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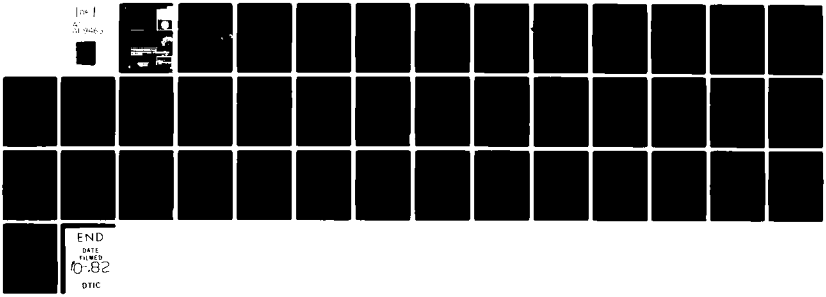

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TRAVEL TIME MODELS FOR AUTOMATED STORAGE/RETRIEVAL SYSTEMS

(12)

by

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TRAVEL TIME MODELS FOR AUTOMATED STORAGE/RETRIEVAL SYSTEMS[1]

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ABSTRACT

Travel time models are developed for automated storage/retrieval (AS/R) machines. The S/R machine travels simultaneously horizontally and vertically as it moves along a storage aisle. For randomized storage conditions expected travel times are determined for both single and dual command cycles. Alternative input/output (I/O) locations are considered. Additionally, various dwell point strategies for the storage/retrieval machine are examined.

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INTRODUCTION

Over the past years, interest in automated storage/retrieval (AS/R) systems has grown tremendously. Such systems have increased considerably in number and probably will continue to do so in the future. The rapid growth in the interest in AS/R systems can be attributed to such benefits as lower building and land cost, labor savings, reduced inventory levels, and an improved throughput level, among others [13].

Much of the early work done to analyze AS/R systems is based on simulation. Bafna [1] developed a design package where the optimum configuration is determined by using simulation in conjunction with a search procedure. A similar approach is presented by Koening [8], where the search for the "optimum" configuration is limited to certain values of the design variables specified by the user. Using simulation in designing such systems has the obvious drawback of added computational effort, plus the possibility of stopping with a sub-optimum solution. The design variables for AS/R systems typically can take on a wide range of values. Thus, simulating each feasible design long enough to assume steady-state behavior is expensive computationally and undesirable at an early design stage where the user is mainly interested in obtaining bench-mark solutions.

A design package based on analytical techniques was developed by White and Bozer [11]. Based on Zollinger's [12] cost model, the mathematical characteristics and relationship of the cost functions were defined corresponding to various elements in the system. Subsequently, the minimum-cost design was determined by performing a Fibonacci search over the number of aisles in the system.

Karasawa, Nakayama, and Dohi [7] developed a cost model of an AS/RS. They considered only single command cycles. Their total cost function included terms for storage racks, S/R machines, building, and land. They assumed the rack cost per opening to be a linear function of the height of the system; the S/R cost per machine was assumed to be constant; and building cost and land cost were assumed to be linearly proportional to the floor area. They solved the nonlinear programming problem using Lagrangian multipliers and then obtained the best neighborhood integer solution.

A number of studies have been performed based on simulation; but they have emphased analysis rather than design. As examples, some studies have addressed the throughput/performance of AS/R systems under different operating and/or storage policies assuming that the design (configuration) of the system is known. Among such studies are those given by [2], [4], [9], and [10].

In this paper, closed-form expressions are developed to determine the expected travel time associated with each trip based on single and dual command cycles. An immediate application of the travel time expressions is in designing AS/R systems and in measuring the performance of AS/RS installations.

EXISTING METHODS

Two analytical approaches have been described for determining the expected travel time. The first approach is given by Graves, et al. in [4] and [6]. Their approach is limited by the need to assume the rack is square-in-time. That is, the dimensions of the rack and the vertical and horizontal speeds of the storage/retrieval (S/R) machine are such that the time to reach the row most distant from the input/output (I/O) point equals the time to reach the most distant column, given that the I/O point is located at the lower left-hand corner of the rack.

Empirical experience indicates that the optimum design for AS/R systems frequently is not square-in-time. For this reason it is desirable to determine the expected travel time for a rack that is not necessarily square-in-time.

An alternate method of estimating the expected travel time is given by the AS/RS Product Section of The Material Handling Institute, Inc. (MHI) [13]. The single command expected travel time is essentially taken to be equal to twice the time required to travel from the I/O point to the storage slot at the center of gravity of the rack. As can be seen in Figure 1, the single command expected travel time is equal to the round-trip travel time between the I/O point and point A. In order to determine the expected round-trip travel time for dual command cycles, it is assumed that a storage is performed at point A and a retrieval is performed at point B, which is assumed to be located at three-fourths of the distance (horizontally and vertically) from the I/O point. As illustrated in Figure 2, the expected round-trip travel time for dual command cycles is equal to the time to travel from the I/O point to A, travel from A to B, and then return to the I/O point.

The MHI travel time model may not provide an accurate representation of the travel of the S/R machine. As an example, when the rack is square-in-time and randomized storage is used, the MHI model underestimates by 25 percent the expected single command travel time.

Another aspect of the MHI model that deserves examination concerns the specification of the retrieval point involved in the dual command cycle. Given that the S/R machine has traveled to the first opening for storage, there is no reason to assume that, in the long-run, the retrieval point will be at the assumed location. Since the retrieval point is assumed to

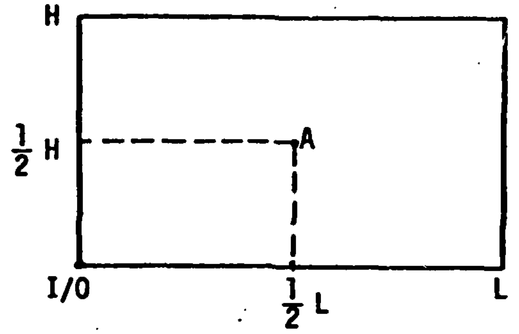

Figure 1: Determining the single command expected travel time using the MHI method.

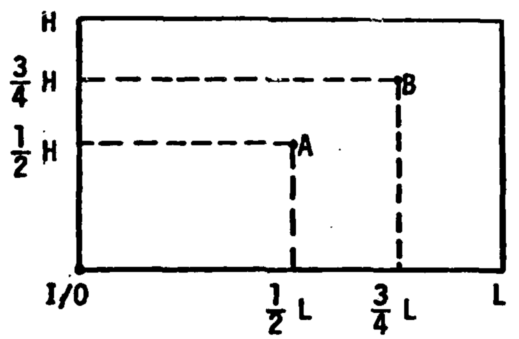

Figure 2: Determining the dual command expected travel time using the MHI method.

be three-fourths of the distance from the I/O point, the expected time required to return to the I/O point will be overestimated. However, as noted above, the expected time to travel from the I/O point to the storage location will be underestimated. Hence, it remains to determine if the total dual command travel time is overestimated or underestimated.

To facilitate the determination of the travel time for an S/R machine, the following notation is introduced.

N = the total number of openings in the rack.

t_{oi} = one-way travel time between the I/O point and the i^{th} opening ($t_{oi} = t_{io}$).

t_{ij} = one-way travel time between the i^{th} opening and the j^{th} opening ($t_{ij} = t_{ji}$).

$E(\mathcal{SC})$ = expected single command round-trip travel time.

$E(\mathcal{DC})$ = expected dual command travel time.

The expected single command travel time can be computed from the following expression.

$$E(\mathcal{SC}) = \frac{1}{N} \sum_{i=1}^{N} 2t_{oi} \tag{1}$$

The expected dual command travel time is given by

$$E(\mathcal{DC}) = \frac{2}{N(N-1)} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} (t_{oi} + t_{ij} + t_{jo}) \tag{2}$$

Computing t_{oi} for each opening may become time consuming for large N. From the double summation involved in Equation 2, if N = 400, then 159,600 terms must be considered.

A STATISTICAL APPROACH

In this section closed-form expressions and the methodology used to develop the travel times will be presented. The following assumptions are made.

1. The rack is considered to be a continuous rectangular pick face rather than an array of discrete openings.
2. The S/R machine operates either on a single or dual command basis, i.e., multiple stops in the aisle are not allowed.
3. The rack length and height, as well as the S/R machine speed in the horizontal and vertical directions, are known.
4. The S/R machine travels simultaneously in the horizontal and vertical directions.
5. In calculating the travel time, constant velocities are used for horizontal and vertical travel.
6. Randomized storage is used. That is, any point within the pick face is equally likely to be selected for storage or retrieval.
7. The I/O point is located at the lower left-hand corner of the pick face. Every trip originates and terminates at the I/O point.
8. Pick-up and deposit (P/D) times associated with load handling are ignored. The P/D times is generally independent of the rack shape and the travel velocity of the S/R machine.

Furthermore, given the load characteristics, the P/D time is usually deterministic. Hence, it is a straightforward matter to include the P/D time after the average travel time has been computed.

As stated earlier, the pick face is a continuous rectangle with known dimensions that can vary from one application to another. To "standardize" the pick face, let

s_h = speed of the S/R machine in the horizontal direction,

s_v = speed of the S/R machine in the vertical direction,

L = length of the rack, and

H = height of the rack.

Now, let t_h represent the horizontal travel time required to go to the farthest column from the I/O station. Likewise, let t_v denote the vertical travel time required to go to the farthest row (level). Then, by definition:

$$t_h = L/s_h$$

$$t_v = H/s_v$$

Let,

$$T = \max \{t_h, t_v\}$$

and

$$b = \min \{t_h/T, t_v/T\}$$

which implies that $0 \leq b \leq 1$. In subsequent discussions, b is referred to as the "shape factor". Note, if the rack dimensions and travel velocities are such that $t_h = t_v$, then $b = 1$ and the rack is said to be "square-in-time". Without loss of generality, assume that $T = t_h$. That is, $b = t_v/T$.

We will consider first the single command travel time. Let the storage (or retrieval) point be represented by (x, y) in time, where $0 \le x \le 1$ and $0 \le y \le b$. Travel from $(0, 0)$ to (x, y), say t_{xy}, will be

$$t_{xy} = \max(x, y)$$

Now, let $F(z)$ denote the probability that travel time to (x, y) ' less than or equal to z. That is,

$$F(z) = Pr(t_{xy} \le z)$$

Assuming the x, y coordinates are independently generated

$$F(z) = Pr(x \le z) \cdot Pr(y \le z)$$

Furthermore, for randomized storage the coordinate locations are assumed to be uniformly distributed. Thus,

$$Pr(x \le z) = z$$

and

$$Pr(y \le z) = \begin{cases} z/b & \text{if } 0 < z \le b \\ 1 & \text{if } b \le z \le 1 \end{cases}$$

Hence,

$$F(z) = \begin{cases} z^2/b & \text{for } 0 \le z \le b \\ z & \text{for } b < z \le 1 \end{cases}$$

Therefore,

$$f(z) = \begin{cases} 2z/b & \text{for } 0 \le z \le b \\ 1 & \text{for } b < z \le 1 \end{cases}$$

Letting E(*SC*) denote the expected travel time under single command for the normalized rack, then

$$\frac{1}{2} E(SC) = \int_{z=0}^{1} z\, f(z)dz = \int_{z=0}^{b} z\, f(z)dz + \int_{z=b}^{1} z\, f(z)dz$$

$$\frac{1}{2} E(SC) = \int_{z=0}^{b} z\, \frac{2z}{b}\, dz + \int_{z=b}^{1} z\, dz$$

$$\frac{1}{2} E(SC) = \frac{1}{6}\, b^2 + \frac{1}{2}$$

Therefore,

$$E(SC) = \frac{1}{3} b^2 + 1 \tag{3}$$

Next, consider the dual command cycle. By definition, each dual command cycle involves two random locations; one representing the storage point, the other representing the retrieval point. To analyze the expected travel time between the two points, recall that any point is represented as (x, y) in time and $0 \le x \le 1$ and $0 \le y \le b$. Let t_B be the time required to travel between the storage and retrieval locations. Also, let

$$F(z) = Pr(t_B \le z)$$

$$F(z) = Pr(|x_1 - x_2| \le z)\ \ Pr(|y_1 - y_2| \le z)$$

where (x_1, y_1) and (x_2, y_2) are the two random points. First consider the term $\Pr(|x_1 - x_2| \le z)$. Recall, if $x_{(1)}, \ldots, x_{(n)}$ are the order statistics of a sample $x_1, \ldots, x_n$, from a population with probability distribution function $f(x)$ and cumulative distribution function $F(x)$, the difference $x_{(n)} - x_{(1)}$ is called the sample range R. Letting $H(r) = \Pr(R \le r)$, in [7] it is shown that

$$H(r) = n \int_{v=-\infty}^{\infty} f(v)[F(v + r) - F(v)]^{n-1}\, dv \tag{4}$$

Differentiating H(r) yields the probability density function

$$h(r) = n(n - 1) \int_{v=-\infty}^{\infty} [F(v + r) - F(v)]^{n-2} f(v)\, f(v + r)\, dv \tag{5}$$

Recall that $0 \le x_1 \le 1$ and $0 \le x_2 \le 1$. With $n = 2$,

$$f(x) = \begin{cases} 1 & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

for

$$F(x) = \begin{cases} 0 & x < 0 \\ x & 0 \le x \le 1 \\ 1 & x > 1 \end{cases}$$

It should be noted that f(x) and F(x) are derived based on the assumption that under randomized storage x_1 and x_2 are uniformly distributed in the interval (0, 1). Since $0 \le R \le 1$, then $0 \le v \le 1 - r$. Letting $n = 2$, from Equation 5

$$h(r) = z(1 - r) \tag{6}$$

Letting

$$F_x(z) = Pr(|x_1 - x_2| \le z)$$

then

$$F_x(z) = Pr(0 \le R \le z)$$

or

$$F_x(z) = 2 \int_0^z (1 - r)dr$$

Therefore,

$$F_x(z) = 2z - z^2 \tag{7}$$

Next consider the term $Pr(|y_1 - y_2|) \le z$, where $0 \le y_i \le b$ for $i = 1, 2$. For this case

$$F(y) = \begin{cases} 0 & y < 0 \\ y/b & 0 \le y \le b \\ 1 & y > b \end{cases}$$

Hence,

$$f(y) = \begin{cases} 1/b & 0 \le y \le b \\ 0 & \text{otherwise} \end{cases}$$

Substituting n = 2 and rewriting Equation 5 gives

$$h(r) = 2 \int_{v=0}^{b-r} f(v)f(v + r)dv$$

$$h(r) = 2 \int_{v=0}^{b-r} \frac{1}{b^2}\, dv$$

$$h(r) = \frac{2}{b^2} (b - r)$$

Letting

$$F_y(z) = Pr(|y_1 - y_2|) \le z \quad \text{where} \quad 0 \le z \le b$$

then

$$F_y(z) = Pr(0 \le R \le z)$$

or

$$F_y(z) = \frac{2}{b^2} \int_0^z (b - r)dr$$

Therefore,

$$F_y(z) = \begin{cases} \dfrac{2z}{b} - \dfrac{z^2}{b^2} & \text{for } 0 \le z \le b \\ 1 & \text{for } b < z \le 1 \end{cases} \tag{8}$$

Recall that $F(z) = Pr(t_B \le z)$ where t_B represents the "travel between" time. Based on Equations 7 and 8, F(z) can be written as:

$$F(z) = F_x(z)\ F_y(z)$$

or

$$F(z) = \begin{cases} (2z - z^2)\left(\frac{2z}{b} - \frac{z^2}{b^2}\right) & \text{for } 0 \le z \le b \\ (2z - z^2) & \text{for } b < z \le 1 \end{cases}$$

Therefore

$$f(z) = \begin{cases} (2 - 2z)(2z/b - z^2/b^2) + (2z - z^2)(2/b - 2z/b^2) & \text{if } 0 \le z \le b \\ 2 - 2z & \text{if } b < z \le 1 \end{cases}$$

Letting $E(TB)$ denote the expected travel time between the two randomly selected points,

$$E(TB) = \int_0^1 z\ f(z)dz = \int_0^b z\ f(z)dz + \int_b^1 z\ f(z)dz$$

or

$$E(TB) = \frac{1}{3} + \frac{1}{6}b^2 - \frac{1}{30}b^3 \tag{9}$$

Let $E(DC)$ denote the expected travel time for a complete dual command cycle with a normalized rack. Then, by definition,

$$E(DC) = E(SC) + E(TB)$$

Hence,

$$E(DC) = \frac{4}{3} + \frac{1}{2} b^2 - \frac{1}{30} b^3 \tag{10}$$

Using Equation 3 and 10, the expected single command and dual command travel times can be calculated for a normalized rack.

<u>EXAMPLE</u>: Suppose the rack dimensions and the S/R machine speed is such that

L = 352 ft.

H = 88 ft.

s_h = 400 fpm

s_v = 90 fpm

Using the approach developed earlier, we have

$$t_h = L/s_h = 352/400 = 0.88 \text{ mins.}$$

and

$$t_v = H/s_v = 88/90 = 0.9778 \text{ mins.}$$

Therefore,

$$T = \max\{0.88, 0.9778\} = 0.9778$$

and

$$b = 0.88/0.9778 = 0.900$$

Hence, the "normalized" rack is 0.900 mins. long in the vertical direction, and 1.0 mins. long in the horizontal direction. Using Equations 3 and 10:

$$E(SC) = \frac{1}{3} b^2 + 1 = 1.27 \text{ mins.}$$

$$E(DC) = \frac{4}{3} + \frac{1}{2} b^2 - \frac{1}{30} b^3 = 1.7140 \text{ mins.}$$

To obtain the results corresponding to the original rack, we "denormalize" the above travel times to obtain

$$E(SC) = \dot{E}(SC)\ T = 1.2418 \text{ mins.}$$

$$E(DC) = \dot{E}(DC)\ T = 1.6759 \text{ mins.}$$

COMPARISON WITH OTHER METHODS

As mentioned previously, the MHI approach underestimates the expected single command cycle time and may overestimate or underestimate the dual command cycle time. First consider the single command cycle time. For any normalized rack with $0 \le b \le 1$, the MHI procedure yields MHI(SC) = 0.50 (2) = 1.00 minute. Using Equation 3 to determine E(SC), the values provided in Table 1 are obtained.

Table 1. Comparison of Single Command Travel Times for a Normalized Rack

b	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
E(SC)	1.000	1.003	1.013	1.030	1.053	1.083	1.120	1.163	1.213	1.270	1.333
MHI(SC)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
% DIFF	0.000	0.300	1.280	2.910	5.030	7.660	10.710	14.020	17.560	21.260	25.000

It is interesting to note that the greatest difference occurs for b = 1, or when the rack is square-in-time. Also, the difference approaches zero as b approaches zero, i.e., storage is one-dimensional.

Next consider the dual command cycle time. For any normalized rack with $0 \le b \le 1$, the MHI method yields MHI(DC) = 0.75 (2) = 1.50 minutes. Using

Equation 10 to determine $E(\mathcal{DC})$, the comparison given in Table 2 was performed.

Table 2. Comparison of Dual Command Travel Times for a Normalized Rack

b	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
$E(\mathcal{DC})$	1.333	1.338	1.353	1.377	1.411	1.454	1.506	1.567	1.636	1.714	1.80
$MHI(\mathcal{DC})$	1.500	1.500	1.500	1.500	1.500	1.500	1.500	1.500	1.500	1.500	1.500
% DIFF	-12.530	-12.110	-10.86	-8.93	-6.310	-3.160	0.400	4.280	8.310	12.490	16.670

In Table 2, a minus sign indicates over-estimation, while a positive percent difference indicates under-estimation of the dual command cycle time. Notice, the maximum under-estimation is 16.67%, while the minimum is 0.40% (for b = 0.60).

From Equations 3 and 10, it can be shown that a square-in-time rack minimizes travel time. Graves, et al. in [4] and [5] developed expressions for a square-in-time rack. Using a different approach, they obtained a value of 7/15 for the expected travel time between two randomly selected points. Letting b = 1 in Equation 9 yields the same value.

Adjusting for Acceleration and Deceleration Rates

Some error is introduced when the acceleration/deceleration rate of the S/R machine is neglected. Suppose values for s_h, s_v and the acceleration/deceleration are known. The total distance required for the S/R machine to start from a stationary position, accelerate to full speed and immediately decelerate down to a full stop can be computed. Suppose the distance is d_1.

Let the time corresponding to d_1 be Δ time units. The impact of neglecting acceleration/deceleration is depicted in Figure 3(a). For any distance less than d_1, the error is less than Δ, and for any distance greater than or equal to d_1, the error will be Δ time units. The total amount of error introduced is represented by the cross-hatched area. From Figure 3(a), $t_1 = t_h$ (or t_v) while t_2 is the time required to travel to the farthest column (or row) from the I/O point, including the acceleration and deceleration rates. One method for reducing the amount of error is to calculate b, $E(\mathcal{SC})$ and $E(\mathcal{DC})$ as before but to use t_2 (instead of t_1) in "denormalizing" the rack; the amount of error will be reduced to the cross-hatched area shown in Figure 3(b). Assuming the acceleration/deceleration rates in the horizontal and vertical directions are approximately equal, t_2 will not be significantly different when computed over L or H.

ALTERNATIVE CONFIGURATIONS FOR THE I/O POINT

In the previous discussion, it was assumed that the I/O point is located at the lower left-hand corner of the rack and every trip originates and terminates at the I/O point. In this section the assumption is relaxed; three alternative configurations are analyzed; and the corresponding expected travel time expressions are developed.

Input and Output at Opposite Ends of the Aisle

For the first configuration to be considered assume that all storage orders are initiated at the input station while all retrieval orders are terminated at the output station. It is also assumed that after each single command storage, the S/R machine returns to the input point; and after each retrieval (which may be initiated at the input point or the output point)

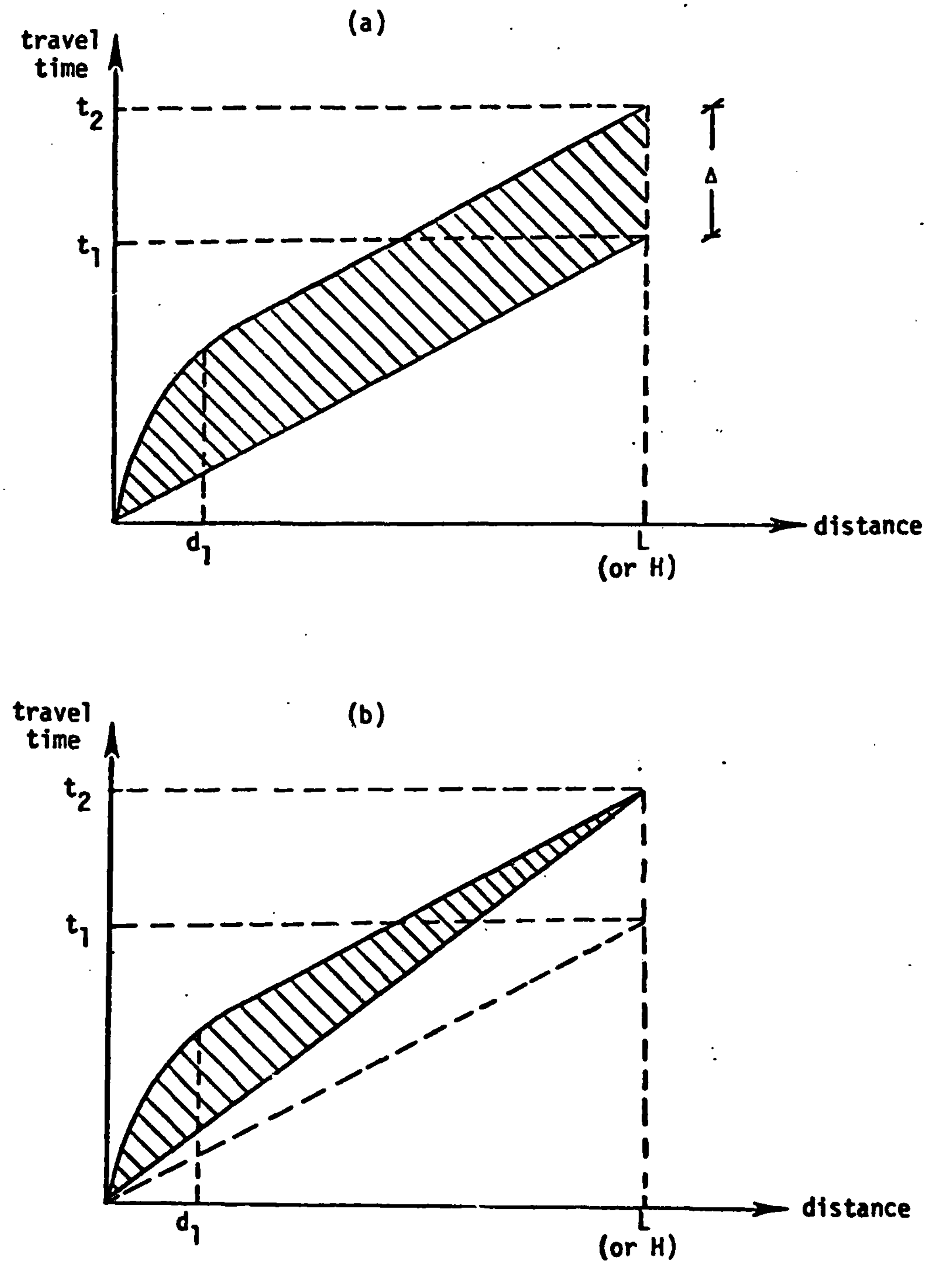

Figure 3: Estimation error generated by neglecting the acceleration/deceleration rate.

the S/R machine travels to the output station and remains there. Consequently, at the time the S/R machine starts a dual cycle, it will be at the input station if the previous trip was a storage; otherwise it will be at the output station. In the first case, the S/R machine can start the dual trip immediately. In the latter case, before the S/R machine starts the dual cycle, it must travel to the input station. The travel of the S/R machine is summarized in Table 3.

Recall, the expected travel time for a single command cycle when the I/O is at (0, 0) is

$$E(SC) = \frac{1}{3} b^2 + 1$$

Dividing the above expression by 2 gives the expected one-way travel time $E(V)$, i.e., the expected travel time from any corner of the rack to a randomly selected point or vice versa. Hence

$$E(V) = \frac{1}{6} b^2 + \frac{1}{2}$$

Also, recall the expected travel time between two randomly selected points is

$$E(TB) = \frac{1}{3} + \frac{1}{6} b^2 - \frac{1}{30} b^3$$

Note that, $E(TB)$ will remain constant regardless of the locations of the input and output stations.

Assume that α% of all the trips are single command cycles and $(1 - \alpha)$% are dual command cycles. Furthermore, since the total number of storages should be equal to the total number of retrievals in the long-run, assume that 50% of all single command cycles are storages, while the remaining 50% are retrievals. Also, assume that all orders are statistically

Table 3. S/R Machine Travel

When the previous cycle was	To perform	S/R machine travels
Single command storage	Single command storage	I→S→I
Single command storage	Single command retrieval	I→R→O
Single command storage	Dual command	I→S→R→O
Single command retrieval	Single command storage	O→I→S→I
Single command retrieval	Single command retrieval	O→R→O
Single command retrieval	Dual command	O→I→S→R→O
Dual command	Single command storage	O→I→S→I
Dual command	Single command retrieval	O→R→O
Dual command	Dual command	O→I→S→R→O

I: input location
O: output location
S: storage location
R: retrieval location

independent. Hence, the probability that a given order is a storage is $\alpha/2$, while the probability that it is a retrieval or dual command order is $1 - (\alpha/2)$.

The above implies that, on the average, $\alpha/2\%$ of the trips will terminate at the input station, and $1 - (\alpha/2)\%$ of the trips will terminate at the output station. If the S/R machine is at the input station, then the expected travel time for a:

(1) storage is equal to $2\ E(V)$

(2) retrieval is equal to $2\ E(V)$

(3) dual trip is equal to $2\ E(V) + E(TB)$

If the S/R machine is at the output station, then the expected travel time for a:

(1) storage is equal to $2\ E(V) + K$

(2) retrieval is equal to $2\ E(V)$

(3) dual trip is equal to $2\ E(V) + E(TB) + K$

where K is the fixed travel time from the output to the input station $(O \to I)$.

Defining an operation as a storage or a retrieval, the expected travel time per operation, say $E_1(T)$ will be:

$$E_1(T) = \frac{\alpha}{2}\left\{\frac{\alpha}{2}\ 2\ E(V) + \frac{\alpha}{2}\ 2\ E(V) + \frac{1}{2}(1-\alpha)[2\ E(V) + E(TB)]\right\}$$

$$+ \left[1 - \frac{\alpha}{2}\right]\left\{\frac{\alpha}{2}(K + 2\ E(V)) + \frac{\alpha}{2}\ 2\ E(V) + \frac{1}{2}(1-\alpha)[K + 2\ E(V) + E(TB)]\right\}$$

which reduces to

$$E_1(T) = \frac{\alpha}{2}\left\{E(V)(1 + \alpha) + \frac{1}{2}E(TB)(1 - \alpha)\right\}$$

$$+ \left[1 - \frac{\alpha}{2}\right]\left\{E(V)(1 + \alpha) + \frac{1}{2}E(TB)(1 - \alpha) + \frac{1}{2}K\right\}$$

Hence,

$$E_1(T) = E(V)\quad(1 + \alpha) + \frac{1}{2}E(TB)(1 - \alpha) + \frac{1}{2}K\left(1 - \frac{\alpha}{2}\right) \tag{11}$$

It is instructive to note that setting $\alpha = 0$ in the above equation will yield:

$$E_1(T) = E(V) + \frac{1}{2}E(TB) + \frac{1}{2}K \tag{12}$$

which is intuitively correct because each dual trip involves two operations (a storage and a retrieval) and total expected travel time is $E(V) + E(TB) + E(V) + K = 2\,E(V) + E(TB) + K$.

Setting $\alpha = 1$ in Equation 9 gives

$$E_1(T) = 2\,E(V) + \frac{1}{4}K \tag{13}$$

which is intuitively correct because 50% of all the orders are storages and the remaining 50% are retrievals; the S/R machine travels from the output station to the input station only if a retrieval is followed by a storage. Note that the S/R machine never travels from the input station directly to the output station.

Suppose the S/R machine is not required to return to the input station after a storage. Instead, assume the following: the S/R machine remains at the point of storage, awaiting the next order. If the next order is a storage or dual command order, the S/R machine returns to the input station. Otherwise, it travels directly to the retrieval point. The travel of the S/R machine is summarized in Table 4.

Table 4. S/R Machine Travel

When the previous cycle was	To perform	S/R machine travels
Single command storage	Single command storage	$X_1 \to I \to X_2$
Single command storage	Single command retrieval	$X_1 \to X_2 \to O$
Single command storage	Dual command	$X_1 \to I \to X_2 \to X_3 \to O$
Single command retrieval	Single command storage	$O \to I \to X$
Single command retrieval	Single command retrieval	$O \to X \to O$
Single command retrieval	Dual command	$O \to I \to X_1 \to X_2 \to O$
Dual command	Single command storage	$O \to I \to X$
Dual command	Single command retrieval	$O \to X \to O$
Dual command	Dual command	$O \to I \to S \to R \to O$

I: input location

O: output location

X: random location in the rack

Let I, O and X denote the states where the S/R machine is at the input station, output station and at some point within the rack, respectively. Then the following transition matrix can be constructed:

	I	O	X
I	0	$1 - \frac{\alpha}{2}$	$\frac{\alpha}{2}$
O	0	$1 - \frac{\alpha}{2}$	$\frac{\alpha}{2}$
X	0	$1 - \frac{c}{2}$	$\frac{\alpha}{2}$

For example, if the S/R machine is in state I, then it will be in state X at the beginning of the next trip only if the current order is a storage. From Table 4 and the above transition matrix it is clear that the S/R machine will never initiate a trip from the input station; after each storage the S/R machine remains within the rack, otherwise it remains at the output station. The steady-state probabilities are as follows:

$$P_I = 0$$
$$P_X = \frac{\alpha}{2}$$
$$P_O = 1 - \frac{\alpha}{2}$$

The steady-state probabilities could be obtained directly from Table 4. However, with complex operating procedures, the construction of a transition matrix may simplify the analysis.

Setting $\alpha = 0$ gives $P_O = 1$ and $P_I = P_X = 0$ which is intuitively correct; if only dual command cycles are performed, the next trip will start from the output station. On the other hand, when $\alpha = 1$, $P_X = P_0 = \frac{1}{2}$ and $P_I = 0$ as expected, since the number of storages is assumed to be equal to the number of retrievals in the long-run.

At the start of a cycle, if the S/R machine is at the output station, then the expected travel time for a:

(1) single command storage is equal to $E(V) + K$

(2) single command retrieval is equal to $2\,E(V)$

(3) dual command is equal to $2\,E(V) + E(TB) + K$.

If the S/R machine is within the rack, then the expected travel time for a:

(1) storage is equal to $2\,E(V)$

(2) retrieval is equal to $E(TB) + E(V)$

(3) dual trip is equal to $3\,E(V) + E(TB)$

Hence, the expected travel time per operation, $E_1(T)$, will be:

$$E_1(T) = \left[1 - \frac{\alpha}{2}\right]\left\{\frac{\alpha}{2}\,(K + E(V) + \frac{\alpha}{2}\quad 2\,E(V) + \frac{1}{2}\,(1 - \alpha)(2\,E(V) + E(TB) + K)\right\}$$

$$+ \frac{\alpha}{2}\left\{\frac{\alpha}{2}\quad 2\,E(V) + \frac{\alpha}{2}\,(E(TB) + E(V)) + \frac{1}{2}\,(1 - \alpha)(3\,E(V) + E(TB))\right\}$$

or

$$E_1(T) = \left(1 - \frac{\alpha}{2}\right)\left\{\frac{1}{2}\,E(V)(\alpha + 2) + \frac{1}{2}\,E(TB)(1 - \alpha) + \frac{1}{2}\,K\right\}$$

$$+ \frac{\alpha}{2}\left\{\frac{3}{2}\,E(V) + \frac{1}{2}\,E(TB)\right\} \qquad (14)$$

It should be pointed out that if all the trips are performed on a dual cycle, the two strategies considered for the first configuration should give identical results. Setting $\alpha = 0$ in Equation 14 gives

$$E_1(T) = E(V) + \frac{1}{2} E(TB) + \frac{1}{2} K \qquad (15)$$

which is identical to Equation 12.

Finally, it is worthwhile to compare the two strategies on an expected travel time basis. The second strategy is expected to perform better, for under the second strategy the S/R machine remains within the rack and travels to the input station only if the next order is a storage. Assuming $\alpha = 1$, from Equation 14 gives

$$E_1(T) = \frac{3}{2} E(V) + \frac{1}{4} K + \frac{1}{4} E(TB) \qquad (16)$$

Hence, from Equations 13 and 16, the following ratio, ϕ, is obtained

$$\phi = \frac{E_1(T) \text{ under strategy 2}}{E_1(T) \text{ under strategy 1}} = \frac{\frac{3}{2} E(V) + \frac{1}{4} K + \frac{1}{4} E(TB)}{2\, E(V) + \frac{1}{4} K} \qquad (17)$$

Assuming $b = 1$, from the equations for $E(V)$ and $E(TB)$ it is seen that $E(V) = \frac{2}{3}$ and $E(TB) = 7/15$. Furthermore, for a "normalized" rack, $K = 1$. Substituting these values into Equation 17 gives $\phi = 0.86$. Hence, for $b = 1$, using the second strategy generates a 14% reduction in the expected travel time for a single command trip.

Input and Output at the Same End of the Aisle, but at Different Elevations

The second configuration to be considered is illustrated in Figure 4. It is assumed that the input station is at the lower left-hand corner of the rack while the output station is located d time units above the input station, where $d < b$. Furthermore, it is assumed that the vertical travel yields the value of b.

Consider the single command cycle first. As shown in Figure 4, the rack can be visualized as being two separate racks (as indicated by the dashed line). Travel from the input station to a random point in the rack can be expressed as

$$E(V) = \frac{1}{6} b^2 + \frac{1}{2} \tag{18}$$

However, on going from the random point to the output station, Equation 18 is not appropriate. The output station may be considered to be located at the corner of racks A and B. Due to symmetry, Equation 18 holds as long as the station is located at one of the corners. Hence, denoting the average return time from section A as $E_A(V)$ and using Equation 18 gives

$$E_A(V) = \frac{(b - d)^2}{6} + \frac{1}{2}$$

or

$$E_A(V) = \frac{b^2 - 2bd + d^2}{6} + \frac{1}{2} \tag{19}$$

Similarly,

$$E_B(V) = \frac{d^2}{6} + \frac{1}{2} \tag{20}$$

Now, let $E_0(V)$ denote expected travel time for returning to the output station.

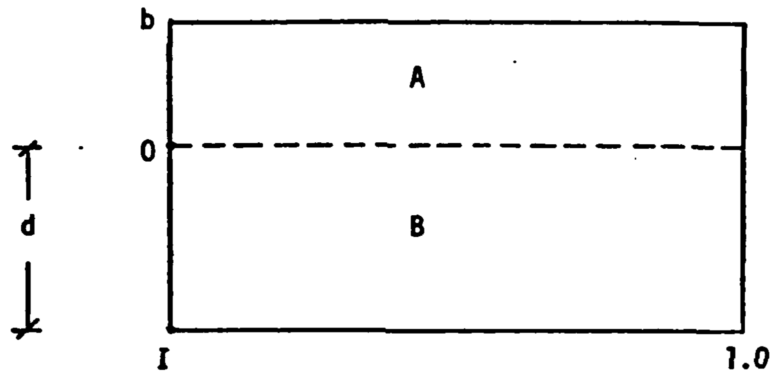

Figure 4: Input and Output at the same end of the aisle, but at different elevations.

Thus,

$$E_0(V) = p_1\,(E_A(V)) + (1 - p_1)(E_B(V)) \tag{21}$$

where

p_1 = probability that the return trip is initiated from section A

Due to randomized storage,

$$p_1 = \frac{\text{area of section A}}{\text{total rack area}} = \frac{b - d}{b}$$

Hence, Equation 21 can be given as

$$E_0(V) = \left(\frac{b - d}{b}\right)\left(\frac{b^2 - 2bd + d^2}{6} + \frac{1}{2}\right) + \frac{d}{b}\left(\frac{d^2}{6} + \frac{1}{2}\right)$$

Simplifying the above expression gives

$$E_0(V) = \frac{1}{6}\,b^2 + \frac{1}{2} - \frac{1}{2}\,d(b-d) \tag{22}$$

or

$$E_0(V) = E(V) - \frac{1}{2}\,d(b-d)$$

Next consider the dual command cycle. By definition, travel time between two randomly selected points in the rack is independent of the location of the input and output stations. Hence, the expression developed earlier for $E(TB)$ is still valid.

Assume that the system operates according to the first strategy defined previously. That is, the S/R machine returns to the input station after each storage and it remains at the output station after each retrieval or dual command cycle. Hence, if the S/R machine is at the input station, then the expected travel time for a:

(1) storage is equal to $2\,E(V)$

(2) retrieval is equal to $E(V) + E_0(V)$

(3) dual trip is equal to $E(V) + E(TB) + E_0(V)$

If the S/R machine is at the output station, then the expected travel time for a:

(1) storage is equal to $d + 2\,E(V)$

(2) retrieval is equal to $2\,E_0(V)$

(3) dual trip is equal to $d + E(V) + E(TB) + E_0(V)$

Thus, expected travel time per operation, $E_2(T)$, will be:

$$E_2(T) = \frac{\alpha}{2}\left\{\frac{\alpha}{2}\quad 2\,E(V) + \frac{\alpha}{2}\,[E(V) + E_0(V)] + \frac{1}{2}\,(1-\alpha)[E(V) + E(TB) + E_0(V)]\right\}$$

$$+ \left(1 - \frac{\alpha}{2}\right)\left\{\frac{\alpha}{2}\,[d + 2\,E(V)] + \frac{\alpha}{2}\quad 2\,E_0(V) + \frac{1}{2}\,(1-\alpha)\right.$$

$$\left.\cdot\,[d + E(V) + E(TB) + E_0(V)]\right\}$$

Simplifying the above expression yields

$$E_2(T) = \frac{\alpha}{2}\left\{\alpha E(V) - \frac{1}{2}\,\alpha E(TB) + \frac{1}{2}\,[E(V) + E(TB) + E_0(V)]\right\}$$

$$+ \left(1 - \frac{\alpha}{2}\right)\left\{\frac{1}{2}\,\alpha\,[E(V) - E(TB) + E_0(V)] + \frac{1}{2}\,[E(V) + E(TB) + E_0(V)] + \frac{1}{2}\,d\right\} \tag{23}$$

In Equation 23, if $\alpha = 0$ (i.e., all trips are dual cycle trips), then, as expected,

$$E_2(T) = \frac{1}{2}\,[E(V) + E(TB) + E_0(V)] + \frac{1}{2}\,d$$

It is also worthwhile to compare the results obtained for the second configuration with those obtained for the first, assuming that the same strategy is used for both configurations. Recall, from Equation 11, $E_1(T)$ can be expressed as

$$E_1(T) = E(V)(1 + \alpha) + \frac{1}{2}\,E(TB)(1 - \alpha) + \frac{1}{2}\,K\,(1 - \frac{\alpha}{2}) \qquad (24)$$

For the purposes of the comparison let $\alpha = 0.50$, $d = 0.50b$, and $b = 1$. By definition, $K = 1$. Thus, $E(V)$, $E(TB)$ and $E_0(V)$ can be determined as follows:

$$E(V) = \frac{1}{6}\,b^2 + \frac{1}{2} = 0.667$$

$$E(TB) = \frac{1}{3} + \frac{1}{6}\,b^2 - \frac{1}{30}\,b^3 = 0.467$$

$$E_0(V) = \frac{1}{6}\,b^2 - \frac{1}{2}\,bd + \frac{1}{2}\,d^2 + \frac{1}{2} = 0.542$$

Thus, from Equation 23:

$$E_2(T) = 1.2188 \text{ minutes}$$

and from Equation 24:

$$E_1(T) = 1.4923 \text{ minutes}$$

Hence, from a travel time standpoint, the second configuration performs

18.33% better than the first configuration. This result was anticipated, because elevating the output station will save some travel time in the vertical direction. Before selecting a configuration involving an elevated output station, however, the costs associated with such a design should be considered.

Input and Output at the Same Elevation, but at a Mid-Point in the Aisle

The third configuration alternative considered is based on the I/O point being located at the center of the rack. Such a configuration can be visualized as having the delivery and take-away conveyors running half-way into the aisle, through a set of rack openings located at the mid-level on either side of the aisle. It is further assumed that vertical travel time $\epsilon(0, b)$, with horizontal travel $\epsilon(0, 1)$. Hence, the I/O point is assumed to be located at $(1/2, b/2)$ for the normalized rack, where $0 \le b \le 1$.

Given randomized storage, the following holds:

$$\Pr(|x - \tfrac{1}{2}| \le z) = \begin{cases} 2z & \text{if } z \le 1/2 \\ 1 & \text{if } z > 1/2 \end{cases}$$

$$\Pr(|y - \tfrac{b}{2}| \le z) = \begin{cases} \frac{2z}{b} & \text{if } z \le b/2 \\ 1 & \text{if } z > b/2 \end{cases}$$

Hence,

$$F(z) = \begin{cases} 4z^2/b & \text{if } 0 \le z \le b/2 \\ 2z & \text{if } b/2 < z \le 1/2 \\ 1 & \text{if } z > 1/2 \end{cases}$$

Consequently,

$$f(z) = \begin{cases} 8z/b & \text{if } 0 \le z \le b/2 \\ 2 & \text{if } b/2 \le z \le 1/2 \\ 0 & \text{otherwise} \end{cases}$$

Denoting expected travel time from the center of the rack to a randomly selected point as $E_M(V)$, then:

$$E_M(V) = \int_0^{1/2} z \ f(z)\, dz = \frac{8}{b} \int_0^{b/2} z^2\, dz + \int_{b/2}^{1/2} 2z\, dz$$

or

$$E_M(V) = \frac{1}{12} b^2 + \frac{1}{4} \tag{25}$$

Note that $E_M(V) = \frac{1}{2} E(V)$.

Next consider the strategy described for the second configuration. Since the input and output stations are coincident for the third configuration, the strategy is equivalent to the case where every trip originates and terminates at the I/O point. Hence, the expected travel time per operation, say $E_3(T)$, will be

$$E_3(T) = \alpha[2\ E_M(V)] + (1 - \alpha)[2\ E_M(V) + E(TB)] \tag{26}$$

It is instructive to briefly compare the results for the second configuration with the results for the third configuration. Due to the convenient location of the I/O point, it is expected that the third configuration will perform better from a travel time standpoint. However, the magnitude of the improvement should be determined in order to economically justify higher conveyor costs and the loss of the rack openings required for the conveyor. Again for comparison's sake assume $\alpha = 0.50$, $d = 0.50b$, and $b = 1$. Thus,

$$E(V) = 0.667$$

$$E(TB) = 0.467$$

$$E_0(V) = 0.542$$

$$E_M(V) = 0.333$$

Hence,

$$E_2(T) = 1.2188 \text{ minutes}$$

and from Equation 26

$$E_3(T) = 0.8995 \text{ minutes}$$

which implies a 26.2% reduction in expected travel time per operation.

Input and Output Elevated at the End of the Aisle

The fourth configuration alternative treated considers the situation where the I/O station has the location (0, d). As before, it is assumed that the maximum horizontal and vertical travel times are 1.0 and b, respectively.

Recall the analysis of the configuration involving input and output stations at the end of the aisle, but at different elevations. From the results obtained, it is straightforward to obtain the following expressions for the expected travel times for single command and dual command cycles:

$$E(SC) = \frac{1}{3} b^2 + 1 - d(b - d) \qquad (27)$$

$$E(DC) = \frac{4}{3} + \frac{1}{2} b^2 - \frac{1}{30} b^3 - d(b - d) \qquad (28)$$

Comparing Equations 27 and 29 with Equations 3 and 10, elevating the I/O station d time units introduces a correction factor of d(b - d) in the computation of cycle times.

DWELL POINT STRATEGIES

The dwell point is referred to as the location of the S/R machine when it becomes idle. In some situations, the determination of the optimum dwell point strategy can be treated as a Markov decision problem. In performing such analyses, the travel time modeling approach used in previous sections should be applicable.

The previous discussion focused on two strategies for locating the S/R machine following completion of storage and retrieval operations:

A. Return to the input station following the completion of a single command storage; remain at the output station following the completion of either a single command retrieval or a dual command cycle; and

B. Remain at the storage location following the completion of a single command storage; remain at the output station following the completion of either a single command retrieval or a dual command cycle.

Obviously, many other strategies could be considered. As examples of additional strategies consider the following:

C. Travel to a mid-point location in the rack following the completion of any cycle; and

D. Travel to the input station following the completion of any cycle.

Strategy C might be appropriate when the next operation might be either a storage or a retrieval; in such a case, for a given probability that the next operation will be a storage an optimum dwell point location can be determined. Strategy D would be appropriate when it is highly probable that the next operation will be either a single command storage or a dual command.

SUMMARY

In summary, a number of travel time models were developed for automated storage/retrieval machines under randomized storage conditions. Expected travel times were determined for both single and dual command cycles. The following I/O locations were addressed: I/O at (0, 0); I at (0, 0) and O at (1, 0); I at (0, 0) and O at (0, d); I/O at (0.5, d); and I/O at (0, d). Several dwell point strategies were considered. Based on the analyses performed, it is felt that a number of insights can be obtained concerning AS/RS design tradeoffs using the travel time models.

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