END

FILMED

DTIC

MICROCOPY RESOLUTION TEST CHART

NATIONAL BUREAU OF STANDARDS-1963-A

UNCLASSIFIED

AD A123294

TITLE: On the Design of Reinforced Hull Panels

Del Proyecto de Paneles Renforzados

AUTHOR(S) AND/OR EDITOR(S): Francisco Gonzalez Gonzales

ORIGINAL LANGUAGE: Spanish

TYPEWRITTEN PAGES: 16

TRANSLATOR: C

NISC TRANSLATION NO. .6918

APPROVED _____

DATE 5 October 1982

DTIC
ELECTE
JAN 1 2 1983
S
D

D

UNCLASSIFIED

88 01 12 053

ON THE DESIGN OF REINFORCED HULL PANELS

Author's Abstract   /78*

The arrangement of transverse stiffeners in rectangular gross panels under combined lateral and in-plane, compressive loads is studied and a reasonable method for optimizing the design is arrived at.

The analysis is based on a combined deflection-strength-weight criterion. Minimum weight compatible with limited stress and deflection is obtained through a combination of simple formulae that take into account possible buckling modes of the structure.

CONTENTS

Accession For	
NTIS GRA&I	☒
DTIC TAB	☐
Unannounced	☐
Justification	
By Per DTIC Form 50	
Distribution/ on File	
Availability Codes	
Dist	Avail and/or Special
A	

1. Introduction

One of the typical structural substructures or subsystems of the hull of a ship is a plate--plane or curved - reinforced by stiffeners welded to it.

The arrangement of the structural stiffeners usually meet design consideratios, adoption of which is related to aspects such as:

--periphery support conditions of the substructure;
--loads on the connection to the rest of the structure;
--own loads of the substructure;
--restrictions imposed by the local geometry and the adjoining structure.

---

*Numbers in right margin indicate pagination in the original text.

Traditionally, the hull of welded steel ships is constructed by selecting one of two fundamental types of stiffener:

--longitudinal;
--transverse.

In the study of the working stresses of each structural element, the loads can be divided into three different groups:

--normal to the edges of the plate and in its plane;
--tangential to the edges of the plate and in its plane;
--normal to the plane of the panel.

In the study of the behavior of the stiffened flat panels, the analysis of aeronautical engineering has made a fundamental contribution; however, in naval architecture the requirements on the panels usually present conditions which complicate the problem of design with other contour conditions and other loads on the structure.

In this work we are going to study the design of a type of panel which has been avoided in the technical literature owing to its admitted complexity. This involves the case of a plate with its four edges fixed at the periphery, subjected to pressure loads normal to its plane and coplanar compressive loads in the direction of its larger dimension and stiffened by reinforcements perpendicular to the coplanar load.

This work arrives at a simple expression of the effectiveness of the design expressed in terms of weight of the structure. This parameter is usually important in the design of warship hulls, a case in which, moreover, the loads contour conditions, and arrangement of the stiffeners adopted in this study are usually presented.

In merchant ship construction, however, the minimum cost is not necessarily synonymous with the lower weight of the hull, if the latter is attained at the expense of inefficient forms or of abnormal arrangements of the elements of the structure.

In order to be able to state the solution in a simple and practical expression, the following assumptions are made:

--uniformly distributed normal pressure load;
--uniformly distributed coplanar compression.

The work of Klitchieff (1) analyzes the case of a panel simply supported at its periphery, and arrives at a simple and neat solution to express the resistance <u>/79</u> to bending. Timoshenko (2) presents the classical method of analysis of this type of structural problem. The solution of the simply supported panel with minimal weight is studied by Gomez and Seide (3) and by Wah (4), who, together with Bleich (5), also presents a valuable collection of the practical results of the investigations from various sources.

In one way this work has followed the analytical focus of Klitchieff (1) to continue on the practical line laid out by Wah (4) and Bleich (5), although with the complexity which the contour conditions adopted impose. In this way, we avoided diverting attention and confusing the discussion with the formulation of

a complete mathematical solution. Since the arrangement of stiffeners in the structure of a ship is usually uniform, this simplification was adopted from the outset of the work.

On the other hand, a greater simplicity of the calculation process is attained if the stiffeners are assumed to be situated in the nodal lines of the deformed plate. In reality, this assumption is in line with that adopted by Klitchieff, and therefore permits reaching a solution with an equal degree of validity. As a special load case, Heller and Jasper (6) are studying the structural resistance of planing ships. In these cases, the normal pressure is dynamic, and cannot be assumed to be uniformly distributed. Nevertheless, a first approximation would permit considering an equivalent uniform pressure which would produce the same deformation or identical level of stress.

Finally, in this study, only elastic and minor deformations are considered, so that the effect of the membrane stresses on the result can be disregarded.

2. Table of Symbols

P Kg/cm^2, coplanar compression at the edge of the plate, in the x-direction

q Kg/cm^2, lateral pressure normal to the plane of the plate

a   Length of the panel, in the x-direction

b   width of the panel, in the y-direction

c   length of the subpanel, or section, in the x-direction

d   proportion of one section - c/b

h   thickness of the plate

u   h/b

r   number of stiffeners

RA   Proportion, ratio of the panel area = a/b

A   area of section of one stiffener

I   moment of flexural inertia of one stiffener

m   load interaction factor

s   stress, strain

f   deformation

D   flexural rigidity of the plate

E   Young's modulus

v   Poisson ratio

3

Subscripts

as   antisymmetrical

cr   critical, - a

e   elastic limit

x, y   coordinates of the plate or panel

bx, by flexure in the x- and y- directions

s   relative to stresses

f   relative to deformations

1, 2   subcases

$(f)_x(^n$ nth partial derivative of f with respect to x.

Note: Numbers in parentheses refer to bibliography at the end of this work.

3. Geometry, Loads, and Contour Conditions

A panel is considered as composed of a flat plate with stiffeners parallel to one of the edges. The proportion of the dimensions or aspect ration of the panel RA = a/b is not limited. The number of sections, that is the number of stiffeners plus one, is taken as greater than four. The proportion of these sections is held variable between 0 and approximately 1.5.

The normal, lateral pressure can act on any of the faces of the panel. The compressive load in the plane of the panel acts in a direction perpendicular to the stiffeners.

The panel is fixed along its four edges. The stiffeners are fixed at their extremities, and are connected to the plate so that each one of the smaller panels, or sections, can behave as if fixed at the connection of the stiffeners.
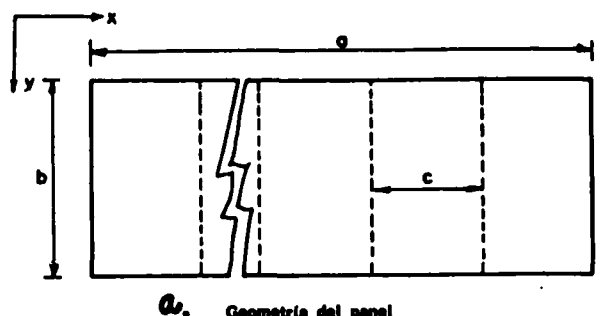
4. Analysis of the Problem

4.1. Deformations

In panels with their simply supported edges, the deformation can normally exceed half of the thickness of the plate. In the case where the four edges are held fixed, this value of the deformation can be considered as a valid upper limit.
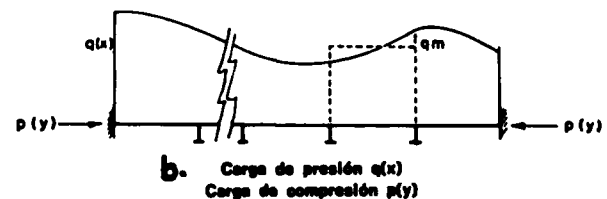
According also to Bleich (5), the effect of superposing the two types of load can be expressed by way of an interaction factor m, such that:

$$f = m \cdot f_0$$

That is, the total deformation due to the combination of a compressive load at edges p and another, normal pressure load q can be evaluated as a function of the deformation f_0 produced by the lateral pressure alone.

4

Fig. 1. Geometry, loads, deformation

a. Geometry of the panel
b. Pressure load q (x)
   Compressive load P (y)
c. Deformations below q (x)

a. Geometría del panel

b. Carga de presión q(x)
   Carga de compresión p(y)
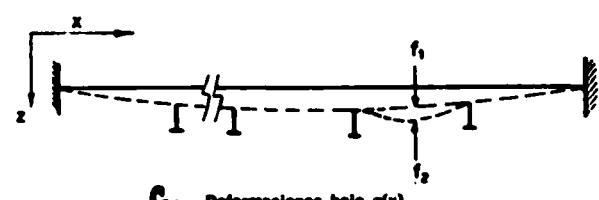
C. Deformaciones bajo q(x)

Figura 1.—Geometría, cargas, deformación.

The factor m takes the value:

$$m = \frac{S_{..}}{S_{.r} - P}$$

where S_{cr} represents the unit bending stress of the reinforced panel.

$$S_{.r} \leq S_c$$
$$f \leq h, 2$$

With these two limitations a linear approximation can be used which is derived from the theory of large deformations. The result of this linearization is the characteristic factor m.

The deformation F_0 is a linear function of the lateral pressure q, provided the material is not stressed beyond its elastic limit.

Timoshenko (7) gives for deformation f_0 an expression of the form

$$f_0 = K \frac{q}{384\,D}$$

in which k is a function of the ratio c/b, whose values are given in Appendix A for the various values of that proportion.

D represents the fluxural rigidity of the plate, assumed isotropic.

If the buckling modes of the panel are defined on the basis of the condition of non-deformed stiffeners, this justifies the selection of a section as a unit of study of the elastic deformation. In reality, the deformation at any point

5

on the panel (x, y) can be expressed by the sum of two terms:

$$f_o = f_1 + f_2$$

f_1 being the primary deformation which results from considering the plate hinged at the reinforcements and infinitely rigid everywhere else, and f_2, the secondary deformation, which appears only on the plate.

In other words, the total deformation can be produced by superposition of what is produced in the hinged rigid plate and that which the plate takes between reinforcements when it loses its rigidity.

In panels with fixed edges, the two addends of the deformation are much smaller than in those which have their edges supported. On the other hand, the magnitude of the deformation can be expected to be distributed in a different proportion between the two, such that f_2 can be considered predominant when infinitely rigid stiffeners welded to the plate are put in place.

4.2. Resistance to bending

In Appendix B is presented the application which was made of the theory of cuts in order to solve the problem of finding an expression of the critical stress of the reinforced panel.

In order to find a value below S_{cr}, only transverse waves are considered. The presence of stiffeners forces the plate to buckle in a definite number of half waves in the direction normal to those stiffeners.

The first mode has (n-1) regularly spaced nodal lines. The second mode has nodal lines uniformly spaced at the center, but with a separation of the extreme lines at the parallel edges, which approaches 1.5 times the separation when n increases. This characteristic of the second mode becomes precise for very large values of n.

If the reinforcements are placed in the nodal lines, it happens that those elements deform exclusively by lateral torsion to follow and to permit the undulated deformation of the plate, assuming that it is rigidly welded to the reinforcements.

Pandeo simétrico

Pandeo antisimétrico
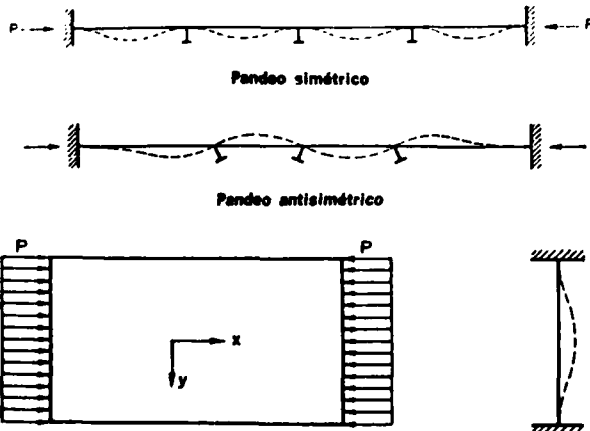
a. Symmetric bending
b. Antisymmetric bending
c. Compressive loads
d. Transverse buckling

e. Fig. 2. Examples of buckling

For both buckling modes, the first symmetric [i] and the second, antisymmetric [as], there are obtained for each values of K_{cr} as a function of a variable d which has the values:

6

$$d_{n} = \frac{RA}{2n+1} = \frac{RA}{r+2}$$

$$d_{ni} = \frac{RA}{n} = \frac{RA}{r+1}$$

4.3. Stresses

According to Bleich (5), the greater values of the composite stress must occur at the central points of the edges of the smaller panels, or sections, (in the extreme sections in the case of antisymmetric buckling). The resulting values of the stresses are:

$$S_x = P + m\, S_{bx}$$

The stress in the [symbol missing?] direction is not considered when the term for compression, P, is not included. The value S_{bx} represents the flexural component in the x-direction which is produced by lateral pressure q.

The coefficient m has the same value set forth above.

4.4. Flexural stress

For any proportion of dimensions of the section, the flexural stress due to lateral pressure q can be expressed by:

$$S_{bx} = K_s \cdot \frac{q}{2} \cdot (b/h)^2$$

This relation is the result of working up the Timoshenko coefficients presented in Appendix A.

Coefficient K is a function of the proportions of the sections.

4.5. Restrictions

As pointed out earlier, there are limitations which must be observed in order to be able to apply the linear theory of small deformations:

$$f = m \cdot f_{...} \text{ smaller than } h\, 2.\, y$$

$$s_. = P + m \cdot S_{...} \text{ smaller than } S.$$

All the variables have been defined on the basis of Appendices A and B. It then becomes easy to obtain an expression for the corresponding limit values of u = h/b:

$$u_i = \left| \frac{P\,2}{F} \cdot \underset{}{\dashv} \; \right| \; \frac{P\,2}{F} + K \cdot \frac{q\,(1-v)}{16\cdot E}$$

$$u_i = \left| \cdot \frac{P}{F} + K_s \cdot \frac{q\,2}{S_. - P} \right.$$
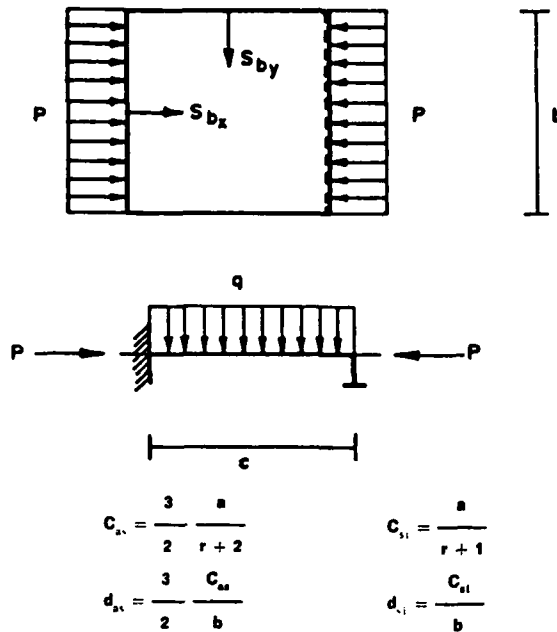
7

Fig. 3. Loads and stresses on extreme sections

These two values limit the thickness-to-width ratio of the panel such that $u = h/b$ must always be greater than U_f and less than U_s.

The value of F is:

$$\frac{E \cdot \pi^2 \cdot K_{cr}}{12 (1 - V^2)}$$

where K_{cr} alternately takes the values of K_{si} and K_{as} in each of the buckling modes discussed.

The calculation conducted for usual values of E, u, P and q permits simplification to

$$u_f \lessapprox u_s .$$

Consequently, the limit value for $U = h/b$ turns out to be u_f, and the construction of lower weight is obtained for $u = u_f$.

The influence of the value of u on the total weight is discussed in the next section.

4.6. Weight of the panel

For any material used in the construction of a reinforced plate, the total weight is proportional to its volume.

$$V = a \cdot b \cdot h + r \cdot A \cdot b ,$$

A being the area of the right section of one of the two reinforcements, and all of the latter are equal.

8

To select the structure of least weight, merely select that of lower weight per unit of area of the panel, or minimize the equivalent mean thickness:

$$h_{eq} = \frac{V}{a \cdot b} = h + \frac{r \cdot A}{a}$$

In naval shipbuilding, the sections of the reinforcements can be expressed in a great number of ways as a function of their inertias.

$$A = K_n \cdot I^{1/n}, \text{ with } n = 2 \text{ ó } 3$$

I being the moment of inertia of the transverse area of the reinforcement with respect to an axis located at the union of the reinforcement with the plate (4):

K_n is a non-dimensional factor which takes the values K_2 and K_3 according to the value of n.

In order to keep this study brief, only the profiles with n = z will be considered, that is, the majority of sections at T and L.

On the other hand the condition which has been imposed on the reinforcements to act at y as nodal lines of the buckling requires a minimum value of l.

This minimum value of the inertia can be expressed as a function of a variable:

$$g = \frac{E \cdot I}{b \cdot D}$$

Klitschieff (1) gives the expression of that variable for the case of panels simply supported at their periphery. In this work, that value of g_{min} is accepted as the lower limit for I. In reality, a different value can be expected for the panel with fixed edges, but a theoretical consideration of the interaction of the plate and its reinforcements with the two contour conditions suggests that a similar distribution of deformation energy can occur between the two elements only slightly affected by the contour conditions.

The expression of (1) can be simplified and substituted by:

$$g_{min} = \frac{2 (1 - d')^2}{d^3 (5 - d')}$$

with a small error, provided $r > 3$.

Hence, it follows that the minimum value l compatible with a buckling mode can be calculated by:

$$I_{min} = \frac{b \cdot D}{E} g_{min}$$

Strictly speaking, the buckling made of the plate is governed by two factors:

--the rigidity of the reinforcements;
--the position of those reinforcements along the a dimension of the panel.

The torsional rigidity of the reinforcements can be disregarded without significant error (1) (3).

In accordance with these considerations, the equivalent thickness can be expressed thus:

$$h_{eq} = h + \frac{K_2 \cdot g^{1/2} \cdot h^{3/2}}{12\,(1 - V^2)^{1/2}} \quad \frac{1}{d\,b^{1/2}}$$

$$= u \cdot b + \frac{b \cdot g^{1/2} \cdot u^{3/2}}{d} \cdot M$$

$$u = h\,b$$

and

$$M = \frac{K_2}{12\,(1 - V^2)^{1/2}}$$

with $d = d_{as}$ or d_{si}, depending on the buckling mode selected.

4.7. Minimum weight solutions

The application of the calculation of variations to h_{eq} as a function of u, b, and d leads to the trivial solutions:

$$b = 0, \text{ de } (h_{eq})_u = 0$$

and

$$u = 0, \text{ de } (h_{eq})_b = 0$$

This suggests the selection of an other evaluation function:

$$w = h_{eq}/b$$

to minimize it, or the function

$$w = u + \frac{g^{1/2} \cdot u^{3/2}}{d} \cdot M$$

The real minimum can be obtained with:

$$u = u_f$$

$$g = g_{min}$$

With that the variable d is left as the only independent one.

Nevertheless, the analytical solution of w_{min} is not direct. The equation which results at d includes functions of the load (p, g) and of the geometry of the panel and its material (M).

The calculation of u and w is performed then as a function of d for the possible values of the rest of the variables of the problem: E, u, S, P, q, M.

4.8. Effective width of the panel

10

In the analysis of this work, and for the loads assumed, it appears logical to consider the whole plate as effectively associated with the reinforcements. This makes the procedure consistent with the first assemption made of a large number of reinforcements.

4.9. Safety factor

In this study, no type of safety margin or factor was taken into account in any of the components or stages. It must be admitted that the adoption of a safety coefficient is a decision of the designer, who takes it as a function of the special circumstances of each problem and by taking into account the reliability of the working up or the theoretical procedure which may have created the formulation which he uses.

By not adopting such a guarantee, the study is kept within the strictly theoretical field in which it was planned.

5. Results

The calculation of the relations proposed for w and u as a function of d can be extended to a practical limit of the variation of the other parameters p, q, M.

The graph representing those parameters is presented by way of illustration:

$$E = 10.3 \times 10^5 \text{ Kg cm}^2 \text{ (alumin).}$$
$$S_e = 2.100 \text{ Kg, cm}^2.$$
$$M = 0.050.$$
$$P = 1.400 \text{ Kg/cm}^2.$$
$$q = 0.7 \text{ Kg/cm}^2.$$

and it gives the values of u as a function of d.

The most prominent characteristics of the calculation can be summarized thus:

a. The two curves (u_{as} and u_{si} are very similar for any combination of values of (p, q, M).

b. The two curves intersect at point A, which divides the variation interval of d into two different zones:

$$\text{-- For } d < d_A, \; u_{as} < u_{si} \text{ and}$$
$$w_{as} < w_{si} \text{ - that is,}$$

the arrangement of the reinforcements for antisymmetric buckling determines a lighter structure.

$$\text{--for } d > d_A, \text{ opposite results are obtained.}$$

c. The least weight solution is obtained for d = o, that is to say, that the closer the reinforcements are, the less the weight of the resulting structure.

d. The location of the point of intersection A changes with the values of p and q:

11

--when p increases, A is displaced toward the right and the zone
of advantage for the antisymmetric arrangement of reinforcements
is larger;

--when q increases, point A is displaced to the lowest zone of d
to the benefit of the advantage of the symmetric arrangement of
reinforcements;

--the zone in which the point of intersection A occurs, for small
values of p and high values of q, is around the value d = 0.40;

--the absence of an absolute minimum for w with d different from d = o <u>/83</u>
leaves the designer with determining the value of the thickness of
the plate based on a permissible minimum value of u, that is, u = u_f;
the section of the corresponding reinforcements is determined by
g = g_{min} for each pair of values (p, q) and a geometry M.
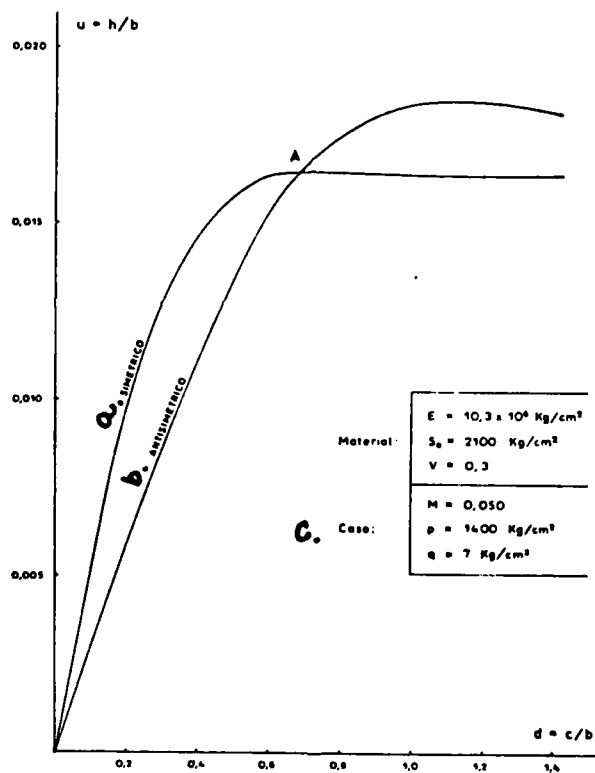
d. Figura 4.—Gráfico de selección.

a. Symmetrical [si]
b. Antisymmetric [as]
c. Case
d. Fig. 4. Selection graph (page 82)

6. Conclusions

The result obtained permits the preparation of design graphs to establish the
dimensions of a panel with minimal weight.

Once the g_{min} is known for a buckling mode, the minimum thickness of the
plate associated with this mode can be determined, as can the scaling of the

12

of the reinforcements which permit that g_{min}.

Since an absolute minimum value was not obtained as a single solution, no value remains in the set of relations obtained to obtain a panel with the least weight starting from u_f and g_{min}.

Finally, the result that a larger number of reinforcements provides a smaller total weight for the panel coincides with the conclusions reached in other works published on this same subject.

7. Recommendations

--It would be desirable to test the validity of the calculations included here with experimental data so as to ratify the practical validity of some of the assumptions of the proposed solution;

--The theory developed contemplates the position of the reinforcements as determining the buckling mode. This procedure ought to be extended to other arrangements of reinforcements which can hold practical interest;

--The final objective must continue to be the development of a simple mathematical expression of the type derived by Klitchieff (1). To that end, some transformations of the elastic deformation equations must be accepted.

8. References

1. KLITCHIEFF, J. M.: «On the stability of plates reinforced by ribs» Journal of Applied Mechanics, March 1949.

2. TIMOSHENKO, S.: «Theory of Elastic Stability. New York. McGraw-Hill, 1956.

3. GOMZA AND SEIDE, P.· NACA-TN-1710.

4. WAH, T. (editor), et al.. «A Guide for the Analysis of Ship Structures», U.S. Dept. of Commerce. 1960.

5. BLEICH, F.: «Buckling Strength of Metal Structures», N. York. McGraw-Hill, 1962.

6. JASPER AND HELLER: «Dynamic Loads in a M.T.B.», Transactions SNAME, N. York. 1961.

7. TIMOSHENKO, S., et al.: «Theory of Plates and Shells», N. York. McGraw-Hill, 1959.

8. DEN HARTOG, J. P.: «Advanced Strength of Materials», N. York. McGraw-Hill, 1952.

9. WEISS, S.: «Zur Stabilität von Platten mit excentrish angeordneten Quersteifen bei elactischer Einspannung der Längsräuder». Schiffstechnik. Mai 1965.

10. MAULBETSCH, J. L.: «Buckling of Compressed Rectangular Plates with built-in Edges», Journal of Applied Mechanichs. 1937

11. EVANS, J. H. (editor): «Ship Structural design Concepts», Cambridge, MD., Cornell Maritime Press, 1975.

12. HUGHES, O. F.: «Design of Laterally loaded plating · uniform pressure loads». Journal of Ship Research. N York. SNAME Jun. 1981.

Appendix A

Reworking of the Timoshenko expressions for deformations and stresses on a small panel due to lateral pressure loads with proportions which vary from 0.2 to 1.4.

1. Antisymmetric buckling:

$$c = \frac{3}{2} \cdot \frac{a}{r+2}$$

for an extreme panel. And:

$$c/b = 3 \, d/2$$

2. Symmetric buckling:

$$d = c, b = RA/(r+1)$$

The following can be expressed with these relations:

$$S_{bx} = K_s \cdot \frac{a}{2 \cdot u^2}$$

$$f = K_t \cdot \frac{q \cdot b^4}{h^3} \cdot \frac{(1 - V^2)}{32 \cdot E}$$

where the Ks include all the parameters which are not explicit.

In the following tables are listed the values of K_s and K_f, which have been *calculated from the Timoshenko curves* and by including the absent variables:

d	K_s	K_f	K_s	K_t
0.2	0.090	0.0081	0.040	0.0016
0.4	0.342	0.0117	0.160	0.0253
0.6	0.567	0.4205	0.342	0.1165
0.8	0.660	0.66	0.512	0.287
1.0	0.684	0.85	0.610	0.48
1.2	0.685	0.94	0.66	0.66
1.4	0.685	0.99	0.68	0.79

antisymmetric mode   symmetric mode

Appendix B

Resistance to buckling of a rectangular plate with its four edges fixed.

The buckling of a column with fixed ends subject to axial compression P is governed by the differential equation (2):

$$(f)_x{}^{t4} + K^2 \cdot (f)_x{}^{t2} = 0$$

with

$$K^2 = P/EI$$

The valid solutions have the form:

$$f = A \sin Kx + B \cos Kx + Cx + D$$

14

The contour conditions corresponding to fixed ends give:

$$(f)_\cdot = 0, \text{ para } x = 0 \text{ y } x = a ,$$

a being the length of the column and f, the deformation:

$$f = 0$$

at both extremes.

In this manner, two values for K can be found, one to define each of the two possible bucking modes:

a. Symmetric buckling

$$\sin \frac{Ka}{2} = 0$$

or

$$Ka = 2\pi \cdot n$$

b. Antisymmetric buckling:

$$\tan \frac{Ka}{2} = \frac{Ka}{2}$$

which, for n 3 can be approximated by:

$$Ka = (2n + 1)\pi$$

without appreciable error.

The first buckling mode defines (n-1) intermediate nodes and the second, (2n-1).

If we assume the panel composed of an infinite number of elementary columns, there will also be two buckling modes in the panel.

The deformation of the panel can be written:

$$f(x, y) = f_1(x) \cdot f_2(y)$$

where $f_1(x)$ has the form of the equation solved above, and $f_2(y)$ is assumed to be in the form of a single transverse wave of the symmetric type.

Once the deformation of the panel is known, its resistance to buckling can be calculated by energy methods (8).

In essence, these methods consist in equalizing the energy stored in the deformation with the work performed by the forces applied during the deformation until a position of neutral equilibrium is reached. The value (or values) of P which verify this condition of equilibrium is called the critical value of the compressive load.

For the symmetric type of deformation this critical value is:

$$S_{\cdot_r} = \frac{D \cdot \pi^2}{h \cdot b^2} \cdot K_{\cdot_r}$$

15

with

$$K_{cr} = \frac{4\,n^2}{RA^2} + \frac{4 \cdot RA^2}{n^2} + \frac{8}{3} = K_{si}$$

with

$$RA = a/b \,.$$

The same expression S_{cr} is verified for the antisymmetric buckling mode, but with:

$$K_{cr} = \frac{(2\,n + 1)^2}{RA^2}\,A + \frac{16}{3} \cdot \frac{RA}{(2\,n + 1)^2} - B + \frac{8}{3} = K_{si}$$

where the constants A and B have the values

$$A = \frac{4 + (Ka)^2}{-4 + (Ka)^2}$$

and

$$B = \frac{5\,(Ka)^2 - 36}{3\,(Ka) - 12}$$

with

$$(Ka) = (2\,n + 1)\,\pi \,.$$

For a large number of nodes, these coefficients do not differ much in value from:

$$A = 1$$
$$B = 5\,3,$$

which can be considered as sufficiently exact approximations in practice.