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RELIABILITY GROWTH OF CONTINUOUS SYSTEMS

BY

D. D. PENROD

FEBRUARY, 1982

U.S. ARMY MISSILE COMMAND
Redstone Arsenal, Alabama 35898

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A three-parameter reliability growth model is developed for continuous systems, which model assumes failure rates are improved only after each failure and times between failures are exponential. The basic model is given by $\lambda_n=\lambda_u+\alpha^n(\lambda_o-\lambda_u)$ where λ_n is the failure rate after the nth failure, λ_o is the initial failure rate, λ_u is the ultimate failure rate, and α is a shape parameter. Both maximum likelihood and Bayesian methods of estimating the parameters from test data were investigated. Use of the model for assessment and prediction are addressed and some comparisons are made with the AMSAA model currently in use.

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RELIABILITY GROWTH OF CONTINUOUS SYSTEMS

by

Darrell D. Penrod

February, 1982

CONTENTS

RELIABILITY GROWTH OF CONTINUOUS SYSTEMS

By

Darrell D. Penrod

ABSTRACT

The prediction and assessment of reliability growth in a test-analysis-and fix program has gained importance in recent years and methodology is still being developed to perform the required analyses. The results of this study of a particular model (based on a stochastic learning model) of the process provide analytical tools appropriate to the underlying model. Some comparisons with existing methods are made and a number of possible applications of the methods are explored.

1. INTRODUCTION

Reliability growth planning and assessment continue to be important topics surrounding development and testing of new complex systems. This report contains the description of a conceptual model of the process of test-analysis-and fix of an evolutionary system and associated mathematical tools for analyzing the progress of a program. The type of system under consideration is one which operates continuously until a failure occurs, at which point the failure is analyzed and, if appropriate, the system is modified to reduce the likelihood of a subsequent failure of the same type. An earlier report [1] dealt with discrete or "one-shot" systems which were tested for success or failure. For such systems, it is the probability of success which changes as the program unfolds. The continuous case, under consideration here, is characterized by mean time between failures (MTBF) and it is this parameter which must be scrutinized.

2. THE MODEL

The model presumes a test-analysis-fix program being carried out on a system which is modifiable at the point of each failure. The intent of such modifications (in design or components) is assumed to be to reduce the probability of a subsequent similar failure. The mathematical description of the process is as follows:

A. Between failures (and fixes), the behavior of system failures is Poisson, λ_n being the failure rate after n failures and fixes.

B. A relationship exists between failure rates of the form

$$\lambda_n - \lambda_u = \alpha\ (\lambda_{n-1} - \lambda_u) \qquad n = 1, 2, \ldots \qquad (2.1)$$

Since most programs of this type will show improvement in MTBF (reduction in λ_n), the most important class of problems will have

$$0 < \alpha < 1 \quad \text{and} \quad \lambda_o > \lambda_u.$$

Notice that

$$\lambda_n - \lambda_u = \alpha^n (\lambda_o - \lambda_u). \qquad (2.2)$$

If $0<\alpha<1$, then λ_n approaches λ_u as a limit. Typical growth curves for the most important cases are shown in Fig. 1.

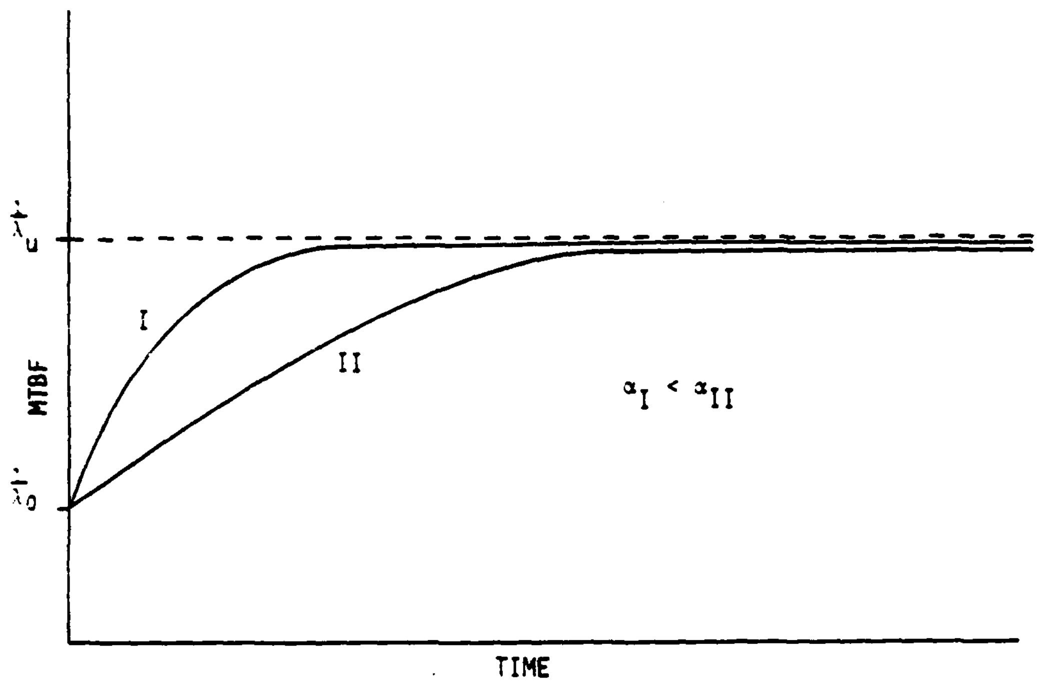

Fig. 1. TYPICAL GROWTH CURVES

The three parameters which determine the curve are λ_0, the failure rate of the system at the start of the program; λ_u, the ultimate failure rate of the mature system; α, a parameter which determines the shape of the curve.

Assessment of a program involves the determination of these three parameters from test data. Planning requires that they be predicted at the start of a test program in order to estimate the proper length of the test phase. Assessment is easier to deal with mathematically and can be approached by classical and Bayesian methods. Planning, without extensive analysis and modelling, is more subjective. In fact, it is safe to say that it is almost always done on the basis of informed judgment. It appears that this is the principal reason that most of the work in reliability growth deals with assessment. This subject will be dealt with first here as well.

3. ASSESSMENT

Two approaches will be formulated. Maximum likelihood will be discussed first followed by a Bayesian analysis.

A. Maximum Likelihood

The assumptions of the model imply exponentially distributed MTBF's with parameter λ_n. Hence the likelihood function for the <u>failure truncated</u> case is

$$L = \lambda_o \lambda_1 \cdot\cdot \lambda_{n-1} \; e^{-\lambda_o T_1} \; e^{-\lambda_1 T_2} \ldots \; e^{-\lambda_{n-1} T_n} \tag{3.1}$$

where T_n is the time elapsing between failure number n-1 and failure number n. The procedure is to find values of the parameters which maximize L (or LnL). Recalling that

$$\lambda_n = \lambda_u + \alpha^n(\lambda_o - \lambda_u) \tag{3.2}$$

and substituting into eq. (3.1) and taking derivatives, one obtains the following equations for an extreme value:

$$\frac{\partial LnL}{\partial \lambda_o} = 0 = \sum_{i=0}^{n-1} \frac{\alpha^i}{\lambda_u + \alpha^i(\lambda_o - \lambda_u)} - \sum_{i=0}^{n-1} \alpha^i T_{i+1} \tag{3.3}$$

$$\frac{\partial LnL}{\partial \lambda_u} = 0 = \sum_{i=0}^{n-1} \frac{(1-\alpha^i)}{\lambda_u + \alpha^i(\lambda_o - \lambda_u)} - \sum_{i=0}^{n-1} (1-\alpha^i) T_{i+1} \tag{3.4}$$

$$\frac{\partial LnL}{\partial \alpha} = 0 = \sum_{i=1}^{n-1} \frac{i\,\alpha^{i-1}\,(\lambda_o - \lambda_u)}{\lambda_u + \alpha^i(\lambda_o - \lambda_u)} - \sum_{i=1}^{n-1} i\alpha^{i-1}(\lambda_o - \lambda_u) T_{i+1} \tag{3.5}$$

The result is three simultaneous nonlinear equations for λ_o, λ_u, and α. There are several standard numerical approaches to the solution of these equations. The method used here is relaxation, that is, solving the equations one at a time, and iterating until convergence. The method of solution of each individual equation is Newton-Raphson. It should be noted that <u>time truncated</u> testing where the sequence does not end with a failure, but rather at a specific time changes the problem only slightly. If the amount of time expired after the $n^{\underline{th}}$ failure is T, then the likelihood function is

$$L = \lambda_o \lambda_1 \lambda_2 \cdots \lambda_{n-1} e^{-\lambda_o T_1} e^{-\lambda_1 T_2} \cdots e^{-\lambda_{n-1} T_n} e^{-\lambda_n T} \quad (3.6)$$

The corresponding numerical problem is essentially the same as the failure truncated case and the solution method is the same as previously indicated.

The growth curve is a plot of the expected value of MTBF against time. In order to make this computation, we introduce

$P_n(t)$ = Probability of exactly n failures by time t.

Since the process is assumed Poisson between failures, $P_n(t)$ satisfies

$$\frac{dP_o}{dt} = -\lambda_o P_o \quad (3.7)$$

$$\frac{dP_n}{dt} = -\lambda_n P_n + \lambda_{n-1} P_{n-1} \qquad n = 1,2,\ldots \quad (3.8)$$

The expected time between failure $\overline{M}(t)$ is, then

$$\overline{M}(t) = \sum_{j=0}^{\infty} \frac{1}{\lambda_j} P_j(t) \quad (3.9)$$

The solution to the system of equations (3.7) and (3.8) is given below for the case where all the λ_n are distinct, and there are no failures at time t = 0. Then,

$$P_o(t) = e^{-\lambda_o t}$$

$$P_1(t) = \frac{\lambda_o}{(\lambda_o - \lambda_1)} e^{-\lambda_1 t} - \frac{\lambda_o}{(\lambda_o - \lambda_1)} e^{-\lambda_o t}$$

and

$$P_n(t) = \sum_{i=o}^{n} A_{in} e^{-\lambda_i t}$$

where

$$A_{in} = \frac{\prod_{k=o}^{n-1} \lambda_k}{\prod_{\substack{k=o \\ k \neq i}}^{n} (\lambda_k - \lambda_i)}$$

This method of solution, although exact, is not the most efficient numerically. A Monte Carlo method, which is described in the appendix, is useful for generating $\overline{M}(t)$.

Sample cases are given below for both time and failure truncation. These were taken from a previous report [2] which used the AMSAA model for deriving the growth curves. Those results are reproduced here for comparison.

Table 1. System Failure Times

Failure Number	Time to Fail	Cumulative Time
1	1.5	1.5
2	1.7	3.2
3	8.6	11.8
4	17.8	29.6
5	24.0	53.6
6	11.6	65.2
7	54.2	119.4
8	145.9	265.3
9	28.7	294.0
10	147.1	441.1
11	24.0	465.1
12	101.9	567.0
13	118.8	685.8
14	145.6	831.4
15	118.3	949.7

These data were used to calculate λ_o, λ_u, and α (using equations 3.3 - 3.5). The results being λ_o= .672 per hour, λ_u = .00919 per hour, α = .476. Table 2 displays the $\overline{M}(t)$ calculations for both methods.

Table 2. MTBF Calculations - Failure Truncated

Time (Hrs)	$\bar{M}(t)$-Current Method	$\bar{M}(t)$-AMSAA
200	78.3	59.3
400	102.3	86.2
600	105.7	107.2
800	107.5	125.2
949.7	107.9	137.3
1000	108.0	141.2

A related, but time truncated case assumes the system just discussed was tested beyond the fifteenth failure until 1000 hours at which point testing ceased. This time truncated case gives

$$\lambda_o = .649, \quad \lambda_u = .00848, \quad \alpha = .487$$

Table 3. MTBF Calculations - Time Truncated

Time (Hrs)	$\bar{M}(t)$-Current Method	$\bar{M}(t)$-AMSAA
200	77.3	61.1
400	104.5	89.5
600	111.7	111.8
800	114.5	130.9
1000	116.9	148.0

The application of the two methods is further illustrated by two cases which were generated by simulation of a system which was Poisson between failures having the following failure rates.

Case 1

$\lambda_0 = .0625$	$\lambda_5 = .02048$	$\lambda_{10} = .0067108$
$\lambda_1 = .05$	$\lambda_6 = .016384$	$\lambda_{11} = .0053686$
$\lambda_2 = .04$	$\lambda_7 = .0131072$	$\lambda_{12} = .0042948$
$\lambda_3 = .032$	$\lambda_8 = .0104857$	$\lambda_{13} = .0034358$
$\lambda_4 = .0256$	$\lambda_9 = .0083885$	$\lambda_{14} = .0027486$
		$\lambda_{15} = .0021988$

The randomly generated failure times are:

Failure Number	Time to Fail	Cumulative Time
1	23.9	23.9
2	21.0	44.9
3	64.8	109.7
4	37.7	147.4
5	81.4	228.8
6	146.0	374.8
7	104.6	479.4
8	35.4	514.8
9	179.4	694.2
10	163.9	858.1
11	292.7	1150.8
12	99.8	1250.6
13	339.4	1590.0
14	603.7	2193.7
15	192.1	2385.8

Application of the current method, the AMSAA model, and the underlying MTBF are given in the next table.

Table 4. Model Generated Case 1

Time	$\overline{M}(t)$-Current Method	$\overline{M}(t)$-AMSAA	$\overline{M}(t)$-Model
300	100.5	114.0	91.0
600	163.0	153.6	166.0
900	206.1	182.0	241.0
1200	247.2	205.6	316.0
1500	291.0	225.5	391.0
1800	321.3	244.0	466.0
2100	356.0	260.3	541.0
2400	381.0	276.6	616.0

Case 2 is produced in exactly the same manner as Case 1, however, different values of λ_j are used.

Case 2

$\lambda_0 = .06$
$\lambda_1 = .048$
$\lambda_2 = .0336$
$\lambda_3 = .02$
$\lambda_4 = .01$
$\lambda_5 = .008$
$\lambda_6 = .007$
$\lambda_7 = .0065$
$\lambda_8 = .0061$
$\lambda_9 = .0058$
$\lambda_{10} = .0056$
$\lambda_{11} - \lambda_{15} = .0055$

The randomly generated failure times are

Failure Number	Time to Fail	Cumulative Time
1	12.2	12.2
2	2.6	14.8
3	.7	15.5
4	4.9	20.4
5	43.1	63.5
6	37.6	101.1
7	74.6	175.7

Failure Number (Continued)	Time to Fail	Cumulative Time
8	16.4	192.1
9	480.3	672.4
10	253.4	925.8
11	19.0	944.8
12	203.8	1148.6
13	26.9	1175.5
14	259.0	1434.5
15	69.7	1504.2

The results of MTBF calculations are given below.

Time	$\overline{M}(t)$-Current Method	$\overline{M}(t)$-AMSAA	$\overline{M}(t)$-Model
300	119.2	90.0	134.6
600	160.0	128.3	159.8
900	179.7	158.0	172.2
1200	188.7	182.7	178.0
1500	194.1	204.9	180.4

B. Bayesian Formulation

This method treats λ_o, λ_u and α as random variables with some prior distribution functions. As test data become available, posterior distributions are calculated by Bayes theorem and the "updated" distributions are used to predict MTBF.

Let λ_o have possible values λ_{oi} for i = 1, 2,..., N_o; λ_u have values λ_{uj} for j = 1, 2,..., N_u and α have values α_k for k = 1,2,..., $N\alpha$. The prior distribution is given by μ_{ijk} = Probability $[\lambda_o = \lambda_{oi}, \lambda_u = \lambda_{uj}, \alpha = \alpha_k]$. The posterior distribution μ^*_{ijk}, given a set of N failure times $T_1, T_2 \ldots, T_n$ is calculated to be

$$\mu^*_{ijk} = \frac{[\lambda_0\lambda_1\cdots\lambda_{n-1}e^{-\lambda_0 T_1}\ e^{-\lambda_1 T_2}\cdots e^{-\lambda_{n-1}T_n}]_{ijk}\ \mu_{ijk}}{\sum_{i=1}^{N_0}\sum_{j=1}^{N_u}\sum_{k=1}^{N_\alpha}[\lambda_0\lambda_1\cdots\lambda_{n-1}\ e^{-\lambda_0 T_1}\ e^{-\lambda_1 T_2}\cdots e^{-\lambda_{n-1}T_n}]_{ijk}\ \mu_{ijk}} \tag{3.10}$$

where the notation

$$[\lambda_0\lambda_1\cdots{}_{n-1}e^{-\lambda_0 T_1}\ e^{-\lambda_1 T_2}\cdots e^{-\lambda_{n-1}T_n}]_{ijk} \tag{3.11}$$

indicates that the λ's in the expression are calculated according to eq. (3.2) with $\lambda_0 = \lambda_{0i}$, $\lambda_u = \lambda_{uj}$ and $\alpha = \alpha_k$. There is a set of state probabilities corresponding to each i, j, k. Namely

$$\frac{dP_{0ijk}}{dt} = -\ \lambda_{ijk}\ P_{0ijk}$$

$$\frac{dP_{nijk}}{dt} = -\ \lambda_{nijk}\ P_{nijk} + \lambda_{n-1\ ijk}\ P_{n-1\ ijk} \tag{3.12}$$

The expected MTBF for these values of i, j, k is

$$\overline{M}_{ijk}(t) = \sum_{n=0}^{\infty} \frac{1}{\lambda_{nijk}}\ P_{nijk}(t) \tag{3.13}$$

and, averaged over all values of i, j, k, yields

$$\overline{M}(t) = \sum_{i=1}^{N_o}\ \sum_{j=1}^{N_u}\ \sum_{k=1}^{N_\alpha}\ \mu^*_{ijk}\ \overline{M}_{ijk}(t) \tag{3.14}$$

It is readily seen that the number of required calculations can be very large since a system of differential equations must be solved for each i, j, and k. The process of finding posterior distributions is illustrated below using, once again, the data from Table 1. λ_o has three possible values .6, .7, .8; λ_u has five possible values .007, .008, .009, .01, .011; and α has four possible values .3, .4, .5, .6. The prior distribution is uniform

$$\mu_{ijk} = 1/60 \quad \text{all } i, j, k.$$

The posterior distribution, given the fifteen failure times in Table 1 is given by three matrices (corresponding to λ_o = .6, .7 and .8).

$\lambda_o = .6$

	$\lambda_u = .007$	$\lambda_u = .008$	$\lambda_u = .009$	$\lambda_u = .01$	$\lambda_u = .011$
$\alpha = .3$	.0026	.0043	.0060	.0075	.0083
$\alpha = .4$	.0123	.0174	.0215	.0237	.0239
$\alpha = .5$	.0323	.0373	.0386	.0367	.0325
$\alpha = .6$	.0087	.0076	.0062	.0047	.0035

$\lambda_o = .7$

$\alpha = .3$	.0036	.0057	.0080	.0097	.0107
$\alpha = .4$	.0159	.0219	.0265	.0288	.0287
$\alpha = .5$	.0318	.0357	.0360	.0335	.0290
$\alpha = .6$	.0036	.0031	.0024	.0018	.0013

$\lambda_o = .8$

$\alpha = .3$	.0045	.0072	.0098	.0118	.0129
$\alpha = .4$	.0186	.0252	.0300	.0321	.0315
$\alpha = .5$	.0278	.0304	.0300	.0274	.0233
$\alpha = .6$	.0013	.0011	.0008	.0006	.0004

Unfortunately, the calculation of $\overline{M}_{ijk}(t)$ and subsequently the final growth curve $\overline{M}(t)$ is very time consuming, a fact which renders this approach undesirable. A modified Bayesian approach assumes that λ_o and α are known, and that λ_u has a distribution. This method is illustrated next, again with data from Table 1. Take λ_o = .672 and α = .476 and let λ_u have five possible values .007, .008, .009, .010, .011 with a uniform prior.

$$\pi_k = .2 \qquad k = 1,\ldots,5$$

Calculation of the posterior distribution using equation (3.10), modified for a fixed value of λ_o and α, yields

	Prob ($\lambda_u = \lambda_{uk}$ | Test data)
λ_{u1}(.007)	.1713
λ_{u2}(.008)	.2047
λ_{u3}(.009)	.2185
λ_{u4}(.010)	.2129
λ_{u5}(.011)	.1925

The corresponding $\overline{M}_k(t)$ are

Time (hrs.)	$\overline{M}_1(t)$	$\overline{M}_2(t)$	$\overline{M}_3(t)$	$\overline{M}_4(t)$	$\overline{M}_5(t)$
100	68.1	67.1	62.2	59.1	58.6
200	103.9	90.8	85.6	79.1	76.6
300	118.0	107.1	96.7	87.7	83.8
400	127.6	114.4	101.5	93.5	87.2
500	133.6	117.4	105.5	96.9	88.5
600	136.1	119.9	107.6	98.3	89.6
700	137.6	120.8	109.4	98.8	90.1
800	139.0	122.0	110.2	99.4	90.4
900	140.4	123.0	110.6	99.7	90.6
1000	141.3	123.5	110.7	99.8	90.7

The composite MTBF $\overline{M}(t)$ is

Time (Hrs)	$\overline{M}(t)$	Time (Hrs)	$\overline{M}(t)$
100	62.9	600	109.5
200	86.7	700	110.6
300	98.1	800	111.4
400	104.1	900	112.1
500	107.6	1000	112.4

This method is tractable and relatively simple to use. It has capability to match the early data (influenced mostly by λ_o and α), and the later data (represented more by λ_u).

C. Sensitivity

A complete sensitivity analysis of the model described herein is beyond the scope of this study. However, some excursions about the base case (Table 1) have been taken to get an indication of how certain parameters are affected by these changes. Specifically, the values of λ_o, α, and λ_u were calculated on the basis of 5, 10, and 15 tests.

<u>5 Tests</u>

λ_o = .817 λ_u = .0215 α = .379

<u>10 Tests</u>

λ_o = .656 λ_u = .00790 α = .487

<u>15 Tests</u>

λ_o = .672 λ_u = .00919 α = .476

In addition, changes were made in T_1 and T_2, in Table 1, which held $T_1 + T_2$ constant. All other times to fail were the same. The results were:

<u>T_1 = .4 , T_2 = 2.8</u>
λ_o = 1.034 λ_u = .00982 α = .407

<u>T_1 = 1.5, T_2 = 1.7</u>
λ_o = .672 λ_u = .00919 α = .487

<u>T_1 = 2.8, T_2 = .4</u>
λ_o = .5199 λ_u = .00879 α = .516

These excursions were chosen because of the intuitive conclusion (based on the underlying model) that λ_u should be more sensitive to late test data while λ_o should be most sensitive to early test data. These results tend to support this conclusion, but in-depth analysis would be required for proof.

D. Block Data

There are situations in many test programs which make it impossible to report exact failure times. In such cases, it may be possible to report the number of failures in a given time. This block reporting modifies the maximum likelihood formulation slightly. Poisson processes have a "memoryless" character. That is, succeeding failures do not depend on the time at which previous failures occurred. Only the rate of failure changes, and that only depends upon how many failures have preceded, not upon when they occurred. Therefore, the likelihood function is:

$$L = P_{n1} (T_1) P_{n2} (T_2). \ . \ . \ . \ P_{nk} (T_k)$$

where $P_{n1}(T_1)$ is the probability that n1 failures occur in time T_1 (with failure rates λ_o, λ_1, λ_2. . .λ_{n1}). $P_{n2}(T_2)$ is the probability that n2 failures occur in time T_2 (with failure rates $\lambda n1+1$, $\lambda n1+2$, . . .$\lambda n1+n2$) etc.

Since each of the λ_k is a function of λ_o, λ_u and α, the likelihood function is also. This leads to an optimization problem in these parameters much like the previous cases.

4. PREDICTION

The problem of prediction of the growth process is probably more important, from the program management point of view, than is assessment. This is so because prediction is required at the outset to establish program goals and a consistent test program. Yet, prediction is less studied and less rigorous than analysis. Historical data and comparative analysis are the basis of an earlier method [3].

The model described herein is an attempt to describe mathematically the *underlying test-analysis-and fix process*. It is this view which also provides insight into the prediction problem.

The development of most new complex systems involves incorporation of a number of subsystems each with a varying degree of maturity of design. The early failures in a development program generally come as a result of immature designs in some subsystems, or, in interfacing previously unrelated subsystems. It is felt that the ultimate failure rate of each subsystem can be estimated rather well by summing the stated failure rates on all the components. Hence λ_u in the model is predicted on the basis of a "parts count" reliability estimate. The two parameters λ_o and α are not estimated directly. Rather, the number of interface and immature design mistakes are estimated, on the basis of complexity and maturity (subjective). These are then used to calculate the predicted growth curve. As an example consider part of a fire control system which includes four major subsystems: radar, computer, power system, communications link. The radar is state-of-the-art and on the basis of experience has an MTBF of 1000 hours. The computer, although state-of-the-art, incorporates completely new packaging and hardening. It has an estimated ultimate MTBF of 500 hours, but a "new environment" type failure is probable (with λ = .1/hr) and an interfacing failure (λ = .01/hr) may also occur. The power system is very reliable with MTBF of 10,000 hrs. and only an interfacing failure of λ = .001/hr. Likewise, the communications link is very reliable (MTBF = 10,000 hrs.) with an interfacing failure estimated as λ = .001/hr. The predicted growth process can now be simulated. As a failure occurs, it is corrected. If it is an interface or immature design failure, it is assumed corrected after it occurs. If a routine failure occurs, no change in any subsystem is assumed. A summary of the subsystems, with their failure types and rates is given below

Subsystem	Failure Type	Failure Rate (λ)
Radar	Routine	.001
Computer	Routine	.002
	Immature Design	.1
	Interface	.01
Power System	Routine	.0001
	Interface	.001
Communications Link	Routine	.0001
	Interface	.001

A test analysis and fix program on this system was simulated with the following results:

Failure Number	Elapsed Time (hrs)	Subsystem	Type of Failure
1	8.9	Computer	Immature Design
2	43.5	Computer	Interface
3	86.0	Computer	Routine
4	43.2	Power System	Interface
5	251.0	Radar	Routine
6	205.4	Communications Link	Interface

Such a program on this hypothetical system starts with an overall system MTBF of 8.7 hours. Because of the failures which would have been discovered and corrected (failures 1, 2, 4, 6), the MTBF at the end of six test-analysis-and-fix steps would be 238 hours. The ultimate MTBF (all discoverable failures corrected) is 312.5 hours. This simulation process would be replicated several times to get an expected growth curve for this system. This would then be used to establish an initial growth curve for the program.

5. CONCLUSIONS

The intent of this study was to model, as tractably and accurately as possible, the test-analysis-and-fix of continuous systems. The model is described, along with the analytical methods required to apply it to assessment or prediction. The results of such calculations on some given data and some simulated data yield encouraging results. Because the model is a three-parameter it is more flexible in fitting a wider range of data than two-parameter models such as Duane. However, some of the numerical analysis required for application of the larger model are likewise more demanding.

Of the two formulations, Maximum Likelihood and Bayesian, the former is more tractable and would be the recommended alternative on the basis of time and effort required to apply the method. However, further work in this area might lead to more practical methods of generating the Bayesian growth curves and this is one areas of future work which warrants consideration. More work is also needed to develop algorithms for block data applications.

Prediction applications require detailed case studies to determine whether interface and immature design failures can be adequately predicted. It should be noted that this is in fact being done when the number and type of tests are specified. It is recommended that this be formally recognized and incorporated into a plan.

The results, then, are:

1. The model appears to have the necessary flexibility (more than some other models) to be applicable in assessment and prediction.
2. Efficient numerical routines are required for full application of the method.
3. The potential in prediction is great and the methodology needs considerable effort.

APPENDIX

1. Monte Carlo Method for Growth Curves

Integration of eqs. (3.8), particularly for long times, uses considerable computer time. The underlying process is, however, Poisson. Therefore MTBF's are exponentially distributed. Consequently it is easy to simulate the underlying process. The steps are as follows:

(a) For a given λ_o, λ_u and α calculate $\lambda_1, \lambda_2, \ldots \lambda_N$.

(b) Generate a random number X_1 (presumably uniformly distributed).

(c) Convert this uniform random number to an exponentially distributed one with mean λ_o by the transformation $T_1 = -1/\lambda_o \ \ln (1-X_1)$. This is the elapsed time until failure number 1.

(d) Repeat steps b and c using λ_1 instead of λ_o to get the time T_2.

(e) Continue generating T_3, T_4, etc.

(f) Terminate the process when the total time reaches or exceeds the period of interest.

These steps provide one realization of the process. Replication of this method yields other realizations, and these can be used to calculate approximate values of $P_n(t)$. These, in turn, are used in the calculation of $\overline{M}(t)$

$$\overline{M}(t) = \sum_{n=0}^{\infty} \frac{1}{\lambda_n} P_n(t)$$

A computer program to accomplish 50 replications has been run on a PDP 11-70 in less than one minute. Solving the exact differential equations for $P_n(t)$ for the same problem and same length of time requires an order of magnitude more computer time.

2. Single Differential Equation Approximation

The difference equation form of eq. (2.1) is

$$\lambda_n - \lambda_{n-1} = -(1-\alpha)\lambda_{n-1} + (1-\alpha)\lambda_u$$

The differential equation approximation to this is

$$\frac{d\lambda}{dn} \doteq -(1-\alpha)\lambda + (1-\alpha)\lambda_u$$

where n is the number of failures. Changing to time as the independent variable, one can write

$$\frac{d\lambda}{dt}\,\frac{dt}{dn} = -(1-\alpha)\lambda(t) + (1-\alpha)\lambda_u.$$

Moreover,

$$\frac{dn}{dt} = \lambda$$

Hence,

$$\frac{1}{\lambda}\,\frac{d\lambda}{dt} = -(1-\alpha)\lambda(t) + (1-\alpha)\lambda_u.$$

Finally, $M(t) = 1/\lambda(t)$

which gives

$$-\frac{dM}{dt} = -(1-\alpha) + (1-\alpha)\lambda_u M.$$

The solution to the previous equation is

$$M(t) = \frac{1}{\lambda_u} + [1/\lambda_o - 1/\lambda_u]\, e^{-(1-\alpha)\lambda_u t}$$

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