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COMPUTER SIMULATION OF ARTILLERY S&A MECHANISM (INVOLUTE GEAR TRAIN AND STRAIGHT-SIDED VERGE RUNAWAY ESCAPEMENT)

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LARGE CALIBER
WEAPON SYSTEMS LABORATORY
DOVER, NEW JERSEY

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This investigation developed a computer simulation which is adaptable to the two configurations of a complete artillery safing and arming mechanism that is driven by a rotor and contains a straight-sided verge runaway escapement and a two-pass involute gear train. The mathematical model treats three motion regimes of the associated escapement; i.e., coupled motion, free motion, and impact. The computer program is well adapted to parametric studies, and it allows the use of pallets with arbitrarily located centers of mass. Further,
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20. ABSTRACT (cont)

it makes it possible to determine the maximum value of the contact forces in the gear train as well as in the escapement. A simulation run with the dimensions of the M739 safing and arming mechanism showed excellent agreement with experimental results when a system coefficient of friction of 0.1 was used.

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INTRODUCTION

A computer simulation was developed for a complete artillery safing and arming (S&A) mechanism containing a straight-sided verge runaway escapement. Figures 1 and 2 show the essentials of two different configurations of such a mechanism which, in addition to the escapement, contains a spin driven rotor and a two-pass involute gear train.

While the mathematical model of the verge runaway escapement represents a new study, the description of the rotor and the gear train, together with the essence of the computer program, is based to a considerable extent on reference 1, which deals with the computer simulation of an S&A mechanism containing a pin pallet runaway escapement.

As in reference 1, the three motion regimes of the runaway escapement (coupled motion, free motion, and impact) are considered. Again, the pallet of the escapement may have an arbitrarily-located center of mass.

Complete derivations are given for the mathematical model, including the configuration variations. A detailed description and operational instructions are given for the associated computer program whose output furnishes instantaneous kinematic conditions of the escapement components and the number of turns to arm, as well as all non-impact contact forces. This program was extensively tested for the configuration no. 2 data (fig. 2) of the M739 fuze S&A mechanism. This mechanism simulation required approximately 26 turns to arm when a system coefficient of friction of 0.1 was used.

DESCRIPTION OF COMPUTER PROGRAM

The following gives the essential steps of program SANDA2V which is listed in appendix A. The mathematical model of the verge runaway escapement naturally differs from that of the pin pallet escapement as shown in reference 1, especially since the differential equations for entrance- and exit-coupled motions in the verge escapement are not the same. However, the programming schemes are essentially identical. Thus, with the exception of controls pertaining to the aforementioned entrance- and exit-coupled motions, the program runs parallel to the one associated with the pin pallet escapment.

The following outlines the essential features of the program without giving specific control details.

The main program starts with reading in and writing of all relevant physical data. This is followed by the computation of gear ratios, fuze body angles, centrifugal forces, gear train constants, and earliest and latest possible values of the gear angles by way of subroutine ALFA, as well as the initialization of the gear angles. The simulation begins with the entrance-coupled motion at a starting angle PHID, which represents that angle ϕ of the escape wheel which is associated with the approximate center of the entrance working surface of the

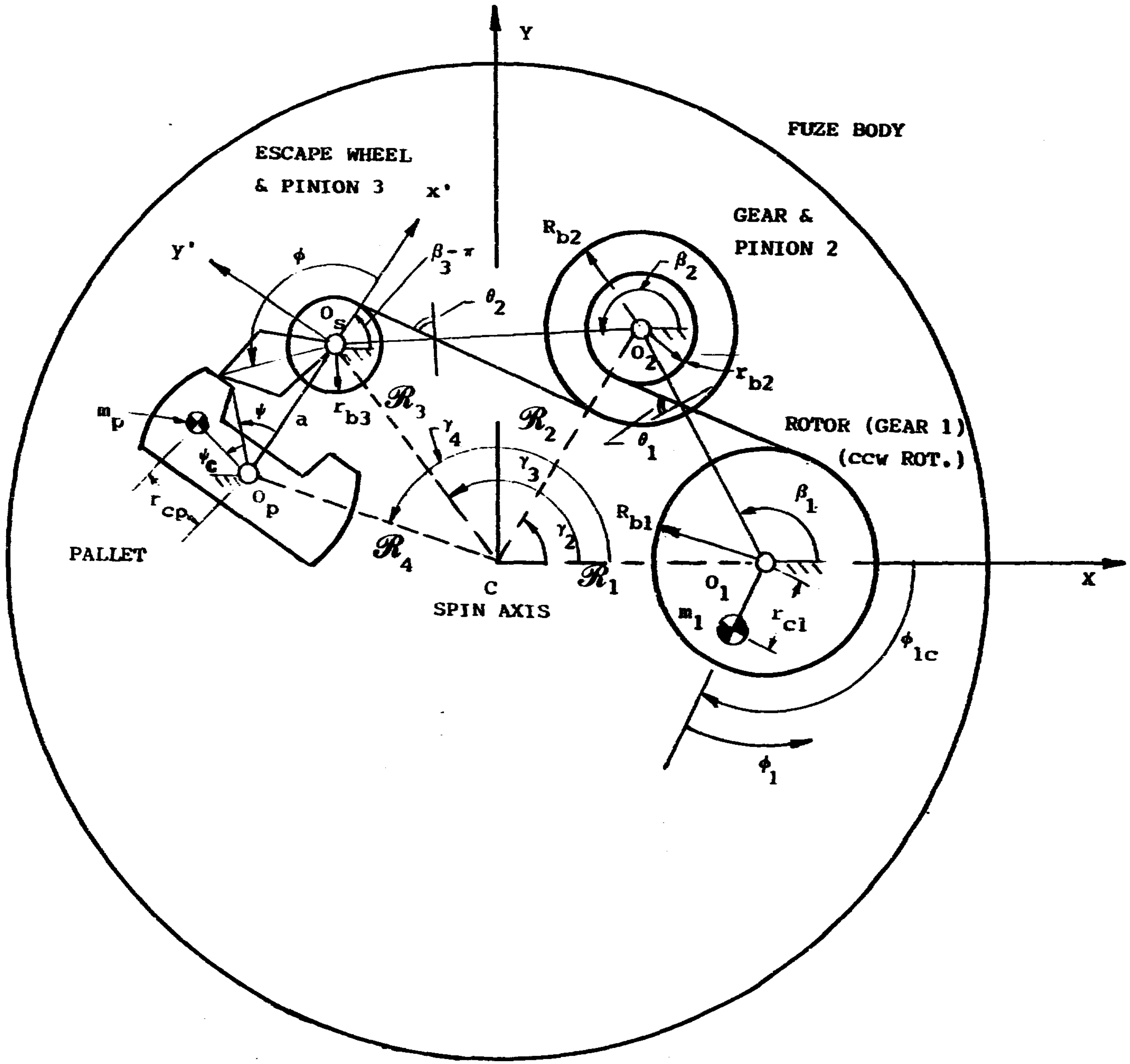

Figure 1. Rotor driven S&A device, configuration no. 1

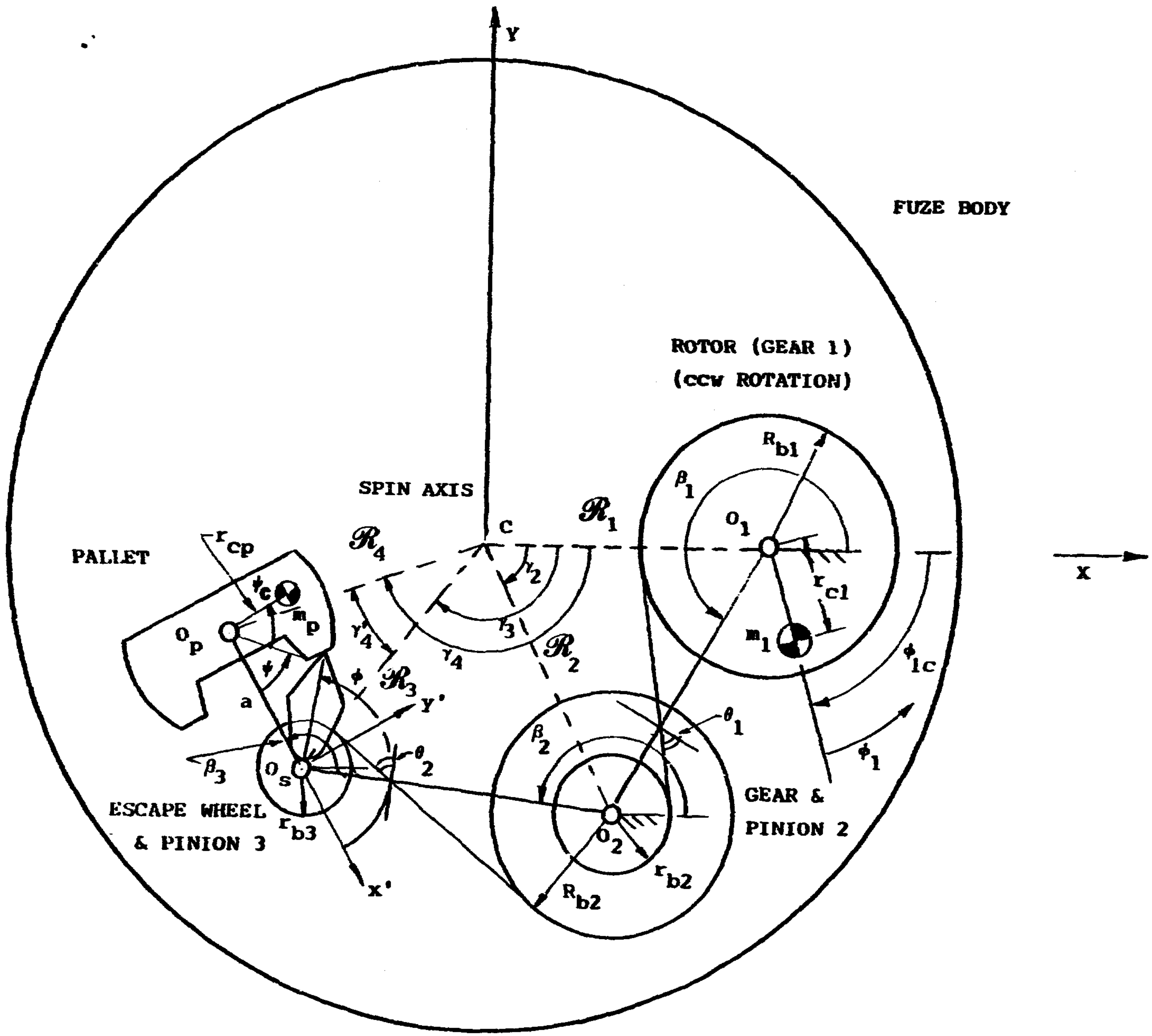

Figure 2. Rotor driven S&A device, configuration no. 2

pallet. This angle then corresponds to a cumulative escape wheel angle PHITOT of 0°.

Coupled Motion (Location 1)

Depending on whether entrance- or exit-coupled motion takes place, differential equation B-100 or equation B-119 of appendix B must be solved. To this end, the main program calls on an available fourth-order Runge-Kutta routine.[1] The main purpose of the subroutine FCT is to present the second-order differential equation in terms of two first-order ones to RKGS. PHI(1) and PHI(2) represent the angle ϕ and the angular velocity $\dot{\phi}$, respectively. The computation of all parameters of the differential equation takes place in subroutine FCT as well as in subroutines IN2 and KINEM.

The subroutine KINEM computes current values of g, ψ and $\dot{\psi}$, as shown in appendix C, as well as of the moment arms A_1', B_1', C_1', and D_1' as shown in appendix D.

Subroutine IN2 computes various gear mesh parameters and instantaneous mesh contact angles, as well as the signum functions s_1, s_2, s_4, and s_5. In addition, the parameters A_1 to A_{44}, listed in appendix B, are obtained. It is to be noted that the parameters A_1, A_5, A_9, A_{12}, A_{17}, and A_{18} depend on whether entrance- or exit-coupled motion is involved.[2] The parameters A_{45} to A_{50} are computed in subroutine FCT. The reason for this division is found in the limited number of arguments permitted in any one subroutine. Finally, the gear-indexing operation, as outlined in reference 2, is performed with the help of the angle ϕ.

The following discusses the manner in which the instantaneous rotor angle $\phi_{1C} + N_{31}\phi_T$ of the coupled motion differential equations must be expressed in subroutine FCT. Recall that ϕ_{1C} is the initial rotor angle, ϕ_T is the total angle of rotation of the escape wheel, and N_{31} stands for the gear ratio between the rotor and the escape wheel. Since the angle ϕ with the Runge-Kutta variable PHI(1) varies between approximately 134° and 144° during entrance- and between 209° and 216° during exit-coupled motion, the total escape wheel angle ϕ_T can only be obtained by continuously adding the increments due to each cycle of Runge-Kutta computations. Thus,

$$\phi_T = \phi_{TOT} + \Delta\phi \qquad (1)$$

---

[1] RKGS Routine, IBM System/360 Scientific Subroutine Package (360A-CM-OX3), Version III.

[2] The program uses the symbols AA1, etc. throughout. This should not be confused with the symbols AA_1 to AA_{18}, which are used in the combined exit-coupled motion differential equation (B-119).

where

ϕ_{TOT} = total escape wheel angle up to a certain Runge-Kutta cycle. (This is represented by PHITOT in the program.)

$\Delta\phi$ = increment of escape wheel during this Runge-Kutta cycle.

The increment $\Delta\phi$ is calculated as the difference between the latest value of PHI(1) and its previous one which has been stored as PHIPR. In this manner, equation 1 becomes

$$\phi_T = \text{PHITOT} + \text{PHI(1)} - \text{PHIPR} \qquad (2)$$

Subroutine FCT also decides on the values of I_{PR} and I_{1R} as required by equations B73 and B74 as well as equations B150 and B151 (app B). The associated conditional statements assign the larger values for these combined moments of inertia whenever the product of the angular velocity and the angular acceleration is positive; i.e., both quantities have the same sign.

The associated suboutine OUTP is responsible for printing out the results ϕ, $\dot{\phi}$, and $\ddot{\phi}$, together with the current values of time, g, ψ, $\dot{\psi}$, and PHITOT. Further, all coupled motion contact forces are calculated according to equations B183 and B186, and the maximum values of these forces during one-rotor cycle are determined.

Free Motion (Location 5)

The differential equations of free motion, as given by equation B187 for the pallet and equation B193 for the combined escape wheel, gear train, and rotor are again solved by the Runge-Kutta routine. To obtain the magnitudes of the variables ϕ and ψ, as well as their derivatives at identical times, the two independent second-order differential equations are transformed into four simultaneous first-order ones. (While only the two first-order equations associated with each of the two variables are actually coupled, the routine treats all four as if they were coupled and thus produces solutions for identical time increments.) These four expressions, which are presented in subroutine FCTF, are of the following form:

$$\text{DX(1)} = \text{X(2)} \; : \; (= \dot{\phi}) \qquad (3)$$

$$\text{DX(3)} = \text{X(4)} \; : \; (= \dot{\psi}) \qquad (4)$$

$$\text{DX(2)} = \frac{1}{A_{55}} [-A_{56}(\text{X(2)})^2 + A_{57}\text{X(2)} + A_{58} + A_{59}\sin(\phi_{1C} + N_{31} (\phi_T + \text{X(1)} - \text{PHIPR}))] \; : \; (= \ddot{\phi}) \qquad (5)$$

$$DX(4) = \frac{1}{A_{51}} [-A_{21} (X(4))^2 - A_{52}X(4) - A_{53} + A_{54}\sin(\gamma_P' - \psi - \psi_c)] \; : \; (=\ddot{\psi}) \qquad (6)$$

The subroutine FCTF also computes the parameters A_{51} to A_{59}[3] and calls on subroutine IN2 for the computation of all gear-related parameters.

The associated subroutine OUTPF computes the free motion contact forces according to equations B199 and B200 and finds their maxima after determining the parameters A_{51} to A_{59}. The computation of these latter variables also requires a call on subroutine IN2. In addition to the above, a continuous count of PHITOT is provided in OUTPF. This angle, time, ϕ, $\dot{\phi}$, ψ, $\dot{\psi}$, and the contact forces are printed out. This subroutine also makes the decision whether or not to remain in free motion. The sensing variables f and g' = GP (equations E4 and E5 of appendix E) are used for this purpose.

Impact (Location 15)

The subroutine IMPACT uses the current values of the angular velocities $\dot{\phi}_i$ and $\dot{\psi}_i$ and computes the post impact angular velocities $\dot{\phi}_f$ and $\dot{\psi}_f$, applying equations B211 and B212 of appendix B. (Note that the moment of inertia of the escape wheel is now expressed according to equation B213 (app B), which refers the rotor as well as the gear train inertias to the escape wheel.)

Reversal of Gear Train Motion Due to Impact

If the impact torque on the escape wheel is sufficiently large, the motion of the gear train may be temporarily reversed; i.e., the escape wheel angular velocity $\dot{\phi}$ may become negative. This would cause the friction forces between the gear teeth and at the various gear pivots to be reversed in direction. (The normal forces between the gear teeth remain unaffected, and the normal bearing forces are obtained in the usual manner.) This change in the direction of the friction forces is expressed for both coupled and free motion by letting the coefficient of friction μ of all gear train components become negative (discussed in app E of ref 1). This is accomplished in subroutine IN2 by the following use of the signum function $\dot{\phi}/|\dot{\phi}|$:

$$MU = ABS(MU) * \dot{\phi}/|\dot{\phi}| \qquad (7)$$

---

[3] Whenever $A_{51} = I_{PR} = 0$, the simulation stops because of the division by zero. Should this occur, FCTF prints "IPR EQUALS ZERO - SIMULATION TERMINATED."

(The coefficient of friction associated with the escapement interface and the pallet pivot is called μ_1 and is read into the program as MU1.) Any motion reversal at these surfaces is accounted for by the signum functions s_4 and s_5, respectively.

Termination of Computations

Computations are terminated whenever the geared motion of the rotor ends. This corresponds to ϕ = PHICUTD. The duration of the subsequent unretarded motion of the rotor is assumed to be negligible.

COMPUTER SIMULATION OF EXAMPLE MECHANISM

This section discusses a computer simulation of a modified S&A device of the M739 fuze. The mechanism has configuration no. 2, as shown in figure 2, and contains a newly-designed involute gear train. While this gear train has the same gear ratio and individual center distances as the original design, each of the meshes now has unity contact ratio.[4] The simulation of this mechanism was accomplished with the help of computer program SANDA2V which, with a sample output, is listed in appendix A. It was run for 30,000 rpm to obtain maximum contact forces.

The following shows the input requirements of the program, explains the various output data, and discusses the manner in which the "number-of-turns-to-arm" is obtained for a given spin velocity.

Input Data (Appendix A)

The first portion of the output repeats all input data, which represent the mechanism parameters of the M739 fuze. These are listed both as computer variables and as symbols, according to appendixes C and F of this report:

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[4] Both meshes were designed with the help of computer programs INVOL11 and GEARPARAM2, originally shown in Progress Report No. 11 of the "Development of Automated Design Optimization Technique for Safety and Arming Devices" (Contract No. DAAK10-79-C-0251, January 15, 1981). Copies of this report may be obtained from either F. R. Tepper, ARRADCOM or G. G. Lowen, The City College of New York.

Escapement Parameters

A = a = 0.226 in. (5.740 mm) = distance between pivots O_p and O_s (fig. 2)

B = b = 0.168 in. (4.267 mm) = escape wheel radius

C = c = 0.13138 in. (3.337 mm) = pallet radius as defined by figure F-1 of appendix F

ALPHEN = α_{en} = 43.5352° = entrance working surface angle

ALPHEX = α_{ex} = 29.2981° = exit working surface angle

NT = 4 = number of escape wheel teeth spanned by verge

CONFIG = 2 = Configuration no. 2 (See section on fuze body configuration no. 2 in appendix B.)

EREST = e_r = 0 = Coefficient of restitution

LAMBDA = λ = 92.93° = angle between entrance and exit pallet radii (figure F-1 in appendix F)

N = 22 = Number of escape wheel teeth

For details of the above nomenclature, see appendixes C, E, and F.

Mass Parameters of Components

M1 = m_1 = 0.3165×10^{-4} lb-sec^2/in. (5.552×10^{-3} kg) = mass of rotor

M2 = m_2 = 0.3275×10^{-5} lb-sec^2/in. (5.745×10^{-4} kg) = mass of gear and pinion no. 2

M3 = m_3 = 0.2631×10^{-5} lb-sec^2/in. (4.615×10^{-4} kg) = mass of escape wheel and pinion no. 3

MP = m_p = 0.1640×10^{-5} lb-sec^2/in. (2.877×10^{-4} kg) = mass of pallet

I1 = I_1 = 0.1967×10^{-5} in.-lb-sec^2 (2.226×10^{-7} kg-m^2) = moment of inertia of rotor

I2 = I_2 = 0.4026×10^{-7} in.-lb-sec^2 (4.556×10^{-9} kg-m^2) = moment of inertia of gear and pinion no. 2

I3 = I_3 = 0.2125×10^{-7} in.-lb-sec^2 (2.405×10^{-9} kg-m^2) = moment of inertia of escape wheel and pinion no. 3

IP = I_p = 0.1950 x 10^{-7} in.-lb-sec^2 (2.207 x 10^{-9} kg-m^2) = moment of inertia of pallet

General Parameters

RC1 = r_{c1} = 0.0576 in. (1.463 mm) = distance from pivot of rotor to its center of mass

RCP = r_{cp} = 0. = pallet eccentricity

RHOP = ρ_p = 0.0227 in. (0.577 mm) = pallet pivot radius

RPM = 30,000 = spin rate

PHI1CD = ϕ_{1c} = -120.134° = rotor angle in starting position (fig. 2)

PSICCD = ψ_c = 0° = eccentricity angle of pallet

PHID = 139° = escape wheel starting angle of initial coupled motion

PHICUTD = 1485° = cumulative escape wheel angle obtained from product of total engaged rotor rotation and gear ratio. The total rotor rotation for the M739 fuze is 46.41°, while the gear ratio is 32. Thus, PHICUTD = 46.41 x 32 = 1485°

MU = μ = 0.10 = coefficient of friction of gear train (pivots and tooth-to-tooth contacts) and escape wheel pivot

MU1 = μ_1 = 0.10 = coefficient of friction of pallet-escape wheel interface and pallet pivot

Gear Parameters

PSUBD1 = P_{d1} = 80 = diametral pitch of mesh no. 1 (rotor and pinion no. 2)

PSUBD2 = P_{d2} = 100 = diametral pitch of mesh no. 2 (gear no. 2 and escape wheel pinion)

NG1 = N_{G1} = 64 = number of teeth of rotor (full gear no. 1)

NG2 = N_{G2} = 36 = number of teeth of gear no. 2

NP2 = N_{P2} = 9 = number of teeth of pinion no. 2

NP3 = N_{P3} = 8 = number of teeth of pinion no. 3 (escape wheel pinion)

CAPRP1 = R_{p1} = 0.41214 in. (10.468 mm) = pitch radius of gear no. 1 (rotor)

CAPRP2 = R_{p2} = 0.19039 in. (4.836 mm) = pitch radius of gear no. 2

RP2 = r_{p2} = 0.05796 in. (1.472 mm) = pitch radius of pinion no. 2

RP3 = r_{p3} = 0.04231 in. (1.075 mm) = pitch radius of pinion no. 3 (escape wheel pinion)

THETA1 = θ_1 = 24.215° = pressure angle of mesh no. 1

THETA2 = θ_2 = 27.326° = pressure angle of mesh no. 2

R1 = $\mathcal{R}_1$ = 0.250 in. (6.350 mm) = distance of rotor pivot from spin axis

R2 = $\mathcal{R}_2$ = 0.317 in. (8.052 mm) = distance of pivot of gear and pinion set no. 2 from spin axis

R3 = $\mathcal{R}_3$ = 0.309 in. (7.849 mm) = distance of pivot of escape wheel from spin axis

R4 = $\mathcal{R}_4$ = 0.304 in. (7.722 mm) = distance of pivot of pallet from spin axis

RHO1 = ρ_1 = 0.03075 in. (0.781 mm) = pivot radius of rotor

RHO2 = ρ_2 = 0.015 in. (0.381 mm) = pivot radius of gear and pinion no. 2

RHO3 = ρ_3 = 0.015 in. (0.381 mm) = pivot radius of escape wheel

CAPRB1 = R_{b1} = 0.37588 in. (9.547 mm) = base radius of gear no. 1

CAPRB2 = R_{b2} = 0.16915 in. (4.296 mm) = base radius of gear no. 2

RB2 = r_{b2} = 0.05286 in. (1.343 mm) = base radius of pinion no. 2

RB3 = r_{b3} = 0.03759 in. (0.955 mm) = base radius of escape wheel pinion

CAPRO1 = R_{O1} = 0.41425 in. (10.522 mm) = outside radius of gear no. 1

CAPRO2 = R_{O2} = 0.19404 in. (4.929 mm) = outside radius of gear no. 2

RO2 = r_{O2} = 0.07670 in. (1.948 mm) = outside radius of pinion no. 2

RO3 = r_{O3} = 0.05580 in. (1.417 mm) = outside radius of escape wheel pinion

J1 = J_1 = 0 = initialization parameter for mesh no. 1 [The zero value corresponds to earliest possible contact of mesh (ref 3).]

J2 = J_2 = 0 = initialization parameter for mesh no. 2

Output Data

The data blocks following the input data represent the results of various computations.

Fuze Geometry

The angles BETA1D = β_1 to BETA3D = β_3 and GAMMA2D = γ_2 to GAMMA4D = γ_4 are printed for checking purposes.

Coupled Motion

The first coupled motion output refers to the entrance side of the verge. For each time T of the coupled motion, the following variables are computed:

PHI = ϕ = instantaneous escape wheel angle (deg)

PHIDOT = $\dot{\phi}$ = escape wheel angular velocity (rad/sec)

G = g = pallet - escape wheel contact position (in.) (equation C15 of appendix C)

PSID = ψ = pallet angle (deg)

PSIDOT = $\dot{\psi}$ = pallet angular velocity (rad/sec)

PHITOT = ϕ_T = cumulative escape wheel angle (deg)

F23 = F_{23} = normal contact force of gear no. 2 on pinion no. 3 (lb)

F12 = F_{12} = normal contact force of gear no. 1 on pinion no. 2 (lb)

PN = P_n = normal contact force between escape wheel and pallet (lb), computed according to equation B185 in appendix B

PNPSI = P_n = normal contact force between escape wheel and pallet (lb), computed according to equation B186 in appendix B (serves for checking)

DPHI2 = $\ddot{\phi}$ = escape wheel angular acceleration (rad/sec^2), Runge-Kutta output

Free Motion

The first free motion on the exit side follows the coupled motion on the entrance side of the verge. For each time T of the free motion, the following variables are evaluated:

PHI = ϕ = instantaneous escape wheel angle (deg)

PHIDOT = $\dot{\phi}$ = escape wheel angular velocity (rad/sec)

PSI = ψ = pallet angle (deg)

PSIDOT = $\dot{\psi}$ = pallet angular velocity (rad/sec)

PHITOT = ϕ_T = cumulative escape wheel angle (deg)

FF12 = F_{F12} = normal contact force of gear no. 1 on pinion no. 2 for free motion (lb)

FF23 = F_{F23} = normal contact force of gear no. 2 on escape wheel pinion for free motion (lb)

Impact

The first exit impact follows the first exit free motion. Just preceding the "IMPACT" label, the program prints the values of VP = V_{TNi} and VS = V_{SNi} which stand for the pre-impact velocity components, normal to the verge face, of both the pallet and escape wheel contact points (equations D13 and D15). Subsequent to the "IMPACT" label, the following variables are evaluated:

PHI = ϕ = instantaneous escape wheel angle (deg), same as before impact

PHIDOT = $\dot{\phi}$ = post-impact escape wheel angular velocity (rad/sec)

PSI = ψ = pallet angle (deg), same as before impact

PSIDOT = $\dot{\psi}$ = post-impact pallet angular velocity (rad/sec)

PHITOT = ϕ_T = cumulative escape wheel angle (deg), same as before impact

VP = V_{TNf} = post-impact normal velocity component of pallet at contact point (equation D15)

VS = V_{SNf} = post-impact normal velocity component of escape wheel tooth at contact point (equation D13)

Note that in the present program, the post-impact VP is equal to VS since the coefficient of restitution is zero.

"Number of Turns-to-Arm" and Maximum Contact Forces

The "number of turns-to-arm" at 30,000 rpm is obtained with the help of that time T_{1485} which corresponds to the escape wheel angle PHICUTD = 1485°. Thus,

$$\text{number of turns-to-arm} = \frac{30000}{60} \times 0.05156 = 25.8$$

The maximum non-impact contact forces for the total cycle, for both coupled and free motion, are listed at the end of the output.

CONCLUSIONS

While it was not the purpose of this investigation to undertake a parametric study of the mechanism for which the program was written, the program was sufficiently tested to confirm that such a study is possible. It may include variations in masses and moments of inertia of all components; variations in the locations of the centers of mass of the verge and the rotor; and variations of gear, escapement, and fuze geometries; as well as various friction and coefficient of restitution conditions.

Previous high-speed motion picture observations of pin pallet escapements showed that the impacts were essentially inelastic and that, therefore, a zero coefficient of restitution was justified. Similar observations made on the detached lever escapement of the M577 fuze timer confirmed this.

The present work reports only on a single test run using the M739 fuze S&A data with a system coefficient of friction of 0.1. This is assumed to be representative of actual test conditions since (1) previous simulations of pin pallet escapements showed that the range of actual experimental results may be reproduced with coefficients of friction between 0.1 and 0.2 and (2) a special lubricant is used in conjunction with the M739 fuze. This choice of coefficient of friction is borne out by the good agreement with experimental results.

Noteworthy is the fact that the mechanism armed in approximately 22 turns with a coefficient of friction of 0.05 and in approximately 31 turns with a coefficient of friction of 0.15.

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APPENDIX A

COMPUTER PROGRAM SANDA2V

```
      PROGRAM SANDA2V(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      COMMON A,B,C,R,ALPHR,PI,ZZ,M1,M2,M3,MP,I1,I2,I3,IP,EREST,LAMBDA,DE
     1LTA,PHITOT,PHIPR,N31,N32,OMEGA,OM2,RC1,PHI1C,TEST1,TEST2,NG1,NG2,N
     2P2,NP3,CAPRB1,CAPRB2,RB2,RB3,THETA1,THETA2,R1,R2,R3,R4,RHO1,RHO2,R
     3HO3,RHOP,J1,J2,GAMMA2,GAMA3P,GAMMA3,GAMA4P,GAMMA4,GAMAPP,DELTA2,DE
     4LTA3,DELTA4,BETA1,BETA2,BETA3,D1,D2,AL1IN,AL1FIN,AL2IN,AL2FIN,ALPH
     5A1,ALPHA2,IN,T2,T3,MU,MU1,RCP,PSIC,S1,S2,S4,S5,A1,A2,DPHI2,DPSI2,F
     623MAX,F12MAX,FF23MAX,FF12MAX,PNMAX,PN,ALPHEN,ALPHEX
      COMMON /ZETA/ PSI,TIME,G,DPSI,GP,PHICUTD
      DIMENSION AUX(8,2), AUX2(8,4), PRMT(5), PHI(2), DPHI(2), X(4), DX(
     14)
      REAL M1,M2,M3,MP,I1,I2,I3,IP,LAMBDA,K,N31,N32,J1,J2,NG1,NG2,NP2,NP
     13,MU,MU1,NT,N
      EXTERNAL FCT,OUTP,FCTF,OUTPF
C
C     READ IN AND WRITE DATA
C
      READ (5,17) A,B,C,ALPHEN,ALPHEX,NT,CONFIG
      WRITE (6,18) A,B,C,ALPHEN,ALPHEX,NT,CONFIG
      READ (5,19) EREST,LAMBDA,N
      WRITE (6,20) EREST,LAMBDA,N
      READ (5,21) M1,M2,M3,MP
      WRITE (6,22) M1,M2,M3,MP
      READ (5,21) I1,I2,I3,IP
      WRITE (6,23) I1,I2,I3,IP
      READ (5,24) RC1,RCP,RHOP,RPM,PHI1CD,PSICCD,PHID,PHICUTD,MU,MU1
      WRITE (6,25) RC1,RCP,RHOP,RPM,PHI1CD,PSICCD,PHID,PHICUTD,MU,MU1
      READ (5,26) PSUBD1,PSUBD2,NG1,NG2,NP2,NP3,CAPRP1,CAPRP2,RP2,RP3,TH
     1ETA1,THETA2
      WRITE (6,29) PSUBD1,PSUBD2,NG1,NG2,NP2,NP3,CAPRP1,CAPRP2,RP2,RP3,T
     1HETA1,THETA2
      READ (5,27) R1,R2,R3,R4
      WRITE (6,30) R1,R2,R3,R4
      READ (5,19) RHO1,RHO2,RHO3
      WRITE (6,31) RHO1,RHO2,RHO3
      READ (5,27) CAPRB1,CAPRB2,RB2,RB3
      WRITE (6,32) CAPRB1,CAPRB2,RB2,RB3
      READ (5,27) CAPRO1,CAPRO2,RO2,RO3
      WRITE (6,33) CAPRO1,CAPRO2,RO2,RO3
      READ (5,28) J1,J2
      WRITE (6,34) J1,J2
C
C     INITIALIZATION OF PARAMETERS AND CONVERSION TO RADIANS
C
      TIME=0.
      PHITOT=0.
      PHIPR=PHID
      DPHI2=0.
      DPSI2=0.
      F23MAX=0.
      F12MAX=0.
      FF23MAX=0.
      FF12MAX=0.
      PNMAX=0.
      PI=3.14159
      ZZ=PI/180.
      OMEGA=RPM*2.*PI/60.
```

```
      OM2=OMEGA*OMEGA
      PHI1C=PHI1CD*ZZ
      PSICC=PSICCD*ZZ
      PSIC=PSICC
      ALPHEN=ALPHEN*ZZ
      ALPHEX=ALPHEX*ZZ
      DELTA=360./N
C
C     COMPUTATION OF GEAR RATIOS
C
      N31=NP2*NP3/(NG1*NG2)
      N32=-NP3/NG2
C
C     DETERMINATION OF SIGNUM FUNCTION S6
C
      IF (CONFIG.EQ.1.) S6=1.
      IF (CONFIG.EQ.2.) S6=-1.
C
C     COMPUTATION OF GAMMAS AND BETAS
C
      GAMMA2=S6*ACOS((R1*R1+R2*R2-(CAPRP1+RP2)**2)/(2.*R1*R2))
      GAMA3P=ACOS((R2*R2+R3*R3-(CAPRP2+RP3)**2)/(2.*R2*R3))
      GAMMA3=GAMMA2+S6*GAMA3P
      GAMA4P=ACOS((R3*R3+R4*R4-A*A)/(2.*R3*R4))
      GAMMA4=GAMMA3+S6*GAMA4P
      GAMMA2D=GAMMA2/ZZ
      GAMMA3D=GAMMA3/ZZ
      GAMMA4D=GAMMA4/ZZ
      DELTA2=ACOS(((CAPRP1+RP2)**2+R1*R1-R2*R2)/(2.*R1*(CAPRP1+RP2)))
      DELTA3=ACOS(((CAPRP2+RP3)**2+R2*R2-R3*R3)/(2.*R2*(CAPRP2+RP3)))
      DELTA4=ACOS((A*A+R3*R3-R4*R4)/(2.*A*R3))
      BETA1=PI-S6*DELTA2
      BETA2=GAMMA2+PI-S6*DELTA3
      BETA3=GAMMA3+PI-S6*DELTA4
      IF (CONFIG.EQ.1.) GAMAPP=DELTA4+GAMA4P
      IF (CONFIG.EQ.2.) GAMAPP=2.*PI-DELTA4-GAMA4P
      BETA1D=BETA1/ZZ
      BETA2D=BETA2/ZZ
      BETA3D=BETA3/ZZ
      WRITE (6,35) BETA1D,BETA2D,BETA3D,GAMMA2D,GAMMA3D,GAMMA4D
C
C     CONVERSION OF PRESSURE ANGLES TO RADIANS
C
      THETA1=THETA1*ZZ
      THETA2=THETA2*ZZ
C
C     COMPUTATION OF CENTRIFUGAL FORCES
C
      T2=M2*R2*OM2
      T3=M3*R3*OM2
C
C     DETERMINATION OF GEAR TRAIN CONSTANTS
C
      TEST1=TAN(THETA1)
      TEST2=TAN(THETA2)
      D1=(CAPRB1+RB2)*TAN(THETA1)
      D2=(CAPRB2+RB3)*TAN(THETA2)
```

```
      C                                                                 A 115
      C     DETERMINATION OF EARLIEST AND LATEST POSSIBLE VALUES OF ALPHAS A 116
      C                                                                 A 117
            CALL ALFA (CAPRB1,RB2,THETA1,CAPRO1,RO2,AL1IN,AL1FIN)       A 118
            CALL ALFA (CAPRB2,RB3,THETA2,CAPRO2,RO3,AL2IN,AL2FIN)       A 119
      C                                                                 A 120
      C     INITIALIZATION OF ALPHAS                                    A 121
      C                                                                 A 122
            ALPHA1=AL1IN+(AL1FIN-AL1IN)*J1                              A 123
            ALPHA2=AL2IN+(AL2FIN-AL2IN)*J2                              A 124
      C                                                                 A 125
      C     DATA FOR RUNGE KUTTA                                        A 126
      C                                                                 A 127
            ALPHR=ALPHEN                                                A 128
            PRMT(2)=3.                                                  A 129
            PRMT(4)=.01                                                 A 130
            NDIM=2                                                      A 131
            NDIM2=4                                                     A 132
            PHI(1)=PHID*ZZ                                              A 133
            PHI(2)=0.                                                   A 134
      C                                                                 A 135
      C     COUPLED MOTION                                              A 136
      C                                                                 A 137
          1 PRMT(1)=TIME                                                A 138
            PRMT(3)=.00001                                              A 139
            DPHI(1)=.5                                                  A 140
            DPHI(2)=.5                                                  A 141
            IF (PHITOT.GT.30..AND.PHITOT.LT.1450.) GO TO 2              A 142
            WRITE (6,36)                                                A 143
          2 CALL RKGS (PRMT,PHI,DPHI,NDIM,IHLF,FCT,OUTP,AUX)            A 144
            IF (PHITOT.GT.PHICUTD) GO TO 16                             A 145
      C                                                                 A 146
      C     TEST FOR ENTRANCE OR EXIT ACTION                            A 147
      C                                                                 A 148
            IF (PN.LE.0.) GO TO 5                                       A 149
            PHID=PHI(1)/ZZ                                              A 150
            IF (PHID.GE.130.00.AND.PHID.LE.160.) GO TO 3                A 151
            GO TO 4                                                     A 152
          3 PHI(1)=PHI(1)+DELTA*ZZ*NT                                   A 153
            PHIPR=PHI(1)/ZZ                                             A 154
            PSI=PSI+2.*PI-LAMBDA*ZZ                                     A 155
            PSIC=PSICC+LAMBDA*ZZ                                        A 156
            ALPHR=ALPHEX                                                A 157
            GO TO 5                                                     A 158
          4 PHI(1)=PHI(1)-DELTA*ZZ*(NT+1.)                              A 159
            PHIPR=PHI(1)/ZZ                                             A 160
            PSI=PSI-2.*PI+LAMBDA*ZZ                                     A 161
            ALPHR=ALPHEN                                                A 162
            PSIC=PSICC                                                  A 163
      C                                                                 A 164
      C     FREE MOTION                                                 A 165
      C                                                                 A 166
          5 PRMT(1)=TIME                                                A 167
            X(1)=PHI(1)                                                 A 168
            X(2)=PHI(2)                                                 A 169
            X(3)=PSI                                                    A 170
            X(4)=DPSI                                                   A 171
```

```
      DX(1)=.25
      DX(2)=.25
      DX(3)=.25
      DX(4)=.25
      PRMT(3)=.00001
      IF (PHITOT.GT.30..AND.PHITOT.LT.1450.) GO TO 6
      WRITE (6,37)
    6 CALL RKGS (PRMT,X,DX,NDIM2,IHLF,FCTF,OUTPF,AUX2)
      IF (PHITOT.GT.PHICUTD) GO TO 16
      PHI(1)=X(1)
      PHI(2)=X(2)
      PSI=X(3)
      DPSI=X(4)
      G=(B*SIN(PHI(1))-C*SIN(PSI))/SIN(PSI+ALPHR)
      PHID=PHI(1)/ZZ
      IF (PHID.LT.160..AND.GP.GT.0.) GO TO 10
      IF (PHID.GT.160..AND.GP.LT.0.) GO TO 8
      IF (PHID.LT.160.) GO TO 7
      PHI(1)=PHI(1)-DELTA*ZZ*(NT+1.)
      PHIPR=PHI(1)/ZZ
      PSI=PSI-2.*PI+LAMBDA*ZZ
      PSIC=PSICC
      GO TO 5
    7 PHI(1)=PHI(1)+DELTA*ZZ*NT
      PHIPR=PHI(1)/ZZ
      PSI=PSI+2.*PI-LAMBDA*ZZ
      PSIC=PSICC+LAMBDA*ZZ
      GO TO 5
C
C     EXIT ACTION
C
C     COMPUTATION OF VELOCITIES VP AND VS FOR EXIT ACTION
C
    8 VP=DPSI*(C*COS(ALPHR)+G)
      VS=PHI(2)*B*COS(PHI(1)-PSI-ALPHR)
      IF (PHITOT.GT.30..AND.PHITOT.LT.1450.) GO TO 9
      WRITE (6,38) VP,VS
C
C     EXIT ACTION TESTS
C
    9 IF (PHI(2).GE.0..AND.DPSI.GE.0.) GO TO 12
      IF (PHI(2).GE.0..AND.DPSI.LE.0..AND.ABS(VP).GT.ABS(VS)) GO TO 5
      IF (PHI(2).GE.0..AND.DPSI.LE.0..AND.ABS(VP).LT.ABS(VS)) GO TO 12
      IF (PHI(2).GE.0..AND.DPSI.LE.0..AND.ABS(VP).EQ.ABS(VS)) GO TO 1
      IF (PHI(2).LE.0..AND.DPSI.GE.0..AND.ABS(VP).GT.ABS(VS)) GO TO 12
      IF (PHI(2).LE.0..AND.DPSI.GE.0..AND.ABS(VP).LT.ABS(VS)) GO TO 5
      IF (PHI(2).LE.0..AND.DPSI.GE.0..AND.ABS(VP).EQ.ABS(VS)) GO TO 1
      IF (PHI(2).LE.0..AND.DPSI.LE.0.) GO TO 5
C
C     COMPUTATION OF VELOCITIES VP AND VS FOR ENTRANCE ACTION
C
   10 VP=DPSI*(C*COS(ALPHR)+G)
      VS=PHI(2)*B*COS(PHI(1)-PSI-ALPHR)
      IF (PHITOT.GT.30..AND.PHITOT.LT.1450.) GO TO 11
      WRITE (6,38) VP,VS
C
C     ENTRANCE ACTION
```

```
      C
       11 IF (PHI(2).GE.0..AND.DPSI.GE.0..AND.ABS(VP).GT.ABS(VS)) GO TO 5
          IF (PHI(2).GE.0..AND.DPSI.GE.0..AND.ABS(VP).EQ.ABS(VS)) GO TO 1
          IF (PHI(2).GE.0..AND.DPSI.GE.0..AND.ABS(VP).LT.ABS(VS)) GO TO 12
          IF (PHI(2).LE.0..AND.DPSI.GE.0.) GO TO 5
          IF (PHI(2).GE.0..AND.DPSI.LE.0.) GO TO 12
          IF (PHI(2).LE.0..AND.DPSI.LE.0..AND.ABS(VP).LT.ABS(VS)) GO TO 5
          IF (PHI(2).LE.0..AND.DPSI.LE.0..AND.ABS(VP).GT.ABS(VS)) GO TO 12
          IF (PHI(2).LE.0..AND.DPSI.LE.0..AND.ABS(VP).EQ.ABS(VS)) GO TO 1
      C
      C   IMPACT
      C
       12 CALL IMPACT (PHI(1),PHI(2),PSI,DPSI)
          IF (TIME.GT.5.0) GO TO 16
      C
      C   TEST FOR EXIT ACTION
      C
          PHID=PHI(1)/ZZ
          IF (PHID.LE.160.0) GO TO 14
      C
      C   EXIT ACTION
      C
      C   COMPUTATION OF VELOCITIES VP AND VS FOR EXIT ACTION
          VP=DPSI*(C*COS(ALPHR)+G)
          VS=PHI(2)*B*COS(PHI(1)-PSI-ALPHR)
          IF (PHITOT.GT.30..AND.PHITOT.LT.1450.) GO TO 13
          WRITE (6,38) VP,VS
       13 IF (ABS(ABS(VP)-ABS(VS)).LT.2.0) GO TO 1
      C
      C   EXIT ACTION TESTS
      C
          IF (PHI(2).GE.0..AND.DPSI.GE.0.) GO TO 1
          IF (PHI(2).GE.0..AND.DPSI.LE.0..AND.ABS(VP).GT.ABS(VS)) GO TO 5
          IF (PHI(2).GE.0..AND.DPSI.LE.0..AND.ABS(VP).LT.ABS(VS)) GO TO 1
          IF (PHI(2).GE.0..AND.DPSI.LE.0..AND.ABS(VP).EQ.ABS(VS)) GO TO 1
          IF (PHI(2).LE.0..AND.DPSI.GT.0..AND.ABS(VP).LT.ABS(VS)) GO TO 5
          IF (PHI(2).LE.0..AND.DPSI.GT.0..AND.ABS(VP).GT.ABS(VS)) GO TO 1
          IF (PHI(2).LE.0..AND.DPSI.GT.0..AND.ABS(VP).EQ.ABS(VS)) GO TO 1
          IF (PHI(2).LE.0..AND.DPSI.LE.0.) GO TO 5
      C
      C   COMPUTATION OF VELOCITIES VP AND VS FOR ENTRANCE ACTION
      C
       14 VP=DPSI*(C*COS(ALPHR)+G)
          VS=PHI(2)*B*COS(PHI(1)-PSI-ALPHR)
          IF (PHITOT.GT.30..AND.PHITOT.LT.1450.) GO TO 15
          WRITE (6,38) VP,VS
       15 IF (ABS(ABS(VP)-ABS(VS)).LT.2.0) GO TO 1
      C
      C   ENTRANCE ACTION TESTS
      C
          IF (PHI(2).GE.0..AND.DPSI.GE.0..AND.ABS(VP).GT.ABS(VS)) GO TO 5
          IF (PHI(2).GE.0..AND.DPSI.GE.0..AND.ABS(VP).LT.ABS(VS)) GO TO 1
          IF (PHI(2).GE.0..AND.DPSI.GE.0..AND.ABS(VP).EQ.ABS(VS)) GO TO 1
          IF (PHI(2).LE.0..AND.DPSI.GE.0.) GO TO 5
          IF (PHI(2).GE.0..AND.DPSI.LE.0.) GO TO 1
          IF (PHI(2).LE.0..AND.DPSI.LE.0..AND.ABS(VP).GT.ABS(VS)) GO TO 1
          IF (PHI(2).LE.0..AND.DPSI.LE.0..AND.ABS(VP).LT.ABS(VS)) GO TO 5
```

```
      IF (PHI(2).LE.0..AND.DPSI.LE.0..AND.ABS(VP).EQ.ABS(VS)) GO TO 1
   16 TURNS=RPM*TIME/60.
      WRITE (6,39) F23MAX,F12MAX,FF23MAX,FF12MAX,PNMAX,TURNS
      STOP
C
C
C
   17 FORMAT (7F10.5)
   18 FORMAT (1H1,5X,2HA=,F13.5,5X,2HB=,F13.5,5X,2HC=,F13.5,5X,7HALPHEN=
     1,F9.4,5X,7HALPHEX=,F9.4//6X,3HNT=,F3.0,5X,8HCONFIG =,F3.0/)
   19 FORMAT (3F10.5)
   20 FORMAT (1H ,5X,6HEREST=,F5.2,3X,7HLAMBDA=,F8.3,3X,3HN =,F4.0/)
   21 FORMAT (4E12.5)
   22 FORMAT (1H ,5X,4HM1 =,E15.5,3X,4HM2 =,E15.5,3X,4HM3 =,E15.5,3X,4HM
     1P =,E15.5/)
   23 FORMAT (1H ,5X,4HI1 =,E15.5,3X,4HI2 =,E15.5,3X,4HI3 =,E15.5,3X,4HI
     1P =,E15.5/)
   24 FORMAT (7F10.4/3F10.4)
   25 FORMAT (6X,5HRC1 =,F7.4,3X,5HRCP =,F7.4,3X,6HRHOP =,F7.4,3X,5HRPM
     1=,F6.0,3X,8HPHI1CD =,F9.4,3X,8HPSICCD =,F9.4,3X,6HPHID =,F9.4//6X,
     29HPHICUTD =,F6.0//6X,4HMU =,F4.2,3X,5HMU1 =,F4.2/)
   26 FORMAT (2F10.4/4F10.0/4F10.5/2F10.4)
   27 FORMAT (4F10.4)
   28 FORMAT (2F10.2)
   29 FORMAT (1H ,5X,8HPSUBD1 =,F5.1,3X,8HPSUBD2 =,F5.1//6X,5HNG1 =,F4.0
     1,3X,5HNG2 =,F4.0,3X,5HNP2 =,F4.0,3X,5HNP3 =,F4.0//6X,8HCAPRP1 =,F8
     2.5,3X,8HCAPRP2 =,F8.5//6X,5HRP2 =,F8.5,3X,5HRP3 =,F8.5//6X,8HTHETA
     31 =,F8.3,3X,8HTHETA2 =,F8.3/)
   30 FORMAT (6X,4HR1 =,F7.5,3X,4HR2 =,F7.5,3X,4HR3 =,F7.5,3X,4HR4 =,F7.
     15/)
   31 FORMAT (6X,6HRHO1 =,F7.5,3X,6HRHO2 =,F7.5,3X,6HRHO3 =,F7.5/)
   32 FORMAT (6X,8HCAPRB1 =,F7.5,3X,8HCAPRB2 =,F7.5,3X,5HRB2 =,F7.5,3X,5
     1HRB3 =,F7.5/)
   33 FORMAT (6X,8HCAPRO1 =,F7.5,3X,8HCAPRO2 =,F7.5,3X,5HRO2 =,F7.5,3X,5
     1HRO3 =,F7.5/)
   34 FORMAT (1H ,5X,4HJ1 =,F4.2,3X,4HJ2 =,F4.2/)
   35 FORMAT (6X,8HBETA1D =,F7.2,3X,8HBETA2D =,F7.2,3X,8HBETA3D =,F7.2/6
     1X,9HGAMMA2D =,F7.2,3X,9HGAMMA3D =,F7.2,3X,9HGAMMA4D =,F7.2/)
   36 FORMAT (1H0,5X,14HCOUPLED MOTION)
   37 FORMAT (1H0,5X,11HFREE MOTION//)
   38 FORMAT (4HOVP=,F8.3,3X,3HVS=,F8.3)
   39 FORMAT (1H0,6X,8HF23MAX =,F6.2/1H0,6X,8HF12MAX =,F6.2/1H0,6X,9HFF2
     13MAX =,F6.2/1H0,6X,9HFF12MAX =,F6.2/1H0,6X,7HPNMAX =,F6.2//6X,23HN
     2UMBER OF TURNS TO ARM=,F8.3)
      END
```

```
      SUBROUTINE IMPACT (PHI,DPHI,PSI,DPSI)
      COMMON A,B,C,R,ALPHR,PI,ZZ,M1,M2,M3,MP,I1,I2,I3,IP,EREST,LAMBDA,DE
     1LTA,PHITOT,PHIPR,N31,N32,OMEGA,OM2,RC1,PHI1C,TEST1,TEST2,NG1,NG2,N
     2P2,NP3,CAPRB1,CAPRB2,RB2,RB3,THETA1,THETA2,R1,R2,R3,R4,RHO1,RHO2,R
     3HO3,PHOP,J1,J2,GAMMA2,GAMA3P,GAMMA3,GAMA4P,GAMMA4,GAMAPP,DELTA2,DE
     4LTA3,DELTA4,BETA1,BETA2,BETA3,D1,D2,AL1IN,AL1FIN,AL2IN,AL2FIN,ALPH
     5A1,ALPHA2,IN,T2,T3,MU,MU1,RCP,PSIC,S1,S2,S4,S5,A1,A2,DPHI2,DPSI2,F
     623MAX,F12MAX,FF23MAX,FF12MAX,PNMAX,PN,ALPHEN,ALPHEX
      REAL I1,I2,I3,IP,LAMBDA,N31,N32,ISTOT,K
      ISTOT=I3+I2*N32*N32+I1*N31*N31
      G=(B*SIN(PHI)-C*SIN(PSI))/SIN(PSI+ALPHR)
      AONE=B*COS(PHI-PSI-ALPHR)
      DONE=C*COS(ALPHR)+G
      DPHIIN=DPHI
      DPHI=(IP*AONE*DPSI+ISTOT*DONE*DPHI+IP*AONE*EREST/DONE*(DPSI*DONE-D
     1PHI*AONE))/(IP*AONE**2/DONE+ISTOT*DONE)
      DPSI=(DPHI*AONE-EREST*(DPSI*DONE-DPHIIN*AONE))/DONE
      PHID=PHI/ZZ
      PSID=PSI/ZZ
      IF (PHITOT.GT.30..AND.PHITOT.LT.1450.) GO TO 1
      WRITE (6,2)
      WRITE (6,3) PHID,DPHI,PSID,DPSI,PHITOT
    1 RETURN
C
C
C
    2 FORMAT (1H0,5X,6HIMPACT)
    3 FORMAT (1H0,18X,4HPHI=,F8.3,3X,7HPHIDOT=,F8.3,3X,4HPSI=,F8.3,3X,7H
     1PSIDOT=,F8.3,3X,8HPHITOT =,F8.3)
      END
```

```
      SUBROUTINE FCT (T,PHI,DPHI)
      COMMON A,B,C,R,ALPHR,PI,ZZ,M1,M2,M3,MP,I1,I2,I3,IP,EREST,LAMBDA,DE
     1LTA,PHITOT,PHIPR,N31,N32,OMEGA,OM2,RC1,PHI1C,TEST1,TEST2,NG1,NG2,N
     2P2,NP3,CAPRB1,CAPRB2,RB2,RB3,THETA1,THETA2,R1,R2,R3,R4,RHO1,RHO2,R
     3HO3,RHOP,J1,J2,GAMMA2,GAMA3P,GAMMA3,GAMA4P,GAMMA4,GAMAPP,DELTA2,DE
     4LTA3,DELTA4,BETA1,BETA2,BETA3,D1,D2,AL1IN,AL1FIN,AL2IN,AL2FIN,ALPH
     5A1,ALPHA2,IN,T2,T3,MU,MU1,RCP,PSIC,S1,S2,S4,S5,A1,A2,DPHI2,DPSI2,F
     623MAX,F12MAX,FF23MAX,FF12MAX,PNMAX,PN,ALPHEN,ALPHEX
      DIMENSION PHI(2), DPHI(2)
      REAL M1,M2,M3,MP,I1,I2,I3,IP,I1R,N31,N32,MU,MU1,IPR
      PHID=PHI(1)/ZZ
      DELPHI=PHID-PHIPR
      PHIT=(PHITOT+DELPHI)*ZZ
      IN=1
      CALL KINEM (A,B,ALPHR,PHI,C,G,P,Q,S,PSI,DPSI,AONE,BONE,CONE,DONE,U
     1,V,VST)
      CALL IN2 (PHI(2),PHIT,DELPHI,VST,PSI,DPSI,AONE,BONE,CONE,DONE,AA1,
     1AA2,AA3,AA4,AA5,AA6,AA7,AA8,AA9,AA10,AA11,AA12,AA13,AA14,AA15,AA16
     2,AA17,AA18,AA19,AA20,AA21,AA22,AA23,AA24,AA25,AA26,AA27,AA28,AA29,
     3AA30,AA31,AA32,AA33,AA34,AA35,AA36,AA37,AA38,AA39,AA40,AA41,AA42,A
     4A43,AA44)
      IF (DPSI*DPSI2.GE.0.) IPR=IP+AA22
      IF (DPSI*DPSI2.LT.0.) IPR=IP-AA22
      IF (PHI(2)*DPHI2.GE.0.) I1R=I1+MU*RHO1*(AA26+AA30)
      IF (PHI(2)*DPHI2.LT.0.) I1R=I1-MU*RHO1*(AA26+AA30)
      IF (I1R.LT.0.) I1R=0.
      IF (IPR.LT.0.) IPR=0.
      AA45=I3*AA18+IPR*AA17*U-(I2*N32-AA42*I1R*N31/AA31)*AA15*AA18/AA44
      AA46=IPR*AA17*V+AA17*AA21*U*U+AA15*AA18*AA34*AA42*N31*N31/(AA31*AA
     144)
      AA47=2.*OM2*AA17*AA20*U/ABS(OMEGA)+AA15*AA18*AA33*AA42*N31/(AA31*A
     1A44)
      AA48=-AA15*AA18*AA32*AA42/(AA31*AA44)-AA15*AA18*AA43*T2/AA44-AA16*
     1AA18*T3-OM2*AA17*AA19
      AA49=AA15*AA18*AA35*AA42/(AA31*AA44)
      AA50=MP*RCP*R4*AA17*OM2
      DPHI(1)=PHI(2)
      DPHI(2)=(-AA46*PHI(2)*PHI(2)-AA47*PHI(2)+AA48-AA49*SIN(PHI1C*N31*(
     1PHITOT*ZZ+PHI(1)-PHIPR*ZZ))+AA50*SIN(GAMAPP-PSI-PSIC))/AA45
      RETURN
      END
```

```
      SUBROUTINE OUTP (T,PHI,DPHI,IHLF,NDIM,PRMT)
      REAL M1,M2,M3,MP,I1,I2,I3,IP,I1R,N31,N32,MU,MU1,IPR
      DIMENSION PHI(2), DPHI(2), PRMT(5)
      COMMON A,B,C,R,ALPHR,PI,ZZ,M1,M2,M3,MP,I1,I2,I3,IP,EREST,LAMBDA,DE
     1LTA,PHITOT,PHIPR,N31,N32,OMEGA,OM2,RC1,PHI1C,TEST1,TEST2,NG1,NG2,N
     2P2,NP3,CAPRB1,CAPRB2,RB2,RB3,THETA1,THETA2,R1,R2,R3,R4,RHO1,RHO2,R
     3HO3,RHOP,J1,J2,GAMMA2,GAMA3P,GAMMA3,GAMA4P,GAMMA4,GAMAPP,DELTA2,DE
     4LTA3,DELTA4,BETA1,BETA2,BETA3,D1,D2,AL1IN,AL1FIN,AL2IN,AL2FIN,ALPH
     5A1,ALPHA2,IN,T2,T3,MU,MU1,RCP,PSIC,S1,S2,S4,S5,A1,A2,DPHI2,DPSI2,F
     623MAX,F12MAX,FF23MAX,FF12MAX,PNMAX,PN,ALPHEN,ALPHEX
      COMMON /ZETA/ PSI,TIME,G,DPSI,GP,PHICUTD
      PHID=PHI(1)/ZZ
      DELPHI=PHID-PHIPR
      PHIPR=PHID
      PHITOT=PHITOT+DELPHI
      PHIT=PHITOT*ZZ
      IN=0
      CALL KINEM (A,B,ALPHR,PHI,C,G,P,Q,S,PSI,DPSI,AONE,BONE,CONE,DONE,U
     1,V,VST)
      CALL IN2 (PHI(2),PHIT,DELPHI,VST,PSI,DPSI,AONE,BONE,CONE,DONE,AA1,
     1AA2,AA3,AA4,AA5,AA6,AA7,AA8,AA9,AA10,AA11,AA12,AA13,AA14,AA15,AA16
     2,AA17,AA18,AA19,AA20,AA21,AA22,AA23,AA24,AA25,AA26,AA27,AA28,AA29,
     3AA30,AA31,AA32,AA33,AA34,AA35,AA36,AA37,AA38,AA39,AA40,AA41,AA42,A
     4A43,AA44)
      IF (DPSI*DPSI2.GE.0.) IPR=IP+AA22
      IF (DPSI*DPSI2.LT.0.) IPR=IP-AA22
      IF (PHI(2)*DPHI2.GE.0.) I1R=I1+ABS(MU)*RHO1*(AA26+AA30)
      IF (PHI(2)*DPHI2.LT.0.) I1R=I1-ABS(MU)*RHO1*(AA26+AA30)
      IF (I1R.LT.0.) I1R=0.
      IF (IPR.LT.0.) IPR=0.
      AA45=I3*AA18+IPR*AA17*U-(I2*N32-AA42*I1R*N31/AA31)*AA15*AA18/AA44
      AA46=IPR*AA17*V+AA17*AA21*U*U+AA15*AA18*AA34*AA42*N31*N31/(AA31*AA
     144)
      AA47=2.*OM2*AA17*AA20*U/ABS(OMEGA)+AA15*AA18*AA33*AA42*N31/(AA31*A
     1A44)
      AA48=-AA15*AA18*AA32*AA42/(AA31*AA44)-AA15*AA18*AA43*T2/AA44-AA16*
     1AA18*T3-OM2*AA17*AA19
      AA49=AA15*AA18*AA35*AA42/(AA31*AA44)
      AA50=MP*RCP*R4*AA17*OM2
      DPHI2=(-AA46*PHI(2)*PHI(2)-AA47*PHI(2)+AA48-AA49*SIN(PHI1C+N31*(PH
     1ITOT*ZZ+PHI(1)-PHIPR*ZZ))+AA50*SIN(GAMAPP-PSI-PSIC))/AA45
      DPSI2=U*DPHI2+V*PHI(2)*PHI(2)
C
C     COMPUTATION OF CONTACT FORCES
C
      F23=(-AA32*AA42/AA31-T2*AA43-AA33*AA42*N31*PHI(2)/AA31-AA42*AA34*N
     131*N31*PHI(2)*PHI(2)/AA31-AA35*AA42*SIN(PHI1C+N31*PHIT)/AA31+(I2*N
     232-AA42*I1R*N31/AA31)*DPHI2)/AA44
      F12=(F23*AA44+T2*AA43-I2*N32*DPHI2)/AA42
      PN=(-I3*DPHI2+F23*AA15-T3*AA16)/AA17
      IF (F23.GT.F23MAX) F23MAX=F23
      IF (F12.GT.F12MAX) F12MAX=F12
      IF (PN.GT.PNMAX) PNMAX=PN
      PNPSI=(IPR*DPSI2+AA21*DPSI*DPSI+2.*OM2*DPSI*AA20/ABS(OMEGA)+OM2*AA
     119-MP*RCP*R4*OM2*SIN(GAMAPP-PSI-PSIC))/AA18
C
C     TEST FOR CONTINUATION OF COUPLED MOTION
```

```
C
      IF (PHID.GT.150.) GO TO 1
      IF (.NOT.(G.GE.0..AND.PN.GT.0.)) PRMT(5)=1.
      GO TO 2
    1 IF (.NOT.(G.LE.0..AND.PN.GT.0.)) PRMT(5)=1.
C
C     WRITE OUTPUT
C
    2 IF (PHITOT.GT.30..AND.PHITOT.LT.1450.) GO TO 3
      PSID=PSI/ZZ
      WRITE (6,5) T,PHID,PHI(2),G,PSID,DPSI,PHITOT,F23,F12,PN,PNPSI,DPHI
     12
    3 IF (PHITOT.LT.PHICUTD) GO TO 4
      PRMT(5)=1.
    4 TIME=T
      RETURN
C
C
C
    5 FORMAT (6X,3HT =,F8.5,3X,5HPHI =,F7.2,3X,8HPHIDOT =,F7.2,3X,3HG =,
     1F6.4,3X,6HPSID =,F7.2,3X,8HPSIDOT =,F8.2,3X,8HPHITOT =,F7.2/20X,5H
     2F23 =,F7.4,3X,5HF12 =,F7.4,3X,4HPN =,F7.4,3X,7HPNPSI =,F7.4,3X,7HD
     3PHI2 =,E12.4)
      END
```

```
      SUBROUTINE FCTF (T,X,DX)
      COMMON A,B,C,R,ALPHR,PI,ZZ,M1,M2,M3,MP,I1,I2,I3,IP,EREST,LAMBDA,DE
     1LTA,PHITOT,PHIPR,N31,N32,OMEGA,OM2,RC1,PHI1C,TEST1,TEST2,NG1,NG2,N
     2P2,NP3,CAPRB1,CAPRB2,RB2,RB3,THETA1,THETA2,R1,R2,R3,R4,RHO1,RHO2,R
     3HO3,RHOP,J1,J2,GAMMA2,GAMA3P,GAMMA3,GAMA4P,GAMMA4,GAMAPP,DELTA2,DE
     4LTA3,DELTA4,BETA1,BETA2,BETA3,D1,D2,AL1IN,AL1FIN,AL2IN,AL2FIN,ALPH
     5A1,ALPHA2,IN,T2,T3,MU,MU1,RCP,PSIC,S1,S2,S4,S5,A1,A2,DPHI2,DPSI2,F
     623MAX,F12MAX,EF23MAX,EF12MAX,PNMAX,PN,ALPHEN,ALPHEX
      DIMENSION X(4), DX(4), PRMT(5)
      REAL M1,M2,M3,MP,I1,I2,I3,IP,I1R,N31,N32,MU,MU1,IPR
      PHID=X(1)/ZZ
      DELPHI=PHID-PHIPR
      PHIT=(PHITOT+DELPHI)*ZZ
      IN=1
      CALL IN2 (X(2),PHIT,DELPHI,0.,X(3),X(4),0.,0.,0.,0.,AA1,AA2,AA3,AA
     14,AA5,AA6,AA7,AA8,AA9,AA10,AA11,AA12,AA13,AA14,AA15,AA16,AA17,AA18
     2,AA19,AA20,AA21,AA22,AA23,AA24,AA25,AA26,AA27,AA28,AA29,AA30,AA31,
     3AA32,AA33,AA34,AA35,AA36,AA37,AA38,AA39,AA40,AA41,AA42,AA43,AA44)
      IF (X(4)*DPSI2.GE.0.) IPR=IP+AA22
      IF (X(4)*DPSI2.LT.0.) IPR=IP-AA22
      IF (X(2)*DPHI2.GE.0.) I1R=I1+MU*RHO1*(AA26+AA30)
      IF (X(2)*DPHI2.LT.0.) I1R=I1-MU*RHO1*(AA26+AA30)
      IF (I1R.LT.0.) I1R=0.
      IF (IPR.LT.0.) IPR=0.
      IF (IPR.EQ.0.) WRITE (6,1)
      AA51=IPR
      AA52=2.*OM2*AA20/ABS(OMEGA)
      AA53=OM2*AA19
      AA54=OM2*MP*RCP*R4
      AA55=I3-AA15*N32*I2/AA44+AA15*AA42*I1R*N31/(AA31*AA44)
      AA56=AA15*AA34*AA42*N31*N31/(AA31*AA44)
      AA57=AA15*AA33*AA42*N31/(AA31*AA44)
      AA58=-AA16*T3-AA15*AA43*T2/AA44-AA15*AA32*AA42/(AA31*AA44)
      AA59=AA15*AA35*AA42/(AA31*AA44)
      DX(1)=X(2)
      DX(3)=X(4)
      DX(2)=(-AA56*X(2)*X(2)-AA57*X(2)+AA58-AA59*SIN(PHI1C+N31*(PHITOT*Z
     1Z+X(1)-PHIPR*ZZ)))/AA55
      DX(4)=(-AA21*X(4)*X(4)-AA52*X(4)-AA53+AA54*SIN(GAMAPP-X(3)-PSIC))/
     1AA51
      RETURN
C
C
C
    1 FORMAT (40H0IPR EQUALS ZERO - SIMULATION TERMINATED)
      END
```

```
      SUBROUTINE OUTPF (T,X,DX,IHLF,NDIM,PRMT)
      COMMON A,B,C,R,ALPHR,PI,ZZ,M1,M2,M3,MP,I1,I2,I3,IP,EREST,LAMBDA,DE
     1LTA,PHITOT,PHIPR,N31,N32,OMEGA,OM2,RC1,PHI1C,TEST1,TEST2,NG1,NG2,N
     2P2,NP3,CAPRB1,CAPRB2,RB2,RB3,THETA1,THETA2,R1,R2,R3,R4,RHO1,RHO2,R
     3HO3,RHOP,J1,J2,GAMMA2,GAMA3P,GAMMA3,GAMA4P,GAMMA4,GAMAPP,DELTA2,DE
     4LTA3,DELTA4,BETA1,BETA2,BETA3,D1,D2,AL1IN,AL1FIN,AL2IN,AL2FIN,ALPH
     5A1,ALPHA2,IN,T2,T3,MU,MU1,RCP,PSIC,S1,S2,S4,S5,A1,A2,DPHI2,DPSI2,F
     6F23MAX,F12MAX,FF23MAX,FF12MAX,PNMAX,PN,ALPHEN,ALPHEX
      REAL M1,M2,M3,MP,I1,I2,I3,IP,I1R,N31,N32,MU,MU1,IPR
      DIMENSION X(4), DX(4), PRMT(5)
      COMMON /ZETA/ PSI,TIME,G,DPSI,GP,PHICUTD
      PHID=X(1)/ZZ
      PSID=X(3)/ZZ
      DELPHI=PHID-PHIPR
      PHITOT=PHITOT+DELPHI
      PHIT=PHITOT*ZZ
      PHIPR=PHID
      IN=0
      CALL IN2 (X(2),PHIT,DELPHI,0.,X(3),X(4),0.,0.,0.,0.,AA1,AA2,AA3,AA
     14,AA5,AA6,AA7,AA8,AA9,AA10,AA11,AA12,AA13,AA14,AA15,AA16,AA17,AA18
     2,AA19,AA20,AA21,AA22,AA23,AA24,AA25,AA26,AA27,AA28,AA29,AA30,AA31,
     3AA32,AA33,AA34,AA35,AA36,AA37,AA38,AA39,AA40,AA41,AA42,AA43,AA44)
      IF (X(4)*DPSI2.GE.0.) IPR=IP+AA22
      IF (X(4)*DPSI2.LT.0.) IPR=IP-AA22
      IF (X(2)*DPHI2.GE.0.) I1R=I1+MU*RHO1*(AA26+AA30)
      IF (X(2)*DPHI2.LT.0.) I1R=I1-MU*RHO1*(AA26+AA30)
      IF (I1R.LT.0.) I1R=0.
      IF (IPR.LT.0.) IPR=0.
      AA51=IPR
      AA52=2.*OM2*AA20/ABS(OMEGA)
      AA53=OM2*AA19
      AA54=OM2*MP*RCP*R4
      AA55=I3-AA15*N32*I2/AA44+AA15*AA42*I1R*N31/(AA31*AA44)
      AA56=AA15*AA34*AA42*N31*N31/(AA31*AA44)
      AA57=AA15*AA33*AA42*N31/(AA31*AA44)
      AA58=-AA16*T3-AA15*AA43*T2/AA44-AA15*AA32*AA42/(AA31*AA44)
      AA59=AA15*AA35*AA42/(AA31*AA44)
      PSI=X(3)
      DPSI=X(4)
      DPSI2=(-AA21*X(4)*X(4)-AA52*X(4)-AA53+AA54*SIN(GAMAPP-X(3)-PSIC))/
     1AA51
      DPHI2=(-AA56*X(2)*X(2)-AA57*X(2)+AA58-AA59*SIN(PHI1C+N31*(PHITOT*Z
     1Z+X(1)-PHIPR*ZZ)))/AA55
C
C     COMPUTATION OF CONTACT FORCES
C
      FF23=(I3*DPHI2+T3*AA16)/AA15
      FF12=(FF23*AA44+T2*AA43-I2*N32*DPHI2)/AA42
      IF (FF23.GT.FF23MAX) FF23MAX=FF23
      IF (FF12.GT.FF12MAX) FF12MAX=FF12
C
C     WRITE OUTPUT
C
      IF (PHITOT.GT.30..AND.PHITOT.LT.1450.) GO TO 1
      WRITE (6,6) T,PHID,X(2),PSID,X(4),PHITOT,FF12,FF23
    1 IF (T.EQ.TIME) GO TO 4
C
```

```
C     CHECK FOR CONTINUED FREE MOTION
C
      F=A*SIN(X(3)+ALPHR)-B*SIN(X(1)-X(3)-ALPHR)-C*SIN(ALPHR)
      GP=A*COS(X(3)+ALPHR)+B*COS(X(1)-X(3)-ALPHR)-C*COS(ALPHR)
      IF (PHID.LT.145..AND.F.GT.0.) GO TO 2
      IF (PHID.GT.145..AND.F.LT.0.) GO TO 3
      PRMT(5)=1.
    2 IF (GP.LT.0.) PRMT(5)=1.
      GO TO 4
    3 IF (GP.GT.0.) PRMT(5)=1.
    4 TIME=T
      IF (PHITOT.LT.PHICUTD) GO TO 5
      PRMT(5)=1.
    5 RETURN
C
C
C
    6 FORMAT (6X,3HT =,F8.5,3X,5HPHI =,F7.2,3X,8HPHIDOT =,F7.2,3X,5HPSI
     1=,F7.2,3X,8HPSIDOT =,F8.2,3X,8HPHITOT =,F7.2/20X,6HFF12 =,F7.3,3X,
     26HFF23 =,F7.3)
      END
```

```
      SUBROUTINE KINEM (A,B,ALPHR,PHI,C,G,P,Q,S,PSI,DPSI,AONE,BONE,CONE,
     1DONE,U,V,VST)
      DIMENSION PHI(2)
      PI=3.14159
      CAPA=A*COS(ALPHR)+B*COS(PHI(1)-ALPHR)
      CAPB=A*SIN(ALPHR)-B*SIN(PHI(1)-ALPHR)
      CAPC=C*SIN(ALPHR)
      PSI=2.*ATAN2((CAPA-SQRT(CAPA**2+CAPB**2-CAPC**2)),(CAPB+CAPC))
      IF (PSI.LT.0.) PSI=2.*PI+PSI
      G=(B*SIN(PHI(1))-C*SIN(PSI))/SIN(PSI+ALPHR)
      P=B*COS(PHI(1)-ALPHR-PSI)
      Q=A*COS(PSI+ALPHR)+B*COS(PHI(1)-ALPHR-PSI)
      U=P/Q
      V=1./Q**3*(A*P**2*SIN(PSI+ALPHR)-B*(P-Q)**2*SIN(PHI(1)-ALPHR-PSI))
      AONE=B*COS(PHI(1)-PSI-ALPHR)
      BONE=B*SIN(PHI(1)-PSI-ALPHR)
      CONE=-C*SIN(ALPHR)
      DONE=C*COS(ALPHR)+G
      DPSI=U*PHI(2)
      VST=-PHI(2)*B*SIN(PHI(1)-PSI-ALPHR)-DPSI*C*SIN(ALPHR)
      RETURN
      END
```

```
      SUBROUTINE IN2 (DPHI,PHIT,DELPHI,VST,PSI,DPSI,AONE,BONE,CONE,DONE, H   1
     1AA1,AA2,AA3,AA4,AA5,AA6,AA7,AA8,AA9,AA10,AA11,AA12,AA13,AA14,AA15, H   2
     2AA16,AA17,AA18,AA19,AA20,AA21,AA22,AA23,AA24,AA25,AA26,AA27,AA28,A H   3
     3A29,AA30,AA31,AA32,AA33,AA34,AA35,AA36,AA37,AA38,AA39,AA40,AA41,AA H   4
     442,AA43,AA44)                                                      H   5
      COMMON A,B,C,R,ALPHR,PI,ZZ,M1,M2,M3,MP,I1,I2,I3,IP,EREST,LAMBDA,DE H   6
     1LTA,PHITOT,PHIPR,N31,N32,OMEGA,OM2,RC1,PHI1C,TEST1,TEST2,NG1,NG2,N H   7
     2P2,NP3,CAPRB1,CAPRB2,RB2,RB3,THETA1,THETA2,R1,R2,R3,R4,RHO1,RHO2,R H   8
     3HO3,RHOP,J1,J2,GAMMA2,GAMA3P,GAMMA3,GAMA4P,GAMMA4,GAMAPP,DELTA2,DE H   9
     4LTA3,DELTA4,BETA1,BETA2,BETA3,D1,D2,AL1IN,AL1FIN,AL2IN,AL2FIN,ALPH H  10
     5A1,ALPHA2,IN,T2,T3,MU,MU1,RCP,PSIC,S1,S2,S4,S5,A1,A2,DPHI2,DPSI2,F H  11
     623MAX,F12MAX,FF23MAX,FF12MAX,PNMAX,PN,ALPHEN,ALPHEX                H  12
      REAL M1,M2,M3,MP,MU,MU1,N31,N32,I1                                 H  13
      IF (DPHI.EQ.0.) GO TO 1                                            H  14
      MU=ABS(MU)*DPHI/ABS(DPHI)                                          H  15
    1 IF (IN.EQ.0) GO TO 2                                               H  16
C                                                                        H  17
C     UPDATE VALUES OF ALPHAS                                            H  18
C                                                                        H  19
      DELAL2=DELPHI*ZZ                                                   H  20
      DELAL1=DELAL2*RB2/CAPRB1                                           H  21
      ALPHA1=ALPHA1+DELAL1                                               H  22
      ALPHA2=ALPHA2+DELAL2                                               H  23
      IF (ALPHA1.GT.AL1FIN) ALPHA1=AL1IN                                 H  24
      IF (ALPHA2.GT.AL2FIN) ALPHA2=AL2IN                                 H  25
C                                                                        H  26
C     DETERMINATION OF SIGNUMS                                           H  27
C                                                                        H  28
    2 IF (ALPHA1.LT.TEST1) S1=1.                                         H  29
      IF (ALPHA2.LT.TEST2) S2=1.                                         H  30
      IF (ALPHA1.EQ.TEST1) S1=0.                                         H  31
      IF (ALPHA2.EQ.TEST2) S2=0.                                         H  32
      IF (ALPHA1.GT.TEST1) S1=-1.                                        H  33
      IF (ALPHA2.GT.TEST2) S2=-1.                                        H  34
      IF (VST.NE.0.) GO TO 3                                             H  35
      S4=1.                                                              H  36
      GO TO 4                                                            H  37
    3 S4=VST/ABS(VST)                                                    H  38
    4 IF (DPSI.NE.0.) GO TO 5                                            H  39
      S5=1.                                                              H  40
      GO TO 6                                                            H  41
    5 S5=DPSI/ABS(DPSI)                                                  H  42
C                                                                        H  43
C     COMPUTATION OF A1 AND A2                                           H  44
C                                                                        H  45
    6 A1=ALPHA1+CAPRB1                                                   H  46
      A2=ALPHA2+CAPRB2                                                   H  47
      DENOM=1.+MU*MU                                                     H  48
      DENOM1=1.+MU1*MU1                                                  H  49
C                                                                        H  50
C     COMPUTATION OF AA1 TO AA44                                         H  51
C                                                                        H  52
      AA2=ABS((MP*(R4*(SIN(GAMAPP)-MU1*S5*COS(GAMAPP))+RCP*(SIN(PSI+PSIC H  53
     1)-MU1*S5*COS(PSI+PSIC)))))/DENOM1)                                 H  54
      AA3=ABS((MP*RCP*(SIN(PSI+PSIC)-MU1*S5*COS(PSI+PSIC)))/DENOM1)      H  55
      AA4=ABS((MP*RCP*(COS(PSI+PSIC)+MU1*S5*SIN(PSI+PSIC)))/DENOM1)      H  56
      AA6=ABS((MP*(-R4*(COS(GAMAPP)+MU1*S5*SIN(GAMAPP))-RCP*(COS(PSI+PSI H  57
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     1C)+MU1*S5*SIN(PSI+PSIC))))/DENOM1)
      AA7=ABS((MP*RCP*(COS(PSI+PSIC)+MU1*S5*SIN(PSI+PSIC)))/DENOM1)
      AA8=ABS((MP*RCP*(SIN(PSI+PSIC)-MU1*S5*COS(PSI+PSIC)))/DENOM1)
      AA10=ABS((-MU*(1.-S2)*SIN(BETA2+THETA2)-(1.+MU*MU*S2)*COS(BETA2+TH
     1ETA2))/DENOM)
      AA11=ABS((SIN(GAMMA3)-MU*COS(GAMMA3))/DENOM)
      AA13=ABS((-(1.+MU*MU*S2)*SIN(BETA2+THETA2)+MU*(1.-S2)*COS(BETA2+TH
     1ETA2))/DENOM)
      AA14=ABS((COS(GAMMA3)+MU*SIN(GAMMA3))/DENOM)
      AA15=RB3-MU*(S2*(D2-A2)+RHO3*(AA10+AA13))
      AA16=MU*RHO3*(AA11+AA14)
      IF (ALPHR.EQ.ALPHEX) GO TO 7
      AA1=ABS((-MU1*(S4+S5)*SIN(PSI+ALPHR)-(1.-S4*S5*MU1*MU1)*COS(PSI+AL
     1PHR))/DENOM1)
      AA5=ABS((MU1*(S4+S5)*COS(PSI+ALPHR)+(1.-S4*S5*MU1*MU1)*SIN(PSI+ALP
     1HR))/DENOM1)
      AA9=ABS(((MU1*S4+MU)*SIN(PSI+ALPHR+BETA3)+(1.-MU1*MU*S4)*COS(PSI+A
     1LPHR+BETA3))/DENOM)
      AA12=ABS(((1.-MU1*MU*S4)*SIN(PSI+ALPHR+BETA3)-(S4*MU1+MU)*COS(PSI+
     1ALPHR+BETA3))/DENOM)
      AA17=AONE-MU1*S4*BONE+MU*RHO3*(AA9+AA12)
      AA18=DONE-CONE*MU1*S4-RHOP*MU1*S5*(AA1+AA5)
      GO TO 8
    7 AA1=ABS((MU1*(-S4+S5)*SIN(PSI+ALPHR)+(1.+S4*S5*MU1*MU1)*COS(PSI+AL
     1PHR))/DENOM1)
      AA5=ABS((MU1*(S5-S4)*COS(PSI+ALPHR)-(1.+S4*S5*MU1*MU1)*SIN(PSI+ALP
     1HR))/DENOM1)
      AA9=ABS(((MU1*S4-MU)*SIN(PSI+ALPHR+BETA3)-(1.+MU1*MU*S4)*COS(PSI+A
     1LPHR+BETA3))/DENOM)
      AA12=ABS((-(1.+MU1*MU*S4)*SIN(PSI+ALPHR+BETA3)-(S4*MU1-MU)*COS(PSI
     1+ALPHR+BETA3))/DENOM)
      AA17=-AONE-MU1*S4*BONE+MU*RHO3*(AA9+AA12)
      AA18=-DONE-CONE*MU1*S4-RHOP*MU1*S5*(AA1+AA5)
    8 AA19=RHOP*MU1*S5*(AA2+AA6)
      AA20=RHOP*MU1*(AA3+AA7)
      AA21=S5*AA20
      AA22=RHOP*MU1*(AA4+AA8)
      AA23=ABS(((1.-MU*MU*S1)*SIN(BETA1-THETA1)-MU*(1.+S1)*COS(BETA1-THE
     1TA1))/DENOM)
      AA24=ABS((M1*(R1+RC1*(COS(PHI1C+N31*PHIT)+MU*SIN(PHI1C+N31*PHIT)))
     1)/DENOM)
      AA25=ABS((M1*RC1*(COS(PHI1C+N31*PHIT)+MU*SIN(PHI1C+N31*PHIT)))/DEN
     1OM)
      AA26=ABS((M1*RC1*(SIN(PHI1C+N31*PHIT)-MU*COS(PHI1C+N31*PHIT)))/DEN
     1OM)
      AA27=ABS((MU*(1.+S1)*SIN(BETA1-THETA1)+(1.-MU*MU*S1)*COS(BETA1-THE
     1TA1))/DENOM)
      AA28=ABS((MU*M1*R1+M1*RC1*(MU*COS(PHI1C+N31*PHIT)-SIN(PHI1C+N31*PH
     1IT)))/DENOM)
      AA29=ABS((M1*RC1*(MU*COS(PHI1C+N31*PHIT)-SIN(PHI1C+N31*PHIT)))/DEN
     1OM)
      AA30=ABS((M1*RC1*(MU*SIN(PHI1C+N31*PHIT)+COS(PHI1C+N31*PHIT)))/DEN
     1OM)
      AA31=CAPRB1-MU*S1*A1+MU*RHO1*(AA23+AA27)
      AA32=OM2*RHO1*MU*(AA24+AA28)
      AA33=OM2/ABS(OMEGA)*2.*ABS(MU)*RHO1*(AA25+AA29)
      AA34=MU*RHO1*(AA25+AA29)
```

```
      AA35=M1*R1*RC1*DM2
      AA36=ABS(((1.-MU*MU*S2)*SIN(BETA2+THETA2)+MU*(1.+S2)*COS(BETA2+THE
     1TA2))/DENOM)
      AA37=ABS(((1.+MU*MU*S1)*SIN(BETA1-THETA1)+MU*(1.-S1)*COS(BETA1-THE
     1TA1))/DENOM)
      AA38=ABS((COS(GAMMA2)-MU*SIN(GAMMA2))/DENOM)
      AA39=ABS((MU*(1.+S2)*SIN(BETA2+THETA2)-(1.-MU*MU*S2)*COS(BETA2+THE
     1TA2))/DENOM)
      AA40=ABS((MU*(1.-S1)*SIN(BETA1-THETA1)-(1.+MU*MU*S1)*COS(BETA1-THE
     1TA1))/DENOM)
      AA41=ABS((MU*COS(GAMMA2)+SIN(GAMMA2))/DENOM)
      AA42=RB2-MU*(S1*(D1-A1)+RHO2*(AA37+AA40))
      AA43=MU*RHO2*(AA38+AA41)
      AA44=CAPRB2-MU*(S2*A2-RHO2*(AA36+AA39))
      RETURN
      END
```

```
SUBROUTINE ALFA        74/74    OPT=1                         FTN 4.8+508          06/08/82  13.52.46          PAGE     1

                    SUBROUTINE ALFA (CAPRB,RB,THETA,CAPRO,RO,ALIN,ALFIN)
              C
              C     THIS SUBROUTINE COMPUTES THE INITIAL AND FINAL VALUES OF ALPHAS
              C
                    ALIN=((CAPRB+RB)*TAN(THETA)-SQRT(RO*RO-RB*RB))/CAPRB
                    ALFIN=SQRT(CAPRO*CAPRO-CAPRB*CAPRB)/CAPRB
                    RETURN
                    END
```

```
A=       .22600     B=        .16800     C=        .13138     ALPHEN=  43.5352     ALPHEX=  29.2981
NT= 4.     CONFIG = 2.
EREST= 0.00   LAMBDA=  92.930    N = 22.
M1 =      .31650E-04   M2 =      .32750E-05   M3 =      .26310E-05   MP =      .16400E-05
I1 =      .19670E-05   I2 =      .40260E-07   I3 =      .21250E-07   IP =      .19500E-07
RC1 =  .0576   RCP = 0.0000   RHOP =  .0227   RPM =30000.   PHI1CD =-120.1340   PSICCD =   0.0000   PHID = 139.0000
PHICUTD = 1485.
MU = .10   MU1 = .10
PSUBD1 = 80.0   PSUBD2 =100.0
NG1 = 64.   NG2 = 36.   NP2 =  9.   NP3 =  8.
CAPRP1 =  .41214   CAPRP2 =  .19039
RP2 =  .05796   RP3 =  .04231
THETA1 =  24.215   THETA2 =  27.326
R1 = .25000   R2 = .31700   R3 = .30900   R4 = .30400
RHO1 = .03075   RHO2 = .01500   RHO3 = .01500
CAPRB1 = .37588   CAPRB2 = .16915   RB2 = .05286   RB3 = .03759
CAPRO1 = .41425   CAPRO2 = .19404   RO2 = .07670   RO3 = .05580
J1 =0.00   J2 =0.00
BETA1D = 218.87   BETA2D = 134.90   BETA3D =  92.11
GAMMA2D =-111.47   GAMMA3D =-155.09   GAMMA4D =-198.34

COUPLED MOTION
T = 0.00000   PHI = 139.00   PHIDOT =    0.00   G = .0222   PSID =  42.08   PSIDOT =   0.00   PHITOT =   -.00
              F23 = 2.4663   F12 = 9.6322   PN =  .3312   PNPSI =  .3312   DPHI2 =  .1568E+07
T =  .00001   PHI = 139.00   PHIDOT =    6.35   G = .0222   PSID =  42.08   PSIDOT =   5.42   PHITOT =    .00
              F23 = 2.5031   F12 = 9.6925   PN =  .3305   PNPSI =  .3305   DPHI2 =  .1212E+07
T =  .00001   PHI = 139.00   PHIDOT =   12.41   G = .0222   PSID =  42.08   PSIDOT =  10.59   PHITOT =    .00
              F23 = 2.5031   F12 = 9.6925   PN =  .3306   PNPSI =  .3306   DPHI2 =  .1212E+07
T =  .00002   PHI = 139.01   PHIDOT =   18.47   G = .0222   PSID =  42.09   PSIDOT =  15.76   PHITOT =    .01
              F23 = 2.5037   F12 = 9.6924   PN =  .3309   PNPSI =  .3309   DPHI2 =  .1213E+07
T =  .00002   PHI = 139.01   PHIDOT =   24.53   G = .0222   PSID =  42.09   PSIDOT =  20.93   PHITOT =    .01
              F23 = 2.5037   F12 = 9.6925   PN =  .3310   PNPSI =  .3310   DPHI2 =  .1213E+07
T =  .00003   PHI = 139.02   PHIDOT =   30.60   G = .0222   PSID =  42.10   PSIDOT =  26.12   PHITOT =    .02
              F23 = 2.5047   F12 = 9.6924   PN =  .3316   PNPSI =  .3316   DPHI2 =  .1215E+07
T =  .00003   PHI = 139.03   PHIDOT =   36.67   G = .0221   PSID =  42.11   PSIDOT =  31.30   PHITOT =    .03
              F23 = 2.5048   F12 = 9.6925   PN =  .3318   PNPSI =  .3318   DPHI2 =  .1214E+07
T =  .00004   PHI = 139.04   PHIDOT =   42.74   G = .0221   PSID =  42.12   PSIDOT =  36.50   PHITOT =    .04
              F23 = 2.5062   F12 = 9.6923   PN =  .3326   PNPSI =  .3326   DPHI2 =  .1218E+07
T =  .00004   PHI = 139.06   PHIDOT =   48.82   G = .0220   PSID =  42.13   PSIDOT =  41.71   PHITOT =    .06
              F23 = 2.5063   F12 = 9.6925   PN =  .3328   PNPSI =  .3328   DPHI2 =  .1217E+07
T =  .00005   PHI = 139.07   PHIDOT =   54.91   G = .0220   PSID =  42.14   PSIDOT =  46.93   PHITOT =    .07
              F23 = 2.5081   F12 = 9.6923   PN =  .3339   PNPSI =  .3339   DPHI2 =  .1221E+07
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T =  .00005   PHI = 139.09   PHIDOT =  61.00   G = .0219   PSID =  42.16   PSIDOT =  52.16   PHITOT =   .09
              F23 = 2.5083   F12 = 9.6926   PN =  .3342   PNPSI =  .3342   DPHI2 =  .1220E+07
T =  .00005   PHI = 139.11   PHIDOT =  67.11   G = .0218   PSID =  42.17   PSIDOT =  57.41   PHITOT =   .11
              F23 = 2.5105   F12 = 9.6924   PN =  .3355   PNPSI =  .3355   DPHI2 =  .1225E+07
T =  .00006   PHI = 139.13   PHIDOT =  73.22   G = .0218   PSID =  42.19   PSIDOT =  62.68   PHITOT =   .13
              F23 = 2.5107   F12 = 9.6926   PN =  .3359   PNPSI =  .3359   DPHI2 =  .1224E+07
T =  .00006   PHI = 139.15   PHIDOT =  79.34   G = .0217   PSID =  42.21   PSIDOT =  67.96   PHITOT =   .15
              F23 = 2.5134   F12 = 9.6924   PN =  .3375   PNPSI =  .3375   DPHI2 =  .1230E+07
T =  .00007   PHI = 139.17   PHIDOT =  85.48   G = .0216   PSID =  42.23   PSIDOT =  73.26   PHITOT =   .17
              F23 = 2.5136   F12 = 9.6927   PN =  .3379   PNPSI =  .3379   DPHI2 =  .1228E+07
T =  .00007   PHI = 139.20   PHIDOT =  91.63   G = .0215   PSID =  42.25   PSIDOT =  78.59   PHITOT =   .20
              F23 = 2.5167   F12 = 9.6925   PN =  .3398   PNPSI =  .3398   DPHI2 =  .1235E+07
T =  .00008   PHI = 139.22   PHIDOT =  97.78   G = .0214   PSID =  42.27   PSIDOT =  83.94   PHITOT =   .22
              F23 = 2.5169   F12 = 9.6929   PN =  .3402   PNPSI =  .3402   DPHI2 =  .1233E+07
T =  .00008   PHI = 139.25   PHIDOT = 103.96   G = .0213   PSID =  42.30   PSIDOT =  89.31   PHITOT =   .25
              F23 = 2.5205   F12 = 9.6925   PN =  .3424   PNPSI =  .3424   DPHI2 =  .1242E+07
T =  .00009   PHI = 139.28   PHIDOT = 110.15   G = .0212   PSID =  42.32   PSIDOT =  94.71   PHITOT =   .28
              F23 = 2.5207   F12 = 9.6930   PN =  .3429   PNPSI =  .3429   DPHI2 =  .1239E+07
T =  .00009   PHI = 139.32   PHIDOT = 116.35   G = .0211   PSID =  42.35   PSIDOT = 100.14   PHITOT =   .32
              F23 = 2.5247   F12 = 9.6927   PN =  .3453   PNPSI =  .3453   DPHI2 =  .1248E+07
T =  .00010   PHI = 139.35   PHIDOT = 122.57   G = .0209   PSID =  42.38   PSIDOT = 105.59   PHITOT =   .35
              F23 = 2.5250   F12 = 9.6932   PN =  .3459   PNPSI =  .3459   DPHI2 =  .1246E+07
T =  .00011   PHI = 139.39   PHIDOT = 128.81   G = .0208   PSID =  42.41   PSIDOT = 111.09   PHITOT =   .39
              F23 = 2.5295   F12 = 9.6928   PN =  .3486   PNPSI =  .3486   DPHI2 =  .1256E+07
T =  .00011   PHI = 139.42   PHIDOT = 135.07   G = .0207   PSID =  42.45   PSIDOT = 116.60   PHITOT =   .42
              F23 = 2.5298   F12 = 9.6934   PN =  .3493   PNPSI =  .3493   DPHI2 =  .1253E+07
T =  .00012   PHI = 139.46   PHIDOT = 141.34   G = .0205   PSID =  42.48   PSIDOT = 122.17   PHITOT =   .46
              F23 = 2.5347   F12 = 9.6930   PN =  .3523   PNPSI =  .3523   DPHI2 =  .1264E+07
T =  .00012   PHI = 139.51   PHIDOT = 147.64   G = .0204   PSID =  42.52   PSIDOT = 127.76   PHITOT =   .51
              F23 = 2.5351   F12 = 9.6936   PN =  .3530   PNPSI =  .3530   DPHI2 =  .1261E+07
T =  .00013   PHI = 139.55   PHIDOT = 153.95   G = .0202   PSID =  42.55   PSIDOT = 133.39   PHITOT =   .55
              F23 = 2.5404   F12 = 9.6932   PN =  .3564   PNPSI =  .3564   DPHI2 =  .1273E+07
T =  .00013   PHI = 139.59   PHIDOT = 160.29   G = .0200   PSID =  42.59   PSIDOT = 139.06   PHITOT =   .59
              F23 = 2.5408   F12 = 9.6939   PN =  .3572   PNPSI =  .3572   DPHI2 =  .1269E+07
T =  .00014   PHI = 139.64   PHIDOT = 166.65   G = .0199   PSID =  42.63   PSIDOT = 144.78   PHITOT =   .64
              F23 = 2.5466   F12 = 9.6935   PN =  .3608   PNPSI =  .3608   DPHI2 =  .1282E+07
T =  .00014   PHI = 139.69   PHIDOT = 173.03   G = .0197   PSID =  42.67   PSIDOT = 150.53   PHITOT =   .69
              F23 = 2.5471   F12 = 9.6943   PN =  .3617   PNPSI =  .3617   DPHI2 =  .1278E+07
T =  .00015   PHI = 139.74   PHIDOT = 179.43   G = .0195   PSID =  42.72   PSIDOT = 156.34   PHITOT =   .74
              F23 = 2.5534   F12 = 9.6938   PN =  .3656   PNPSI =  .3656   DPHI2 =  .1292E+07
T =  .00015   PHI = 139.79   PHIDOT = 185.86   G = .0193   PSID =  42.76   PSIDOT = 162.19   PHITOT =   .79
              F23 = 2.5539   F12 = 9.6947   PN =  .3666   PNPSI =  .3666   DPHI2 =  .1288E+07
T =  .00015   PHI = 139.85   PHIDOT = 192.32   G = .0191   PSID =  42.81   PSIDOT = 168.09   PHITOT =   .85
              F23 = 2.5607   F12 = 9.6942   PN =  .3709   PNPSI =  .3709   DPHI2 =  .1303E+07
T =  .00016   PHI = 139.90   PHIDOT = 198.79   G = .0189   PSID =  42.86   PSIDOT = 174.04   PHITOT =   .90
              F23 = 2.5612   F12 = 9.6951   PN =  .3720   PNPSI =  .3720   DPHI2 =  .1298E+07
T =  .00016   PHI = 139.96   PHIDOT = 205.30   G = .0187   PSID =  42.91   PSIDOT = 180.05   PHITOT =   .96
              F23 = 2.5685   F12 = 9.6946   PN =  .3766   PNPSI =  .3766   DPHI2 =  .1314E+07
T =  .00017   PHI = 140.02   PHIDOT = 211.82   G = .0184   PSID =  42.96   PSIDOT = 186.11   PHITOT =  1.02
              F23 = 2.5691   F12 = 9.6957   PN =  .3778   PNPSI =  .3778   DPHI2 =  .1309E+07
T =  .00017   PHI = 140.08   PHIDOT = 218.39   G = .0182   PSID =  43.02   PSIDOT = 192.23   PHITOT =  1.08
              F23 = 2.3204   F12 = 9.6984   PN =  .3795   PNPSI =  .3795   DPHI2 =  .1306E+07
T =  .00018   PHI = 140.14   PHIDOT = 224.92   G = .0180   PSID =  43.07   PSIDOT = 198.37   PHITOT =  1.14
              F23 = 2.3210   F12 = 9.6995   PN =  .3808   PNPSI =  .3808   DPHI2 =  .1301E+07
T =  .00018   PHI = 140.21   PHIDOT = 231.39   G = .0177   PSID =  43.13   PSIDOT = 204.48   PHITOT =  1.21
              F23 = 2.3245   F12 = 9.7052   PN =  .3800   PNPSI =  .3800   DPHI2 =  .1282E+07
T =  .00019   PHI = 140.28   PHIDOT = 237.81   G = .0175   PSID =  43.19   PSIDOT = 210.58   PHITOT =  1.28
              F23 = 2.3251   F12 = 9.7064   PN =  .3814   PNPSI =  .3814   DPHI2 =  .1276E+07
T =  .00019   PHI = 140.35   PHIDOT = 244.15   G = .0172   PSID =  43.25   PSIDOT = 216.65   PHITOT =  1.35
              F23 = 2.3287   F12 = 9.7123   PN =  .3804   PNPSI =  .3804   DPHI2 =  .1256E+07
T =  .00020   PHI = 140.42   PHIDOT = 250.44   G = .0169   PSID =  43.32   PSIDOT = 222.72   PHITOT =  1.42
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                F23 = 2.3294   F12 = 9.7136    PN =  .3819   PNPSI =  .3819   DPHI2 =   .1250E+07
T =  .00020     PHI = 140.45   PHIDOT = 253.37   G = .0168    PSID =  43.35   PSIDOT =  225.57   PHITOT =   1.45
                F23 = 2.5807   F12 = 9.7315    PN =  .3681   PNPSI =  .3681   DPHI2 =   .1162E+07
T =  .00020     PHI = 140.49   PHIDOT = 256.26   G = .0167    PSID =  43.38   PSIDOT =  228.41   PHITOT =   1.49
                F23 = 2.5811   F12 = 9.7322    PN =  .3688   PNPSI =  .3688   DPHI2 =   .1159E+07
T =  .00021     PHI = 140.53   PHIDOT = 259.16   G = .0165    PSID =  43.41   PSIDOT =  231.26   PHITOT =   1.53
                F23 = 2.5857   F12 = 9.7319    PN =  .3718   PNPSI =  .3718   DPHI2 =   .1169E+07
T =  .00021     PHI = 140.56   PHIDOT = 262.07   G = .0164    PSID =  43.45   PSIDOT =  234.13   PHITOT =   1.56
                F23 = 2.5861   F12 = 9.7326    PN =  .3725   PNPSI =  .3725   DPHI2 =   .1166E+07
T =  .00021     PHI = 140.60   PHIDOT = 264.99   G = .0163    PSID =  43.48   PSIDOT =  237.02   PHITOT =   1.60
                F23 = 2.5908   F12 = 9.7323    PN =  .3756   PNPSI =  .3756   DPHI2 =   .1176E+07
T =  .00021     PHI = 140.64   PHIDOT = 267.92   G = .0161    PSID =  43.51   PSIDOT =  239.92   PHITOT =   1.64
                F23 = 2.5912   F12 = 9.7330    PN =  .3763   PNPSI =  .3763   DPHI2 =   .1172E+07
T =  .00022     PHI = 140.68   PHIDOT = 270.85   G = .0160    PSID =  43.55   PSIDOT =  242.85   PHITOT =   1.68
                F23 = 2.5961   F12 = 9.7327    PN =  .3795   PNPSI =  .3795   DPHI2 =   .1182E+07
T =  .00022     PHI = 140.72   PHIDOT = 273.79   G = .0158    PSID =  43.58   PSIDOT =  245.79   PHITOT =   1.72
                F23 = 2.5965   F12 = 9.7334    PN =  .3803   PNPSI =  .3803   DPHI2 =   .1179E+07
T =  .00022     PHI = 140.76   PHIDOT = 276.75   G = .0157    PSID =  43.62   PSIDOT =  248.75   PHITOT =   1.76
                F23 = 2.6015   F12 = 9.7332    PN =  .3835   PNPSI =  .3835   DPHI2 =   .1189E+07
T =  .00022     PHI = 140.80   PHIDOT = 279.70   G = .0155    PSID =  43.65   PSIDOT =  251.72   PHITOT =   1.80
                F23 = 2.6019   F12 = 9.7339    PN =  .3844   PNPSI =  .3844   DPHI2 =   .1185E+07
T =  .00023     PHI = 140.84   PHIDOT = 282.67   G = .0154    PSID =  43.69   PSIDOT =  254.72   PHITOT =   1.84
                F23 = 2.6070   F12 = 9.7337    PN =  .3877   PNPSI =  .3877   DPHI2 =   .1196E+07
T =  .00023     PHI = 140.88   PHIDOT = 285.65   G = .0152    PSID =  43.73   PSIDOT =  257.74   PHITOT =   1.88
                F23 = 2.6075   F12 = 9.7344    PN =  .3886   PNPSI =  .3886   DPHI2 =   .1192E+07
T =  .00023     PHI = 140.92   PHIDOT = 288.63   G = .0151    PSID =  43.76   PSIDOT =  260.77   PHITOT =   1.92
                F23 = 2.6127   F12 = 9.7342    PN =  .3920   PNPSI =  .3920   DPHI2 =   .1203E+07
T =  .00023     PHI = 140.96   PHIDOT = 291.62   G = .0149    PSID =  43.80   PSIDOT =  263.83   PHITOT =   1.96
                F23 = 2.6132   F12 = 9.7350    PN =  .3929   PNPSI =  .3929   DPHI2 =   .1199E+07
T =  .00024     PHI = 141.00   PHIDOT = 294.63   G = .0147    PSID =  43.84   PSIDOT =  266.90   PHITOT =   2.00
                F23 = 2.6185   F12 = 9.7347    PN =  .3965   PNPSI =  .3965   DPHI2 =   .1209E+07
T =  .00024     PHI = 141.04   PHIDOT = 297.63   G = .0146    PSID =  43.88   PSIDOT =  270.00   PHITOT =   2.04
                F23 = 2.6190   F12 = 9.7355    PN =  .3974   PNPSI =  .3974   DPHI2 =   .1205E+07
T =  .00024     PHI = 141.09   PHIDOT = 300.65   G = .0144    PSID =  43.92   PSIDOT =  273.12   PHITOT =   2.09
                F23 = 2.6245   F12 = 9.7353    PN =  .4010   PNPSI =  .4010   DPHI2 =   .1216E+07
T =  .00024     PHI = 141.13   PHIDOT = 303.68   G = .0142    PSID =  43.96   PSIDOT =  276.25   PHITOT =   2.13
                F23 = 2.6250   F12 = 9.7362    PN =  .4020   PNPSI =  .4020   DPHI2 =   .1212E+07
T =  .00025     PHI = 141.17   PHIDOT = 306.71   G = .0141    PSID =  44.00   PSIDOT =  279.41   PHITOT =   2.17
                F23 = 2.6306   F12 = 9.7359    PN =  .4058   PNPSI =  .4058   DPHI2 =   .1223E+07
T =  .00025     PHI = 141.22   PHIDOT = 309.75   G = .0139    PSID =  44.04   PSIDOT =  282.59   PHITOT =   2.22
                F23 = 2.6311   F12 = 9.7368    PN =  .4068   PNPSI =  .4068   DPHI2 =   .1219E+07
T =  .00025     PHI = 141.26   PHIDOT = 312.81   G = .0137    PSID =  44.08   PSIDOT =  285.79   PHITOT =   2.26
                F23 = 2.6369   F12 = 9.7366    PN =  .4106   PNPSI =  .4106   DPHI2 =   .1230E+07
T =  .00025     PHI = 141.31   PHIDOT = 315.86   G = .0136    PSID =  44.12   PSIDOT =  289.01   PHITOT =   2.31
                F23 = 2.6374   F12 = 9.7375    PN =  .4117   PNPSI =  .4117   DPHI2 =   .1225E+07
T =  .00026     PHI = 141.35   PHIDOT = 318.93   G = .0134    PSID =  44.16   PSIDOT =  292.26   PHITOT =   2.35
                F23 = 2.6433   F12 = 9.7373    PN =  .4157   PNPSI =  .4157   DPHI2 =   .1237E+07
T =  .00026     PHI = 141.40   PHIDOT = 322.01   G = .0132    PSID =  44.20   PSIDOT =  295.53   PHITOT =   2.40
                F23 = 2.6439   F12 = 9.7382    PN =  .4168   PNPSI =  .4168   DPHI2 =   .1232E+07
T =  .00026     PHI = 141.45   PHIDOT = 325.09   G = .0130    PSID =  44.25   PSIDOT =  298.81   PHITOT =   2.45
                F23 = 2.3861   F12 = 9.7413    PN =  .4172   PNPSI =  .4172   DPHI2 =   .1224E+07
T =  .00026     PHI = 141.49   PHIDOT = 328.15   G = .0129    PSID =  44.29   PSIDOT =  302.10   PHITOT =   2.49
                F23 = 2.3866   F12 = 9.7423    PN =  .4184   PNPSI =  .4184   DPHI2 =   .1219E+07
T =  .00027     PHI = 141.54   PHIDOT = 331.18   G = .0127    PSID =  44.33   PSIDOT =  305.37   PHITOT =   2.54
                F23 = 2.3891   F12 = 9.7465    PN =  .4178   PNPSI =  .4178   DPHI2 =   .1205E+07
T =  .00027     PHI = 141.59   PHIDOT = 334.19   G = .0125    PSID =  44.38   PSIDOT =  308.64   PHITOT =   2.59
                F23 = 2.3896   F12 = 9.7475    PN =  .4189   PNPSI =  .4189   DPHI2 =   .1200E+07
T =  .00027     PHI = 141.64   PHIDOT = 337.18   G = .0123    PSID =  44.42   PSIDOT =  311.91   PHITOT =   2.64
                F23 = 2.3922   F12 = 9.7517    PN =  .4183   PNPSI =  .4183   DPHI2 =   .1185E+07
T =  .00027     PHI = 141.68   PHIDOT = 340.14   G = .0121    PSID =  44.47   PSIDOT =  315.16   PHITOT =   2.68
                F23 = 2.3927   F12 = 9.7528    PN =  .4194   PNPSI =  .4194   DPHI2 =   .1180E+07
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T =  .00028   PHI = 141.73   PHIDOT = 343.08   G = .0120   PSID =  44.51   PSIDOT =  318.41   PHITOT =   2.73
              F23 = 2.3953   F12 = 9.7571   PN =  .4188    PNPSI =  .4188   DPHI2 =   .1165E+07
T =  .00028   PHI = 141.78   PHIDOT = 345.99   G = .0118   PSID =  44.56   PSIDOT =  321.66   PHITOT =   2.78
              F23 = 2.3958   F12 = 9.7581   PN =  .4199    PNPSI =  .4199   DPHI2 =   .1160E+07
T =  .00028   PHI = 141.83   PHIDOT = 348.68   G = .0116   PSID =  44.60   PSIDOT =  324.71   PHITOT =   2.83
              F23 = 2.6506   F12 = 9.7759   PN =  .4043    PNPSI =  .4043   DPHI2 =   .1066E+07
T =  .00028   PHI = 141.88   PHIDOT = 351.33   G = .0114   PSID =  44.65   PSIDOT =  327.74   PHITOT =   2.88
              F23 = 2.6512   F12 = 9.7769   PN =  .4054    PNPSI =  .4054   DPHI2 =   .1061E+07
T =  .00029   PHI = 141.93   PHIDOT = 353.99   G = .0112   PSID =  44.70   PSIDOT =  330.79   PHITOT =   2.93
              F23 = 2.6578   F12 = 9.7768   PN =  .4099    PNPSI =  .4099   DPHI2 =   .1073E+07
T =  .00029   PHI = 141.98   PHIDOT = 356.65   G = .0110   PSID =  44.74   PSIDOT =  333.87   PHITOT =   2.98
              F23 = 2.6583   F12 = 9.7779   PN =  .4110    PNPSI =  .4110   DPHI2 =   .1068E+07
T =  .00029   PHI = 142.04   PHIDOT = 359.33   G = .0108   PSID =  44.79   PSIDOT =  336.98   PHITOT =   3.04
              F23 = 2.6651   F12 = 9.7777   PN =  .4156    PNPSI =  .4156   DPHI2 =   .1080E+07
T =  .00029   PHI = 142.09   PHIDOT = 362.01   G = .0106   PSID =  44.84   PSIDOT =  340.10   PHITOT =   3.09
              F23 = 2.6657   F12 = 9.7788   PN =  .4168    PNPSI =  .4168   DPHI2 =   .1075E+07
T =  .00030   PHI = 142.14   PHIDOT = 364.70   G = .0104   PSID =  44.89   PSIDOT =  343.26   PHITOT =   3.14
              F23 = 2.6725   F12 = 9.7787   PN =  .4215    PNPSI =  .4215   DPHI2 =   .1087E+07
T =  .00030   PHI = 142.19   PHIDOT = 367.40   G = .0102   PSID =  44.94   PSIDOT =  346.43   PHITOT =   3.19
              F23 = 2.6732   F12 = 9.7798   PN =  .4228    PNPSI =  .4228   DPHI2 =   .1081E+07
T =  .00030   PHI = 142.24   PHIDOT = 370.11   G = .0100   PSID =  44.99   PSIDOT =  349.64   PHITOT =   3.24
              F23 = 2.6801   F12 = 9.7797   PN =  .4276    PNPSI =  .4276   DPHI2 =   .1094E+07
T =  .00030   PHI = 142.30   PHIDOT = 372.82   G = .0098   PSID =  45.04   PSIDOT =  352.86   PHITOT =   3.30
              F23 = 2.6808   F12 = 9.7809   PN =  .4289    PNPSI =  .4289   DPHI2 =   .1088E+07
T =  .00031   PHI = 142.35   PHIDOT = 375.55   G = .0096   PSID =  45.09   PSIDOT =  356.12   PHITOT =   3.35
              F23 = 2.6879   F12 = 9.7808   PN =  .4338    PNPSI =  .4338   DPHI2 =   .1100E+07
T =  .00031   PHI = 142.40   PHIDOT = 378.28   G = .0094   PSID =  45.14   PSIDOT =  359.40   PHITOT =   3.40
              F23 = 2.6886   F12 = 9.7820   PN =  .4352    PNPSI =  .4352   DPHI2 =   .1094E+07
T =  .00031   PHI = 142.46   PHIDOT = 381.02   G = .0092   PSID =  45.19   PSIDOT =  362.71   PHITOT =   3.46
              F23 = 2.6958   F12 = 9.7820   PN =  .4403    PNPSI =  .4403   DPHI2 =   .1107E+07
T =  .00031   PHI = 142.51   PHIDOT = 383.76   G = .0089   PSID =  45.25   PSIDOT =  366.05   PHITOT =   3.51
              F23 = 2.6965   F12 = 9.7832   PN =  .4417    PNPSI =  .4417   DPHI2 =   .1100E+07
T =  .00032   PHI = 142.57   PHIDOT = 386.52   G = .0087   PSID =  45.30   PSIDOT =  369.41   PHITOT =   3.57
              F23 = 2.7039   F12 = 9.7832   PN =  .4469    PNPSI =  .4469   DPHI2 =   .1113E+07
T =  .00032   PHI = 142.62   PHIDOT = 389.28   G = .0085   PSID =  45.35   PSIDOT =  372.80   PHITOT =   3.62
              F23 = 2.7047   F12 = 9.7845   PN =  .4484    PNPSI =  .4484   DPHI2 =   .1106E+07
T =  .00032   PHI = 142.68   PHIDOT = 392.05   G = .0083   PSID =  45.40   PSIDOT =  376.22   PHITOT =   3.68
              F23 = 2.7122   F12 = 9.7845   PN =  .4537    PNPSI =  .4537   DPHI2 =   .1119E+07
T =  .00032   PHI = 142.74   PHIDOT = 394.83   G = .0081   PSID =  45.46   PSIDOT =  379.66   PHITOT =   3.74
              F23 = 2.7130   F12 = 9.7858   PN =  .4552    PNPSI =  .4552   DPHI2 =   .1112E+07
T =  .00033   PHI = 142.79   PHIDOT = 397.61   G = .0079   PSID =  45.51   PSIDOT =  383.14   PHITOT =   3.79
              F23 = 2.4523   F12 = 9.7856   PN =  .4610    PNPSI =  .4610   DPHI2 =   .1126E+07
T =  .00033   PHI = 142.85   PHIDOT = 400.40   G = .0076   PSID =  45.57   PSIDOT =  386.64   PHITOT =   3.85
              F23 = 2.4530   F12 = 9.7870   PN =  .4626    PNPSI =  .4626   DPHI2 =   .1119E+07
T =  .00033   PHI = 142.91   PHIDOT = 403.18   G = .0074   PSID =  45.62   PSIDOT =  390.15   PHITOT =   3.91
              F23 = 2.4562   F12 = 9.7923   PN =  .4619    PNPSI =  .4619   DPHI2 =   .1100E+07
T =  .00033   PHI = 142.97   PHIDOT = 405.93   G = .0072   PSID =  45.68   PSIDOT =  393.66   PHITOT =   3.97
              F23 = 2.4569   F12 = 9.7937   PN =  .4634    PNPSI =  .4634   DPHI2 =   .1093E+07
T =  .00034   PHI = 143.02   PHIDOT = 408.64   G = .0070   PSID =  45.74   PSIDOT =  397.16   PHITOT =   4.02
              F23 = 2.4601   F12 = 9.7990   PN =  .4627    PNPSI =  .4627   DPHI2 =   .1075E+07
T =  .00034   PHI = 143.08   PHIDOT = 411.33   G = .0067   PSID =  45.79   PSIDOT =  400.66   PHITOT =   4.08
              F23 = 2.4608   F12 = 9.8005   PN =  .4643    PNPSI =  .4643   DPHI2 =   .1067E+07
T =  .00034   PHI = 143.14   PHIDOT = 413.98   G = .0065   PSID =  45.85   PSIDOT =  404.14   PHITOT =   4.14
              F23 = 2.4640   F12 = 9.8059   PN =  .4636    PNPSI =  .4636   DPHI2 =   .1049E+07
T =  .00034   PHI = 143.20   PHIDOT = 416.60   G = .0063   PSID =  45.91   PSIDOT =  407.62   PHITOT =   4.20
              F23 = 2.4648   F12 = 9.8074   PN =  .4652    PNPSI =  .4652   DPHI2 =   .1041E+07
T =  .00035   PHI = 143.23   PHIDOT = 417.80   G = .0062   PSID =  45.94   PSIDOT =  409.26   PHITOT =   4.23
              F23 = 2.7282   F12 = 9.8250   PN =  .4482    PNPSI =  .4482   DPHI2 =   .9531E+06
T =  .00035   PHI = 143.26   PHIDOT = 418.99   G = .0060   PSID =  45.97   PSIDOT =  410.89   PHITOT =   4.26
              F23 = 2.7286   F12 = 9.8257   PN =  .4489    PNPSI =  .4489   DPHI2 =   .9495E+06
T =  .00035   PHI = 143.29   PHIDOT = 420.18   G = .0059   PSID =  46.00   PSIDOT =  412.53   PHITOT =   4.29
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                F23 = 2.7326   F12 = 9.8258   PN = .4519   PNPSI = .4519   DPHI2 =  .9558E+06
T =  .00035     PHI = 143.32   PHIDOT = 421.37   G = .0058   PSID = 46.03   PSIDOT = 414.17   PHITOT =  4.32
                F23 = 2.7330   F12 = 9.8265   PN = .4527   PNPSI = .4527   DPHI2 =  .9521E+06
T =  .00035     PHI = 143.35   PHIDOT = 422.56   G = .0057   PSID = 46.06   PSIDOT = 415.83   PHITOT =  4.35
                F23 = 2.7371   F12 = 9.8265   PN = .4556   PNPSI = .4556   DPHI2 =  .9585E+06
T =  .00035     PHI = 143.38   PHIDOT = 423.75   G = .0056   PSID = 46.09   PSIDOT = 417.49   PHITOT =  4.38
                F23 = 2.7376   F12 = 9.8273   PN = .4564   PNPSI = .4564   DPHI2 =  .9547E+06
T =  .00035     PHI = 143.41   PHIDOT = 424.95   G = .0054   PSID = 46.12   PSIDOT = 419.16   PHITOT =  4.41
                F23 = 2.7417   F12 = 9.8274   PN = .4594   PNPSI = .4594   DPHI2 =  .9611E+06
T =  .00035     PHI = 143.44   PHIDOT = 426.14   G = .0053   PSID = 46.15   PSIDOT = 420.83   PHITOT =  4.44
                F23 = 2.7421   F12 = 9.8281   PN = .4602   PNPSI = .4602   DPHI2 =  .9573E+06
T =  .00036     PHI = 143.47   PHIDOT = 427.34   G = .0052   PSID = 46.18   PSIDOT = 422.51   PHITOT =  4.47
                F23 = 2.7463   F12 = 9.8282   PN = .4633   PNPSI = .4633   DPHI2 =  .9636E+06
T =  .00036     PHI = 143.50   PHIDOT = 428.54   G = .0051   PSID = 46.21   PSIDOT = 424.20   PHITOT =  4.50
                F23 = 2.7467   F12 = 9.8289   PN = .4641   PNPSI = .4641   DPHI2 =  .9597E+06
T =  .00036     PHI = 143.54   PHIDOT = 429.74   G = .0050   PSID = 46.24   PSIDOT = 425.90   PHITOT =  4.54
                F23 = 2.7509   F12 = 9.8290   PN = .4672   PNPSI = .4672   DPHI2 =  .9660E+06
T =  .00036     PHI = 143.57   PHIDOT = 430.94   G = .0048   PSID = 46.27   PSIDOT = 427.61   PHITOT =  4.57
                F23 = 2.7514   F12 = 9.8290   PN = .4681   PNPSI = .4681   DPHI2 =  .9621E+06
T =  .00036     PHI = 143.60   PHIDOT = 432.14   G = .0047   PSID = 46.30   PSIDOT = 429.32   PHITOT =  4.60
                F23 = 2.7556   F12 = 9.8299   PN = .4712   PNPSI = .4712   DPHI2 =  .9684E+06
T =  .00036     PHI = 143.63   PHIDOT = 433.35   G = .0046   PSID = 46.33   PSIDOT = 431.04   PHITOT =  4.63
                F23 = 2.7561   F12 = 9.8307   PN = .4720   PNPSI = .4720   DPHI2 =  .9644E+06
T =  .00036     PHI = 143.66   PHIDOT = 434.55   G = .0045   PSID = 46.36   PSIDOT = 432.77   PHITOT =  4.66
                F23 = 2.7604   F12 = 9.8308   PN = .4752   PNPSI = .4752   DPHI2 =  .9707E+06
T =  .00036     PHI = 143.69   PHIDOT = 435.76   G = .0044   PSID = 46.39   PSIDOT = 434.51   PHITOT =  4.69
                F23 = 2.7608   F12 = 9.8316   PN = .4761   PNPSI = .4761   DPHI2 =  .9666E+06
T =  .00037     PHI = 143.72   PHIDOT = 436.97   G = .0042   PSID = 46.42   PSIDOT = 436.25   PHITOT =  4.72
                F23 = 2.7651   F12 = 9.8317   PN = .4793   PNPSI = .4793   DPHI2 =  .9729E+06
T =  .00037     PHI = 143.75   PHIDOT = 438.18   G = .0041   PSID = 46.45   PSIDOT = 438.00   PHITOT =  4.75
                F23 = 2.7656   F12 = 9.8325   PN = .4802   PNPSI = .4802   DPHI2 =  .9687E+06
T =  .00037     PHI = 143.78   PHIDOT = 439.39   G = .0040   PSID = 46.49   PSIDOT = 439.76   PHITOT =  4.78
                F23 = 2.7637   F12 = 8.9806   PN = .4824   PNPSI = .4824   DPHI2 =  .9705E+06
T =  .00037     PHI = 143.82   PHIDOT = 440.60   G = .0039   PSID = 46.52   PSIDOT = 441.52   PHITOT =  4.82
                F23 = 2.7642   F12 = 8.9813   PN = .4833   PNPSI = .4833   DPHI2 =  .9663E+06
T =  .00037     PHI = 143.85   PHIDOT = 441.81   G = .0037   PSID = 46.55   PSIDOT = 443.29   PHITOT =  4 85
                F23 = 2.7636   F12 = 8.9801   PN = .4858   PNPSI = .4858   DPHI2 =  .9689E+06
T =  .00037     PHI = 143.88   PHIDOT = 443.01   G = .0036   PSID = 46.58   PSIDOT = 445.07   PHITOT =  4.88
                F23 = 2.7641   F12 = 8.9808   PN = .4868   PNPSI = .4868   DPHI2 =  .9646E+06
T =  .00037     PHI = 143.91   PHIDOT = 444.22   G = .0035   PSID = 46.61   PSIDOT = 446.85   PHITOT =  4.91
                F23 = 2.7635   F12 = 8.9796   PN = .4893   PNPSI = .4893   DPHI2 =  .9672E+06
T =  .00037     PHI = 143.94   PHIDOT = 445.42   G = .0034   PSID = 46.65   PSIDOT = 448.63   PHITOT =  4.94
                F23 = 2.7640   F12 = 8.9804   PN = .4902   PNPSI = .4902   DPHI2 =  .9629E+06
T =  .00038     PHI = 143.97   PHIDOT = 446.62   G = .0032   PSID = 46.68   PSIDOT = 450.43   PHITOT =  4.97
                F23 = 2.7634   F12 = 8.9791   PN = .4928   PNPSI = .4928   DPHI2 =  .9654E+06
T =  .00038     PHI = 144.01   PHIDOT = 447.83   G = .0031   PSID = 46.71   PSIDOT = 452.22   PHITOT =  5.01
                F23 = 2.7639   F12 = 8.9799   PN = .4937   PNPSI = .4937   DPHI2 =  .9610E+06
T =  .00038     PHI = 144.04   PHIDOT = 449.03   G = .0030   PSID = 46.74   PSIDOT = 454.03   PHITOT =  5.04
                F23 = 2.7633   F12 = 8.9787   PN = .4963   PNPSI = .4963   DPHI2 =  .9635E+06
T =  .00038     PHI = 144.07   PHIDOT = 450.23   G = .0028   PSID = 46.77   PSIDOT = 455.84   PHITOT =  5.07
                F23 = 2.7638   F12 = 8.9795   PN = .4973   PNPSI = .4973   DPHI2 =  .9590E+06
T =  .00038     PHI = 144.10   PHIDOT = 451.42   G = .0027   PSID = 46.81   PSIDOT = 457.65   PHITOT =  5.10
                F23 = 2.7632   F12 = 8.9782   PN = .4999   PNPSI = .4599   DPHI2 =  .9614E+06
T =  .00038     PHI = 144.14   PHIDOT = 452.62   G = .0026   PSID = 46.84   PSIDOT = 459.47   PHITOT =  5.14
                F23 = 2.7638   F12 = 8.9790   PN = .5009   PNPSI = .5009   DPHI2 =  .9568E+06
T =  .00038     PHI = 144.17   PHIDOT = 453.82   G = .0025   PSID = 46.87   PSIDOT = 461.30   PHITOT =  5.17
                F23 = 2.7632   F12 = 8.9778   PN = .5035   PNPSI = .5035   DPHI2 =  .9592E+06
T =  .00038     PHI = 144.20   PHIDOT = 455.01   G = .0023   PSID = 46.91   PSIDOT = 463.13   PHITOT =  5.20
                F23 = 2.7637   F12 = 8.9786   PN = .5045   PNPSI = .5045   DPHI2 =  .9546E+06
T =  .00039     PHI = 144.23   PHIDOT = 456.20   G = .0022   PSID = 46.94   PSIDOT = 464.97   PHITOT =  5.23
                F23 = 2.4987   F12 = 8.9794   PN = .5041   PNPSI = .5041   DPHI2 =  .9438E+06
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T =  .00039   PHI = 144.27   PHIDOT = 457.38   G = .0021   PSID =  46.97   PSIDOT =  466.80   PHITOT =   5.27
              F23 = 2.4891   F12 = 8.9802   PN =  .5051   PNPSI =  .5051   DPHI2 =   .9391E+06
T =  .00039   PHI = 144.30   PHIDOT = 458.55   G = .0019   PSID =  47.01   PSIDOT =  468.63   PHITOT =   5.30
              F23 = 2.4863   F12 = 8.9816   PN =  .5039   PNPSI =  .5039   DPHI2 =   .9247E+06
T =  .00039   PHI = 144.33   PHIDOT = 459.70   G = .0018   PSID =  47.04   PSIDOT =  470.45   PHITOT =   5.33
              F23 = 2.4867   F12 = 8.9824   PN =  .5050   PNPSI =  .5050   DPHI2 =   .9200E+06
T =  .00039   PHI = 144.36   PHIDOT = 460.85   G = .0017   PSID =  47.07   PSIDOT =  472.27   PHITOT =   5.36
              F23 = 2.4839   F12 = 8.9838   PN =  .5037   PNPSI =  .5037   DPHI2 =   .9055E+06
T =  .00039   PHI = 144.40   PHIDOT = 461.98   G = .0015   PSID =  47.11   PSIDOT =  474.08   PHITOT =   5.40
              F23 = 2.4843   F12 = 8.9847   PN =  .5048   PNPSI =  .5048   DPHI2 =   .9008E+06
T =  .00039   PHI = 144.43   PHIDOT = 463.10   G = .0014   PSID =  47.14   PSIDOT =  475.89   PHITOT =   5.43
              F23 = 2.4814   F12 = 8.9860   PN =  .5035   PNPSI =  .5035   DPHI2 =   .8863E+06
T =  .00039   PHI = 144.46   PHIDOT = 464.21   G = .0013   PSID =  47.18   PSIDOT =  477.70   PHITOT =   5.46
              F23 = 2.4819   F12 = 8.9869   PN =  .5045   PNPSI =  .5045   DPHI2 =   .8815E+06
T =  .00040   PHI = 144.50   PHIDOT = 465.30   G = .0012   PSID =  47.21   PSIDOT =  479.49   PHITOT =   5.50
              F23 = 2.4790   F12 = 8.9882   PN =  .5033   PNPSI =  .5033   DPHI2 =   .8670E+06
T =  .00040   PHI = 144.53   PHIDOT = 466.39   G = .0010   PSID =  47.24   PSIDOT =  481.29   PHITOT =   5.53
              F23 = 2.4795   F12 = 8.9891   PN =  .5043   PNPSI =  .5043   DPHI2 =   .8622E+06
T =  .00040   PHI = 144.56   PHIDOT = 467.34   G = .0009   PSID =  47.28   PSIDOT =  482.95   PHITOT =   5.56
              F23 = 2.5664   F12 = 9.8047   PN =  .4552   PNPSI =  .4552   DPHI2 =   .6466E+06
T =  .00040   PHI = 144.60   PHIDOT = 468.24   G = .0008   PSID =  47.31   PSIDOT =  484.57   PHITOT =   5.60
              F23 = 2.5669   F12 = 9.8056   PN =  .4561   PNPSI =  .4561   DPHI2 =   .6422E+06
T =  .00040   PHI = 144.63   PHIDOT = 469.04   G = .0006   PSID =  47.35   PSIDOT =  486.09   PHITOT =   5.63
              F23 = 2.5714   F12 = 9.8059   PN =  .4594   PNPSI =  .4594   DPHI2 =   .6479E+06
T =  .00040   PHI = 144.67   PHIDOT = 469.84   G = .0005   PSID =  47.38   PSIDOT =  487.63   PHITOT =   5.67
              F23 = 2.5719   F12 = 9.8067   PN =  .4603   PNPSI =  .4603   DPHI2 =   .6436E+06
T =  .00040   PHI = 144.70   PHIDOT = 470.65   G = .0003   PSID =  47.42   PSIDOT =  489.17   PHITOT =   5.70
              F23 = 2.5764   F12 = 9.8070   PN =  .4637   PNPSI =  .4637   DPHI2 =   .6493E+06
T =  .00040   PHI = 144.73   PHIDOT = 471.45   G = .0002   PSID =  47.45   PSIDOT =  490.71   PHITOT =   5.73
              F23 = 2.5769   F12 = 9.8079   PN =  .4646   PNPSI =  .4646   DPHI2 =   .6449E+06
T =  .00041   PHI = 144.77   PHIDOT = 472.26   G = .0001   PSID =  47.49   PSIDOT =  492.27   PHITOT =   5.77
              F23 = 2.5815   F12 = 9.8082   PN =  .4680   PNPSI =  .4680   DPHI2 =   .6505E+06
T =  .00041   PHI = 144.80   PHIDOT = 473.07   G =-.0001   PSID =  47.52   PSIDOT =  493.83   PHITOT =   5.80
              F23 = 2.5820   F12 = 9.8091   PN =  .4689   PNPSI =  .4689   DPHI2 =   .6461E+06

FREE MOTION

T =  .00041   PHI = 210.25   PHIDOT = 473.07   PSI = 314.59   PSIDOT =  493.93   PHITOT =   5.80
              FF12 =  9.439   FF23 =  2.359
T =  .00041   PHI = 210.32   PHIDOT = 480.56   PSI = 314.66   PSIDOT =  491.92   PHITOT =   5.87
              FF12 =  9.414   FF23 =  2.399
T =  .00041   PHI = 210.39   PHIDOT = 488.17   PSI = 314.74   PSIDOT =  490.02   PHITOT =   5.94
              FF12 =  9.415   FF23 =  2.390
T =  .00041   PHI = 210.46   PHIDOT = 495.84   PSI = 314.81   PSIDOT =  488.11   PHITOT =   6.01
              FF12 =  9.412   FF23 =  2.159
T =  .00042   PHI = 210.53   PHIDOT = 503.55   PSI = 314.88   PSIDOT =  486.20   PHITOT =   6.08
              FF12 =  9.412   FF23 =  2.160
T =  .00042   PHI = 210.61   PHIDOT = 511.24   PSI = 314.94   PSIDOT =  484.29   PHITOT =   6.15
              FF12 =  9.419   FF23 =  2.164
T =  .00042   PHI = 210.68   PHIDOT = 518.91   PSI = 315.01   PSIDOT =  482.39   PHITOT =   6.23
              FF12 =  9.419   FF23 =  2.164
T =  .00042   PHI = 210.76   PHIDOT = 526.55   PSI = 315.08   PSIDOT =  480.48   PHITOT =   6.30
              FF12 =  9.426   FF23 =  2.168
T =  .00043   PHI = 210.83   PHIDOT = 534.16   PSI = 315.15   PSIDOT =  478.57   PHITOT =   6.38
              FF12 =  9.426   FF23 =  2.168
T =  .00043   PHI = 210.87   PHIDOT = 537.79   PSI = 315.19   PSIDOT =  477.62   PHITOT =   6.42
              FF12 =  9.454   FF23 =  2.409
T =  .00043   PHI = 210.91   PHIDOT = 541.40   PSI = 315.22   PSIDOT =  476.66   PHITOT =   6.45
              FF12 =  9.454   FF23 =  2.409
T =  .00043   PHI = 210.95   PHIDOT = 545.03   PSI = 315.25   PSIDOT =  475.71   PHITOT =   6.49
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                FF12 =  9.451   FF23 =  2.412
   T =  .00043  PHI = 210.99    PHIDOT = 548.67   PSI = 315.29   PSIDOT =  474.76   PHITOT =   6.53
                FF12 =  9.451   FF23 =  2.412
   T =  .00043  PHI = 211.03    PHIDOT = 552.33   PSI = 315.32   PSIDOT =  473.80   PHITOT =   6.57
                FF12 =  9.449   FF23 =  2.416
   T =  .00043  PHI = 211.07    PHIDOT = 556.00   PSI = 315.36   PSIDOT =  472.85   PHITOT =   6.61
                FF12 =  9.449   FF23 =  2.416
   T =  .00044  PHI = 211.11    PHIDOT = 559.69   PSI = 315.39   PSIDOT =  471.90   PHITOT =   6.65
                FF12 =  9.446   FF23 =  2.419
   T =  .00044  PHI = 211.15    PHIDOT = 563.38   PSI = 315.42   PSIDOT =  470.94   PHITOT =   6.69
                FF12 =  9.446   FF23 =  2.419
   T =  .00044  PHI = 211.19    PHIDOT = 567.10   PSI = 315.46   PSIDOT =  469.99   PHITOT =   6.73
                FF12 =  9.444   FF23 =  2.422
   T =  .00044  PHI = 211.23    PHIDOT = 570.83   PSI = 315.49   PSIDOT =  469.04   PHITOT =   6.77
                FF12 =  9.444   FF23 =  2.422
   T =  .00044  PHI = 211.27    PHIDOT = 574.58   PSI = 315.52   PSIDOT =  468.08   PHITOT =   6.81
                FF12 =  9.441   FF23 =  2.426
   T =  .00044  PHI = 211.31    PHIDOT = 578.34   PSI = 315.56   PSIDOT =  467.13   PHITOT =   6.86
                FF12 =  9.441   FF23 =  2.426
   T =  .00044  PHI = 211.35    PHIDOT = 582.12   PSI = 315.59   PSIDOT =  466.18   PHITOT =   6.90
                FF12 =  9.438   FF23 =  2.429
   T =  .00044  PHI = 211.39    PHIDOT = 585.91   PSI = 315.62   PSIDOT =  465.22   PHITOT =   6.94
                FF12 =  9.439   FF23 =  2.429
   T =  .00045  PHI = 211.44    PHIDOT = 589.72   PSI = 315.66   PSIDOT =  464.27   PHITOT =   6.98
                FF12 =  9.436   FF23 =  2.433
   T =  .00045  PHI = 211.48    PHIDOT = 593.54   PSI = 315.69   PSIDOT =  463.31   PHITOT =   7.02
                FF12 =  9.436   FF23 =  2.433
   T =  .00045  PHI = 211.52    PHIDOT = 597.38   PSI = 315.72   PSIDOT =  462.36   PHITOT =   7.07
                FF12 =  9.433   FF23 =  2.437
   T =  .00045  PHI = 211.56    PHIDOT = 601.24   PSI = 315.76   PSIDOT =  461.41   PHITOT =   7.11
                FF12 =  9.433   FF23 =  2.437
   T =  .00045  PHI = 211.61    PHIDOT = 605.11   PSI = 315.79   PSIDOT =  460.45   PHITOT =   7.15
                FF12 =  9.430   FF23 =  2.440
   T =  .00045  PHI = 211.65    PHIDOT = 609.00   PSI = 315.82   PSIDOT =  459.50   PHITOT =   7.20
                FF12 =  9.430   FF23 =  2.440
   T =  .00045  PHI = 211.69    PHIDOT = 612.91   PSI = 315.86   PSIDOT =  458.55   PHITOT =   7.24
                FF12 =  9.427   FF23 =  2.444
   T =  .00045  PHI = 211.74    PHIDOT = 616.83   PSI = 315.89   PSIDOT =  457.59   PHITOT =   7.28
                FF12 =  9.428   FF23 =  2.444
   T =  .00046  PHI = 211.78    PHIDOT = 620.77   PSI = 315.92   PSIDOT =  456.64   PHITOT =   7.33
                FF12 =  9.424   FF23 =  2.206
VP=  47.109   VS= -71.706

   IMPACT

              PHI= 211.782   PHIDOT= 113.856   PSI= 315.922   PSIDOT=-127.482   PHITOT =   7.327
VP= -13.152   VS= -13.152

   COUPLED MOTION
   T =  .00046  PHI = 211.78    PHIDOT = 113.86    G =-.0115   PSID = 315.90   PSIDOT = -127.59   PHITOT =   7.33
                F23 = 2.4000    F12 = 9.7741   PN =  .3925   PNPSI =  .3925   DPHI2 =   .1124E+07
   T =  .00047  PHI = 211.85    PHIDOT = 125.13    G =-.0113   PSID = 315.82   PSIDOT = -139.53   PHITOT =   7.40
                F23 = 2.4015    F12 = 9.7764   PN =  .3882   PNPSI =  .3882   DPHI2 =   .1126E+07
   T =  .00048  PHI = 211.93    PHIDOT = 136.50    G =-.0111   PSID = 315.74   PSIDOT = -151.39   PHITOT =   7.47
                F23 = 2.4002    F12 = 9.7742   PN =  .3863   PNPSI =  .3863   DPHI2 =   .1141E+07
   T =  .00049  PHI = 212.01    PHIDOT = 147.94    G =-.0108   PSID = 315.65   PSIDOT = -163.12   PHITOT =   7.55
                F23 = 2.4021    F12 = 9.7769   PN =  .3812   PNPSI =  .3812   DPHI2 =   .1142E+07
   T =  .00050  PHI = 212.09    PHIDOT = 159.49    G =-.0105   PSID = 315.55   PSIDOT = -174.74   PHITOT =   7.64
                F23 = 2.4006    F12 = 9.7743   PN =  .3790   PNPSI =  .3790   DPHI2 =   .1159E+07
   T =  .00050  PHI = 212.14    PHIDOT = 164.97    G =-.0103   PSID = 315.50   PSIDOT = -180.14   PHITOT =   7.69
                F23 = 2.6589    F12 = 9.7892   PN =  .3619   PNPSI =  .3619   DPHI2 =   .1096E+07
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T =  .00051   PHI = 212.19   PHIDOT = 170.45   G =-.0102   PSID = 315.45   PSIDOT = -185.48   PHITOT =   7.73
              F23 = 2.6580   F12 = 9.7878   PN =  .3607   PNPSI =  .3607   DPHI2 =  .1105E+07
T =  .00051   PHI = 212.24   PHIDOT = 176.02   G =-.0100   PSID = 315.39   PSIDOT = -190.86   PHITOT =   7.78
              F23 = 2.6633   F12 = 9.7856   PN =  .3629   PNPSI =  .3629   DPHI2 =  .1129E+07
T =  .00052   PHI = 212.29   PHIDOT = 181.67   G =-.0098   PSID = 315.34   PSIDOT = -196.26   PHITOT =   7.84
              F23 = 2.6623   F12 = 9.7841   PN =  .3617   PNPSI =  .3617   DPHI2 =  .1138E+07
T =  .00052   PHI = 212.34   PHIDOT = 187.41   G =-.0096   PSID = 315.28   PSIDOT = -201.69   PHITOT =   7.89
              F23 = 2.6679   F12 = 9.7817   PN =  .3640   PNPSI =  .3640   DPHI2 =  .1165E+07
T =  .00053   PHI = 212.40   PHIDOT = 193.23   G =-.0095   PSID = 315.22   PSIDOT = -207.14   PHITOT =   7.94
              F23 = 2.6669   F12 = 9.7801   PN =  .3626   PNPSI =  .3626   DPHI2 =  .1175E+07
T =  .00053   PHI = 212.45   PHIDOT = 199.16   G =-.0093   PSID = 315.16   PSIDOT = -212.63   PHITOT =   8.00
              F23 = 2.6728   F12 = 9.7775   PN =  .3650   PNPSI =  .3650   DPHI2 =  .1203E+07
T =  .00054   PHI = 212.51   PHIDOT = 205.17   G =-.0091   PSID = 315.10   PSIDOT = -218.14   PHITOT =   8.06
              F23 = 2.6717   F12 = 9.7757   PN =  .3636   PNPSI =  .3636   DPHI2 =  .1214E+07
T =  .00054   PHI = 212.57   PHIDOT = 211.30   G =-.0089   PSID = 315.04   PSIDOT = -223.68   PHITOT =   8.12
              F23 = 2.6779   F12 = 9.7729   PN =  .3660   PNPSI =  .3660   DPHI2 =  .1244E+07
T =  .00055   PHI = 212.63   PHIDOT = 217.52   G =-.0087   PSID = 314.97   PSIDOT = -229.25   PHITOT =   8.18
              F23 = 2.6767   F12 = 9.7710   PN =  .3644   PNPSI =  .3644   DPHI2 =  .1256E+07
T =  .00055   PHI = 212.70   PHIDOT = 223.86   G =-.0085   PSID = 314.91   PSIDOT = -234.86   PHITOT =   8.24
              F23 = 2.6832   F12 = 9.7679   PN =  .3670   PNPSI =  .3670   DPHI2 =  .1288E+07
T =  .00056   PHI = 212.76   PHIDOT = 230.30   G =-.0082   PSID = 314.84   PSIDOT = -240.49   PHITOT =   8.31
              F23 = 2.6819   F12 = 9.7659   PN =  .3653   PNPSI =  .3653   DPHI2 =  .1301E+07
T =  .00056   PHI = 212.83   PHIDOT = 236.88   G =-.0080   PSID = 314.77   PSIDOT = -246.16   PHITOT =   8.37
              F23 = 2.6888   F12 = 9.7625   PN =  .3679   PNPSI =  .3679   DPHI2 =  .1336E+07
T =  .00057   PHI = 212.90   PHIDOT = 243.56   G =-.0078   PSID = 314.70   PSIDOT = -251.85   PHITOT =   8.44
              F23 = 2.6874   F12 = 9.7603   PN =  .3661   PNPSI =  .3661   DPHI2 =  .1350E+07
T =  .00057   PHI = 212.97   PHIDOT = 250.38   G =-.0075   PSID = 314.63   PSIDOT = -257.58   PHITOT =   8.51
              F23 = 2.6947   F12 = 9.7566   PN =  .3688   PNPSI =  .3688   DPHI2 =  .1388E+07
T =  .00058   PHI = 213.04   PHIDOT = 257.32   G =-.0073   PSID = 314.55   PSIDOT = -263.33   PHITOT =   8.58
              F23 = 2.6932   F12 = 9.7543   PN =  .3667   PNPSI =  .3667   DPHI2 =  .1402E+07
T =  .00058   PHI = 213.11   PHIDOT = 264.41   G =-.0070   PSID = 314.47   PSIDOT = -269.13   PHITOT =   8.66
              F23 = 2.4310   F12 = 9.7544   PN =  .3647   PNPSI =  .3647   DPHI2 =  .1419E+07
T =  .00059   PHI = 213.19   PHIDOT = 271.56   G =-.0068   PSID = 314.40   PSIDOT = -274.88   PHITOT =   8.74
              F23 = 2.4296   F12 = 9.7519   PN =  .3625   PNPSI =  .3625   DPHI2 =  .1434E+07
T =  .00059   PHI = 213.27   PHIDOT = 278.75   G =-.0065   PSID = 314.32   PSIDOT = -280.55   PHITOT =   8.82
              F23 = 2.4314   F12 = 9.7546   PN =  .3573   PNPSI =  .3573   DPHI2 =  .1436E+07
T =  .00060   PHI = 213.35   PHIDOT = 285.99   G =-.0062   PSID = 314.24   PSIDOT = -286.16   PHITOT =   8.90
              F23 = 2.4299   F12 = 9.7521   PN =  .3549   PNPSI =  .3549   DPHI2 =  .1452E+07
T =  .00060   PHI = 213.37   PHIDOT = 287.71   G =-.0062   PSID = 314.22   PSIDOT = -287.45   PHITOT =   8.92
              F23 = 2.6899   F12 = 9.7713   PN =  .3359   PNPSI =  .3359   DPHI2 =  .1362E+07
T =  .00060   PHI = 213.39   PHIDOT = 289.41   G =-.0061   PSID = 314.19   PSIDOT = -288.72   PHITOT =   8.94
              F23 = 2.6895   F12 = 9.7707   PN =  .3353   PNPSI =  .3353   DPHI2 =  .1366E+07
T =  .00060   PHI = 213.41   PHIDOT = 291.12   G =-.0060   PSID = 314.17   PSIDOT = -289.99   PHITOT =   8.96
              F23 = 2.6916   F12 = 9.7696   PN =  .3361   PNPSI =  .3361   DPHI2 =  .1377E+07
T =  .00060   PHI = 213.43   PHIDOT = 292.85   G =-.0060   PSID = 314.15   PSIDOT = -291.26   PHITOT =   8.98
              F23 = 2.6912   F12 = 9.7690   PN =  .3355   PNPSI =  .3355   DPHI2 =  .1381E+07
T =  .00060   PHI = 213.46   PHIDOT = 294.58   G =-.0059   PSID = 314.13   PSIDOT = -292.54   PHITOT =   9.00
              F23 = 2.6933   F12 = 9.7679   PN =  .3363   PNPSI =  .3363   DPHI2 =  .1392E+07
T =  .00060   PHI = 213.48   PHIDOT = 296.32   G =-.0058   PSID = 314.11   PSIDOT = -293.82   PHITOT =   9.02
              F23 = 2.6929   F12 = 9.7673   PN =  .3357   PNPSI =  .3357   DPHI2 =  .1396E+07
T =  .00060   PHI = 213.50   PHIDOT = 298.07   G =-.0058   PSID = 314.09   PSIDOT = -295.10   PHITOT =   9.04
              F23 = 2.6951   F12 = 9.7661   PN =  .3365   PNPSI =  .3365   DPHI2 =  .1407E+07
T =  .00061   PHI = 213.52   PHIDOT = 299.83   G =-.0057   PSID = 314.07   PSIDOT = -296.38   PHITOT =   9.06
              F23 = 2.6946   F12 = 9.7655   PN =  .3359   PNPSI =  .3359   DPHI2 =  .1412E+07
T =  .00061   PHI = 213.54   PHIDOT = 301.60   G =-.0056   PSID = 314.05   PSIDOT = -297.66   PHITOT =   9.09
              F23 = 2.6968   F12 = 9.7644   PN =  .3367   PNPSI =  .3367   DPHI2 =  .1423E+07
T =  .00061   PHI = 213.56   PHIDOT = 303.37   G =-.0055   PSID = 314.03   PSIDOT = -298.95   PHITOT =   9.11
              F23 = 2.6964   F12 = 9.7637   PN =  .3361   PNPSI =  .3361   DPHI2 =  .1427E+07
T =  .00061   PHI = 213.58   PHIDOT = 305.16   G =-.0055   PSID = 314.00   PSIDOT = -300.24   PHITOT =   9.13
              F23 = 2.6986   F12 = 9.7625   PN =  .3369   PNPSI =  .3369   DPHI2 =  .1439E+07
T =  .00061   PHI = 213.61   PHIDOT = 306.96   G =-.0054   PSID = 313.98   PSIDOT = -301.53   PHITOT =   9.15
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            F23 = 2.9576  F12 =10.7202  PN =  .3459  PNPSI =  .3459  DPHI2 =   .1762E+07
T =  .05146 PHI = 214.20  PHIDOT = 378.21  G =-.0034  PSID = 313.41  PSIDOT = -355.72  PHITOT =1482.48
            F23 = 2.9634  F12 =10.7166  PN =  .3478  PNPSI =  .3478  DPHI2 =   .1795E+07
T =  .05147 PHI = 214.26  PHIDOT = 382.70  G =-.0032  PSID = 313.36  PSIDOT = -358.52  PHITOT =1482.53
            F23 = 2.9621  F12 =10.7144  PN =  .3459  PNPSI =  .3459  DPHI2 =   .1807E+07
T =  .05147 PHI = 214.31  PHIDOT = 387.25  G =-.0030  PSID = 313.31  PSIDOT = -361.33  PHITOT =1482.59
            F23 = 2.9679  F12 =10.7106  PN =  .3478  PNPSI =  .3478  DPHI2 =   .1841E+07
T =  .05147 PHI = 214.37  PHIDOT = 391.85  G =-.0028  PSID = 313.25  PSIDOT = -364.14  PHITOT =1482.64
            F23 = 2.9666  F12 =10.7084  PN =  .3458  PNPSI =  .3458  DPHI2 =   .1853E+07
T =  .05147 PHI = 214.43  PHIDOT = 396.52  G =-.0026  PSID = 313.20  PSIDOT = -366.97  PHITOT =1482.70
            F23 = 2.9655  F12 = 9.7768  PN =  .3468  PNPSI =  .3468  DPHI2 =   .1883E+07
T =  .05148 PHI = 214.48  PHIDOT = 401.23  G =-.0024  PSID = 313.15  PSIDOT = -369.78  PHITOT =1482.76
            F23 = 2.9641  F12 = 9.7747  PN =  .3447  PNPSI =  .3447  DPHI2 =   .1896E+07
T =  .05148 PHI = 214.54  PHIDOT = 406.00  G =-.0023  PSID = 313.09  PSIDOT = -372.61  PHITOT =1482.81
            F23 = 2.9603  F12 = 9.7683  PN =  .3453  PNPSI =  .3453  DPHI2 =   .1924E+07
T =  .05148 PHI = 214.60  PHIDOT = 410.81  G =-.0021  PSID = 313.04  PSIDOT = -375.42  PHITOT =1482.87
            F23 = 2.9588  F12 = 9.7661  PN =  .3432  PNPSI =  .3432  DPHI2 =   .1938E+07
T =  .05148 PHI = 214.66  PHIDOT = 415.69  G =-.0019  PSID = 312.99  PSIDOT = -378.24  PHITOT =1482.93
            F23 = 2.9550  F12 = 9.7595  PN =  .3437  PNPSI =  .3437  DPHI2 =   .1967E+07
T =  .05149 PHI = 214.72  PHIDOT = 420.61  G =-.0017  PSID = 312.93  PSIDOT = -381.05  PHITOT =1482.99
            F23 = 2.9535  F12 = 9.7573  PN =  .3415  PNPSI =  .3415  DPHI2 =   .1981E+07
T =  .05149 PHI = 214.78  PHIDOT = 425.59  G =-.0015  PSID = 312.88  PSIDOT = -383.86  PHITOT =1483.05
            F23 = 2.9495  F12 = 9.7505  PN =  .3419  PNPSI =  .3419  DPHI2 =   .2011E+07
T =  .05149 PHI = 214.84  PHIDOT = 430.62  G =-.0013  PSID = 312.82  PSIDOT = -386.67  PHITOT =1483.11
            F23 = 2.9479  F12 = 9.7482  PN =  .3396  PNPSI =  .3396  DPHI2 =   .2025E+07
T =  .05149 PHI = 214.90  PHIDOT = 435.71  G =-.0010  PSID = 312.77  PSIDOT = -389.47  PHITOT =1483.18
            F23 = 2.9438  F12 = 9.7413  PN =  .3401  PNPSI =  .3401  DPHI2 =   .2056E+07
T =  .05150 PHI = 214.97  PHIDOT = 440.86  G =-.0008  PSID = 312.71  PSIDOT = -392.27  PHITOT =1483.24
            F23 = 2.9422  F12 = 9.7389  PN =  .3377  PNPSI =  .3377  DPHI2 =   .2071E+07
T =  .05150 PHI = 215.03  PHIDOT = 446.07  G =-.0006  PSID = 312.65  PSIDOT = -395.07  PHITOT =1483.30
            F23 = 2.6482  F12 = 9.7316  PN =  .3382  PNPSI =  .3382  DPHI2 =   .2104E+07
T =  .05150 PHI = 215.09  PHIDOT = 451.33  G =-.0004  PSID = 312.60  PSIDOT = -397.85  PHITOT =1483.37
            F23 = 2.6467  F12 = 9.7291  PN =  .3357  PNPSI =  .3357  DPHI2 =   .2119E+07
T =  .05150 PHI = 215.16  PHIDOT = 456.63  G =-.0002  PSID = 312.54  PSIDOT = -400.61  PHITOT =1483.43
            F23 = 2.4741  F12 =10.5969  PN =  .3045  PNPSI =  .3045  DPHI2 =   .1957E+07
T =  .05151 PHI = 215.22  PHIDOT = 461.94  G = .0000  PSID = 312.48  PSIDOT = -403.33  PHITOT =1483.50
            F23 = 2.4727  F12 =10.5942  PN =  .3021  PNPSI =  .3021  DPHI2 =   .1972E+07

FREE MOTION

T =  .05151 PHI = 133.41    PHIDOT = 461.94  PSI =  45.41  PSIDOT = -403.33  PHITOT =1483.50
            FF12 = 10.356   FF23 =  2.342
T =  .05151 PHI = 133.47    PHIDOT = 470.16  PSI =  45.36  PSIDOT = -401.59  PHITOT =1483.56
            FF12 = 10.384   FF23 =  2.6[illegible]
T =  .05151 PHI = 133.54    PHIDOT = 478.37  PSI =  45.30  PSIDOT = -399.85  PHITOT =1483.63
            FF12 = 10.383   FF23 =  2.625
T =  .05151 PHI = 133.61    PHIDOT = 486.65  PSI =  45.24  PSIDOT = -398.10  PHITOT =1483.70
            FF12 = 10.378   FF23 =  2.631
T =  .05152 PHI = 133.68    PHIDOT = 494.97  PSI =  45.18  PSIDOT = -396.36  PHITOT =1483.77
            FF12 = 10.378   FF23 =  2.631
T =  .05152 PHI = 133.75    PHIDOT = 503.37  PSI =  45.13  PSIDOT = -394.62  PHITOT =1483.84
            FF12 = 10.373   FF23 =  2.637
T =  .05152 PHI = 133.82    PHIDOT = 511.80  PSI =  45.07  PSIDOT = -392.88  PHITOT =1483.92
            FF12 = 10.372   FF23 =  2.637
T =  .05152 PHI = 133.90    PHIDOT = 520.3[illegible]  PSI =  45.01  PSIDOT = -391.14  PHITOT =1483.99
            FF12 = 10.367   FF23 =  2.644
T =  .05153 PHI = 133.97    PHIDOT = 528.87  PSI =  44.96  PSIDOT = -389.40  PHITOT =1484.06
            FF12 = 10.367   FF23 =  2.644
T =  .05153 PHI = 134.05    PHIDOT = 537.51  PSI =  44.90  PSIDOT = -387.66  PHITOT =1484.14
            FF12 = 10.361   FF23 =  2.651
```

```
T =  .05153   PHI = 134.13   PHIDOT = 546.19   PSI =  44.85   PSIDOT = -385.92   PHITOT =1484.22
              FF12 = 10.360   FF23 =  2.651
T =  .05153   PHI = 134.21   PHIDOT = 554.92   PSI =  44.79   PSIDOT = -384.18   PHITOT =1484.30
              FF12 = 10.365   FF23 =  2.395
T =  .05154   PHI = 134.29   PHIDOT = 563.63   PSI =  44.74   PSIDOT = -382.44   PHITOT =1484.38
              FF12 = 10.365   FF23 =  2.395
T =  .05154   PHI = 134.37   PHIDOT = 572.30   PSI =  44.68   PSIDOT = -380.70   PHITOT =1484.46
              FF12 = 10.373   FF23 =  2.400
T =  .05154   PHI = 134.45   PHIDOT = 580.93   PSI =  44.63   PSIDOT = -378.96   PHITOT =1464.54
              FF12 = 10.373   FF23 =  2.400
T =  .05154   PHI = 134.53   PHIDOT = 589.44   PSI =  44.57   PSIDOT = -377.22   PHITOT =1484.63
              FF12 = 10.406   FF23 =  2.665
T =  .05155   PHI = 134.62   PHIDOT = 597.57   PSI =  44.52   PSIDOT = -375.48   PHITOT =1484.71
              FF12 = 10.406   FF23 =  2.665
T =  .05155   PHI = 134.71   PHIDOT = 605.78   PSI =  44.47   PSIDOT = -373.74   PHITOT =1484.80
              FF12 = 10.399   FF23 =  2.673
T =  .05155   PHI = 134.79   PHIDOT = 614.04   PSI =  44.41   PSIDOT = -372.00   PHITOT =1484.88
              FF12 = 10.399   FF23 =  2.673
T =  .05155   PHI = 134.88   PHIDOT = 622.40   PSI =  44.36   PSIDOT = -370.26   PHITOT =1484.97
              FF12 = 10.392   FF23 =  2.681
T =  .05156   PHI = 134.97   PHIDOT = 630.81   PSI =  44.31   PSIDOT = -368.51   PHITOT =1485.06
              FF12 = 10.392   FF23 =  2.681

 F23MAX =  3.24

 F12MAX = 11.41

 FF23MAX =  2.93

 FF12MAX = 10.94

 PNMAX =   .66

NUMBER OF TURNS TO ARM=  25.779
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APPENDIX B

DYNAMICS OF ROTOR DRIVEN S&A MECHANISM WITH A TWO-PASS INVOLUTE GEAR TRAIN AND A VERGE (PLATE PALLET) RUNAWAY ESCAPEMENT

Description of Systems and Outline of Derivations

This appendix gives derivations for the complete mathematical model of an S&A mechanism, consisting of a spin driven rotor, a two pass, step-up involute gear train and a verge-type runaway escapement. Figures B-1 and B-2 show the two types of configurations to which this model may be adapted.

This work uses geometric and kinematic equations for the verge escapement developed in appendixes C through F. In addition, the dynamics of fuze gear trains is based on the work reported in reference B-1. As in previous efforts of Lowen and Tepper on the pin pallet escapement (refs B-1 and B-2), the following three motion regimes of the mechanism are considered:

Coupled Motion

The tip of the escape wheel is in constant contact with either the entrance or the exit working surface of the verge while it is driven by the rotor (gear no. 1) through the gear and pinion set no. 2. The entrance- as well as the exit-coupled motion differential equations, which differ from each other to some extent, are expressed in the escape wheel variable ϕ and are obtained by combining the solutions to the Newtonian force and moment expressions for the individual mechanism components.

Free Motion

The pallet and the escape wheel, gear train, rotor systems move independently of each other in this phase of the motion. Thus, there results one differential equation for the pallet in the variable ψ, and another one for the combined system in the escape wheel variable ϕ.

Impact

The formulation of the impact regime is similar to that shown in reference B-1 for the pin pallet escapement. Again, the moment of inertia of the escape wheel and pinion no. 3 also contains the referred mass properties of the rotor and gear and pinion set no. 2. This impact simulation is based on the classical angular impulse - momentum model, where a coefficient of restitution is used to account for the energy losses. Since it is assumed that the effect of the impact force between the escape wheel and the pallet is much greater than the effects due to the driving torque of the rotor and the various retarding torques due to friction, the latter are disregarded.

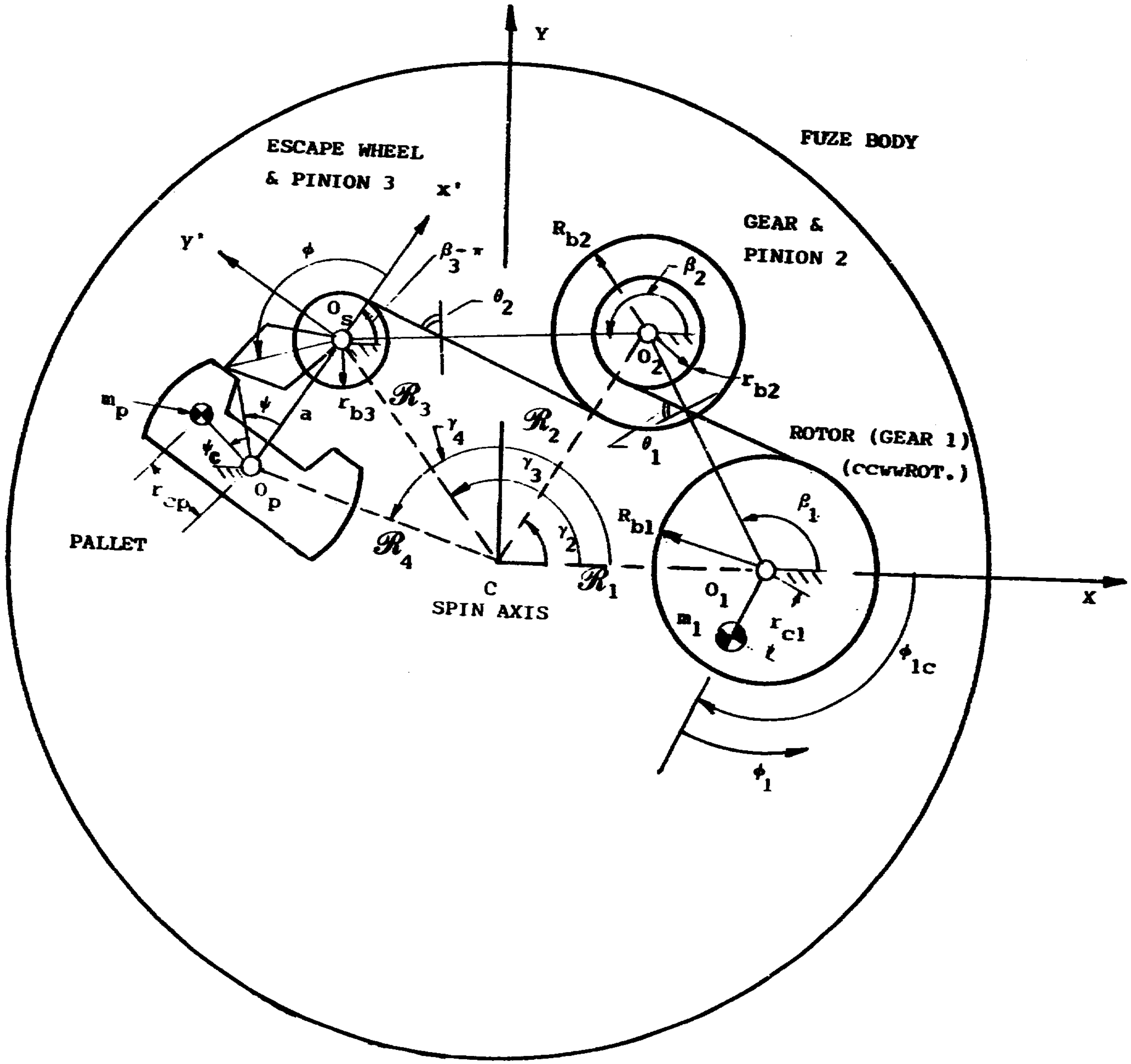

Figure B-1. Rotor driven S&A device, configuration no. 1

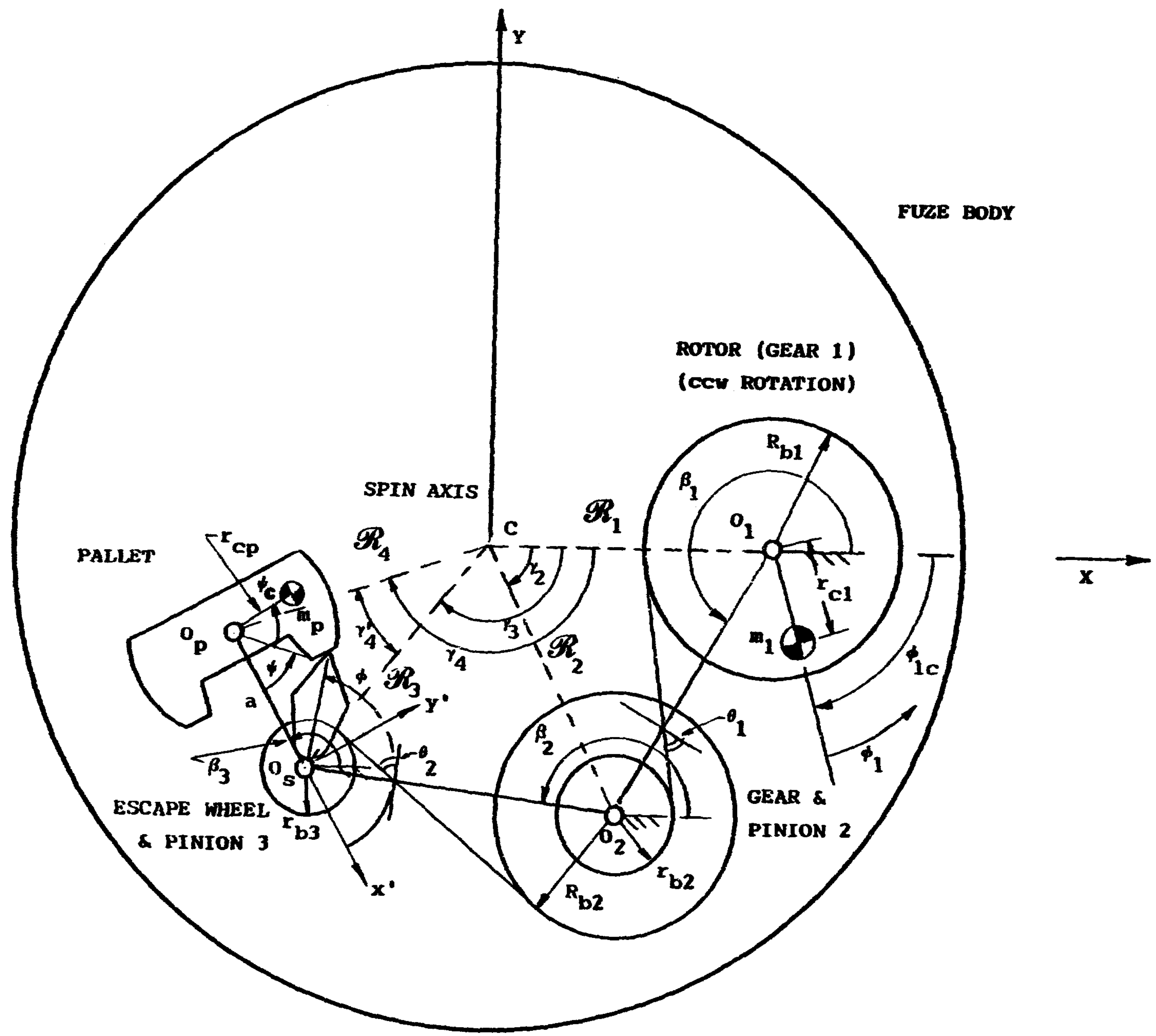

Figure B-2. Rotor driven S&A device, configuration no. 2

Treatment of Friction Forces

The influence of friction forces is considered both in the coupled and the free motion regimes. There is friction at the interface between the escape wheel and the pallet during coupled motion, and there is friction between the teeth of the gears at all of the pivots during both of these types of motions. As in reference B-1, the individual pivot friction torques are obtained by the algebraic addition of the two friction moments due to the x and y components of the normal bearing forces, rather than by the direct use of the resulting normal forces. This conservative approach to friction is necessary to avoid the difficulties which the presence of a square root term introduces into the solutions of the various differential equations.

Appendix E of reference B-1 proves that a change in sign of the coefficient of friction, both in the coupled and the free motion simulations of the gear train, including the rotor and the escape wheel (with the exception of the escape wheel-pallet interface), is sufficient to account for the changed directions of the friction forces encountered during a possible motion reversal of the gear train following an impact.

Geometry of Fuze Body Configurations

The following section contains derivations of expressions for the various fuze body angles associated with the pivot holes of both configurations of the two pass, step-up gear mechanisms. In addition, relationships between the unit vectors of the body-fixed X-Y and x'-y' systems (figs. B-1 and B-2) are given to describe certain escapement forces in terms of the primed coordinate system. (Note that the x'-axis is oriented along the escapement center-line O_p-O_s.) Finally a signum function is introduced so that common programming expressions for both configurations can be devised.

Fuze Body Configuration No. 1

The following relationships for the angles γ_i, δ_i, and β_i for configuration no. 1 are indicated by figure B-3:

<u>Angles γ_i</u>

$$\gamma_2 = \cos^{-1} \left[\frac{{}_1^2 + {}_2^2 - (R_{p1} + r_{p2})^2}{2R_1R_2}\right] \qquad (B1)$$

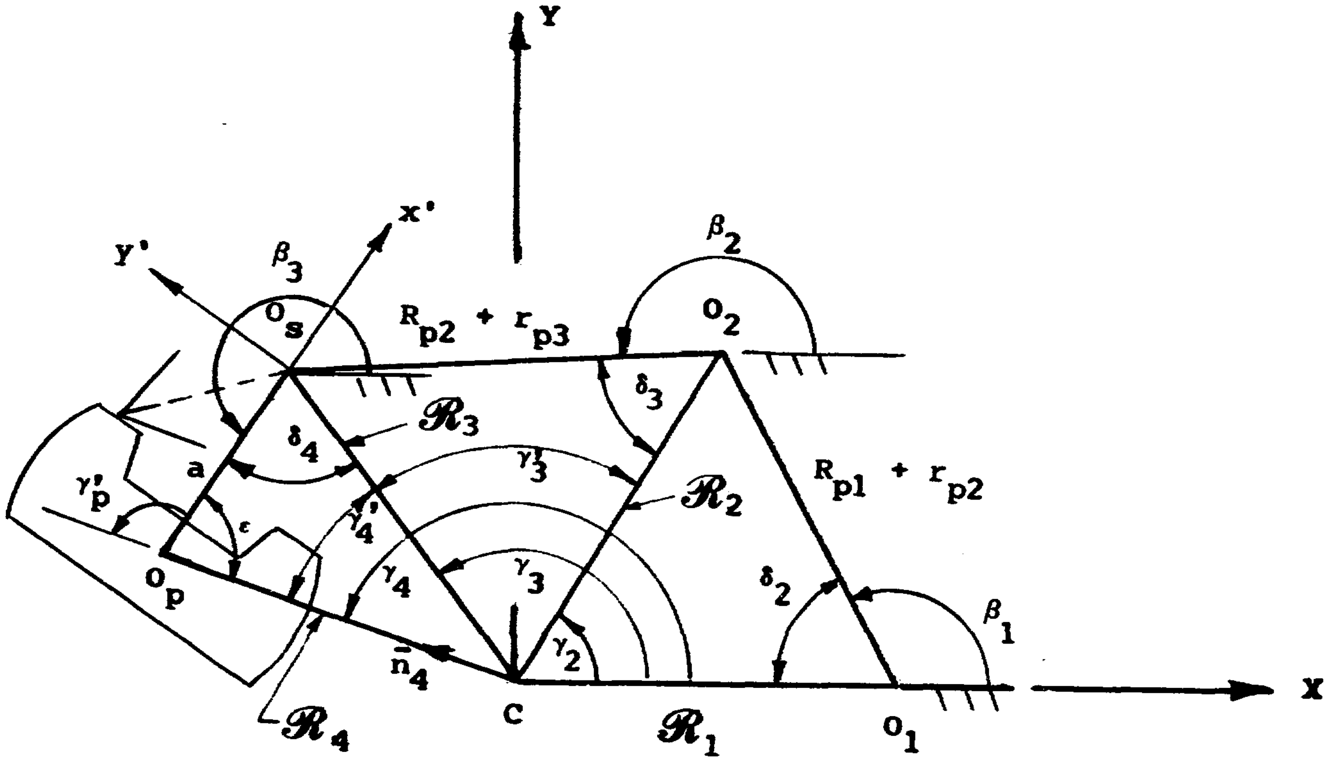

Figure B-3. Fuze body configuration no. 1

Further,

$$\gamma_3' = \cos^{-1}\left[\frac{\mathcal{R}_2^2 + \mathcal{R}_3^2 - (R_{p2} + r_{p3})^2}{2\mathcal{R}_2\mathcal{R}_3}\right] \tag{B2}$$

Finally,

$$\gamma_3 = \gamma_2 + \gamma_3' \tag{B3}$$

The angle γ_4' is given by:

$$\gamma_4' = \cos^{-1}\left[\frac{\mathcal{R}_3^2 + \mathcal{R}_4^2 - a^2}{2\mathcal{R}_3\mathcal{R}_4}\right] \tag{B4}$$

Then,

$$\gamma_4 = \gamma_3 + \gamma_4' \tag{B5}$$

<u>Angles δ_i</u>. The angles δ_2 and δ_3, as shown in figure B-3, become:

$$\delta_2 = \cos^{-1}\left[\frac{(R_{p1} + r_{p2})^2 + \mathcal{R}_1^2 - \mathcal{R}_2^2}{2\mathcal{R}_1 (R_{p1} + r_{p2})}\right] \tag{B6}$$

and

$$\delta_3 = \cos^{-1}\left[\frac{(R_{p2} + r_{p3})^2 + \mathcal{R}_2^2 - \mathcal{R}_3^2}{2\mathcal{R}_2 (R_{p2} + r_{p3})}\right] \tag{B7}$$

The angle δ_4 must take the pivot-to-pivot distance a of the escapement into account. Therefore,

$$\delta_4 = \cos^{-1}\left[\frac{a^2 + \mathcal{R}_3^2 - \mathcal{R}_4^2}{2a\mathcal{R}_3}\right] \tag{B8}$$

<u>Angles β_i</u>. The angles β_1 and β_2 become respectively:

$$\beta_1 = \pi - \delta_2 \tag{B9}$$

and

$$\beta_2 = \gamma_2 + \pi - \delta_3 \tag{B10}$$

The angle β_3 is found with the help of angle δ_4 of equation B8. Then,

$$\beta_3 = \gamma_3 + \pi - \delta_4 \quad \text{(B11)}$$

Figure B-3 shows the angle γ'_p between the positive x'-axis and the unit vector $\bar{n}_4$. It is given by:

$$\gamma'_p = \pi - \varepsilon \quad \text{(B12)}$$

where ε is obtained with the help of equations B4 and B8:

$$\varepsilon = \pi - \delta_4 - \gamma'_4, \quad \text{(B13)}$$

and therefore:

$$\gamma'_p = \delta_4 + \gamma'_4 \quad \text{(B14)}$$

The unit vector $\bar{n}_4$ is expressed in terms of the primed coordinate system as follows:

$$\bar{n}_4 = \cos \gamma'_p \bar{i}' + \sin \gamma'_p \bar{j}' \quad \text{(B15)}$$

Further, the unit vectors $\bar{I}'$ and $\bar{J}'$, when expressed in the X-Y system, become:

$$\bar{i}' = \cos (\beta_3 - \pi)\bar{i} + \sin (\beta_3 - \pi)\bar{j}$$

or

$$\bar{i}' = -\cos\beta_3\bar{i} - \sin\beta_3\bar{j} \quad \text{(B16)}$$

and

$$\bar{j}' = \bar{k}' \times \bar{i}' = \sin\beta_3\bar{i} - \cos\beta_3\bar{j} \quad \text{(B17)}$$

Fuze Body Configuration No. 2

The angles γ_i, δ_i, and β_i of configuration no. 2 are defined in figure B-4.

Angles γ_i. Since these angles are defined in the clockwise direction with respect to the body-fixed X-axis, their values must be negative.

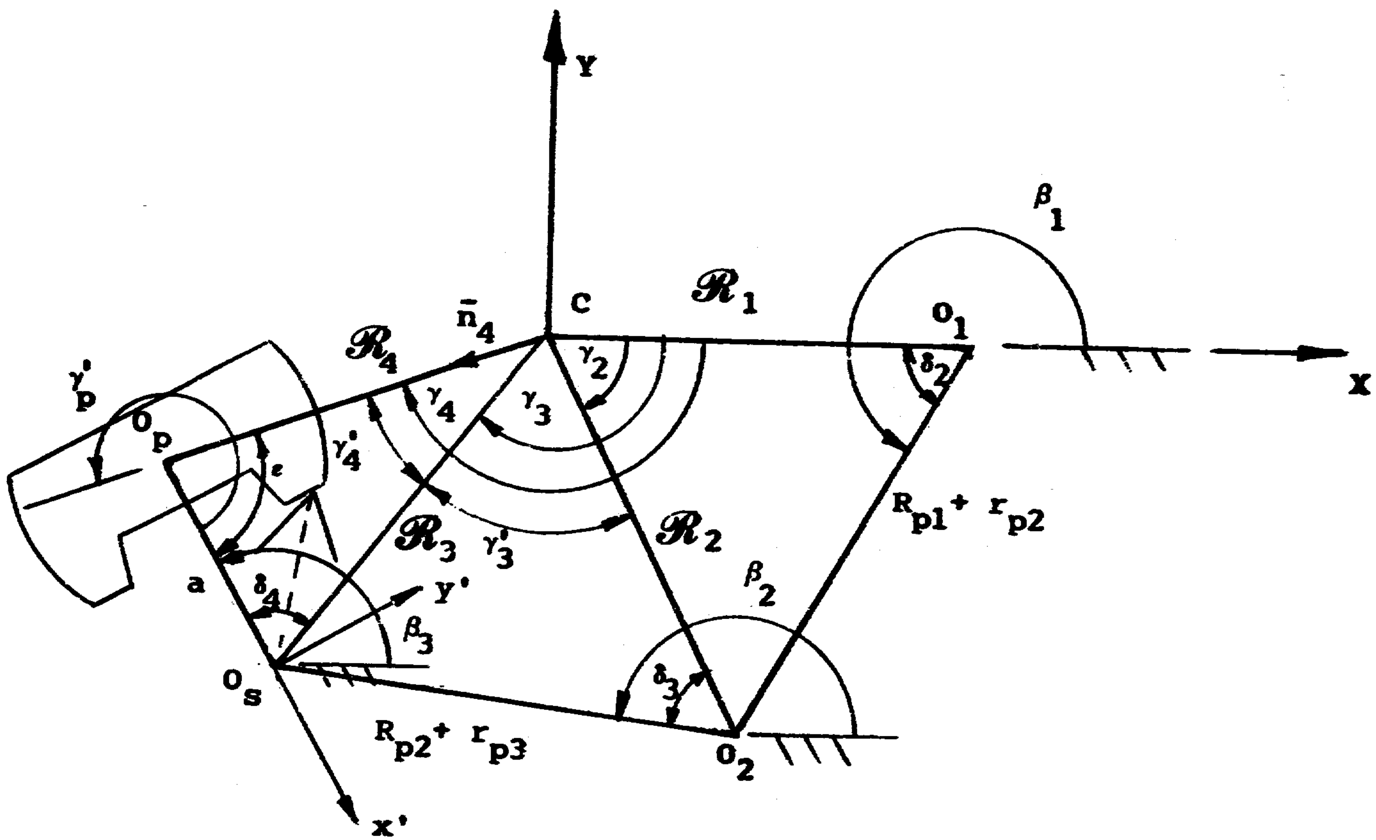

Figure B-4. Fuze body configuration no. 2

Then,

$$\gamma_2 = -\cos^{-1}\left[\frac{\mathcal{R}_1^2 + \mathcal{R}_2^2 - (R_{p1} + r_{p2})^2}{2\mathcal{R}_1\mathcal{R}_2}\right] \tag{B18}$$

Angle γ_3' is given by equation B2 and with that the following expression is obtained for γ_3:

$$\gamma_3 = \gamma_2 - \gamma_3' \tag{B19}$$

The negative sign for γ_3' is necessary in order to make γ_3 negative.

The angle γ_4' is given by equation B4. It allows the following expression for angle γ_4:

$$\gamma_4 = \gamma_3 - \gamma_4' \tag{B20}$$

Again, γ_4' must have a negative sign to make γ_4 negative.

Angles δ_i. The angles δ_2, δ_3, and δ_4 can be taken directly from equations B6, B7, and B8, respectively.

Angles β_i. Angle β_1 becomes:

$$\beta_1 = \pi + \delta_2 \tag{B21}$$

Further

$$\beta_2 = \gamma_2 + \pi + \delta_3 \tag{B22}$$

Finally,

$$\beta_3 = \gamma_3 + \pi + \delta_4 \tag{B23}$$

The angle γ_p' between the positive x'-axis and the unit vector $\bar{n}_4$ is shown in figure B-4. This angle is given by:

$$\gamma_p' = \pi + \varepsilon \tag{B24}$$

where

$$\varepsilon = \pi - \delta_4 - \gamma_4' \tag{B25}$$

Therefore

$$\gamma_p' = 2\pi - \delta_4 - \gamma_4' \tag{B26}$$

The unit vector $\bar{n}_4$ can again be expressed in terms of the primed coordinate system as

$$\bar{n}_4 = \cos \gamma_p' \bar{i}' + \sin \gamma_p' \bar{j}' \tag{B27}$$

The unit vectors $\bar{i}'$ and $\bar{j}'$ are expressed in the X-Y system as follows:

$$\bar{i}' = -\cos \beta_3 \bar{i} - \sin \beta_3 \bar{j} \tag{B28}$$

$$\bar{j}' = \sin \beta_3 \bar{i} - \cos \beta_3 \bar{j} \tag{B29}$$

Common Computational Expressions for Both Configurations

To find common programming expressions for all angles of both configurations (with the exception of the angles γ_p' of equations B14 and B26) the following signum function s_6 is introduced:

$$s_6 = +1 \text{ for configuration no. 1} \tag{B30}$$

$$s_6 = -1 \text{ for configuration no. 2} \tag{B31}$$

This leads to the following expressions:

According to equations B1 and B18:

$$\gamma_2 = s_6 \cos^{-1} \left[\frac{\mathcal{R}_1^2 + \mathcal{R}_2^2 - (R_{p1} + r_{p2})^2}{2\mathcal{R}_1 \mathcal{R}_2}\right] \tag{B32}$$

According to equation B2:

$$\gamma_3' = \cos^{-1} \left[\frac{\mathcal{R}_2^2 + \mathcal{R}_3^2 - (R_{p2} + r_{p3})^2}{2\mathcal{R}_2 \mathcal{R}_3}\right] \tag{B33}$$

According to equations B3 and B19:

$$\gamma_3 = \gamma_2 + s_6 \gamma_3' \tag{B34}$$

According to equation B4:

$$\gamma_4' = \cos^{-1} \left[\frac{\mathcal{R}_3^2 + \mathcal{R}_4^2 - a^2}{2\mathcal{R}_3 \mathcal{R}_4}\right] \tag{B35}$$

According to equations B5 and B20:

$$\gamma_4 = \gamma_3 + s_6 \gamma_4' \quad \text{(B36)}$$

According to equations B6, B7, and B8, respectively

$$\delta_2 = \cos^{-1} \left[\frac{(R_{p1} + r_{p2})^2 + \mathcal{R}_1^2 - \mathcal{R}_2^2}{2\mathcal{R}_1 (R_{p1} + r_{p2})}\right] \quad \text{(B37)}$$

$$\delta_3 = \cos^{-1} \left[\frac{(R_{p2} + r_{p3})^2 + \mathcal{R}_2^2 - \mathcal{R}_3^2}{2\mathcal{R}_2 (R_{p2} + r_{p3})}\right] \quad \text{(B38)}$$

$$\delta_4 = \cos^{-1} \left[\frac{a^2 + \mathcal{R}_3^2 - \mathcal{R}_4^2}{2a\mathcal{R}_3}\right] \quad \text{(B39)}$$

Dynamics of Pallet and Escape Wheel in Coupled Motion

This section shows derivations for both entrance- and exit-coupled motion differential equations. While the resulting combined differential equations are similar, they differ in a number of terms because of the inherent differences of the $\bar{n}_t - \bar{n}_n$ coordinate systems for the entrance and exit sides. While these derivations are given for configuration no. 1, they are also applicable to configuration no. 2 with the help of the signum function s_6 introduced in the previous section.

The analysis of the pallet is performed in the x' - y' system for convenience, while that of the escape wheel uses the X - Y system. An appropriate coordinate transformation makes it possible to combine the systems.

The coefficient of friction μ_1 is used at the escape wheel-pallet interface as well as at the pallet pivot, while the coefficient of friction μ serves to express the friction forces at the escape wheel pivot as well as at the escape wheel-pinion no. 2 tooth interface. (μ is also used in all other gear meshes and gear pivots.)

Signum Functions and Acceleration of Center of Mass of Pallet

To define the direction of the friction force of the escape wheel point S on the coincident pallet point T, the signum function s_4 is introduced. Thus,

$$s_4 = \frac{V_{S/T}}{|V_{S/T}|} \quad \text{(B40)}$$

where $V_{S/T}$ is given by equation C25 of appendix C. The signum function s_5 serves for the determination of the direction of the friction forces acting on the pallet pivot. Then

$$s_5 = \frac{\dot{\psi}}{|\dot{\psi}|} \tag{B41}$$

With the constant spin ω of the fuse body, the acceleration $\bar{A}_{cp}$ of the center of mass of the pallet is given by (figs. B-1 and B-5):

$$\begin{aligned}\bar{A}_{cp} = -\omega^2 R_4 \bar{n}_4 - (\omega + \dot{\psi})^2 r_{cp}[\cos(\psi + \psi_c)\bar{i}' + \sin(\psi + \psi_c)\bar{j}'] \\ + \ddot{\psi}\, r_{cp}\, [-\sin(\psi + \psi_c)\bar{i}' + \cos(\psi + \psi_c)\bar{j}']\end{aligned} \tag{B42}$$

Dynamics of Pallet and Escape Wheel for Entrance-Coupled Motion

A detailed free body diagram of the pallet for entrance-coupled motion is shown in figure B-5. The free body diagram of the escape wheel is simplified and presented for orientation only. A complete free body diagram of the escape wheel which includes all pivot and contact forces is given by figure B-6.

Force Equation of Pallet. Substitution of all forces into Newton's Law, together with the acceleration of the center of mass according to equation B42 furnishes:

$$\begin{aligned}P_n\bar{n}_n + \mu_1 s_4 P_n \bar{n}_t + F'_{xp}\bar{i}' - \mu_1 s_5 F'_{yp}\bar{i}' + F'_{yp}\bar{j}' + \mu_1 s_5 F'_{xp}\bar{j}' \\ = m_p \{-\omega^2 R_4 \bar{n}_4 - (\omega + \dot{\psi})^2 r_{cp}\, [\cos(\psi + \psi_c)\bar{i}' + \sin(\psi + \psi_c)\bar{j}'] \\ + \ddot{\psi}\, r_{cp}\, [-\sin(\psi + \psi_c)\bar{i}' + \cos(\psi + \psi_c)\bar{j}']\}\end{aligned} \tag{B43}$$

(For nomenclature, consult figures B-1 and B-5, as well as reference B-1.)

Moment Equation of Pallet. The moment equation of the pallet must be written with the understanding that pivot point O_p is an accelerated point. Thus,

$$\bar{M}_{op} = -\ddot{\bar{r}}_{op} \times m_p\, \bar{r}_{cp} + \dot{\bar{H}}_{op} \tag{B44}$$

where

$\bar{M}_{op}$ = sum of all external moments with respect to pivot O_p

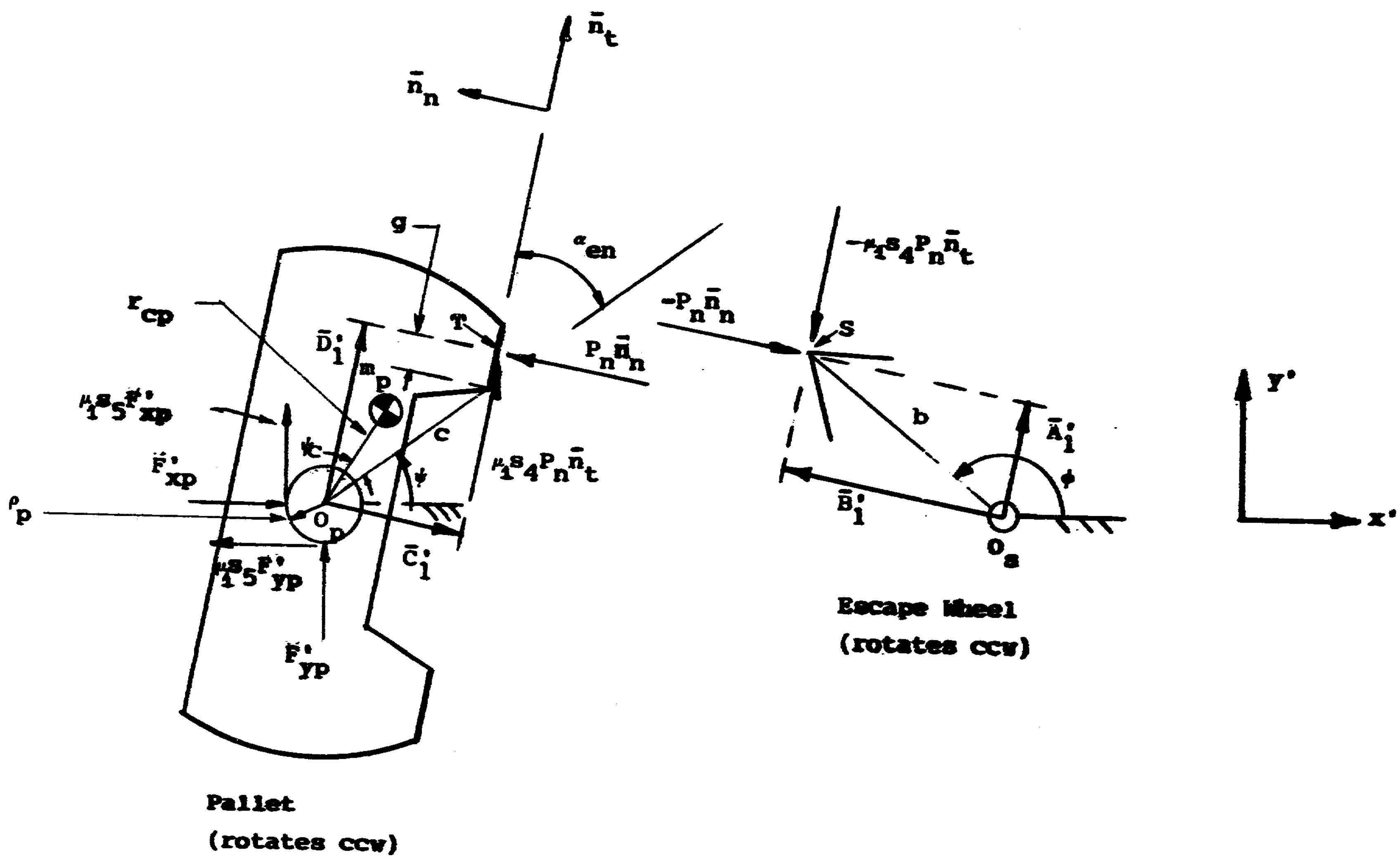

Figure B-5. Free body diagram of pallet in entrance-coupled motion

$\ddot{\bar{r}}_{op}$ = absolute acceleration of point O_p

$\dot{\bar{H}}_{op}$ = time rate of change of angular momentum of the pallet with respect to point O_p.

Since the spin angular velocity $\bar{\omega}$ of the fuze body is constant, equation B44 takes the following form:

$$\bar{M}_{op} = -(-\omega^2 R_4 \bar{n}_4) \times m_p r_{cp} [\cos(\psi + \psi_c)\bar{i}' + \sin(\psi + \psi_c)\bar{j}'] + I_p \ddot{\psi}\bar{k} \quad \text{(B45)}$$

If $\bar{n}_4$ is substituted according to equation B27, the above becomes in scalar terms:

$$M_{op} = I_p \ddot{\psi} - m_p r_{cp} R_4 \omega^2 \sin(\gamma_p' - \psi - \psi_c) \quad \text{(B46)}$$

To find the value of $\bar{M}_{op}$ consider figure B-5.

$$\bar{M}_{op} = D_1' \bar{n}_t \times P_n \bar{n}_n + C_1' \bar{n}_n \times \mu_1 s_4 P_n \bar{n}_t - \rho_p \mu_1 s_5 (\tilde{F}_{xp} + \tilde{F}_{yp})\bar{k} \quad \text{(B47)}$$

or, in scalar terms

$$M_{op} = D_1' P_n - C_1' \mu_1 s_4 P_n - \rho_1 \mu_1 s_5 (\tilde{F}_{xp} + \tilde{F}_{yp}) \quad \text{(B48)}$$

When equation B48 is substituted into equation B46, the final moment expression for the pallet is obtained:

$$P_n [D_1' - C_1' \mu_1 s_4] - \rho_p \mu_1 s_5 (\tilde{F}_{xp} + \tilde{F}_{yp}) = I_p \ddot{\psi} - m_p R_4 r_{cp} \omega^2 \sin(\gamma_p' - \psi - \psi_c) \quad \text{(B49)}$$

As originally discussed in an ARRADCOM report (ref B-3), $\tilde{F}_{xp}$ and $\tilde{F}_{yp}$ represent conservatively evaluated pivot force components which assure that the pivot friction moments are opposed to the rotation at all times.

The pivot friction components F_{xp}' and F_{yp}' are first obtained from the following component expressions of equation B43. Subsequently, they are transformed to become $\tilde{F}_{xp}$ and $\tilde{F}_{yp}$, respectively.

$$-P_n \sin(\psi + \alpha) + \mu_1 s_4 P_n \cos(\psi + \alpha) + F_{xp}' - \mu_1 s_5 F_{yp}' = m_p [-\omega^2 R_4 \cos\gamma_p' - (\omega + \dot{\psi})^2 r_{cp} \cos(\psi + \psi_c) - \ddot{\psi} r_{cp} \sin(\psi + \psi_c)] \quad \text{(B50)}$$

and

$$P_n\cos(\psi + \alpha) + \mu_1 s_4 P_n \sin(\psi + \alpha) + \mu_1 s_5 F'_{xp} + F'_{yp} \tag{B51}$$

$$= m_p\,[-\omega^2 \mathcal{R}_4 \sin\gamma'_p - (\omega + \dot{\psi})^2 r_{cp}\sin(\psi + \psi_c) + \ddot{\psi} r_{cp}\cos(\psi + \psi_c)]$$

Simultaneous solution of the above furnishes:

$$\tilde{F}_{yp} = P_n A_1 + \omega^2 A_2 \pm 2\frac{\omega^2}{|\omega|}\dot{\psi}A_3 \pm \dot{\psi}^2 A_3 \pm \ddot{\psi}A_4 \tag{B52}$$

$$\tilde{F}_{xp} = P_n A_5 + \omega^2 A_6 \pm 2\frac{\omega^2}{|\omega|}\dot{\psi}A_7 \pm \dot{\psi}^2 A_7 \pm \ddot{\psi}A_8 \tag{B53}$$

where

$$A_1 = \left|\frac{-\mu_1(s_4 + s_5)\sin(\psi + \alpha) - (1 - s_4 s_5 \mu_1^2)\cos(\psi + \alpha)}{1 + \mu_1^2}\right| \tag{B54}$$

$$A_2 = \left|\frac{m_p\,[\mathcal{R}_4(\sin\gamma'_p - \mu_1 s_5\cos\gamma'_p) + r_{cp}(\sin(\psi + \psi_c) - \mu_1 s_5 \cos(\psi + \psi_c))]}{1 + \mu_1^2}\right| \tag{B55}$$

$$A_3 = \left|\frac{m_p r_{cp}[\sin(\psi + \psi_c) - \mu_1 s_5 \cos(\psi + \psi_c)]}{1 + \mu_1^2}\right| \tag{B56}$$

$$A_4 = \left|\frac{m_p r_{cp}[\cos(\psi + \psi_c) + \mu_1 s_5 \sin(\psi + \psi_c)]}{1 + \mu_1^2}\right| \tag{B57}$$

$$A_5 = \left|\frac{\mu_1(s_4 + s_5)\cos(\psi + \alpha) + (1 - s_4 s_5 \mu_1^2)\sin(\psi + \alpha)}{1 + \mu_1^2}\right| \tag{B58}$$

$$A_6 = \left|\frac{m_p[-\mathcal{R}_4(\cos\gamma'_p + \mu_1 s_5 \sin\gamma'_p) - r_{cp}(\cos(\psi + \psi_c) + \mu_1 s_5 \sin(\psi + \psi_c))]}{1 + \mu_1^2}\right| \tag{B59}$$

$$A_7 = \left| \frac{m_p r_{cp}[\cos(\psi + \psi_c) + \mu_1 s_5 \sin(\psi + \psi_c)]}{1 + \mu_1^2} \right| \quad \text{(B60)}$$

$$A_8 = \left| \frac{m_p r_{cp}[\sin(\psi + \psi_c) - \mu_1 s_5 \cos(\psi + \psi_c)]}{1 + \mu_1^2} \right| \quad \text{(B61)}$$

The factor $\omega^2/|\omega|$ is introduced in place of ω to assure that the quantity is positive regardless of direction of spin. (This too, assures that the friction moments oppose rotation. The driving moment is proportional to ω^2 and, therefore, independent of spin direction.) To make the final decision concerning the signs in equations B52 and B53, these forces are now substituted into moment equation B49, and the influence of the direction of rotation on each of the resulting moment computations is explored:

$$\begin{aligned} &P_n[D_1' - C_1'\mu_1 s_4 - \rho_p \mu_1 s_5(A_1 + A_5)] \pm \rho_p \mu_1 s_5 \omega^2 (A_2 + A_6) \\ &\pm 2\rho_p \mu_1 s_5 \frac{\omega^2}{|\omega|}\dot{\psi}(A_3 + A_7) \pm \rho_p \mu_1 s_5 \dot{\psi}^2 (A_3 + A_7) \pm \rho_p \mu_1 s_5 \ddot{\psi}(A_4 + A_8) \\ &= I_p \ddot{\psi} - m_p r_{cp} R_5 \omega^2 \sin(\gamma_p' - \psi - \psi_c) \end{aligned} \quad \text{(B62)}$$

With s_5 positive for positive rotation, and vice versa, and with all other parameters positive at all times, the following moment components of equation B62 must have negative signs during positive rotation:

$$-P_n \rho_p \mu_1 s_5 (A_1 + A_5) \quad \text{(B63)}$$

$$-\rho_p \mu_1 s_5 \omega^2 (A_2 + A_6) \quad \text{(B64)}$$

$$-\rho_p \mu_1 s_5 \dot{\psi}^2 (A_3 + A_7) \quad \text{(A65)}$$

The sign of the term containing $\dot{\psi}$ is negative and is controlled by the sign of $\dot{\psi}$. Therefore, the signum operator s_5 is omitted and the term becomes:

$$-2\rho_p \mu_1 \frac{\omega^2}{|\omega|}\dot{\psi}(A_3 + A_7) \quad \text{(B66)}$$

The choice of sign for the friction moment term in equation B62, which is proportional to the pallet angular acceleration $\ddot{\psi}$, is discussed in detail in appendix F of reference B-1. This work leads to the computational rules of equations B73 and B74 below which deal with the sign in the expressions for the effective moment of inertia I_{PR}. (The signum function s_5 has now been omitted.)

With these considerations, equation B62[B-1] now becomes

$$P_n A_{18} - \omega^2 A_{19} - 2\frac{\omega^2}{|\omega|}\dot{\psi} A_{20} - \dot{\psi}^2 A_{21} = I_{PR}\ddot{\psi} - m_p r_{cp} R_4 \omega^2 \sin(\gamma_p' - \psi - \psi_c) \tag{B68}$$

where[B-2]

$$A_{18} = D_1' - C_1'\mu_1 s_4 - \rho_p \mu_1 s_5 (A_1 + A_5) \tag{B69}$$

$$A_{19} = \rho_p \mu_1 s_5 (A_2 + A_6) \tag{B70}$$

$$A_{20} = \rho_p \mu_1 (A_3 + A_7) \tag{B71}$$

$$A_{21} = \rho_p \mu_1 s_5 (A_3 + A_7) \tag{B72}$$

$$I_{PR} = I_p + A_{22}, \text{ when } \dot{\psi} \text{ and } \ddot{\psi} \text{ have identical signs} \tag{B73}$$

$$I_{PR} = I_p - A_{22}, \text{ when } \dot{\psi} \text{ and } \ddot{\psi} \text{ have opposite signs}^{B-3} \tag{B74}$$

$$A_{22} = \rho_p \mu_1 (A_4 + A_8) \tag{B75}$$

Equation B68 is now used to find an expression for the contact force P_n. This expression will later make it possible to establish a single differential equation for the escapement in coupled motion. Thus,

$$P_n = \frac{I_{PR}\ddot{\psi} + A_{21}\dot{\psi}^2 + 2\frac{\omega^2}{|\omega|} A_{20}\dot{\psi} + \omega^2 A_{19} - m_p r_{cp} R_4 \omega^2 \sin(\gamma_p' - \psi - \psi_c)}{A_{18}} \tag{B76}$$

B-1 Note that there is no equation B67.

B-2 The parameters A_n are not sequential at this point. A_9 to A_{17} may be found in equations B90 to B95 and B97 to B99 in conjunction with the work on the escape wheel.

B-3 Care must be taken that $I_p - A_{22}$ does not become negative. If this occurs, I_{PR} must be set equal to zero.

It is now necessary to write equation B76 in terms of the angular velocity $\dot{\phi}$ and the angular acceleration $\ddot{\phi}$ of the escape wheel so that it may later be equated to a similar expression for the escape wheel.

Thus, with the help of equations C19 and C26 of appendix C, the contact force becomes:

$$P_n = \frac{1}{A_{18}} [I_{PR}U\ddot{\phi} + (A_{21}U^2 + I_{PR}V)\dot{\phi}^2 + 2\frac{\omega^2}{|\omega|} A_{20}U\dot{\phi} \tag{B77}$$

$$+ \omega^2 A_{19} - m_p r_{cp} R_4 \omega^2 \sin(\gamma_p' - \psi - \psi_c)]$$

Escape Wheel Analysis. A detailed free body diagram of the escape wheel and pinion no. 3 in entrance-coupled motion is shown in figure B-6. The pivot forces F_{x3} and F_{y3} as well as the contact force F_{23} and the force T_3 are now defined in the general X-Y system. This makes it necessary to transform the unit vectors $\bar{n}_t$ and $\bar{n}_n$ from the x' - y' system to the X-Y system. Since

$$\bar{i}' = \cos(\beta_3 - \pi)\bar{i} + \sin(\beta_3 - \pi)\bar{j} \tag{B78}$$

or

$$\bar{i}' = -\cos\beta_3\bar{i} - \sin\beta_3\bar{j}, \tag{B79}$$

and

$$\bar{j}' = \sin\beta_3\bar{i} - \cos\beta_3\bar{j} \tag{B80}$$

the unit vectors $\bar{n}_t$ and $\bar{n}_n$ can now be determined with the help of equations C5 and C6 of appendix C. Thus,

$$\bar{n}_t = -\cos(\psi + \alpha + \beta_3)\bar{i} - \sin(\psi + \alpha + \beta_3)\bar{j} \tag{B81}$$

$$\bar{n}_n = \sin(\psi + \alpha + \beta_3)\bar{i} - \cos(\psi + \alpha + \beta_3)\bar{j} \tag{B82}$$

Parallel work is given in appendix B of reference B-1.

Force Equation of Escape Wheel. Force equilibrium is obtained with the help of the following expression, based on figure B-6:

$$-P_n\bar{n}_n - \mu_1 s_4 P_n\bar{n}_t + F_{23}\bar{n}_{23} + \mu s_2 F_{23}\bar{n}_{N23} + \bar{T}_3 \tag{B83a}$$

$$+ F_{x3}\bar{i} + \mu F_{y3}\bar{i} + \mu F_{x3}\bar{j} - F_{y3}\bar{j} = 0$$

CCW Rotation

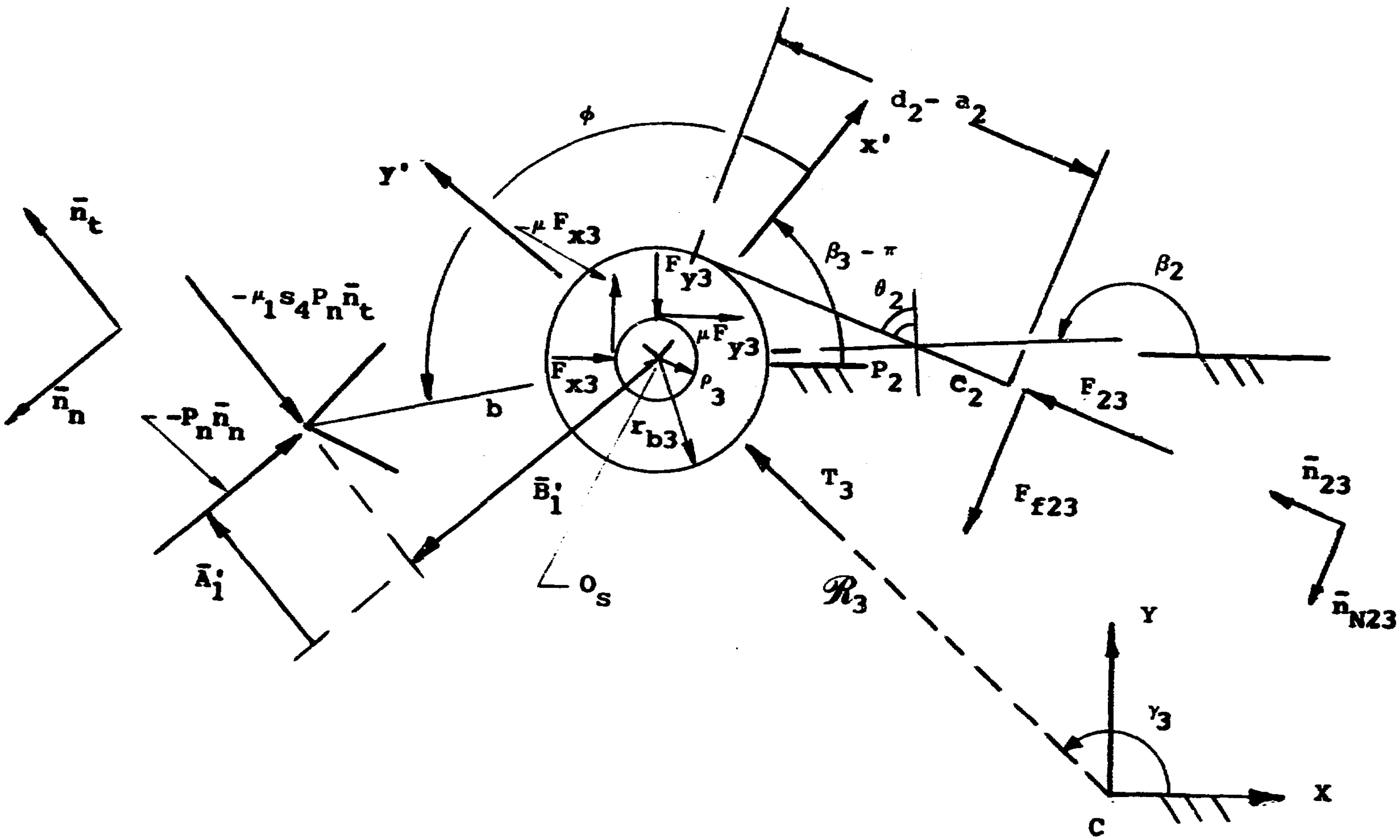

Figure B-6. Free body diagram of escape wheel and pinion 3 in entrance-coupled motion

The signum function s_2 is defined in in reference B-3. Further, according to reference B-1, the D'Alembert force $\bar{T}_3$ is given by

$$\bar{T}_3 = m_3 R_3 \omega^2 (\cos\gamma_3 \bar{i} + \sin\gamma_3 \bar{j}) \tag{B83b}$$

Moment Equation of Escape Wheel. Again, based on the nomenclature of figure B-6, the moment equation of the escape wheel is given by:

$$A_1' \bar{n}_t' x(-P_n)\bar{n}_n + B_1' \bar{n}_n' \times (-\mu_1 s_4 P_n)\bar{n}_t$$
$$-\mu\rho_3(\tilde{F}_{x3} + \tilde{F}_{y3})\bar{k} + r_{b3}F_{23}\bar{k} - \mu s_2(d_2 - a_2)F_{23}\bar{k} = I_3\ddot{\phi}\bar{k} \tag{B84}$$

This becomes in scalar form:

$$-P_n(A_1' - B_1'\mu_1 s_4) - \mu\rho_3(\tilde{F}_{x3} + \tilde{F}_{y3}) + r_{b3}F_{23}$$
$$-\mu s_2(d_2 - a_2)F_{23} = I_3\ddot{\phi} \tag{B85}$$

The pivot force components F_{x3} and F_{y3}, which eventually become $\tilde{F}_{x3}$ and $\tilde{F}_{y3}$, are now determined with the help of the following scalar component expressions of equation B83:

$$-P_n \sin(\psi + \alpha + \beta_3) + \mu_1 s_4 P_n \cos(\psi + \alpha + \beta_3) + T_3 \cos\gamma_3$$
$$+ F_{23}\sin(\beta_2 + \theta_2) + \mu s_2 F_{23}\cos(\beta_2 + \theta_2) + F_{x3} + \mu F_{y3} = 0 \tag{B86}$$

and

$$P_n \cos(\psi + \alpha + \beta_3) + \mu_1 s_4 P_n \sin(\psi + \alpha + \beta_3) + T_3 \sin\gamma_3$$
$$-F_{23}\cos(\beta_2 + \theta_2) + \mu s_2 F_{23}\sin(\beta_2 + \theta_2) + \mu F_{x3} - F_{y3} = 0 \tag{B87}$$

Simultaneous solution of the above yields the following "conservative" pivot force components:

$$\tilde{F}_{y3} = P_n A_9 + F_{23}A_{10} + T_3 A_{11} \tag{B88}$$

$$\tilde{F}_{x3} = P_n A_{12} + F_{23}A_{13} + T_3 A_{14} \tag{B89}$$

Since there are no kinematic terms, it is most conservative to sum all terms and use the absolute values of A_9-A_{14} as follows:

$$A_9 = \left| \frac{(\mu_1 s_4 + \mu)\sin(\psi + \alpha + \beta_3) + (1-\mu\mu_1 s_4)\cos(\psi + \alpha + \beta_3)}{1 + \mu^2} \right| \quad (B90)$$

$$A_{10} = \left| \frac{-\mu(1-s_2)\sin(\beta_2 + \theta_2) - (1 + \mu^2 s_2)\cos(\beta_2 + \theta_2)}{1 + \mu^2} \right| \quad (B91)$$

$$A_{11} = \left| \frac{\sin\gamma_3 - \mu\cos\gamma_3}{1 + \mu^2} \right| \quad (B92)$$

$$A_{12} = \left| \frac{(1 - \mu_1\mu s_4)\sin(\psi + \alpha + \beta_3) - (s_4\mu_1 + \mu)\cos(\psi + \alpha + \beta_3)}{1 + \mu^2} \right| \quad (B93)$$

$$A_{13} = \left| \frac{-(1 + \mu^2 s_2)\sin(\beta_2 + \theta_2) + \mu(1 - s_2)\cos(\beta_2 + \theta_2)}{1 + \mu^2} \right| \quad (B94)$$

$$A_{14} = \left| \frac{\cos\gamma_3 + \mu\sin\gamma_3}{1 + \mu^2} \right| \quad (B95)$$

Substitution of equations B88 and B89 into the moment equation B85 and subsequent rearrangement lead to the following expression for the contact force P_n in terms of escape wheel quantities:

$$P_n = \frac{-I_3\ddot{\phi} + F_{23}A_{15} - T_3A_{16}}{A_{17}} \quad (B96)$$

where

$$A_{15} = r_{b3} - \mu[s_2(d_2 - a_2) + \rho_3(A_{10} + A_{13})] \quad (B97)$$

$$A_{16} = \mu\rho_3\,(A_{11} + A_{14}) \quad (B98)$$

$$A_{17} = A_1' - \mu_1 s_4 B_1' + \mu\rho_3(A_9 + A_{12}) \quad (B99)$$

Combined Entranced-Coupled Motion Differential Equation. Equations B77 and B96 are now set equal to each other. This furnishes the following combined motion differential equation for the escapement:

$$[I_3A_{18} + I_{PR}A_{17}U]\ddot{\phi} + [I_{PR}A_{17}V + A_{17}A_{21}U^2]\dot{\phi}^2 + 2\frac{\omega^2}{|\omega|}A_{17}A_{20}U\dot{\phi} \tag{B100}$$

$$= A_{15}A_{18}F_{23} - A_{16}A_{18}T_3 - A_{17}\omega^2[A_{19} - m_p r_{cp}\mathcal{R}_4\sin(\gamma_p' - \psi - \psi_c)]$$

Dynamics of Pallet and Escape Wheel for Exit-Coupled Motion

A free body diagram of the pallet for exit-coupled motion, together with a simplified representation of the escape wheel is shown in figure B-7. When this figure is compared with figure B-5, which depicts entrance-coupled motion, it is noted that now

$$\bar{P}_n = -P_n\bar{n}_n \tag{B101}$$

This sign change is reflected both in the force and in the moment equation.

Force Equations of Pallet. When the scalar force equations B50 and B51 are altered to account for the above change, they have the following form:

$$P_n\sin(\psi + \alpha) + \mu_1 s_4 P_n\cos(\psi + \alpha) + F'_{xp} - \mu_1 s_5 F'_{yp} \tag{B102}$$

$$= m_p[-\omega^2\mathcal{R}_4\cos\gamma_p' - (\omega + \dot{\psi})^2 r_{cp}\cos(\psi + \psi_c) - \ddot{\psi}r_{cp}\sin(\psi + \psi_c)]$$

$$-P_n\cos(\psi + \alpha) + \mu_1 s_4 P_n\sin(\psi + \alpha) + \mu_1 s_5 F'_{xp} + F'_{yp} \tag{B103}$$

$$= m_p[-\omega^2\mathcal{R}_4\sin\gamma_p' - (\omega + \dot{\psi})^2 r_{cp}\sin(\psi + \psi_c) + \ddot{\psi}r_{cp}\cos(\psi + \psi_c)]$$

Simultaneous solution of the above shows that only the factors A_1 and A_5 as given by equations B54 and B58 need be changed. These will now be called AA_1 and AA_5. Then,

$$AA_1 = \left| \frac{\mu_1(s_5 - s_4)\sin(\psi + \alpha) + (1 + \mu_1^2 s_4 s_5)\cos(\psi + \alpha)}{1 + \mu_1^2} \right| \tag{B104}$$

and

$$AA_5 = \left| \frac{\mu_1(s_5 - s_4)\cos(\psi + \alpha) - (1 + \mu_1^2 s_4 s_5)\sin(\psi + \alpha)}{1 + \mu_1^2} \right| \tag{B105}$$

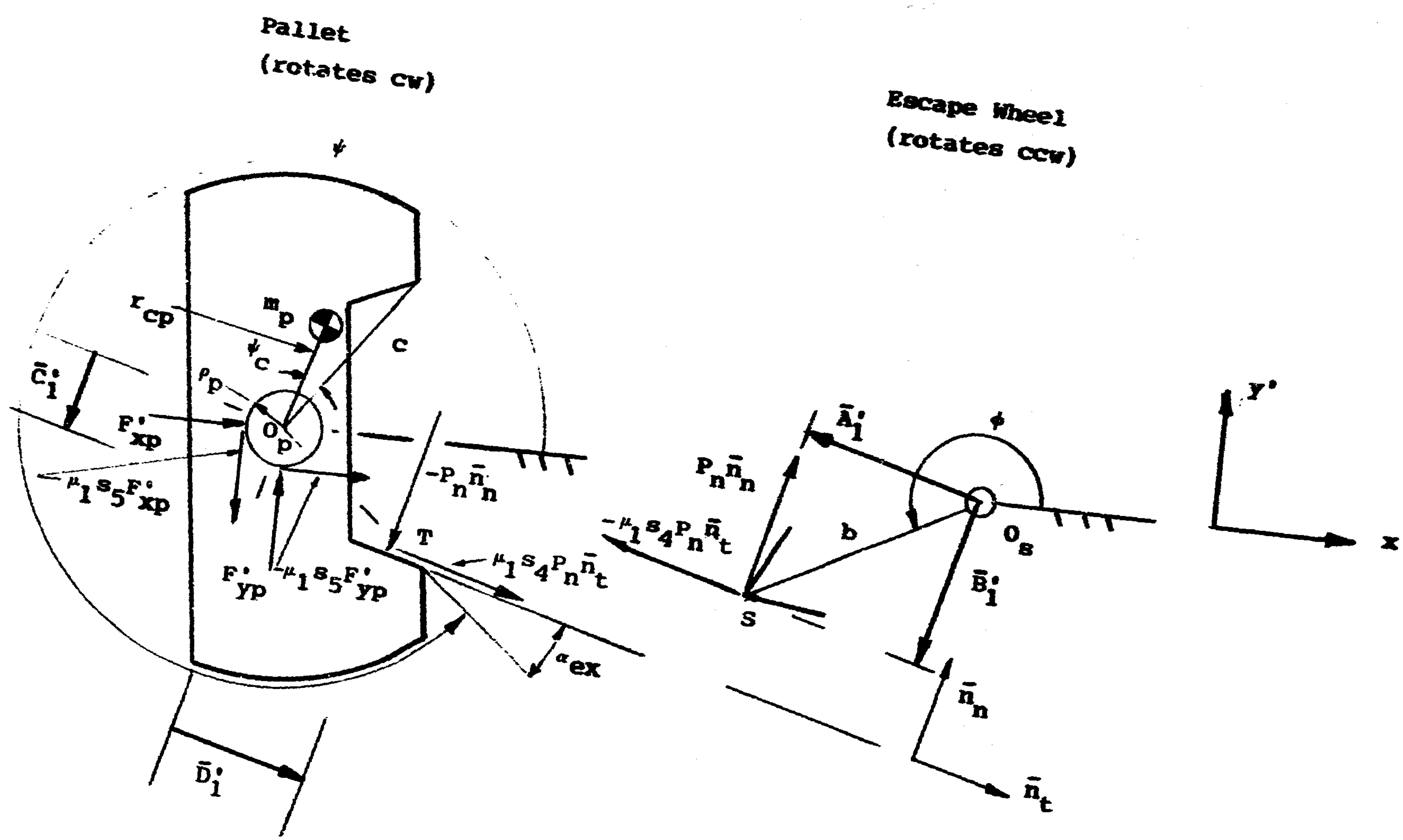

Figure B-7. Free body diagram of pallet in exit-coupled motion

Moment Equation of Pallet. The sign change of the contact force P_n reflects itself in the scalar moment equation B49 as follows:

$$P_n[-D_1' - C_1'\mu_1 s_4] - \rho_p \mu_1 s_5(\tilde{F}_{xp} + \tilde{F}_{yp})$$
$$= I_p\ddot{\psi} - m_p R_4 r_{cp}\omega^2 \sin(\gamma_p' - \psi - \psi_c) \quad \text{(B106)}$$

(Note change of sign of D_1'.)

Following the same path as for the entrance contact, it can be shown that, with the exception of the factor A_{18}, equation B76 for the contact force P_n remains unchanged. Then,

$$P_n = \frac{I_{PR}\ddot{\psi} + A_{21}\dot{\psi}^2 + 2\frac{\omega^2}{|\omega|}A_{20}\dot{\psi} + \omega^2 A_{19} - m_p r_{cp} R_4 \omega^2 \sin(\gamma_p' - \psi - \psi_c)}{AA_{18}} \quad \text{(B107)}$$

where

$$AA_{18} = -[D_1' + C_1'\mu_1 s_4 + \rho_p \mu_1 s_5(AA_1 + AA_5)] \quad \text{(B108)}$$

Finally, the expression equivalent to equation B77 becomes:

$$P_n' = \frac{1}{AA_{18}}[I_{PR}U\ddot{\phi} + (A_{21}U^2 + I_{PR}V)\dot{\phi}^2 + 2\frac{\omega^2}{|\omega|}A_{20}U\dot{\phi}$$
$$+ \omega^2 A_{19} - m_p r_{cp} R_4 \omega^2 \sin(\gamma_p' - \psi - \psi_c)] \quad \text{(B109)}$$

Force Equations of Escape Wheel. A free body diagram of the escape wheel in exit-coupled motion is shown in figure B-8. When the scalar force equations B86 and B87 for the escape wheel in entrance-coupled motion are adjusted to account for this condition, the following expressions are obtained:

$$P_n \sin(\psi + \alpha + \beta_3) + \mu_1 s_4 P_n \cos(\psi + \alpha + \beta_3) + T_3 \cos\gamma_3$$
$$+ F_{23}\sin(\beta_2 + \theta_2) + \mu s_2 F_{23}\cos(\beta_2 + \theta_2) + F_{x3} + \mu F_{y3} = 0 \quad \text{(B110)}$$

and

$$-P_n \cos(\psi + \alpha + \beta_3) + \mu_1 s_4 P_n \sin(\psi + \alpha + \beta_3) + T_3 \sin\gamma_3$$
$$-F_{23}\cos(\beta_2 + \theta_2) + \mu s_2 F_{23}\sin(\beta_2 + \theta_2) + \mu F_{x3} - F_{y3} = 0 \quad \text{(B111)}$$

CCW Rotation

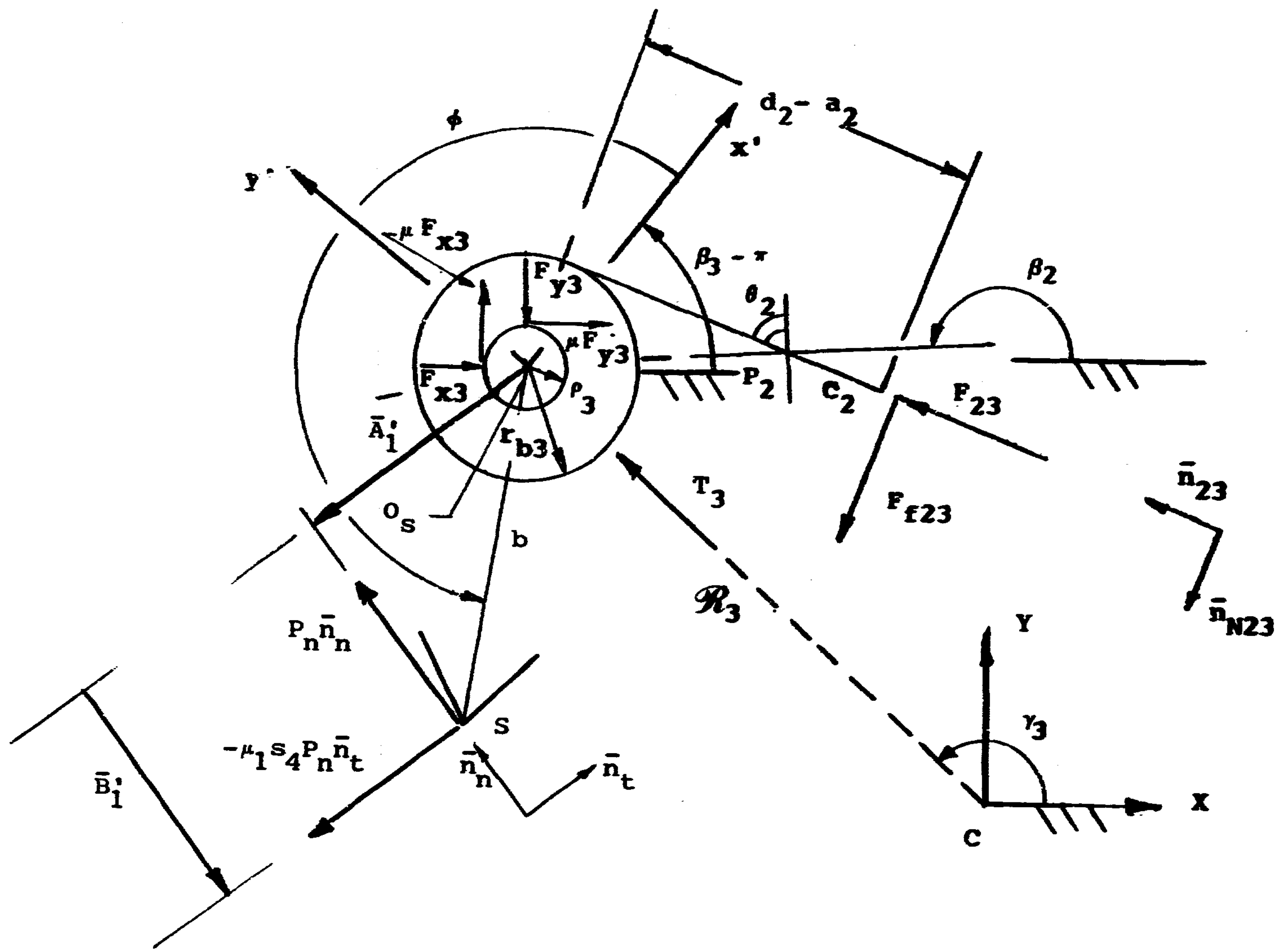

Figure B-8. Free body diagram of escape wheel and pinion 3 in exit-coupled motion

Simultaneous solution of the above for F_{x3} and F_{y3} shows that certain coefficients of these pivot forces will differ from those given in the solutions for entrance-coupled motion. Thus,

$$\tilde{F}_{y3} = P_n AA_9 + F_{23} A_{10} + T_3 A_{11} \tag{B112}$$

$$\tilde{F}_{x3} = P_n AA_{12} + F_{23} A_{13} + T_3 A_{14} \tag{B113}$$

where A_{10}, A_{11}, A_{13}, and A_{14} are given by equations B91, B92, B94, and B95, respectively. The new coefficients AA_9 and AA_{12} are defined as follows:

$$AA_9 = \left| \frac{(\mu_1 s_4 - \mu)\sin(\psi + \alpha + \beta_3) - (1 + s_4 \mu\mu_1)\cos(\psi + \alpha + \beta_3)}{1 + \mu^2} \right| \tag{B114}$$

$$AA_{12} = \left| \frac{-(1 + \mu\mu_1 s_4)\sin(\psi + \alpha + \beta_3) + (\mu - \mu_1 s_4)\cos(\psi + \alpha + \beta_3)}{1 + \mu^2} \right| \tag{B115}$$

Moment Equation of Escape Wheel. Modification of the escape wheel moment equation B84 for exit contact gives:

$$A_1' \bar{n}_t \times P_n \bar{n}_n + \bar{B}_1' \times (-\mu_1 s_4 P_n)\bar{n}_t - \mu\rho_3(\tilde{F}_{x3} + \tilde{F}_{y3})\bar{k} + r_{b3} F_{23}\bar{k} - \mu s_2(d_2 - a_2)F_{23}\bar{k} = I_3 \ddot{\phi}\bar{k} \tag{B116}$$

Substitution of equations B112 and B113 into the scalar form of equation B116 above furnishes the following expression for P_n:

$$P_n = \frac{-I_3\ddot{\phi} + F_{23}A_{15} - T_3 A_{16}}{AA_{17}} \tag{B117}$$

where A_{15} and A_{16} are given by equations B97 and B98, respectively. The factor AA_{17} is new; i.e.,

$$AA_{17} = -A_1' - \mu_1 s_4 B_1' + \mu\rho_3(AA_9 + AA_{12}) \tag{B118}$$

Combined Exit-Coupled Motion Differential Equation. Equations B109 and B117 are now equated to each other. This furnishes the combined exit-coupled motion differential equation:

$$[I_3 AA_{18} + I_{PR} AA_{17} U]\ddot{\phi} + [I_{PR} AA_{17} V + AA_{17} A_{21} U^2]\dot{\phi}^2$$

$$+ 2 \frac{\omega^2}{|\omega|} AA_{17} A_{20} U\dot{\phi}$$

$$= A_{15} AA_{18} F_{23} - A_{16} AA_{18} T_3 - AA_{17} \omega^2 [A_{19} - m_p r_{cp} R_4 \sin(\gamma_p' - \psi - \psi_c)] \quad \text{(B119)}$$

Dynamics of Rotor (Gear No. 1)
(Applicable to both configurations with appropriate choice of s_6. See also reference B-1.)

A free body diagram of the rotor of configuration no. 1, which moves in a counterclockwise direction, is shown in figure B-9. The acceleration of its center of mass is given by equation A122 of appendix A of reference B-1.

Since the motion of the rotor must be expressed in terms of the escape wheel variable ϕ, use must be made of the following relationships:

$$\dot{\phi}_1 = N_{31}\dot{\phi} \quad \text{(B120)}$$

and

$$\ddot{\phi}_1 = N_{31}\ddot{\phi} \quad \text{(B121)}$$

where

$$N_{31} = \frac{N_{P3} \times N_{P2}}{N_{G2} \times N_{G1}} \quad \text{(B122)}$$

The rotor angle $\phi_{1C} + \phi_1$ is expressed in terms of the total escape wheel rotation ϕ_T:

$$\phi_{1C} + \phi_1 = \phi_{1C} + N_{31}\ \phi_T \quad \text{(B123)}$$

(For details, see Program Description.)

Newton's force equation may now be written in the form of equation B63 of reference B-1. The tooth forces

$$\bar{F}_{21} = -F_{12}\ \bar{n}_{12} \quad \text{(B124)}$$

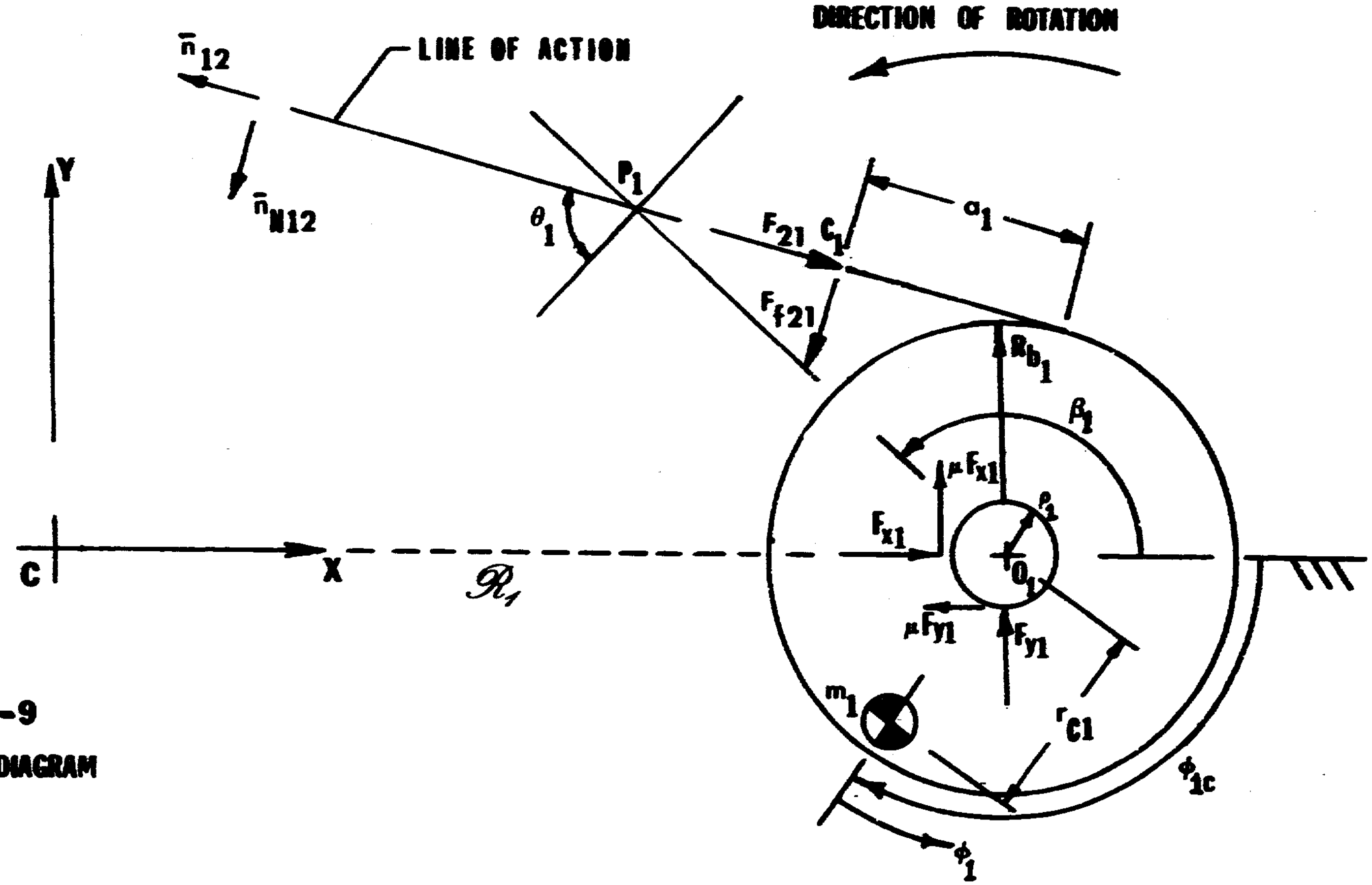

FIGURE E-9

FREE BODY DIAGRAM OF ROTOR

Figure B-9. Free body diagram of rotor

and

$$\bar{F}_{f21} = \mu s_1 F_{12} \bar{n}_{N12} \tag{B125}$$

have the appropriate directions to account for friction. (The signum function s_1 is defined in reference B-1.)

Thus:

$$\begin{aligned} &-F_{21}\bar{n}_{12} + \mu s_1 F_{12}\bar{n}_{N12} + F_{x1}\bar{i} - \mu F_{y1}\bar{i} + F_{y1}\bar{j} + \mu F_{x1}\bar{j} \\ &= m_1\{-\omega^2 R_1\bar{i} - (\omega + N_{31}\dot{\phi})^2 r_{c1}[\cos(\phi_{1c} + N_{31}\phi_T)\bar{i} + \sin(\phi_{1c} + N_{31}\phi_T)\bar{j}] \\ &+ N_{31}\ddot{\phi} r_{c1}[-\sin(\phi_{1c} + N_{31}\phi_T)\bar{i} + \cos(\phi_{1c} + N_{31}\phi_T)\bar{j}]\} \end{aligned} \tag{B126}$$

where

$$\bar{n}_{12} = -\sin(\beta_1 - \theta_1)\bar{i} + \cos(\beta_1 - \theta_1)\bar{j} \tag{B127}$$

and

$$\bar{n}_{N12} = -\cos(\beta_1 - \theta_1)\bar{i} - \sin(\beta_1 - \theta_1)\bar{j} \tag{B128}$$

The moment equation must be written with respect to the accelerated point O_1. The pivot reactions $\tilde{F}_{x1}$ and $\tilde{F}_{y1}$ are written such that the associated friction moments retard the counterclockwise rotation of the rotor. This leads to:

$$\begin{aligned} &-F_{12}R_{b1} + \mu s_1 a_1 F_{12} - \mu\rho_1(\tilde{F}_{x1} + \tilde{F}_{y1}) \\ &= m_1\omega^2 R_1 r_{c1}\sin(\phi_{1c} + N_{31}\phi_T) + I_1 N_{31}\ddot{\phi} \end{aligned} \tag{B129}$$

The forces $\tilde{F}_{x1}$ and $\tilde{F}_{y1}$ are obtained after the simultaneous solution of the following component expressions of equation B63 of reference B-1 for F_{x1} and F_{y1}:

$$\begin{aligned} &F_{12}\sin(\beta_1 - \theta_1) - \mu s_1 F_{12}\cos(\beta_1 - \theta_1) + F_{x1} - \mu F_{y1} \\ &= m_1[-\omega^2 R_1 - (\omega + N_{31}\dot{\phi})^2 r_{c1}\cos(\phi_{1c} + N_{31}\phi_T) - N_{31}\ddot{\phi} r_{c1}\sin(\phi_{1c} + N_{31}\phi_T)] \end{aligned} \tag{B130}$$

and

$$\begin{aligned} &-F_{12}\cos(\beta_1 - \theta_1) - \mu s_1 F_{12}\sin(\beta_1 - \theta_1) + F_{y1} + \mu F_{x1} \\ &= m_1[-(\omega + N_{31}\dot{\phi})^2 r_{c1}\sin(\phi_{1c} + N_{31}\phi_T) + N_{31}\ddot{\phi} r_{c1}\cos(\phi_{1c} + N_{31}\phi_T)] \end{aligned} \tag{B131}$$

Simultaneous solution of the above furnishes:

$$\tilde{F}_{x1} = F_{12}A_{23} \pm \omega^2 A_{24} \pm 2\frac{\omega^2}{|\omega|} N_{31}\dot{\phi}A_{25} \pm (N_{31}\dot{\phi})^2 A_{25} \pm N_{31}\ddot{\phi}A_{26} \quad \text{(B132)}$$

$$\tilde{F}_{y1} = F_{12}A_{27} \pm \omega^2 A_{28} \pm 2\frac{\omega^2}{|\omega|} N_{31}\dot{\phi}A_{29} \pm (N_{31}\dot{\phi})^2 A_{29} \pm N_{31}\ddot{\phi}A_{30} \quad \text{(B133)}$$

where

$$A_{23} = \left| \frac{(1 - \mu^2 s_1)\sin(\beta_1 - \theta_1) - \mu(1 + s_1)\cos(\beta_1 - \theta_1)}{1 + \mu^2} \right| \quad \text{(B134)}$$

$$A_{24} = \left| \frac{m_1[\mathcal{R}_1 + r_{c1}(\cos(\phi_{1c} + N_{31}\phi_T) + \mu\sin(\phi_{1c} + N_{31}\phi_T))]}{1 + \mu^2} \right| \quad \text{(B135)}$$

$$A_{25} = \left| \frac{m_1 r_{c1}[\cos(\phi_{1c} + N_{31}\phi_T) + \mu\sin(\phi_{1c} + N_{31}\phi_T)]}{1 + \mu^2} \right| \quad \text{(B136)}$$

$$A_{26} = \left| \frac{m_1 r_{c1}[\sin(\phi_{1c} + N_{31}\phi_T) - \mu\cos(\phi_{1c} + N_{31}\phi_T)]}{1 + \mu^2} \right| \quad \text{(B137)}$$

$$A_{27} = \left| \frac{\mu(1 + s_1)\sin(\beta_1 - \theta_1) + (1 - \mu^2 s_1)\cos(\beta_1 - \theta_1)}{1 + \mu^2} \right| \quad \text{(B138)}$$

$$A_{28} = \left| \frac{\mu m_1 \mathcal{R}_1 + m_1 r_{c1}[\mu\cos(\phi_{1c} + N_{31}\phi_T) - \sin(\phi_{1c} + N_{31}\phi_T)]}{1 + \mu^2} \right| \quad \text{(B139)}$$

$$A_{29} = \left| \frac{m_1 r_{c1}[\mu\cos(\phi_{1c} + N_{31}\phi_T) - \sin(\phi_{1c} + N_{31}\phi_T)]}{1 + \mu^2} \right| \quad \text{(B140)}$$

$$A_{30} = \left| \frac{m_1 r_{c1}[\mu\sin(\phi_{1c} + N_{31}\phi_T) + \cos(\phi_{1c} + N_{31}\phi_T)]}{1 + \mu^2} \right| \quad \text{(B141)}$$

Equations B132 and B133 are now substituted into the moment equation B129.

$$-F_{12}R_{b1} + \mu s_1 a_1 F_{12} - \mu\rho_1[F_{12}(A_{23} + A_{27}) \pm \omega^2(A_{24} + A_{28})$$

$$\pm 2\frac{\omega^2}{|\omega|} N_{31}\dot{\phi}(A_{25} + A_{29}) \pm (N_{31}\dot{\phi})^2(A_{25} + A_{29}) \pm N_{31}\ddot{\phi}(A_{26} + A_{30})]$$

$$= m_1\omega^2 R_1 r_{c1}\sin(\phi_{1c} + N_{31}\phi_T) + I_1 N_{31}\ddot{\phi} \qquad \text{(B142)}$$

In order to have the pivot friction moment negative, the following terms on the left hand side of equation B142 must be negative:

$$\mu\rho_1 F_{12} (A_{23} + A_{27})$$

$$\mu\rho_1 \omega^2 (A_{24} + A_{28})$$

$$\mu\rho_1 (N_{31}\dot{\phi})^2 (A_{25} + A_{29})$$

$$2|\mu|\rho_1 \frac{\omega^2}{|\omega|} N_{31}\dot{\phi} (A_{25} + A_{29})$$ [B-4]

The term $\mu\rho_1 N_{31}\ddot{\phi}(A_{26} + A_{30})$ is treated in the same manner as was shown in connection with equation A147 of reference B-1. (The discussion in appendix A of reference B-1 is also of interest for the determination of the effective moment of inertia I_{1R}.)

The above considerations give the moment equation the following form:

$$-F_{12}[R_{b1} - \mu s_1 a_1 + \mu\rho_1(A_{23} + A_{27})] - \omega^2\mu\rho_1(A_{24} + A_{28})$$

$$- \frac{2\mu\rho_1\omega^2 N_{31}\dot{\phi}}{|\omega|} (A_{25} + A_{29}) - \mu\rho_1(N_{31}\dot{\phi})^2(A_{25} + A_{29})$$

$$= m_1\omega^2 R_1 r_{c1}\sin(\phi_{1c} + N_{31}\phi_T) + [I_1 \pm \mu\rho_1(A_{26} + A_{30})]N_{31}\ddot{\phi} \qquad \text{(B143)}$$

[B-4] With $N_{31}\dot{\phi}$ = positive, must be absolute. See also discussion following equation A147 of reference B-1.

Finally, the above expression is solved for F_{12}, using the effective moment of inertia I_{1R}:

$$F_{12} = \frac{-A_{32} - A_{33}N_{31}\dot{\phi} - A_{34}(N_{31}\dot{\phi})^2 - A_{35}\sin(\phi_{1c} + N_{31}\phi_T) - I_{1R}N_{31}\ddot{\phi}}{A_{31}} \quad \text{(B144)}$$

where

$$A_{31} = R_{b1} - \mu s_1 a_1 + \mu\rho_1(A_{23} + A_{27}) \quad \text{(B145)}$$

$$A_{32} = \mu\rho_1\omega^2(A_{24} + A_{28}) \quad \text{(B146)}$$

$$A_{33} = 2|\mu|\rho_1 \frac{\omega^2}{|\omega|} (A_{25} + A_{29}) \quad \text{(B147)}$$

$$A_{34} = \mu\rho_1 (A_{25} + A_{29}) \quad \text{(B148)}$$

$$A_{35} = m_1\mathcal{R}_1 r_{c1}\omega^2 \quad \text{(B149)}$$

$I_{1R} = I_1 + |\mu|\rho_1(A_{26} + A_{30})$, when $\dot{\phi}$ and $\ddot{\phi}$ have the same signs. (B150)

$I_{1R} = I_1 - |\mu|\rho_1 (A_{26} + A_{30})$, when $\dot{\phi}$ and $\ddot{\phi}$ have opposite signs. (B151)

Dynamics of Gear and Pinion Set No. 2 (applicable to both configurations with appropriate choice of s_6)

The free body diagram of gear and pinion set no. 2 is shown in figure B-10. Its force equation includes the D'Alembert force

$$T_2 = m_2\mathcal{R}_2\omega^2 \quad \text{(B152)}$$

Thus,

$$-F_{23}\bar{n}_{23} - s_2\mu F_{23}\bar{n}_{N23} + F_{12}\bar{n}_{12} - \mu s_1 F_{12}\bar{n}_{N12} + F_{x2}\bar{i} + \mu F_{y2}\bar{i} \quad \text{(B153)}$$

$$+ F_{y2}\bar{j} - \mu F_{x2}\bar{j} + T_2(\cos\gamma_2\bar{i} + \sin\gamma_2\bar{j}) = 0$$

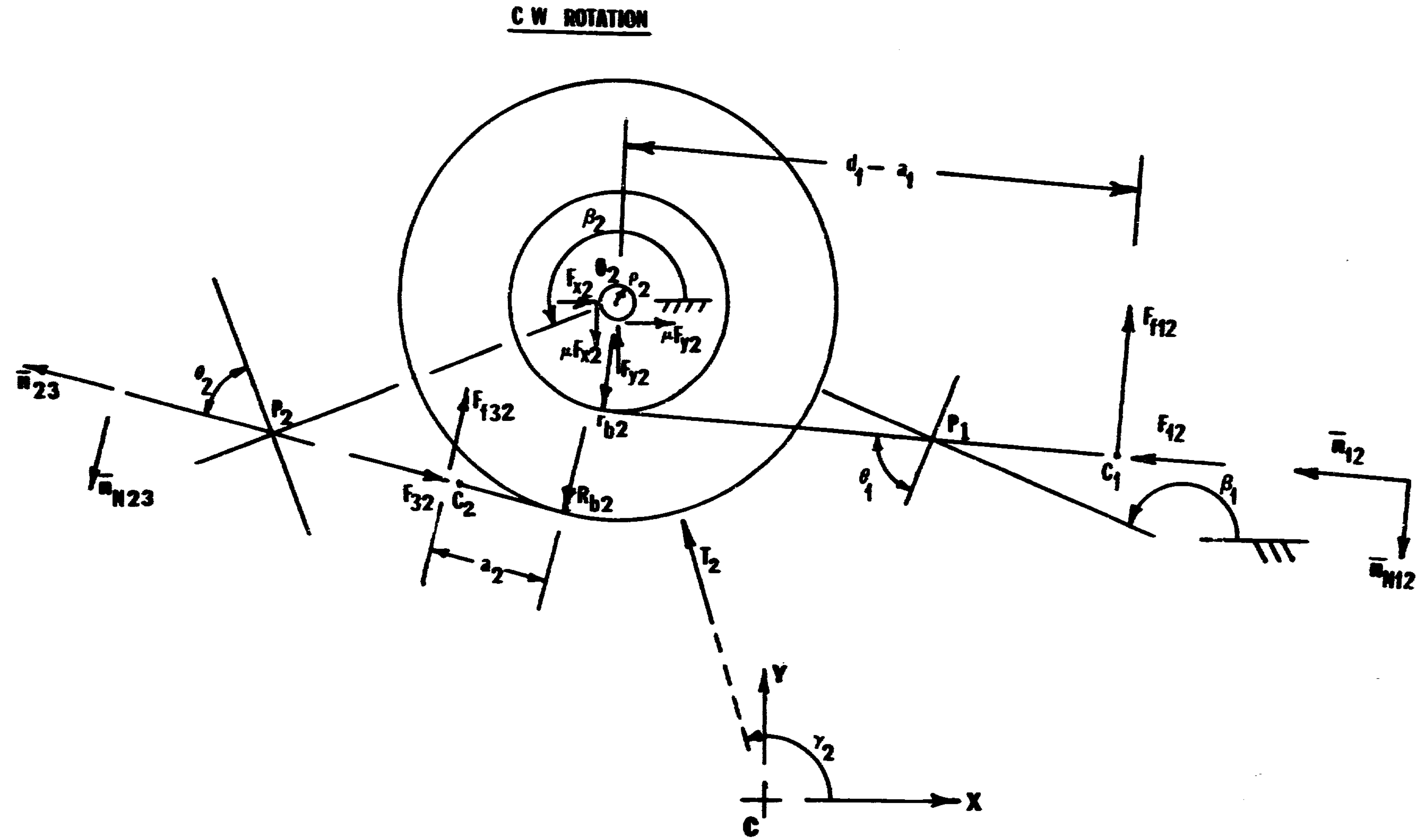

Figure B-10. Free body diagram of gear and pinion set no. 2

where

$$\bar{n}_{23} = \sin(\beta_2 + \theta_2)\bar{i} - \cos(\beta_2 + \theta_2)\bar{j} \qquad \text{(B154)}$$

and

$$\bar{n}_{N23} = \cos(\beta_2 + \theta_2)\bar{i} + \sin(\beta_2 + \theta_2)\bar{j} \qquad \text{(B155)}$$

The unit vectors $\bar{n}_{12}$ and $\bar{n}_{N12}$ were given by equations B127 and B128. Hence

$$\bar{F}_{32} = -F_{23}\,\bar{n}_{23} \qquad \text{(B156)}$$

$$\bar{F}_{f32} = -s_2\mu F_{23}\,\bar{n}_{N23} \qquad \text{(B157)}$$

To write the moment equation, let

$$\ddot{\phi}_2 = N_{32}\ddot{\phi} \qquad \text{(B158)}$$

where

$$N_{32} = -\frac{N_{P3}}{N_{G2}} \qquad \text{(B159)}$$

Then,

$$F_{23}R_{b2} - \mu s_2 F_{23} a_2 - F_{12} r_{b2} + \mu s_1 F_{12}(d_1 - a_1) + \mu\rho_2(\tilde{F}_{x2} + \tilde{F}_{y2}) = I_2 N_{32}\ddot{\phi} \qquad \text{(B160)}$$

The conservative bearing forces $\tilde{F}_{x2}$ and $\tilde{F}_{y2}$ are obtained from the simultaneous solution of the following component equations of the force expression B153 for F_{x2} and F_{y2}:

$$-F_{23}\sin(\beta_2 + \theta_2) - F_{23}\mu s_2\cos(\beta_2 + \theta_2) - F_{12}\sin(\beta_1 - \theta_1) + \mu s_1 F_{12}\cos(\beta_1 - \theta_1) + F_{x2} + \mu F_{y2} + T_2\cos\gamma_2 = 0 \qquad \text{(B161)}$$

and

$$F_{23}\cos(\beta_2 + \theta_2) - \mu s_2 F_{23}\sin(\beta_2 + \theta_2) + F_{12}\cos(\beta_1 - \theta_1) + \mu s_1 F_{12}\sin(\beta_1 - \theta_1) + F_{y2} - \mu F_{x2} + T_2\sin\gamma_2 = 0 \qquad \text{(B162)}$$

This leads to:

$$\tilde{F}_{x2} = F_{23}A_{36} + F_{12}A_{37} + T_2 A_{38} \qquad \text{(B163)}$$

$$\tilde{F}_{y2} = F_{23}A_{39} + F_{12}A_{40} + T_2A_{41} \qquad \text{(B164)}$$

(All signs are left positive in order to furnish positive friction moments in equation B160.) In the above:

$$A_{36} = \left| \frac{(1 - \mu^2 s_2)\sin(\beta_2 + \theta_2) + \mu(1 + s_2)\cos(\beta_2 + \theta_2)}{1 + \mu^2} \right| \qquad \text{(B165)}$$

$$A_{37} = \left| \frac{(1 + \mu^2 s_1)\sin(\beta_1 - \theta_1) + \mu(1 - s_1)\cos(\beta_1 - \theta_1)}{1 + \mu^2} \right| \qquad \text{(B166)}$$

$$A_{38} = \left| \frac{\cos\gamma_2 - \mu\sin\gamma_2}{1 + \mu^2} \right| \qquad \text{(B167)}$$

$$A_{39} = \left| \frac{\mu(1 + s_2)\sin(\beta_2 + \theta_2) - (1 - \mu^2 s_2)\cos(\beta_2 + \theta_2)}{1 + \mu^2} \right| \qquad \text{(B168)}$$

$$A_{40} = \left| \frac{\mu(1 - s_1)\sin(\beta_1 - \theta_1) - (1 + \mu^2 s_1)\cos(\beta_1 - \theta_1)}{1 + \mu^2} \right| \qquad \text{(B169)}$$

$$A_{41} = \left| \frac{\mu\cos\gamma_2 + \sin\gamma_2}{1 + \mu^2} \right| \qquad \text{(B170)}$$

Substitution of equations B163 and B164 into the moment equation B160 and subsequent solution for the contact force F_{23} give:

$$F_{23} = \frac{F_{12}A_{42} - T_2A_{43} + I_2N_{32}\ddot{\phi}}{A_{44}} \qquad \text{(B171)}$$

where

$$A_{42} = r_{b2} - \mu[s_1(d_1 - a_1) + \rho_2(A_{37} + A_{40})] \qquad \text{(B172)}$$

$$A_{43} = \mu\rho_2\,(A_{38} + A_{41}) \qquad \text{(B173)}$$

$$A_{44} = R_{b2} - \mu[s_2a_2 - \rho_2(A_{36} + A_{39})] \qquad \text{(B174)}$$

Dynamics of Combined System in Coupled Motion (applicable to both configurations)

To obtain a single differential equation for the total system in coupled motion in terms of the escape wheel angle ϕ, an appropriate expression for the contact force F_{23}, which also contains the contribution of the rotor, must be substituted into either equation B100 or equation B119 for entrance or exit-coupled motion, respectively.

Thus, first substitute equation B144 for F_{12} into equation B171, which is the above expression for F_{23}:

$$F_{23} = \frac{1}{A_{44}} \Big[\frac{-A_{42}A_{32}}{A_{31}} - T_2A_{43} - \frac{A_{42}A_{33}}{A_{31}} N_{31}\dot{\phi} - \frac{A_{42}A_{34}}{A_{31}} N_{31}^2 \dot{\phi}^2 - \frac{A_{42}A_{35}}{A_{31}} \sin(\phi_{1c} + N_{31}\phi_T) + (I_2N_{32} - \frac{A_{42}}{A_{31}} I_{1R}N_{31})\ddot{\phi} \Big] \qquad \text{(B175)}$$

Now, equation B175 is substituted into equation B100 or equation B119 and the final differential equation of coupled motion results:

$$A_{45}\ddot{\phi} + A_{46}\dot{\phi}^2 + A_{47}\dot{\phi} = A_{48} - A_{49}\sin(\phi_{1c} + N_{31}\phi_T) + A_{50}\sin(\gamma_p' - \psi - \psi_c) \qquad \text{(B176)}$$

where[B-5]

$$A_{45} = I_3A_{18} + I_{PR}A_{17}U - \frac{A_{15}A_{18}}{A_{44}} (I_2N_{32} - \frac{A_{42}}{A_{31}} I_{1R}N_{31}) \qquad \text{(B177)}$$

$$A_{46} = I_{PR}A_{17}V + A_{17}A_{21}U^2 + \frac{A_{15}A_{18}A_{34}A_{42}}{A_{31}A_{44}} N_{31}^2 \qquad \text{(B178)}$$

$$A_{47} = 2 \frac{\omega^2}{|\omega|} A_{17}A_{20}U + \frac{A_{15}A_{18}A_{33}A_{42}}{A_{31}A_{44}} N_{31} \qquad \text{(B179)}$$

[B-5] The coefficients A_{45} to A_{50} are only valid for entrance contact; i.e., for equation B100. If exit-coupled motion is in operation, equation B119 is applicable and A_{17} becomes AA_{17}, and A_{18} becomes AA_{18}.

$$A_{48} = \frac{-A_{15}A_{18}A_{32}A_{42}}{A_{31}A_{44}} - \frac{A_{15}A_{18}A_{43}}{A_{44}} T_2 - A_{16}A_{18}T_3 - \omega^2 A_{17}A_{19} \tag{B180}$$

$$A_{49} = \frac{A_{15}A_{18}A_{35}A_{42}}{A_{31}A_{44}} \tag{B181}$$

$$A_{50} = m_p r_{cp} \mathcal{R}_4 A_{17} \omega^2 \tag{B182}$$

Contact Force Equations for Coupled Motion (applicable to both configurations)

The contact force F_{23} is given by equation B175:

$$F_{23} = \frac{1}{A_{44}} \Big[\frac{-A_{32}A_{42}}{A_{31}} - T_2 A_{43} - \frac{A_{33}A_{42}}{A_{31}} N_{31}\dot{\phi} - \frac{A_{42}A_{34}}{A_{31}} N_{31}^2 \dot{\phi}^2 - \frac{A_{35}A_{42}}{A_{31}} \sin(\phi_{1c} + N_{31}\phi_T) + (I_2 N_{32} - \frac{A_{42}}{A_{31}} I_{1R} N_{31})\ddot{\phi} \Big] \tag{B183}$$

The contact force F_{12} is found with the help of equation B171:

$$F_{12} = \frac{F_{23}A_{44} + T_2 A_{43} - I_2 N_{32}\ddot{\phi}}{A_{42}} \tag{B184}$$

The contact force P_n is given either by equation B96 or equation B117:

$$P_n = \frac{-I_3\ddot{\phi} + F_{23}A_{15} - T_3 A_{16}}{A_{17}} \tag{B185}$$

Note that for exit-coupled motion; i.e., equation B117, the term A_{17} becomes AA_{17}. If this force is desired in terms of the pallet variable ψ, equation B76 or equation B107 must be used:

$$P_n = \frac{I_{PR}\ddot{\psi} + A_{21}\dot{\psi}^2 + 2\frac{\omega^2}{|\omega|} A_{20}\dot{\psi} + \omega^2 A_{19} - m_p r_{cp} \mathcal{R}_4 \omega^2 \sin(\gamma_p' - \psi - \psi_c)}{A_{18}} \tag{B186}$$

Again, note that for exit motion A_{18} becomes AA_{18}.

Differential Equations for Free Motion (applicable to both configurations)

Pallet Free Motion

By letting $P_n = 0$ in equation B186, the single free motion differential equation for the pallet, which is now independent of exit or entrance contact, is obtained.

$$A_{51}\ddot{\psi} + A_{21}\dot{\psi}^2 + A_{52}\dot{\psi} = -A_{53} + A_{54}\sin(\gamma_p' - \psi - \psi_c) \tag{B187}$$

where

$$A_{51} = I_{PR} \tag{B188}$$

$$A_{52} = 2\frac{\omega^2}{|\omega|}A_{20} \tag{B189}$$

$$A_{53} = \omega^2 A_{19} \tag{B190}$$

$$A_{54} = \omega^2 m_p r_{cp} R_4 \tag{B191}$$

Escape Wheel - Gear Train - Rotor Free Motion

By setting $P_n = 0$ in equation B96 or equation B117, the common differential equation of the system without the pallet is obtained (again independent of entrance or exit contact). This leads to:

$$I_3\ddot{\phi} = A_{15}F_{23} - T_3A_{16} \tag{B192}$$

Equation B175 is then substituted into the above for F_{23}. The resulting differential equation is given by:

$$A_{55}\ddot{\phi} + A_{56}\dot{\phi}^2 + A_{57}\dot{\phi} = A_{58} - A_{59}\sin(\phi_{1c} + N_{31}\phi_T) \tag{B193}$$

where

$$A_{55} = I_3 - \frac{A_{15}}{A_{44}}N_{32}I_2 + \frac{A_{15}A_{42}}{A_{31}A_{44}}N_{31}I_{1R} \tag{B194}$$

$$A_{56} = \frac{A_{15}A_{34}A_{42}}{A_{31}A_{44}} N_{31}^2 \quad \text{(B195)}$$

$$A_{57} = \frac{A_{15}A_{33}A_{42}}{A_{31}A_{44}} N_{31} \quad \text{(B196)}$$

$$A_{58} = -A_{16}T_3 - \frac{A_{15}A_{43}}{A_{44}} T_2 - \frac{A_{15}A_{32}A_{42}}{A_{31}A_{44}} \quad \text{(B197)}$$

$$A_{59} = \frac{A_{15}A_{35}A_{42}}{A_{31}A_{44}} \quad \text{(B198)}$$

Contact Force Equations for Free Motion (the subscript F stands for free motion) (applicable to both configurations)

Equation B192 may be solved for the free motion contact force F_{F23}, once $\ddot{\phi}$ is known:

$$F_{F23} = \frac{I_3\ddot{\phi} + T_3A_{16}}{A_{15}} \quad \text{(B199)}$$

Equation B171, which was derived from the force and moment equations of gear and pinion set no. 2, may be modified to obtain the free motion contact force F_{F12}:

$$F_{F12} = \frac{F_{F23}A_{44} + T_2A_{43} - I_2N_{32}\ddot{\phi}}{A_{42}} \quad \text{(B200)}$$

It must be understood that in both equations B199 and B200 the angular acceleration of free motion $\ddot{\phi}$ as obtained from the solution of equation B193 is used.

Impact

While the following will show that the general form of the impact equation is the same for entrance and exit contact, it must be kept in mind that the expressions for the moment arms A_1', B_1', C_1' and D_1' depend on whether the angle α_{EN} or the angle α_{EX} is involved. (See equations D4, D5, D8, and D9 in appendix D.)

Entrance Impact

The general condition of entrance impact is illustrated in figure B-11. As in reference B-2, it is assumed that the only acting impulse on the pallet and the escape wheel is represented by the mutual normal impulse P_n between these mechanism components. Experience has shown that the frictional impulse μP_n may be disregarded. Under these circumstances, the angular impulse on the pallet becomes:

$$\bar{J}_p = D_1'\bar{n}_t \times P_n\bar{n}_n = P_n D_1'\bar{k} \tag{B201}$$

The angular impulse on the escape wheel is given by:

$$\bar{J}_3 = A_1'\bar{n}_t \times (-P_n)\bar{n}_u = -P_n A_1'\bar{k} \tag{B202}$$

If the above equations are made part of the usual angular momentum relationships, the components can be expressed as

$$I_p(\dot{\psi}_f - \dot{\psi}_i) = P_n D_1' \tag{B203}$$

and

$$I_3(\dot{\phi}_f - \dot{\phi}_i) = -P_n A_1' \tag{B204}$$

where I_p and I_3 represent the polar mass moments of inertia of the pallet and the escape wheel, respectively. The subscript i indicates the angular velocities before impact while f indicates the same quantities after impact. When P_n is simultaneously eliminated from equations B203 and B204, the following expression results:

$$I_p A_1'\dot{\psi}_f + I_3 D_1'\dot{\phi}_f = I_p A_1'\dot{\psi}_i + I_3 D_1'\dot{\phi}_i \tag{B205}$$

Exit Impact

The conditions of exit impact are shown in figure B-12. With the same assumptions as in part a above, the expression for the angular impulse on the pallet becomes:

$$\bar{J}_p = D_1'\bar{n}_t \times (-P_n)\bar{n}_n = -P_n D_1'\bar{k} \tag{B206}$$

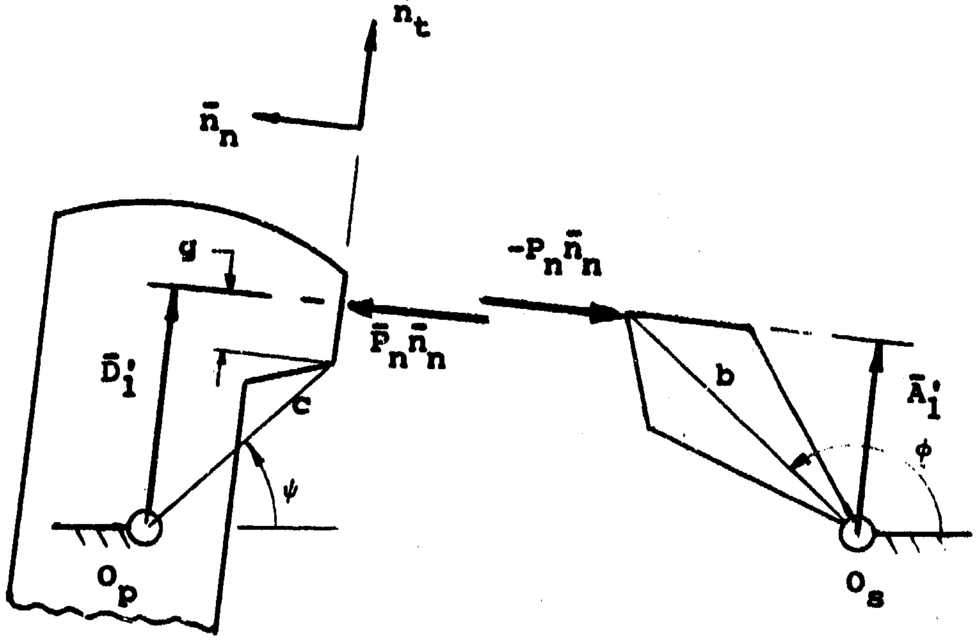

Figure B-11. Entrance impact

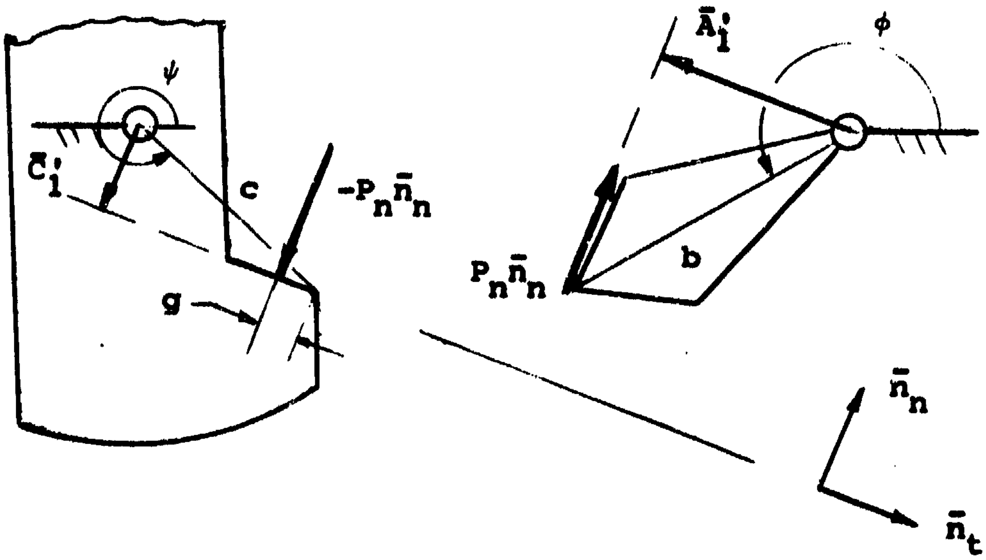

Figure B-12. Exit impact

The angular impulse on the escape wheel now becomes:

$$\bar{J}_3 = A_1'\bar{n}_t \times (P_n)\bar{n}_n = P_n A_1'\bar{k} \tag{B207}$$

Again, these expressions are used in the angular momentum formulations of both components, as follows:

$$I_p(\dot{\psi}_f - \dot{\psi}_i) = -P_n D_1' \tag{B208}$$

and

$$I_3(\dot{\phi}_f - \dot{\phi}_i) = P_n A_1' \tag{B209}$$

When P_n is now simultaneously eliminated from equations B208 and B209, equation B205 results again. Thus, identical formulations may be used for entrance and exit contact.

Impact Equations for Escapement

In order to obtain expressions for the post impact angular velocities $\dot{\phi}_f$ and $\dot{\psi}_f$, the restitution equation

$$e_r = \frac{V_{TNf} - V_{SNf}}{V_{TNi} - V_{SNi}} \tag{B210}$$

must be solved simultaneously with equation B205. V_{TNi} and V_{SNi} are defined by equations D13 and D15 in appendix D. The velocities V_{TNf} and V_{SNf} represent the normal contact point velocities after impact. After appropriate substitutions, the simultaneous solution of these expressions furnishes:

$$\dot{\phi}_f = \frac{\dot{\phi}_i(I_3 D_1'^2 - e_r I_p A_1'^2) + \dot{\psi}_i I_p A_1'(1 + e_r)}{I_p A_1'^2 + I_3 D_1'^2} \tag{B211}$$

and

$$\dot{\psi}_f = \frac{\dot{\phi}_f A_1' - e_r(\dot{\psi}_i D_1' - \dot{\phi}_i A_1')}{D_1'} \tag{B212}$$

Changes in Impact Equation due to Presence of the Rotor and Gear and Pinion No. 2

The presence of the rotor and gear and pinion no. 2 is accounted for by referring their moments of inertia to the escape wheel shaft. Thus, the total escape wheel moment of inertia is given by:

$$I_{STOT} = I_3 + I_2 N_{32}^2 + I_1 N_{31}^2 \quad \text{(B213)}$$

Equations B122 and B159 contain the above transmission ratios.

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APPENDIX C

KINEMATICS OF COUPLED MOTION

Nomenclature

The following derivations for the kinematics of coupled motion are applicable to both entrance and exit motion. It must be remembered that for entrance motion, according to figure C-1:

$$\alpha = \alpha_{en} \tag{C1}$$

and

$$g = g_{en} \tag{C2}$$

while for exit motion:

$$\alpha = \alpha_{ex} \tag{C3}$$

and

$$g = g_{ex} \tag{C4}$$

Further, from figure C-1

ϕ = Escape wheel angle, measured with respect to positive x' - axis.

ψ = Verge angle, measured with respect to positive x' - axis. For entrance condition, defined by line O_pV. For exit condition, defined by line O_pW (measured ccw with respect to the positive x' - axis).

a = Distance between pivot points O_p and O_s

b = Escape wheel radius

c = For entrance condition, length O_pV. For exit condition, length O_pW (usually identical).

Unit Vectors

Tangential ard normal unit vectors, which are associated both with the entrance and the exit working surfaces, are shown in figure C-1. When expressed without distinction between these surfaces, they become:

$$\bar{n}_t = \cos(\psi + \alpha)\bar{i}' + \sin(\psi + \alpha)\bar{j}' \tag{C5}$$

and

$$\bar{n}_n = -\sin(\psi + \alpha)\bar{i}' + \cos(\psi + \alpha)\bar{j}' \tag{C6}$$

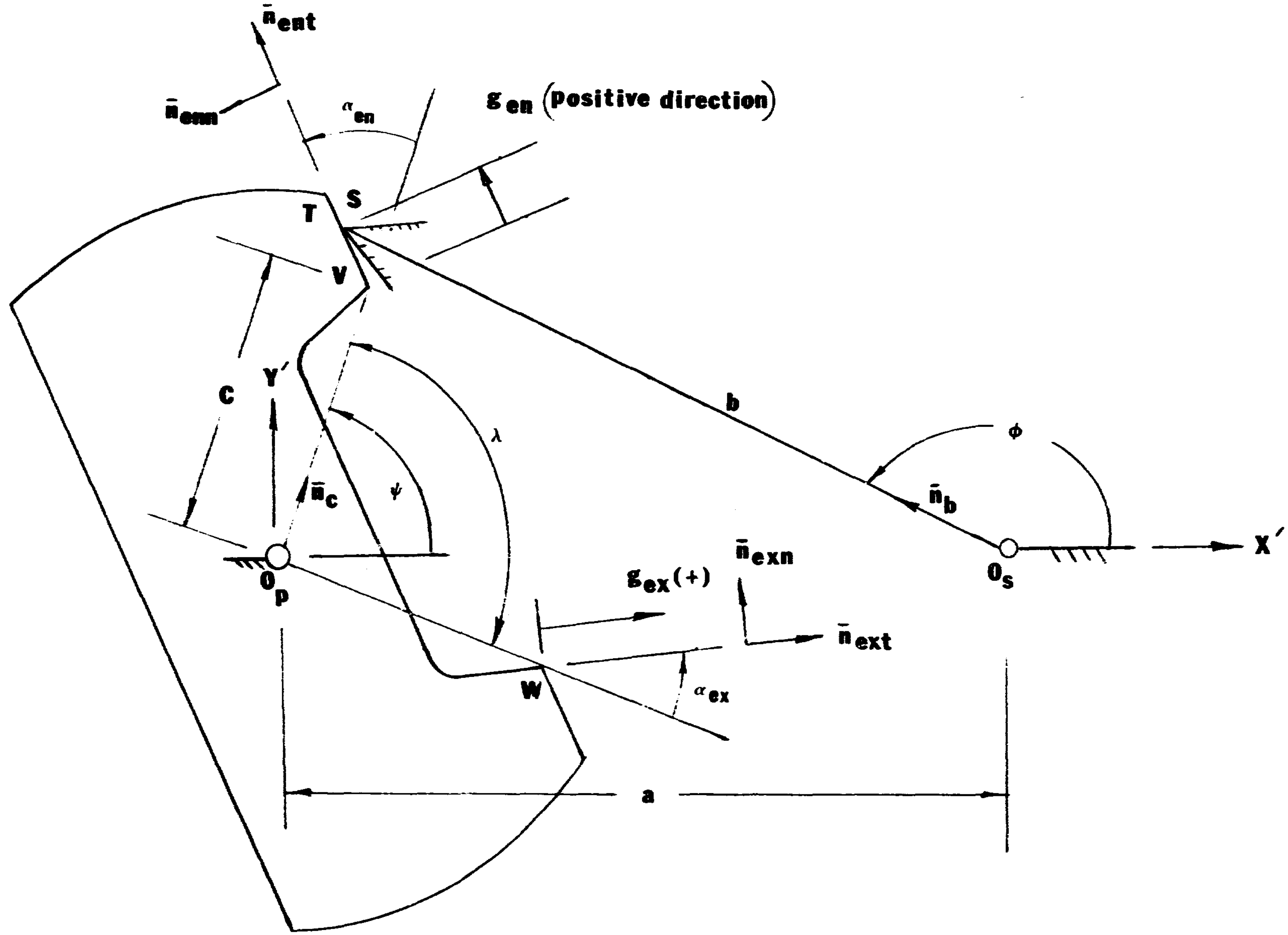

Figure C-1. Coupled motion nomenclature

$$\bar{n}_n = -\sin(\psi + \alpha)\bar{i}' + \cos(\psi + \alpha)\bar{j}' \qquad (C6)$$

Further, the unit vectors in directions O_sS and O_pV (or O_pW), respectively, are given by

$$\bar{n}_b = \cos\phi\ \bar{i}' + \sin\phi\ \bar{j}' \qquad (C7)$$

and

$$\bar{n}_c = \cos\psi\ \bar{i}' + \sin\psi\ \bar{j}' \qquad (C8)$$

Loop Equation and Input-Output Relationship

The loop equation for coupled motion is given below. Contact point S is at the tip of the escape wheel tooth. The coincident point on the pallet is point T (fig C-1).

$$\overline{O_pV} + \overline{VT} + \overline{SO_s} + \overline{O_sO_p} = 0 \qquad (C9)$$

Substitution of the appropriate dimensions and unit vectors into the above furnishes:

$$c\bar{n}_c + g\bar{n}_t - b\bar{n}_b - a\bar{i}' = 0 \qquad (C10)$$

This becomes with the help of equations C5 to C8:

$$c(\cos\psi\bar{i}' + \sin\psi\bar{j}') + g(\cos(\psi + \alpha)\bar{i}' + \sin(\psi + \alpha)\bar{j}') \qquad (C11)$$
$$-b(\cos\phi\bar{i}' + \sin\phi\bar{j}') - a\bar{i}' = 0$$

Appropriate ordering of equation C11 leads to component expressions which may be solved for g and ψ:

$$b\cos\phi + a - c\cos\psi - g\cos(\psi + \alpha) = 0 \qquad (C12)$$

and

$$b\sin\phi - c\sin\psi - g\sin(\psi + \alpha) = 0 \qquad (C13)$$

Both of these component equations may be solved for g. Thus,

$$g = \frac{b\cos\phi + a - c\cos\psi}{\cos(\psi + \alpha)} \tag{C14}$$

and

$$g = \frac{b\sin\phi - c\sin\psi}{\sin(\psi + \alpha)} \tag{C15}$$

g may be eliminated by setting equation C14 and C15 equal to each other. This leads to

$$(b\cos\phi + a - c\cos\psi)\sin(\psi + \alpha) = (b\sin\phi - c\sin\psi)\cos(\psi + \alpha) \tag{C16}$$

After expansion and rearrangement, a four-bar linkage type expression is obtained which may be solved for the angle ψ according to the method shown by R. Hartenberg and J. Denavit.*

$$A\sin\psi + B\cos\psi = C \tag{C17}$$

where

$$A = a\cos\alpha + b\cos(\phi - \alpha)$$

$$B = a\sin\alpha - b\sin(\phi - \alpha)$$

$$C = c\sin\alpha$$

The solution to equation C17 is of the form:

$$\psi = 2\tan^{-1}\frac{A \pm \sqrt{A^2 + B^2 - C^2}}{B+C} \tag{C18}$$

Once the correct ψ has been obtained for a given α_{en} or α_{ex}, the associated g_{en} or g_{ex} may be found with the help of equation C14 or equation C15. For the given directions of the associated unit vectors g_{en} will be a positive quantity while g_{ex} will be a negative one.

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* Kinematic Synthesis of Linkages, McGraw-Hill Publishing Co., New York, 1964.

Angular Velocity of Pallet

Implicit differentiation of equation C17 leads to the following expression for the angular velocity of the pallet when driven in coupled motion by the escape wheel:

$$\dot{\psi} = \dot{\phi}\,\frac{P}{Q} \tag{C19}$$

where

$\dot{\phi}$ = Angular velocity of the escape wheel

$$P = b\cos(\phi - \alpha - \psi) \tag{C20}$$

$$Q = a\cos(\psi + \alpha) + b\cos(\phi - \alpha - \psi) \tag{C21}$$

Relative Velocity of Contact Point S on the Escape Wheel with Respect to Contact Point T on the Pallet

With point S the tip of the escape wheel tooth and point T the coincident contact point of the pallet, the relative velocity $\bar{V}_{S/T}$ in the direction of the unit vector $\bar{n}_t$ is found from (fig. C-1):

$$\bar{V}_{S/T} = [\bar{V}_{S/O_s} \cdot \bar{n}_t - \bar{V}_{T/O_p} \cdot \bar{n}_t]\,\bar{n}_t \tag{C22}$$

The velocity $\bar{V}_{S/O_s}$ of point S with respect to the escape wheel pivot is obtained from

$$\bar{V}_{S/O_s} = \dot{\phi}\bar{k} \times b\bar{n}_b \tag{C23}$$

while $\bar{V}_{T/O_p}$, the velocity of point T with respect to the pallet pivot, is given by:

$$\bar{V}_{T/O_p} = \dot{\psi}\bar{k} \times (c\bar{n}_c + g\bar{n}_t) \tag{C24}$$

Substitution of equations C23 and C24, together with equations C5, C7, C8, into C22 leads, after the appropriate vector manipulations, to:

$$\bar{V}_{S/T} = -[\dot{\phi}b\sin(\phi - \psi - \alpha) + \dot{\psi}c\sin\alpha]\bar{n}_t \tag{C25}$$

It can be shown that the above expression also represents the vector $\dot{\bar{g}} = \dot{g}\bar{n}_t$, as obtained from the rate of change of the quantity $\bar{g}$, which was defined by equations C14 and C15.

Angular Acceleration of the Pallet

Differentiation of equation C19 with respect to time results in the following expression for the pallet angular acceleration:

$$\ddot{\psi} = U\ddot{\phi} + V\dot{\phi}^2 \qquad \text{(C26)}$$

where, with equations C20 and C21

$$U = \frac{P}{Q} \qquad \text{(C27)}$$

and

$$V = \frac{1}{Q^3}\,[aP^2\sin(\psi + \alpha) - b(P - Q)^2\sin(\phi - \alpha - \psi)] \qquad \text{(C28)}$$

APPENDIX D

MOMENT ARMS AND NORMAL VELOCITIES OF CONTACT POINTS

Moment Arms

The following derivations for the moment arm vectors, which find use in both coupled motion and impact calculations, are valid for both entrance and exit conditions (figs. D-1 and D-2) as long as the appropriate angle α_{en} or α_{ex} is used. The variables ϕ and ψ must be available from the free motion computations.

The following loop for the determination of vectors $\bar{A}_1'$ and $\bar{B}_1'$ is shown by figure D-1:

$$A_1'\bar{n}_t + B_1'\bar{n}_n - b\bar{n}_b = 0 \tag{D1}$$

Substitution of the unit vectors given by equations C5 through C7 and subsequent separation of component expressions leads to the following two equations which may be solved for A_1' and B_1':

$$A_1'\cos(\psi + \alpha) - B_1'\sin(\psi + \alpha) = b \cos \phi \tag{D2}$$

and

$$A_1'\sin(\psi + \alpha) + B_1'\cos(\psi + \alpha) = b \sin \phi \tag{D3}$$

Simultaneous solution of the above expressions for A_1' and B_1' leads to

$$\bar{A}_1' = b\cos(\phi - \psi - \alpha)\bar{n}_t \tag{D4}$$

and

$$\bar{B}_1' = b\sin(\phi - \psi - \alpha)\bar{n}_n \tag{D5}$$

Figure D-1 also serves for the determination of the vectors $\bar{C}_1'$ and $\bar{D}_1'$. The loop equation

$$D_1'\bar{n}_t + C_1'\bar{n}_n - c\bar{n}_c - g\bar{n}_t = 0$$

furnishes, with the help of equations C5, C6, and C8 for the unit vectors, the following set of component expressions:

$$(D_1' - g)\cos(\psi + \alpha) - C_1'\sin(\psi + \alpha) = c \cos \psi \tag{D6}$$

and

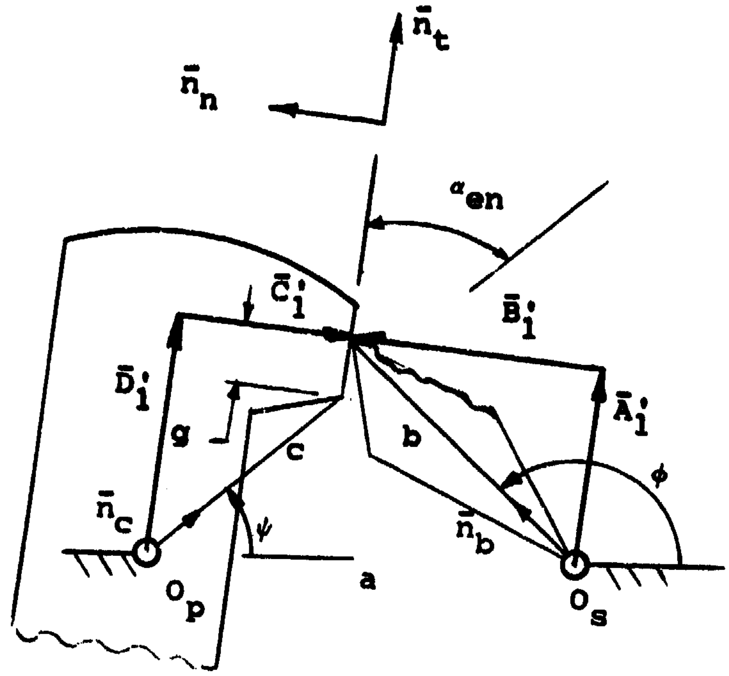

Figure D-1. Moment arms (entrance side)

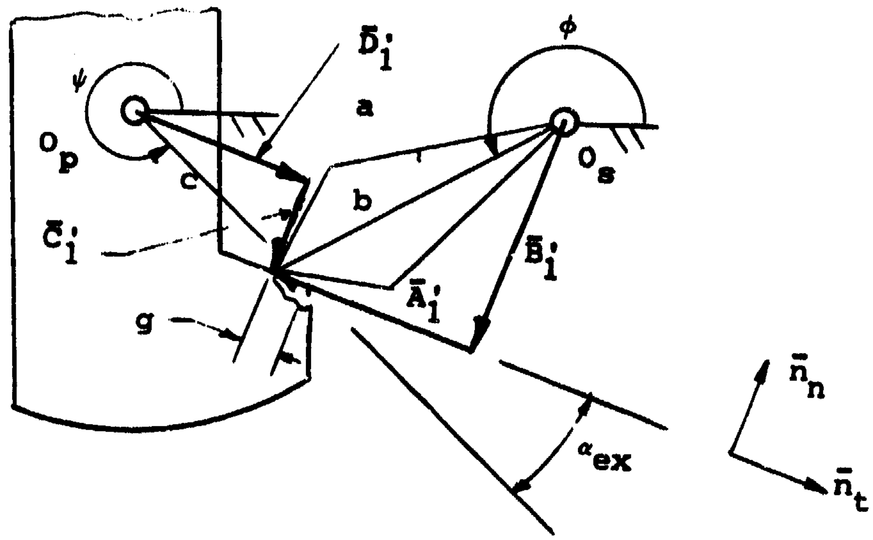

Figure D-2. Moment arms (exit side)

$$(D_1' - g)\sin(\psi + \alpha) + C_1'\cos(\psi + \alpha) = c \sin \psi \quad \text{(D7)}$$

Simultaneous solution for C_1' and D_1' gives:

$$\bar{C}_1' = -c\sin\alpha\bar{n}_n \quad \text{(D8)}$$

and

$$\bar{D}_1' = (c\cos\alpha + g)\bar{n}_t \quad \text{(D9)}$$

Again, it must be remembered to substitute the appropriate entrance or exit values for α and g.

Normal Velocities of Contact Points Before and After Impact

The contact point S on the escape wheel and the contact point T on the pallet are shown in figure D-3. At the instant before impact, the respective escape wheel and pallet angular velocities are given by:

$$\bar{\dot{\phi}}_i = \dot{\phi}_i\bar{k}' \quad \text{(D10)}$$

and

$$\bar{\dot{\psi}}_i = \dot{\psi}_i\bar{k}' \quad \text{(D11)}$$

The associated velocity component $\bar{V}_{SNi}$ of the escape wheel contact point S is obtained from:

$$\bar{V}_{SNi} = \bar{\dot{\phi}}_i \times \bar{A}_1' \quad \text{(D12)}$$

This velocity component is normal to the pallet face. This becomes, with the help of equations D10 and D4:

$$\bar{V}_{SNi} = b\dot{\phi}_i\cos(\phi - \psi - \alpha)\bar{n}_n \quad \text{(D13)}$$

In a similar manner the velocity component $\bar{V}_{TNi}$ of the pallet contact point T becomes:

$$\bar{V}_{TNi} = \bar{\dot{\psi}}_i \times \bar{D}_1' \quad \text{(D14)}$$

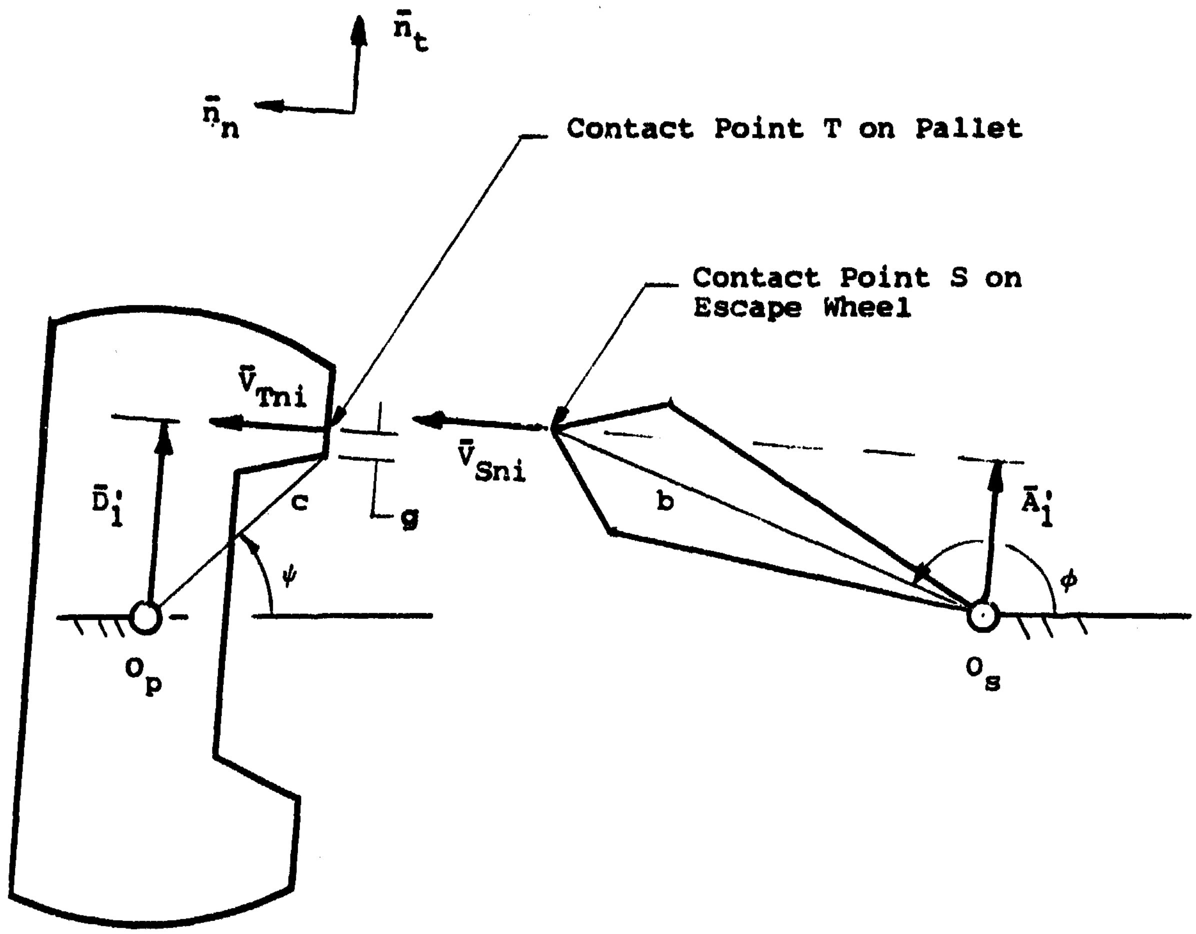

Figure D-3. Normal velocities of contact points S and T

This velocity component is also normal to the pallet face. Substitution of equations D9 and D11 into the above furnishes:

$$\bar{V}_{TNi} = \dot{\psi}_i(c\cos\alpha + g)\bar{n}_n \qquad \text{(D15)}$$

When equations D13 and D15 are adapted to the post-impact normal velocities $\bar{V}_{SNf}$ and $\bar{V}_{TNf}$, the respective angular velocities $\dot{\phi}_f$ and $\dot{\psi}_f$ must be substituted (impact section of appendix B).

APPENDIX E

KINEMATICS OF FREE MOTION: CONTACT SENSING EQUATIONS

The contact sensing quantities f and g' for entrance and exit conditions are defined by figures E-1 and E-2, respectively. It is assumed that the angles ϕ and ψ are known at every instant of free motion. To determine the entrance side sensing quantities, the following loop equation is written:

$$\overline{O_sS} + \overline{ST} + \overline{TV} + \overline{VO_p} + \overline{O_pO_s} = 0 \tag{E1}$$

This becomes with the help of the appropriate unit vectors

$$b\bar{n}_b + f\bar{n}_n - g'\bar{n}_t - c\bar{n}_c + a\bar{i}' = 0 \tag{E2}$$

When a similar expression is written for the exit side (fig. E-2), the following relationship is obtained:

$$b\bar{n}_b - f\bar{n}_n + g'\bar{n}_t - c\bar{n}_c + a\bar{i}' = 0 \tag{E3}$$

The quantities f and g' have opposite signs in equations E2 and E3. If equation E2 is solved for f and g' with the help of the unit vectors of equations C5 to C8, then for the entrance side:

$$f = a\sin(\psi + \alpha) - b\sin(\phi - \psi - \alpha) - c\sin\alpha \tag{E4}$$

and

$$g' = a\cos(\psi + \alpha) + b\cos(\phi - \psi - \alpha) - c\cos\alpha \tag{E5}$$

The above expressions will yield positive numerical quantities. When the same expressions are used for the exit side (with the appropriate angle α_{ex}), the resulting numerical quantities will be negative because of the aforementioned sign differences between equations E2 and E3.

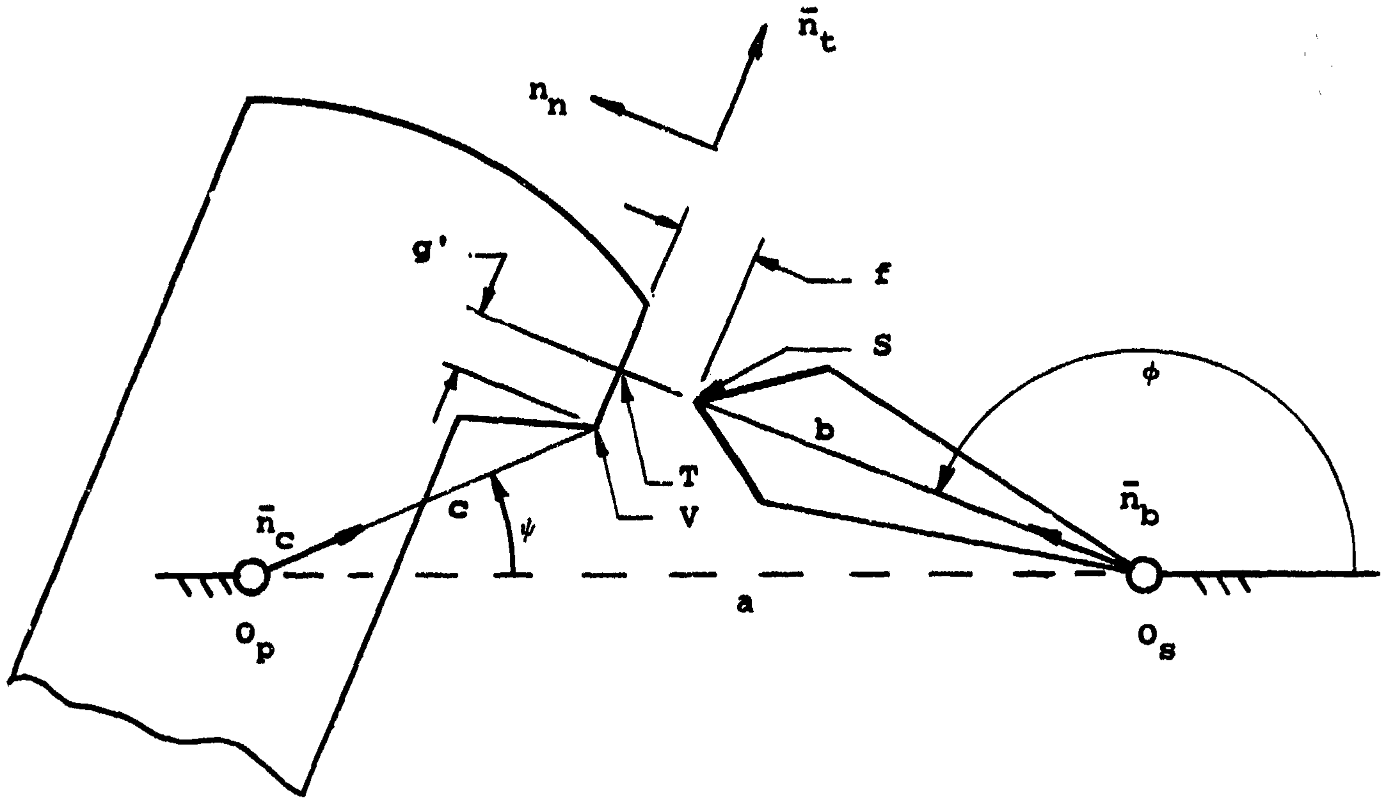

Figure E-1. Contact sensing on entrance side (center distance not to scale)

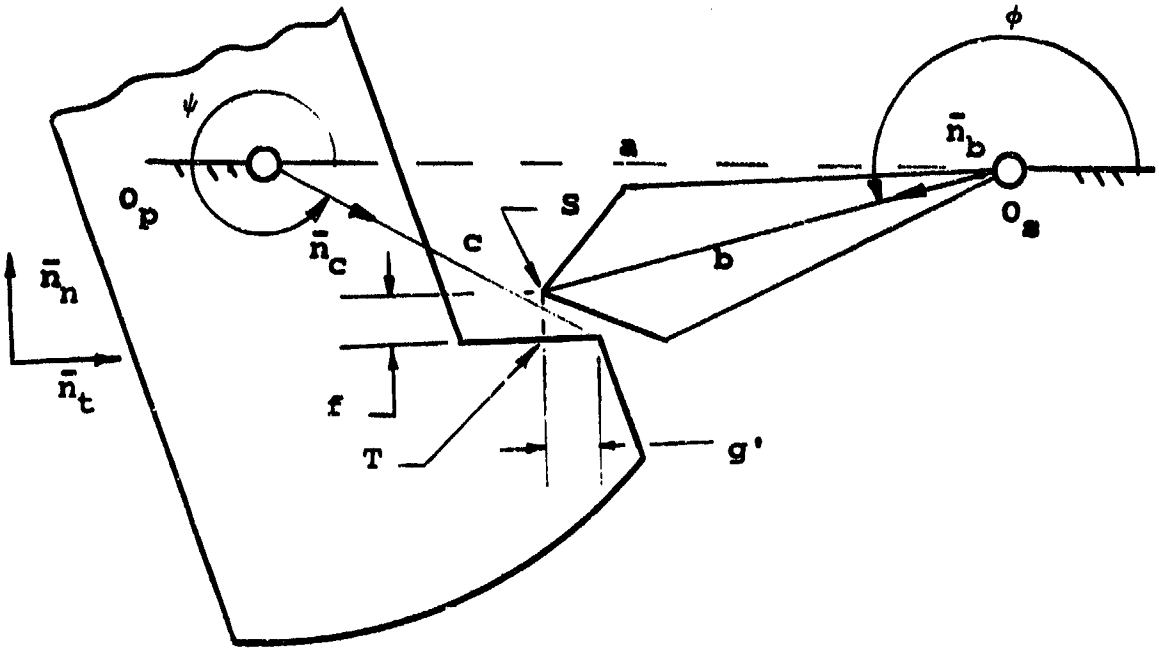

Figure E-2. Contact sensing on exit side (center distance not to scale)

APPENDIX F

GEOMETRY OF STRAIGHT-SIDED VERGE

Angles λ, α_{en}, and α_{ex}

The nomenclature of the indicated straight sided verge is defined in figure F-1. The angle λ is obtained as follows:

$$\lambda = 2\tan^{-1} \frac{h}{2L'} \qquad \text{(F1)}$$

With the above, and angle η, the angles α_{en} and α_{ex} become, respectively:

$$\alpha_{en} = \pi - \frac{\lambda}{2} \qquad \text{(F2)}$$

$$\alpha_{ex} = \frac{\lambda}{2} - \eta \qquad \text{(F3)}$$

The angle η is a basic design dimension.

Lengths of Working Surfaces

Figure F-2 is used to find the distance p_{en}; i.e., the length of the straight portion of the entrance working surface, as well as the corresponding length p_{ex} of the exit working surface of the verge.

The length p_{en} = LV is determined from the difference in lengths

$$p_{en} = y(L') - \frac{h}{2} \qquad \text{(F4)}$$

where

$y(L')$ = the y-coordinate of the point of intersection of the circle $x^2 + y^2 = r^2$, and the straight line $x = L'$

Then:

$$L'^2 + y^2 = r^2$$

and

$$y = \sqrt{r^2 - L'^2}$$

Equation F4 becomes:

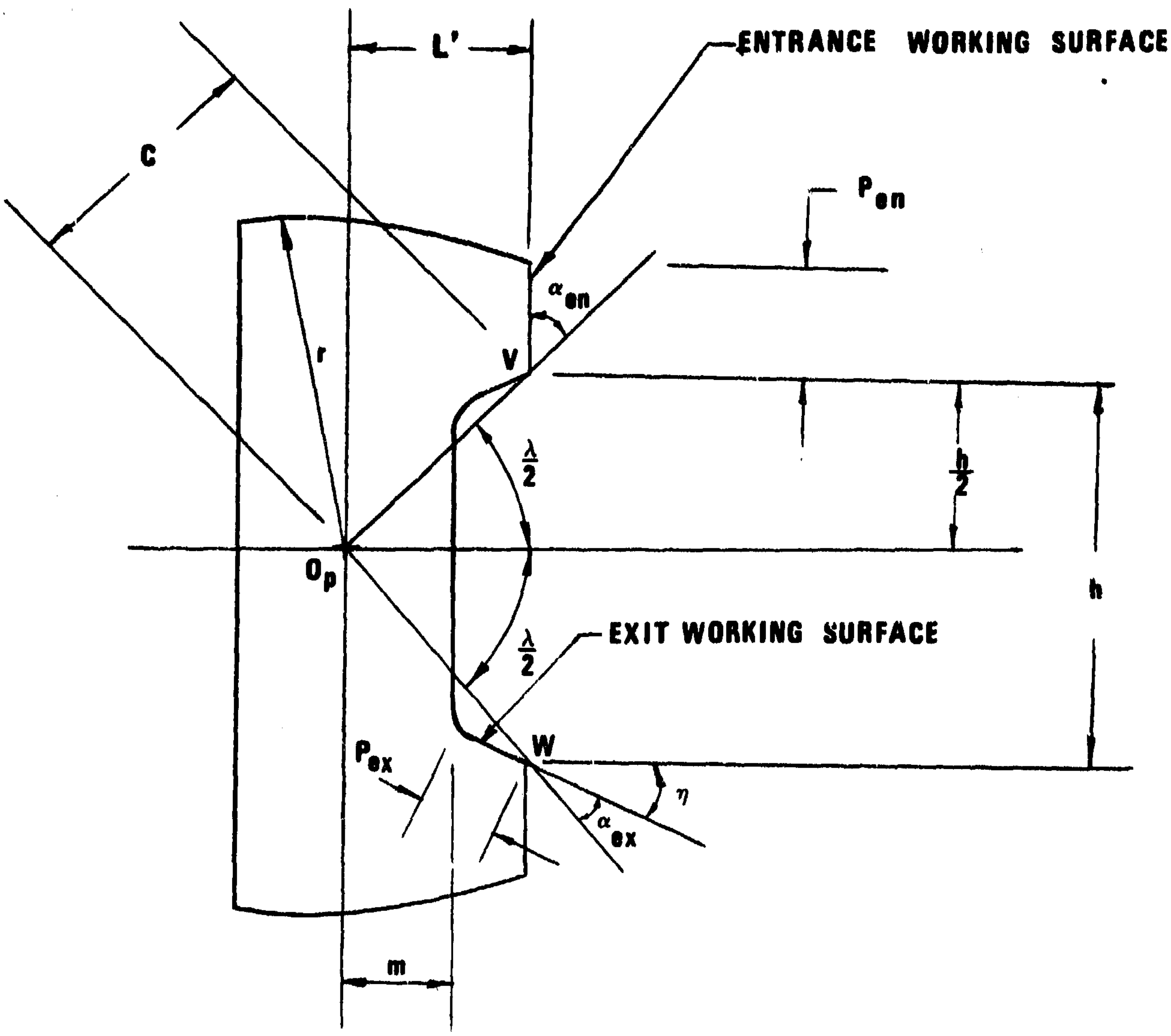

Figure F-1. Geometry of verge

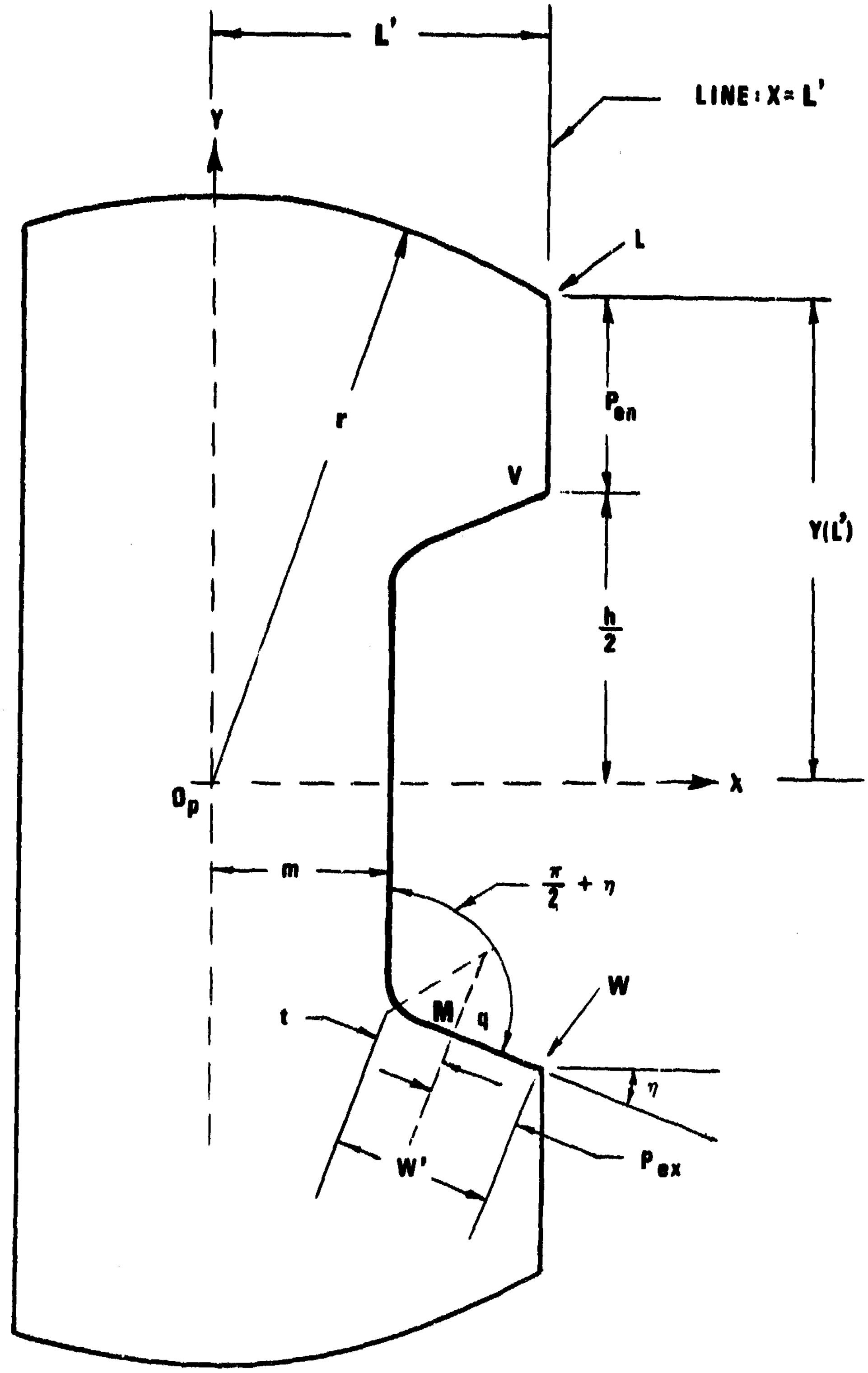

Figure F-2. Determination of p_{en} and p_{ex}

$$p_{en} = \sqrt{r^2 - L'^2} - \frac{h}{2} \tag{F5}$$

The length p_{ex} = MW is obtained with the help of

$$p_{ex} = W' - t \tag{F6}$$

where point M represents the blend point between the fillet of radius q and the straight line of the exit working surface. The length W' is obtained from

$$W' = \frac{L' - m}{\cos\eta} \tag{F7}$$

while t is given by:

$$t = \frac{q}{\tan\left(\frac{\frac{\pi}{2} + \eta}{2}\right)} \tag{F8}$$

where $\frac{\pi}{2} + \eta$ is the angle spanned by the fillet. With the above, equation F6 becomes

$$p_{ex} = \frac{L' - m}{\cos\eta} - \frac{q}{\tan\left(\frac{\frac{\pi}{2} + \eta}{2}\right)} \tag{F9}$$

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