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UNITED STATES AIR FORCE

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Wright-Patterson Air Force Base, Ohio

AFIT/GOR/MA/79D-8

INCORPORATION OF ASYMPTOTIC NORMALITY PROPERTIES OF THE BINOMIAL DISTRIBUTION INTO A MONTE CARLO TECHNIQUE FOR ESTIMATING LOWER CONFIDENCE LIMITS ON SYSTEM RELIABILITY

THESIS

AFIT/GOR/MA/79D-8

Roy E. Rice
Capt USAF

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THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
in Partial Fulfillment of the
Requirements for the Degree of
Master of Science

by
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Preface

Many methods of obtaining confidence limits for system reliability exist in both practice and literature. The understanding and use of these techniques are often quite difficult. Because of this, I embarked on an effort to devise a technique that was theoretically sound, easy to understand, and simple to use. This report discusses that technique and its accuracy.

Sincere gratitude is extended to Dr. Albert H. Moore for his guidance, expertise, and motivation toward completion of this study. Thanks also goes to Mr. Joseph J. Meli, ASD/ENADC, who was my sponsor.

Roy E. Rice

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Abstract

Several methods for estimating lower confidence limits were examined. A proposed technique was developed and used to obtain limits for selected systems. These limits were compared to those obtained by other methods. Then the proposed method was tested to verify its accuracy. Results indicate that the proposed method is simple to understand, easy to implement, and accurate.

INCORPORATION OF ASYMPTOTIC NORMALITY PROPERTIES OF THE BINOMIAL DISTRIBUTION INTO A MONTE CARLO TECHNIQUE FOR ESTIMATING LOWER CONFIDENCE LIMITS ON SYSTEM RELIABILITY

I. Introduction

In the late 1920's and early 1930's, prominent statisticians began to realize that a single point estimate for the probability of occurrence of an event was not sufficient. Initial studies by Fischer, Neyman, Clopper, Pearson, and many others show the necessity and practicality of placing probabilistic bounds on these estimates. The application of these intervals injects confidence into the accuracy of these point estimates. Through these pioneering efforts we arrived at the concept of confidence intervals.

This analysis intends to introduce some of the more important historical works that address the calculation of confidence limits. It contains considerably detailed discussions of the evolution of various techniques for deriving confidence limits with major emphasis on intervals for binomial distributed parameters. There are discussions of techniques involving Bayesian Limits using Mellin Transforms (Ref 22:611), Approximately Optimum Limits (Ref 18: 335-347), Monte Carlo simulation (Ref 19:459), and others. There is then a discussion of the proposed method of incorporating the asymptotic normality assumption into a Monte

Carlo technique. Finally, a verification technique is included that shows how accurate the confidence limits are.

Component test data is obtained where the number of failures is assumed to be distributed binomially. Then the components are arranged in a logically complex system and an estimate of the reliability and confidence limit is made. These estimates from the proposed method are then compared with estimates from the other methods. A final effort is made to verify the accuracy of the limits by simulating the testing of the components and determining the number of times the limits cover the true reliability.

Of particular interest is the treatment of test data in which no failures occur. Many of the methods are not valid when the data shows zero failures. Others simply disregard the samples with zero failures. Still others manipulate the data to circumvent the problem. Almost all of this results in a biasing effect on the estimates. This study explores another way of handling samples that exhibit no failures.

Coupled with the improvement of these estimating techniques is the advancing computer technology. High speed computers make it increasingly easier and more efficient to make detailed computations and simulations. Thus, it is becoming more practical to apply these methods to systems exhibiting pass/fail characteristics from the simple (switches, pins, relays) to the very complex (missile flights, docking systems, targeting).

In the following study there are several basic assumptions. It is assumed that the reader is familiar with the concept of confidence limits, the binomial distribution, and has a basic understanding of statistics and reliability pertaining to both components and systems.

II. Background of Estimating Techniques

In one of the first studies in this area, Springer and Thompson (Ref 22:611) examined the posterior density functions of $R_i (i=1,2,...,N)$, the binomial parameters of N independent subsystems, when the prior distribution is uniform or Beta. The Bayes posterior density function of R_i in the uniform prior case is

$$f_i(R_i) = \frac{(n_i+1)!}{m_i!(n_i-m_i)!} R_i^{m_i}(1-R_i)^{n_i-m_i}$$

$$(0 \leq R_i \leq 1, i=1,2,...,N)$$

where m_i = number of successes in n_i trials.

Epstein gives the Mellin integral transform of $f_i(R_i)$ as

$$M\{f_i(R_i)|s\} = \frac{(n_i+1)!}{m_i!} \frac{(s+m_i)}{(s+n_i+1)}, \ R_e(s) > m_i .$$

Huntington shows that the Mellin convolution of $f_1(R_1)$ and $f_2(R_2)$ is exactly the probability density function of the product R_1R_2 and that the repeated convolution of the probability density functions is equivalent in the transform to repeated multiplication of the Mellin transforms. From this, Springer and Thompson derive a closed form of the posterior distribution function for N independent subsystems in series, H(R). At this point, they emphasize that, for large N, computation of H(R) is tedious for hand computations and extensive using computers. Thus, they conclude that some asymptotic method might warrant investigation. For more detailed investigation of their method, see Reference 22.

Later, in a study by Mann and Grubbs (Ref 18:335-347), confidence bounds for system reliability are derived based on component test data when the components are assumed to be (1) exponentially distributed with censoring or truncation of tests for a fixed number of failures, (2) exponentially distributed with truncation of tests at fixed times, and (3) binomially distributed (pass-fail) with fixed but different sample sizes and random numbers of failures for subsystem tests. With regard to the binomial distribution, Mann arrives at approximate expressions for m_s and v_s, the mean and variance of $-\ln R_s$ which corresponds to these optimum nonrandomized confidence bounds for series systems. These are

$$m_s = \frac{.5(1+\frac{1}{a})}{n^o} + \frac{\hat{P}}{(1-.5\hat{P})}$$

and

$$v_s = \frac{.5(1-\frac{1}{a})m_s}{n^o} ,$$

where $n^o = n_{(1)}(1-.5\hat{P}^2)(1-.5\hat{P})$

$n_{(1)}$ is the smallest of the $n_1, n_2, \ldots, n_k$.

$\hat{P} = 1 - \prod_{j=1}^{k} \hat{R}_j$ is the max-likelihood estimate of the probability of failure of the series system.

$a = n_{(1)} \Sigma(1/n_j)$ which is a restricted sum.

The n_j's for all zero-failure subsystems are omitted in calculating the value of $a = n_{(1)} \Sigma(1/n_j)$ except for a single zero-failure subsystem with its sample size equal to $n_{(1)}$. Mann and Grubbs point out that this subsystem is also ignored if at least one of the other subsystems which exhibit some failures has the smallest sample size $n_{(1)}$, or if the lower confidence bound obtained by not ignoring it is larger than that obtained by ignoring it.

> "The exclusion of some subsystems in calculating series-system confidence bounds also is necessitated when all sample sizes are large and most of the failures pertain to a single subsystem. In this case the bound obtained based on all subsystem failures can be larger than that based on the single subsystem exhibiting nearly all the failures." (Ref 18:337)

In their analysis they compare their bounds with those based on Buehler's exact methods, those based on maximum likelihood normal approximations, the modified maximum likelihood procedure of Easterling, and the likelihood ratio method of Madansky. These last three procedures cannot be applied effectively when one or more subsystems exhibit zero-failures. This comparison, along with the estimates from the proposed method, is shown in Tables I, II, III, and IV.

Another technique that is of extreme importance is that developed by Gatliffe (Ref 7). Mathematically, Gatliffe begins by stating that, for systems with k components in series,

$$R_s = \prod_{i=1}^{k} P_i \text{ , } P_i \text{ is the reliability of the } i^{th} \text{ component}$$

$$Q_i = 1 - P_i \text{ .}$$

Let $$s = -\ln R_s = -\sum_{i=1}^{k} \ln(1 - Q_i)$$

$$s = \sum_{i=1}^{k} (Q_i + \frac{Q_i^2}{2} + \frac{Q_i^3}{3} + \ldots)$$

$$\approx \sum_{i=1}^{k} (Q_i + \frac{Q_i^2}{2}) = \sum_{i=1}^{k} T_i \text{ , } T_i = (Q_i + \frac{Q_i^2}{2}) \text{ .}$$

The unbiased estimator, $\hat{T}_i = A_i\hat{Q}_i + B_i(\hat{Q}_i^2/2)$

$$A_i = \frac{(2N_i - 3)}{(2N_i - 2)}$$

$$B_i = \frac{N_i}{N_i - 1} \text{ ,}$$

$$Q_i = \frac{F_i}{N_i} \text{ , } F_i = \text{the number of failures resulting from } N_i \text{ mission tests conducted on } i^{th} \text{ component.}$$

He establishes another unbiased estimator,

$$\hat{s} = \sum_{i=1}^{k} \hat{T}_i$$

with
$$\mathrm{Var}(\hat{s}) = \sum_{i=1}^{k} (T_i/N_i) \ .$$

He intuitively assumes the probability distribution of $\hat{s}$ is gamma with parameters r and θ,

$$f_{\hat{s}}(x,r,\theta) = \begin{cases} \frac{1}{\Gamma_{r} \cdot \theta^{r}} x^{r} e^{-x/\theta} & , x \geq 0 \\ 0 & , x < 0 \end{cases}$$

with mean = $r\theta$, variance = $r\theta^2$.

$$E(\hat{s}) = r\theta = \sum_{i=1}^{k} T_i$$

$$\mathrm{Var}(\hat{s}) = r\theta^2 = \sum_{i=1}^{k} (T_i/N_i)$$

$$r = \frac{(\Sigma T_i)^2}{\Sigma \frac{T_i}{N_i}}$$

$$\theta = \frac{\Sigma T_i/N_i}{\Sigma T_i}$$

$$\hat{r} = \frac{(\Sigma \hat{T}_i)^2}{\Sigma \hat{T}_i/N_i} \ .$$

Since $\hat{s}$ is distributed as gamma $(r, \hat{\theta})$, $\frac{2\hat{s}}{\theta}$ will be $\chi^2_{(2r)}$. From

$$1 - \alpha = P\left[\frac{2\hat{s}}{\theta} \geq \chi^2_{(2r,1-\alpha)}\right] \quad \text{and}$$

$$1 - \alpha = P\left[\theta \leq \frac{2\hat{s}}{\chi^2_{(2r,1-\alpha)}}\right] \quad \text{and since } \theta = \frac{E(\hat{s})}{r} \text{ and}$$

$E(\hat{s}) = -\ln(R_s)$, he has obtained a $100(1-\alpha)\%$ lower confidence limit, $R_s(\alpha)$, for R_s

$$R_s^*(\alpha) = \exp\left[\frac{-2r\hat{s}}{\chi^2_{(2r,1-\alpha)}}\right].$$

A $100(1-\alpha)\%$ lower confidence limit estimate, $\hat{R}_s^*(\alpha)$, for R_s would be

$$\hat{R}_s^*(\alpha) = \exp\left[\frac{-2\hat{r}\hat{s}}{\chi^2_{(2r,1-\alpha)}}\right].$$

It is apparent that a problem arises in the event of no failures in any components. This yields an $\hat{s}$ value of zero, thus, a system reliability confidence limit estimate of one (1).

Gatliffe outlines his procedure to deal with such an event as follows:

(1) Find the series component with the smallest actual or equivalent number of trails, N or N'.

(2) Change the number of component or subassembly failures to F' by the following scheme:

α	F'
.2	.37
.1	.25
.15	.16

(See derivation in Appendix A.)

(3) Recompute the lower confidence limit estimate using the basic log-gamma procedure with the revised failure data. This estimate is approximately equivalent to $(\alpha)^{1/N}$.

Gatliffe's scheme to arrive at equivalent failures was used in the proposed method. It is an amazingly simple, yet effective tool; and, for this reason, it is recommended that the reader refer to Appendix A.

The techniques discussed in this chapter were not used directly in the proposed method. Yet, they give different approaches that are of historical and analytical importance. They are primarily doorways that open up into a vast and intriguing area of study.

III. Proposed Method

The essence of this study is to examine a series-system with components that experience failures distributed binomially and derive a simple, yet accurate, method for estimating confidence limits for system reliability. The method of obtaining lower confidence limits that is proposed in this study draws upon two important concepts; the asymptotic normality properties of the binomial distribution and Monte Carlo simulation. Both warrant discussion.

It has been shown that, when a pass-fail component is tested a sufficient number of times (20 or more), its failures are distributed normally with a mean of $\hat{p}$ and a variance of $\frac{\hat{p}\hat{q}}{n}$. In this treatment, $\hat{p}$ is defined as the probability of success, $\hat{q}$ is $1-\hat{p}$, and n is the number of trials. So for any component i,

$$\hat{p}_i = 1 - \frac{F_i}{n_i}, \quad F_i = \text{the number of failures of } i^{th} \text{ component in } n_i \text{ trials},$$

$$\hat{q}_i = 1 - \hat{p}_i .$$

Note that if there are no failures, $\hat{p}_i = 1$ and the variance equals zero. The existence of components with a reliability absolutely equal to 1 is, to say the least, a rarity. To eliminate this problem, an equivalent number of failures, F_i', is assigned according to Gatliffe's method (see

Appendix A). Once these initial estimates have been obtained, they can be used in a Monte Carlo simulation.

This Monte Carlo method begins by assigning failures or equivalent failures to each component in a series system. From this a $\hat{p}_i$ is obtained for each component. Now, using the asymptotic normality property, a random variable from a normal distribution, with mean = 0 and variance = 1, is drawn. If $\frac{\hat{p}_i\hat{q}_i}{n_i}$ is the asymptotic variance, then the square root of this value is the standard deviation. Since a N(0,1) random variable is

$$Z = \frac{\hat{p}_i - \mu}{\delta} ,$$

one can take this random variable, multiply it by the asymptotic standard deviation, and add it to the mean, $\hat{p}_i$. What is obtained is a second estimate of p_i,

$$\hat{\hat{p}}_i = N(\hat{p}_i, \frac{\hat{p}_i\hat{q}_i}{n_i}) .$$

If this is done for each component, one can obtain an estimate of the system reliability for k components in series,

$$\hat{R}_s = \prod_{i=1}^{k} \hat{\hat{p}}_i .$$

A Monte Carlo simulation (Ref 9,10,15,19,20,21) is used to generate a number of these system reliability estimates. These estimates are then ranked in ascending order.

The ordered reliabilities partition the interval [0,1] into n+1 equally probable intervals. For simplicity, let n+1 equal 1000. From the ordered simulation reliabilities one can obtain any desired lower confidence limit by extracting the 100(1-α) percentile value. For instance, with 999 ordered reliability estimates, the 90 percent lower confidence limit would be the 99th ordered estimate.

The outline of this procedure is as follows:

(1) For each component in turn assign the number of failures, F_i, or equivalent failures, F_i',

(2) Calculate the first estimate

$$\hat{p}_i = 1 - \frac{F_i}{n_i} \text{ or } 1 - \frac{F_i'}{n_i} ,$$

$$\hat{q}_i = 1 - \hat{p}_i ,$$

$$\text{asymptotic variance} = \frac{\hat{p}_i \hat{q}_i}{n_i} ,$$

(3) Draw a random variable from N(0,1) for each component,

(4) Obtain a second estimate, $\hat{\hat{p}}_i \sim N(\hat{p}_i, \frac{\hat{p}_i \hat{q}_i}{n_i})$ by taking the N(0,1) random variable, multiply by the asymptotic standard deviation, and add it to $\hat{p}_i$ for each component,

(5) With the components in a logically complex system, arrive at a system reliability estimate, $\hat{R}_s = \prod_{i=1}^{k} \hat{\hat{p}}_i$,

(6) Repeat steps (3) through (5) 999 times,

(7) Rank order these 999 estimates of R_s,

(8) Extract the $100(1-\alpha)$ percentile value to obtain a $100(1-\alpha)$% lower confidence limit.

This procedure was applied to systems of 2 and 3 components in series. The computer program used (see Appendix B) shows that, with minor adjustments, larger systems can be examined. In these 2 and 3 component systems, various numbers of failures and tests were analyzed. These particular values were chosen so as to give a basis of comparison (see Tables I, II, III, and IV). The proposed method produces values that compare extremely well with those used in the Mann and Grubbs study (Ref 18:345).

Several points in the tables deserve note. First, the values obtained in this method most closely compare with those from the maximum likelihood method. Secondly, in every case, they are slightly higher than the "exact" values which tends to indicate upward biasness in the estimates. This latter aspect is what led to further investigation as to the accuracy of the estimates. The next chapter examines this area.

TABLE I

90% Limits, Two Components

No. of Components=2	No. of Failures		90% LCL					
	f_1	f_2	AN/MC	ML	LR	OPT	AO	MMLI
n=10	1	1	.655	.655	.629	.607	.606	.585
	1	2	.542	.545	.529	.497	.493	.489
	2	2	.458	.456	.451	.445	.430	.441
	1	4	.337	.347	.350	.344	.335	.318
	2	3	.372	.373	.375	.354	.353	.362
n=20	1	2	.754	.756	.739	.716	.728	.709
	2	2	.700	.701	.687	.683	.678	.669
	1	3	.693	.697	.683	.660	.675	.655
	2	3	.646	.647	.636	.622	.628	.619
	3	3	.599	.599	.591	.585	.582	.570

AN/MC = Proposed Method (Asymptotic Normality/Monte Carlo)
ML = Maximum Likelihood
LR = Likelihood Ratio (Madansky)
OPT = Exact Lower Bound
AO = Approximation Optimum (Mann)
MMLI = Modified Max Likelihood (Easterling)

TABLE II

95% Limits, Two Components

No. of Components=2	No. of Failures		95% LCL					
	f_1	f_2	AN/MC	ML	LR	OPT	AO	MMLI
n=10	1	1	.614	.611	.571	.548	.552	.530
	1	2	.495	.495	.473	.443	.435	.436
	2	2	.414	.405	.397	.392	.382	.391
	1	4	.290	.292	.301	.298	.293	.271
	2	3	.328	.320	.326	.304	.307	.315
n=20	1	2	.724	.728	.700	.677	.693	.671
	2	2	.669	.670	.647	.643	.643	.631
	1	3	.663	.665	.643	.620	.639	.616
	2	3	.612	.614	.597	.582	.593	.580
	3	3	.565	.565	.551	.544	.548	.532

TABLE III

90% Limits, Three Components

No. of Components=3	No. of Failures			90% LCL					
	f_1	f_2	f_3	AN/MC	ML	LR	OPT	AO	MMLI
n=20	1	1	1	.757	.760	.743	.747	.741	.721
	1	1	2	.704	.704	.690	.693	.689	.669
	1	2	2	.653	.654	.643	.639	.643	.619
	1	2	3	.608	.605	.596	.595	.597	.587
n=30	1	2	3	.724	.723	.714	.705	.716	.669
	1	1	1	.833	.835	.822	.825	.820	.803
	2	2	2	.722	.725	.715	.712	.717	.703
n=50	1	2	4	.806	.805	.798	.789	.800	.788
	1	1	2	.873	.874	.865	.861	.870	.852
n=100	1	1	2	.935	.936	.931	.929	.932	.923
	2	3	5	.866	.866	.861	.858	.865	.856

TABLE IV

95% Limits, Three Components

No. of Components=3	No. of Failures			95% LCL					
	f_1	f_2	f_3	AN/MC	ML	LR	OPT	AO	MMLI
n=20	1	1	1	.725	.732	.705	.709	.708	.684
	1	1	2	.667	.673	.651	.644	.657	.631
	1	2	2	.613	.621	.604	.598	.609	.580
	1	2	3	.567	.571	.557	.544	.554	.549
n=30	1	2	3	.693	.698	.683	.674	.691	.638
	1	1	1	.810	.816	.794	.796	.698	.775
	2	2	2	.694	.700	.685	.681	.692	.672
n=50	1	2	4	.784	.788	.776	.767	.781	.766
	1	1	2	.856	.860	.845	.841	.854	.833
n=100	1	1	2	.926	.929	.920	.918	.923	.913
	2	3	5	.852	.855	.848	.844	.854	.842

IV. Verification of Accuracy

To merely derive an estimating technique and its subsequent estimates is not sufficient. Analysis must be conducted to verify the accuracy of the method. As discussed in Chapter I of this study, the concept of confidence intervals and limits evolved from the need to know the accuracy of point estimates. But it is obvious upon examining Tables I, II, III, and IV that, depending on which method is used, values of these confidence limits vary considerably. This leads to an effort to verify the accuracy of the proposed method of obtaining lower confidence limits.

Basic to the theory of confidence limits is the fact that, if a system of known reliability is tested a number of times, the estimates of a $100(1-\alpha)\%$ confidence interval should include the true system reliability $100(1-\alpha)\%$ of the time. From this fact came the motivation for the subsequent analysis.

Given a system with known reliability that is made up of components with established probabilities of success, one can apply the procedures described in Chapter III to estimate desired lower confidence limits and compare these limits with the true system reliabilities. Then, by generating many of these limits, it can be determined how often the true reliability is in the interval [LCL,1]. For example, if 1000 80% lower confidence limits are generated, it is expected, theoretically, that 800 of the limits are lower than the true reliability.

Monte Carlo simulation, again, is the tool. Given k components with known probabilities of success, p_i, connected in series, a true system reliability is calculated,

$$R_s = \prod_{i=1}^{k} p_i .$$

Next, one must specify the number of tests that each component is to be subjected to, NT_i. To simulate the testing of each component, simply draw a uniform random variable in the interval [0,1] for each test. Now compare each of these NT_i random variables with p_i. If the random variable is greater than p_i, a failure is recorded. By doing this for each component, one obtains the first estimate of p_i,

$$\hat{p}_i = 1 - \frac{F_i}{NT_i} ,$$

$$\hat{q}_i = 1 - \hat{p}_i ,$$

$$\text{asymptotic variance} = \frac{\hat{p}_i \hat{q}_i}{NT_i} .$$

As done in the previous chapter, if no failures are experienced, Gatliffe's method is used to assign equivalent failures. This is done for each component. The proposed method then produces a second estimate of p_i by drawing a random variable from a N(0,1), multiplying by the asymptotic standard deviation, and adding this to $\hat{p}_i$. These estimates, $\hat{\hat{p}}_i$, are then combined to produce an estimate of R_s,

$$\hat{R}_s = \prod_{i=1}^{k} \hat{\hat{p}}_i \ .$$

Again, 999 of these estimates, $\hat{R}_s$, are generated and ranked in ascending order. Then the $100(1-\alpha)$ percentile value is extracted to yield a $100(1-\alpha)\%$ lower confidence limit estimate, $\hat{R}_{s_j}(\alpha)$.

Now, to test the accuracy of this method, many more repetitions of this testing and limit estimating are done. This yields a number of lower confidence limits that can be compared to the true system reliability. As stated before, one would expect $100(1-\alpha)\%$ of these $100(1-\alpha)\%$ lower confidence limits to include the true system reliability.

The outline of this procedure is as follows:

(1) Establish the value of p_i for each component and the number of tests to subject each component to, NT_i,

(2) For each component in turn, draw NT_i random variables from Uniform [0,1],

(3) For each component, compare these NT_i random variables to p_i; for each random variable that is greater than p_i, record a failure,

(4) Calculate the first estimate

$$\hat{p}_i = 1 - \frac{F_i}{NT_i} \text{ or } 1 - \frac{F_i'}{NT_i} \ ,$$

$$\hat{q}_i = 1 - \hat{p}_i \ ,$$

$$\text{asymptotic variance} = \frac{\hat{p}_i \hat{q}_i}{NT_i} ,$$

(5) Draw a random variable from N(0,1) for each component,

(6) Obtain a second estimate, $\hat{\hat{p}}_i$, by taking the N(0,1) random variable, multiply by the asymptotic standard deviation, and add it to $\hat{p}_i$ for each component,

(7) With the components in a logically complex system, calculate a system reliability estimate, $\hat{R}_s = \prod_{i=1}^{k} \hat{\hat{p}}_i$,

(8) Repeat steps (5) through (7) 999 times,

(9) Rank order these 999 estimates of R_s,

(10) Extract the 100(1-α) percentile value to obtain a 100(1-α)% lower confidence limit,

(11) Repeat steps (2) through (10) 1000 times to obtain 1000 of these 100(1-α)% lower confidence limits,

(12) Determine how many of these lower confidence limits are covering the true system reliability, $R_s = \prod_{i=1}^{k} p_i$.

The above procedure was applied to systems containing 2 and 3 components in series. The computer program used (see Appendix C) shows that, with minor adjustments, larger systems can be examined. In these 2 and 3 component systems, known probabilities of success, p_i, of 0.6, 0.8, 0.9, and 0.95 were examined to find 80%, 90%, and 95% lower confidence limits. Table V and Table VI show the number of lower confidence limits that contained the true system reliability for each of the systems with various values of p_i, α, and number of tests.

TABLE V

Two Component System

	No. of Tests	% of Limits Covering R_s		No. of Tests	% of Limits Covering R_s
$\boxed{P_i=.6}$ $\alpha=.2$	20 50 100	80 80 90	$\boxed{P_i=.8}$ $\alpha=.2$	20 50 100	81 84 82
$\alpha=.1$	20 50 100	90.3 91 91	$\alpha=.1$	20 50 100	89.5 90 90
$\alpha=.05$	20 50 100	95.1 95 96	$\alpha=.05$	20 50 100	94 94 96
$\boxed{P_i=.9}$ $\alpha=.2$	20 50 100	83 80 81	$\boxed{P_i=.95}$ $\alpha=.2$	20 50 100	86.8 76.2 78.2
$\alpha=.1$	20 50 100	88 90 91	$\alpha=.1$	20 50 100	86.8 89.1 88.1
$\alpha=.05$	20 50 100	95 92.6 93.8	$\alpha=.05$	20 50 100	87.9 88.6 91.6

TABLE VI

Three Component System

	No. of Tests	% of Limits Covering R_s		No. of Tests	% of Limits Covering R_s
$\boxed{P_i=.6}$ $\alpha=.2$	20 50 100	90 81 85	$\boxed{P_i=.8}$ $\alpha=.2$	20 50 100	79 84 82
$\alpha=.1$	20 50 100	94 90 94	$\alpha=.1$	20 50 100	92 91 90.4
$\alpha=.05$	20 50 100	97 95.4 96	$\alpha=.05$	20 50 100	94.1 97 95.5
$\boxed{P_i=.9}$ $\alpha=.2$	20 50 100	80 80 82	$\boxed{P_i=.95}$ $\alpha=.2$	20 50 100	83.2 77.5 78.7
$\alpha=.1$	20 50 100	90.3 89 94	$\alpha=.1$	20 50 100	94.2 88.7 89.1
$\alpha=.05$	20 50 100	93 97 94.3	$\alpha=.05$	20 50 100	95.8 90.5 93.5

Close examination of Table V and Table VI points out that accuracy declines as p_i gets larger and as α gets smaller. These results verify, as Mann and Grubbs mentioned (Ref 18:343), that an upward biasness might exist.

By studying the computer program in Appendix C, the reader should note several points. At higher values of p_i, some tests will exhibit no failures. Hence, it is possible that $\hat{\hat{p}}_i$ can be greater than unity. To eliminate this event, $\hat{\hat{p}}_i$ is chosen to be the minimum $[N(\hat{p}_i, \frac{\hat{p}_i\hat{q}_i}{NT_i}),1]$. Also, the reader should note that, on output, he is provided with an average lower confidence limit. In the generation of the 1000 lower confidence limits, they are averaged to produce the most accurate estimate.

To illustrate the utility and simplicity of this technique, consider the mission profile of an air-to-ground missile. Assume that it has eight independent mission stages with associated probabilities of success:

(1) Release from aircraft, $p_1 = 0.95$

(2) Air start, $p_2 = 0.95$

(3) Deploy flight control surfaces, $p_3 = 0.9$

(4) Arrive at battle area, $p_4 = 0.95$

(5) Terrain following/avoidance, $p_5 = 0.85$

(6) Penetrate defenses, $p_6 = 0.75$

(7) Hit target, $p_7 = 0.95$

(8) Detonate, $p_8 = 0.95$

These stages are analogous to a system with eight components in series. By multiplying all of the p_i's, the true system reliability (probability of killing the target) is determined to be 0.42059. Exercising the proposed method with 100 tests per component with the above p_i's, a 90% lower confidence limit for system reliability is 0.3665. In this particular case, 91.3% of the 1000 limits cover the true reliability. Therefore, we would say that it is a very accurate estimate.

Another example of the usefulness and simplicity of this technique is shown by estimating the lower confidence limits for the two simple parallel-series systems shown in Figures 1 and 2. In analyzing the three component system shown in Figure 1, the probability of success for each component, p_i, was chosen to be 0.9; and each component was tested 100 times. This design yields a system reliability of 0.891. The estimating technique provides an 80% lower confidence limit of 0.86842 with the lower limits covering the system reliability in 75% of the 1000 Monte Carlo repetitions. This would indicate that the LCL estimate is slightly high.

The five component system in Figure 2 was assigned the following p_i's:

(1) $p_1 = 0.95$

(2) $p_2 = 0.75$

(3) $p_3 = p_4 = 0.90$

(4) $p_5 = 0.80$

Calculations yield a system reliability of 0.9006 and a 90% lower confidence limit of 0.8671. These limits cover the

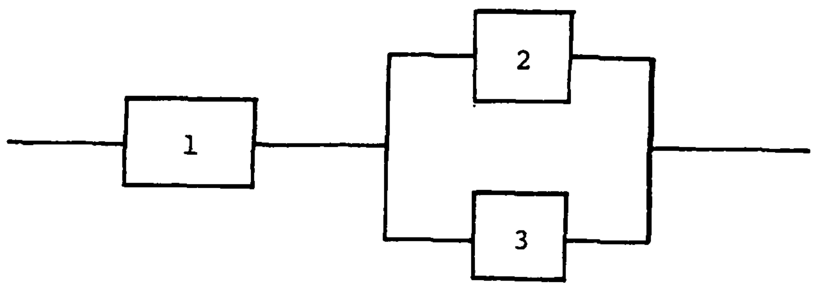

Figure 1. Three Component System

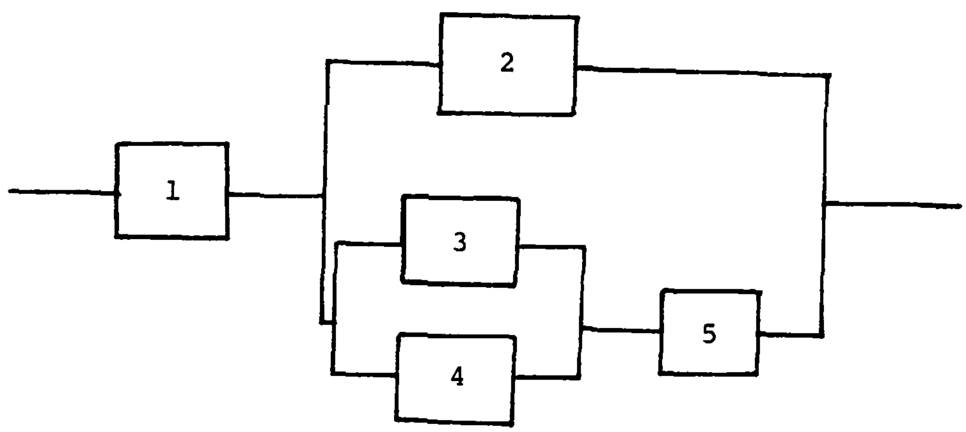

Figure 2. Five Component System

system reliability in 87.8% of the 1000 cases which indicates again that the LCL estimate is slightly optimistic.

V. Conclusions and Recommendations

Stated very simply, it has been shown that this method of estimating lower confidence limits is accurate, practical, and simple to use. One need only supply the component reliabilities, the number of component tests, and the desired level of confidence; and he obtains, not only an estimated lower confidence limit of the system reliability, but also an indication of how accurate this estimate is.

It is worth re-emphasizing that, where most of the other techniques are not valid in the case of zero-failures, this method accurately and easily accommodates such a situation. This method is also not restricted to series systems; it can easily handle parallel configurations. A logical next step would be to incorporate such a method into a system where other components exhibit different failure distributions as in the work done by Moore, Harter, and Snead (Ref 20).

This next generation of systems to be analyzed obviously provokes one to imagine all the possible applications. These methods could be applied to an entire spectrum of systems from simple hardware items to strategic mission planning.

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Appendix A

Case Where $\hat{s}=0$

In Gatliffe's method (Ref 7), he must deal with the case of $\hat{s}=0$. In his study, the number of mission successes, W, can be related to mission reliability, R_s, by

$$\sum_{j=W}^{N} \binom{N}{j} R_s^j (1-R_s)^{N-j} = \text{Prob (\# successes} \geq W) ,$$

where N = number of mission trials. The 100(1-α)% Lower Confidence Limit estimate for system reliability would be the solution for R* in

$$\sum_{j=W}^{N} \binom{N}{j} (R^*)^j (1-R^*)^{N-j} = \alpha$$

which, when W=N, reduces to

$$(R^*)^N = \alpha \quad \text{or} \quad R^* = (\alpha)^{1/N} .$$

Now, $$\hat{R}^*(\alpha) = \exp\left[\frac{-2\hat{r}\hat{s}}{\chi^2_{(2\hat{r},1-\alpha)}}\right] = (\alpha)^{1/N}$$

where $$\hat{s} = \Sigma T_i, \quad \hat{r} = \max\left[1, \frac{(\Sigma T_i)^2}{\Sigma(\frac{T_i}{N_i})}\right]$$

which reduces to $\hat{s} = \hat{T}_i$ and $\hat{r} = \max(1, \hat{T}_i N_i)$, since only one component contributes non-zero failures.

Now if $\hat{r} = 1.0$, then $\hat{T}_i N_i \leq 1$ implies that

$$\hat{T}_i \leq \frac{1}{N_i}$$

$$\hat{T}_i = A_i \hat{Q}_i + B_{i/2}\, \hat{Q}_i^2$$

$$A_i = \frac{2N_i - 3}{2N_i - 2}\ ,\ B_i = \frac{N_i}{N_i - 1}\ ,\ \hat{Q}_i = \frac{F_i}{N_i}\ .$$

Thus, $\left(\frac{B_i}{2N_i^2}\right) F_i^2 + \left(\frac{A_i}{N_i}\right) F_i - \hat{T}_i = 0$ or

$$\left(\frac{N_i}{2(N_i-1)(N_i^2)}\right) F_i^2 + \left(\frac{2N_i-3}{2(N_i-1)(N_i)}\right) F_i - \frac{1}{N_i} \leq 0$$

$$F_i^2 + (2N_i-3)F_i - 2(N_i-1) \leq 0$$

$$(F_i-1)(F_i + 2(N_i-1)) \leq 0\ .$$

The only positive values for F_i that can satisfy this equation are between zero (0) and one (1). Also, for any size N_i, $2\hat{r} = 2$. Hence,

$$\exp\left[\frac{-2\hat{r}\hat{s}}{\chi^2_{(2\hat{r},1-\alpha)}}\right] = (\alpha)^{1/N}$$

$$\hat{s} = \frac{\chi^2_{(2\hat{r},1-\alpha)}}{2N} (\ln\alpha) = \frac{A_i F_i^*}{N} + \frac{B_i (F_i^*)^2}{2N^2}$$

which implies $(F_i^*)^2 + (2N-3)F_i^* + (N-1) \ln\alpha(\chi^2_{(2\hat{r},1-\alpha)}) = 0.$ Solving for F_i^* as a function of N and α, Gatliffe obtains

N	α=.2	α=.1	α=.05
5	.3888	.2670	.1713
10	.3721	.2531	.1610
20	.3652	.2475	.1570
100	.3603	.2435	.1542
1000	.3593	.2427	.1536

Appendix B

Computer Program to Determine Limits

The computer program contained herein determines the 100(1-α)% lower confidence limits on system reliability. It is written in FORTRAN IV and utilizes AFIT and IMSL subroutines. At the beginning, each variable is defined. Throughout the program, major sections are introduced with comments that explain what is being done so as to enhance the flow of the program.

Input: The first data card is in free field form containing:

(1) the number of components in the system,

(2) the confidence level (α = 0.05 means a 95% level),

(3) DSEED which is the double precision random number seed used in the subroutines,

(4) the number of simulations to be performed (999).

Following this card is a card for each component, in format, that establishes the number of tests and number of failures for that component.

Output: On output is displayed:

(1) the number of components,

(2) the number of tests and failures for each component,

(3) the confidence level,

(4) the 100(1-α)% lower confidence limit.

```
      PROGRAM RIMREL(INPUT=/80,OUTPUT)
      DIMENSION PIHAT(50),RSYSJ(1000),PIHAT2(50),FITEST(50),NITEST(50),
     1QIHAT(50),SIGIHAT(50),R(50)
      DOUBLE PRECISION DSEED
C
C     NITEST= # OF COMPONENT I TESTED
C     FITEST= # OF COMPONENT I FAILED IN TEST
C     PIHAT= FIRST ESTIMATE OF P FOR COMPONENT I
C     PIHAT2= SECOND ESTIMATE OF P FOR COMPONENT I
C     RSYSJ= RELIABILITY OF SYSTEM ON MONTE CARLO SIM J
C     NCOM= # OF COMPONENTS IN THE SYSTEM
C     ALPHA= 100(1-ALPHA)% CONFIDENCE LIMIT
C     FI= EQUIVALENT FAILURES
C     QIHAT= FIRST ESTIMATE OF Q FOR COMPONENT I:Q=1-P
C     SIGIHAT= ESTIMATE OF VARIANCE
C     DSEED= SEED FOR NORMAL R.V. GENERATION
C     R= OUTPUT VECTOR OF NORMAL R.V.S
C     NSIM= # OF MONTE CARLO SIMULATIONS
C     KALPHA= THAT VALUE CORRESPONDING TO LIMIT
C
2     FORMAT (1X,I4,1X,F7.3)
3     FORMAT (F8.5)
5     READ*, NCOM,ALPHA,DSEED,NSIM
      IF(EOF(5LINPUT).NE.0.) STOP "END OF PROGRAM"
C
C     NO. OF COMPONENTS TESTED AND NO. OF FAILURES
      DO 10 I=1,NCOM
      READ 2, (NITEST(I),FITEST(I))
10    CONTINUE
      PRINT*,"INPUT CHECK...NO. OF COMP= ",NCOM," NO. OF TESTS AND FAIL"
      PRINT*," "
      PRINT 2,(NITEST(I),FITEST(I),I=1,NCOM)
C
C     ASSIGNING EQUIVALENT FAILURES WITH SATLIFFE'S METHOD
      IF (ALPHA.EQ.0.2) FI=.760
      IF (ALPHA.EQ.0.1) FI =.245
      IF (ALPHA.EQ.0.05) FI =.155
      DO 20 I=1,NCOM
      IF (FITEST(I).EQ.0) FITEST(I)=FI
20    CONTINUE
C
C     CALCULATING P, Q, AND VARIANCE
      DO 30 I=1,NCOM
      PIHAT(I)=1-(FITEST(I))/(NITEST(I))
      QIHAT(I)=1-PIHAT(I)
      SIGIHAT(I)=PIHAT(I)*QIHAT(I)/NITEST(I)
30    CONTINUE
C
C     GENERATING NORMAL RANDOM VARIABLES AND SECOND ESTIMATE OF P
      DO 50 J=1,NSIM
      CALL GGNML(DSEED,NCOM,R)
      DO 40 I=1,NCOM
      PIHAT2(I)=R(I)*SQRT(SIGIHAT(I))+PIHAT(I)
40    CONTINUE
C
C     CALCULATING SYSTEM RELIABILITY (SERIES)
      DO 45 I=2,NCOM
```

```
      RIHAT2(I)=RIHAT2(I)*RTHAT2(I-1)
      CONTINUE
      RSYSJ(J)=RIHAT2(NCOM)
30    CONTINUE
C
C     ARRANGING SYSTEM RELIABILITY IN ASCENDING ORDER
      CALL SORT(NSIM,RSYSJ)
      IF(ALPHA.EQ.0.1) KALPHA=100
      IF(ALPHA.EQ. 0.05) KALPHA=50
      PRINT*," "
      PRINT*,"NO. OF COMPONENTS = ",NCOM
      DO 50 I=1,NCOM
      PRINT*,"NO. OF COMPONENT ",I," TESTED=",NITEST(I)
      PRINT*,"NO. OF FAILURES FOR THIS COMFONENT WAS ",FITEST(I)
50    CONTINUE
      T=1.0-ALPHA
      PRINT*,"LOWER LIMIT OF ",T," IS "
      PRINT 3, (RSYSJ(KALPHA))
      GO TO 5
      END
```

Appendix C

Computer Program to Verify Accuracy

The computer program contained herein determines the $100(1-\alpha)\%$ lower confidence limit on system reliability and determines how accurate this limit is. It is written in FORTRAN IV and utilizes AFIT and IMSL subroutines. At the beginning, each variable is defined. Throughout the program, major sections are introduced with comments that explain what is being done so as to enhance the flow of the program.

Input: The first data card is in free field form containing:

(1) the number of components in the system,

(2) the confidence level ($\alpha = 0.05$ means a 95% level),

(3) DSEED which is the double precision random number seed used in the subroutines,

(4) the number of simulations to be performed (999),

(5) the number of lower limits to be generated (1000).

Following this card is a card for each component, in format, that establishes the true probability of success and number of tests for that component.

Output: On output is displayed:

(1) the true probability of success and number of tests for each component,

(2) the true system reliability,

(3) the confidence level and number of limits,

(4) the averaged $100(1-\alpha)\%$ lower confidence limit,

(5) the accuracy of the limit (the percent of the total limits that covered the true system reliability).

```
      PROGRAM RINREL1 (INPUT,OUTPUT)
      DIMENSION PIHAT(10),RSYSJ(1000),R(1000),FITEST(10),NITEST(10),
     1QIHAT(10),SIGIHAT(10),PIHAT2(3,1000),TRUEP(10),RTEST(1000)
      DOUBLE PRECISION DSEED
C
C*************************************************************************
C     NITEST= # OF COMPONENT I TESTED:
C     FITEST= # OF COMPONENT I FAILED IN TEST
C     PIHAT= FIRST ESTIMATE OF P FOR COMPONENT I
C     PIHAT2= SECOND ESTIMATE OF P FOR COMPONENT I
C     RSYSJ= RELIABILITY OF SYSTEM ON MONTE CARLO SIM J
C     NCOM= # OF COMPONENTS IN THE SYSTEM
C     ALPHA= 100(1-ALPHA)% CONFIDENCE LIMIT
C     FI= EQUIVALENT FAILURES
C     QIHAT= FIRST ESTIMATE OF Q FOR COMPONENT I:Q=1-P
C     SIGIHAT= ESTIMATE OF VARIANCE
C     DSEED= SEED FOR NORMAL R.V. GENERATION
C     R= OUTPUT VECTOR OF NORMAL R.V.S
C     NSIM= # OF MONTE CARLO SIMULATIONS
C     KALPHA= THAT VALUE CORRESPONDING TO LIMIT
C     NOVER=NO OF LCL OVER THE TRUE RELIABILITY
C     NLIM=NUMBER OF LOWER CONFIDENCE LIMITS
C     TRSYS=THE TRUE SYSTEM RELIABILITY
C     RTEST=ARRAY CONTAINING TEST RESULTS OF THE PI'S
C      REL= RELIABILITY OF THE SYSTEM FOR THAT SIMULATION
C     CUMLM= ADDS ALL THE LCL'S
C     AVGLIM= AVERAGES THESE LIMITS OVER THE TOTAL NO. GENERTTED
C*************************************************************************
C
2     FORMAT (2X,F6.4,1X,I3)
3     FORMAT (F8.6)
5     READ*, NCOM,ALPHA,DSEED,NSIM,NLIM
      IF (EOF(5LINPUT).NE.0.0) STOP "END OF PROGRAM"
C
C*************************************************************************
C  TRUE PROB OF SUCCESS AND NO. OF TESTS FOR EACH COMPONENT
C
      TRSYS=1.0
      DO 10 I=1,NCOM
      READ 2, (TRUEP(I),NITEST(I))
      TRSYS=TRSYS*TRUEP(I)
      PRINT*,"THE TRUE VALUE OF P FOR COMPONENT ",I," = ",TRUEP(I)
      PRINT*,"NUMBER OF TESTS ON COMPONENT ",I," = ",NITEST(I)
10    CONTINUE
      PRINT*," "
      PRINT*,"THE TRUE SYSTEM RELIABILITY IS ",TRSYS
C
C*************************************************************************
C  NOW TESTING COMPONENTS WITH UNIFORM FUNCTION
C
      NOVER=0
      CUMLM=0.0
      DO 100 LIMITS=1,NLIM
      DO 20 M=1,NCOM
      N=NITEST(M)
      CALL GGUBS(DSEED,N,RTEST)
C
```

```
C***************************************************************************
C  FTEST CONTAINS TEST VALUES FOR COMPONENTS
C
      FITEST(M)=0.0
      DO 30  K=1,N
        IF(FTEST(K).GT.TRUEP(M)) FITEST(M)=FITEST(M)+1
30    CONTINUE
C
C***************************************************************************
C  ASSIGNING EQUIVALENT FAILURES WITH SATLIFFE'S METHOD
C
         IF  (ALPHA.EQ.0.2) FI=.360
         IF  (ALPHA.EQ.0.1)  FI=.245
         IF  (ALPHA.EQ.0.05) FI=.155
        IF  (FITEST(M).EQ.0.0)  FITEST(M)=FI
C
C***************************************************************************
C  CALCULATING ESTIMATES OF P, Q, AND VARIANCE
C
      PIHAT(M)=1.0-(FITEST(M))/(NITEST(M))
      QIHAT(M)=1.0-PIHAT(M)
      SIGIHAT(M)=PIHAT(M)*QIHAT(M)/NITEST(M)
20    CONTINUE
C
C***************************************************************************
C  GENERATING NORMAL RANDOM VARIABLES AND SECOND ESTIMATE OF P
C  CALCULATING SYSTEM RELIABILITY (SERIES)
C
      DO 50  I=1,NCOM
      CALL GGNML(DSEED,NSIM,R)
      DO 40  J=1,NSIM
      PIHAT2(I,J)=PIHAT(I)-(R(J)*SQRT(SIGIHAT(I)))
      IF(PIHAT2(I,J).GT.1.0)  PIHAT2(I,J)=1.0
40    CONTINUE
50    CONTINUE
      DO 60  J=1,NSIM
      REL=1.0
      DO 70  I=1,NCOM
      REL=REL*PIHAT2(I,J)
70    CONTINUE
      RSYSJ(J)=REL
60    CONTINUE
C
C***************************************************************************
C  ARRANGING SYSTEM RELIABILITY IN ASCENDING ORDER
C
      CALL  SORT(NSIM,RSYSJ)
      KALPHA=ALPHA*NSIM
      T=1.0-ALPHA
        IF(RSYSJ(KALPHA).GT.TRSYS)  NOVER=NOVER+1
      CUMLM=CUMLM+RSYSJ(KALPHA)
100   CONTINUE
      AVGLIM=CUMLM/NLIM
      PERCENT=NLIM-NOVER
      PERCNT=(PERCENT/NLIM)*100.0
      PRINT*,"NUMBER OF LOWER LIMITS AT ALPHA= ",T," IS ",NLIM
      PRINT*,"AVERAGE LOWER LIMIT OF SYSTEM RELIABILITY IS ",AVGLIM
```

```
PRINT*,"PERCENT OF LOWER LIMITS COVERING THE TRUE REL IS ",PERCNT
PRINT*," "
PRINT*," "
GO TO  5
END
```

VITA

Roy Eugene Rice was born on 24 January 1953 in Little Rock, Arkansas. He graduated from high school in Lonoke, Arkansas, in 1971 and attended the United States Air Force Academy. Upon graduation in June 1975, he received a Bachelor of Science degree in Mathematics and a commission in the U. S. Air Force. He was then placed on active duty at Tinker Air Force Base, Oklahoma, where he was a Reliability Engineer. Some of the systems on which he worked were the B-1 Bomber, B-52, KC-135, E-3A (AWACS), E-4, and the Advanced Aerial Refueling Boom. After approximately three years at Tinker, he entered the School of Engineering, Air Force Institute of Technology in June 1978.

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Several methods for estimating lower confidence limits were examined. A proposed technique was developed and used to obtain limits for selected systems. These limits were compared to those obtained by other methods. Then the proposed method was tested to verify its accuracy. Results indicate that the proposed method is simple to understand, easy to impliment, and accurate.

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