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ROBUST AND LEAST SQUARES ORTHOGONAL MAPPING: METHODS FOR THE ST--ETC(U)

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ROBUST AND LEAST SQUARES ORTHOGONAL MAPPING:  
METHODS FOR THE STUDY OF  
CEPHALOFACIAL FORM AND GROWTH

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## Abstract

A method is presented for the description and analysis of cephalofacial form and growth using two-dimensional coordinate data. The procedure permits the identification of shape differences at specific cephalofacial coordinate locations without reliance upon conventional cephalometric landmarks. The resulting size-standardized coordinates can be analyzed by statistical methods for further data exploration. Two methods of shape transformation--least squares and robust fitting--are described and compared. An example of the utility of the technique for cephalofacial growth studies is provided.

The characterization and analysis of form and its component parts, size and shape, has long been a central issue in morphological comparisons. Recent advances in many fields have provided powerful new tools for the description and analysis of biological shape. Since the classic work of D'Arcy Thompson ('17), two-dimensional Cartesian coordinates have been widely used to analyze size and shape. Cartesian coordinates have the advantage of measuring in a precise and balanced manner the shape of the feature under consideration. This avoids the criticisms that might be leveled at the standard use of linear measurements and angles based upon conventional cephalometric landmarks.

Sneath ('67) proposed a two-stage procedure to study cranial form using size-standardized Cartesian coordinates analyzed by means of trend-surface analysis. Walker and Kowalski ('71, '72) presented a general method for the recording and analysis of a large number of two-dimensional coordinates that describe craniofacial morphology. The use of their cephalometric data acquisition system, in conjunction with methods to determine growth trajectories, allows the description, analysis and prediction of craniofacial growth.

Brower and Veinus ('78) provided a useful example of how two-dimensional coordinate data can be used to analyze multivariate allometry. Lestrel ('76) has described how Fourier analysis can be used to fit a curve to a complex form and partition it into size and shape components. Bookstein ('78,

'80) has advanced a new method of studying Thompson's transformation grids using "biorthogonal grids." Tobler ('78) has applied nonlinear grid transformations to the comparison of plane figures. Todd et al. ('80) have used geometric transformations such as cardioidal strain and affine shear to analyze the perception of human cranial growth.

Work has also proceeded on three-dimensional analysis of the cranium. Studies by Benfer ('77), Herron ('73), Oyen and Walker ('77), Scheibengraber ('79), McHenry and Corruccini ('78), Creel and Preuschoft ('76) and Huber ('80) should be noted in this regard. While these studies have demonstrated the importance of the quantification and analysis of craniofacial form in three-dimensional space, the two-dimensional cephalogram remains the most common and economical source of data for craniofacial growth studies.

Orthogonal mapping is a method of quantifying shape differences based on the initial procedures of Sneath ('67) and was first described in detail by Huffman et al. ('78). The use of two-dimensional coordinates recorded from cephalograms and the application of the orthogonal mapping method make it possible to easily describe and analyze craniofacial shape and shape changes during growth. Orthogonal mapping has the following advantages: 1) utilization of Cartesian coordinates, 2) the use of cephalometric landmarks as reference points is not necessary, 3) a uniform removal of size effects, 4) the easy identification of shape differences at a particular x,y coordinate point or for a group of coordinates, and 5) the

resulting size-standardized coordinates can be used in a variety of multivariate statistical procedures.

The orthogonal mapping approach essentially involves determination of the transformation that yields the closest fit between analogous points on two objects (Huffman et al., '78), where transformations consist of three parts. The first is a rigid (orthogonal) rotation of one set of coordinate points, removing the effect of initial orientation. Second, to remove the difference in size, one set of coordinate points is scaled by an overall magnification or shrinking factor. Third, both objects are referred to the same origin by a linear translation of one set of points.

There are now two general methods for choosing the best transformation, and each is "best" in a different way. The traditional method is least squares, related to the methods of Huffman et al. ('78), Gower ('75), and Sneath ('67). The least squares solution chooses the combination of rotation, scale and translation that minimizes the sum of squared differences between the coordinates of one specimen and the transformed analogous coordinates of the second specimen.

The second method is an application of the ideas of statistical robustness. While these methods may also minimize some objective function, they may be better thought of as methods that prefer a close correspondence throughout as much of the specimens as possible even at the expense of a poor correspondence in a small part of the specimens. If a localized region of poor correspondence is found, this is often

very useful in identifying regions of difference. The special robust method used here is based on repeated medians and chooses rotation, scale and translation values as nested medians of estimates based on all possible corresponding pairs of analogous points, two from each specimen. We will show the results of least squares and robust fitting. Technical details of both methods are described in Siegel and Benson ('82), and the computer program may be found in Siegel ('82).

A useful property of orthogonal mapping is that as long as both objects are in the same coordinate system, the position of the origin for each object can be independent. This means that traditional cephalometric landmarks are not necessary for reference and orientation. The orthogonal mapping procedure produces a vector of residual values in which each vector element represents the shape difference at that x,y location. By plotting these residual values one can easily determine the locations that markedly differ in shape between the two objects. Another useful value is the scale factor which is the summary measure of the overall size difference between the objects under consideration. Comparisons between objects can also be made by performing orthogonal mapping with each object compared against a standard or reference set of coordinates (e.g. the set of grand means of each point for the sample). The resulting adjusted (size-standardized) coordinates can then be treated as variables in statistical analyses such as cluster, factor and discriminant analysis.

As an example of the utility of the orthogonal mapping



procedure in cephalofacial growth studies, shape changes in the cephalofacial complex of a male Macaca nemestrina were examined using orthogonal mapping. Seventy-two points were defined by superimposing a polar coordinate grid on the cephalogram using the grid center aligned with sella turcica and the supraorbital point. Each point was then defined as the intersection of each vector (at 5° intervals) and the cephalofacial outline (fig. 1). The x and y coordinates were recorded for each point using an electronic digitizer. For this particular specimen, 10 such sets of x and y coordinates were recorded, each representing the age at which a cephalogram was obtained (about every three months for the first three years then biannually for the remainder of the growth period). The results presented here represent the growth period from 0.90 to 5.77 years of age. Figure 2 shows the superimposition of the two coordinate sets, one for the youngest age (0.90) and the other the oldest (5.77) age. The analysis summarizes the entire growth changes in the cephalofacial complex for this specimen.

Figure 3 is a plot of the residuals (shape differences) as determined by least squares orthogonal mapping between the two specimens. By referring back to figure 2 one can now see that relative to other groups of points, the coordinate locations between 25 and 35 and between 55 and 60 have undergone the most change in shape during the growth of the monkey. Figure 4 shows the coresponding plot for the repeated median orthogonal mapping. This is largely consistent with the least squares picture, but some features are indicated more strongly. The

peak between 55 and 60 is present in both figures, but is much more pronounced by the repeated median technique. The peak around 30 is present in both.

Figure 5 is another means of displaying the shape differences obtained by the least squares method. The residuals have been drawn as arrows from each point on the younger outline to the corresponding point on the oldest outline after least squares fit. The length of the arrow indicates the amount of shape change. The inward direction of the arrows indicates areas where the older form was mapped inside the younger outline. The areas of shape change shown in figures 3 and 5 reflect growth in the occipital and anterior facial regions.

Figure 6 shows the results of the fit by repeated median orthogonal mapping. The repeated median method has also indicated shape differences in the facial and occipital areas. However, the robust fit has emphasized major shape change in the facial region and to a much lesser degree in the occipital portion. The robust method has provided a more uniform fit of the entire outline by reducing the effects of the poor fit in the neurocranium.

Of course, the results presented here deal only with a single monkey; however, curve fitting procedures can be used to describe size and shape changes during growth with a large cephalofacial growth series. We suggest using both the least squares and repeated medians methods when performing a shape analysis because each method provides useful clues for the

description of shape change. Generally, however, a robust fit will be more effective than least squares in the identification of a localized change in shape.

In their review of craniofacial growth and development in Old World monkeys and apes, Sirianni and Swindler ('79) point out that nonhuman primate data have not been analyzed in a sophisticated manner. As an attempt to resolve this problem, research is currently underway using the orthogonal mapping procedure to examine various aspects of cephalofacial growth and development in Macaca nemestrina and Papio cynocephalus (Olshan and Swindler, n.d.).

It is hoped that through the use of techniques such as orthogonal mapping, cephalofacial form and growth in primates can be described and analyzed in a straightforward and detailed fashion so as to further our understanding of the basic processes underlying form and growth.

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## LEGENDS TO FIGURES

- Fig. 1 The 72 coordinates used to describe cephalofacial morphology.
- Fig. 2 Comparison of cephalofacial growth of a male Macaca nemestrina studied at 0.90 years (inside) and 5.77 years (outside)
- Fig. 3 Computer plot of the lengths of shape difference vectors as determined by least squares orthogonal mapping. The plot summarizes cephalofacial growth for a single macaque from age 0.90 to 5.77 years.
- Fig. 4 Shape differences for coordinate points based on repeated median orthogonal mapping.
- Fig. 5 Shape differences as determined by least squares method. Length of the arrow at each specific coordinate location indicates amount of shape difference.
- Fig. 6 Shape differences as determined by robust method.

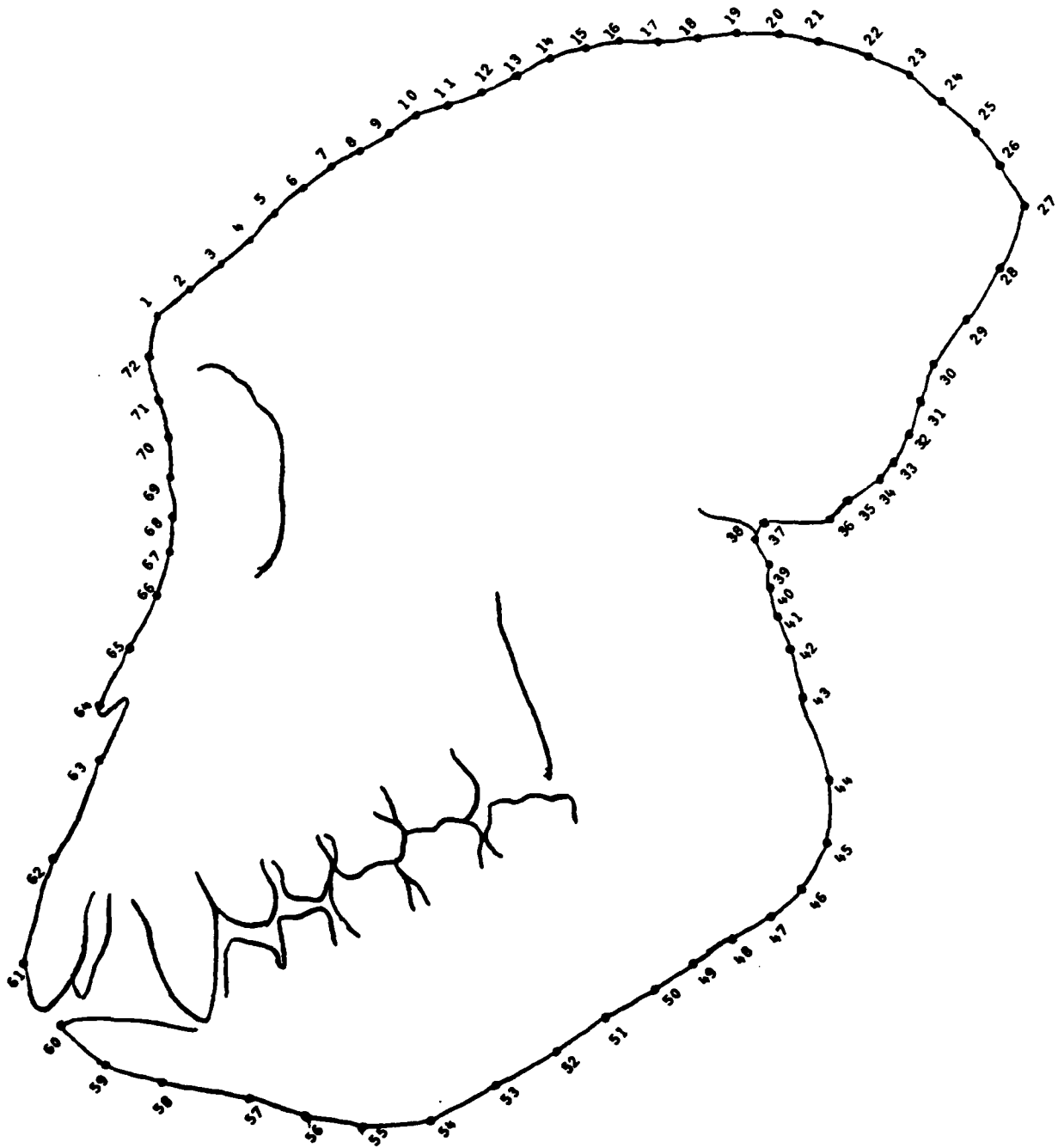
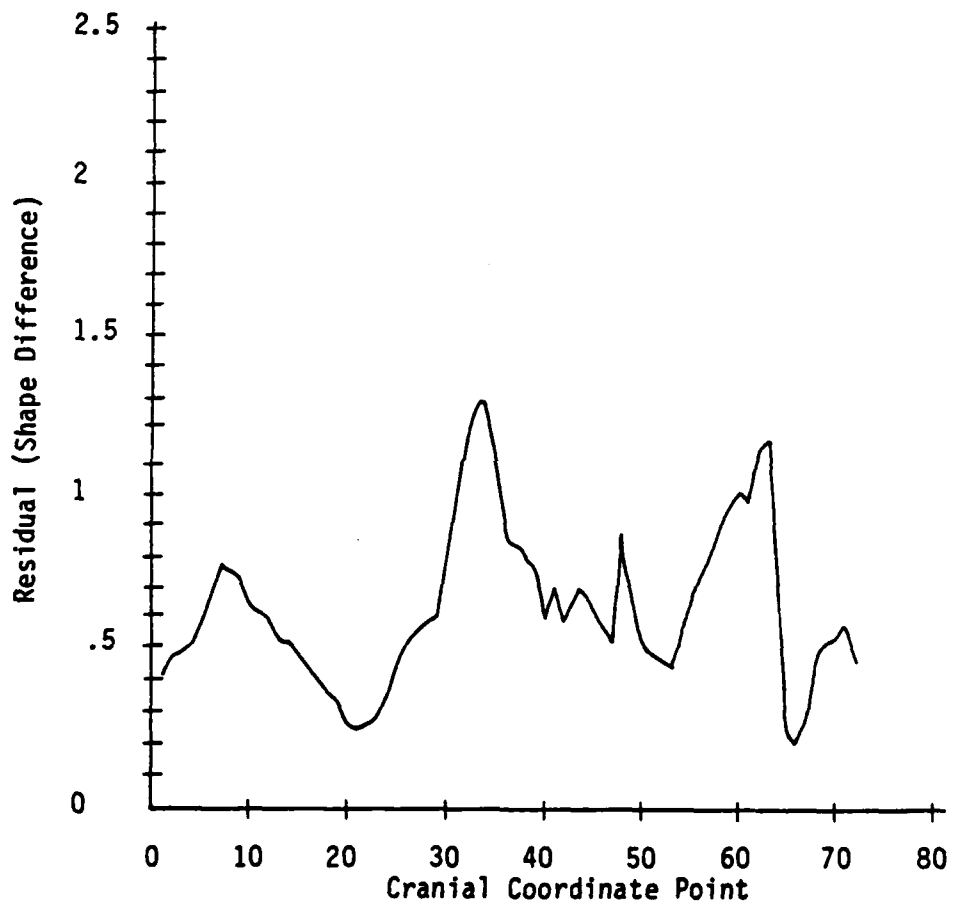


Figure 1



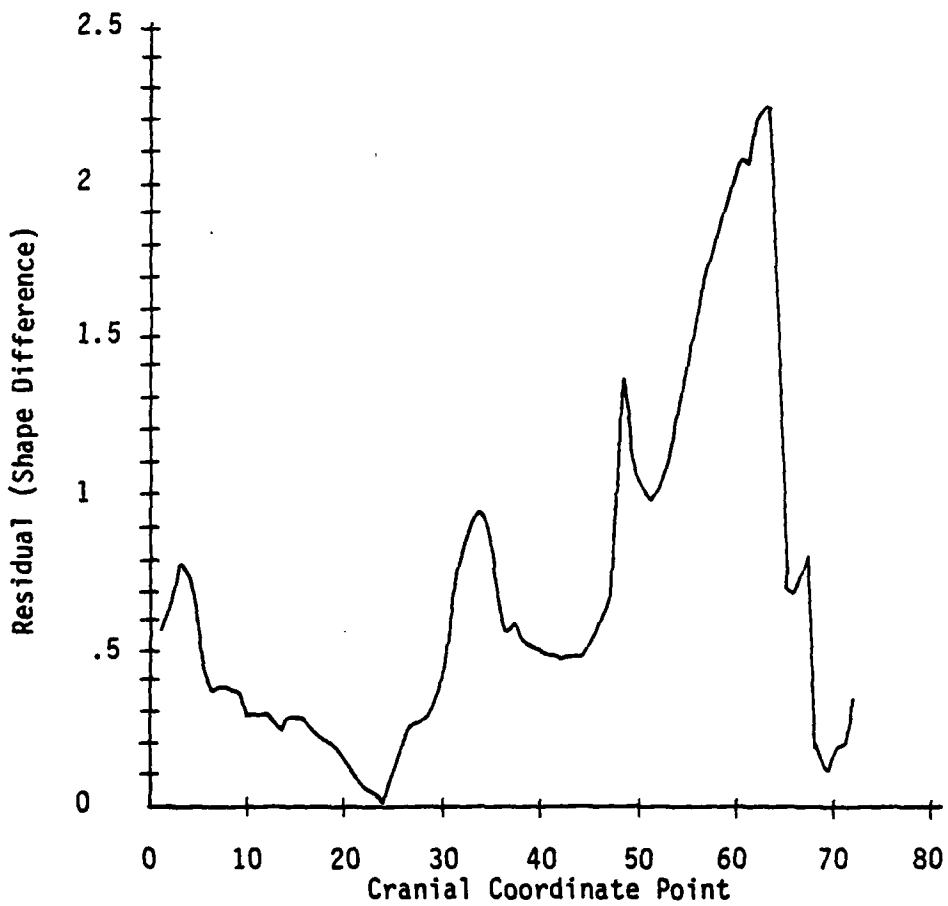
Figure 2





LEAST SQUARES FIT

Figure 3



RESISTANT FIT

Figure 4

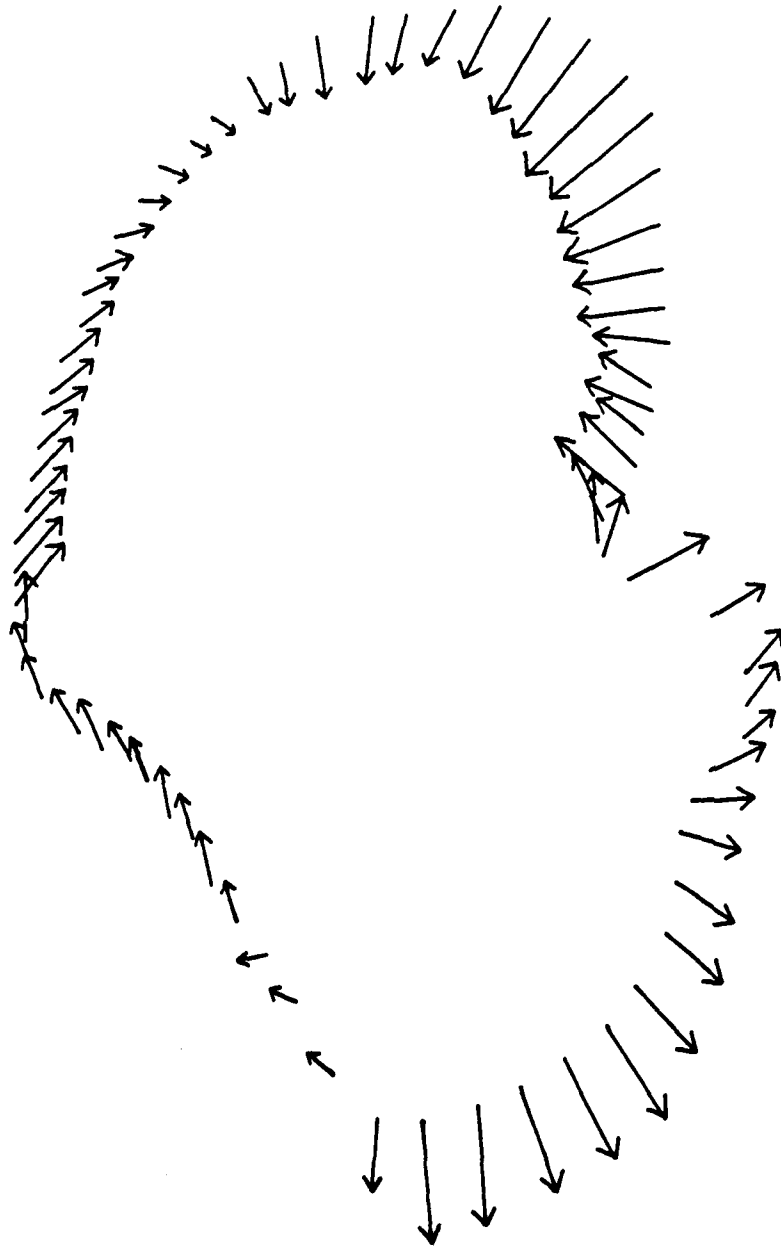


Figure 5

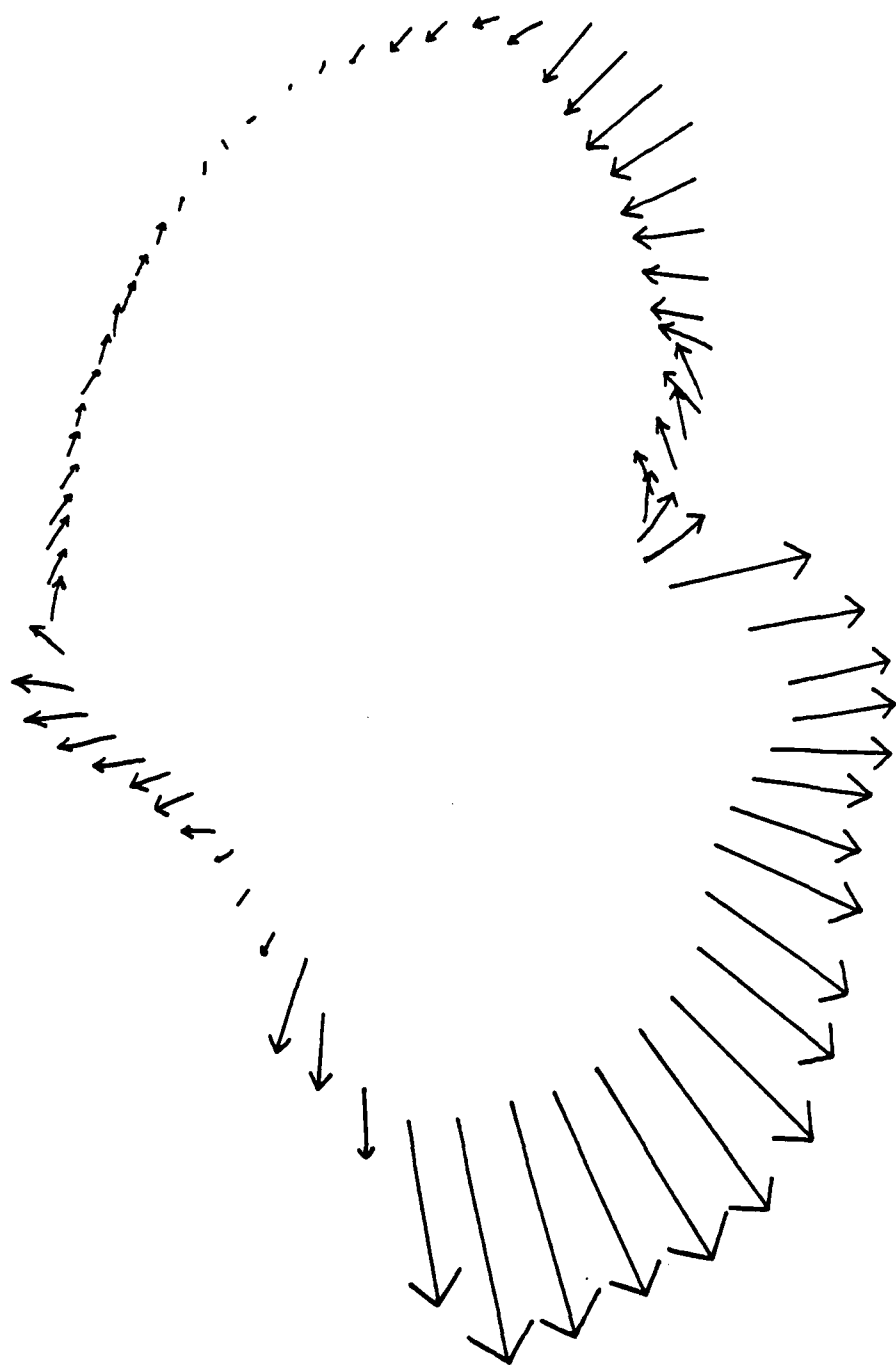


Figure 6

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