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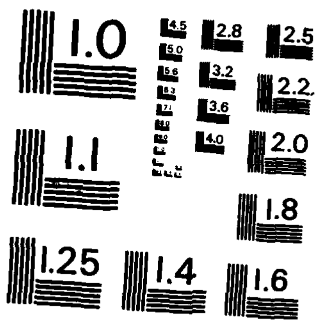

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NOTE ON SHEAR INTERACTION BETWEEN TWO FIBERS

S. B. Batdorf

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Application of shear lag theory to find the stress distribution in damaged composites has been handicapped by lack of knowledge concerning the effective shear stiffness of the matrix coupling adjacent fibers. This note gives an exact solution for a limiting case, namely two infinitely long rigid rods immersed in an infinite elastic matrix and displaced axially with respect to each other.

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Introduction

The shear lag approximation for stress distributions in damaged composites employed by Hedgepeth and Van Dyke [1], and later by others, employs an effective shear stiffness Gh to describe the role of the matrix in transferring direct stress from a fiber to a neighboring fiber.

In their treatment it was assumed that the force per unit length transferred between fibers 1 and 2 is given by

$$F' = A_f \frac{d\sigma}{dz} = Gh\ (w_2 - w_1)/d \tag{1}$$

where A_f is fiber area, σ is fiber stress, G is the matrix shear modulus, d is distance between fiber centers, and $w_2 - w_1$ is the relative axial displacement. Shear lag was originally devised as a technique for obtaining stress distributions in aircraft sheet-stiffener combinations. In that context d is the distance between rivet lines and h is sheet thickness. In extending the analysis to a 3D composite d remains well-defined, but loses physical significance, since $(w_2 - w_1)/d$ no longer serves as a direct measure of shear strain. But h is undefined, and loses physical significance as well.

Hedgepeth and Van Dyke said nothing about how to evaluate h. For their purposes its magnitude did not matter. They only used shear lag theory to evaluate stress concentration factors in cracked unidirectionally reinforced composites having fibers in square or hexagonal arrays, and they found that for this problem the results are independent of the value of h. However, the main reason for wanting to know stress concentration factors is to evaluate the probability of crack extension. This requires knowing not only the stress concentration factor but also the entire stress distribution in fibers bordering a crack, and stress distribution does depend on h. Thus the theoretical analysis of damage accumulation requires being able to evaluate h. While lack of knowledge of h is most serious in the case of 3D composites, it is also a handicap in the study of 2D composites.

Different authors have used different tactics in studying damage accumulation and comparing theory and experiment without knowing h. For instance, Rosen and Zweben [2,3] assumed the stress concentration factor is constant over virtually the entire overload region in a fiber neighboring a crack. In calculating the extent of this region they used an analysis in which a broken fiber is embedded in a cylinder having the properties of the matrix and both are surrounded by an infinite orthotropic elastic medium having the average properties of the composite [4]. Goree and Gross found the stress distribution in neighboring fibers using a modified form of shear lag, and in an early study [5] assumed in effect that h and d are equal. This cannot be true, since it implies that the load transferred between fibers is independent of their separation, whereas it is easily shown that the load transfer approaches infinity as the distance of closest approach goes to zero. In later studies Goree and colleagues have evaluated the effective shear transfer stiffness for a given composite by comparing the theoretically calculated and the experimentally measured crack opening displacement [6,7]. Batdorf and Ghaffarian [8] did not use h at all, but found the effective size of the overloaded region in fibers adjacent to cracks in a given composite by comparing a theory for ultimate strength with experimental data. The last two approaches can only be applied to composites that have already been fabricated and with which the appropriate experiments have been carried out.

It seems likely that in the 3D case, h depends on fiber size, fiber separation, and composite geometry [square, hexagonal or some other array]. An approximate value for it could probably be obtained using the methods of [4]. A more accurate value can be found experimentally using an electric analogue [9]. A thorough evaluation using the analogue will require some time, however. The present paper gives an exact analytical treatment of h for the limiting case of a single pair of fibers. This will provide a standard against which to measure the accuracy of the electric analogue approach and may shed some light on the value of h for a composite.

Theory

The theory for shear transfer in the case of two infinitely long cylinders bonded to an infinite elastic matrix closely parallels problems with a similar geometry in such fields as electrostatics, fluid mechanics, potential theory, and complex variables. Those already familiar with such problems may prefer to go directly to (4). Here we offer an elementary solution specifically relating to the problem in hand.

Consider a line in an infinite elastic medium. The line passes through the point (-a,o) in the x-y plane and extends to $\pm\infty$ in the z-direction. If it is loaded with an axial force F' per unit length, there will be a displacement w (x,y) in the surrounding elastic medium. Since the axial load transmitted from the inside to the outside of any coaxial cylindrical shell must be independent of radius,

$$F' = 2\pi r \tau = -2\pi r\, G \frac{dw}{dr} \qquad (2)$$

The negative sign is used so that w and F' will have the same sign convention.

From this

$$w(r) = -\frac{F'}{2\pi G} \ell n \left(\frac{r}{R_o}\right) \qquad (3)$$

Here r is the distance of the field point from the line load, and G is the shear modulus of the medium. R_o is a large distance, approaching infinity, at which w is essentially zero.

If an equal and opposite line load is applied at (a,o) the resulting axial displacement at (x,y) will be

$$w(x,y) = -\frac{F'}{2\pi G}\left(\ell n \frac{r1}{R_o} - \ell n \frac{r2}{R_o}\right)$$
$$= -\frac{F'}{2\pi G} \ell n \left(r_1/r_2\right) \qquad (4)$$

Here r_1 and r_2 are the distances of the field point (x,y) from the load points (-a,o) and (a,o) respectively.

The loci

$$r_1/r_2 = c = \text{constant} \qquad (5)$$

are cylinders, the traces of which in the x,y plane are as shown in Figure 1. On each cylindrical surface the displacement w is constant. Accordingly the elastic materials inside the cylinder can be replaced by a rigid rod without altering the stress distribution outside of the cylinder. The cylinders

$$r_1/r_2 = c \qquad (6a)$$

and

$$r_2/r_1 = c \qquad (6b)$$

are mirror images of each other with respect to the xz plane. They will represent for our purposes two equal and parallel rigid rods, mutually displaced axially a distance such that the load transferred per unit length is F', namely,

$$w_2 - w_1 = \frac{F'}{\pi G} \ln c \tag{7a}$$

or

$$F' = \pi G \,(w_2 - w_1)/\ln c \tag{7b}$$

We now seek to use the above analysis to evaluate h in Hedgepeth's equation (1). We note that in terms of the set-up in Figure 1, the radius of a circle described by (5) is

$$r = \left(\frac{2c}{c^2-1}\right) a \tag{8}$$

Two such circles have their centers at

$$x = \pm \frac{c^2 + 1}{c^2 - 1}\, a \tag{9}$$

Thus the distance between centers is

$$d = 2\frac{(c^2 + 1)}{(c^2 - 1)}\, a \tag{10}$$

Comparing (1) and (7b)

$$h/d = \pi/\ell n\ c \tag{11}$$

The distance of the closest approach can be shown to be given by

$$s = 2\left(\frac{c-1}{c+1}\right) a \tag{12}$$

Thus

$$s/d = \frac{(c^2 - 1)\ (c - 1)}{(c^2 + 1)\ (c + 1)} = \frac{(c-1)^2}{c^2+1} \tag{13}$$

(11) and (13) may be regarded as parametric equations relating h/d to s/d. Alternatively (13) may be solved for c as a function of s/d and the result substituted in (11), with the somewhat cumbersome result

$$h/d = \pi/\ell n\ [(1 \pm \sqrt{1-(1-s/d)^2}\ )/(1-s/d)] \tag{14}$$

This relationship is shown graphically in Figure 2.

Discussion

It is obvious from Fig. 2 that h/d is not a constant, but varies with the ratio s/d. As $s \rightarrow 0$, $h/d \rightarrow \infty$ as it should, and for $s/d \rightarrow 1$, $h/d \rightarrow 0$. For $s/d \cong 0.2$, the value for a composite having a fiber volume fraction of 0.5, $h/d \cong 4.5$.

Equation (14) represents an exact solution of the 2-fiber problem. The approach employed cannot be extended to a larger number of circular fibers because for that case the w = constant contours are no longer circular. The electric analogue approach can be used to obtain very accurate solutions for arbitrarily chosen values of the geometric parameters, but of course does not yield an analytic solution such as that given here for the two-fiber case.

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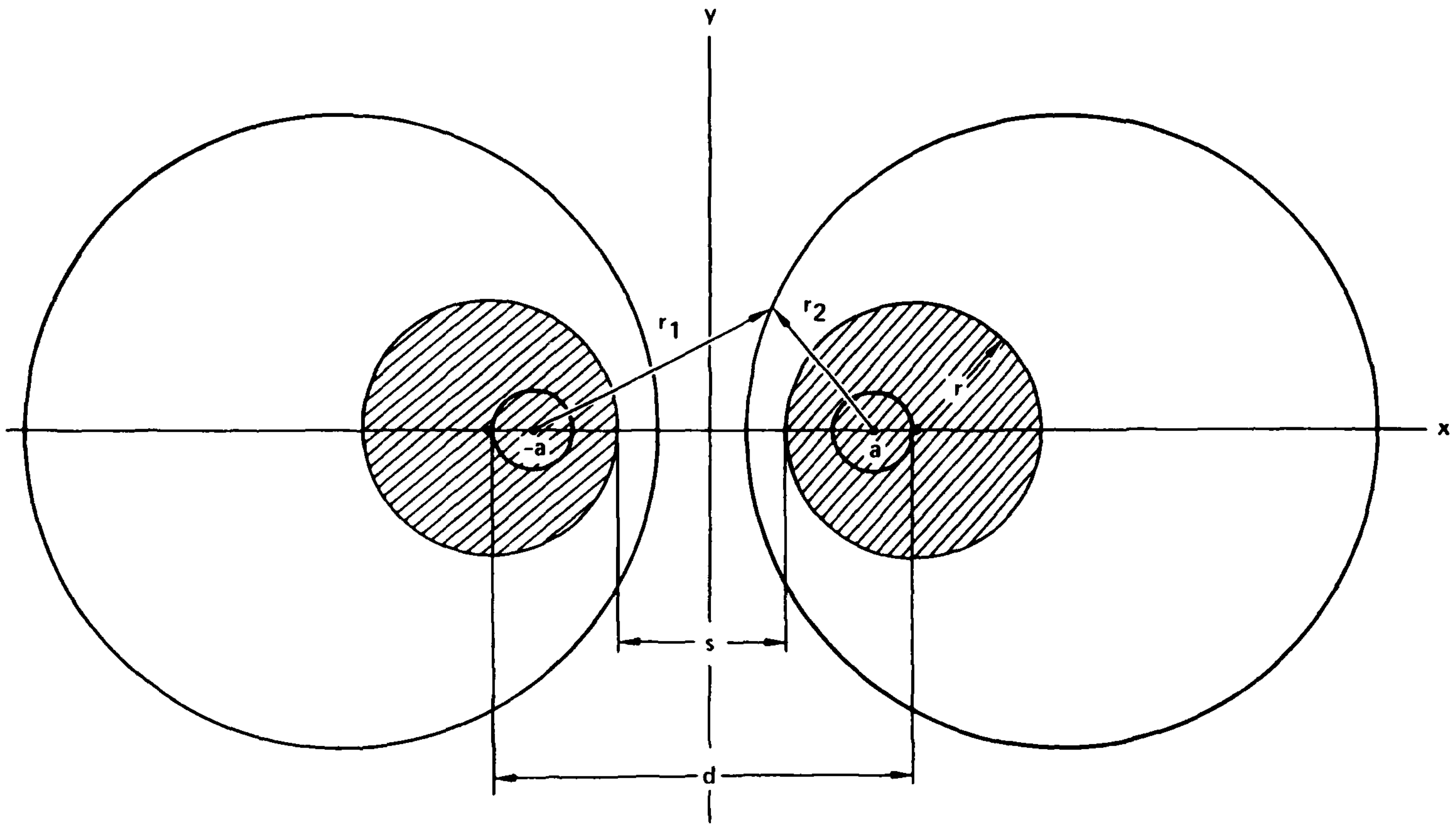

Fig. 1 - Shaded regions represent two parallel rigid rods in infinite elastic medium subjected to equal and opposite axial loads. Circles are contours of constant axial displacement.

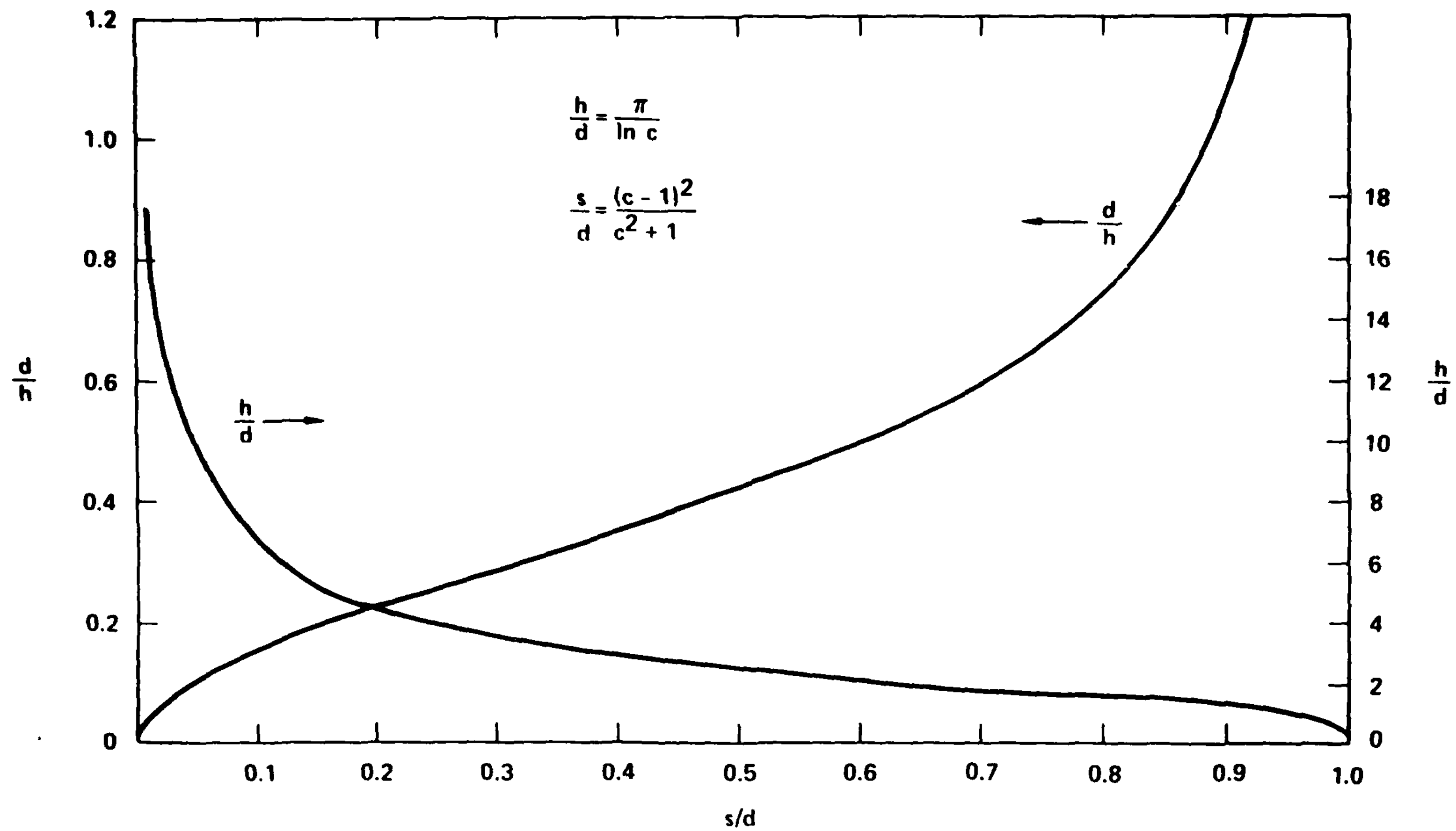

Fig. 2 - Plot of h/d vs. s/d

References

1. Hedgepeth, J., and VanDyke, P., "Local Stress Concentrations in Imperfect Filamentary Composite Materials," J. of Composite Materials, Vol. 1, 1967, pp. 294-309.

2. Rosen, B.W., "Tensile Failure of Fibrous Composites," AIAA J., Vol. 2, No. 12, 1964, pp. 1985-1991.

3. Zweben, C., and Rosen, B.W., "A Statistical Theory of Composite Strength with Application to Composite Materials," J. Mech. Phys. Solids, Vol. 18, pp. 189-206.

4. Dow, N.F., "Study of Stresses Near a Discontinuity in a Filament - Reinforced Composite Metal," Space Mechanics Memo #102, G.E. Space Sciences Lab, January, 1961.

5. Goree, J.G., and Gross, R.S., "Stresses in a Three-Dimensional Unidirectional Composite Containing Broken Fibers," Eng. Fr. Mech., Vol. 13, 1980, pp. 395-405.

6. Goree, J.G., and Gross, R.S., "Analysis of a Unidirectional Composite Containing Broken Fibers and Matrix Damage," Eng. Fr. Mech., Vol. 13, 1980, pp. 563-578.

7. Goree, James G., Dharaim, L.R., and Jones, Walter F., "Mathematical Modelling of Damage in Unidirectional Composites," NASA Contractor Report 3453, August, 1981.

8. Batdorf, S.B. and Ghaffarian, Reza, "Tensile Strength of Unidirectionally Reinforced Composites-II," J. Reinf. Pl. and Comp., Vol. 1, (April, 1982).

9. Batdorf, S.B., "Experimental Determination of Stresses in Damaged Composites Using an Electric Analogue." To be published in J. Appl. Mech.