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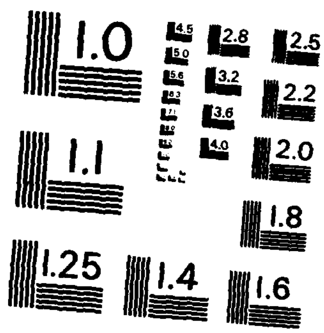

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The Interactive Fractal Analysis System (IFAS) allows the user to measure fractal dimensions of curves and surfaces. This is accomplished interactively through the use of virtual maps, character commands and responses, a graphic cursor, and an audible bell. With either a curve or surface, the user selects the most appropriate fractal dimension by entering a sampling interval and examining the generated scatterplot, correlation coefficient, and table. On a real-time basis, the user also has the capability of determining the fractal dimension for a portion of a curve or surface, editing features, (Over)

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windowing, and creating a perspective view of a surface. Several examples demonstrate that IFAS is able to closely approximate the fracticality of curves and surfaces.

IFAS: A PROGRAM TO MEASURE FRACTAL DIMENSIONS OF CURVES AND SURFACES

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ABSTRACT

The Interactive Fractal Analysis System (IFAS) allows the user to measure fractal dimensions of curves and surfaces. This is accomplished interactively through the use of virtual maps, character commands and responses, a graphic cursor, and an audible bell. With either a curve or surface, the user selects the most appropriate fractal dimension by entering a sampling interval and examining the generated scatterplot, correlation coefficient, and table. On a real-time basis, the user also has the capability of determining the fractal dimension for a portion of a curve or surface, editing features, windowing, and creating a perspective view of a surface. Several examples demonstrate that IFAS is able to closely approximate the fracticality of curves and surfaces.

INTRODUCTION

The problem of describing the forms of curves and surfaces has vexed researchers over the years. For example, a coastline is neither straight, nor circular, nor elliptic and therefore Euclidean lines cannot adequately describe most real world features. Imagine attempting to describe the boundaries of clouds or outlines of complicated coastlines in terms of classical geometry. An intriguing concept proposed by Mandelbrot (1967, 1977, 1982) is to use fractals to fill the void caused by the absence of suitable geometric representations. A fractal characterizes curves and surfaces in terms of their complexity by treating dimension as a continuum. Normally, dimension is an integer number (1 for curves, 2 for areas, and 3 for volumes); however, fractal dimensions may vary anywhere between 1 and 2 for a curve and 2 and 3 for a surface depending upon the irregularity of the form. Although individual fractals have been around since the 1900's, Mandelbrot was the first to recognize their applications outside of mathematics.

This paper discusses the Interactive Fractal Analysis System (IFAS) designed to measure the fracticality of a curve or surface. Several examples show the overall capabilities of the system. The software was developed and implemented on the Ohio State University computer system.

BACKGROUND

Definition of Fractals and Self-Similarity

In Euclidean geometry every curve has a dimension of 1 and every plane has a dimension of 2. This is generally referred to as the topological dimension (Dt). These dimensions remain constant no matter how complex or irregular a curve or plane may be. For example, the west coast of Great Britain contains many irregularities, but the topological dimension remains 1.

In the fractal domain a curve's dimension may be between 1 and 2 according to its complexity. The more contorted a straight line becomes, the higher its fractal dimension. Similarly, a plane's dimension may be a non-integer value between 2 and 3. The fractal dimension for any curve or surface is denoted by (D) and within this framework: $D > Dt$. Mandelbrot (1977) proposes the following definition for a fractal: "A fractal will be defined as a set for which the Hausdorff-Besicovitch dimension strictly exceeds the topological dimension."

Central to the concept of fractals is the notion of self-similarity. Self-similarity means that for any curve or surface a portion of the curve or surface can be considered a reduced image of the whole. However, seldom in nature (crystals are one exception) does self-similarity occur and therefore a statistical form of self-similarity is often encountered. In other words, if a curve or surface is examined at any scale it will resemble the whole in a statistical sense; therefore, D will remain constant. Brownian motion is an excellent example of statistical self-similarity. Because of this principle, a curve can be decomposed into $N=r$ nonoverlapping parts and each subsegment has a length of $1/r=1/N$. Similarly, a unit square can be divided into $N=r^2$ squares, where the similarity ratio is $r(N) = 1/r = 1/N^{1/2}$. In either case the following equation applies:

$$D=\log N/\log (1/r) \qquad (1)$$

and could be called the shape's similarity dimension. D can also be expressed as:

$$D=\log (N/N_0)/\log (\lambda_0/\lambda) \qquad (2)$$

where λ_0 and λ are two sampling intervals and N_0 and N are the number of such intervals contained. If a curve resembles a straight line then when the sampling interval is halved, N doubles and the proportion equals 1. The majority of cartographic curves are not straight lines and therefore N will more than double causing D to be greater than 1. The principle of self-similarity is dismissed by Goodchild (1980), Hakanson (1978), and Scheidegger (1970). Hakanson, for example, points out the absurdity of postulating the validity of self-similarity down to the size of the pebbles on the coastline and at the molecular interstices of those pebbles. Goodchild demonstrates that although Richardson (1961) found the west coast of Britain to have a constant D of 1.25 over sampling intervals between 10 and 1000km., he found the east coast to vary between 1.15 and 1.31 for a similar sampling interval. This suggests that whatever created the irregularities on the coastline acted at specific scales. Goodchild states that since self-similarity is only one aspect of the fractal approach, it would be unwise to reject the entire concept.

Interactive Cartography

The virtues of interactive cartography have been extensively noted by Moellering (1977, 1980) and several others. Moellering's general theme is that the real power behind interactive cartography lies in the virtual map and its manipulability by the user. Because the virtual map, displayed on the CRT, is in a transient and flexible state, the user can easily manipulate and edit it. With the realization that interactive programming and computer graphics not only allows real-time control over the mapping process, but is also very useful in solving problems, IFAS was placed in an interactive setting. The results demonstrate that interactive cartography enhanced the development of appropriate analytical and numerical techniques.

DESCRIPTION OF THE INTERACTIVE FRACTAL ANALYSIS SYSTEM

The goal of IFAS is to allow the user to determine the fractal dimension for an entire curve or surface or a portion of the curve or surface. By entering the proper sampling interval, which is suggested by ancillary information, the result is a number which approximates the complexity of a feature.

The theory on which this system operates is based upon the empirical work performed by Richardson (1961) and later extended by Mandelbrot (1967).

Richardson measured the lengths of several frontiers by manually walking a pair of dividers along the outline so as to count the number of steps. The opening of the dividers (n) was fixed in advance and fractional side was estimated at the end of the walk. The main purpose in this section of Richardson's research was to study the broad variation of Σn with n.

Richardson produced a scatterplot in which he plotted log total length against log step size for five land frontiers and a circle. Mandelbrot (1967) discovered a relationship between the slope (β) of the lines and fractal dimension (D). To Richardson the slope had no theoretical meaning, but to Mandelbrot it could be used as an estimate of 1-D, which leads to:

$$D=1-\beta \qquad (3)$$

In computing the fractal dimension of a curve, an algorithm was designed by Shelberg (1982) which simulates walking a pair of dividers along a curve and counts the number of steps. The algorithm ultimately uses Equation 3 to calculate D. Shelberg, Moellering and Lam (1982) detail the development of this algorithm and present several examples.

The fractal dimension of a surface is computed by an algorithm that is modeled after the research performed by Goodchild (1980). Since all the surfaces he examined were self-similar, he was able to use the length of the mean isarithm line to calculate D. Shelberg (1982) modified Goodchild's algorithm so that it could be used for nonself-similar surfaces and be used in an interactive setting. By inputing the proper isarithm interval, a number of isarithm lines are used to capture the overall complexity of a surface.

For the fractal dimension to be properly calculated, the user must be supplied with additional information. In the case of a curve, the user is informed of the average segment length. This information suggests at what length the initial chord length should be set. It would be meaningless to choose a chord length many times shorter than the shortest line segment. If an extremely short chord length is selected, an attempt to examine the fractal character of a curve would extend beyond the primitive subelements used to represent the geometry of the resulting form. In other words, beyond this lower limit of primitive subelements, the curve's fractal dimension behaves as if it is a straight line. In order to verify the selection of the proper initial chord length, the user may check the r^2 value or view the scatterplot or table summary.

In the case of a surface, the user is given no prior indication of what sampling interval to select. The strategy is to choose the lowest isarithm interval with no isarithm lines being eliminated. For example, after an isarithm interval is selected and D is calculated, the user is informed of the number of isarithm lines included and excluded in the calculations. The user can then use this information and that of the r^2 value, scatterplot and table summary to best approximate the fractal dimension of a surface.

IFAS is implemented on a large computer system at the Ohio State University. At Ohio State, the central processing unit is an Amdahl 470 V6, while the time sharing system used is IBM TSO (Time Sharing Option). IFAS utilizes the Tektronix 4012 and 4014 storage CRT terminals for interactive graphics and the Tektronix 4631 Hardcopy Unit for real map output. IFAS is written in FORTRAN IV and the Tektronix Interactive Graphics Library is used to perform all graphics work.

IFAS SYSTEM OPERATION

The command structure in Table 1 allows the user a flexible approach in calculating D and performing other functions for either a curve or a surface.

Curve

With a system prompt of IFAS>, the user may enter CIFAS indicating a curve session is desired along with the name of the curve. For example, if the West Coast of Great Britain is to be examined, then the previously established filename of GBRIT is entered. With the issuing of the CURVE command, the outline of the curve is drawn on the CRT. Also printed on the screen, in the map area and above the curve, is the title, scale, date, time, and the number of digitized points. Figure 1 depicts a typical session. First, the user enters in FR, which is an abbreviated form of the command FRACTL, indicating D is to be computed. The program responds with FRACTL> signifying the level at which the user is in the system. It is important to note that all NAME> prompts are designed to aid the user in knowing what level in the command hierarchy he is currently operating. The system also responds with the average segment length and with the suggested initial chord length. This suggested initial chord length is based on the sampling theorem and is one-half the average segment length. The program then asks if the chords are to be drawn and the user responds with a NO answer. Next, the user is asked to input the initial chord length and the number of maximum solution steps. After these numbers are input, D is calculated and is printed in the map area below the outline. Also printed in this section is the length of the initial chord, the actual number of solution steps, the r^2 value, and the original length of the feature. Directly above this bottom legend, in the lower left corner, is a graphic display of the initial chord length.

After D is calculated, the program returns to the CURVE> level, and the user is able to verify the initial chord length selection by issuing SCPLT for a scatterplot (Figure 2) or a SUMRY command for a printed summary. Also, within the CURVE> level, the user is able to find the area (AREA) of a closed curve using the trapezoidal method. The user may edit (EDIT) and save (SAVE) the edited curve for subsequent processing. The fractal dimension for a portion of the curve can be computed with the FWINDOW command. The

System Prompt

SIFAS
CIFAS

Commands

CURVE (C)
SURFACE (S)

Subcommands

CURVE	SURFACE
AREA (A)	DISGRID (D)
EDIT (E)	EDIT (E)
FRACTL (FR)	FRACTL (FR)
FWINDOW (F)	FWINDOW (F)
REDISP (RE)	PERSURF (P)
RESETW (R)	REDISP (RE)
SAVE (SA)	RESETW (R)
SCPLT (S)	RSINT (RS)
SUMRY (SU)	SAVE (SA)
WINDOW (W)	SCPLT (S)
END	SUMRY (SU)
	WINDOW (W)
	END

Subcommands

Edit

CHANGE (C)
END (E)

Table 1. Command Structure

```
  CURVE>
FR
  FRACTL>
  AVERAGE SEGMENT LENGTH
  EQUALS: 0.06510
  SUGGESTED INITIAL CHORD
  LENGTH EQUALS: 0.03255
  DRAW CHORDS?
N
  INPUT INITIAL CHORD
  LENGTH
?
.06510
  INPUT MAXIMUM NUMBER OF
  SOLUTION STEPS
?
10
  CURVE>
```

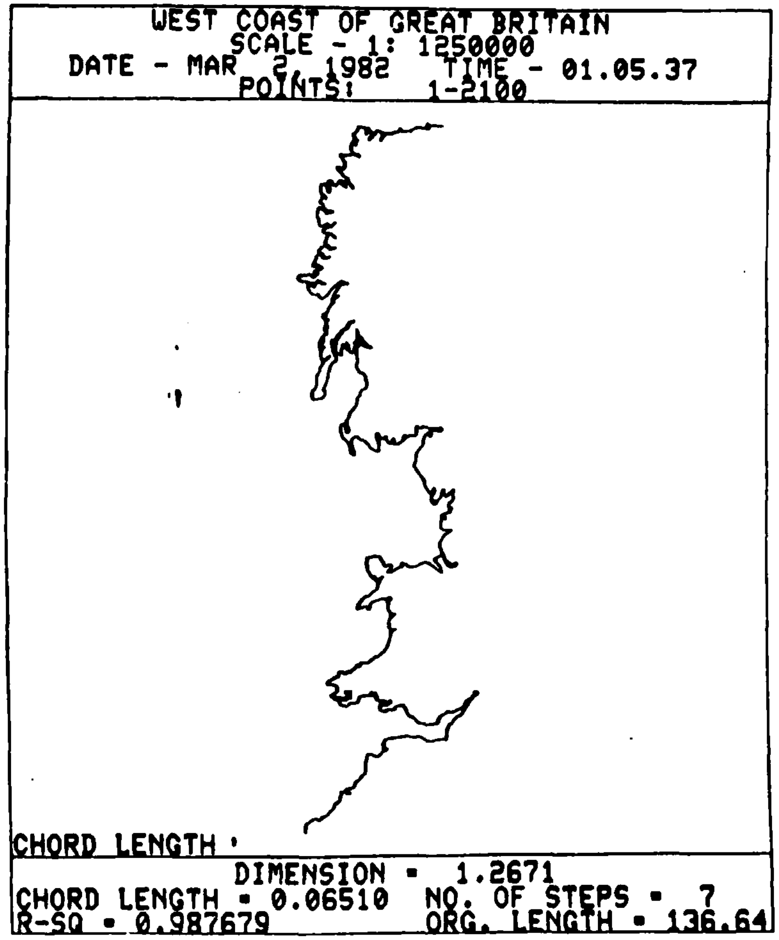

Figure 1. Curve Session

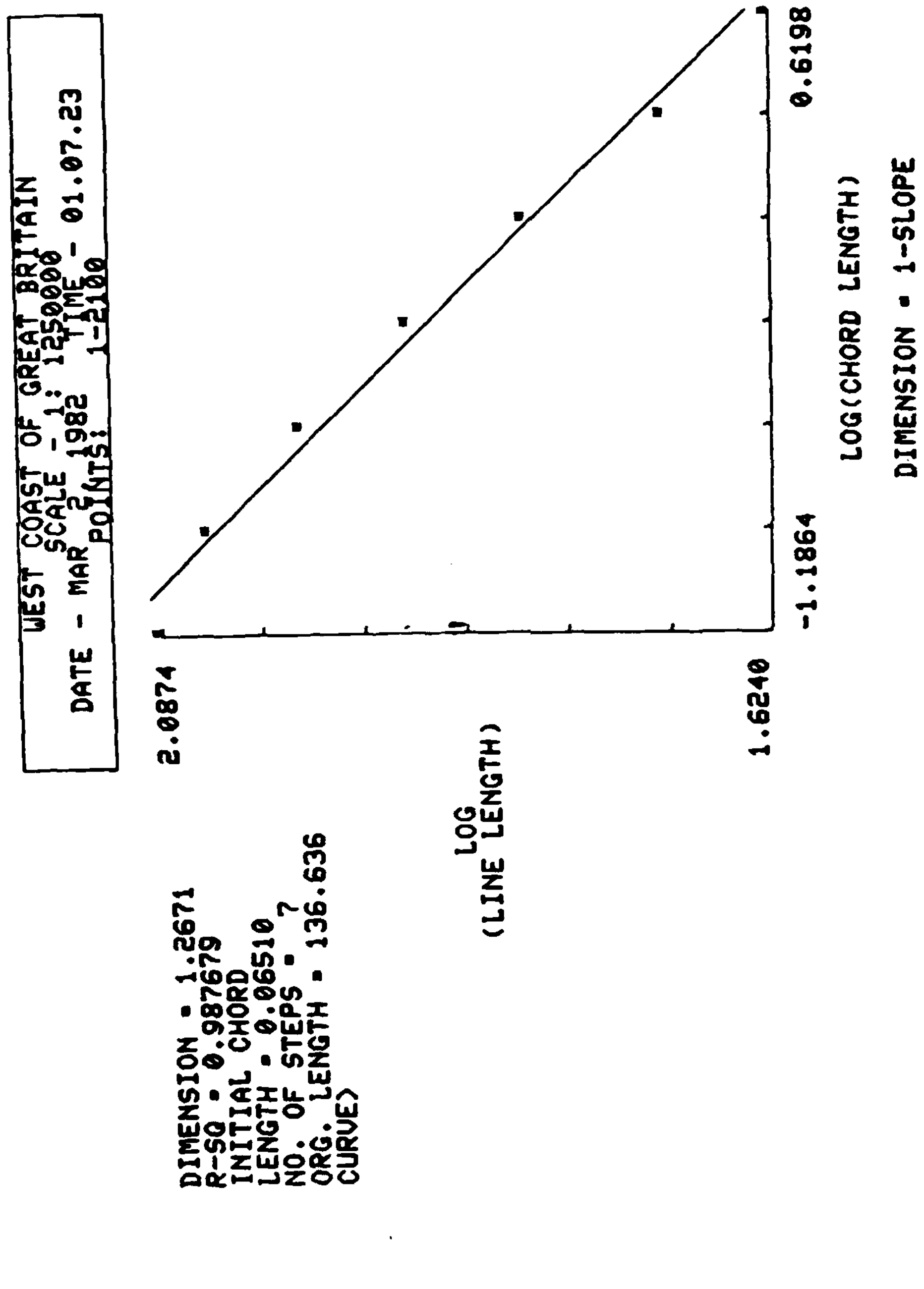

Figure 2. Scatterplot

current image may be redisplayed (REDISP) or have the screen image set and all internal values back to the original curve using the RESETW command. Finally, a portion of the curve can be zoomed in with the WINDOW command.

Surface

If a surface session is desired, the user enters SIFAS, after the system prompt of IFAS>, and then enters the filename for a specific surface. With the issuing of the SURFACE command the legend above the map display area is printed. This legend contains the title, scale, date, time, and minimum and maximum Z-heights. In Figure 3, the user enters a PERSURF command which indicates a perspective surface is to be drawn. The program asks for the mode, which is an option which allows for the printing of a legend, straight line or histogram connections. The input of -3 indicates that the legend is not to be printed and the surface will be shown as straight line connections with hidden points and lines drawn to them deleted. The user is then asked to input the angle between the observer's line of sight and the Z-axis measured in degrees. The next input is the angle between the projection of the observer's line of sight to the x-y plane and the X-axis measured in degrees. After the surface is drawn, control is then returned to the SURFACE> level. Next, the user issues a FRACTL command in which the program responds with FRACTL>, indicating the level, and asks if rows or columns are desired. By entering either rows or columns, this provides the ability to capture the maximum trend in the surface in the x or y directions. In the example, rows are selected after which the program asks for the isarithm interval. Upon entering the maximum cell size, the fractal dimension is computed and printed out in the legend below the map display area. The legend also contains the r^2 value, isarithm interval, maximum cell size and the rows and columns displayed. In the user communication area, the program also responds with the number of isarithm lines included and excluded from the calculations. Control is then returned to the SURFACE> level.

In the SURFACE> level, the user may also display the surface with the actual Z heights or as dots with every tenth position being a plus mark (DISGRID). This type of display is designed to accomplish editing (EDIT) by being able to pick each location with the available crosshairs. The fractal dimension may be determined for a portion of the surface with the FWINDOW command. The REDISP command redisplays the current screen image and the RESETW command resets the screen image and all internal values back to the original surface. A scatterplot, similar to that in Figure 2, can be generated (SCPLT) for each isarithm line, along with a summary (SUMRY) in table form also for each isarithm line. And like for a curve, the user has the capability to zoom in on a portion of the surface with the WINDOW command.

SUMMARY AND CONCLUSIONS

The interactive computer cartographic program IFAS allows the cartographer to measure the fractal dimensions of complicated curves and surfaces. This fractal dimension provides a method in which to quantify the irregularity of a curve or the roughness of a surface. In computing the fractal dimension for a feature, a sampling interval must be chosen. The recommended choice of this sampling interval is based on the average segment length for a curve and the number of isarithm lines included versus the number of lines excluded from the calculations for a surface. The user is able to verify the sampling interval selection with the issuing of various commands to view the scatterplot and summary report.

Results from the research indicate the accuracy of the fractal dimension is dependent upon the amount of self-similarity a feature possesses and the sampling interval selection. The variations in the fractal dimension, over a number of sampling intervals, reflect a need to examine the effects of self-similarity, or lack of it on a feature's fracticality. A possible enhancement to IFAS would be to automatically determine the proper sampling interval to closely approximate D. Finally, this research exemplifies the power and application of interactive computer graphics in the cartographic field.

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```
  PERSURF>
  INPUT THE MODE
?
-3
  INPUT THE VIEWING ANGLE
  FROM THE Z-AXIS
?
45
  INPUT THE VIEWING ANGLE
  FROM THE X-AXIS
?
45
  SURFACE>
fr
  FRACTL>
  USE ROWS OR COLUMNS
r
  ENTER ISARITHM INTERVAL
?
5
  INPUT MAXIMUM CELL SIZE
?
5
  NO. OF ISARITHM LINES
  INCLUDED =  40
  NO. OF ISARITHM LINES
  NOT INCLUDED =   1
  SURFACE>
```

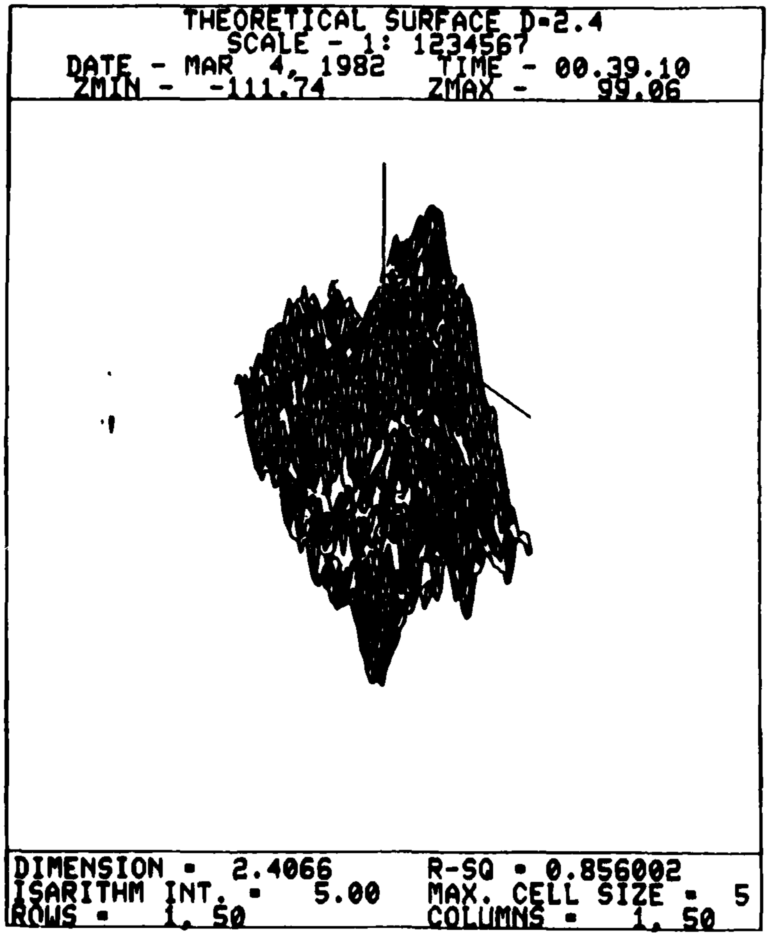

Figure 3. Surface Session

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