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A NEW CLASS OF HERMITE-BIRKHOFF QUADRATURE FORMULAS OF GAUSSIAN TYPE

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ABSTRACT

It is shown how to combine incidence matrices, which admit Hermite-Birkhoff quadrature formulas of Gaussian type for any positive measure, in such a way that the resulting matrix also admits Gaussian type quadratures for any positive measure. Moreover, the uniqueness property and the extremal property of the formulas corresponding to the submatrices are transferred to the formula admitted by the composed matrix.

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SIGNIFICANCE AND EXPLANATION

Methods for approximating the integral of a function, given the values of the function and/or some of its derivatives at several points in the interval of integration, are investigated. The integral is approximated by a weighted sum of the given data--a quadrature formula.

It is shown that for a wide class of different data configurations, there exist appropriate points of evaluation and weights such that the resulting quadrature formula is exact for all polynomials of the maximal possible degree. The well-known Gaussian quadrature formulas represent the particular case in which only function values (and no derivatives) are employed.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the author of this report.

A NEW CLASS OF HERMITE-BIRKHOFF QUADRATURE FORMULAS OF GAUSSIAN TYPE

Nira Dyn*

1. Introduction

Recently results concerning the existence of quadrature formulas of Gaussian type related to Hermite-Birkhoff interpolation problems have been obtained by several authors [5], [1], [2], [3]. Given the incidence matrix $E = \{e_{ij}\}_{i=0,j=0}^{m+1\ n-1}$ with entries consisting of zeroes and ones, a Hermite-Birkhoff quadrature formula of Gaussian type (HB-GQF) is defined as a formula of the form

$$\int_a^b p\,d\sigma = \sum_{e_{ij}=1} a_{ij} p^{(j)}(x_i), \qquad p \in \Pi_{n-1}, \tag{1.1}$$

such that the number of parameters in (1.1) equals the dimension of Π_{n-1} (the space of polynomials of degree $\leq n - 1$):

$$n = \sum_{i=0}^{m+1} \sum_{j=0}^{n-1} e_{ij} + m . \tag{1.2}$$

In (1.1) $d\sigma$ is a non-negative measure supported on more than m points in (a,b), $(x_0,\ldots,x_{m+1}) \in S^m$ where

$$S^m = \{y = (y_0,\ldots,y_{m+1}) | a = y_0 < y_1 < \cdots < y_m < y_{m+1} = b\}$$

and $\sum_{j=0}^{n-1} e_{ij} > 0$, $1 \leq i \leq m$, $\sum_{j=0}^{n-1} e_{ij} \geq 0$, $i = 0$ or $m + 1$.

This notion of HB-GQF extends the classical notion of Gaussian quadrature formulas (GQF):

$$\int_a^b p\,d\sigma = \sum_{i=1}^{m} a_i p(x_i), \qquad p \in \Pi_{2m-1}$$

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where $a < x_1 < \cdots < x_m < b$ and $a_i > 0$, $i = 1,\ldots,m$, and also the notion of the multiple nodes GQF [6], [7].

The problem of characterizing incidence matrices admitting HB-GQF is posed in [4], and the following necessary condition on such matrices is proved:

Result A. Let $E = \{e_{ij}\}_{i=0,j=0}^{m+1\ n-1}$ admits a quadrature formula exact for Π_{n-1}, for some non-negative measure $d\sigma$ supported on more than m points of $[a,b]$, and let r be the minimal number of ones which must be added to E to obtain a matrix $\tilde{E} = \{\tilde{e}_{ij}\}$ without odd sequences in rows $1,\ldots,m$. Then $\tilde{E}$ is a Pólya matrix (the number of ones in any first ℓ columns exceeds $\ell - 1$), and

$$n \leq \sum_{i=0}^{m+1} \sum_{j=0}^{n-1} e_{ij} + r . \tag{1.3}$$

In [2] the existence of two classes of HB-GQF satisfying (1.3) with equality, has been claimed. The first class is related to incidence matrices with rows $1,\ldots,m$ consisting of odd Hermite sequences and even non-Hermite sequences. The existence of this class of HB-GQF is proved in [1].

The present paper is concerned with the proof of the existence of the second class of HB-GQF, corresponding to incidence matrices which can be decomposed vertically into several submatrices, each admitting an HB-GQF with equality in (1.3). It is also proved here that such a formula is unique and/or has an extremal property in case each of the HB-GQF corresponding to the submatrices is unique and/or has an extremal property of the following type:

Definition 1: The HB-GQF (1.1) related to $E = \{e_{ij}\}_{i=0,j=0}^{m+1\ n-1}$ has an extremal property, if for any f satisfying $f^{(n)} \geq 0$ on $[a,b]$:

$$\min_{p \in P(E,f)} [(-1)^{\mu} \int_a^b (f - p)d\sigma] = (-1)^{\mu}[\int_a^b f d\sigma - \sum_{e_{ij}=1} a_{ij} f^{(j)}(x_i)] \tag{1.4}$$

where $\mu = \sum_{j=0}^{n-1} e_{m+1,j}$ and

$$P(E,f) = \{p | p \in \Pi_{n-1}, p^{(j)}(v_i) = f^{(j)}(v_i), \tilde{e}_{ij} = 1, Y \in S^m\},$$

with $\tilde{E} = (\tilde{e}_{ij})$ the matrix of Result A related to E.

The existence of another interesting class of HB-GQF has been established in [3]. These formulas are related to incidence matrices of pyramidal type:

(a) Each interior row, i, contains one sequence only which is of odd length ℓ_i starting at column k_i.

(b) There exist $1 \leq I \leq J \leq m$ such that

$k_i + \ell_i \geq k_{i-1} \geq k_i$, $i = 2,\ldots,I$, $k_i = 0$, $i = I,\ldots,J$, $k_i + \ell_i \geq k_{i+1} \geq k_i$, $i = J,\ldots,m-1$

(c) The two rows $i = 0$, $i = m+1$ have non-zero entries in arbitrary positions $k \geq k_1$ and $k \geq k_m$ respectively.

The proof of the existence in [3] is based on the extremal property of Definition 1, characterizing several other classes of HB-GQF, in particular the classical GQF and the multiple nodes GQF [6], [7]. This class of matrices, as well as the class in [1], can be used in composing new HB-GQF of the type discussed in Section 2.

2. Main Results

Given an incidence matrix $E = \{e_{ij}\}_{i=0,j=0}^{m+1\ n-1}$ with n ones, and a set of points $X \in S^m$, we denote by $p(E,X,f)$ the interpolating polynomial from Π_{n-1} to f at the data (E,X):

$$(2.1) \qquad p(E,X,f)^{(j)}(x_i) = f^{(j)}(x_i), \qquad e_{ij} = 1 .$$

Lemma 1: Let E be a Pólya matrix with even sequences only in rows $1,\ldots,m$, and let $\mu = \sum_{j=0}^{n-1} e_{m+1,j} \geq 0$. Then $[f - p(E,X,f)](-1)^{\mu} \geq 0$ on $[a,b]$ whenever $f^{(n)} \geq 0$ on $[a,b]$.

Proof: It is sufficient to consider functions satisfying $f^{(n)} > 0$ on $[a,b]$. The case $f^{(n)} \geq 0$ on $[a,b]$ is obtained by taking the limit of such functions.

Since E has only even sequences in rows $1,\ldots,m$, all the Rolle's zeroes of the derivatives of $g = f - p(E,X,f)$ do not coincide with those prescribed by E. This together with the Pólya conditions

$$(2.2) \qquad \sum_{j=0}^{k-1} \sum_{i=0}^{m+1} e_{ij} \geq k, \quad 1 \leq k \leq n-1, \quad \sum_{j=0}^{n-1} \sum_{i=0}^{m+1} e_{ij} = n$$

imply that the numbers of Rolle's zeroes of $g^{(k)}$ is at least $\left[\sum_{j=0}^{k-1} \left(\sum_{i=0}^{m+1} e_{ij} - 1\right)\right]$. Therefore $g^{(n-1)}$ has at least one $\left(\sum_{j=0}^{n-1} \sum_{i=0}^{m} e_{ij} - (n-1) = 1\right)$ zero, which is the maximal possible number since $g^{(n)} \geq 0$. Thus the zeroes of g and its derivatives are only the Rolle's extension of (E,X), namely those prescribed by (E,X) and the corresponding Rolle's zeroes. In particular all the interior zeroes of g are of even multiplicities, and therefore g does not change sign in $[a,b]$.

To determine the sign of g, observe that if $e_{m+1,k} = 0$ then the right most zero of $g^{(k)}$ in $[a,b]$, δ_k, is equal or greater than the right most change of sign of $g^{(k+1)}$ in (a,b), ϵ_{k+1}. Therefore $g^{(k)}(x)g^{(k+1)}(x) \geq 0$ in $(b - \delta_k, b)$, while if $g^{(k)}(b) = 0$ $(e_{m+1},k = 1)$ then $g^{(k)}(x)g^{(k+1)}(x) \leq 0$ in $(b - \epsilon_{k+1}, b)$. This together with $g^{(n)} \geq 0$ implies that $(-1)^{\mu} g(x) \geq 0$ for $x \in [a,b]$.

As a direct consequence of Lemma 1, we obtain:

Corollary 1: Under the conditions of Lemma 1, the Peano kernel $K(t,x) \equiv K(t,x|E,X)$ for the error in the interpolation prescribed by (E,X):

$$f(x) - p(E,X,f)(x) = \int_a^b f^{(n)}(t)K(t,x|E,X)dt \tag{2.3}$$

satisfies

$$(-1)^{\mu}K(t,x) \geq 0, \quad (t,x) \in [a,b]^2 . \tag{2.4}$$

With these preliminary results the existence of HB-GQF corresponding to vertically decomposable matrices can be obtained.

Theorem 1: Let $E = \{e_{ij}\}_{i=0,j=0}^{m+1\ n-1}$ be an incidence matrix which can be vertically decomposed in ℓ submatrices, each consisting of m_i rows and n_i columns respectively:

$$E = E(n_1) \oplus E(n_2) \oplus \cdots \oplus E(n_\ell), \quad \sum_{i=1}^{\ell} n_i = n , \tag{2.5}$$

such that all the non-zero entries of row i, $1 \leq i \leq m$, belong to one of the submatrices only. If each submatrix $E(n_i)$ admits a HB-GQF exact for Π_{n_i-1}, for any positive measure, and if each $E(n_i)$ satisfies Result A with equality in (1.3), then E admits a HB-GQF exact for Π_{n-1}, for any positive measure, with nodes $X = (x_0,\ldots,x_{m+1}) \in \Omega^m$, where

$$\Omega^m = \{X = (X^{(1)} \cup X^{(2)} \cup \cdots \cup X^{(\ell)}) | X^{(j)} \in S^{m_j}, 1 \leq j \leq \ell\} . \tag{2.6}$$

Proof: It is sufficient to consider the case $\ell = 2$. Let $\tilde{E} = \tilde{E}(n_1) \oplus \tilde{E}(n_2)$ be related to $E = E(n_1) \oplus E(n_2)$ by Result A, let $Y \in S^{m_1}$ be the nodes of the HB-GQF for $E(n_1)$ and the measure $d\sigma$, and let $Z \in S^{m_2}$ be the nodes of the HB-GQF for $E(n_2)$ and the measure $\omega(t)dt$ with

$$\omega(t) = \int_a^b d\sigma(x)K(t,x|\tilde{E}(n_1),Y) . \tag{2.7}$$

That $\omega(t)$ is of constant sign follows from Corollary 1.

Consider now the set of fundamental polynomials for interpolation at $(\tilde{E},X)$ with $X = Y^{(1)} \cup Z^{(1)}$:

$$q_{ij}^{(k)}(x_\nu) = \delta_{i\nu}\delta_{jk}, \quad \tilde{e}_{k\nu} = 1 . \tag{2.8}$$

Then

$$p(\tilde{E},X,f)(x) = \sum_{\tilde{e}_{ij}=1} f^{(j)}(x_i)q_{ij}(x) \tag{2.9}$$

and for any $p \in \Pi_{n-1}$

$$p(x) = \sum_{\tilde{e}_{ij}=1} p^{(j)}(x_i)q_{ij}(x) , \tag{2.10}$$

$$\int_a^b p(x)d\sigma(x) = \sum_{\tilde{e}_{ij}=1} p^{(j)}(x_i) \int_a^b q_{ij}(x)d\sigma(x) . \tag{2.11}$$

Now $\{q_{ij} | \tilde{e}_{ij} = 1, j < n_1\}$ is a basis for Π_{n_1-1}, since $q_{ij}^{(n_1)} \in \Pi_{n_2-1}$ vanishes on $(\tilde{E}(n_2),Z)$, and therefore is identically zero. Using the HB-GQF with nodes $Y \in S^{m_1}$ and coefficients $\{a_{ij} | e_{ij} = 1, j < n_1\}$ admitted by $E(n_1)$, which is exact for Π_{n_1-1}, we obtain:

$$\int_a^b q_{ij}(x)d\sigma(x) = \sum_{\substack{e_{k\nu}=1 \\ \nu<n_1}} a_{k\nu}q_{ij}^{(\nu)}(y_k) = 0 \text{ if } e_{ij} = 0 \text{ and } \tilde{e}_{ij} = 1, 0 \le j < n_1 . \tag{2.12}$$

Consider next the set $\{q_{ij} | \tilde{e}_{ij} = 1, n_1 \le j < n\}$, consisting of n_2 polynomials in Π_{n-1} vanishing on $(\tilde{E}(n_1),Y)$. These polynomials can be represented as

$$q_{ij}(x) = \int_a^b q_{ij}^{(n_1)}(t)K(t,x|\tilde{E}(n_1),Y)dt . \tag{2.13}$$

Hence

$$\int_a^b q_{ij}(x)d\sigma(x) = \int_a^b dt\, q_{ij}^{(n_1)}(t) \int_a^b K(t,x)|\tilde{E}(n_1),Y)d\sigma(x) = \int_a^b q_{ij}^{(n_1)}(t)\omega(t)dt . \tag{2.14}$$

But $q_{ij}^{(n_1)} \in \Pi_{n_2-1}$ for $e_{ij} = 1$, $n_1 \leq j < n$, and therefore the application of the HB-GQF admitted by $E(n_2)$ for the measure $\omega(t)dt$, with nodes $Z \in S^{m_2}$ and coefficients $\{b_{ij} | e_{i,j+n_1} = 1, 0 \leq j < n_2\}$, yields

$$(2.15) \quad \int_a^b q_{ij}^{(n_1)} \omega(t)dt = \sum_{\substack{e_{k\nu}=1 \\ n_1 \leq \nu < n}} b_{k,\nu-n_1} q_{ij}^{(\nu)}(z_k) = 0 \text{ if } e_{ij} = 0 \text{ and } \tilde{e}_{ij} = 1,\ n_1 \leq j < n.$$

Combining (2.11) with (2.12) and (2.15) we conclude that

$$(2.16) \qquad \int_a^b p d\sigma = \sum_{e_{ij}=1} [\int_a^b q_{ij}(x) d\sigma(x)] p^{(j)}(x_i), \quad p \in \Pi_{n-1} .$$

Using the same arguments it is easy to show that

<u>Theorem 2</u>: $E = E(n_1) \oplus \cdots \oplus E(n_\ell)$ admits a HB-GQF at $X = X^{(1)} \cup \cdots \cup X^{(\ell)}$ for *the positive measure* $d\sigma$, *if and only if* $E(n_i)$ *admits a HB-GQF at* $X^{(i)}$ *corresponding to the positive measure* $\omega_i(t)dt$ *with*

$$(2.17) \qquad \omega_i(t) = |\int_a^b K(t,x|E(n_1) \oplus \cdots \oplus E(n_{i-1}), X^{(1)} \cup \cdots \cup X^{(i-1)}) d\sigma(x)| .$$

A direct conclusion of Theorem 2 is

<u>Corollary 2</u>: The HB-GQF admitted by $E = E(n_1) \oplus \cdots \oplus E(n_\ell)$ is unique for any positive measure, if and only if the HB-GQF admitted by $E(n_i)$, $1 \leq i \leq \ell$, is unique for any positive measure.

Another property of the HB-GQF admitted by $E(n_i)$, $i = 1,\ldots,\ell$, which is transferred to the HB-GQF admitted by $E(n_1) \oplus \cdots \oplus E(n_\ell)$, is the extremal property. Note that (1.4) can be formulated also as

$$\min_{p \in P(E,f)} [(-1)^\mu \int_a^b (f - p) d\sigma] = (-1)^\mu \int_a^b [f - p(\tilde{E},X,f)] d\sigma .$$

Theorem 3: Let E satisfy the conditions of Theorem 1. If for any positive measure each $E(n_k)$, $1 \leq k \leq \ell$, admits a HB-GQF of the form (1.1) with the extremal property of Definition 1, then for any positive measure E admits a HB-GQF with the extremal property of Definition 1, but with Ω^m replacing S^m in the definition of $P(E,f)$.

Proof: It is sufficient to consider the case $\ell = 2$. Let $f^{(n)} \geq 0$, let $\xi = \xi^{(1)} \cup \xi^{(2)}$ be arbitrary in Ω^m, and define $g_1 = f - p(\tilde{E},\xi,f)$, where $\tilde{E}$ is related to E by Result A. Since $g_1^{(n_1)}$ is the error in the interpolation of $f^{(n_1)}$ at $(\tilde{E}(n_2),\xi^{(2)})$, it follows from Corollary 1 that

$$g_1^{(n_1)}(x) = \int_a^b g_1^{(n_1+n_2)}(t)K(t,x|\tilde{E}(n_2),\xi^{(2)})dt$$

$$= (-1)^{\mu_2} \int_a^b f^{(n)}(t)|K(t,x|\tilde{E}(n_2),\xi^{(2)})|dt$$

with $\mu_2 = \sum_{n_1 \leq j < n} e_{m+1,j}$, and therefore

$$(-1)^{\mu_2} g_1^{(n_1)}(x) \geq 0, \qquad x \in [a,b] \ . \tag{2.18}$$

Now $0 \in P(E(n_1),(-1)^{\mu_2} g_1)$, and hence by the extremal property of the HB-GQF admitted by $E(n_1)$ with nodes Y, we obtain

$$(-1)^{\mu_1} \int_a^b (-1)^{\mu_2} g_1 d\sigma \geq (-1)^{\mu_1} \int_a^b (-1)^{\mu_2}[g_1 - q_1]d\sigma \tag{2.19}$$

with $\mu_1 = \sum_{j=0}^{n_1-1} e_{m+1,j}$ and $q_1 = p(\tilde{E}(n_1),Y,g_1) \in \Pi_{n_1-1}$. In view of the definition of g_1 and q_1, (2.19) becomes:

$$(-1)^{\mu} \int_a^b [f - p(\tilde{E},\xi,f)]d\sigma \geq (-1)^{\mu} \int_a^b [f - p(\tilde{E},Y \cup \xi^{(2)},f]d\sigma \ , \tag{2.20}$$

with $\mu = \mu_1 + \mu_2$. To complete the proof we use again Corollary 1 and conclude that

$$(-1)^{\mu} \int_a^b [f - p(\tilde{E}, Y \cup \xi^{(2)}, f)]d\sigma$$

(2.21)
$$= (-1)^{\mu} \int_a^b \int_a^b [f - p(\tilde{E}, Y \cup \xi^{(2)}, f)]^{(n_1)}(t)K(t,x|\tilde{E}(n_1), Y)dtd\sigma(x)$$

$$= (-1)^{\mu} \int_a^b \omega(t)[f - p(\tilde{E}, Y \cup \xi^{(2)}, f]^{(n_1)}(t)dt$$

where $\omega(t)$ is defined by (2.7), and satisfy $(-1)^{\mu_1}\omega(t) > 0$ on $[a,b]$. Applying the extremal property of the HB-GQF, admitted by $E(n_2)$ for $(-1)^{\mu_1}\omega(t)dt$ at the nodes Z, we obtain for $g_2 = f^{(n_1)} - p^{(n_1)}(\tilde{E}, Y \cup \xi^{(2)}, f)$, satisfying $g_2^{(n_2)} = f^{(n)} > 0$, the inequality

(2.22)
$$(-1)^{\mu_2} \int_a^b [(-1)^{\mu_1}\omega(t)][f^{(n_1)} - p^{(n_1)}(\tilde{E}, Y \cup \xi^{(2)}, f)]dt >$$

$$> (-1)^{\mu_2} \int_a^b [(-1)^{\mu_1}\omega(t)][f^{(n_1)} - q_2](t)dt$$

where $q_2 = p(\tilde{E}(n_2), Z, f^{(n_1)}) \in \Pi_{n_2-1}$. Observing that $[f - p(\tilde{E}, Y \cup Z, f)]^{(n_1)} = f^{(n_1)} - q_2$, we finally derive from (2.20), (2.21) and (2.22) that

$$(-1)^{\mu} \int_a^b [f - p(\tilde{E}, \xi, f)]d\sigma > (-1)^{\mu_2} \int_a^b [f - p(\tilde{E}, Y \cup Z, f)]^{(n_1)}(-1)^{\mu_1}\omega dt$$

$$= (-1)^{\mu} \int_a^b [f - p(\tilde{E}, Y \cup Z, f)]d\sigma ,$$

indicating the optimal property of the HB-GQF admitted by E at $Y \cup Z$.

The following two matrices are examples of matrices which admit HB-GQF in view of Theorem 1 and the results in [1] and [3]:

$$E_1 = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$E_2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The matrix E_2 admits a HB-GQF with the extremal property of Theorem 3, in view of the results in [3], [6] concerning the two submatrices of E_2.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

It is shown how to combine incidence matrices, which admit Hermite-Birkhoff quadrature formulas of Gaussian type for any positive measure, in such a way that the resulting matrix also admits Gaussian type quadratures for any positive measure. Moreover, the uniqueness property and the extremal property of the formulas corresponding to the submatrices are transferred to the formula admitted by the composed matrix.

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