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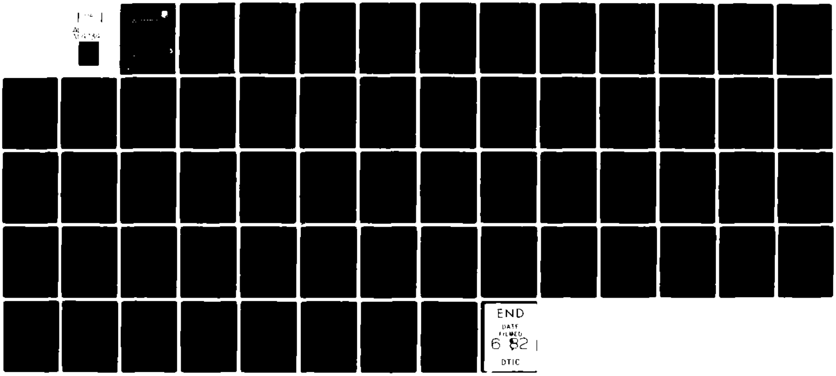

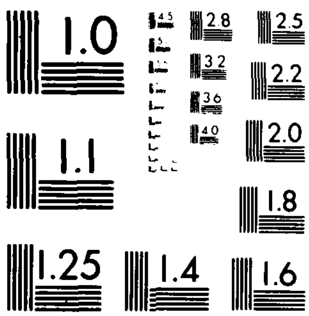
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LINEAR AND NONLINEAR PULSE PROPAGATION IN OPTICAL WAVEGUIDES

The City University of New York

Narkis Tzoar

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ROME AIR DEVELOPMENT CENTER
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APPROVED: Richard W. Picard

RICHARD PICARD
Project Engineer

APPROVED: James F. Hines

JAMES F. HINES, Major, USAF
Assistant Director
Solid State Sciences Division

FOR THE COMMANDER: John P. Huss

JOHN P. HUSS
Acting Chief, Plans Office

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SYNOPSIS

During the period of this contract, our effort was mainly directed toward investigations of nonlinear as well as linear wave propagation in optical wave guides. We have investigated nonlinear pulse propagation (solitons) and self-phase modulation, as well as the nonlinear mode mixing in optical fibers. ~~We have~~ It also investigated the effects of longitudinal inhomogeneities on linear propagation in optical wave guides and calculated radiation loss due to tapers.

Our investigation has resulted in the following publications:

1) "Propagation of Nonlinear Optical Pulses in Inhomogeneous Media (with M. Jain) J. Applied Physics 49 4649 (1978)

2) "Nonlinear Pulse Propagation in Optical Fibers" (with M. Jain) Optics Lett. 3, 202 (1978)

3) "Propagation of Nonlinear Optical Pulses in Fibers" (with M. Jain) Proceedings on "Fiber Optics" Edited by B. Bendow and S.S. Mitra Plenum (1979).

4) "Theory of Nonlinear Pulse Propagation in Optical Waveguides (with B. Bendow, P. D. Gianino and M. Jain) J. Opt. Soc. Am. 70, 539 (1980).

5) "Propagation of Electromagnetic Radiation in Longitudinally Inhomogeneous Optical Waveguides" (with R. Pascone) in "Physics of Fiber Optics" Edited by B. Bendow and S. Mitra, Proceedings of Conference on Fiber Optics sponsored by the American Ceramic Society, (1981).

6) "Self-Phase Modulation in Long Geometry Optical Waveguides" (with M. Jain), Phys. Rev. A23, 1266 (1981).

7) "Theory of CW Beam Propagation In Nonlinear Optical Waveguides" (with B. Bendow and P. D. Gianino) J. Opt. Soc. Am. 71, 656 (1981).

8) "Radiation Loss in Tapered Waveguides" (with R. Pascone), J. Opt. Soc. Am. 71, 1107 (1981).

9) "High-Frequency High-Temperature Conductivity of Metals", J. Opt. Soc. Am 71, 1094 (1981).

10) "Theory of Nonlinear Pulse Propagation in Inhomogeneous Dispersive Media: Application to Optical Waveguides" (with B. Bendow and P.D. Gianino), Proceeding of AGARD meeting on "Special Topics in Optical Propagation", Monterey, Cal., U.S.A., 6-10 April 1981.

11) "Theory of Nonlinear Beam Propagation in Optical Waveguides" (with B. Bendow and P.D. Gianino) Proceeding of a Conference on "Laser Induced Damage in Optical Materials" NBS Special Publication 620 (1981).

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LINEAR AND NONLINEAR PULSE PROPAGATION IN OPTICAL WAVEGUIDES

1. INTRODUCTION

The possibility of transmitting undistorted pulses with high peak powers guided by fibers could prove useful and find application in various fields. Intense, ultrashort light pulses guided by fibers could be used to achieve ultrahigh data rates in communications, to perform surgery in medicine, for cutting and welding in industrial processes, etc. The nonlinearity which governs the propagation characteristics arises from the real part of the guide refractive index, although dissipative nonlinear processes could in principle affect the pulse as well.

The theoretical problem of transmission of nonlinear pulses in optical waveguides is both interesting and challenging, since it involves the simultaneous interplay of transverse confinement, dispersion and nonlinearity. Solutions in many areas of physics have already been obtained (SCOTT, A.C. et al., 1973, for example) and considerable knowledge has been gained for the one-dimensional pulse propagation case, while two- or three-dimensional nonlinear propagation is still in its infancy stage (GERSTEN,J.I., et al., 1975; KAW, P.K., et al., 1975; WILCOX, J.Z., et al., 1975; ZAKHROV,, V.E., et al., 1979).

A critical limitation in the realization of the full-bandwidth capability of optical transmission systems is pulse broadening resulting from dispersion effects (GLOGE, C., 1971; KECK, D., 1976, for example). Although dispersion can be minimized by an appropriate choice of guide materials, geometry and operating frequency, it is apparent that dispersion effects will be detrimental for sufficiently narrow pulsewidths. To overcome this problem the nonlinear dependence of the refractive index on pulse intensity may be used. Here the dispersion effects result in the broadening of the pulses, while nonlinearity tends to sharpen it. It is the appropriate interplay of these opposite effects which can lead to a stable soliton solution for the optical pulse. But nonlinearity can also introduce a host of other effects (CHIAO, R.Y., et al., 1964; HASEGAWA, A., et al., 1973; TZOAR, N., et al., 1976; SHIMIZU, F., 1967) such as self-focusing and self-phase modulation. However, each of these effects is dominant only in certain regimes characterized by the dispersion, peak intensity and nonlinear parameters.

Self-focusing, which occurs for finite-sized beams, requires a very high critical intensity. This phenomenon is an instability which causes a finite-sized beam to collapse to a point, due to an effective focusing lens induced by its own intensity. The condition for self-focus is independent of the dispersion parameter and represents a competition between nonlinearity and diffraction effects. Indeed, for Gaussian beams the condition for self-focusing to occur is given by

$$k^2 \rho_0^2 \geqslant \epsilon_0 / \epsilon_{NL} E_0^2 \qquad (1)$$

Here $k = 2\pi/\lambda$, where λ is the free space wavelength: ϵ_0 (ϵ_{NL}) is the linear (nonlinear) dielectric function; and ρ_0 and E_0 are, respectively, the radius and on-axis field intensity of the beam. This condition is not affected by the guide's parameters. The guide, by balancing diffraction, provides stable trapping of the beam at a finite size even for fields below the critical field (the critical field is defined by Eq. (1) when the equal sign is taken). For glasses one finds the critical power for self focusing to be of the order of 10^7- 10^8 W/cm^2, much greater than the power needed for soliton propagation.

The soliton problem in optical waveguides appears to have been considered first by HASEGAWA and TAPPERT (HT) (1973) but these authors did not take detailed account of the transverse inhomogeneity in their analysis. Subsequently, JAIN and TZOAR (JT) (1978) introduced an approach that accounted for the transverse inhomogeneity of the waveguide in an average fashion, and were thereby able to demonstrate that in fact typical fiberguides could indeed support "bright" solitons. This contrasts with the results of HT who found that anomalous material dispersion or very large core-clad differences ($\Delta n \sim 0.5$) are required for "bright" solitons to exist. The approach of JT has been generalized by BENDOW, GIANINO, TZOAR and JAIN (BGTJ) (1980) to include variational analysis and to treat longitudinal inhomogeneities, and by CROSIGNANI, PAPAS AND DI PORTO (1980) to consider the role of intensity fluctuations on nonlinear soliton propagation in optical waveguides.

2. THEORY OF PULSE PROPAGATION

Consider an optical waveguide having a dispersive and nonlinear refractive index of the form

$$n(\vec{r},\omega,\vec{E}) = n_1(\vec{r},\omega) + n_2|\vec{E}|^2 \tag{2}$$

where ω is the frequency, $\vec{E}$ is the electric field and n_2 the nonlinear coefficient, taken to be independent of frequency. Moreover, n_1 is represented by its local approximation, i.e.

$$n_1(\vec{r},\omega) = n_1(\omega)\, f(\vec{r}) \tag{3}$$

In what follows we consider propagation of pulses which are narrowly centered about a given frequency ω_0 and assume that $n_1(\omega)$ is a slowly varying function of ω in the vicinity of ω_0. The electric field is taken to be

$$\vec{E}(\vec{r},t) = \hat{e}\, A(\vec{r},t)\, e^{i(qz-\omega_0 t)} \tag{4}$$

Where $\hat{e}$ is a unit vector in the transverse direction, $\vec{r} = (\vec{\rho}, z)$, and z is the direction of propagation. We then obtain for A the wave equation:

$$\Big[\nabla_\rho^2 + \frac{\partial^2}{\partial z^2} - q^2 + 2iq\frac{\partial}{\partial z} + f(\vec{r})k_0^2 + 2ik_0k_0' f(\vec{r})\frac{\partial}{\partial t} - (k_0k_0'' + k_0'^2) f(\vec{r})\frac{\partial^2}{\partial t^2} + (2n_2k_0^2/n_0)\left|A(\vec{r},t)\right|^2\Big]A(\vec{r},t) = 0 \tag{5}$$

In Eq. (5) $k(\omega) = n_1(\omega)\omega/c$ and the primes indicate derivatives with respect to ω at $\omega = \omega_0$. The appearance of the time and frequency derivative only to second order is a consequence of using the slowly varying envelope approximation in deriving Eq. (5).

For the waveguide problem f is always inhomogeneous in the transverse direction, i.e., $f(\vec{r}) = f(\vec{\rho})$. Under these conditions it is extremely difficult to obtain time-dependent solutions to Eq. (5), either analytically or numerically. We therefore seek an approximate solution to Eq. (5), based on the observation that the material dispersion in optical waveguides is generally very small. Our previous discussion regarding the balance between dispersion and nonlinearity suggests that the nonlinear effects will also be small in the guide case. We thus expect a negligible effect from the nonlinearity on the mode structure of the guide. Physically, the spatial trapping of the beam and its localization within the guide is dominated by the transverse inhomogeneity. If nonlinearity were absent, the pulse in the guide would experience broadening due to dispersion. The correspondingly small effect induced by the nonlinearity is capable of balancing this longitudinal dispersion. Moreover, it seems reasonable on general grounds to assume that transverse inhomogeneity has only a weak effect on the longitudinal propagation of a pulse along a narrow guide. Rather, the main effect of the transverse inhomogeneity is to determine the mode structure of the linear guide, with the longitudinal propagation characteristics of the modes subsequently being modified by the nonlinearity. This approach, which was first introduced by JT, leads to an approximate solution

$$A(\vec{r},t) = \phi(\vec{\rho})\,\theta(z,t) \tag{6}$$

where the longitudinal characteristics of the pulse are found by taking the transverse average of Eq. (5). Taking any of the linear mode solutions for $A(\vec{r},t)$, multiplying Eq. (5) by $\phi(\vec{\rho})$, using Eq. (6) for $A(\vec{r},t)$ and integrating over the cross section of the guide, results in the following wave equation for $\theta(z,t)$:

$$\Big[d_1 - q^2 + \bar{f}(z)k_0^2 + \frac{\partial^2}{\partial z^2} - (k_0 k_0'' + k_0'^2)\bar{f}(z)\frac{\partial^2}{\partial t^2} + d_2|\theta|^2 + 2i\Big(q\frac{\partial}{\partial z} + k_0 k_0' \bar{f}(z)\frac{\partial}{\partial t}\Big)\Big]\theta(z,t) = 0 \tag{7}$$

In eq. (7) we define the fuction $\bar{f}(z)$ as

$$\bar{f}(z) = \int d\vec{\rho}\,\phi(\vec{\rho})\,f(\vec{\rho},z)\,\phi(\vec{\rho}) \tag{8}$$

and

$$d_1 = \int d\vec{\rho}\,\phi(\vec{\rho})\,\nabla_\rho^2\,\phi(\vec{\rho}) \tag{9}$$

$$d_2 = (2n_2 k_0^2/n_0)\int d\vec{\rho}\,|\phi(\vec{\rho})|^4 \tag{10}$$

The electric field can now be written as

$$\vec{E} = \hat{e}\,\phi(\vec{\rho})\,\theta(z,t)\,e^{i(qz-\omega_0 t)} \tag{11}$$

The effect of the waveguide is now included solely by means of the parameters d_1, d_2 and $\bar{f}(z)$.

We now consider the longitudinally homogeneous guide. Here $\bar{f}(z) = \bar{f}$ and Eq. (7) supports a particular solitary solution for $\theta(z,t) \equiv \theta(t-z/v)$, where

$$v = q / k_0 k_0' \bar{f} \tag{12}$$

and

$$\theta = \theta_0 \operatorname{sech}\left[\gamma(t - z/v)\right] \tag{13}$$

with the conditions:

$$q^2 = k_0^2 \bar{f} + d_1 + d_2 \theta_0^2/2 \tag{14a}$$

$$\left\{\left[(k_0 k_0' \bar{f})^2/q^2\right] - (k_0 k_0'' + k_0'^2)\bar{f}\right\}\gamma^2 = d_2 \theta_0^2/2 \tag{14b}$$

The significant difference of our result (give in Eqs. (14) from the homogeneous medium case, is the factor d_1 above, which is a direct consequence of the transverse inhomogeneity, i.e., the waveguiding mechanism. for example, we consider the condition for the "bright" soliton solution, given in Eq. (13) when the nonlinear factor d_2 is larger than zero. The requirement $\gamma > 0$ implies that

$$\left[1 + (d_1 + \tfrac{1}{2} d_2 \theta_0^2)/k_0 \bar{f}\right]^{-1}\left[1 + (k_0 k_0''/k_0'^2)\right]^{-1} > 1 \tag{15}$$

For the present case, in which d_1 will be negative, Eq. (15) may be satisfied even when $k_0'' > 0$. This contrasts with previous predictions and with the one dimensional soliton solutions which predict that "bright" solitons will exist only if $k_0'' < 0$.

We point out that Eq. (7) admits a "dark" soliton solution, first realized by HT. Here a solution of the form $\theta = \theta_0\{1 - \theta_1^2 \operatorname{sech}^2[\gamma(t-z/v)]\}^{1/2}$ exists for $\theta_1 \leq 1$. For the particular case of dark solitons, where θ_1 is take to be unity, one obtains

$$\theta = \theta_0 \tanh\left[\gamma(t - z/v)\right] \tag{16}$$

and the corresponding conditions given in ref. 10.

For plane waves $k_o'' = 0$ indicates the zero dispersion condition which determines the frequency at which a pulse of "zero" amplitude will propagate without distortion. In our case the zero-dispersion condition becomes

$$\left[1 + (d_1/k_o^2 \bar{f})\right]\left[1 + (k_o k_o''/k_o'^2)\right] = 1 \quad , \tag{17}$$

for either type of soliton. Since $d_1 < 0$ require $k_o'' > 0$ for the zero -dispersion condition to be realized. To illustrate these results we consider the truncated quadratic profile

$$f(\bar{\rho}) = \begin{cases} 1 - \rho^2/L^2 & ; \quad \rho \leq \rho_o \\ 1 - \rho_o^2/L^2 & ; \quad \rho > \rho_o \end{cases} \tag{18}$$

We approximate $\phi(\bar{\rho})$ by the transverse wave solution for the linear problem when $\rho_o \to \infty$, in order to avoid numerical computations. This approximation is very good for the lowest modes. Following our procedures described above, we obtain for the bright soliton,

$$\begin{aligned} \theta &= \theta_o \operatorname{sech}\left[\gamma(t - z/v)\right] \\ q^2 &= k_o^2\left(\bar{f} - \frac{1}{k_o L}\right) + \frac{1}{2}\frac{n_2}{n_o} k_o^2 \theta_o^2 \\ \gamma^2 &= \left\{\left[(k_o k_o' \bar{f})^2/q^2\right] - \left[k_o k_o'' + k_o'^2\right]\bar{f}\right\} = \frac{1}{2}\frac{n_2}{n_o} k_o^2 \theta_o^2 \\ \bar{f} &= (k_o L)^{-1}\left(e^{-k_o \rho_o^2/L} - 1\right) + 1 \end{aligned} \tag{19}$$

The zero-dispersion condition becomes $(k_o k_o''/k_o'^2) \simeq (k_o L \bar{f})^{-1}$

For 1 μm light in fused silica we then have $k_o L \sim 2 \times 10^2$. If $\rho_o^2/L^2 = 2\%$, for example, $\rho_o \sim 4.5\,\mu$m, while if $\rho_0^2/L^2 = 1\%$, $\rho_o \sim 3.2\,\mu$m. These estimates become increasingly inaccurate as ρ_o/L decreases. JT have obtained similar results in a previous paper, where regimes of bright and dark solitons as a function of L and ω_o are indicated graphically as well. JT also present calculations for step-index fibers, utilizing the transverse function of the linear problem (i. e. the Bessel function). We may use their graphical results to estimate the soliton pulse power density for typical cases. We find, e.g., for the lowest-order mode in a silica fiber with core radius 5 μm and carrier wavelength 1 μm, that the power density $P \sim 10^5$ W/cm^2 for a pulsewidth of $\tau \sim 1$ ns. For longitudinally inhomogeneous waveguides the nonlinear pulse propagation becomes further complicated when the guide refractive index varies as a function of z. Logitudinal inhomogeneity is, in fact, quite common and stems from compositional and diametral variations, as well as microbending. Such variations can be modelled, at least approximately, by retaining a longitudinal dependence in the effective guide refractive index, f=f(z), in Eq. (7). Unfortunately, there are no known methods of solving Eq. (7) in this instance. On the other hand, methods for treating slowly varying variations have been developed, within the adiabatic approximation, for one-dimensional nonlinear propagation in the absence of frequency dispersion (CHEN, H.H., et al., 1976 and 1978). We are thus led to consider an approximation in which dispersion and longitudinal inhomogeneity are assumed decoupled; specifically, we assume

$$\frac{\partial^2}{\partial t^2} = \left.\frac{\partial^2}{\partial t^2}\right|_{\bar{f}(z)=\bar{f}(0)} \tag{20}$$

i.e., the dispersive terms are replaced by their values in the homogeneous limit. Thus, we retain interactions between transverse inhomogeneity, dispersion and nonlinearity, on the one hand, and between longitudinal inhomogeneity and nonlinearity on the other.

The solution of Eq. (7) in this instance is straighforward but tedious. The reader intersted in details is directed to BENDOW et al. (1979 and 1980); we will here just state the results. As in the longitudinally homogeneous case, E is given formally by Eq. (11), but now takes the form

$$\theta(z,t) = \theta_0 \operatorname{Sech}\left[\tilde{\gamma}(z - v_e t)\right] e^{i(q_e - q)z} e^{-i(\omega_e - \omega_0)t} \tag{21}$$

Where $\tilde{\gamma} = \gamma k_0 k_0' \bar{f}/q$, and q_e, ω_e and v_e are time dependent effective wave vector, frequency and soliton group velocity, of the longitudinaly inhomogeneous waveguide, respectively. These are given by (GIANINO, P.D., et al., 1980)

The principal result of the above development is that, remarkably, the soliton retains its shape in the presence of weak longitudinal inhomogeneity, although its velocity change with time. The time-dependent velocity in this case contrasts markedly with that of the longitudinally homogeneous case where $v = q/k_0 k_0' \bar{f}$ is a constant. For example, when the inhomogeneity is nearly linear, i.e., when $\Delta n = 2\alpha z$, then

$$v_e = \frac{q}{k_0 k_0' \bar{f}} - \alpha \left(\frac{k_0 k_0' \bar{f}}{2} \right)^{-1/2} t \tag{22}$$

so that the soliton acquires a constant acceleration proportional to α. When the inhomogeneity varies quadratically, $\Delta n = \alpha^2 z^2$, then

$$v_e = \frac{q}{k_0 k_0' \bar{f}} \left(2 - \frac{\sin 2\alpha t}{2\alpha t} \right) \tag{23}$$

i.e., the velocity varies sinusiodally as a function of t. Not surprisingly, the soliton executes oscillatory motion as a function of time, characteristic of a particle trapped in a harmonic well. Formal solutions may be written down for a variety of other cases as well, such as $\Delta n \sim \sin^2 \alpha z$, in which case v_e involves elliptic integrals (GIANINO, P.D., et al., 1980). In this instanced, one again finds that the soliton executes a complicated oscillatory motion as a function of time.

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SELF-PHASE MODULATION IN OPTICAL WAVEGUIDES

Self-phase modulation was first observed by Shimizu[1] when a modulated spectrum appeared after self-focusing had taken place in a liquid-filled cell and was explained as phase modulation due to the intensity-dependent refractive index. It has since been observed[2] in the absence of self-focusing or self-trapping and at low powers by using liquid-filled glass fibers. Recently, some measurements of frequency broadening of mode-locked laser pulses due to self-phase modulation in single-mode silicacore fibers have been reported.[3] The theory used in ref. 3 is the one developed first by Shimizu for short samples, and shows symmetic broadening of the spectia due to self-phase modulation.

We present calculations for the frequency broadening due to self-phase modulation which show an asymmetry in the output spectrum for symmetric initial pulse. Comparison with results of Ref. 3 is difficult to make at present, firstiy due to large asymmetry of the incoming pulse and secondly due to dispersion effects which will be discussed later in this paper. We thus limit ourselves to the development of the theory and determine the realistic conditions undr which comparison between theory and observation is easy to make.

We limit ourselves to a one-dimensional wave propagation of an optical pulse propagating in a glass characterised by a nonlinear refractive index n. The electric field $\vec{E}$ (z,t) is given by the wave equation[4,5]

$$\frac{\partial^2 \vec{E}}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 \vec{D}_L}{\partial t^2} = \frac{2n_2 n_0}{c^2}\frac{\partial^2}{\partial t^2}\left(|\vec{E}|^2 E\right) \qquad (1)$$

where n_2 represednts the nonlinear part of the refractive index

$$n(\omega,\vec{E}) = n(\omega) + n_2 |\vec{E}|^2$$

with n_0=n(ω_0), where ω_0 is the frequency of the electric field and $\vec{D_L}$ is the linear displacement.

We write the electric field in terms of a slowly varying envelope

$$\vec{E}(z,t) = \hat{e}\, A(z,t)\, \exp i\,(qz - \omega_0 t) \tag{2}$$

where q is the propagation constant and find[5] that A(z,t) obeys the equation

$$\left[\frac{\partial^2}{\partial z^2} + 2iq\frac{\partial}{\partial z} - q^2 + \left(k_0^2 + 2ik_0k_0'\frac{\partial}{\partial t} - (k_0'^2 + k_0k_0'')\frac{\partial^2}{\partial t^2}\right)\right]A(z,t)$$
$$= \frac{2n_2n_0}{c^2}\, e^{i\omega_0 t}\, \frac{\partial^2}{\partial t^2}\left[|A|^2 A\, e^{-i\omega_0 t}\right] \tag{3}$$

where

$$k^2 = \omega^2 n^2(\omega)/c^2\;;\quad k_0^2 = \omega_0^2 n_0^2/c^2\;;\quad k_0' = \partial k/\partial\omega\Big|_{\omega_0}\;;\quad k_0'' = \partial^2 k/\partial\omega^2\Big|_{\omega_0} \tag{4}$$

In Eq. (3) we retain only the first and second time derivatives of A. Here we make the usual assumption that our dielectric is weakly dispersive. This is an extremely good approximation for SiO_2 glasses for our operating frequency which is much smaller than the electronic resonance frequency and much larger than the ionic resonance frequency of SiO_2 .

To first order we take $q=k_0$ as for the plane-wave situation, and identify k_0' to be the reciprocal of the group velocity v_g. Thus we are left with the following equation:

$$\left[\frac{\partial^2}{\partial z^2} + 2ik_0\frac{\partial}{\partial z} + 2ik_0k_0'\frac{\partial}{\partial t} - (k_0'^2 + k_0k_0'')\frac{\partial^2}{\partial t^2}\right]A(z,t) = \frac{2n_2n_0}{c^2}e^{i\omega_0 t}\frac{\partial^2}{\partial t^2}\left(|A|^2Ae^{-i\omega_0 t}\right) \tag{5}$$

In treating self-phase modulation one usually replaces the right-hand side of Eq. (5) by $-2(n_2/n_0)k_0^2|A|^2A$ as the dominant contribution, assuming that $\omega_0 T \gg 1$. Here T represents the width of the pulse. However, as we shall see shortly, this assumption is correct only for short enough samples. For long samples, as for the case of self-phase modulation in optical waveguides, we have to consider the corrections arising from the time derivatives of $|A|^2A$. We will treat this effect perturbatively and thus will retain only the first-order term, i.e., the first derivative of $|A|^2A$. Equation (5) is therefore approximated by

$$\left[\frac{\partial^2}{\partial z^2} + 2ik_0\frac{\partial}{\partial z} + 2ik_0k_0'\frac{\partial}{\partial t} - (k_0'^2 + k_0k_0'')\frac{\partial^2}{\partial t^2}\right]A(z,t) = -\frac{2n_2}{n_0}k_0^2|A|^2A - \frac{4in_2}{c}k_0\frac{\partial}{\partial t}\left(|A|^2A\right) \tag{6}$$

We now make a coordinate transformation to a moving coordinate system defined by

$$\xi = z \quad ; \quad \tau = t - z/v_g$$

Equation (6) transforms to yield

$$\left\{\left[\left(\frac{\partial}{\partial \xi} - 2k_0'\frac{\partial}{\partial \tau}\right) + 2ik_0\right]\frac{\partial}{\partial \xi} - k_0k_0''\frac{\partial^2}{\partial \tau^2}\right\}A(\xi,\tau) = -\frac{2n_2}{n_0}k_0^2|A|^2A - \frac{4in_2}{c}k_0\frac{\partial}{\partial \tau}\left(|A|^2A\right) \tag{7}$$

To further simplify Eq. (7), we realized that without the dispersion (k_0'') and the nonlinearity (n_2) the solution for A is given by an arbitrary function of τ, say, $F[(z - v_g t)/\bar{z}]$, where $\bar{z}$ is the legth of the pulse. It then becomes obvious that while $\partial/\partial\xi$ and $k_0'\,\partial/\partial\tau$ are of the order of $\bar{z}^{-1}$, the term k_0 is proportional to λ_0^{-1}. We can now use the slowly changing envelope approximation which required $\bar{z}$ to be much larger than λ_0 so that $(\partial/\partial\xi - 2k_0'\,\partial/\partial\tau)\,A$ is neglected relative to $(k_0 A)$; i.e., the effects of back-scattered radiation are neglected. This implies that the changes in A per wavelength are extremely small. This condition is compatible with our suggested experimental situation, where changes in A are only observed after the pulse propagates hundreds of meters in the guide. We next define the constants

$$\alpha = -\frac{1}{2}k_0'' \quad , \quad \lambda = \frac{n_2}{n_0}k_0 \quad , \quad \gamma = \frac{2n_2}{c} \tag{8}$$

and obtain our final equation

$$i\frac{\partial A}{\partial \xi} + \alpha\frac{\partial^2 A}{\partial \tau^2} + \lambda|A|^2 A + i\gamma\frac{\partial}{\partial\tau}\left(|A|^2 A\right) = 0 \tag{9}$$

The detailed solution for A is given in the paper by Tzoar et all (Phys Rev A 23 1266, 1981). We present here only the approximate result for the frequency spectrum of the electric field having a Gaussian envelope.

$$\bar{E}(\xi,\omega) = \hat{e}\,A_0\,e^{i(k_0\xi - (\omega_0\xi/v_g))}\int F\,d\tau \tag{10}$$

where

$$F = \exp i\left\{(\omega-\omega_0)\tau + \xi\lambda A_0^2 e^{-2\tau^2/T^2}\left[1 + \frac{8\xi\gamma\tau}{T^2}A_0^2 e^{-2\tau^2/T^2} + \frac{4}{3}\xi^2\frac{\lambda\alpha}{T^2}\left(1 - 12\frac{\tau^2}{T^2}\right)A_0^2 e^{-2\tau^2/T^2}\right] - \frac{2\xi\alpha}{T^2}\left(1 - \frac{2\tau^2}{T^2}\right)\right\} \tag{11}$$

In the above expansion, terms proportional to α arise from dispersion effects and are seen to give rise to a symmetric modulation of the phase. The term proportional to γ, however, is proportional to τ and thus results in asymmetric phase modulation.

To analyze the expression in Eq. (18) we note that in our problem we have two fundamental lengths, the nonlinear length $\xi_0 = (\lambda A_0^2)^{-1}$ and the dispersive length $\xi_1 = T^2/|\alpha|$, and also the dimensionaless parameter $\omega_0 T$. We rewrite Eq. (11) in terms of these parameters as

$$F = \exp i \left\{(\omega-\omega_0)\tau + \frac{\xi}{\xi_0} e^{-2\tau^2/T^2}\left[1 + \frac{\xi}{L_1}\frac{\tau}{T} e^{-2\tau^2/T^2} + \frac{4}{3}\left(\frac{\xi}{L_2}\right)^2\left(1 - 12\frac{\tau^2}{T^2}\right)e^{-2\tau^2/T^2}\right] - \frac{2\xi}{\xi_1}\left(1 - 2\frac{\tau^2}{T^2}\right)\right\} \tag{12}$$

Here $L_1 = \omega_0 T \xi_0/16$ and $L_2 = (\xi_1 \xi_0)^{1/2}$. The effect of the linear dispersion can be omitted in many cases since it is easy to make $\xi_1 \gg \xi_0$. The correction terms depend on L_1 and L_2. The L_1 term is the correction arising from the finite duration of the pulse and is the only term that contributes in our theory to asymmetric spectrum. To observe the asymmetric effects, we want the L_1 term to be the dominant correction. We thus need L_2 to be larger than L_1.

We note that the ratio $\rho = L_1/L_2$ should be, ideally, smaller than unity for our purposes. Here

$$\rho = \frac{1}{16}\left(\frac{c^2 k_0 k_0''}{2 n_0 n_2 A_0^2}\right)^{1/2}$$

is independent of T and becomes small either at high beam intensity or for vanishingly small dispersion, i.e., $k_0'' \simeq 0$.

The dispersion effects can be reduced by operating at a frequency where the dispersion is negligible[6] as has been recently demonstrated.

Thus. in Eq. (17), the integrand reduces to

$$F = \exp i\left[(\omega-\omega_0)\tau + \xi\lambda A_0^2 e^{-2\tau^2/T^2}\left(1+\frac{8\xi\gamma\tau}{T^2}A_0^2 e^{-2\tau^2/T^2}\right)\right] \tag{13}$$

In order to make an estimate of the asymmetric effect, we calculate the value of the coefficient $8\xi\gamma\tau A_0^2/T^2$, using values that can be attained in the laboratory. Taking ξ =1 km, $\tau \sim T$=5ps, n_2 =1.4 10^{13} esu, and A_0 =500 S V/cm,[7] we calculate $8\xi\gamma\tau A_0^2/T^2 \sim 0.25$.
Thus the effect of the asymmetric term compared to 1 is about 25% for the parameters chosen above and should be experimentally measurable.
We would like to point out that when information transmision depends on the phase of the carrier, self-phase modulation must be considered as a noise source that may bleach the signal. In this work we have developed a theory which is better suited for long ($\sim$1 km) optical waveguides. We have shown that self-phase modulation will result in asymmetric power, and similaly, in asymmetric intensity spectra.

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CONTINUOUS-WAVE PROPAGATION IN NONLINEAR OPTICAL WAVEGUIDES

1. INTRODUCTION

Transmission of intense light beams through waveguides is of interest in a wide variety of potential applications, such as integrated optics (four-wave mixing, optical bistable devices), medical procedures (surgery, cauterization), industrial processing (cutting, welding), and power transmission (for electrical hazard zones, say). The nonlinearity of the refractive index of optical waveguides affects both the spatial and the temporal characteristics of intense beams propagating along the guide. Here we investigate certain spatial characteristics of cw beams in waveguides possessing a real nonlinear dielectric constant. In other work various temporal effects in nonlinear waveguides, including soliton propagation[1] and self-phase modulation,[2] have been analyzed. Various phenomena associated with the imaginary part of the nonlinear dielectric constant have been discussed previously.[3]

The inhomogenous, nonlinear wave equation describing beam propagation in waveguides is not, in general, amenable to analytic solutions, nor is it especially well suited to numerical methods of solution. For these reasons, we have chosen to introduce several simplifying approximations commonly employed in the literature that make the wave equation analytically tractable. Specifically, we restrict consideration to Gaussian beams and employ the widely used parabolic or paraxial approximation.[4] This is essentially a ray treatment, which is most accurate in the vicinity of the beam axis. Such an approach has been employed extensively in previous work to investigate nonlinear focusing in homogenous media,[5-7] as well as linear propagtion in inhomogeneous media.[4].

In the present investigation we extend the paraxial approximation to encompass simultaneously inhomogeneity as well as nonlinearity, but restrict ourselves to the effects of nonlinearity on the focal characteristics and on the mode mixing of Gaussian beams that are incident on-axis. Use of these approximations and simplifications allows us to examine some of the principal physical effects associated with nonlinear beam propagation and helps to reduce mathematical complexity to a minimum.

2. FOCAL CHARACTERISTICS OF GAUSSIAN BEAMS

Here we derive an expression for the electric field of a cw Gaussian beam in a nonlinear waveguide by utilizing the paraxial approximation. The results obtained here also incorporate guide attenuation and longitudinal dielectric constant variations. Since it is not possible to solve the governing equations analytically in the general case, we subsequently choose to specialize in the case of lossless, longitudinally uniform guides. In this instance, one is able to obtain simple analytic solutions that clearly reveal the effects of waveguiding and nonlinearity on cw beam propagation. We developed our theory for an azimuthally symmetric waveguide with the z axis along the length of the guide and ρ the radial coordinate in the transverse direction. We choose a model dielectric constant of the form

$$\epsilon(\rho, z, E) = \epsilon_o(z) - \epsilon_R(z)\rho^2 + \epsilon_{NL}|E|^2 - i\,\epsilon_I \tag{1}$$

In Eq. (1), waveguiding is introduced through the quadratic term in ρ^2. This widely used approximation assumes that the beams under consideration are tightly bound within the core of the guide. The nonlinearity is introduced through a term in ϵ proportional to the square of the electric field E, i.e., to the field intensity.The imaginary term $i\epsilon_I$ accounts for guide attenuation (from absorption and scattering, say). Finally, we have allowed for longitudinal variations in the dielectric constant by incorporating a z dependence into ϵ_0 and ϵ_R. Thus, despite the simplicity of this model for the dielectric constant, one can account, at least approximately, for any combination of inhomogeneity (both transverse and longitudinal), nonlinearity, and attenuation.

If we assume a time-harmonic dependence of the electric field, E $e^{i\omega t}$, then the time-independent scalar wave equation, neglecting gradient terms, takes the form[4] ($k_0 = \omega/c$)

$$\left[\nabla^2 + k_0^2 \epsilon(\rho, z, E) \right] E = 0 \quad (2)$$

We look for azimuthally symmetric solutions of the form

$$E(\rho, z) = A(\rho, z) \left[\frac{\epsilon_0(0)}{\epsilon_0(z)} \right]^{1/4} \exp\left[-i \int k(z) dz \right] \quad (3)$$

Where $k^2(z) = \epsilon_0(z)\omega^2/c^2$. By substituting Eq. (3) into Eq. (2) and neglecting[4] the terms proportional to $\partial^2 A/\partial z^2$, $(d \ln \epsilon_0/dz)^2$ and $\frac{d}{dz}(d \ln \epsilon_0/dz)$, one obtains

$$\left(\frac{1}{k}\frac{dk}{dz} + 2ik \right) \frac{\partial A}{\partial z} = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial A}{\partial \rho} \right) - \frac{k^2}{\epsilon_0(z)} \left[\epsilon_R(z)\rho^2 - \epsilon_{NL}|E|^2 + i\epsilon_I \right] A \quad (4)$$

Equation (4) constitutes the starting point for the paraxial approximation, in which variations in A as a function of z are assumed to be negligible compared with a wavelength. It reduces to the same starting equation employed by various other authors[4-7] when ϵ_R, ϵ_{NL}, and/or ϵ_I are set equal to zero.

In order to proceed with the solution of the wave equation, one further expresses A in terms of a real amplitude A_0 and phase S as

$$A = A_0(\rho, z) \exp\left[-ik\, S(\rho, z)\right] \tag{5}$$

and substitutes Eq. (5) into Eq. (4) One then obtains two simultaneous equations for A_0 and S:

$$\frac{2S}{k}\frac{dk}{dz} + 2\frac{\partial S}{\partial z} + \left(\frac{\partial S}{\partial \rho}\right)^2 = \frac{1}{k^2 A_0}\left(\frac{\partial^2 A_0}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial A_0}{\partial \rho}\right) + \frac{\epsilon_{NL}|E|^2}{\epsilon_0(z)} - \frac{\epsilon_R(z)}{\epsilon_0(z)}\rho^2 \tag{6a}$$

$$\frac{\partial A_0^2}{\partial z} + \frac{\partial S}{\partial \rho} + A_0^2\left(\frac{\partial^2 S}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial S}{\partial \rho}\right) + k\frac{\epsilon_I}{\epsilon_0}A_0^2 = 0 \tag{6b}$$

These equations become soluble if we employ for the nonlinear term the quadratic approximation introduced by Ref. 8; namely, we take

$$|A|^2 = |A|_0^2 + \frac{\partial |A|^2}{\partial \rho^2}\rho^2 \tag{7}$$

Where the subscript 0 indicates evaluation at $\rho = 0$. One then finds, in analogy with Refs. 4-7, that Eq. (6) admits the Gaussian beam solution ($k_I = \frac{1}{2}k\,\epsilon_I/\epsilon_0$),

$$A_0 = f^{-1}(z)\, E_0 \exp(-k_I z) \exp\left[-\rho^2/2\rho_0^2 f^2(z)\right] \tag{8a}$$

$$S = \left[2 f(z)\right]^{-1}\left[df(z)/dz\right]\rho^2 + \phi(z) \tag{8b}$$

Where f and ϕ satisfy

$$\frac{d^2 f}{dz^2} + \frac{1}{2\epsilon_0(z)}\frac{df}{dz} = \left(R_d^{-2} - R_{NL}^{-2} e^{-2k_I z}\right) f^{-3}(z) - \left[\epsilon_R(z)/\epsilon_0(z)\right] f \tag{9a}$$

$$\frac{d\phi}{dz} + \frac{1}{2\epsilon_0(z)}\frac{d\epsilon_0(z)}{dz}\phi = \rho_0^2 f^{-2}(z)\left[\frac{1}{2}R_{NL}^{-2} e^{-2k_I z} - R_d^{-2}\right] \tag{9b}$$

and

$$R_d^2 = k^2\rho_o^4 = \omega^2\epsilon_o(z)\rho_o^4/c^2$$

$$R_{NL}^2 = \left[\rho_o^2\epsilon_o(z)/\epsilon_{NL}E_o^2\right]\left[\epsilon_o(z)/\epsilon_o(0)\right]^{1/2} \qquad (10)$$

If the incident beam is a plane wave [4], then the boundary conditions accomplanying Eqs. (9a) and (9b) are

$$\left.\frac{df}{dz}\right|_{z=0} = 0 \quad , \quad f(0) = 1 \quad , \quad \phi(0) = \phi_o \qquad (11)$$

The complete solution for E is then

$$E = \frac{E_o}{f(z)} \exp(-k_I z) \exp\left[-\rho^2/2\rho_o^2 f^2(z)\right]\left[\epsilon_o(0)/\epsilon_o(z)\right]^{1/4}$$
$$\exp\left\{-ik\left[\frac{\rho^2}{2f(z)}\frac{df(z)}{dz} + \phi(z)\right]\right\} \exp\left[i\omega t - i\int k(z)dz\right] . \qquad (12)$$

Where f and ϕ are determined from Eqs. (9) and (11). This set of equations encompasses a complete formal solution, within the paraxial approximation, for the electric field of cw Gaussian beams in a nonlinear wavequide described by the model dielectric constant of Eq. (1).

Certain general characteristics of the propagation are evident directly from inspection of Eqs (9)-(12). For example, the spatial profile of the beam is observed to be a Gaussian profile whose focal parameter varies with distance along the guide z. The z dependence of the focal parameter is influenced by the transverse and longitudinal variations of the dielectric constant of the guide, the nonlinearity, and the loss. Equation (12) indicates that, because of the loss term in ϵ, the field amplitude decreases exponentially with propagation distance in a familiar Beers law manner. Finally, note that the phase of the wave acquires a somewhat complicated z dependence.

The specific characteristics of the propagation are obtained once the differential equations for f and ϕ have been solved. Usually, the primary quantity of interest is the beam intensity, in which case phase factors are irrelevant and only the solution for f(z) is required. Clearly, the solution of the nonlinear inhomogeneous differential equation for f [Eq. (9a)] cannot, in general, be obtained by analytic means but requires the use of tedious numerical techniques instead. For this reason, we choose not to treat the general case here; readers interested in the application of numerical techniques to equations of the form of Eq. (9a) are directed to Ref. 7. Rather, in the remainder of this report, we specialize in the simplified limit of a lossless, longitudinally uniform waveguide. In this limit, is is possible to obtain explicit analytic solutions for f(z) that clearly reveal the effects of waveguiding and nonlinearity on cw beam propagation.

Although in the following we especialize to lossless, longitudinally uniform guides, it should be pointed out that Eqs. (9)-(12) provide a convenient starting point for treating more general cases. The solution of the wave equation (2) in the simultaneous presence of nonlinearity, transverse and longitudinal inhomogeneity, and loss is prohibitively difficult. If the paraxial approximation is employed, the problem is reduced, essentially, to the solution of a single second-order ordinary differential equation [Eq. (9a)].

Solutions for Longitudinally Uniform, Lossless Waveguides:

For a longitudinally uniform lossless guide, we set $\epsilon_o(z) = \epsilon_o, \epsilon_R(z) = \epsilon_R$, and $\epsilon_I = 0$. Then the focal parameter f(z) satisfies a simplified equation of the form

$$\frac{d^2 f}{dz^2} = \frac{1}{R_e^2}\frac{1}{f^3} - \frac{\epsilon_R}{\epsilon_o} f \quad ; \quad R_e^{-2} = R_d^{-2} - R_{NL}^{-2} \qquad (13)$$

Here R_e represents an effective radius determined by the relaive sizes of diffraction and nonlinearity.

Consider first the case in which $R_e^2 > 0$ ($R_d < R_{NL}$). The solution for f yields:

$$f^2 = \frac{1}{2}\left[(1+C) + (1-C)\cos 2\zeta\right] \quad ; \; C > 0$$
$$C = \epsilon_o/\epsilon_R R_e^2 \qquad ; \; \zeta = (\epsilon_R/\epsilon_o)^{1/2} z \tag{14}$$

Thus, for sufficiently small nonlinearity, the beam remains "trapped" in the waveguide and continues to oscillate with spatial period $\pi(\epsilon_o/\epsilon_R)^{1/2}$, despite the nonlinearity of the medium. At this level of approximation, then, the guide effect totally dominates the diffraction effect, whereas nonlinearity influences only the amplitude of the oscillation but not its period.

Several observations are useful: First, it is easily shown that for the above cases ($R_e > 0$), f never vanishes, i.e., the beam never collapses. Second, we note from Eq. (21) that, if C=1, then f=1 independently of z. From Eqs. (13) and (21) this occurs when

$$\frac{1}{\rho_o^4} = \frac{\epsilon_R k^2}{\epsilon_o} + \frac{k^2 \epsilon_{NL} E_o^2}{\epsilon_o \rho_o^2} \equiv \frac{1}{W_1^4} \tag{15}$$

Here w_1 is, in fact, just the spot size of the lowest-order eigenmode of an approximate quasilinear wave equation[one obtained by approximating, the nonlinear term by using Eq. (7) at z=0]. Thus, within this approximation, if the spot size of the incident beam matches that of tfhe lowest-order mode of the guide at z=0, then one predicts (not surprisingly) that the beam will propagate unchanged as a function of z.

Now consider the case in which $R_e^2 < 0$ (equivalently, $C < 0$). In this instance we obtain a singular, self-focusing-type solution for f, similar to the corresponding homogeneous nonlinear medium case irrespective of the waveguide effect. In particular, in this case we find that

$$f^2 = \frac{1}{2}\left[(1-|c|) + (1+|c|)\cos 2\zeta\right] \tag{16}$$

Here we define a self-focusing length (z_F; ζ_F in dimensionless units) determined by the condition $f^2(\zeta) = 0$, whence

$$\zeta_F = \frac{1}{2}\cos^{-1}\left[\frac{|c|-1}{|c|+1}\right] \quad , \quad Z_F = \left(\frac{\epsilon_0}{\epsilon_R}\right)^{1/2}\zeta_F \quad . \tag{17}$$

One obtains a periodic solution for f^2, which may be extended to the region $\zeta_F < \zeta \leq 3\zeta_F$ as

$$f^2 = \frac{1}{2}\left[(1-|c|) + (1+|c|)\cos 2(\zeta - 2\zeta_F)\right] \quad . \tag{18}$$

At $\zeta = \zeta_F$ and $\zeta = 3\zeta_F$ we obtain $f^2=0$, and the derivative of f^2 is discontinuous. The solution for f^2 may be summarized as follows: f^2 is given by Eq. (16) for $0 \leqslant \zeta \leqslant \zeta_F$, and

$$f^2 = \frac{1}{2}\left[(1-|c|) + (1+|c|)\cos 2(\zeta - j\zeta_F)\right]; \quad j\zeta_F \leqslant \zeta \leqslant (j+2)\zeta_F \tag{19}$$

for j=1 1,3,5... This solution indicates that $f^2 = 1$ for even multiples of ζ_F and that $f^2=0$ (beam collapse) for odd nultiples of ζ_F. In this case, nonlinearity dominates both waveguiding and diffraction, resulting in self-focusing.

Just as in the homogeneous medium case, the focusing of the beam to a point ($f \rightarrow 0$) is, in the waveguide case, a consequence of the instability induced by nonlinearity. The critical power for self-focusing [see Eq. (27) below], as determined from the condition $R_d = R_{NL}$ and the oscillatory nature of the solution both remain unaltered by the waveguide. The main effect of the guide is to modify the functional form of f and alter the focal length z_F. But, overall, effects that are due to nonlinearity dominate over those that are due to waveguiding.

In order to estimate the power level P at which the influence of nonlinearity becomes significant, we first reexpress E_0^2 in esu as

$$E_0^2 = 8P/c\,\epsilon_0^{1/2}\rho_0^2 \quad .$$

As in the nonguided case,[5-7] we can consider the critical power P for which nonlinearity exactly balances diffraction. This occurs when C=0, i.e., the f^{-3} term in Eq. (13) vanishes identically, whence the beam propagation is determined exclusively by waveguiding (depending only on ϵ_R/ϵ_0). Setting C= 0 yields (in esu) for P_c :

$$P_c = c\,\lambda_0^2\,\epsilon_0^{1/2}/8(2\pi)^2\epsilon_{NL} \tag{21}$$

which is independent of the initial beam radius. By using Eqs. (27) and (28), it is possible to reexpress the condition for waveguiding as $P < P_c$ whereas the condition for self-focusing becomes $P > P_c$. Clearly, the influence of nonlinearity on the focal parameter is greatest for $P \gtrsim P_c$ with only small changes occurring for $P \ll P_c$.

The actual value of the critical power P_c varies considerably from case to case, depending on the propagation medium and the operating wavelength. For $\lambda_0 \sim 1\ \mu m$, for example, $P_c \sim \epsilon_0^{1/2}\epsilon_{NL}^{-1} \sim 10^{11}$ esu [10 kW] for materials with a large third-order nonlinear susceptibility (various semiconductors and liquids, such as CS_2); on the other hand, $P_c \sim 10^{13}$ esu [1 MW] for weakly nonlinear materials (such as silicate glasses).

3. NONLINEARITY-INDUCED MODE MIXING

Consider the case of a cw Gaussian beam propagating in a lossless, longitudinally uniform waveguided. We found that, for beam powers below the critical power, the oscillation period of the focal parameter remains unchanged, but its amplitude becomes a function of the incident beam power. In applications in which the spatial mode structure of beams is significant, such as phase conjugation of images in fiber guides[11] by four-wave mixing, it is useful to determine the mode mixing (i.e., variation in the modal content of the beam as a function of z) that accompanies the dependence of f on beam power. A description in terms of mode mixing makes the most sense below the critical power, where stable waveguided propagation occurs (although one may formally calculate the mode mixing accompanying self-focusing as well).

We choose for consistency to define mode mixing in terms of an approximate initial set of modes obtained within the spirit of the paraxial formulation.Specifically, if we utilize Eq. (7) and (8) to evaluate the nonlinear term, then we obtain what will be referred to as a quasi-nonlinear wave equation for the transverse eigenfunction $u_n(\rho)$ and corresponding propagation constants k_n at the fixed point z=0:

$$\left\{\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial}{\partial\rho} - \frac{k^2}{\epsilon_0}\left[\epsilon_R + \frac{\epsilon_{NL}E_0^2}{\rho_0^2}\right]\rho^2 + \frac{k^2\epsilon_{NL}E_0^2}{\epsilon_0}\right\}u_n(\rho) = (k_n^2 - k^2)u_n(\rho) \tag{22}$$

where we have used f(0) =1. The eigensolutions of this equation are[10] [writing ϵ_0 for $\epsilon_0(0)$ and ϵ_R for $\epsilon_R(0)$]

$$u_n(\rho) = W_1^{-1}\pi^{-1/2}\exp(-\rho^2/2W_1^2)\,L_n(\rho^2/W_1^2)$$
$$W_1^{-4} = W_0^{-4} + k^2\epsilon_{NL}E_0^2/\epsilon_0\rho_0^2\;;\quad W_0^{-4} = \epsilon_R k^2/\epsilon_0 \tag{23}$$

where L_n is the Laguerre polynomial and n takes on integral values,

$$n = \frac{1}{4}\alpha W_1^2 - \frac{1}{2} \tag{24}$$

$$k_n^2 = k^2(1+\epsilon_{NL}E_0^2/\epsilon_0) - \alpha$$

For the limit of a linear guide on simply sets $\epsilon_{NL}=0$, in which case Eqs. (3) and (31) reduce to the well-known expressions for azimuthally symmetric transverse modes of a linear longitudinally homogeneous guide.[10]

In analogy to Ref. (4), we expand the expression for E/E_0 in Eq. (12) in terms of the u_n's as

$$E(\rho,z) = E_0 \sum_{n=0}^{\infty} B_n(z)\,u_n(\rho) \tag{25}$$

where $u_n(\rho)$ is given by Eq. (23) Employing the orthonormality of the u_n's yields after much algebra the desirable result.

By defining a dimensionless mixing coefficient as:

$$g(n,z) = |B_n(z)|^2/4\pi\rho_0^2 \tag{26}$$

one obtains for the quasilinear expansion set

$$g(n,z) = (\sigma_1+\sigma_2)^{1/2}\frac{\left\{[(\sigma_1+\sigma_2)^{1/2}-1]^2+\sigma_1(f-f^{-1})^2\right\}^n}{\left\{[(\sigma_1+\sigma_2)^{1/2}+1]^2+\sigma_1(f-f^{-1})^2\right\}^{n+1}} \tag{27}$$

Where:

$$\sigma_1 = \frac{k^2\rho_0^4}{R_{NL}^2} = \frac{P}{P_c} \quad ; \quad \sigma_2 = \frac{k^2\rho_0^4\epsilon_R}{\epsilon_0} \tag{28}$$

Several general observations regarding mode mixing in nonlinear waveguides follow directly from an inspection of Eq. (37). For example, we see that mode mixing is generally present. Mode mixing is absent only in the special case in which C =1 (equivalently, $\sigma_1+\sigma_2=1$), in which instance f(z) = constant. Physically, this represents the case in which the incident beam spot size exactly matches that of the lowest-order eigenmode of the guide at z=0 .

For fixed σ_1 and σ_2 , the mode mixing is observed to increase with increasing mode number n. The dependence on σ_1 and σ_2 is somewhat more complicated. Clearly, for large values of σ_2 ($\sigma_2 \gg \sigma_1$), mode mixing increases as the nonlinearity σ increases (i.e., as $P \rightarrow P_c$). However, another determining factor, as discussed above, is the proximity of $\sigma_1 + \sigma_2$ to unity. When $\sigma_1 + \sigma_2 = 1$, mode mixing vanishes, but, as the parameters depart from this condition, the mode mixing increase.

As is the case for P_c , the parameters σ_1 and σ_2 can take on a wider range of values depending on the operating wavelegth, the propagation medium, and the geometrical parameters of the beam and waveguide. To obtain some indication of typical values, let us again consider λ_0=1μm and utilize the range of values indicated previously for P_c (10^4-10^6 W). Then

$$\sigma_1 \sim (10^{-4} - 10^{-6}) P$$

where P is the number of watts.

Typical ranges of values for the waveguide and beam parameters are $\epsilon_0 \rho_0 \sim 10^{-3}-10^{-5}$ and $(k\rho_0)^2/\epsilon_0 \sim 10^3-10^5$, respectively. The corresponding range for σ_2 is then $\sigma_2 \sim 10^{-2}-10^{+2}$, although the lower portion of this range is more typical of standard fiber guides.

4. SUMMARY AND DISCUSSION

In this section we have examined various aspects of propagation of cw beam in nonlinear optical waveguides and contrasted the results with those for linear waveguides and homogeneous nonlinear media. Analytic expressions for the dependence of the focal parameter f of a Gaussian beam as a function of distance z are obtained within the paraxial approximation. We have demonstrated the existence of two distinct regimes. For powers below the critical power for self-focusing in a homogeneous medium, the waveguide effect dominates; the beam becomes trapped and merely varies sinusoidally with z. Above the critical power, nonlinearity dominates the propagation; the beam becomes unstable, and oscillatory self-focusing behavior is predicted, in strict analogy to a homogeneous medium. The detailed solutions reveal that in the waveguide-dominated case, the functional form for f and the self-focusing distance z_F are modified because of the waveguide. One must carry the approximations to higher order in order to obtain changes in oscillation period or critical power because of interactions between waveguiding and nonlinearity.

We have defined mode mixing in terms of the quasilinear modes (which have been corrected for nonlinearity in an approximate fashion). We find that mode mixing is always present in nonlinear guides (except for the trivial case in which the beam focal parameter is a constant). These results contrast strongly with those for the linear guide, in which mode mixing occurs only in longitudinally inhomogeneous guides. We obtain analytic expressions for the mixing coefficient g(n,z) for longitudinally homogeneous guides. The results indicate, in general, increased mixing with increasing mode number and with increasing beam power.

Certain aspects of the present work may be useful in connection with other nonlinear problems. For example, consider the calculation of nonlinear pulse propagation in waveguides. All treatments of the time-dependent case to date have essentially decoupled the longitudinal and transverse motions by averaging over the transverse coordinates of the guide.The present work suggests that, for appropriate choices of the guide parameters (e.g. for sufficiently low beam power), the transverse motion is indeed strongly waveguide dominated, with only negligible modifications because of nonlinearity. Moreover, the mixing of modes of sufficiently low order may be small, in which case the concept of individual solitons associated with the separate modes of a multimode guide may be valid to some level of approximation. In any case, the present formulation may help assess under what circumstarces it is reasonable to decouple transverse and longitudinal effects in nonlinear pulse propagation.

Another promblem for which the present results may be useful is that of multiple-wave mixing in waveguides. Most treatments to date assume that the pump beams are propagated in the lowest-order transverse mode of the guide. If these beams do, in fact, undergo mode mixing, then the spatial characteristics of the signal beams may become altered as well. The present work provides useful input regarding the extent of mode mixing as a function of the relevant system parameters.

We comment briefly on the possibility of experimentally observing the phenomena discussed in this paper. In this connection, one must distinguish between the waveguiding ($P < P_c$) and self-focusing ($P > P_c$) regimes. The relatively small changes induced by nonlinearity for $P < P_c$ are significant primarily with respect to mode mixing. They will therefore be evident only in processes that are highly sensitive to mode structure, such as modulation of spatial signals by multiple-wave mixing, proximity coupling, or guided-wave microbending loss sensing. On the other hand, the self-focusing that is predicted for $P > P_c$ will be a large effect, provided that it is not overshadowed by other competing nonlinear processes. Certain related nonlinear phenomena can easily be excluded or distinguished by their special characteristics. For example,soliton propagation requires pulsed as opposed to cw operation and, in any case, is a relatively low-power phenomenon that serves to balance the usually smalll dispersion encountered in typical waveguides. On the other hand, various parametric processes, such as stimulated Raman and Brillouin emission and multiple-wave mixing, can, in fact, compete or coexist with self-focusing, depending on the operating frequency, propagation medium, and other system parameters. In most instances, such processes can be distinguished for self-focusing because they possess different dependences on propagation distance. The growth of parametric processes is usually assumed to vary as $\exp(gPz/A)$, where g is a gain coefficient and A is the cross-sectional area of the light beam. Neglecting all other factors, let us define a minimum distance z_p for parametric processes be setting $gPz_p/A \sim 1$. For self-focusing, on the other hand, when $P \gg P_c$, and $\sigma_2 \ll \sigma_1$ one can show that $z_F \sim A\lambda(P_c/P)$. Thus we find that

$$\frac{z_F}{z_p} \sim \frac{P_c g}{\lambda}\left(\frac{P}{P_c}\right)^{1/2} .$$

Consider, for example, fused silica at $\lambda \sim 1\ \mu m$ and $P/P_c \sim 4$, say. For stimulated Raman scattering, a prominent nonlinear effect in waveguides, one obtains $z_F / z_p \sim 10^{-1}$. This value represents an upper-bound estimated for z_F/z_p because the value of z_p actually depends on the initial density of spontaneous photons; moreover, note that z_F/z_p will decrease as one moves to longer wavelengths (see Ref. 13). Thus the condition $z_F < z_p$ will be satisfied in a variety of circumstances, implying that self-focusing should be observable before parametric effects. Even when $z_F \sim z_p$, parametric processes may be distinguished from self-focusing because their observation requires overcoming linear losses (i.e., $g\ P/A > \alpha$, where α is the waveguide loss); self-focusing, on the other hand, is an instability, which to a first approximation is insensitive to loss. Various other parametric processes may be suppresssed or distinguished from self-focusing in other ways. Stimulated Brillouin scattering, for example, can be suppressed by controlling the dopant profile of the waveguide and in extreme cases could be eliminated entirely by using a source with a frequency bandwidth larger than typical acoustic phonon frequencies. Multiple-wave parametric processes will be large only for special "phase-matched" frequencies; moreover, tight tolerance on variations in diameter and/or in numerical aperture along the fiber must be satisfied. These considerations serve to distinguish self-focusing from other nonlinear effects operative in waveguides.

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RADIATION LOSS IN TAPERED WAVEGUIDES

1. INTRODUCTION

The role of junctions between large-core multimode optical waveguides and small-core single-mode guides in integrated optical devices has led to interest in tapering between fiber waveguides. This interest has included experimental[1,2] and theoretical-numerical[3-7] studies of tapered structures. An important theoretical basis for several of these studies[4-5] has been provided by the step-approximation method developed by Marcuse[8,9] for tapers in step waveguides. This approach is derived from usual perturbation methods by regarding arbitrary waveguide deformations as a succession of infinitely many infinitesimal steps. It has provided improvements in calculating tapering losses beyond the initial development of the perturbation theory of small-wall distortions.[10]

We use an alternative approach, the eikonal approximation method,[11,13] for the calculation of the total radiation loss of a guided mode that is due to a symmetrically narrowing linear taper. This approach has been developed in the engineering literature for optical dielectric waveguides as the concept of local normal modes (LNM's) or tapered modes,[16-18,21] paralleling earlier work on metallic waveguides. However, the word eikonal has also been used by physicists to describe a similar approximation scheme involving a Wentzel-Kramers-Brillouin (WKB) approximation for the initial state in a scattering matrix element.[13,19]

We present the eikonal or LNM approach by beginning with first principles for TE wave propagation in optical slab waveguides, indicating both the small parameter of the method and its region of applicability. The radiation-loss calculation is based on a weak-coupling approximation in which a propagating undepleted symmetric pump mode couples with radiation modes in the tapering region. This represents no inherent limitation on the eikonal (LNM) method but does facilitate comparison with perturbation theory and with the step-approximation method.

The total radiation-loss result is reduced to similar results for the step-approximation method by keeping only the more important of two matrix elements on which the coupling theory is based. Radiation-loss calculations for tapers have been made previously by using other techniques,[15,20] but the eikonal (LNM) method has not been directly employed for this purpose.

2. RADIATION LOSS OF A SYMMETRICAL LINEAR TAPER

From the time-independent wave equation for TE modes and refractive index n(r,z), with longitudinal and transverse variables z and r, respectively, the electric field is expanded in terms of longitudinally dependent coefficients $A_m(z)$ and transverse functions $\phi_m(r,z)$.[11] The latter are parametrically dependent on z and are defined to satisfy eigenvalue equations of the form

$$\left[\nabla_\perp^2 + K_o^2 n^2(\underline{r},z)\right]\phi_m(\underline{r},z) = \beta_m^2(z)\,\phi_m(\underline{r},z) \tag{1}$$

Where $\beta_m(z)$ is the corresponding eigenvalue that is also assumed to be parametrically dependent on z and $K_o = w/c$. Use of the expansion

$$E(\underline{r},z) = \sum_m A_m(z)\,\phi_m(\underline{r},z)$$

in the wave equation and mode orthogonality result in an infinite set of coupled equations

$$\frac{d^2 A_n}{dz^2} + \beta_n^2(z) A_n + \sum_m 2 G_{nm} \frac{dA_m}{dz} + \sum_m F_{nm} A_m = 0 \tag{2}$$

We have employed the shorthand notation

$$G_{nm}(z) = \left\langle \phi_n \left| \frac{\partial}{\partial z} \right| \phi_m \right\rangle = \int_{\underline{r}} d\underline{r}\, \phi_n^* \frac{\partial}{\partial z} \phi_m \tag{2a}$$

$$F_{nm}(z) = \left\langle \phi_n \left| \frac{\partial^2}{\partial z^2} \right| \phi_m \right\rangle = \int_{\underline{r}} d\underline{r}\, \phi_n^* \frac{\partial^2}{\partial z^2} \phi_m \tag{2b}$$

Equations (2) provided the basis for analysis of physical processes such as reflection amd mode coupling. The only limitations on the above theory are that guide changes must be gradual enough so that the adiabatic assumption of the parametric z dependence applies. This is equivalent to the condition

$$\frac{\lambda}{n} \frac{dn(\underline{r},z)}{dz} << 1 \tag{3}$$

where λ is the wavelength of the radiation in the medium.

In order to apply the above general theory to our tapering problem, we first consider the propagation of an initially strong mode $E_\ell = A_\ell \phi_\ell$ which is undepleted on interaction with the other modes. Equation (2) then becomes

$$\frac{d^2 A_\ell}{dz^2} + \left(\beta_\ell^2(z) + F_{\ell\ell}(z) \right) A_\ell(z) \approx 0 \tag{4a}$$

which, for
$$k_\ell^2 = \beta_\ell^2(z) + F_{\ell\ell}(z) \,, \tag{4b}$$

and has the solution[12]

$$A_\ell(z) = A_{\ell_0}\, k_\ell^{-1/2}(z)\, e^{i\int_0^z k_\ell(z')\,dz'} \tag{4c}$$

where terms of the order (d^2k_1/dz^2) and $(dk_1/dz)^2$ have been neglected. Equation (4c) is valid away from turning points.

A reflection coefficient can be defined for pump mode $E_I = A_{\ell_I}\phi_\ell$ by using the field expansion $E = E_I + E_R = (A_{\ell I} + A_{\ell R})\phi_\ell$ in Eq. (2) giving

$$\frac{d^2 A_{\ell R}}{d z^2} + k_\ell^2(z)\, A_{\ell R} = \frac{A_{\ell_0}}{2}\, e^{i\int_0^z k_\ell(z')\,dz'} \frac{d}{dz}\left(\frac{k_\ell'(z)}{k_\ell^{3/2}(z)}\right) \tag{5}$$

Here $k_\ell'(z) = dk_\ell/dz$, and the effect of the neglected second-order derivatives for $A_{\ell_I}(z)$ in Eq. (4c) on $A_{\ell_R}(z)$ has been retained in the right-hand side of Eq. (5). In Eq. (5) the fast and slow dependence of $A_{\ell R}(z)$ can be separated out by focusing on $a_{\ell R}(z)$:

$$A_{\ell R} = a_{\ell R}(z)\, e^{-i\int_0^z k_\ell(z')\,dz'} = a_{\ell R}(z)\, e^{-i\theta_\ell(z)} \tag{6}$$

where $\theta_\ell(z) = \int_0^z k_\ell\,(z')\,dz'$.

By substituting Eq. (6) into Eq. (5) and neglecting the second derivative of $a_{\ell R}(z)$ in a slowly varying amplitude approximation, integration of the result for a disturbance located between 0 and z gives the reflection coefficient

$$R = \left| \frac{a_{\ell R}(0)}{A_{\ell 0} / k_\ell^{1/2}(0)} \right|^2 = \frac{1}{4} \left| \int_0^\infty \frac{1}{k_\ell(z')} \frac{d}{dz'} k_\ell(z') \, e^{2i\theta_\ell(z')} dz' \right|^2 \tag{7}$$

A similar analysis for coupling between two modes with dissimilar transverse structure, a pump mode ℓ and a mode n with forward- and backward-moving slowly varying components,

$$A_n(z) = a_n^+(z) \, e^{+i\theta_n(z)} + a_n^-(z) \, e^{-i\theta_n(z)}$$

results in forward and backward and coupling, given by

$$\left.\begin{array}{l} k_n^{1/2}(z) \, a_n^+(z) \\ k_n^{1/2}(0) \, a_n^-(0) \end{array}\right\} = \frac{i A_{\ell 0}}{2} \int_0^z \left[\frac{F_{n\ell}(z')}{(k_\ell k_n)^{1/2}} + 2i \left(\frac{k_\ell}{k_n} \right)^{1/2} G_{n\ell}(z') \right] e^{i(\theta_\ell \mp \theta_n)} dz' \tag{8}$$

The mode conversion or forward scattering of a propagating, guided mode to radiation modes that is due to a linear taper is an extension of the above. The geometry of this problem is a longitudinally symmetric linear narrowing of a step-index slab guide with infinite cladding. The longitudinal direction is divided into three regions for which the guide half-width L(z) assumes the values ($L_- > L_+$):

$$L(z) = \begin{cases} L_- & z \leq z_o - \Delta z \\ L_T(z) = L_- - \frac{\Delta L}{\Delta z}(z - z_o + \Delta z) & z_o - \Delta z \leq z \leq z_o \\ L_+ & z_o \leq z \end{cases} \tag{9}$$

Within the guide, for $|x| \leq L(z)$, the index of refraction is n_1 whereas outside, for $|x| > L(z)$, it is n_2, where $n_1 > n_2$ is assumed. Note that $\Delta L = L_- - L_+$.

For TE modes, the wave equation is simply $(\partial^2 E/\partial x^2) + (\partial^2 E/\partial z^2) + n^2 k_o^2 E = 0$, where $n = n_1, n_2,$ in the appropriate transverse regions. From the field expansion $E(x,z) = \sum_m A_m(z)\,\phi_m(x,z)$, the eigenvalue equation for $\phi_m(x,z)$ becomes

$$\frac{\partial^2 \phi_m}{\partial x^2} + n^2 K_o^2 \phi_m = \beta_m^2(z)\,\phi_m \tag{10}$$

where $\beta_m(z)$ is equivalent to the longitudinal wave number. When we define

$$K_m^2 = n_1^2 K_o^2 - \beta_m^2 \qquad |x| \leq L \tag{11a}$$

$$\gamma_m^2 = \beta_m^2 - n_2^2 K_o^2 \qquad |x| > L \tag{11b}$$

the symmetric normalized (orthogonal) eigenfunctions take the form

$$\phi_m(x,z) = \begin{cases} N_m(z)\cos K_m(z)\,x & |x| \leq L \quad (12a) \\ N_m(z)\cos K_m(z)\,L(z)\,e^{-\gamma_m(z)(|x|-L(z))} & |x| > L \quad (12b) \end{cases}$$

where the normalization factor is $N_m(z) = [\, L(z) + \gamma_m^{-1}(z)\,]^{-1/2}$. Since the linear tapering presents a symmetric disturbance to the propagating guided modes, it provides coupling from symmetric modes to other symmetric guided modes and between antisymmetric and other antisymmetric guided modes. We simplify the analysis by treating only the former coupling. Note also that the z dependence of $K_m(z)$, $\gamma_m(z)$ in Eqs. (12) is due to the matching of the x components of the magnetic field, resulting in the (symmetric mode) eigenvalue equation[14]

$$\tan K_m(z)\, L(z) = \gamma_m(z) \big/ K_m(z)$$

If we consider once more the propagation of a single pump mode $E_\ell = A_\ell [\cdots \phi_\ell(x,z)]$, we find that the longitudinal coefficients of this mode satisfy Eq. (4a) with the solution from Eq. (4c):

$$A_\ell(z) \approx A_{\ell_0}\, k_\ell^{-1/2}(z)\, e^{i\theta_\ell(z)} \quad . \qquad (13)$$

This relation is valid away from turning points, provided that

$$k_\ell^{-1}\, dk_\ell / dz \ll n(z)\, \omega / c \quad .$$

It will prove advantageous to discuss the coupling of the above guided mode to a continuum of radiation modes by using discrete notation. This is physically equivalent to surrounding the tapered step guide with an ideal conductor at $x = \pm R$ assumed to be large. We will recover the continuous notation as the latter distance becomes infinite. The TE field expansion $E_{RAD}(x,z) = \sum_q B_q(z)\, \psi_q(x,z)$ in the wave equation, together with the wave-number definitions

$$\sigma_q^2 = n_1^2 K_o^2 - \beta_q^2 \qquad |x| \leq L \qquad (14a)$$

$$\rho_q^2 = n_2^2 K_o^2 - \beta_q^2 \qquad |x| > L \qquad (14b)$$

leads to the following transverse eigenfunction structure for the symmetric radiation modes[14]:

$$\Psi_q(z) = \begin{cases} M_q(z) \cos \sigma_q x & |x| \leq L \qquad (15a) \\ 2\,|m_q(z)|\,\cos\left(\rho_q |x| - \eta_q(z)\right) & |x| \geq L \qquad (15b) \end{cases}$$

The radiation modes are assumed to exist for $z \geqslant z_o - \Delta z$, and the continuity of the related magnetic-field components at the transverse boundaries gives

$$m_q(z) = |m_q(z)|\, e^{i\eta_q(z)} = \frac{M_q(z)}{2} e^{i\rho_q L(z)} \left[\cos \sigma_q L(z) - i \frac{\sigma_q}{\rho_q} \sin \sigma_q L(z)\right] \qquad (16)$$

By considering the slowly varying amplitudes $b_q(z)$, where $B_q = b_q \exp(+i\beta_q z)$, and neglecting the second derivative of $b_q(z)$, we can obtain

$$b_q\,(z \geqslant z_o) = b_q = -\frac{1}{2i\beta_q} \int_{z_o-\Delta z}^{z_o} \left[2\,\frac{dA_{\ell I}}{dz'}\,G_{q\ell} + A_{\ell I}\,F_{q\ell}\right] e^{-i\beta_q z'} dz' \qquad (17)$$

The total forward-scattered radiation loss can be evaluated from Eq. (17) in the limit of the continuum by

$$\begin{array}{l}\text{Total radiation} \\ \text{mode loss}\end{array} = \lim_{R \to \infty} \sum_{q} \frac{|b_q|^2}{\left(A_{\ell 0}^2 / k_{\ell -}\right)} \tag{18}$$

In this limit, the radiation mode wave numbers of Eq. (14) become

$$\beta_q^2 \longrightarrow \beta^2(\rho) = n_2^2 K_0^2 - \rho^2 \tag{19a}$$

$$\sigma_q^2 \dashrightarrow \sigma^2(\rho) = \left(n_1^2 - n_2^2\right) K_0^2 + \rho^2 \equiv K_c^2 + \rho^2 \tag{19b}$$

where Eq. (19b) defines K_c^2. Since we must sum over all forward-propagating radiation modes, we restrict the range of integration on ρ to $0 \leqslant \rho \leqslant n_2 K_0$ to avoid imaginary β_q. With these, Eq. (18) becomes, as $b_q \longrightarrow b(\rho)$,

$$\begin{array}{l}\text{Total radiation} \\ \text{mode loss}\end{array} = \lim_{R \to \infty} \int_0^{n_2 K_0} \frac{|b(\rho)|^2}{\left(A_{\ell 0}^2 / k_{\ell -}\right)} \frac{R}{\pi} \, d\rho \tag{19}$$

Also, it can be shown form the definition [Eq. (2a)] that

$$G_{q\ell}(z) = \frac{2 M_q(z) N_\ell(z)}{k_\ell^2(z) - \sigma_q} \left(K_c K_\ell \frac{\Delta L}{\Delta z} \right) \cos \sigma_q L(z) \tag{20}$$

Furthermore, $F_{q\ell}(z)$ of Eq. (2b) provides higher-order corrections in $\frac{\Delta L}{\Delta z}$ to $G_{q\ell}(z)$ and hence may be neglected in the following first-order calculation. Further approximating $k_\ell(z)$ by $\beta_\ell(z)$ from Eq. (4b), we obtain our main result after some algebra

$$\begin{array}{l}\text{Total fractional forward-scattered} \\ \text{radiation loss}\end{array}$$

$$= \int_0^{n_2 K_0} \frac{d\rho}{\pi} \frac{4 K_c^2 \left(\frac{\Delta L}{\Delta z}\right)^2}{\left[\frac{1}{4(\Delta z)^2} + \beta^2(\rho)\right]} \left| \int_{z_0 - \Delta z}^{z_0} dz' \left[\beta_\ell \beta_\ell\right]^{1/2} \frac{\rho K_\ell \cos \sigma L \, e^{i \int_{z_0 - \Delta z}^{z'} \beta_1(z'') dz'' - i\beta(\rho) z'}}{(K_\ell^2 - \sigma^2)(\rho^2 \cos^2 \sigma L + \sigma^2 \sin^2 \sigma L)^{1/2}} \frac{1}{\left(1 + \frac{1}{\gamma_\ell}\right)^{1/2}} \right|^2 \tag{21}$$

For convenience, we summarize the locations of the definitions of the symbols of the fractional loss, Eq. (21): K_ℓ, β_ℓ, γ_ℓ are defined by Eqs. (11); $\beta(\rho)$, $\sigma(\rho)$, ρ are related by Eqs. (19); $K_C^2=(n_1^2-n_2^2)K_0^2$ is defined in Eq. (19b); $L(z)$, Δz, $\Delta L/\Delta z$ are defined in Eq. (9). Also β_ℓ (z) is found after obtaining K_ℓ (z) from the eigenvalue equation with L =L(z) from Eq. (9). It is only slightly different from β_{ℓ_-}, which is obtained by using L=L-.

The fractional forward-scattering radiation-loss result of Eq. (21) has been limited only by the first-order approximation, including only $G_{q\ell}(z)$. However, the relatively mild approximations involved are still insufficient for analytical evaluation. We may obtain the main effects by approximating the integrand of Eq. (21) to zero order in the longitudinal dependence for the amplitude and to first order in the phase. This would be reasonably accurate for weak tapers. The essence of the eikonal approach is that it correctly takes into account overall phase changes of the wave while propagating through a disturbance. This is the main distinction of the eikonal approach as compared with perturbation theory.

Furthermore, we expect the strongest coupling to the radiation modes to come from higher-order guided modes since the latter are less confined to the core and have stronger cladding fields. For such guided modes, the value of K_ℓ (z) L(z) can be seen to lie near $\ell\pi$ from a graphical evaluation of the eigenvalue equation. With this, the zero-order value for $K_\ell(z)$ is $L^{-1}(z)\ell\pi \approx \ell\pi(L_-)^{-1}$ and the values are similar for the other variables related to K_ℓ (z) from Eqs. (11). The lower-order guided modes, which are more tightly confined, have $K_\ell(z)$ near $(2l+1)/2$ $[\pi/L(z)] \approx (2l+1)/2\ (\pi/L_-)$, and hence the matrix element $G_{q\ell}(z)$ of Eq. (20) becomes small. For higher-order guided modes, to zero order in the z variation,

$$K_{\ell_0} \equiv \ell\pi / L_- \tag{22a}$$

$$\beta_{\ell_0} = \left[n_1^2 K_0^2 - \left(\frac{\ell\pi}{L_-}\right)^2 \right]^{1/2} \approx \beta_{\ell_-} \equiv \left[n_1^2 K_0^2 - K_{\ell_-}^2 \right]^{1/2} \tag{22b}$$

$$\gamma_{\ell_0} = \left[\left(n_1^2 - n_2^2\right) K_0^2 - \left(\frac{\ell\pi}{L_-}\right)^2 \right] = \left[K_c^2 - \left(\frac{\ell\pi}{L_-}\right)^2 \right]^{1/2} \tag{22c}$$

Recall that K_{ℓ_-} is the value of $K_\ell(z)$ from the eigenvalue equation evaluated for $L(z) = L_-$. Also, in the denominator of the integrand in Eq. (21), the following rather complicated factor will be approximated as

$$\left[\rho \cos^2\sigma(\rho)\, L(z) + \sigma^2(\rho) \sin^2\sigma(\rho)\, L(z)\right]^{1/2} = \left[\sigma^2 - \left(\sigma^2 - \rho^2\right)\cos^2\sigma L\right]^{1/2} = \left[\sigma^2 - K_c^2 \cos^2\sigma L\right]^{1/2} \approx \left[\sigma^2 - \frac{K_c^2}{2}\right]^{1/2} \tag{22d}$$

where we have neglected the oscillation that is due to $\cos^2 \sigma\, L(z)$ over the transition region.

We now use the guide width variation to define the variable

$$\alpha(z) = \frac{\Delta L}{L_-} \quad \frac{z - z_0 + \Delta z}{\Delta z}$$

$$L(z) = L_T(z) = L_- - \frac{\Delta L}{\Delta z}\left(z - z_0 + \Delta z\right) = L_-\left(1 - \alpha(z)\right).$$

when the above approximations are combined, Eq. (21) becomes

$$\text{Total fractional forward-scattered radiation loss} = \int_0^{n_2 K_0} \left\{ \frac{4K_c^4 \left|\frac{\Delta L}{\Delta z}\right|^2 \beta_{l_0}^2}{\beta^2(\rho) + \frac{1}{4(\Delta z)^2}} \; \frac{\rho^2}{\left(K_{l_0}^2 - \sigma^2(\rho)\right)^2} \; \frac{1}{\sigma^2(\rho) - \frac{K_c^2}{2}} \; \frac{\gamma_{l_0}}{\gamma_{l_0} L_- + 1} \right\} |I(\rho)|^2 d\rho \tag{23}$$

where
$$I(\rho) = \frac{1}{2\sqrt{\pi}} \int_{z_0-\Delta z}^{z_0} dz' \left\{ e^{-i\sigma L_-} \exp\left[i\sigma L_- \alpha(z') - i\beta z' + i\int_{z_0-\Delta z}^{z'} \beta_{l_0}\left(1 - \frac{K_{l_0}^2}{\beta_{l_0}^2}\alpha(z'')\right) dz'' \right] + e^{+i\sigma L_-} \exp\left[-i\sigma L_- \alpha(z') - i\beta z' + i\int_{z_0-\Delta z}^{z'} \beta_{l_0}\left(1 - \frac{K_{l_0}^2}{\beta_{l_0}^2}\alpha(z'')\right) dz'' \right] \right\}$$

The resulting quadratic z dependence in the exponentials in Eq(23) permits the z integration to be expressed in terms of Fresnel integrals

$$S(x) = \int_0^x \sin\frac{\pi}{2} W^2 dW, \quad C(x) = \int_0^x \cos\frac{\pi}{2} W^2 dW \tag{24}$$

With these, the result of the z integration in Eq. (23) is

$$|I(\rho)|^2 = \frac{(\Delta z)^2}{8u_0}\left\{f(\rho)\right\} = \frac{(\Delta z)^2}{8u_0}\Big\{ (\Delta C_u)^2 + (\Delta S_u)^2 + (\Delta C_v)^2 + (\Delta S_v)^2 + 2\cos\gamma\left(\Delta C_u \Delta C_v + \Delta S_u \Delta S_v\right) + 2\sin\gamma\left(\Delta C_v \Delta S_u - \Delta C_u \Delta S_v\right)\Big\} \tag{25}$$

Here
$$\Delta C_u = C(u_2) - C(u_1), \quad \Delta C_v = C(v_2) - C(v_1)$$
$$\Delta S_u = S(u_2) - S(u_1), \quad \Delta S_v = S(v_2) - S(v_1), \tag{25a}$$

and we have defined the following dimensionless parameters as

$$u_0 = \left[\beta_{l_0} \Delta z \frac{K_{l_0}^2}{\beta_{l_0}^2} \frac{\Delta L}{2L} \right] \tag{25b}$$

$$U_2 = \left[\frac{2u_0}{\pi}\right]^{1/2}\left[\frac{(\beta_{l_0}-\beta(\rho))\Delta z + \sigma(\rho)\Delta L}{(\beta_{l_0}\Delta z)\frac{\kappa_{l_0}^2}{\beta_{l_0}^2}\frac{\Delta L}{L_-}} + 1\right], \quad U_1 = \left[\frac{2u_0}{\pi}\right]^{1/2}\left[\frac{(\beta_{l_0}-\beta(\rho))\Delta z + \sigma(\rho)\Delta L}{(\beta_{l_0}\Delta z)\frac{\kappa_{l_0}^2}{\beta_{l_0}^2}\frac{\Delta L}{L_-}}\right]$$

$$V_2 = \left[\frac{2u_0}{\pi}\right]^{1/2}\left[\frac{(\beta_{l_0}-\beta(\rho))\Delta z - \sigma(\rho)\Delta L}{(\beta_{l_0}\Delta z)\frac{\kappa_{l_0}^2}{\beta_{l_0}^2}\frac{\Delta L}{L_-}} + 1\right], \quad V_1 = \left[\frac{2u_0}{\pi}\right]^{1/2}\left[\frac{(\beta_{l_0}-\beta(\rho))\Delta z - \sigma(\rho)\Delta L}{(\beta_{l_0}\Delta z)\frac{\kappa_{l_0}^2}{\beta_{l_0}^2}\frac{\Delta L}{L_-}}\right]$$

(25c)

and
$$\gamma = 2\,\sigma(\rho)\,L_-\left[\left(\frac{\beta_{l_0}-\beta(\rho)}{\beta_{l_0}}\right) - 1\right]$$ (25d)

By using Eqs. (25) in Eq.(23), we obtain

Total forward-scattered
radiation loss =
$$\int_0^{n_2 k_0} d\rho \left\{\beta_{l_0}^3 \kappa_c^2 \frac{\Delta L}{\Delta z}\left(\frac{\gamma_{l_0} L_-}{\gamma_{l_0} L_- + 1}\right)\right\}\left\{\frac{\rho^2\,\{f(\rho)\}}{\left(\beta^2(\rho)+\frac{1}{4(\Delta z)^2}\right)\left(\kappa_{l_0}^2-\sigma^2(\rho)\right)^2\left(\sigma^2(\rho)-\frac{\kappa_c^2}{2}\right)}\right\}$$ (26)

where $\{f(\rho)\}$ denotes the ρ dependence within the Fresnel integrals as indicated by the part of Eq.(25) within the braces. Because of the nature of the approximation in Eq. (22a) ,Eq. (26) as written is limited to higher-order pump modes. However, if the dependence on $\kappa_{l_0}, \gamma_{l_0}, \beta_{l_0}$ is changed to $\kappa_{l_{-1}}, \gamma_{l_{-1}}, \beta_{l_-}$, in Eqs.(25) and (26), the latter are also suitable for describing the radiation loss of the lower-order modes.

The graph in Fig. 1 illustrates the fractional radiation loss for four cases of taper step size using Eq. (26) for λ_0=1μm free-space guided mode wavelength. The four cases, ΔL = 7.5,5,2.5, and 1 um, correspond to tapering angles of approximately 8.5°, 6°, 3°, and 1°, respectively, for an initial guide half-width of L- =25 and a tapering region length Δz =50 μm. The refractive-index values chosen were n_1=1.05 and n_2=1.00. These parameter values permit only 11 of the 15 modes that propagate in the initial wider guide to propagate through all four tapers.

These four cases show a rapid increase in total radiative loss as the mode number approaches the cuttoff value, particularly for the smaller taper angles. This tendency is in agreement with similar results observed in Ref.5. Note that the near-linear behavior of the total radiation loss on a logarithmic scale for the smaller angles indicates a roughly exponential dependence on mode number ℓ, RAD $\sim e^{(\text{const.})\ell}$. Analogous results are seen in Fig. 2 in which the same parameters are used for plots of radiation versus step size for the modes indicated. Again $RAD \sim e^{(\text{const.})\ell}$. These trends could serve as useful rules in making comparative estimates of the radiative loss in such structures. Of course the accuracy of this approach decreases as both ΔL and the mode number ℓ increase since they both contribute to an increase in the left-hand side of Eq. (3). Similar plots obtained using perturbation theory indicated higher total radiation loss for the higher modes as compared with the plots of Fig. 1. Modes beyond those shown are totally reflected and must be treated by a different analysis because of the breakdown of the WKB approximation inherent in the theory through Eqs.(4).

3. DISCUSSION

The suitability of the eikonal or LNM prodecure to tapering problems has suggested the main calculation of this section, a derivation of the total radiation loss of a guided mode that is due to a symmetrical linear taper. We have limited our results to a weak-mode-coupling theory in which an undepleted pump mode transfers energy to radiation modes and the coupling equations are not solved in a self-consistent manner.

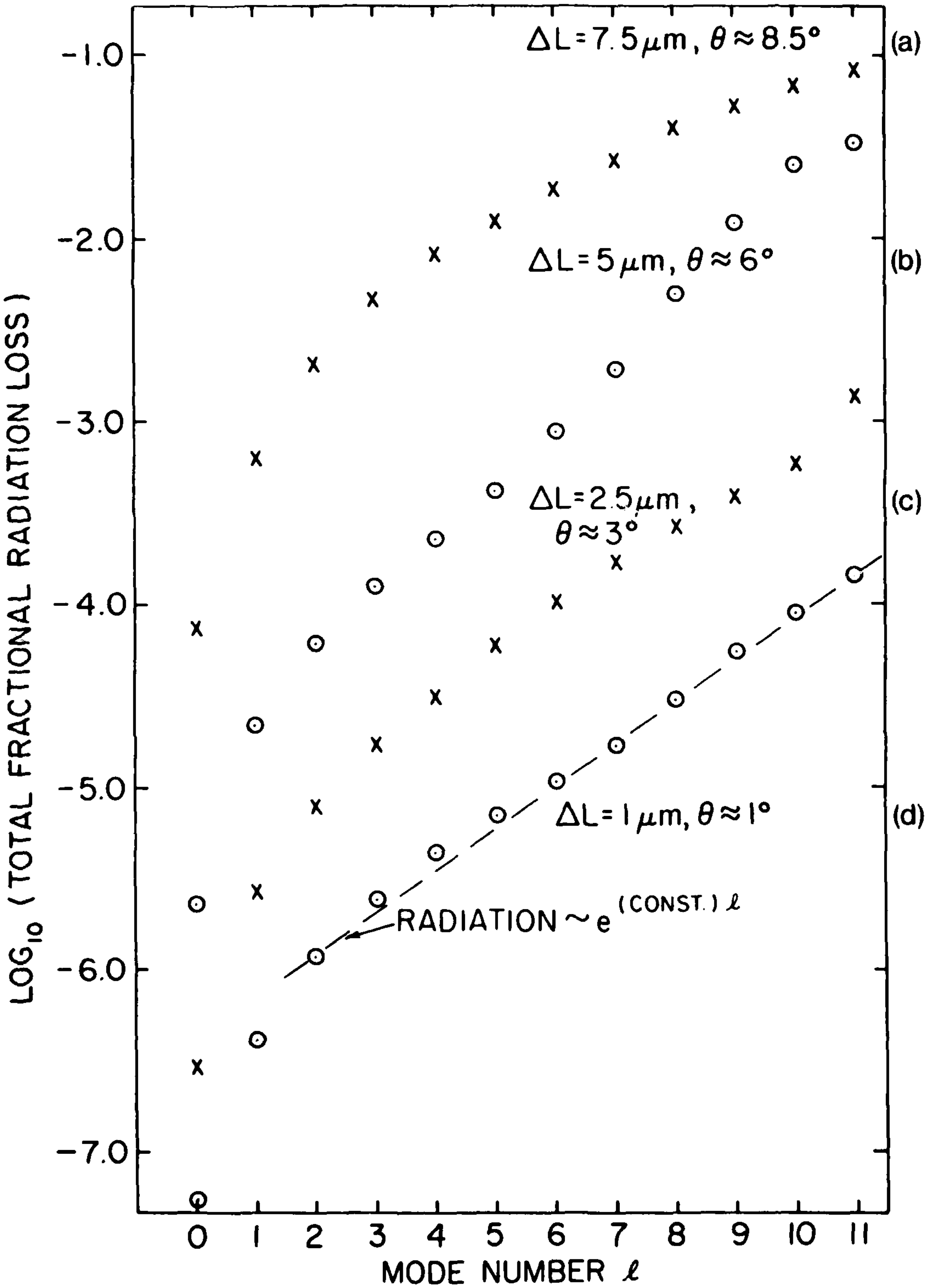
An evaluation of Eq. (21) with zero-order amplitude and first-order phase approximations leads to Eq. (26), which involves Fresnel integrals. This shows the total radiation loss to be essentially proportional to the gradient ($\Delta L / \Delta z$). Analytical evaluation of the final result of the integrations of Eq.(21) is difficult, but it would serve as an appropriate starting point for further numerical studies. In contrast, the approximate expression to Eq. (21), Eq. (26), requires only a relatively simple one-dimensional evaluation. The plots obtained numerically from Eq. (26) illustrate the application of the theory and provide quantitative results for the total fractional radiative coupling of guided modes in the undepleted pump approximation. As expected, Figs. 1 and 2 show a strong increase in radiative loss for increasing ΔL and for modes approaching cutoff. The approximate exponential dependences noted in the figures have not been discussed before. Furthermore, the total radiation loss for tapers has not been previously calculated by using the eikonal or LNM method, and this quantity particularly facilitates comparison with experiment.

The simplicity and general applicability of the eikonal or LNM method under the proviso of small gradients have been indicated in the development of the theory. The approach is particularly well suited to investigating tapered structures but is also readily adaptable to any gradual longitudinal disturbance.

FIGURE CAPTIONS

FIG. 1 Total fractional radiation loss vs. mode number from eq. (26) for radiation $\lambda_0 = 1\mu$ in a slab waveguide of initial halfwidth $L_- = 25\mu$ and tapering region $\Delta z = 50\mu$, tapering angle $\theta = \frac{\Delta L}{\Delta z}$, $n_1 = 1.05$, $n_2 = 1.00$: (a) $\Delta L = 7.5\mu$ (b) $\Delta L = 5\mu$ (c) $\Delta L = 2.5\mu$ (d) $\Delta L = 1\mu$. An approximate exponential dependence on mode number is noted for $\Delta L = 1\mu$.

FIG. 2 Total fractional radiation loss vs taper step size from eq. (26) for the same parameters as in Fig. 1. (a) mode 9 (b) mode 4 (c) mode 1 (d) mode 0.



ΔL=7.5μm, θ≈8.5°
(a)
ΔL=5μm, θ≈6°
(b)
ΔL=2.5μm, θ≈3°
(c)
ΔL=1μm, θ≈1°
(d)
RADIATION ~ e (CONST.) ℓ
LOG10 (TOTAL FRACTIONAL RADIATION LOSS)
-1.0
-2.0
-3.0
-4.0
-5.0
-6.0
-7.0
0 1 2 3 4 5 6 7 8 9 10 11
MODE NUMBER ℓ

FIGURE 1

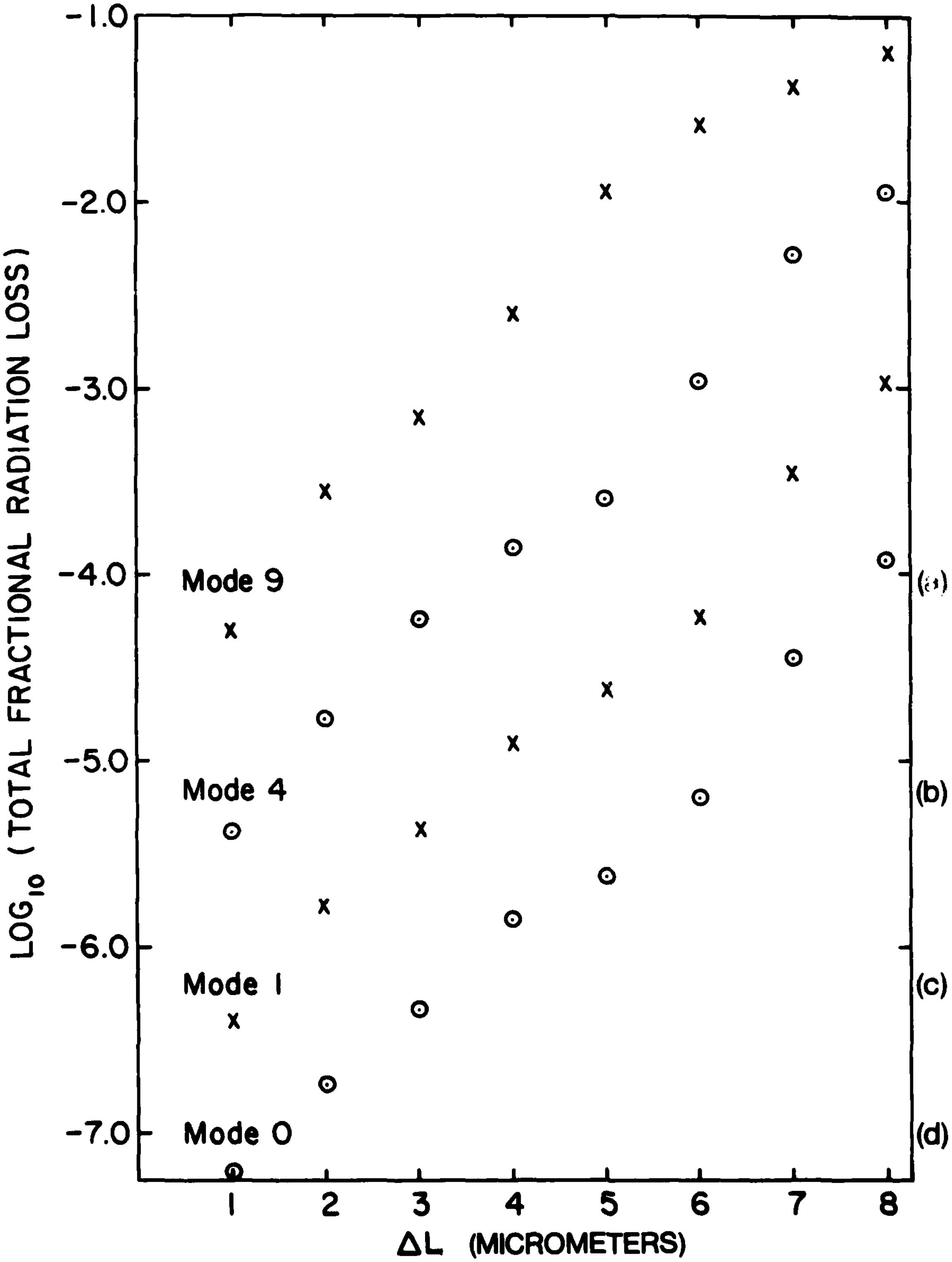

-1.0
-2.0
-3.0
-4.0
-5.0
-6.0
-7.0
LOG10 (TOTAL FRACTIONAL RADIATION LOSS)
Mode 9
Mode 4
Mode 1
Mode 0
(a)
(b)
(c)
(d)
1
2
3
4
5
6
7
8
ΔL (MICROMETERS)

FIGURE 2

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15. See Erratum mentioned in reference 8.

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