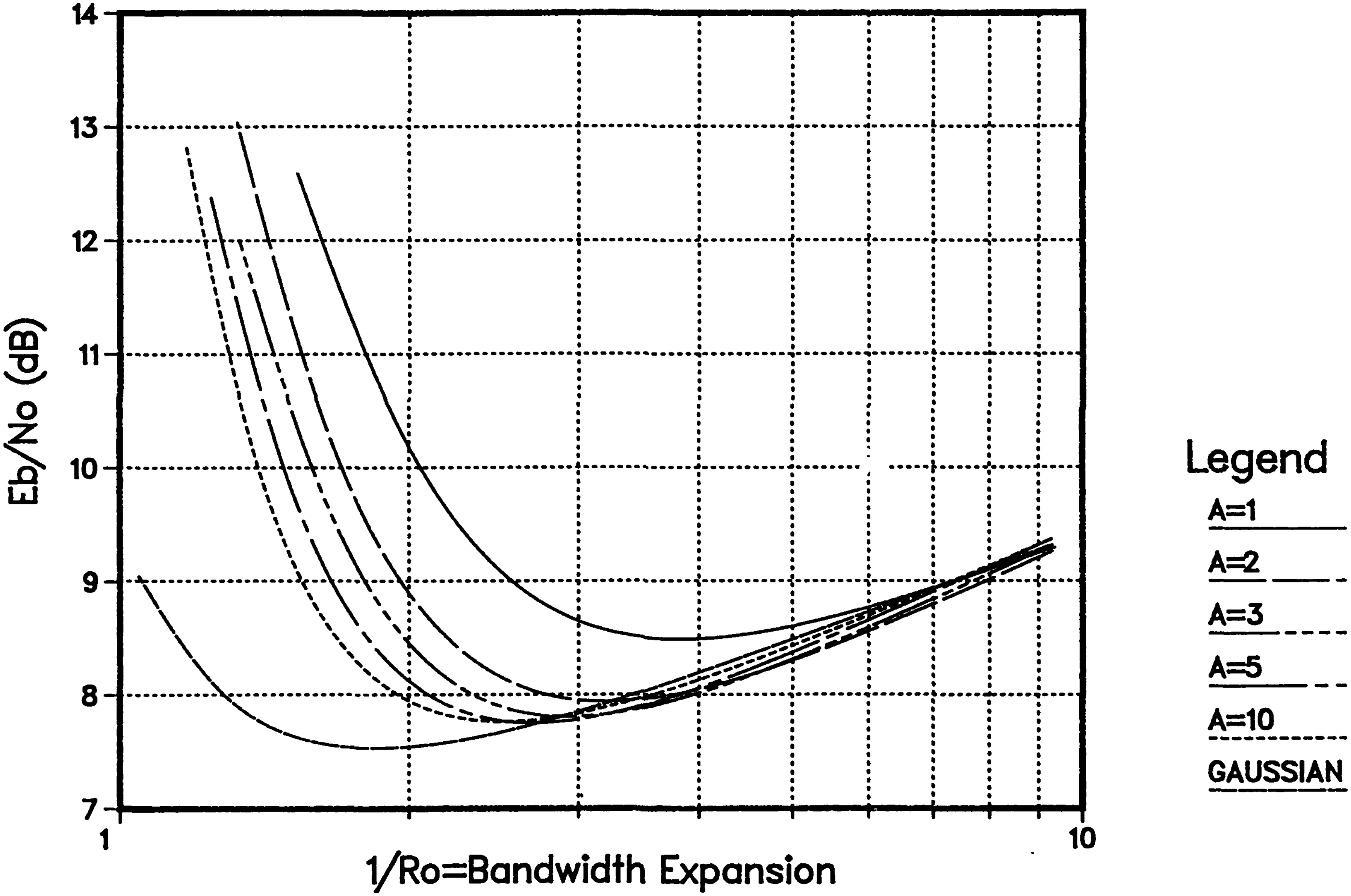


Figure 6. Worst Case Partial–band Jamming
Unfaded
8–ary FSK

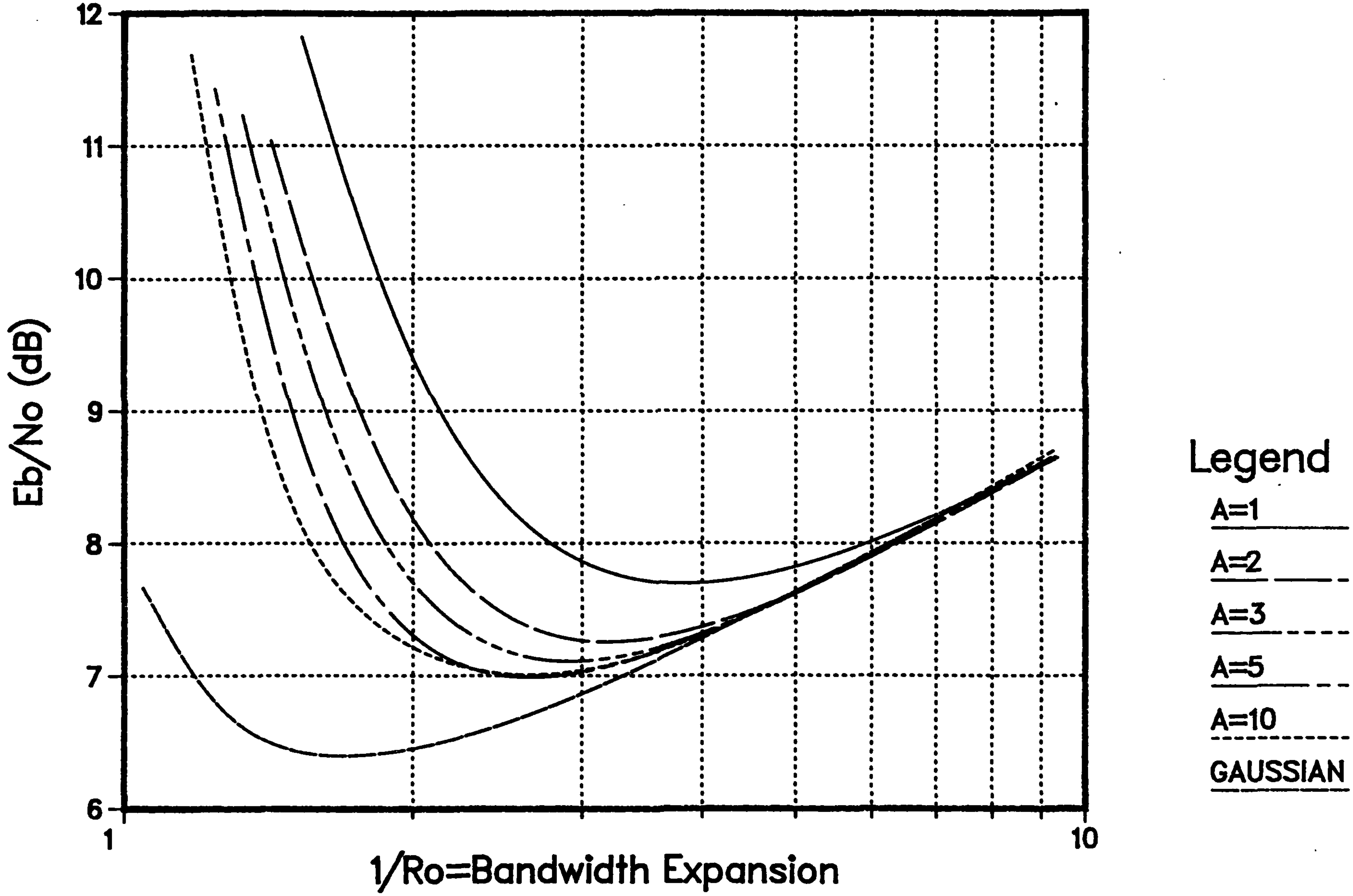


Figure 7. Worst Case Partial-band Jamming Unfaded 16-ary FSK

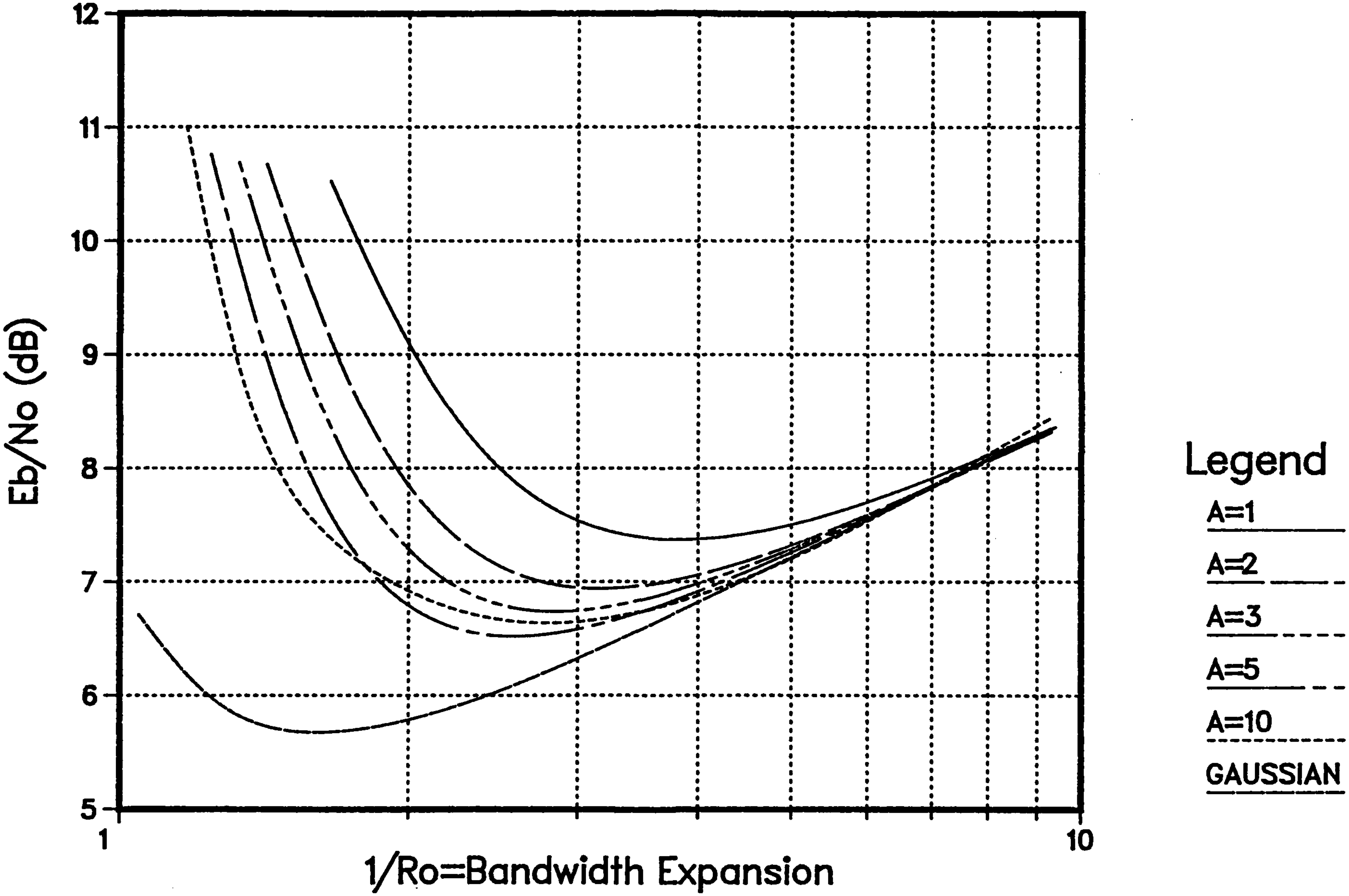
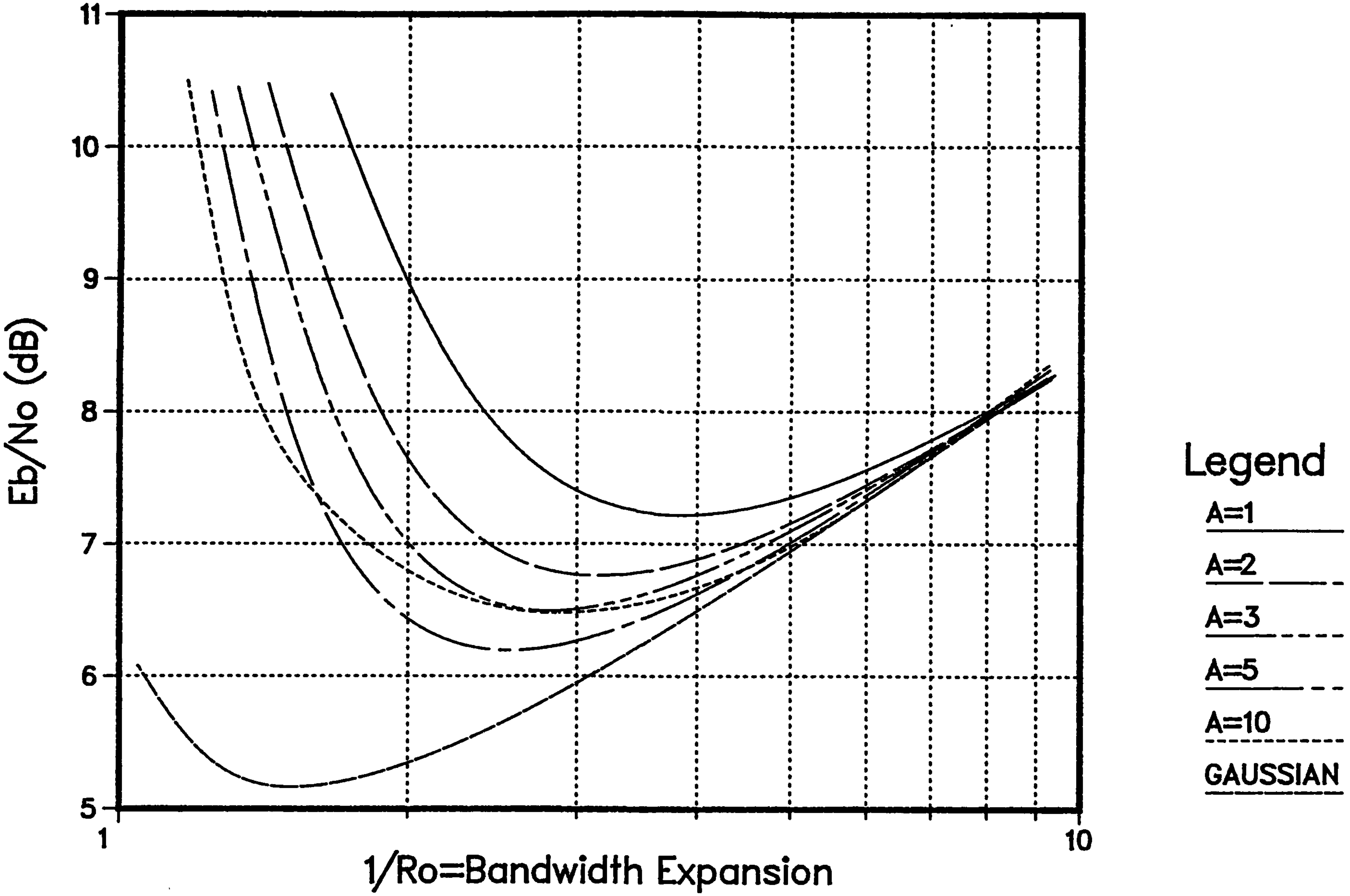


Figure 8. Worst Case Partial–band Jamming Unfaded 32–ary FSK



4. PERFORMANCE OF FREQUENCY HOPPED MFSK IN RAYLEIGH FADING CHANNELS

Rayleigh fading converts both signal and tone jamming interference into narrow-band Gaussian noise. This follows from the fact that if the signal amplitude is Rayleigh distributed and the phase is uniformly distributed, the signal in-phase and quadrature baseband components are independent Gaussian processes. This implies then that for Rayleigh fading channels, tone jamming effects are exactly the same as those of partial band noise jamming.

The effect of partial band jamming on a Rayleigh channel is the same as for an unfaded channel (eqs. 17 and 20) but with E_s replaced by $a^2 E_s$ where a is the Rayleigh distributed random variable with

$$p(a) = a\, e^{-a^2/2}$$

Thus the average every per symbol is

$$\bar{E}_s = \int a^2 E_s p(a)\,da = E_s \int a^3\, e^{-a^2/2}\, da = 2E_s$$

and $f(\theta)$ in (17) and (20) replaced by

$$g(\theta) = \int a\, e^{-a^2/2}\, f(\theta)\,d\theta$$

$$= \sum_{k=1}^{M-1} \frac{(-1)^{k-1}\binom{M-1}{k}}{1+k\theta} \int_0^\infty e^{-Ea^2\, k\theta/(1+k\theta)}\, a\, e^{-a^2/2} da$$

$$= \sum_{k=1}^{M-1} \frac{(-1)^{k-1}\binom{M-1}{k}}{1+k\theta(1+\bar{E}/N_o)} \qquad (21)$$

where $E = L\ E_s = r\ L\ E_b$

Using $g(\theta)$ in place of $f(\theta)$ in (17) and (20) yields the desired performance in Rayleigh fading. In all cases it is found that $\rho=1$ (full-band) yields the worst performance. The results for $\rho=1$ and A = 1, 3, 5, 10 and M = 2, 4, 8, 16 and 32 are shown in Figures 9 through 13. The use of the additional quality bit, derived by the mitigation technique, is not as dramatically successful as for unfaded channels with partial-band jamming, but performance improvements on the order of 1 to 2 dB are achieved at r=1/2.

Figure 9. Worst Case Partial—band Jamming
Faded
2—ary FSK

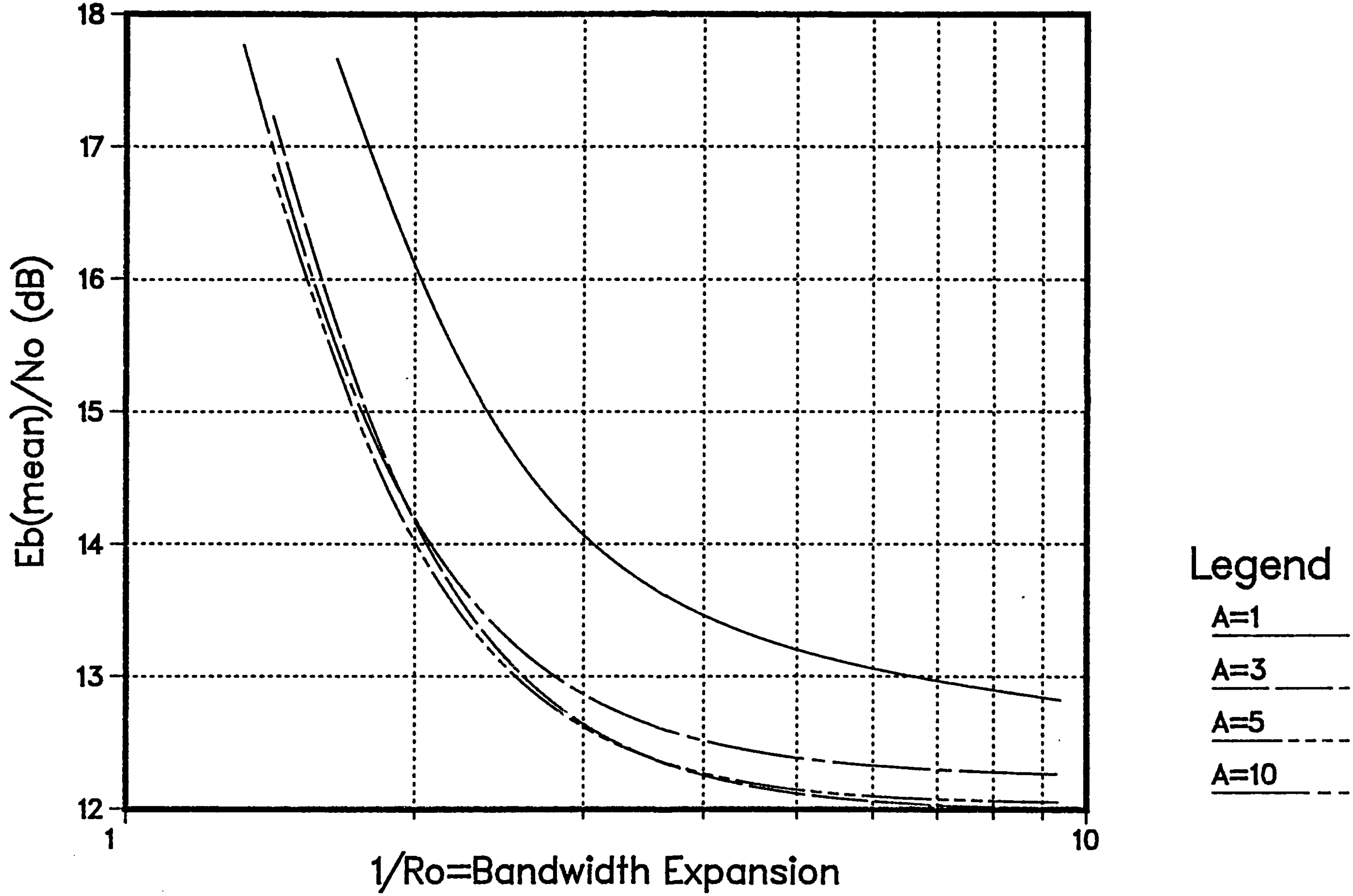


Figure 10. Worst Case Partial–band Jamming
Faded
4–ary FSK

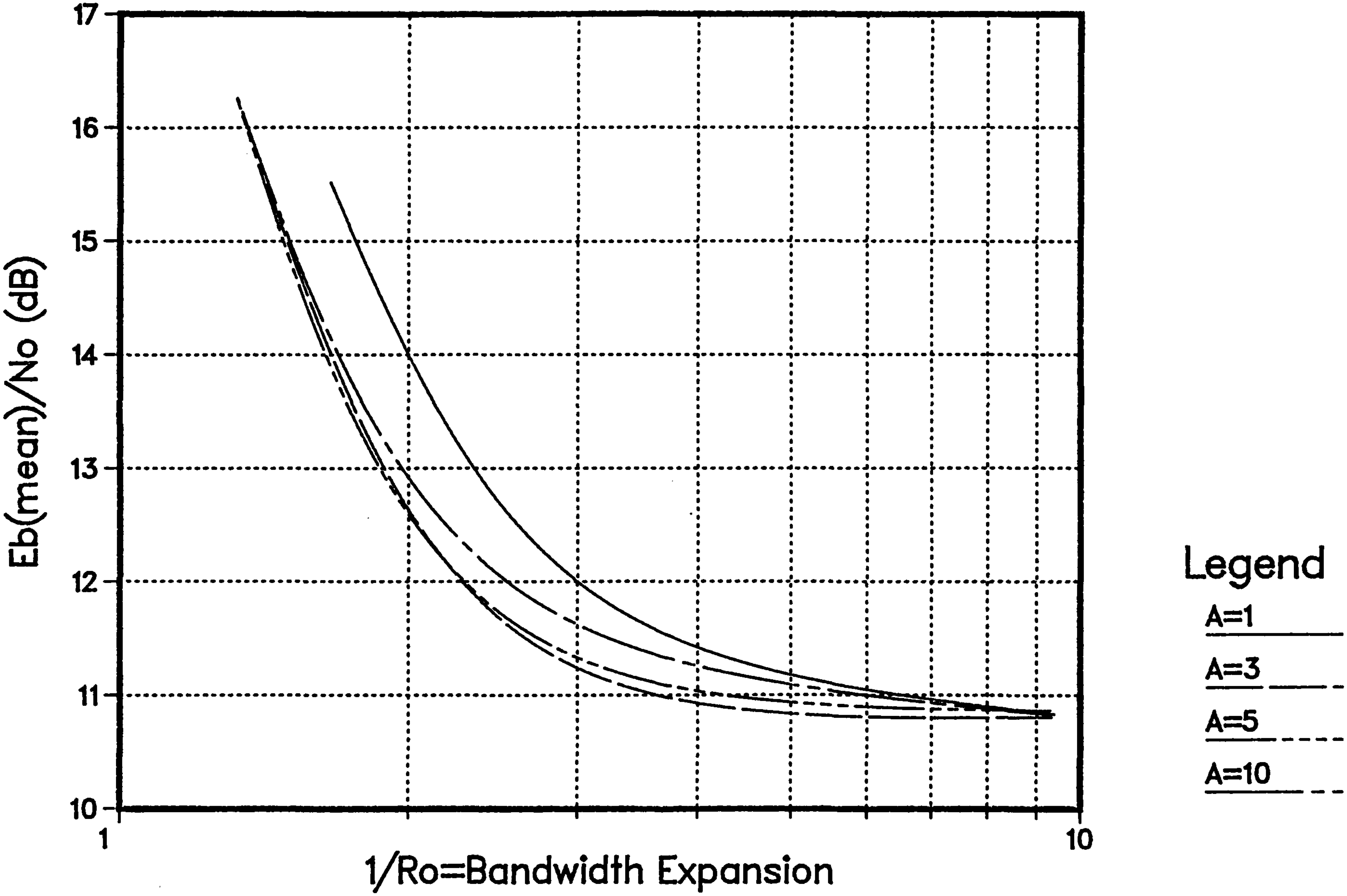


Figure 11. Worst Case Partial-band Jamming
Faded
8-ary FSK

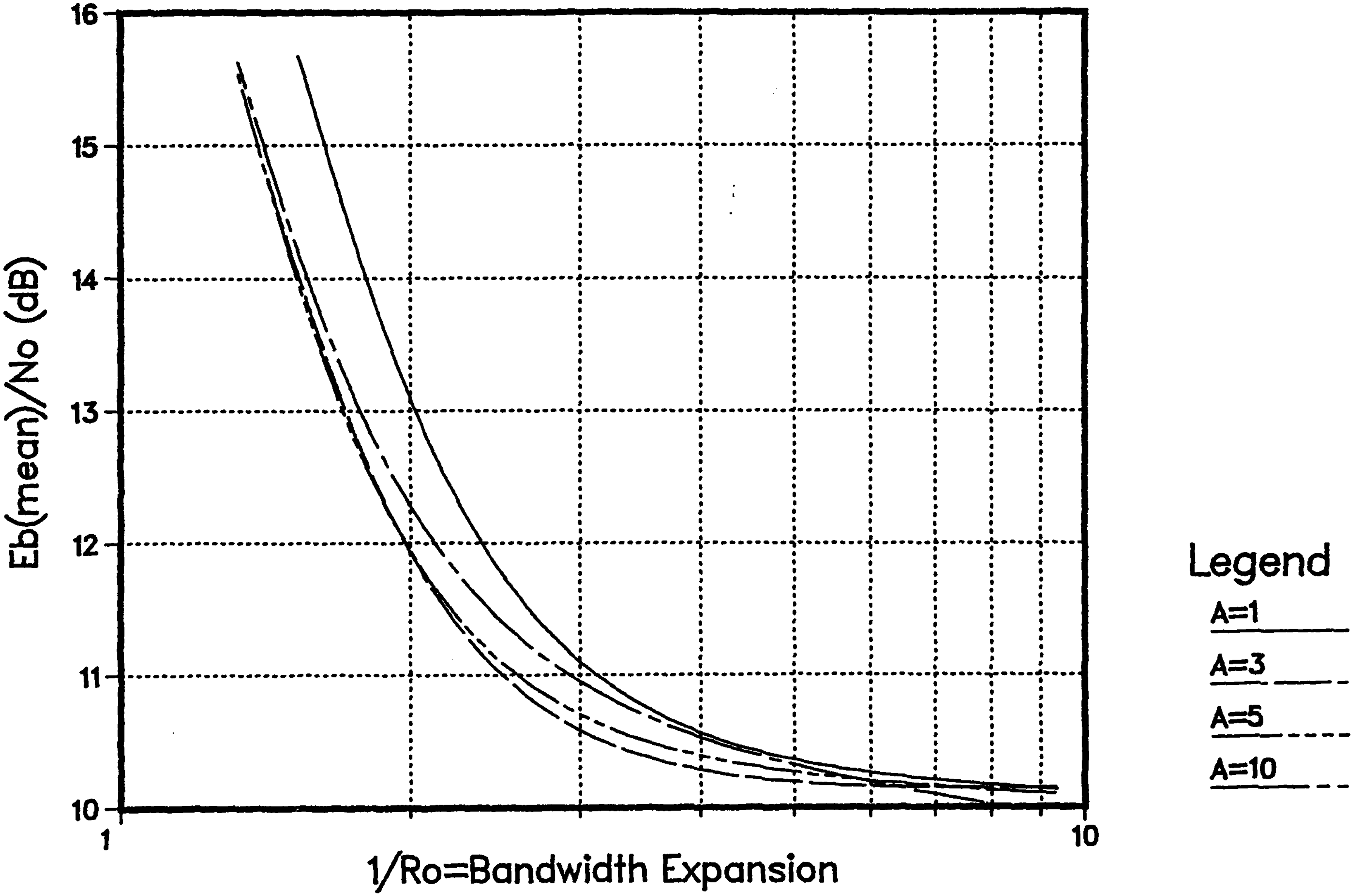


Figure 12. Worst Case Partial-band Jamming
Faded
16-ary FSK

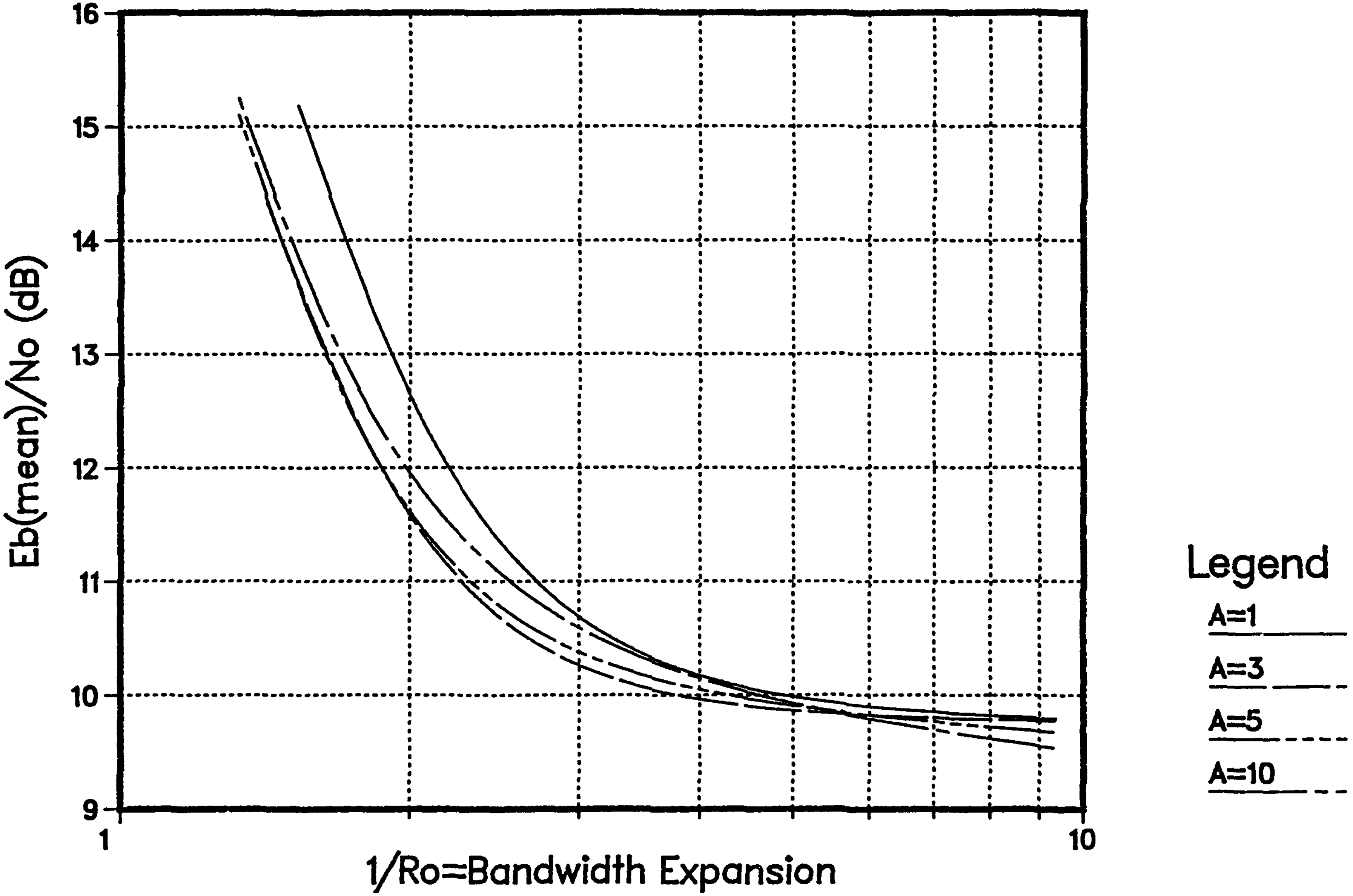
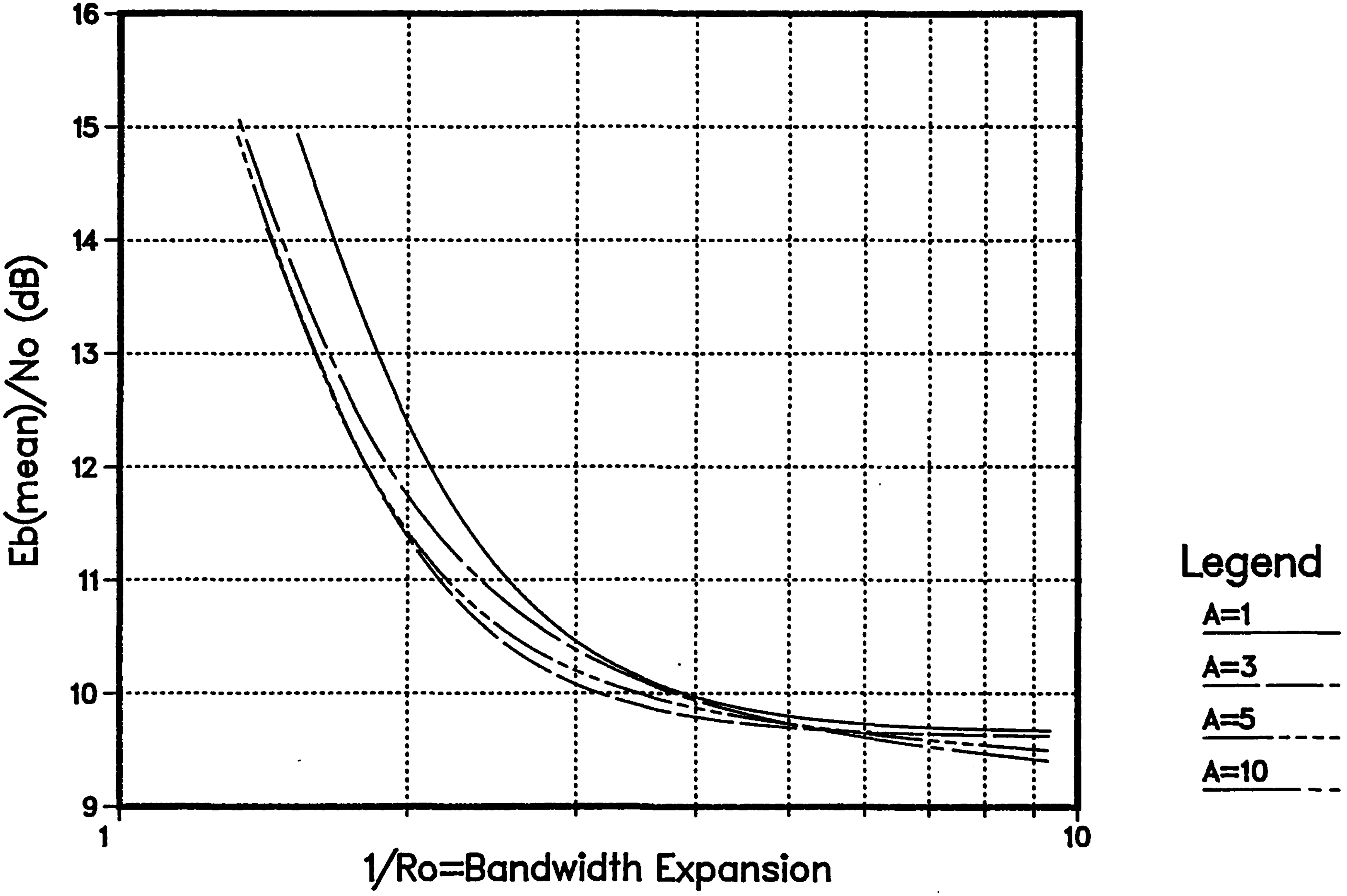


Figure 13. Worst Case Partial-band Jamming
Faded
32-ary FSK



5. PERFORMANCE OF FREQUENCY HOPPED DPSK MODULATION

Differentially coherent PSK (DPSK) is a modulation technique which, unlike coherent PSK, does not require accurate phase tracking. Rather, each symbol's reference is derived from the preceding symbol and the signal is modulated by the binary data as either a change or no change in phase between symbols. It is readily shown (Ref. 1) that performance is identical to that of binary FSK but with an advantage of a factor of 2(3 dB) in E_b/N_o. The only overhead is imposed by the fact that a single unmodulated symbol must be sent to serve as a reference for the first data symbol (transition).

Assuming then that L symbols are transmitted per hop, the effective symbol energy is

$$E_s = \left(\frac{L}{L+1}\right) r_o E_b$$

Thus for partial-band jamming in both unfaded and faded channels, DPSK performs exactly as binary FSK (Figs. 4 and 9) but with E_b/N_o reduced by the factor $(L+1)/2L$ or $10\log((L+1)/2L)$ dB.

6. THEORETICAL PERFORMANCE COMPARISONS AND SIMULATED PERFORMANCE WITH A PRACTICAL DECODER

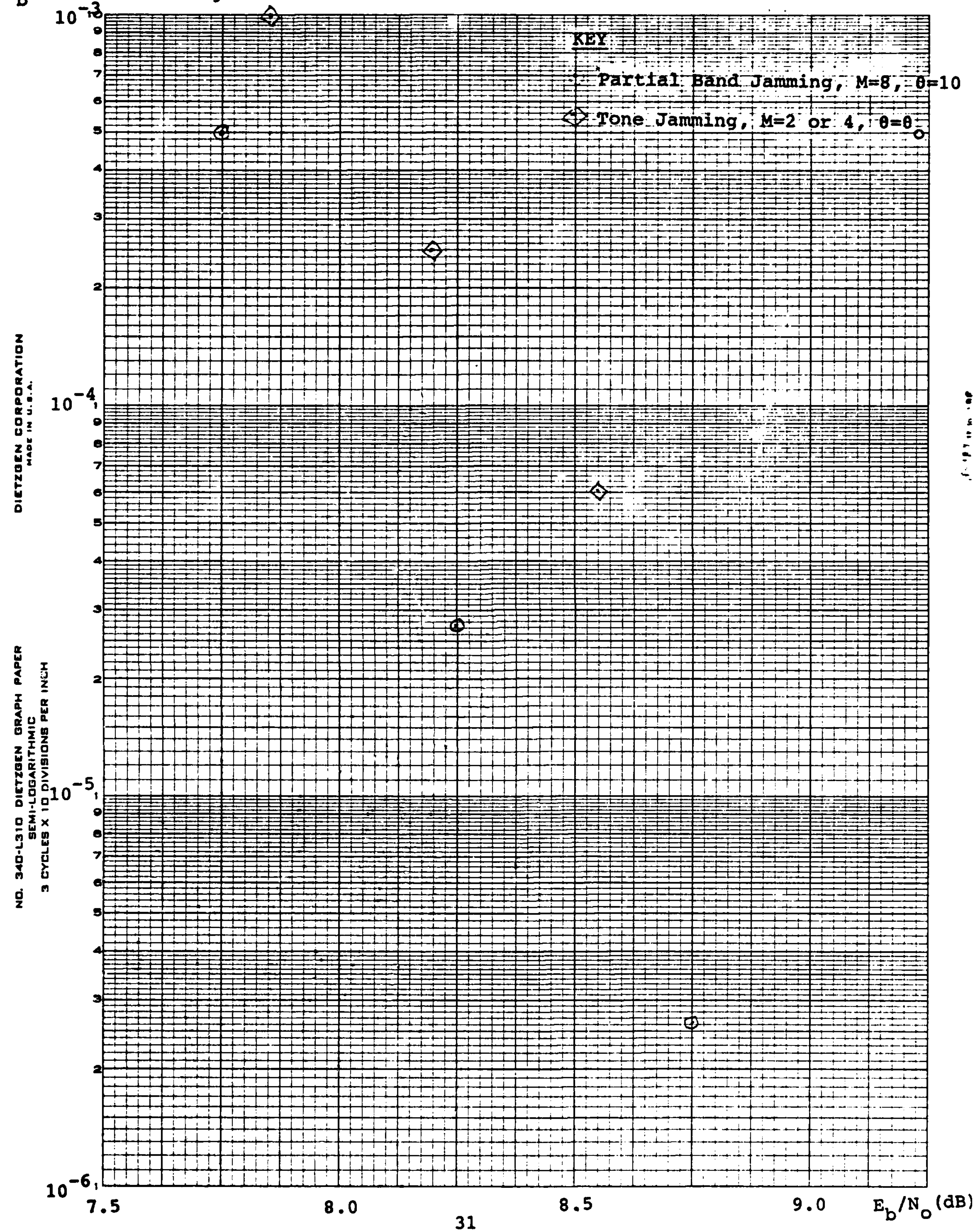
The results obtained in Sec. 3 for Tone and Partial-Band Jamming are partially summarized in Table I for MFSK with M=2, 4 and 8, which gives the minimum theoretical E_b/N_o (based on $r'=r_o$) for r=1/2. The first two columns are without mitigation and the next two are with mitigation at a threshold $\theta=3.7$ chosen to optimize performance for tone jamming. The last column is for full band jamming (white noise) without mitigation. All tone jamming results, obtained by using eqs. (13) and (14), are increased by 3 dB to remove the jammer's advantage of frequency knowledge in each slot.

A set of simulations was performed using a practical sequential decoder, the LINKABIT LS56 wherein the algorithm section is implemented as a single LSI circuit. The decoder, which has a clock rate of 1.5 MHz was operated at a data rate of 100 Kbps. The decoder was presented with a digital stream of soft decisions in the form of random sequences of independent quaternary random variables with the statistics of the transition diagram of Fig. 2. Statistics of both tone jammers (Sec. 3.1) and partial-band jammers (Sec. 3.2) were computed and fed to the sequence generator which produced the input for the simulation. The results for partial-band jamming were obtained for M=8 and $\theta=10$, $\rho=0.275$, $N_1=2.55$, $N_2=0.41$ which are essentially the minimax solution over the range of E_b/N_o considered. For the tone jamming cases, M=2 or 4 and the optimum $\theta=\theta_o$ (eq. 12) was used. The latter results are reduced by 3 dB as discussed above. For M=8 and tone jamming, E_b/N_o must be increased by 4/3 (1.25 dB). All results are plotted in Fig. 14. It appears from the simulated results that good performance (bit error rates below 10^{-5}) can be achieved in all cases considered at E_b/N_o values only 1 dB above the theoretical results of Table I.

Table I. E_b/N_o Performance Summary for Rate 1/2 Coding

M \ Jammer	Unmitigated (θ=1)		Mitigated (θ=3.7)		White Noise (θ=1)
	Tone	Partial Band Noise	Tone	Partial Band Noise	
2	13.5	12.1	7.8	9.9	9.8
4	13.5	10.1	7.8	8.3	7.5
8	14.7	9.4	9.0	7.5	6.5

Figure 14. Simulated Performance of Sequential Decoder



References

1. A. J. Viterbi *Principles of Coherent Communication* McGraw-Hill, New York, 1966.

APPENDIX A

MINIMAX SOLUTION FOR TONE JAMMING WITH MITIGATION

The worst-case jammer will use the levels indicated in Figure 2, since any higher level will not further degrade performance. Without loss of generality, let the transition probabilities of Figure 2 be relabelled:

$$P_E = \alpha\rho_1/2 \qquad P_{EX} = (1-\alpha)\ \rho_2/2$$
$$P_C = \alpha(1-\rho_1/2) \qquad P_{CX} = (1-\alpha)\ (1-\rho_2/2) \tag{A1}$$

Then

$$\bar{J}/S = \alpha\rho_1\theta + (1-\alpha)\ [\rho_2 + (1-\rho_2)/\theta] \tag{A2}$$

while

$$r_o = 1-\log_2\ (1+2\sqrt{P_E P_C} + 2\sqrt{P_{EX}P_{CX}})$$
$$= 1-\log_2\ [1+\alpha\sqrt{\rho_1(2-\rho_1)} + (1-\alpha)\sqrt{\rho_2(2-\rho_2)}] \tag{A3}$$

The jammer wishes to minimize r_o subject to the constraint (A2). Equivalently he wishes to maximize

$$A \stackrel{\Delta}{=} 2^{1-r_o}-1 = \alpha\sqrt{\rho_1(2-\rho_1)} + (1-\alpha)\sqrt{\rho_2(2-\rho_2)} \tag{A4}$$

subject to (A2). The communicator must choose θ to minimize the maximum of (A4). Suppose the communicator makes the choice (to be justified later) of $\theta = \theta_o$ such that

$$\rho_2 + (1-\rho_2)/\theta_o = \rho_2\ \theta_o \tag{A5}$$

Then the jammer must maximize A subject to the constraint:

$$\bar{J}/S = \theta_o\ [\alpha\rho_1 + (1-\alpha)\ \rho_2] \tag{A6}$$

Because of the convexity of $f(\rho) = \sqrt{\rho(2-\rho)}$ it follows that

$$A = \alpha\sqrt{\rho_1(2-\rho_1)} + (1-\alpha)\sqrt{\rho_2(2-\rho_2)}$$

$$= \overline{f(\rho)} \leq f(\bar{\rho}) = \sqrt{\bar{\rho}(2-\bar{\rho})}$$

where $\bar{\rho} = \alpha\rho_1+(1-\alpha)\rho_2 = \bar{J}/(S\theta_o)$

Then if the jammer chooses $\rho_1 = \rho_2$, the choice of α is irrelevant and he maximizes A while satisfying (A-6).

This justifies the choice of either of the channels of Figure 3. The choice of θ is established by (A5) which corresponds to equating (10a) and (10b), since $\rho_1=\rho_2=P_H$. To justify this choice, suppose θ were chosen otherwise; then since (11a) is a decreasing function while (11b) is an increasing function of θ, the use of the channel of Figure 3a for $\theta<\theta_o$ or of the channel of Figure 3b for $\theta>\theta_o$ would result in greater values of E_b/N_o than that obtained by using the solution θ_o of (A5). This establishes that the choice $\rho_1=\rho_2=P_H$ and $\theta=\theta_o$, the solution of (A5), is a minimax solution to the tone jamming channel with mitigation.

APPENDIX B

TRANSITION PROBABILITIES FOR PARTIAL-BAND JAMMING OF MFSK SIGNALS

The error expressions P_E and P_{EX} of (17) are all <u>bit</u> transition probabilities and consequently equal the corresponding event probabilities scaled by (M/2)/(M-1).

Thus

$$P_E = \frac{M/2}{M-1}\,\phi_0 \tag{B1}$$

$$P_{EX} = \frac{M/2}{M-1}\,(\phi_1 - \phi_0) \tag{B2}$$

where $\phi_0 = \Pr\left(\frac{\text{any incorrect filter energy}}{\text{correct filter energy}} > A\right) \triangleq f(\theta{=}A)$

$$\phi_1 - \phi_0 = \Pr\left(A > \frac{\text{any incorrect filter energy}}{\text{correct filter energy}} > 1\right)$$

$$= f(\theta{=}1) - f(\theta{=}A)$$

and

$$f(A) = \Pr\left[\text{any incorrect filter energy} > A(\text{correct filter energy})\right]$$

$$= 1 - \Pr\left[A(\text{correct filter energy}) > \text{every incorrect filter energy}\right]$$

$$= 1 - \int_0^\infty x e^{-(x^2+2E)/2}\, I_0(\sqrt{2E}\,x)\left[\int_0^{\sqrt{A}\,x} y e^{-y^2/2}\,dy\right]^{M-1} dx$$

where $E \triangleq L\,E_s/N_0 = r\,L\,E_b/N_0$

The expression can be written as a finite sum:

$$1 - f(\theta) = e^{-E}\int_0^{\infty} xe^{-x^{2}/2}\; I_0(\sqrt{2E}\,x) \sum_{r=0}^{M-1} (-1)^k \binom{M-1}{k} e^{-kx^2A/2}\, dx$$

$$= e^{-E} \sum_{k=0}^{M-1} (-1)^k \binom{M-1}{k} \int_0^{\infty} x\, e^{-x^2(kA+1)/2}\; I_0(\sqrt{2E}\; x)\; dx$$

$$= e^{-E} \sum_{k=0}^{M-1} (-1)^k \binom{M-1}{k} \int_0^{\infty} \frac{ze^{-z^{2}/2}}{1+kA}\; I_0\left(\sqrt{\frac{2E}{1+kA}}\; z\right) dz$$

(where $z^2 = (1+kA)x^2$)

$$= e^{-E} \sum_{k=0}^{M-1} (-1)^k \binom{M-1}{k} \frac{e^{E/(1+kA)}}{1+kA}$$

Thus

$$f(\theta) = -\sum_{k=1}^{M-1} \frac{(-1)^k \binom{M-1}{k}}{1+kA}\; e^{E/(1+kA)}\; e^{-E}$$

$$= \sum_{k=1}^{M-1} \frac{(-1)^{k-1} \binom{M-1}{k}}{1+kA}\; e^{-E\; kA/(1+kA)} \qquad \text{(B3)}$$

The probability of correct decision with good quality is just

$$P_C = \Pr\left(\frac{\text{correct filter energy}}{\text{any incorrect filter energy}} > A\right) = 1 - \phi_2$$

where $\phi_2 = \Pr\left(\frac{\text{any incorrect filter energy}}{\text{correct filter energy}} > \frac{1}{A}\right)$

or $P_C = 1-\phi_2$

where $\phi_2 = f(\theta=1/A)$ (B4)

It follows that the fourth transition probability is

$$P_{CX} = 1 - P_C - P_E - P_{EX}$$

$$= \frac{M-2}{2(M-1)} (\phi_1 - \phi_0) + (\phi_2 - \phi_1) \qquad \text{(B5)}$$

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