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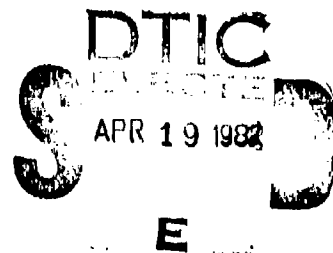# USERS CODE FOR THE FINITE-DIFFERENCE TIME-DOMAIN METHOD

IIT Research Institute

Dr. Allen Taflove
Dr. Korada Umashankar

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Electromagnetic penetration and scattering problems are difficult to treat with many analytical or numerical methods because of the inability of these methods to simply deal with the effects of structure materials, apertures, curvatures, corners, and internal contents. In previous programs, a new approach for the direct modeling of very complex electromagnetic interaction problems was studied: the finite-difference, time-domain (FD-TD) solution of Maxwell's equations. The FD-TD method has key

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advantages relative to available modeling approaches. These advantages permit it to accurately treat complex problems that are beyond the scope of solution by any other method.

The goals of the present research program included the development of specific algorithms of high importance to help provide a flexible, simple-to-use, and highly accurate user-oriented FD-TD computer program. Five key improvements in the FD-TD algorithm were tested during this effort, including the following:

(1) Total-field/scattered-field lattice division,
(2) Variable angle of incidence,
(3) Second-order accurate radiation condition,
(4) Magnitude and phase computation condition for the sinusoidal steady state, and
(5) Near-to-far field transformation.

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REPORT SUMMARY AND DISCUSSION

Electromagnetic penetration and scattering problems are difficult to treat with many analytical or numerical methods because of the inability of these methods to simply deal with the effects of structure materials, apertures, curvatures, corners, and internal contents. In two earlier RADC contracts, F30602-77-C-0163 and F30602-79-C-0039, IIT Research Institute (IITRI) investigated the application of a new approach for the direct modeling of very complex electromagnetic interaction problems: the finite-difference, time-domain (FD-TD) solution of Maxwell's equations. The FD-TD method has key advantages relative to available modeling approaches. These advantages permit it to accurately treat complex problems that are beyond the scope of solution by any other method. The ultimate aim of research in this area is to develop an accurate, easily-used, general computer program solving for either electromagnetic field penetration, scattering, or radiation for arbitrary metal/dielectric structures spanning up to 10 or more wavelengths in three dimensions with a spatial resolution better than 0.1 wavelength.

In order to more fully determine the usefulness of the FD-TD method, RADC thought it is desirable to distribute this technique to as wide a range of users as possible so that it can be tested by actual implementation. The overall objectives of algorithm development in this case are to allow RADC to write a user-oriented computer program for the FD-TD technique.

The goals of the present IITRI research effort for RADC, Contract F30602-80-C-0302, included the development of specific algorithms of high importance to help provide a flexible, simple-to-use, and highly accurate user-oriented FD-TD computer program. To meet these goals, IITRI tested five key improvements in the FD-TD algorithm during this effort, including the following:

1. Total-field/scattered-field lattice division.

    This permits a very high computational dynamic range to accurately
    model fields within shadow zones or cavities. This further permits
    programming of variable angle of incidence and the second-order
    correct radiation condition, summarized below.

2. Variable angle of incidence.

    For two-dimensional problems, this permits a single data card or
    Fortran statement to specify in a very accurate manner the angle
    of incidence of a plane wave illuminating a structure. For three-
    dimensional problems, both the angle of incidence and polarization
    could be specified. There is no requirement to rotate the geom-
    etry of the interacting structure in the FD-TD lattice.

3. Second-order accurate radiation condition.

    This reduces the uncertainty of the final computed results by as
    much as ten-to-one. FD-TD computations using this radiation con-
    dition now have estimated field-magnitude uncertainties of better
    than $\pm 2.5\%$ (± 0.2 dB) versus previous uncertainties of $\pm 10\%$-$\pm 15\%$
    (± 1 dB).

4. Magnitude and phase computation condition for the sinusoidal
    steady state

    This permits accurate determination of the magnitude and phase
    of FD-TD computed fields at any desired points for later use in
    computations involving scattering, radiation, or coupling to wires.
    This approach avoids any ambiguity due to either a possible DC
    offset of the fields or the repetitive nature of the sinusoidal
    waveform.

5. Near-to-far field transformation.

    This permits the far scattered fields and radar cross section of
    arbitrary structures modeled by the FD-TD method to be easily and
    accurately determined. Observed accuracy of the radar cross section
    using this feature is in the order of $\pm 1\%$ (± 0.09 dB).

These FD-TD algorithm improvements are documented in this report. In addition to the algorithm improvements tested by IITRI, this report also summarizes a FD-TD feature which has recently appeared in the literature that permits computation of the coupling of currents to thin wires and struts.

The conclusions of this report are as follows:

1. The accuracy of the pure FD-TD method for electromagnetic interaction problems can reach the high levels previously attained only by method-of-moments (MOM) approaches when the second-order accurate radiation condition is used in the FD-TD algorithm. The FD-TD method retains its significant advantages over MOM in terms of the much larger electrical size and greater complexity of the structures that can be modeled.

2. The specification of variable angle of incidence and polarization of an illuminating wave can be achieved with the FD-TD method using only a single data card or Fortran statement.[1]

   The total-field/scattered-field regional division of the FD-TD lattice can be successfully implemented and offers the significant advantage of a high computational dynamic range. In addition, this lattice division provides a framework for programming variable wave incidence and polarization, improved radiation conditions, and the near-field to far-field transformation for scattering problems.

4. The near-to-far field transformation along a rectangular virtual surface surrounding a scatterer makes it possible to use the FD-TD method to compute the far scattered fields and radar cross section of complex, arbitrary structures with great precision.

It is the opinion of the authors of this report that the FD-TD method deserves additional investigation to probe just what are the limits of application of this extremely promising approach to accurately model electromagnetic penetration, scattering, and radiation problems.

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[1] This incident wave specification is now as simple for the FD-TD method as it has been with MOM. However, the FD-TD approach requires re-running the entire problem for each new incident wave angle. With MOM, only a single inversion of the system matrix is required. Subsequently, arbitrary wave excitation is treated as a simple matrix multiplication of the equivalent excitation vector. MOM therefore permits a conceptually simpler treatment of the variable wave incidence problem.

PREFACE

IIT Research Institute (IITRI) is pleased to submit this Final Report on "User's Code for the Finite-Difference Time-Domain Method" to Rome Air Development Center (RADC/RBCT). The report covers work performed by IITRI under Air Force Contract No. F30602-80-C-0302, designated as IITRI Project No. E6502. This report covers details of the technical work, including relevant theory and numerical results. Appendix A provides a listing of the Fortran computer program used to obtain the results of this report.

The principal investigator on this program was Dr. Allen Taflove. The co-investigator was Dr. Korada Umashankar. The project duration was 1 September 1980 to 31 August 1981.

Respectfully submitted,

IIT RESEARCH INSTITUTE

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IITRI Project No. E6502
1 September 1930 - 31 August 1981
FINAL REPORT

1.0 INTRODUCTION

Electromagnetic penetration and scattering problems are difficult to treat with many analytical or numerical methods because of the inability of these methods to simply deal with the effects of structure materials, apertures, curvatures, corners, and internal contents. Usually, only relatively simple geometries are studied in an attempt to gain insight into the key interaction mechanisms and to allow an indirect estimate of the interaction for more complicated problems.

In earlier RADC Contracts F30602-77-C-0163 and F30602-79-C-0039, IITRI investigated the application of a new approach for the direct modeling of very complex electromagnetic interaction problems: the finite-difference, time-domain (FD-TD) solution of Maxwell's equations. The FD-TD method treats the illumination of a structure as an initial-value problem. At $t = 0$, a plane-wave source of frequency, f, is assumed to be turned on. The propagation of waves from this source is simulated by solving a finite-difference analog of the time-dependent Maxwell's equations on a lattice of cells, including the structure. Time-stepping is continued until the sinusoidal steady state is achieved at each cell. The field envelope, or maximum absolute value, during the final wave-cycle of time-stepping is observed to obtain the magnitude and phase of the steady-state field at any point.

This method has two key advantages relative to available modeling approaches. First, it is simple to implement for complicated metal/dielectric structures because arbitrary electrical parameters can be assigned to each lattice cell using a data card deck. Second, its computer memory and running time requirement is not prohibitive for many complex structures

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of interest. In the RADC work, IITRI has shown the FD-TD method to be capable of accurately solving for more than one million unknown field components within a few minutes on an array-processing computer. Consistently, a $\pm$ 1-dB accuracy relative to known analytical and experimental bench marks has been achieved for a variety of dielectric and metal geometries.

The objective of IITRI's previous RADC studies was to evaluate the suitability of the FD-TD method to determine the amount of electromagnetic coupling through an aperture into an enclosed conducting container and the interaction and coupling of the penetrating fields with internal electronics. Two specific container models were used for the evaluation. The first, a simple conducting cylinder with one open end. The other, the complex guidance section of a missile. Each of these two configurations was modeled to calculate the electromagnetic field coupled into the structure.

The ultimate aim of research in this area is two-fold. First, develop an accurate, easily-used, general code solving for either electromagnetic field penetration, scattering, or radiation for arbitrary metal/dielectric structures spanning up to 10 or more wavelengths in three dimensions with a spatial resolution better than 0.1 wavelength. Second, develop a more sophisticated intuitive understanding of basic wave interaction mechanisms in time domain, such as transient propagation through beyond-cutoff cavity interiors, field build-up at edges, convergence to the sinusoidal steady state, scattering, and radiation.

In order to more fully determine the usefulness of the FD-TD method, RADC thought that it is desirable to distribute this technique to as wide a range of users as possible so that it can be tested by actual implementation. However, the FD-TD computer programs previously developed by IITRI for RADC were research oriented rather than user oriented, i.e., they were written to apply to fairly specific types of interaction structures rather than completely general structures. Further, these FD-TD programs were optimized for a specific vector array processing computer, the Control Data

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Cyber 203, in order to minimize the cost of purchased computer time. The Fortran used for these programs was specialized to benefit from the machine-specific features of the Cyber 203, and is not directly usable by common scalar-processing computers such as the Honeywell 6000 series, CDC 6600, CDC Cyber 76, or IBM 370 series.

The overall objectives of algorithm development in this case are to allow RADC to write a user-oriented program for the FD-TD technique. Such a program would ideally buffer the user from the complexities involved in specifying an interaction geometry in the form needed by the main program. Further, such a program would be suitable for arbitrary shaped bodies, material parameters, incident-wave angle of incidence and direction of polarization, and time dependence on the incident wave. Finally, such a computer program would be machine-independent, i.e., written in a standard language such as Fortran IV, so that implementation on a very wide variety of computers would be easily possible. However, the program would still be structured to make the best economy of computer resources such as memory storage and program execution time.

The goals of the present IITRI research effort for RADC include the development of specific algorithms of high importance to help provide a flexible, simple-to-use, and highly accurate user-oriented FD-TD computer program. IITRI has tested five key improvements in the FD-TD algorithm during this effort, and reports on these developments in this document. The following is a listing of these algorithm developments, including comments indicating the usefulness of each development.

1. Total-field/scattered-field lattice division.

    This permits a very high computational dynamic range to accurately model fields within shadow zones or cavities. This further permits programming of variable angle of incidence and the second-order correct radiation condition, summarized below.

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2. **Variable angle of incidence.**

   For two-dimensional problems, this permits a single data card
   or Fortran statement to specify in a very accurate manner the
   angle of incidence of a plane wave illuminating a structure.
   For three-dimensional problems, both the angle of incidence and
   polarization could be specified. There is no requirement to
   rotate the geometry of the interacting structure in the FD-TD
   lattice.

3. **Second-order accurate radiation condition.**

   This reduces the uncertainty of the final computed results by
   as much as ten-to-one. FD-TD computations using this radiation
   condition now have estimated field-magnitude uncertainties of
   better than $\pm2.5\%$ (±0.2 dB) versus previous uncertainties of
   $\pm10\%$–$\pm15\%$ (±1 dB).

4. **Magnitude and phase computation condition for the sinusoidal
   steady state**

   This permits determination of the magnitude and phase of FD-TD
   computed fields at any desired points for later use in computations
   involving scattering, radiation, or coupling to wires. This
   approach avoids any ambiguity due to either a possible DC offset
   of the fields or the repetitive nature of the sinusoidal waveform.

5. **Near-to-far field transformation.**

   This permits the far scattered fields and radar cross section of
   arbitrary structures modeled by the FD-TD method to be easily and
   accurately determined. Observed accuracy of the radar cross section
   using this feature is in the order of $\pm1\%$ (±0.09 dB).

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In addition to the five algorithm improvements tested by IITRI, this report will also summarize a FD-TD feature which has recently appeared in the literature that permits computation of the coupling of currents to thin wires and struts.

Section 2.0 of this report will provide a complete summary of the theory and basic algorithms of the FD-TD method, including the five new features tested by IITRI and the thin-wire coupling model. Sub-sections which contain discussion of these new features will be denoted by a double asterisk (**) appearing before the headings. Section 3.0 of this report will provide examples of computed results which illustrate each of the key features of the FD-TD method examined to date. Again, examples of new features will be denoted by a double asterisk appearing before the appropriate heading. Appendix A provides a standard Fortran listing of a two-dimensional FD-TD computer program which illustrates the key features tested by IITRI during this research effort.

2.0 THEORY AND BASIC ALGORITHMS OF THE FD-TD METHOD

2.1 Ideas Behind the FD-TD Method

The FD-TD method is a direct solution of Maxwell's time-dependent curl equations. As shown in Figure 1, the goal is to model the propagation of an electromagnetic wave into a volume of space containing a dielectric or conducting structure. By time-stepping, i.e., repeatedly implementing a finite-difference analog of the curl equations at each cell of the corresponding space lattice, the incident wave is tracked as it first propagates to the structure and then interacts with it via surface-current excitation, diffusion, penetration, and diffraction. Wave-tracking is completed when the desired late-time or sinusoidal steady-state behavior is observed at each lattice cell. The rationale for this procedure is that it achieves simplification by analyzing the interaction of the wavefront with portions of the structure surface at a given instant in time, rather than attempting a simultaneous solution of the entire problem.

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Lattice Truncation Plane
(Invisible To All Waves)

$y = J\delta$

Scattered
Wave

Arbitrary
Structure

Incident
Plane Wave

$y = 0$

$x = 1/2\delta$ $x = (I + 1/2)\delta$
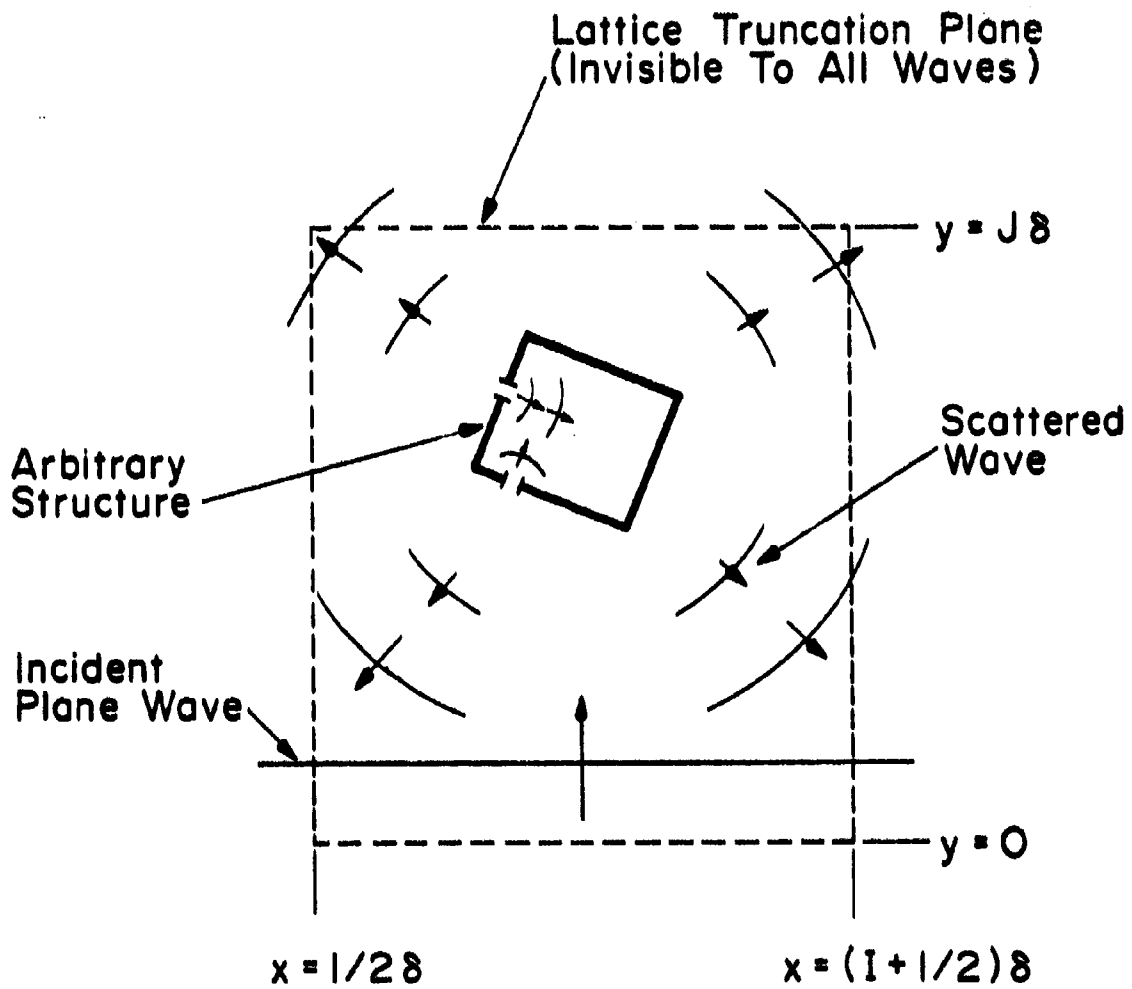
Figure 1.   TIME-DOMAIN WAVE-TRACKING CONCEPT OF THE FD-TD METHOD

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Time-stepping for the FD-TD method is accomplished by an explicit finite-difference procedure due to Yee [1]. For a cubic-cell space lattice, this procedure involves positioning the components of $\vec{E}$ and $\vec{H}$ about a unit cell of the lattice, as shown in Figure 2, and evaluating $\vec{E}$ and $\vec{H}$ at alternate half-time steps. In this manner, centered difference expressions can be used for both the space and time derivatives to attain second-order accuracy in the space and time increments without requiring simultaneous equations to compute the fields at the latest time step.

The finite-difference formulation of the FD-TD method allows the straightforward modeling of the surfaces and interiors of arbitrary dielectric or conducting structures. The structure of interest is mapped into the space lattice by first choosing the space increment and then employing a data card deck to assign values of permittivity and conductivity to each component of $\vec{E}$. No special handling of electromagnetic boundary conditions at media interfaces is required because the curl equations generate these conditions in a natural way by themselves. Therefore, the basic computer program need not be modified to change from structure to structure. In this manner, inhomogeneities or fine details of the structure can be modeled with a maximum resolution of one unit cell; thin surfaces can be modeled as stepped-edge sheets. Figure 3 shows an arbitrary three-dimensional scatterer embedded in a FD-TD lattice.

The explicit formulation of the FD-TD method is particularly suited for programming with minimum storage and execution time using recently developed array-processing computers. First, the required computer storage and running time increases only linearly with N, the total number of unknown field components. Computer techniques (such as the method-of-moments) which require the solution of simultaneous equations usually have a storage requirement proportional to N^2 and a running time proportional to N^2-N^3 [2]. Second, since all FD-TD operations are explicit and can be performed in parallel, rapid array-processing techniques can be readily applied. As will be demonstrated later, these can be employed to solve for more than 10^6 field components in a single FD-TD problem, as opposed to a maximum of about 10^3 field components for conventional approaches using simultaneous-equation solutions [2].
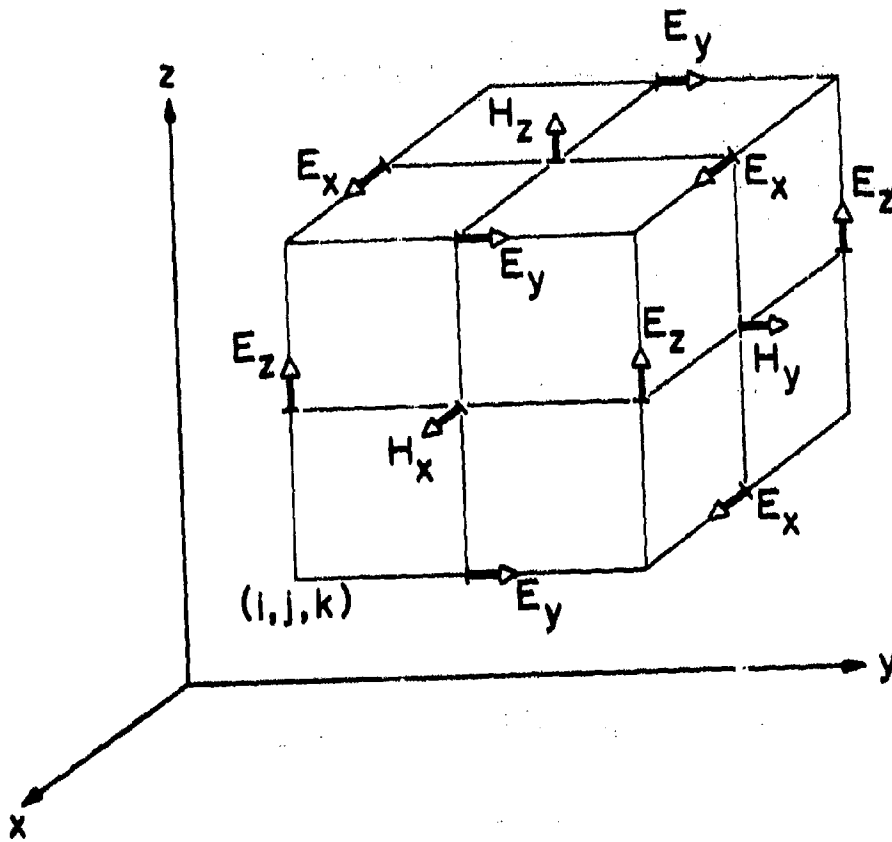
Figure 2. POSITIONS OF THE FIELD COMPONENTS ABOUT A UNIT CELL
OF THE YEE LATTICE [1]

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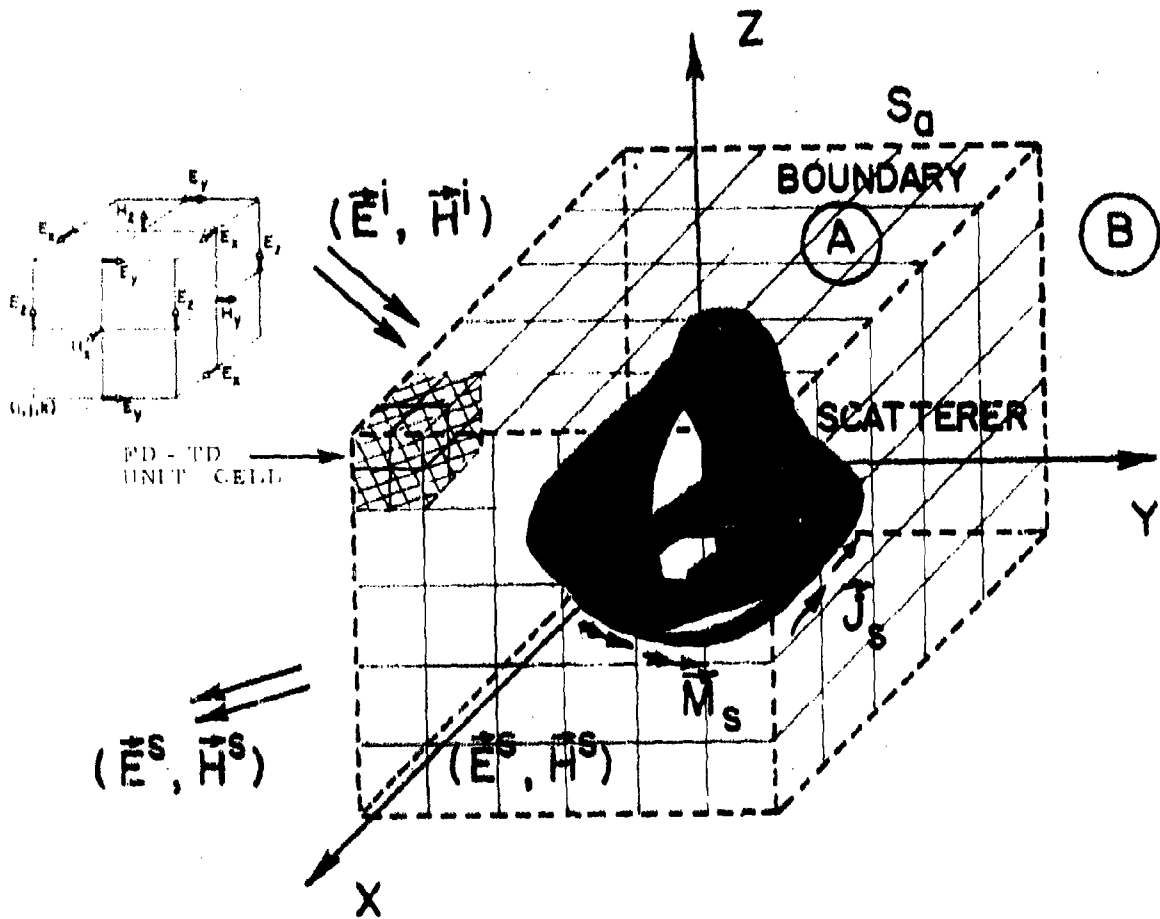
Figure 3. ARBITRARY THREE-DIMENSIONAL SCATTERER EMBEDDED IN A FD-TD LATTICE

Yec applied the FD-TD method to compute the waveforms of TM and TE
pulses scattered from infinitely long, rectangular cross section, con-
ducting cylinders [1]. Other workers investigated electromagnetic-pulse
(EMP) interactions in time-varying inhomogeneous media [3]; with metallic
bodies of revolution [4]; with detailed models of aircraft [5], [6]; with
lossy dielectric stuctures [7]; and with thin struts and wires [8]. Four
distinct problems emerged in the process of adapting the FD-TD method to
model realistic situations:

1) Lattice Truncation Conditions. The field components at the lattice
truncation planes cannot be determined directly from the Maxwell's-equations
analog and must be computed using an auxiliary truncation condition. However,
care must be exercised because this condition must not cause excessive
spurious reflection of waves scattered outward by the structure modeled.
The goal is to formulate truncation planes as close as possible to the struc-
ture (to minimize computer storage), and yet achieve virtual invisibility
of these planes to all possible waves within the lattice.

2) Incident Plane-Wave Source Condition. The simulation of either a
plane-wave pulse or single-frequency plane wave at arbitrary angles of
incidence should not take excessive storage nor cause spurious wave reflec-
tions. The former would occur if the incident wave is programmed as an
initial condition; the latter would occur if the incident wave is programmed
as a fixed field excitation along a single lattice plane.

3) Sinusoidal Steady-State Information. Such data can be obtained
either by a) directly programming a single-frequency incident plane wave
or b) performing a separate Fourier transformation step on the pulse wave-
form response. Both methods require time-stepping to a maximum time equal
to several wave periods at the desired frequency. The second method has
two additional requirements. First, a short-rise-time pulse suffers from
accumulating waveform error due to overshoot and ringing as it propagates
through the space lattice. This leads to a numerical noise component which
should be filtered before Fourier transformation. Second, Fourier trans-

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formation of many lattice-cell field-versus-time waveforms (each probably extending over many hundreds of time steps) would significantly add to the total requirements for computer storage and execution time.

4) Total-Field versus Scattered-Field Formulation. A choice exists in whether to finite-difference only the scattered field instead of the total field (at each lattice cell). The scattered-field approach may lead to a relatively superior lattice truncation condition [4]. However, the total-field approach is more useful in determining the fields penetrating structures having shielding properties, or the fields in the shadow regions of scatterers, where the total field can diminish to levels far below the incident. Scattered-field codes have traditionally run into numerical "noise" problems for such cases since they achieve interior or shadow-zone field reduction by the subtraction of nearly equal scattered and incident field quantities. Computed shielding or shadowing of more than 30 dB may be difficult to achieve in this manner because of a residual "noise" floor inherent in this subtraction process. A total-field approach does not suffer from the subtraction-noise problem and hence is suitable for computing field penetration within shielded structures or into shadow zones.

Previous work by IITRI described initial efforts to solve the first three problems above for the case of a total-field FD-TD program employing a cubic-unit-cell space lattice [9] - [13]. Simple truncation conditions were developed for two and three-dimensional lattices that reduced the reflection coefficient of closely positioned truncation planes to the order of 0.1 for waves of arbitrary incidence. A plane-wave source condition was described that allowed generation of an arbitrary pulsed or sinusoidal wave (at a fixed angle of incidence) without requiring any additional storage and without causing spurious wave reflections. Finally, it was shown that sinusoidal steady-state data could be computed using the FD-TD method by directly programming a single-frequency incident plane wave and time-stepping to the steady-state over several periods of the incident wave. Observed accuracy was ± 1 dB at points of electric-field maxima, and ± 1 lattice

cell in locating electric-field maxima and minima, when the lattice-cell size was chosen to be less than 0.05 λ, as discussed in detail in [12] and [13].

The following sub-sections will provide a full review of the latest formulation of the FD-TD method, including the features developed during the current research effort as well as the thin-wire coupling model. These new developments will be indicated by a double-asterisk (**) appearing before the sub-section heading.

2.2 Computational Details for a Uniform, Cubic Lattice

2.2.1 Basic System of Equations

Using the MKS system of units, and assuming that the material parameters, μ, ε and σ are independent of time, the following system of scalar equations is equivalent to Maxwell's equations in the rectangular coordinate system (x, y, z):

$$\frac{\partial H_x}{\partial t} = \frac{1}{\mu} \left(\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y} \right) \tag{1a}$$

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu} \left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} \right) \tag{1b}$$

$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu} \left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} \right) \tag{1c}$$

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon} \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - \sigma E_x \right) \tag{1d}$$

$$\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon} \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} - \sigma E y \right) \tag{1e}$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon} \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - \sigma E_z \right) \tag{1f}$$

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Yee [1] originally introduced a set of finite-difference equations for the system of Equations (1a) - (1f). Following Yee's notation, we denote a space point in a cubic lattice as

$$(i,j,k) = (i\delta, j\delta, k\delta) \tag{2}$$

and any function of space and time as

$$F^n(i,j,k) = F(i\delta, j\delta, k\delta, n\delta t), \tag{3}$$

where $\delta = \delta x = \delta y = \delta z$ is the space increment, δt is the time increment, and i, j, k, and n are integers. Yee used centered finite-difference expressions for the space and time derivatives that are both simply programmed and second-order accurate in δ and in δt, respectively:

$$\frac{\partial F^n(i,j,k)}{\partial x} = \frac{F^n(i+\frac{1}{2},j,k) - F^n(i-\frac{1}{2},j,k)}{\delta} + O(\delta^2) \tag{4}$$

$$\frac{\partial F^n(i,j,k)}{\partial t} = \frac{F^{n+\frac{1}{2}}(i,j,k) - F^{n-\frac{1}{2}}(i,j,k)}{\delta t} + O(\delta t^2) \tag{5}$$

To achieve the accuracy of Equation (4), and to realize all of the space derivatives of Equations (1a) - (1f), Yee positioned the components of $\vec{E}$ and $\vec{H}$ about a unit cell of the lattice as shown in Figure 2. To achieve the accuracy of Equation (5), he evaluated $\vec{E}$ and $\vec{H}$ at alternate half time steps.

Many electromagnetic interaction problems involve nonpermeable media and can be approached using a fixed time step and space increment. For such problems, the quantity $\delta t / \mu(i,j,k)\delta$ is constant for all (i,j,k) of the lattice, and the Yee system of equations can be simplified to reduce computer time in the following manner. We define the constants:

$$R = \delta t / 2\varepsilon_0 \tag{6a}$$

$$R_a = \delta t^2 / (\delta^2 \mu_0 \varepsilon_0) \tag{6b}$$

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$$R_b = \delta t / \mu_0 \delta \tag{6c}$$

$$C_a(m) = \frac{1 - R\sigma(m)/\varepsilon_r(m)}{1 + R\sigma(m)/\varepsilon_r(m)} \tag{6d}$$

$$C_b(m) = \frac{R_a}{\varepsilon_r(m) + R\sigma(m)} \tag{6e}$$

where m is an integer denoting a particular dielectric or conducting medium in the space to be modeled. We also define the proportional electric-field vector

$$\tilde{E} = R_b \vec{E}. \tag{7}$$

Using the definitions of Equations (6a) - (6e) and (7), we reformulate Yee's finite-difference system for the $\vec{H}$ time-stepping equations as:

$$H_x^{n+\frac{1}{2}}(i,j+\frac{1}{2},k+\frac{1}{2}) = H_x^{n-\frac{1}{2}}(i,j+\frac{1}{2},k+\frac{1}{2}) + \tilde{E}_y^n(i,j+\frac{1}{2},k+1) - \tilde{E}_y^n(i,j+\frac{1}{2},k)$$
$$+ \tilde{E}_z^n(i,j,k+\frac{1}{2}) - \tilde{E}_z^n(i,j+1,k+\frac{1}{2}) \tag{8a}$$

$$H_y^{n+\frac{1}{2}}(i+\frac{1}{2},j,k+\frac{1}{2}) = H_y^{n-\frac{1}{2}}(i+\frac{1}{2},j,k+\frac{1}{2}) + \tilde{E}_z^n(i+1,j,k+\frac{1}{2}) - \tilde{E}_z^n(i,j,k+\frac{1}{2})$$
$$+ \tilde{E}_x^n(i+\frac{1}{2},j,k) - \tilde{E}_x^n(i+\frac{1}{2},j,k+1) \tag{8b}$$

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$$H_z^{n+\frac{1}{2}}(i+\tfrac{1}{2},j+\tfrac{1}{2},k) = H_z^{n-\frac{1}{2}}(i+\tfrac{1}{2},j+\tfrac{1}{2},k) + \tilde{E}_x^n(i+\tfrac{1}{2},j+1,k) - \tilde{E}_x^n(i+\tfrac{1}{2},j,k)$$

$$+ \tilde{E}_y^n(i,j+\tfrac{1}{2},k) - \tilde{E}_y^n(i+1,j+\tfrac{1}{2},k) \tag{8c}$$

Assuming m = MEDIA(i,j,k) denotes the type of medium at an electric field component location, $E_{x,y,z}(i,j,k)$, the $\vec{E}$ time-stepping equations are:

$$\tilde{E}_x^{n+1}(i+\tfrac{1}{2},j,k) = C_a(m)\tilde{E}_x^n(i+\tfrac{1}{2},j,k) + C_b(m)[H_z^{n+\frac{1}{2}}(i+\tfrac{1}{2},j+\tfrac{1}{2},k) - H_z^{n+\frac{1}{2}}(i+\tfrac{1}{2},j-\tfrac{1}{2},k)$$

$$+ H_y^{n+\frac{1}{2}}(i+\tfrac{1}{2},j,k-\tfrac{1}{2}) - H_y^{n+\frac{1}{2}}(i+\tfrac{1}{2},j,k+\tfrac{1}{2})] \tag{8d}$$

$$\tilde{E}_y^{n+1}(i,j+\tfrac{1}{2},k) = C_a(m)\tilde{E}_y^n(i,j+\tfrac{1}{2},k) + C_b(m)[H_x^{n+\frac{1}{2}}(i,j+\tfrac{1}{2},k+\tfrac{1}{2}) - H_x^{n+\frac{1}{2}}(i,j+\tfrac{1}{2},k-\tfrac{1}{2})$$

$$+ H_z^{n+\frac{1}{2}}(i-\tfrac{1}{2},j+\tfrac{1}{2},k) - H_z^{n+\frac{1}{2}}(i+\tfrac{1}{2},j+\tfrac{1}{2},k)] \tag{8e}$$

$$\tilde{E}_z^{n+1}(i,j,k+\tfrac{1}{2}) = C_a(m)\tilde{E}_z^n(i,j,k+\tfrac{1}{2}) + C_b(m)[H_y^{n+\frac{1}{2}}(i+\tfrac{1}{2},j,k+\tfrac{1}{2}) - H_y^{n+\frac{1}{2}}(i-\tfrac{1}{2},j,k+\tfrac{1}{2})$$

$$+ H_x^{n+\frac{1}{2}}(i,j-\tfrac{1}{2},k+\tfrac{1}{2}) - H_x^{n+\frac{1}{2}}(i,j+\tfrac{1}{2},k+\tfrac{1}{2})] \tag{8f}$$

This reformulation eliminates the three multiplications needed by Yee to compute H_x, H_y, and H_z. Further, it eliminates the need for computer storage of separate ε and σ arrays. Now, only a MEDIA array which specifies the type-integer of the dielectric or conducting medium at the location of each electric field component in the lattice need be stored. In addition, the

ε_r and σ of each medium can now be changed without having to re-punch a large data card deck, if the basic structure geometry is unchanged. Such a change involves only the recalculation of the few values of $C_a(m)$ and $C_b(m)$.

With the system of Equations (8a) - (8f), the new value of a field vector component at any lattice point depends only on its previous value and on the previous values of the components of the other field vector at adjacent points. Therefore, at any given time step, the computation of a field vector may proceed either one point at a time, or, with a parallel processing computer, at many points at a time.

The choice of δ and δt is motivated by the reasons of accuracy and algorithm stability, respectively. To insure the accuracy of the computed spatial derivatives of the electromagnetic fields, δ must be small compared to a wavelength (usually $\leq \lambda/10$). Further, to insure that the cubic lattice approximation to the surfaces of the structure modeled is not too coarse, δ must be small compared to the overall dimensions of the structure.

To insure the stability of the time-stepping algorithm of Equations (8a) - (8f), δt is chosen to satisfy the inequality

$$\delta t \leq \left(\frac{1}{\delta x^2} + \frac{1}{\delta y^2} + \frac{1}{\delta z^2} \right)^{-\frac{1}{2}} c_{max}^{-1} = \frac{\delta}{c_{max}\sqrt{3}} \text{ (for a cubic lattice)} \qquad (9)$$

where c_{max} is the maximum wave phase velocity within the model [9]. The corresponding stability criterion set forth by Yee in Equations (7) and (8) of his paper [1] is incorrect.

**2.2.2 Lattice Regions and Plane Wave Source Condition [14]-[16]

As shown in Figure 4a, the latest formulation of the FD-TD lattice involves the division of the computation space into two distinct regions, separated by a rectangular surface which serves to connect fields in each region. In two dimensions, the surface has four faces; in three dimensions, the surface has six faces.
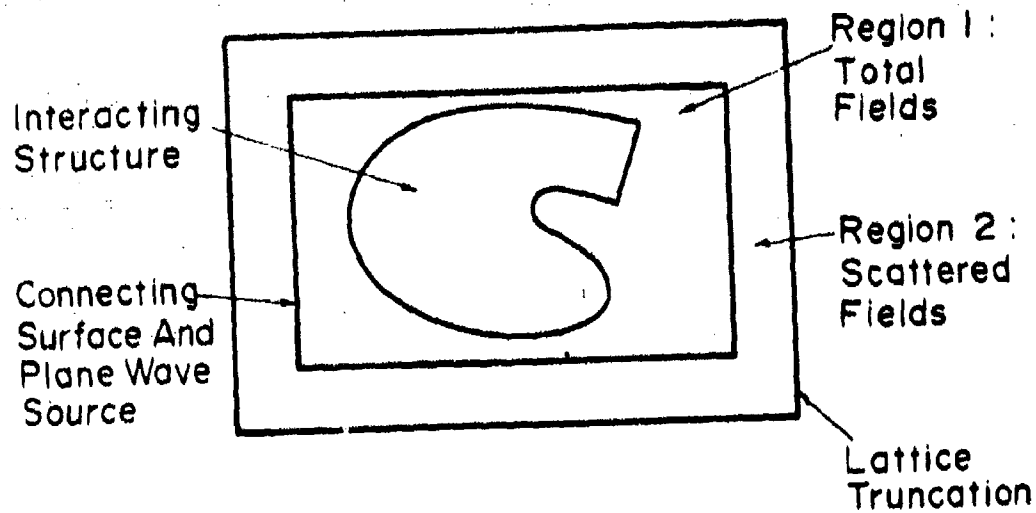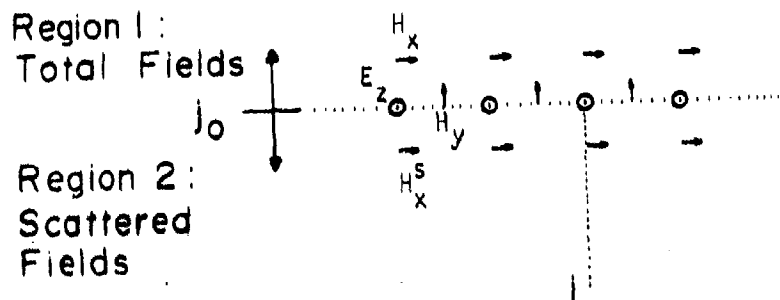
Figure 4. DIVISION OF FD-TD LATTICE INTO TOTAL-FIELD
AND SCATTERED-FIELD REGIONS. (a) Lattice
division; (b) Field component geometry at
connecting plane $y = j_0 \delta$

Region 1 of the FD-TD lattice is denoted as the total-field region. Here, it is assumed that all computed field quantities are comprised of the sum of the incident wave and the scattered field. The interacting structure of interest is embedded within this region.

Region 2 of the FD-TD lattice is denoted as the scattered-field region. Here, it is assumed that all computed field quantities are comprised only of the scattered field. The outer lattice planes bounding Region 2, called the lattice truncation planes, serve to implement the free-space radiation condition, discussed in the next subsection.

The rectangle faces comprising the boundary between Regions 1 and 2 contain E and H field components which, according to the system of equations (8a) - (8f), require the formulation of various field-component differences across the boundary planes for proceeding one time step. Typical FD-TD computations at these boundary points are as follows, using the spatial coordinates shown in Figure 4b.

$$E_z^{n+1}(i,J_o) = E_z^{n+1}(i,J_o)\big|_{\text{Eqn. (8f)}} + C_b(m)H_x^{i\,n+\frac{1}{2}}(i,J_o-\frac{1}{2}) \qquad (10a)$$

$$H_x^{n+\frac{1}{2}}(i,J_o-\frac{1}{2}) = H_x^{n+\frac{1}{2}}(i,J_o-\frac{1}{2})\big|_{\text{Eqn. (8a)}} + E_z^{i\,n}(i,J_o) \qquad (10b)$$

Here, $E_z^{n+1}(i,J_o)$ is the usual FD-TD value of the total E_z component evaluated at point (i,J_o) and time step $n+1$. Similarly, $H_x^{n+\frac{1}{2}}(i,J_o-\frac{1}{2})$ is the usual FD-TD value of the scattered H_x^s component evaluated at point $(i,J_o-\frac{1}{2})$ and time step $n+\frac{1}{2}$. $C_b(m)$ denotes a proportionality factor defined in Equation (6e). The superscript "i" denotes the known incident field component value. These computations assure consistency of the subtraction operations of field components across the Region 1/Region 2 boundary. In effect, total-field quantities are always subtracted from similar total-field quantities; and scattered-field quantities are always subtracted from

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similar scattered-field quantities. This enforcement of consistency serves to precisely connect the total-field and scattered-field regions. Further, the inclusion of arbitrary values of $\tilde{E}^i$ and H^i in the consistency relations permits the precise specification of the desired plane wave of arbitrary angle of incidence and arbitrary polarization.

There are a number of key advantages to this methodology: (a) The high-dynamic-range, total-field formalism is retained for the entirety of the interacting structure, permitting accurate computations of low-level fields penetrating into cavities through apertures, and in the shadow regions of scatterers. (b) The scattered-field formalism is retained for the lattice truncation region, permitting a very accurate simulation of the radiation condition. (c) The incident wave contribution need be computed or stored only for the field components at the rectangular surface connecting Regions 1 and 2. This results in much less computation or storage than if the incident field were to be computed at all points within the interacting structure to implement a pure scattered-field formalism. (d) The scattered near field in Region 2 can be easily integrated to derive the far-field scattering and radar cross section, as discussed later.

**2.2.3 Lattice Truncation Conditions

A basic consideration with the FD-TD approach to solve electromagnetic field problems is that most such problems are usually considered to be "open" problems where the domain of the computed field is ideally unbounded. Clearly, no computer can store an unlimited amount of data, and the field computation zone must be limited. The computation zone must be large enough to enclose the structure of interest, and a suitable boundary condition on the outer perimeter of the computation zone must be used to simulate the extension of the computation zone to infinity. Outer boundary conditions of this type have been called either radiation conditions, absorbing boundary conditions, or lattice truncation conditions by various recent workers in this area.

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Let us now consider this problem in more detail. Inspection of Equations (8a) - (8f) indicates that the values of field components at lattice truncation planes cannot be determined from the system of finite-difference equations because of the centered nature of the spatial derivatives. Therefore, these values must be computed using an auxiliary truncation condition. However, care must be taken because this condition must not cause the spurious back-reflection of outgoing scattered waves (as observed by Yee), and must not cause numerical instability. The goal of formulating the truncation condition is to make the lattice truncation planes invisible to all possible waves propagating within the lattice, as shown in Figure 1.

A desirable truncation condition relates in a simple way the values of the field components at the truncation planes to field component values at points one or more δ within the lattice. We now review several possible approaches which have been recently published.

1. _First-order approximations._ In three dimensions, an outgoing scattered-wave field component, F^S (either an electric or magnetic field) has a (r, θ, ϕ) variation of the type [17].

$$F^S = F_0 e^{j(\omega t - k_0 r)} \left[\frac{A(\theta, \phi)}{r} + \frac{B(\theta, \phi)}{r^2} + \ldots \right] \qquad (11)$$

Here, the bracketed infinite series represents in effect a multipole expansion of the scattered field, where A, B, . . . are initially unknown functions of angular position.

First-order simulations of the outer lattice boundary condition approximate the $A(\theta, \phi)/r$ dependence only. These approximations include those of: a) Taylor et al [3], using a radial field extrapolation method; b) Taflove and Brodwin [9], [12], using simulation of near-field outgoing waves with propagation delay, a field averaging process, and the possible use of electric or magnetic loss in the scattered-field zone; and c) Merewether [4] and Kunz and Lee [6] simulating the radiation condition at large distances from the center of the scatterer.

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2. Second and higher-order approximations. These simulate the $B(\theta,\phi)/r^2$ and higher-order r-dependent behavior of F^S in addition to the $A(\theta,\phi)/r$ term. These approximations include those of: a) Engquist and Majda [18]; and b) Kriegsmann and Morawetz [19]. In general, these approaches involve the construction of mixed radial and time-derivative operators, D_i on F^S, which result in $D_i(F^S) = \mathcal{O}(r^{-i-1})$ so that arbitrary precision at any r can be obtained by increasing the order, i, of D_i (here $\mathcal{O}$ represents the order of a function).

A second-order approximation of this type at truncation plane x = 0 is given by [18]

$$\left(\frac{1}{c} \frac{\partial^2 F^S}{\partial x \partial t} - \frac{1}{c^2} \frac{\partial^2 F^S}{\partial t^2} + \frac{1}{2} \left(\frac{\partial^2 F^S}{\partial y^2} + \frac{\partial^2 F^S}{\partial z^2} \right) \right)_{\substack{at \\ x=0}} = 0 + \mathcal{O}\left(\frac{1}{r^3} \right) \quad (12)$$

for the radiation condition at the lattice truncation plane x = 0. (Here c is the free-space phase velocity). A typical FD-TD computation realizing this radiation condition is given by [16]:

$$\tilde{E}_z^{n+1}(0,j,k+\tfrac{1}{2}) = -\tilde{E}_z^{n-1}(1,j,k+\tfrac{1}{2})$$

$$+ \frac{c\delta t - \delta}{c\delta t + \delta} \left[\tilde{E}_z^{n+1}(1,j,k+\tfrac{1}{2}) + \tilde{E}_z^{n-1}(0,j,k+\tfrac{1}{2}) \right]$$

$$+ \frac{2\delta}{c\delta t + \delta} \left[\tilde{E}_z^n(0,j,k+\tfrac{1}{2}) + \tilde{E}_z^n(1,j,k+\tfrac{1}{2}) \right] \quad (13)$$

$$+ \frac{(c\delta t)^2}{2\delta(c\delta t + \delta)} \begin{bmatrix} \tilde{E}_z^n(0,j+1,k+\tfrac{1}{2}) - 2\tilde{E}_z^n(0,j,k+\tfrac{1}{2}) + \tilde{E}_z^n(0,j-1,k+\tfrac{1}{2}) \\ +\tilde{E}_z^n(1,j+1,k+\tfrac{1}{2}) - 2\tilde{E}_z^n(1,j,k+\tfrac{1}{2}) + \tilde{E}_z^n(1,j-1,k+\tfrac{1}{2}) \\ +\tilde{E}_z^n(0,j,k+\tfrac{3}{2}) - 2\tilde{E}_z^n(0,j,k+\tfrac{1}{2}) + \tilde{E}_z^n(0,j,k-\tfrac{1}{2}) \\ +\tilde{E}_z^n(1,j,k+\tfrac{3}{2}) - 2\tilde{E}_z^n(1,j,k+\tfrac{1}{2}) + \tilde{E}_z^n(1,j,k-\tfrac{1}{2}) \end{bmatrix}$$

In Equation (13), $\tilde{E}_z^{n+1}$ (0,j,k+½) is the desired truncation-plane value of $\tilde{E}_z$ at point (0,j,k+½); δ is the lattice cell size; and δt is the time step. An analogous condition can be written for E_y at x = 0, as well as for E_z and E_y at truncation plane x_{max}; E_z and E_x at truncation planes y = 0 and y_{max}; and E_x and E_y at truncation planes z = 0 and z_{max} (in three-dimensional problems).

**2.2.4 Sinusoidal Steady-State Magnitude and Phase Information

Such data can be obtained either by (a) directly programming a single-frequency incident plane wave, or (b) performing a separate Fourier transformation step on the pulse waveform response. Both methods require time-stepping to a maximum time equal to several wave periods at the desired frequency. The second method has two additional requirements. First, a short-rise-time pulse suffers from accumulating waveform error due to over-shoot and ringing as it propagates through the space lattice. This leads to a numerical noise component which must be filtered before Fourier transformation. Second, Fourier transformation of many lattice-cell field-versus-time waveforms would significantly add to the total requirements for computer storage and execution time [3,4].

Recent IITRI work has shown that very accurate magnitude and phase information for sinusoidal steady-state FD-TD problems can be obtained by method (a) above and observing the peak positive and negative-going excursions of the fields over a complete cycle of the incident wave (after having time-stepped through 2-5 cycles of the transient period following the beginning of time stepping). For certain two and three-dimensional scattering problems, a dc offset of particular computed field components can be possible. This leads to the following requirements to obtain correct magnitude and phase data:

1. The peak-to-peak value of the sinusoidal response at any point must be observed to eliminate the effects of any dc offset upon the computation of the phasor magnitude.

2. The zero-crossing of the field waveform may not be useful in determining relative phase. Rather, it may be necessary to locate the zero-derivative points of the waveform for this purpose.

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The following is an algorithm to locate the zero-derivative points of a field-vs-time waveform computed using the FD-TD method. This particular algorithm has an uncertainty of one time step or less, and operates by noting where the sign of the waveform time derivative changes from positive to negative, or negative to positive.

For ease of understanding the algorithm and relating it to the Fortran program of Appendix A, the algorithm is given here in the form of a simplified Fortran program. The symbols used here are defined as follows:

NCYCS = the number of complete cycles of the sinusoidal incident wave that the FD-TD program is time-stepped;

FREQ = the frequency of the sinusoidal incident wave (in Hz);

DT = the time step of the FD-TD program (in seconds);

NMIN = the number of time steps spanning one complete cycle of the sinusoidal incident wave;

NMAX = the total number of time steps that the FD-TD program is time-stepped;

NMINA = the number of time steps spanning one-half cycle of the sinusoidal incident wave;

N = absolute time-step number (from beginning of time-stepping), ranging from 1 to NMAX;

NA = number of time steps since the start of the present full-wave-cycle observation period for zero-derivative points, ranging from 0 to NMIN-1;

F = field component (either E_x, E_y, E_z, H_x, H_y, or H_z);

DF, DFNU = time derivatives of the F waveform;

ENF = stored value of F, taken at the time of each zero-derivative event of F;

IPHF = stored value of NA, taken at the time of a zero-derivative event of F coupled with a change of sign of the F derivative from positive to negative.

The algorithm is as follows:

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```
      NMIN = 1.0/FREQ/DT
      NMID = NMIN
      NMAX = NCYCS * NMIN                              ⎫
      NMINA = FLOAT(NMIN)/2.0                          ⎬  Define constants
      NMIDA = NMINA                                    ⎪
      NMAXA = NCYCS * 2 * NMINA                        ⎭
      DO 21 N=1, NMAX
      FNMOD = FLOAT(N)/FLOAT(NMID)
      NA = N - IFIX(FNMOD)*NMID
      DO zzz J = jlimits                               ⎫
      DO zzz I = ilimits                               ⎪
      STORE = F(I,J)                                   ⎪
      F(I,J) = new F computed using Eqns.(8a-8f)       ⎪
      DFNU = F(I,J) - STORE                            ⎪  Test field component
      IF(DFNU*DF(I,J).GT.0.0) GO TO xxx                ⎬  F(i,j) for change of
      ENF(I,J) = F(I,J)                                ⎪  sign of time derivative
      IF (DFNU.LT.0.0) IPHF(I,J)= NA                   ⎪
xxx   DF(I,J) = DFNU                                   ⎪
zzz   CONTINUE                                         ⎭


      DO 12 K=NMINA,NMAXA,NMIDA                         ⎫  Test for half-cycle
      IF(N.EQ.K) GO TO 13                               ⎬  intervals of time-
 12   CONTINUE                                          ⎭  stepping
      GO TO 20
 13   Fortran statements which first print out
      the ENF array and then reset the ENF array
      to zero.
            ⋮
 20   DO 512 K=NMIN,NMAX,NMID                           ⎫  Test for full-cycle
      IF (N.EQ.K) GO TO 513                             ⎬  intervals of time-
512   CONTINUE                                          ⎭  stepping
      GO TO 520
513   Fortran statements which first print out
      the IPHF array, and then reset the IPHF array
      to zero.
```

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```
520 CONTINUE
 21 CONTINUE                              ___   End of main time-
    STOP                                        stepping loop
    END
```

Every time step, the above algorithm tests the desired field waveform,
F, for a change of sign of the time derivative. If a sign change is de-
tected, the latest computed value of F is stored in the array ENF. Note
that this value can be either positive or negative. If, further, the new
sign of the time derivative is negative, indicating that a "peak" of the
F waveform has just occurred, the time-step number, NA, is stored in the
array IPHF.

The above algorithm also tests to determine when half-cycle and full-
cycle intervals of time stepping have passed. Each half-cycle, the stored
values of ENF are first printed out and then reset to zero. Each full-
cycle, the stored values of IPHF are first printed out and then reset to
zero. Therefore, the desired zero-to-peak value of F is determined by

subtracting the last printed value of ENF from the immediately-preceding
printed value of ENF, taking the absolute value, and then dividing by two.
Also, the desired relative phase of F can be obtained by subtracting
values of IPHF (as given in the last printout of IPHF) from some fixed
reference value at a specified point. This methodology avoids any ambiguity
due to either a possible DC offset of the F waveform or the repetitive
nature of the F waveform.

   **2.2.5 Far-Field Scattering Information via the Near-to-Far
            Field Transformation

In principle, the electromagnetic scattering by an arbitrary conducting
body can be determined by solving an integral equation for the induced
electric currents on the surface of the body. Then, the induced currents
can be integrated to calculate the near or the far fields. However, the
body surface may have a complex shape or may be loaded with dielectrics in
some way as to make implementation of the needed integral equation and
surface integrals very difficult, and in fact, a unique problem for each
scatterer. A useful alternative would be to obtain the scattered-field

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information from off-surface, near field data, rather than surface-current data. Here, the near field information could be computed using the FD-TD method, which can easily account for the complexities introduced by the object's shape and composition. Further, the near-field data could be integrated along arbitrary, planar, virtual surfaces which completely surround the object of interest. In this manner, accounting of the data would be simple and would be independent of the precise shape of the object which resides within the integration surfaces. Fortunately, this near-field to far-field transformation can be shown to be feasible by using powerful electromagnetic equivalence relations.

Figure 5 demonstrates a procedure to implement the electromagnetic equivalence needed to obtain far-field data from FD-TD-computed near-field information at an arbitrary, closed, virtual boundary, S_a, fully enclosing a scatterer. S_a separates Region A (FD-TD computation zone) from Region B (exterior scattered-field zone). S_a is optimally a rectangular surface which conforms with distinct planes of the FD-TD lattice located in the scattered-field region (Region 2) of Figure 4.

To apply the hybrid FD-TD method, the tangential components of the scattered fields, $\vec{E}^S$ and $\vec{H}^S$, are first obtained at S_a using FD-TD. Then, as indicated in Figure 5b, an equivalent problem is set up external to S_a which is completely valid for Region B. The new excitation data are $\vec{J}_{S_{eq}}$ and $\vec{M}_{S_{eq}}$, the equivalent surface electric and magnetic currents, respectively, on S_a which are obtained according to [20]:

$$\vec{J}_{S_{eq}}(\vec{r}) = \hat{n} \times \vec{H}^S(\vec{r}) \tag{14a}$$

$$\vec{M}_{S_{eq}}(\vec{r}) = -\hat{n} \times \vec{E}^S(\vec{r}) \tag{14b}$$

where $\hat{n}$ is the outward unit normal vector at the surface S_a.

The equivalent surface currents on S_a produce the same scattered field ($\vec{E}^S$, $\vec{H}^S$) external to S_a as in the original problem. Region A is now made empty with zero fields and no sources, and also forced to have the same medium characteristics as Region B.
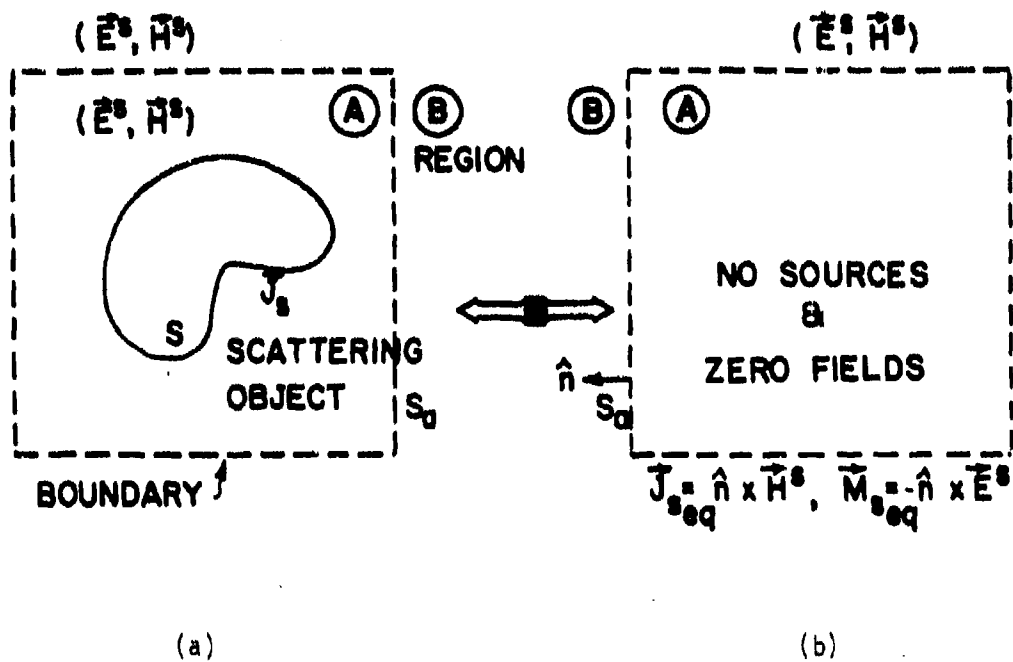
Figure 5. NEAR-TO-FAR FIELD TRANSFORMATION GEOMETRY
(a) Original problem; (b) Equivalent
problem external to the virtual surface, S_a

The scattered far fields are thus given by the transform of the equivalent currents of Eqns. (14) over the free-space Green's function [17]. If $(\mu_0, \epsilon_0, \sigma_0)$ are the Region-B medium characteristics, and if we define the relation $s = j\omega$, we have the following general scattered-field expressions [17] in three dimensions for $\vec{r} > \vec{r}_a$:

$$\vec{H}^S(\vec{r},s) = -\frac{s}{\gamma_0^2} \{ \nabla[\nabla \cdot \vec{A}_m(\vec{r},s)] - \gamma_0^2 \vec{A}_m(\vec{r},s) \} +$$

$$\frac{1}{\mu_0} \nabla \times \vec{A}(\vec{r},s) \tag{15a}$$

$$\vec{E}^S(\vec{r},s) = \frac{s}{\gamma_0^2} \{ \nabla[\nabla \cdot \vec{A}(\vec{r},s)] - \gamma_0^2 \vec{A}(\vec{r},s) \} +$$

$$\frac{s}{(\sigma_0 + s\epsilon_0)} \nabla \times \vec{A}_m(\vec{r},s) \tag{15b}$$

In Eqns. (15a) and (15b), the electric vector potential is given by

$$\vec{A}_m(\vec{r},s) = \frac{(\sigma_0 + s\epsilon_0)}{4\pi s} \iint_{S_a} \vec{M}_{s_{eq}}(\vec{r}_a{}',s) \, G(\vec{r},\vec{r}_a{}',s) \, dS_a{}' \tag{16a}$$

$$G(\vec{r},\vec{r}{}',s) = \frac{e^{-\gamma_0|\vec{r} - \vec{r}_a{}'|}}{|\vec{r} - \vec{r}_a{}'|} \tag{16b}$$

$$|\vec{r} - \vec{r}_a{}'| = [(x - x_a{}')^2 + (y - y_a{}')^2 + (z - z_a{}')^2]^{\frac{1}{2}} \tag{16c}$$

and the magnetic vector potential is given by

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$$\acute{A}(\acute{r},s) \; = \; \frac{\mu_o}{4\pi} \; \iint\limits_{S_a} \; \acute{J}_{s_{eq}}(\acute{r}_a\acute{}\,,s) \; G(\acute{r},\acute{r}_a\acute{}\,,s) \; dS_a\acute{} \qquad\qquad (16d)$$

where the propagation constant is

$$\gamma_o \; = \; [\; s\mu_o(\sigma_o + s\varepsilon_o)\;]^{\frac{1}{2}} \; = \; jk_o \; . \qquad\qquad (16e)$$

Taking the limit $r \rightarrow \infty$ gives the required far-field scattering distribution [17].

In order to validate the feasibility of the proposed hybrid method, the scattering of a transverse-magnetic (TM) polarized plane wave by a two-dimensional conducting cylinder of arbitrary cross section was considered during the present research program. This canonical problem, shown in Figure 6, is well documented [17, 21, 22] and therefore serves as a good test of feasibility and accuracy of the hybrid method.

The near-field to far-field transformation discussed above is now specialized for this two-dimensional canonical problem. Assuming that k_o is the free-space propagation constant and n_o is the intrinsic impedance equal to $\sqrt{\mu_o/\varepsilon_o}$, we have in the far-field region:

$$E_z^S \; = \; -j\omega\mu_o\Psi_{z_{eq}} \; + \; jk_o[-F_{x_{eq}}\sin\phi + F_{y_{eq}}\cos\phi] \qquad\qquad (17a)$$

where

$$\Psi_{z_{eq}} \; \sim \; n_o k_o K \int\limits_{S_a} J_{s_{eq_z}}(x\acute{}\,, y\acute{}) \; e^{+jk(x\acute{}\cos\phi + y\acute{}\sin\phi)} \; d\ell\acute{} \qquad\qquad (17b)$$

$$F_{x,y_{eq}} \; \sim \; n_o k_o K \int\limits_{S_a} M_{s_{eq_{x,y}}}(x\acute{}\,, y\acute{}) \; e^{+jk(x\acute{}\cos\phi + y\acute{}\sin\phi)} \; d\ell\acute{} \qquad (17c)$$

$$K \; = \; \frac{e^{-jk\rho}}{\sqrt{8\pi k\rho}} \; e^{-j\frac{3\pi}{4}} \qquad\qquad (17d)$$

29
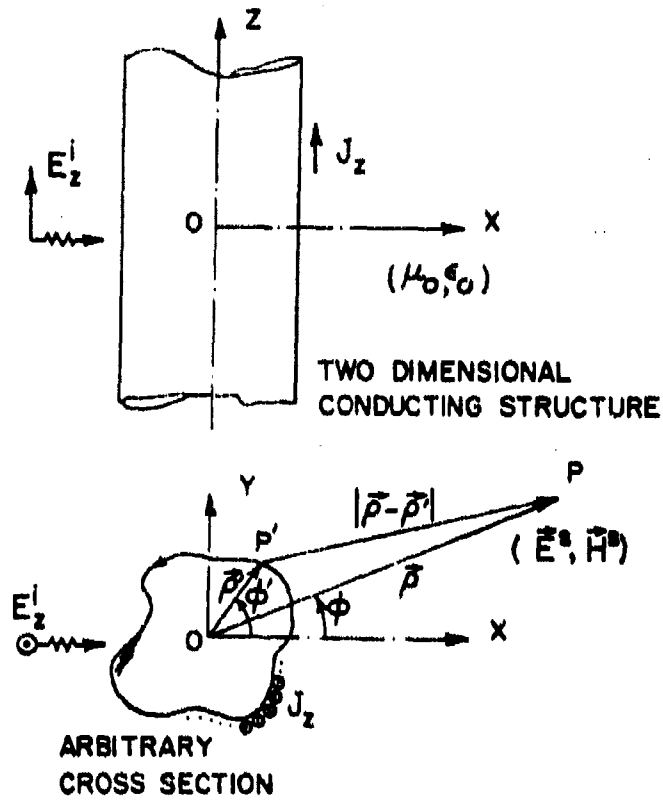
Figure 6. GEOMETRY OF TWO-DIMENSIONAL SCATTERING, TM POLARIZATION CASE

and the scattering cross section based on the equivalent currents is given by:

$$RCS_{eq} = \lim_{\rho \to \infty} \{ 2\pi\rho \ |E^S(\phi)/E^i|^2 \} . \tag{17e}$$

For comparison, the far fields and scattering cross section obtained by the usual surface electric currents (J_z) approach is based upon solving the following integral equation using the method of moments (MOM):

$$E_z^i(\vec{\rho}) = \frac{k_o \eta_o}{4} \int_C J_z(\vec{\rho}') \ H_o^{(2)}(k_o|\vec{\rho} - \vec{\rho}'|) \ d\ell' \quad \text{for} \ \vec{\rho} \in C \tag{18a}$$

$$E_z^i(\vec{\rho}) = E_o \ e^{-jk_o\rho \ \cos(\phi - \phi^i)} \tag{18b}$$

In Eqns. (18a) and (18b), ϕ^i is the angle of incidence (angle between the x axis and the direction of propagation), and $H_o^{(2)}$ is the Hankel function of the second kind and zero order. Now, the far-field distribution is obtained directly from the induced cylinder currents, J_z, by:

$$E_z^S(\phi) \sim \frac{k_o \eta_o}{\sqrt{8\pi k_o \rho}} \ e^{-j(k\rho + \frac{3\pi}{4})} \int_C J_z(x', y') \ e^{+jk(x'\cos\phi + y'\sin\phi)} \ d\ell \tag{19}$$

The scattering cross section is again given by Equation (17e).

2.2.6 Penetrating Near-Field Information via the Schelkunoff Aperture Electric Current Equivalence Principle [13]

The analysis of the electromagnetic excitation of an aperture on an arbitrarily shaped object is a complex problem [23]. This problem has been given special attention in the broad area of EMP penetration and simulation studies [24] with great efforts being applied to assess coupling to objects present behind apertures on finite, metallic, hollow scattering bodies.

The problem of the penetration and coupling of electromagnetic energy through an aperture has been studied extensively by many investigators using

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analytical and method-of-moments (MOM) numerical approaches [21] to characterize the behavior of simple aperture shapes in a conducting screen [25] - [29] or in a finite, scattering body [30, 31]. However, the analysis becomes very complex if there are other scattering objects in the vicinity which are coupled to the aperture [23], [32, 33] wherein the mutual interaction must be fully taken into account.

One powerful alternate approach is the pure FD-TD method previously discussed in References [12] and [13] which allows computation of the penetrated internal electromagnetic fields by direct modeling of the complete object of interest, including apertures and internal contents. However, the pure FD-TD method is best suited for modeling localized regions. When such a region is part of a larger structure that significantly affects the electromagnetic excitation, it becomes difficult to account for the physics of the entire coupling problem using a single FD-TD model having a constant space lattice cell size. In fact, electromagnetic coupling problems involving the need to account simultaneously for the effects of both large and small spatial details (the "global-local problem") have been difficult to structure with any single analytical or numerical technique.

In order to treat such coupling problems more effectively, a new hybrid MOM/FD-TD technique was developed based on a novel use of a field equivalence theorem due to Schelkunoff [13], [20], [34, 35]. This hybrid technique, described in this section, is basically an equivalent aperture excitation method. This allows analysis of the coupling problem in two distinct steps:

Step 1- Analysis of the relatively simple exterior problem using MOM to compute the equivalent excitation currents in the apertures leading to the interior region. This can be done independent of any knowledge of the contents of the interior region.

Step 2- Use of the FD-TD method to analyze the relatively complex interior region, assuming as an excitation the equivalent currents found in Step 1.

In this way, each analysis method is applied in the range of structure size and complexity that it is best suited for, allowing an overall solution that is accurate for large, simple structures that are penetrated by apertures leading to complex, interior cavities.

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Figure 7a illustrates the classical problem of a perfectly conducting scatterer, S_C, with an aperture, S_A. External to the scatterer in Region 1, an incident field $(\vec{E}^i, \vec{H}^i)$ illuminates the aperture, and part of the energy penetrates into the cavity (Region 2) through the aperture. To compute the total field $(\vec{E}_2, \vec{H}_2)$ in Region 2, many analytical and MOM numerical approaches studied in the literature have employed integral-equation formulations treating Regions 1 and 2 simultaneously. These approaches characterize either S_A or S_C in terms of tangential fields or equivalent surface currents [20], [21].

An alternative is to employ a field equivalence theorem due to Schelkunoff (Theorem no. 3 in [20]) to permit sequential treatment of Region 1 and Region 2. This theorem is based upon an equivalent aperture electric current formulation which connects the exterior and interior regions. This formulation expresses fields in a region as the superposition of the so-called short-circuit fields $(\vec{E}_{sc}, \vec{H}_{sc})$ with the aperture not present (shorted) plus the aperture field contribution, $(\vec{E}^A, \vec{H}^A)$, maintaining the required continuity of the fields across the aperture. The first partial field, $(\vec{E}_{sc}, \vec{H}_{sc})$, is simply equal to zero in the Region 2. The second partial field, designated $(\vec{E}^A, \vec{H}^A)$, is generated by the non-physical current distribution, $\vec{J}^A$, flowing through the empty space of the open-circuit aperture locus, where $\vec{J}^A = -\vec{J}_0^A$, the short-circuit aperture current distribution.

In Figures 7b and 7c, the MOM/FD-TD hybrid method is illustrated schematically, and is summarized below as a four-step computation.

Region 1: MOM Technique

(a) The aperture region, S_A, is short-circuited, and the straight-forward exterior problem is solved via MOM to obtain the induced electric current distribution, $\vec{J}_0^A$, in the short-circuited aperture region (Figure 7b).

(b) $\vec{J}_0^A$ is now placed in the open aperture region with a sign change (Figure 7c) to account for the continuity of the fields in the aperture region. The electric current, $-\vec{J}_0^A$, non-physically placed in the open aperture region acts as an equivalent source for the fields in Regions 1 and 2.

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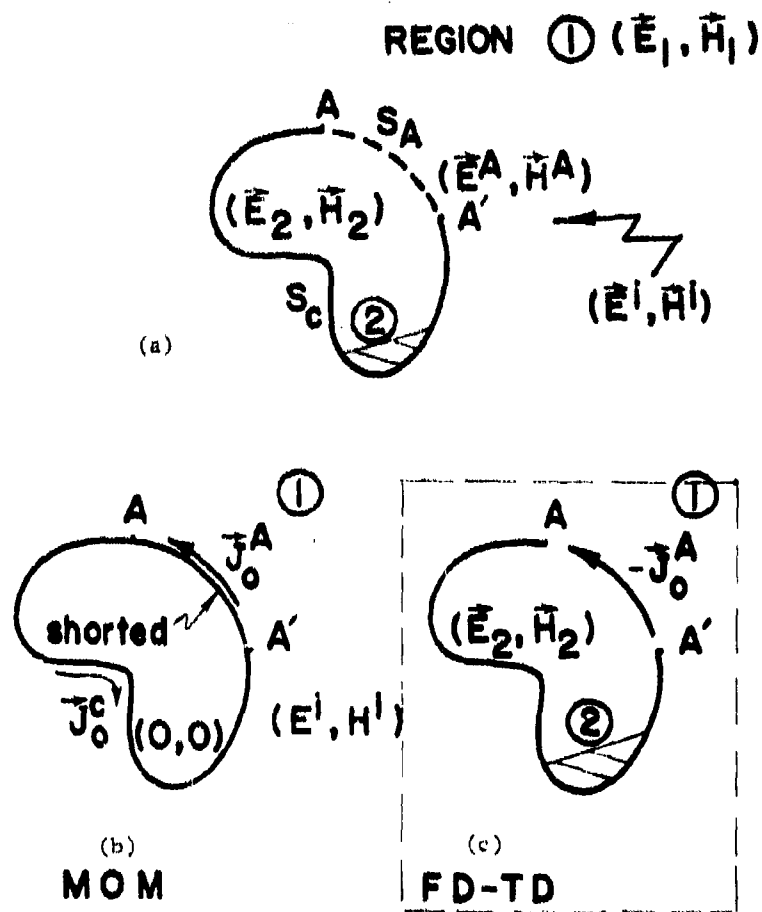REGION ① $(\vec{E}_1, \vec{H}_1)$

(a)

(b)
MOM

(c)
FD-TD

Figure 7. HYBRID MOM/FD-TD TECHNIQUE FOR COMPUTING PENETRATION
INTO CAVITIES. (a) Original problem; (b) MOM solution
of exterior problem for shorted-aperture case; (c)
FD-TD solution of interior problem using equivalent
aperture-current excitation

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Region 2: FD-TD Technique

(c) We note that the MOM technique gives the equivalent current source distribution, $-\vec{J}_o^A$, in the frequency domain as a spatial distribution of phasor quantities having both magnitude (relative to the incident fields) and phase (relative to some phase reference, normally at the origin of the coordinate system). With the FD-TD method being a time-domain technique, the phase distribution of $-\vec{J}_o^A$, is interpreted as a time-delay distribution with respect to the original phase reference location. The magnitude distribution is taken intact without interpretation or modification. In this manner, the FD-TD aperture equivalent current source distribution assumes sinusoidal steady-state quantities starting at the very beginning of time-stepping, with the proper time delay to account for phase shift.

(d) Using the FD-TD approach, the interior fields $(\vec{E}_2, \vec{H}_2)$ are computed directly by using $-\vec{J}_o^A$ as a source term distribution in the $\nabla \times H$ difference equations. For example, if $-J_{o_x}$ is a source term, Eqn. (8d) is re-written for the aperture points as:

$$m = \text{MEDIA } (i+\tfrac{1}{2}, j, k)$$

$$\tilde{E}_x^{n+1}(i+\tfrac{1}{2},j,k) = C_a(m)\tilde{E}_x^n(i+\tfrac{1}{2},j,k) + C_b(m)[H_z^{n+\tfrac{1}{2}}(i+\tfrac{1}{2},j+\tfrac{1}{2},k)$$

$$-H_z^{n+\tfrac{1}{2}}(i+\tfrac{1}{2},j-\tfrac{1}{2},k) + H_y^{n+\tfrac{1}{2}}(i+\tfrac{1}{2},j,k-\tfrac{1}{2}) - H_y^{n+\tfrac{1}{2}}(i+\tfrac{1}{2},j,k+\tfrac{1}{2})$$

$$-J_{o_x}^{n+\tfrac{1}{2}}(i+\tfrac{1}{2},j,k)] \tag{20}$$

Here, specifying J_{o_x} in the aperture is equivalent to specifying the addition of a discontinuity in the z-directed tangential magnetic field, H_z, across the aperture source plane. Namely, we have added

$$J_{o_x} = H_z^- - H_z^+ , \tag{21}$$

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where H_z^- and H_z^+ are tangential magnetic fields located an infinitesimal distance to either side of the equivalent aperture source plane. This procedure is consistent with the partial-field approach discussed by Schelkunoff [20].

Note that, unlike possible hybrid MOM/MOM methods, the hybrid MOM/FD-TD method does not require computation of an equivalent aperture electric field excitation, $\vec{E}^A$. This is because the interior-region FD-TD solution easily accepts the non-physical aperture electric current distribution, $-\vec{J}_o^A$, as the excitation. Thus, there is no need to set up and solve for the mutual interactions of the cavity contents and the apertures, and no need to compute the cavity Green's function. The hybrid MOM/FD-TD method easily permits the modeling of the cavity interior to as fine an extent as desired without any additional numerical complication. For realistic, general cavities having Green's functions that are difficult or impossible to compute, the hybrid MOM/FD-TD approach may be the only way to calculate the penetrated interior fields.

2.2.7 Thin-Wire Coupling Model

2.2.7.1 Simplified Model for Total-Field FD-TD Formulation

A very simple way of approximating electromagnetic coupling to a round, thin wire having a diameter, $2r_w$, less than the FD-TD lattice-cell size, δ, is indicated in Reference [13]. This method can be easily used when the wire of interest is embedded in a total-field modeling region of the FD-TD lattice. It is based upon modeling the thin wire as a thicker, square-cross section wire occupying exactly one cell of the lattice. Simply, the electrical conductivity, σ_m, assigned to the E components tangential to the model-wire space cells is adjusted so that the high-frequency internal impedance of the model wire equals that of the actual thin wire of interest. For example, the high-frequency internal impedance of a thin wire is given by [36]

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$$Z_{i_{wire}} = \left(\frac{\pi f \mu_w}{\sigma_w}\right)^{\frac{1}{2}} \frac{(1+j)}{2\pi r_w} \quad \text{ohms/meter} \tag{22a}$$

where f is the frequency of the wave excitation, μ_w is the permeability of the wire material, σ_w is the conductivity of the wire material, and r_w is the radius of the wire. The high-frequency internal impedance of a model wire having a square cross section occupying one lattice cell is well approximated by

$$Z_{i_{\substack{model \\ wire}}} = \left(\frac{\pi f \mu_0}{\sigma_m}\right)^{\frac{1}{2}} \frac{(1+j)}{4\delta} \quad \text{ohms/meter} \tag{22b}$$

where μ_0 is the vacuum permeability, σ_m is the conductivity of the model wire material, and δ is the width of the model wire (one lattice cell size). Equating (22a) and (22b), the desired conductivity of the model wire is given by

$$\sigma_m = \frac{\mu_0}{\mu_w} (\pi r_w/2\delta)^2 \sigma_w \quad \text{mhos/m} \tag{22c}$$

It should be pointed out that this procedure achieves extreme simplicity at the expense of neglecting the exact geometrical relation of the thin wire and its surroundings. The internal impedance of the thin wire is adequately modeled here, but the precise capacitance and inductance linking the thin wire to adjacent structures is somewhat in error due the larger effective radius used for the model wire. The following discussion summarizes a procedure for taking into account the radius of the thin wire with regard to these external impedances. This procedure is considerably more complicated than the above, and is usable mainly for a scattered-field version of the FD-TD method. The relative accuracy of the two approaches remains to be determined.

**2.2.7.2 Holland-Simpson Model [8] for Scattered-Field
FD-TD Formulation

Holland and Simpson [8] have recently described a scattered-field-only FD-TD code (which they name "THREDE") that permits the modeling of electromagnetic coupling to thin wires having a diameter of less than one space cell of the FD-TD lattice. This sub-section summarizes the theoretical background and the thin-wire coupling algorithm published by these authors.

Figure 8 shows a cross section of a FD-TD cell with a wire running perpendicularly to the plane of the figure. Holland and Simpson separate the electromagnetic field into an incident and a scattered part

$$\vec{E} = \vec{E}^{s} + \vec{E}^{i} \tag{23a}$$

$$\vec{H} = \vec{H}^{s} + \vec{H}^{i} \tag{23b}$$

They then write Maxwell's equations in polar coordinates about the wire center. The θ component of the curl-$\vec{E}$ equation is given by

$$\frac{\partial E_r^s}{\partial z} + \frac{\partial E_r^i}{\partial z} - \frac{\partial E_z^s}{\partial r} - \frac{\partial E_z^i}{\partial r} = -\mu_0 \left(\frac{\partial H_\theta^s}{\partial t} + \frac{\partial H_\theta^i}{\partial t} \right) . \tag{24}$$

At $r = a$, there is the boundary condition

$$E_z^s + E_z^i = 0 \tag{25}$$

They integrate Equation (24) out to some radius r to yield

$$E_z^s(r) + E_z^i(r) = \mu_0 \frac{\partial}{\partial t} \int_a^r (H_\theta^s + H_\theta^i) d\rho$$

$$+ \frac{\partial}{\partial z} \int_a^r (E_r^s + E_r^i) d\rho \tag{26}$$

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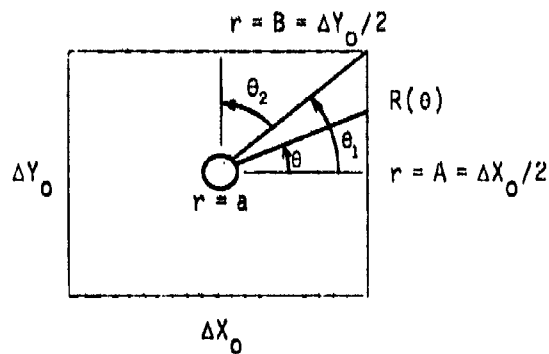
Figure 8.   GEOMETRY FOR COMPUTING THE IN-CELL INDUCTANCE
            OF A WIRE IN A CELL OF RECTANGULAR CROSS SECTION [8]
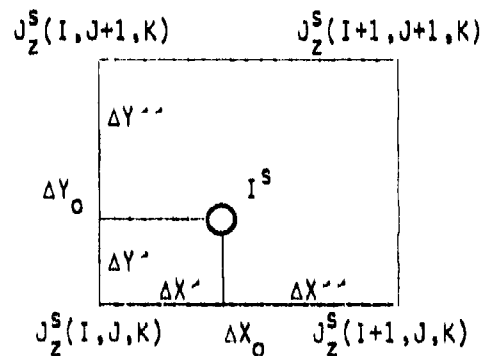
$$J_z^S(I,J,K) = (\Delta X''/\Delta X_0)(\Delta Y''/\Delta Y_0)\ I^S/(\Delta X_0 \Delta Y_0)$$

$$J_z^S(I+1,J,K) = (\Delta X'/\Delta X_0)(\Delta Y''/\Delta Y_0)\ I^S/(\Delta X_0 \Delta Y_0)$$

$$J_z^S(I,J+1,K) = (\Delta X''/\Delta X_0)(\Delta Y'/\Delta Y_0)\ I^S/(\Delta X_0 \Delta Y_0)$$

$$J_z^S(I+1,J+1,K) = (\Delta X'/\Delta X_0)(\Delta Y'/\Delta Y_0)\ I^S/(\Delta X_0 \Delta Y_0)$$

Figure 9.   PARTITIONING I^S INTO CURRENT DENSITIES J_z^S AT THE
            FOUR CLOSEST E_z^S MESH POINTS [8]

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By hypothesis, the cell sizes, and thus all r's of interest, are small compared to the shortest wavelength present. Then, in an average sense around θ, Holland and Simpson write

$$H_\theta{}^S + H_\theta{}^i = \frac{I^S + I^i}{2\pi r} \tag{27}$$

$$E_r{}^S + E_r{}^i = \frac{Q^S}{2\pi r\epsilon_0} \tag{28}$$

where I is the current on the wire and Q is the charge per unit length on the wire. If the wire is driven by current injection, I^i locally represents the relevant injected current. (Normally, a scatterer would not be excited by an incident field and current injection in the same event. Thus, Holland and Simpson usually have either $E^i = 0$ or $I^i = 0$). The I^S and Q^S represent the scattered current and charge per unit length on the wire. Holland and Simpson use Equations (27) and (28) to re-express Equation (26) as

$$E_z{}^S(r) + E_z{}^i(r) = \frac{\partial(I^S + I^i)}{\partial t} \cdot \mu_0 \frac{\ln(r/a)}{2\pi}$$

$$+ \frac{\partial Q^S}{\partial z} \cdot \frac{\ln(r/a)}{2\pi\epsilon_0} \tag{29}$$

They provide similar treatment of the r component of the curl $\vec{H}$ equation, resulting in

$$\frac{\partial Q^S}{\partial t} + \frac{\partial I^S}{\partial z} = \frac{\partial I^i}{\partial z} . \tag{30}$$

(Integrating over θ from 0 to 2π to obtain the average around θ causes the $(1/r)(\partial H_z/\partial\theta)$ term in the curl-$\vec{H}$ equation to go to zero. Equation (30) alternatively can be interpreted as merely a statement of conservation of charge on the wire.)

Holland and Simpson point out that the incident quantities E^i, H^i, and I^i are known, externally-defined variables and are input quantities to their THREDE FD-TD system. However, they note that the scattered quantities E^s, H^s, Q^s, and I^s are unknowns which are solved for by means of their FD-TD program.

They next re-write Equation (29) as

$$E_z{}^s(r) + E_z{}^i(r)$$

$$= \begin{cases} \left[\mu_0 \dfrac{\partial(I^s + I^i)}{\partial t} + \dfrac{1}{\epsilon_0} \dfrac{\partial Q^s}{\partial z} \right] \dfrac{\ln\,(r/a)}{2\pi}, & r > a \\[2em] 0, & r \leq a \end{cases} \qquad (31)$$

They state that most of the uncertainty in their modeling of the coupling to a thin wire depends on the next operation performed on Equation (31). Their common manipulation is to average Equation (31) over the area bounded by $r = R$ where $\pi R^2 =$ the cell area, $\Delta X_0 \cdot \Delta Y_0$. Averaging removes the r dependence of the equation and yields

$$L \left(\frac{\partial(I^s + I^i)}{\partial t} + \frac{1}{\epsilon_0 \mu_0} \frac{\partial Q^s}{\partial z} \right) = <E_z{}^s> + <E_z{}^i> \qquad (32)$$

where L has the dimensions of inductance per unit length and is given by

$$L = \frac{\mu_0}{2\pi} \frac{\displaystyle\int_a^R \ln(r/a) r\,dr}{\displaystyle\int_0^R r\,dr} = \frac{\mu_0}{2\pi} \left(\ln\,(R/a) - \frac{1}{2} + \frac{a^2}{2R^2} \right). \qquad (33)$$

Holland and Simpson found empirically [8] that the lower limit of r=0 on the denominator integral of Equation (33) results in a better fit of their computed results to known analytic solutions of simple problems than does a choice of r=a. Thus, the limit choice used in Equation (33) is more pragmatic than rigorous. The numerator integral of this equation must begin at r=a because $E_z^S + E_z^i$ vanishes for $r \leq a$ as Equation (31) indicates.

Holland and Simpson implement their thin-wire coupling model by applying a finite-difference analog of Equations (30) and (32) as well as Maxwell's equations. Thus, a cell with a wire running through it requires eight quantities (six field components and Q^S and I^S) to be advanced each program time step. For a wire segment parallel to the z axis, they found that it is easiest computationally to place I^S mesh points in the planes of E_z^S mesh points, and Q^S mesh points in the planes of H_z^S mesh points.

The Holland-Simpson thin-wire coupling algorithm therefore proceeds as follows [8]:

1) Evaluate $< \bar{E}^i >$ and $\partial I^i / \partial t = \dot{I}^i$ at the I^S mesh points.

2) Evaluate $< \bar{E}^S >$ at the I^S mesh points. For a wire along z, it is desirable to run the wire through the E_z^S mesh points. Then $< E_z^S >$ is just E_z^S at that point. If the wire cannot conveniently be run through the E_z^S mesh point, $< E_z^S >$ should be interpolated from the four closest mesh points, as shown in Figure 8 or 9.

3) Advance I^S according to the following finite-difference analog of Equation (32):

$$I^S(K)^n = I^S(K)^{n-1} - \Delta t \dot{I}^i(K)^{n-\frac{1}{2}}$$

$$+ \frac{\Delta t}{L} \left(< E_z^S(K)^{n-\frac{1}{2}} + < E_z^i(K)^{n-\frac{1}{2}} \right.$$

$$\left. - L \frac{Q^S(K+1)^{n-\frac{1}{2}} - Q^S(K)^{n-\frac{1}{2}}}{\varepsilon_0 \mu_0 \Delta Z_0} \right) . \tag{34}$$

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4) Advance $\bar{H}^S$ in the usual way from the curl-$\bar{E}$ equation.

5) Advance time by $\Delta t/2$.

6) Partition $I^S + I^{\dagger}$ among the four closest $E_z{}^S$ mesh points according to Figure 9. If the wire runs through the $E_z{}^S$ mesh point, as preferred, all $I^S + I^{\dagger}$ are associated with that mesh point, $J_z{}^S(I,J,K) = (I^S(K) + I^{\dagger}(K))/(\Delta X_o \cdot \Delta Y_o)$.

7) Advance $\bar{E}^S$ using the curl-$\bar{H}$ equation with $\bar{J}^S$ as determined from partioning the $I^S + I^{\dagger}$ according to step 6.

8) Advance Q^S according to the following finite-difference analog of Equation (30):

$$Q^S(K)^{n+\frac{1}{2}} = Q^S(K)^{n-\frac{1}{2}}$$

$$- \Delta t \left(\frac{I^S(K)^n + I^{\dagger}(K)^n - I^S(K-1)^n - I^{\dagger}(K-1)^n}{\Delta Z} \right) . \qquad (35)$$

9) Advance time by $\Delta t/2$ and go back to step 1.

Holland and Simpson [8] also describe their treatment of wire junctions; evaluation of the in-cell inductance for non-cubic cells; boundary conditions for wires ending either in free space or at a conducting surface; resistive wires; and applications to a linear antenna and a circular loop antenna. The reader is referred to their paper for the details.

3.0 EXAMPLES OF COMPUTED RESULTS OF THE FD-TD METHOD

3.1 Pure FD-TD Method, Three-Dimensional Penetration Problems

3.1.1 Empty Cylindrical Metal Cavity, Broadside Incidence, Transverse Electric (TE) Polarization Case [13].

This problem involved the computation of the fields within a 19.0-cm diameter, 68.5-cm long, circular aluminum cylinder with one open end. The incident plane wave was assumed to have a frequency of 300 MHz, have the

field components $E_{z_{inc}}$ and $H_{x_{inc}}$, and propagate perpendicular to the cylinder axis with a transverse electric (TE) polarization. Since the cutoff frequency of the cylinder (as a waveguide) is above 900 MHz, the interior fields are expected to decay with distance from the aperture.

The geometry of the cylinder model relative to the 160 x 63 x 24-cell (1,450,000 unknown field components) FD-TD problem lattice is depicted in Figure 10 at lattice symmetry plane k = 24 and in Figure 11 at a typical i = constant plane cutting through the cylinder transversely. Like the axial-incidence case discussed in [11], δ = 0.5 cm = $\lambda_0/200$ was used; δt = 8.33 psec; and aluminum σ = 3.7 · 10^7 mho/m. First-order correct radiation conditions were available at the time; thus, an air conductivity, σ = 0.01 mho/m, was used to attenuate scattered waves and accelerate convergence of the interior fields. 2.5 · 10^6 words of memory and 9.4 minutes of central processor time were required on the Control Data Star 100 for a complete run of 800 time steps (2.0 periods of the incident wave at 300 MHz).[1]

Figure 12 compares the FD-TD computed $|H_x/H_{x_{inc}}|$ along the cylinder axis with computed results by D. Wilton and A. Glisson using a frequency-domain, MOM, body-of-revolution computer program [37]. The FD-TD results, shown as a solid curve, are after 800 time steps; the MOM results are graphed as a dashed curve. Excellent agreement (within 0.5 dB) of the results of the two approaches is seen over the first 15 cm of field penetration into the cylinder. Over this span, the total H_x field decay is of the order of 50-55 dB. The rate of decay of H_x computed by either technique equals 3.3 dB/cm, which compares favorably with the 3.46 dB/cm rate predicted by simple waveguide mode theory for this beyond-cutoff case.

Figure 13 graphs contour maps of the FD-TD computed field components at the horizontal symmetry plane of the cylinder in an attempt to indicate the level of detail achieved.

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[1]The computer memory and running time remains unchanged if the cavity is loaded by arbitrary metal and dielectric contents, for a fixed number of time steps.

Figure 11. TRANSVERSE CROSS SECTION
OF CYLINDER MODEL OF FIGURE 10

Figure 10. FD-TD MODEL GEOMETRY OF
OPEN-ENDED ALUMINUM CYLINDER AT
HORIZONTAL SYMMETRY PLANE

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Figure 12.   COMPARISON OF RESULTS FOR THE LONGITUDINAL
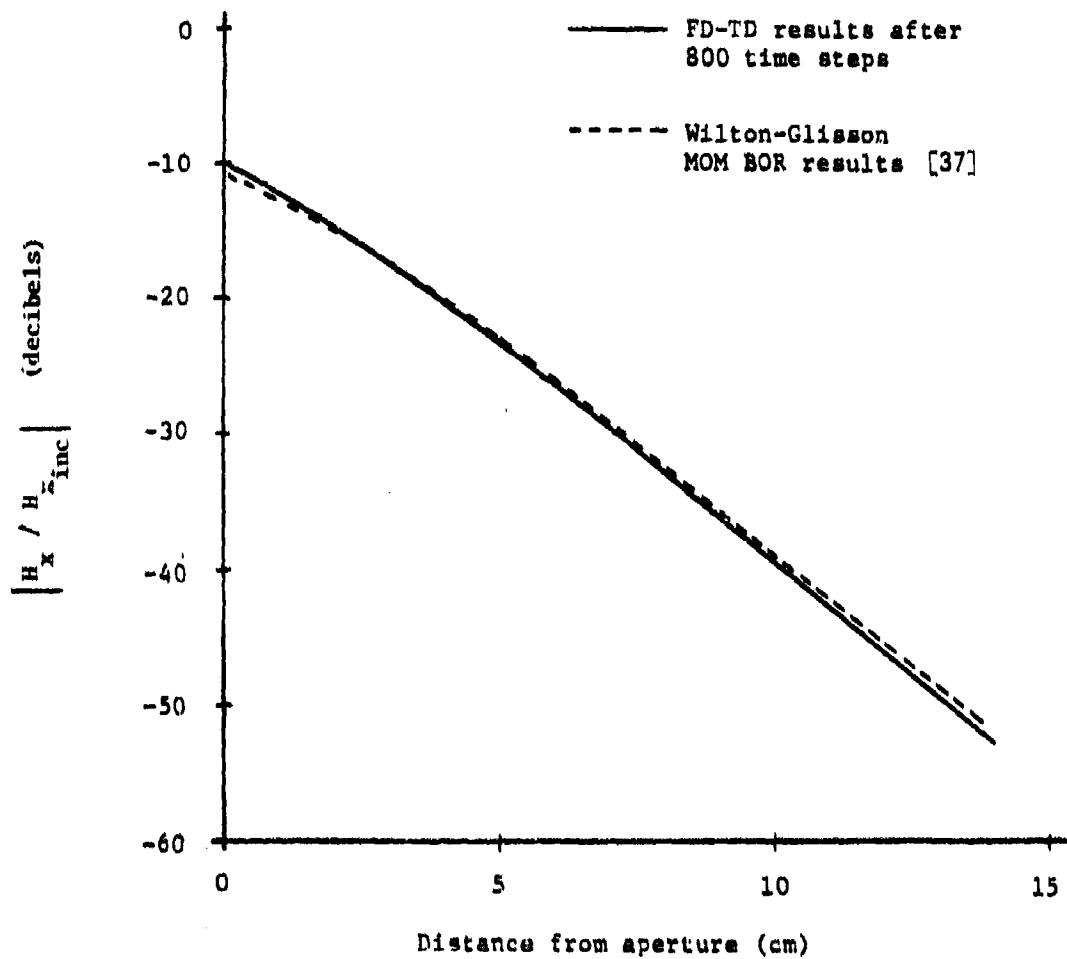MAGNETIC FIELD ALONG THE CYLINDER AXIS OF
FIGURES 10 AND 11

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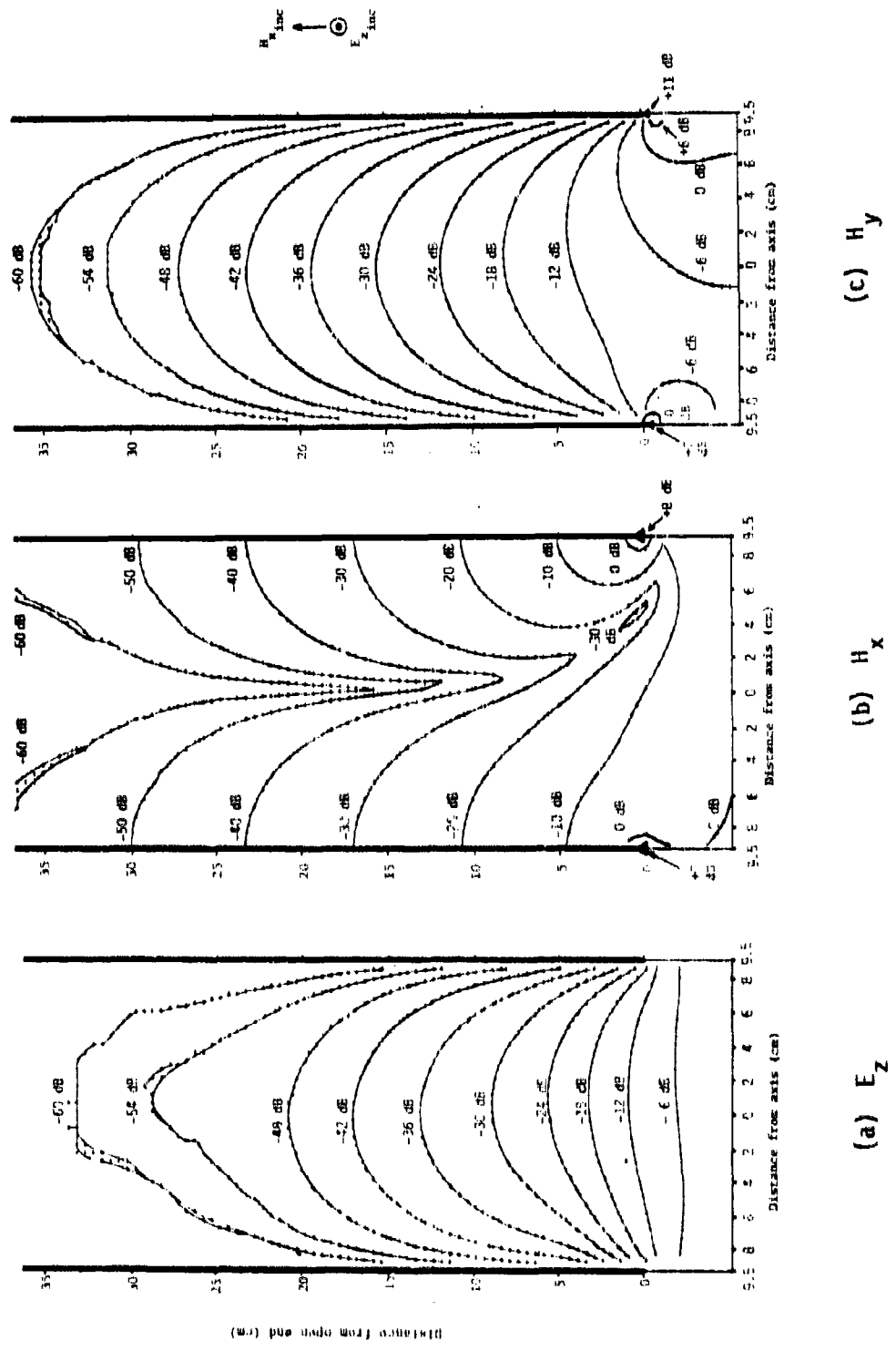Figure 13. FD-TD COMPUTED FIELD CONTOURS IN HORIZONTAL SYMMETRY PLANE OF CYLINDER OF FIGURES 10 AND 11

(a) E_z

(b) H_x

(c) H_y

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3.1.2 Loaded Missile Guidance Section, Axial Incidence Case [13].

This problem involved the computation of the fields within a detailed model of a metal-coated missile guidance section, as shown in cross-section in Figures 14 and 15, illuminated at axial incidence by the 300 MHz plane wave, $(E_{z_{inc}}, H_{x_{inc}})$.

Two apertures were modeled: a circular one in the nose (behind the magnesium fluoride infrared dome); and a circumferential sleeve fitting 23 cm aft. The missile body beyond the sleeve fitting was assumed to continue to infinity with a constant cross-section shape. The following metal and dielectric interior components were modeled:

1) Head coil assembly (assumed solid metal);

2) Cooled detector unit - CDU (assumed solid metal);

3) Phenolic ring around the CDU;

4) Preamp can (metal);

5) Wire[1] connecting the CDU to the preamp can;

6) Wire[1] connecting the preamp can to the metal backplane; and

7) Longitudinal metal support rods.

The following electrical parameters were assumed for the media comprising the model:

Medium	Relative Permittivity, ϵ'_r	Conductivity σ (mhos/m)
Aluminum	1.0	$3.7 \cdot 10^7$
Fiberglas	5.5	$2.4 \cdot 10^{-3}$
Phenolic	4.5	$8.0 \cdot 10^{-4}$
Magnesium fluoride	5.3	0.0

---

[1] The two wires were really idealizations of a more complex situation in which two multiconductor wire bundles extended between the structures mentioned. By using a simple, single-wire model for each bundle, only the bulk current of each bundle (net metal-to-metal current) was modeled using the FD-TD method. The wire model was that of Section 2.2.7.1 of this report.
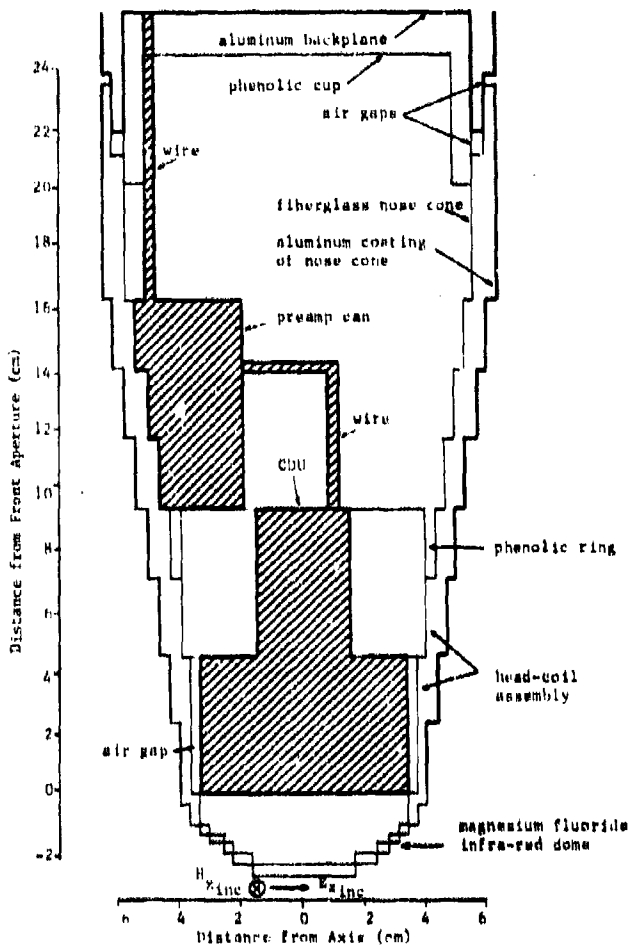
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Figure 14. GEOMETRY OF GUIDANCE SECTION
MODEL AT VERTICAL SYMMETRY PLANE,
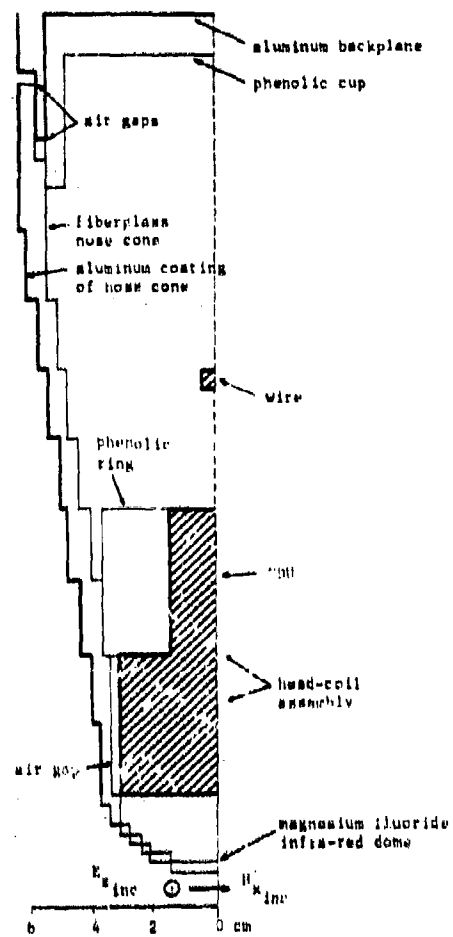SHOWING COMPONENT MATERIALS

Figure 15. GEOMETRY OF GUIDANCE
SECTION MODEL AT HORIZONTAL
OBSERVATION PLANE

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Since only first-order correct radiation conditions were available at the time, an air conductivity, $\sigma = 0.025$ mho/m, was used to attenuate scattered waves. To accelerate convergence of the interior fields, an air conductivity, $\sigma = 0.025$ mho/m, was applied to the total fields within the guidance section.

The model was implemented on a 24 x 100 x 48-cell (690,000 unknown field components) FD-TD problem lattice. A lattice resolution $\delta = 1/3$ cm $= \lambda_o/300$ was used, with a time step $\delta t = \delta/2c = 5.55$ ps. $1.6 \cdot 10^6$ words of memory and 7.0 min of Control Data Star-100 central processor time were required to complete an 1800 time-step run (3.0 cycles of the incident wave).

Figure 16 graphs contour maps of the FD-TD computed field components at the vertical symmetry plane of the guidance section. An important observation is that the wires connecting the cooled detector unit, preamp can, and metal backplane are paralleled by high-level magnetic field contours (Figure 16b). This is indicative of substantial, uniform current flow along each wire. Such current flow would generate locally a magnetic field looping around the wire which, when "cut" by the vertical symmetry plane, shows up as parallel field contours spaced equally on each side of the wire. Using a simple Ampere's law argument, I_{wire} can be computed as being equal to $H \cdot 2\pi r_o$, where H is the magnitude of the parallel magnetic field contour and r_o is its separation from the wire center. In this manner, Table 1 lists the predicted values of I_1 (current in the wire from the cooled detector unit to the pre-amp can) and I_2 (current in the wire from the pre-amp can to the backplane). For the case of I_1, it is assumed that $H = H_{inc}$ and $r_o = 1.5 \cdot \delta = 0.005$ m. For the case of I_2, it is assumed that $H = 3.16 H_{inc}$ and $r_o = 1.5 \cdot \delta = 0.005$ m.
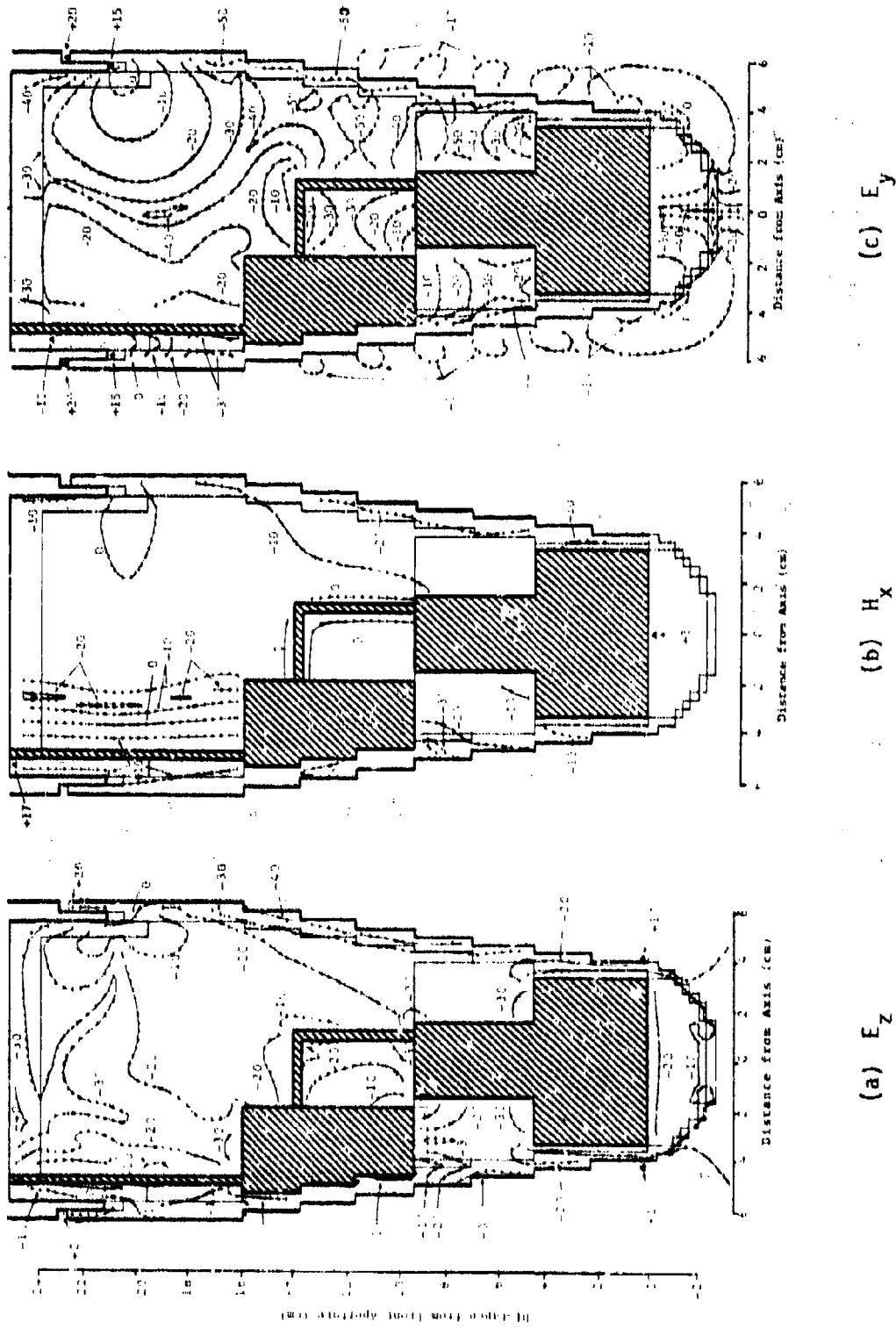
Figure 16. FD-TD COMPUTED FIELD CONTOURS IN VERTICAL SYMMETRY PLANE OF GUIDANCE SECTION OF FIGS. 14 AND 15

(a) E_z

(b) H_x

(c) E_y

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TABLE 7

PREDICTED GROUND WIRE CURRENTS

E_{inc}(V/m)	H_{inc}(A/m)	P_{inc}(μW/cm^2)	I_1(μA)	I_2(μA)
0.1	$2.65 \cdot 10^{-4}$	$2.65 \cdot 10^{-3}$	8.3	26.3
1.0	$2.65 \cdot 10^{-3}$	0.265	83	263
10.0	$2.65 \cdot 10^{-2}$	26.5	830	2630

The missile guidance section model demonstrates the capability of the pure FD-TD method to map fields penetrating into a complex structure that has multiple apertures and interior dielectric and metallic materials. Determination of the accuracy bound for this method, as applied to such complex structures, awaits the results of future experimental programs since alternative numerical approaches cannot deal with this level of complexity.

3.2 Hybrid MOM/FD-TD Method. Three-Dimensional Penetration Problem--Loaded Missile Guidance Section, Axial Incidence Case [13], [34], [35]

The MOM/FD-TD hybrid model for the axial-incidence case of the loaded missile guidance section (as shown in Figures 17 and 14, 15) was run for 1800 time steps (3.0 cycles of the incident wave at f = 300 MHz). The excitation consisted of data for the magnitude and phase of the electric current distribution over the loci of the short-circuited nose aperture and sleeve-fitting aperture (as required by Schelkunoff's third theorem) computed using a MOM body-of-revolution code [37]. The results were compared to pure FD-TD results presented above in an attempt to establish the consistency of the MOM/FD-TD model for this case.

Figure 18 plots the comparison of the hybrid MOM/FD-TD results for the H_x field contours in the vertical symmetry plane with the pure FD-TD results already presented in Figure 16b. It is seen that, for both methods stepped to 1800 time steps, there is an excellent agreement for the 0 dB contours

Axial incidence: $H_{x_{inc}}$ ⊙ ⟶ $E_{z_{inc}}$ $f = 300$ MHz

$E_{z_{inc}} = 1$ v/m

k=0   k=48

j=0 ⟶

Nose aperture
(Equivalent
source plane)

FD-TD lattice plane
i = 24.5

23.4
cm

Sleeve aperture
(Equivalent
source ring)

j=100 ⟶

1.1 m

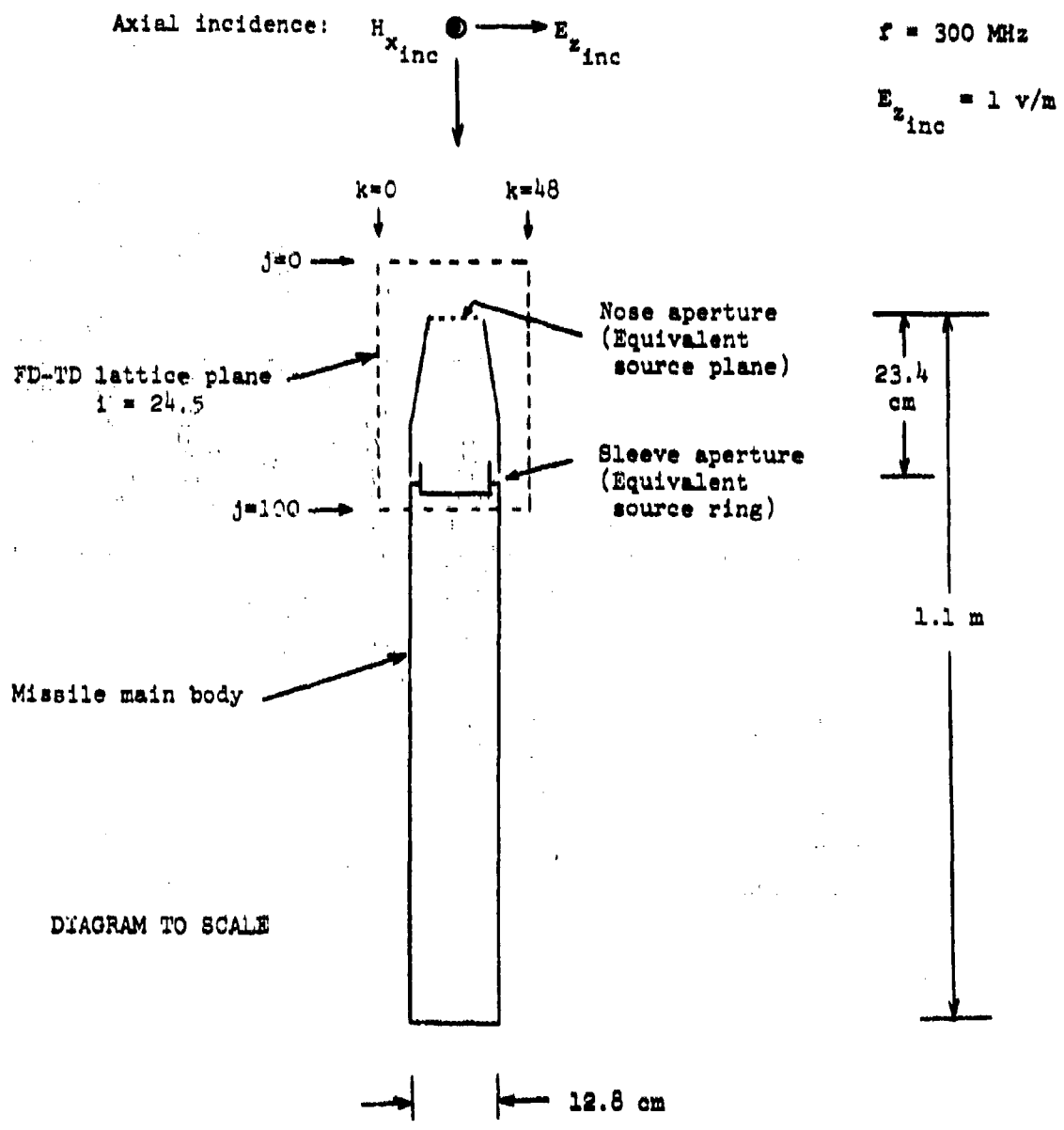Missile main body

DIAGRAM TO SCALE

12.8 cm

Figure 17. HYBRID MOM/FD-TD MODEL OF LOADED MISSILE GUIDANCE
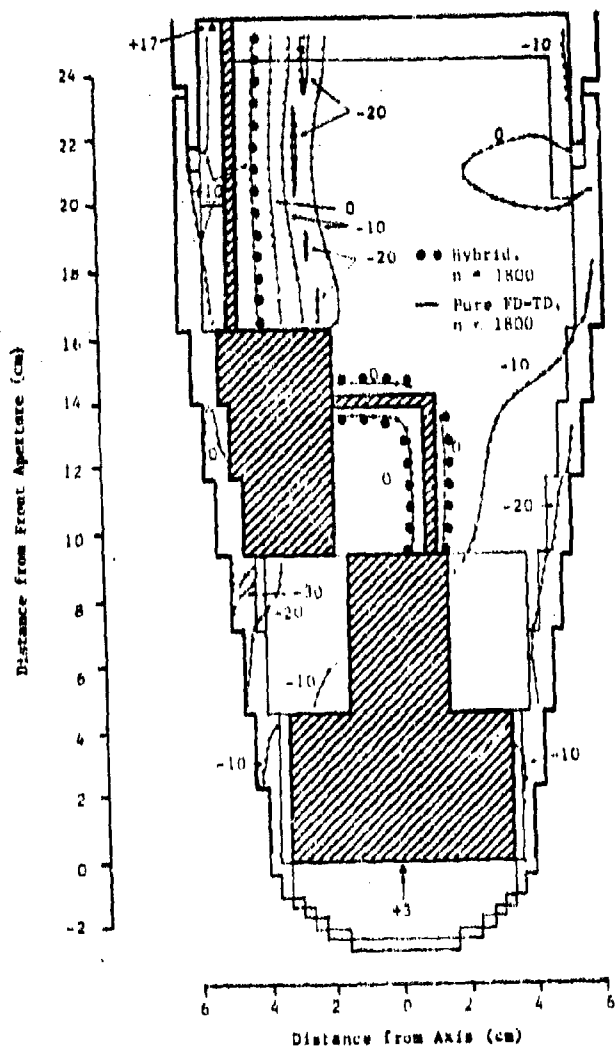SECTION FOR AXIAL INCIDENCE CASE

53

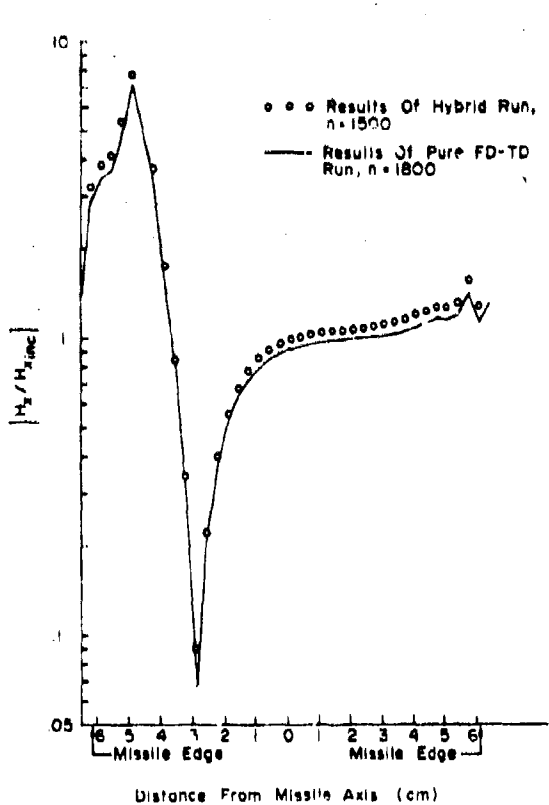Figure 18. COMPARISON OF FD-TD AND HYBRID MOM/FD-TD DATA FOR H_x CONTOURS IN VERTICAL SYMMETRY PLANE

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near the wire connecting the cooled detector unit to the pre-amp can. For this contour, the maximum spatial shift is only about 0.16 cm ($\sim 0.5\delta$) in a direction further out from the wire. Further, there is observed to be excellent agreement of the +10 dB contour near the wire connecting the pre-amp can to the metal backplane. For this contour, the maximum spatial shift is only about 0.1 cm ($\sim 0.3\delta$) in a direction further out from the wire. This implies that the currents in these two major wires are predicted to be almost the same by both the pure FD-TD method and the hybrid MOM/FD-TD method.

Figure 19 plots the comparison of the hybrid results for the H_x and E_z fields along with a vertical cut through the center of the guidance section at a point 21 cm in back of the nose aperture (about 2 cm in front of the sleeve fitting, at the point where the circumferential slot opens into the interior of the nose cone). The hybrid run results are after 1500 time steps, while the pure FD-TD results are after 1800 time steps. For this case, a very high level of agreement is observed between the two sets of data at all points of comparison. The worst-case difference between the results is only 1 dB, with most results consistent within only fractions of a decibel.[1]

This case study shows that the hybrid MOM/FD-TD approach yields results for the missile guidance section wire current and electromagnetic fields which are consistent with the pure FD-TD data. The study implies that a wire passing very close to an aperture, and strongly coupled to that aperture (as for the case of the pre-amp can-to-backplane wire near the sleeve-fitting aperture), can be consistently modeled using the pure FD-TD approach

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[1] Comparison of the data sets for the 1800 time-step case for each set results in slightly lessened agreement, such that the worst-case difference is about 1.7 dB. This may result from the hybrid program progressing to the sinusoidal steady state at a slightly faster rate than the pure FD-TD program, since a sinusoidal steady-state equivalent aperture current excitation is employed from the very beginning of the hybrid program, rather than an aperture excitation which must build to the steady state.
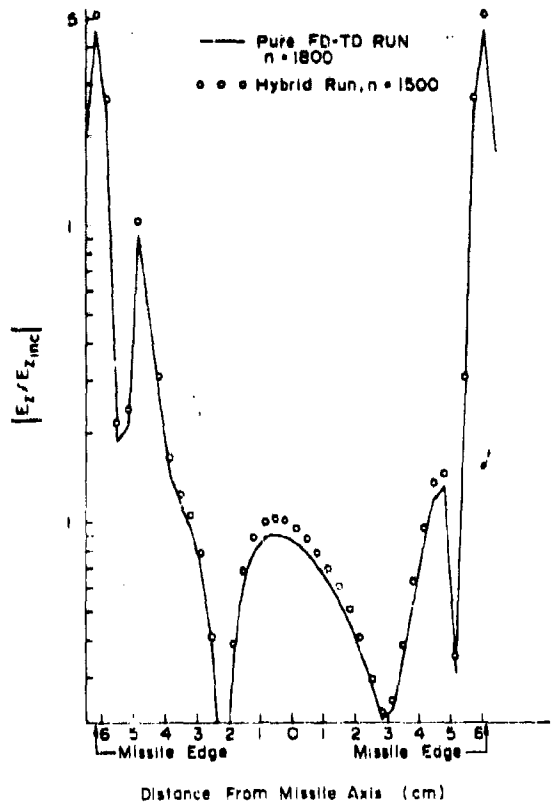
Figure 19.   COMPARISON OF FD-TD AND HYBRID MOM/FD-TD DATA
FOR FIELDS ALONG VERTICAL CUT THROUGH CENTER
OF GUIDANCE SECTION NEAR SLEEVE-FITTING APERTURE

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and the hybrid MOM/FD-TD approach. In the latter, it must be remembered that the MOM-derived equivalent aperture excitation takes into account none of the interior details of the structure. Finally, the study implies that complex sub-sections of large, simply-shaped structures are prime candidates for detailed modeling of the interior penetrating fields via the hybrid MOM/FD-TD method.

**3.3 Hybrid FD-TD Method, Two-Dimensional Scattering Problems

In order to validate the feasibility of the new formulation of the FD-TD lattice regions (Section 2.2.2), the new variable-angle plane wave source condition (Section 2.2.2), the new second-order correct lattice truncation conditions (Section 2.2.3), the new sinusoidal steady-state magnitude and phase computation (Section 2.2.4), and the new far-field scattering computation via the near-to-far field transformation (Section 2.2.5), several canonical, two-dimensional, conducting and dielectric structures were studied during the present research program. The numerical results[1] of the FD-TD computed surface electric current distribution and near electric and magnetic fields are presented for the case of a two-dimensional, square metal cylinder subject to TM-polarized illumination at normal and oblique incidence. These electric currents and near fields are compared to the MOM-computed results [21, 22]. The scattered-field pattern and the corresponding radar cross section (RCS) are derived from the near-to-far field transformation of the FD-TD data. These are then compared to the results obtained by using the MOM. Additional RCS results for the case of circular metal and dielectric cylinders are also presented. It is shown that a very high degree of correspondence is obtained using this method.

**3.3.1 Square Metal Cylinder, Normal (Broadside) Incidence, TM Polarization of Incident Wave

We first consider the example of the scattering of a plane wave by a square conducting cylinder. The cylinder has the electrical size $k_o s = 2$, where s is the width of the side of the cylinder. The plane-wave excitation

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[1] The numerical results were obtained using the FD-TD computer program listed in Appendix A. Standard Fortran is used for this program.

is TM polarized, with field components E_z^i and H_x^i, and propagates in the +y direction so that it is at normal incidence to one side of the cylinder ($\phi^i = 0°$). An 84-point MOM solution of Equation (18a) is used as the benchmark for comparison with all FD-TD results, with pulse current expansion and point matching [21, 22].

For the FD-TD analysis, the square cylinder is embedded in a two-dimensional lattice as shown in Figure 4. Each side of the cylinder spans 20 lattice-cell divisions. The connecting virtual surface between the FD-TD total-field and scattered-field regions is located at a uniform distance of 5 cells from the cylinder surface. Figure 20 shows the geometry of the square cylinder, the incident plane wave, and the loci $\overline{abcd}$ (cylinder surface) and $\overline{ABCD}$ (off-surface, near-scattered field) along which comparative FD-TD and MOM results will be graphed.

Figures 21a and 21b graph the comparative results for the FD-TD and MOM analyses of the magnitude and phase of the cylinder surface electric current distribution for this case. Here, the FD-TD computed surface current is taken as $\hat{n} \times \vec{H}_{tan}$, where $\hat{n}$ is the unit normal vector at the cylinder surface, and $\vec{H}_{tan}$ is the FD-TD value of the magnetic field parallel to the cylinder surface, but at a distance of 0.5 space cell from the surface. (The displacement of the H component from the cylinder surface is a consequence of the spatially-interleaved nature of the E and H components of the FD-TD lattice, indicated in Figure 2). In Figure 21a, the magnitude of the FD-TD computed surface current agrees with the 84-point MOM solution to better than ±1% (0.09 dB) at all comparison points more than 2 cells from the cylinder edges (current singularities). In Figure 21b, the phase of the FD-TD solution agrees with the MOM solution to within ±3° at all comparison points, including the shadow region. The uncertainty bars shown in this figure indicate the present level of imprecision in using the FD-TD method to locate the constant-phase points of the computed time-domain $\vec{H}_{tan}$ waveform (equivalent to ±1 time step). This imprecision can be reduced in future FD-TD programs by incorporating a simple interpolation algorithm to achieve fractional time-step resolution of points of constant phase.

Figures 22a, 22b, 23a and 23b show the comparison of the magnitude and
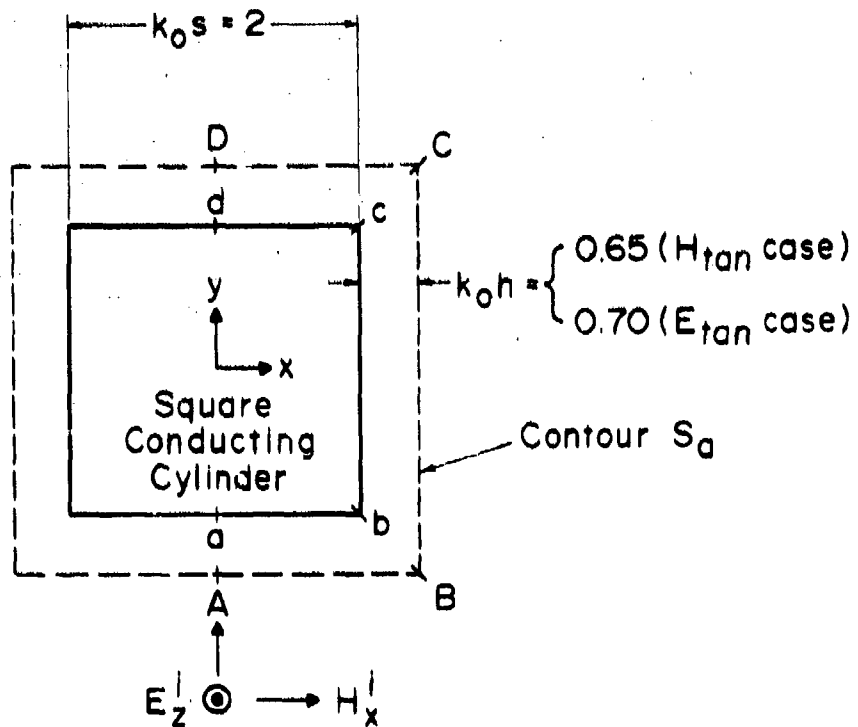
Fig. 20. GEOMETRY FOR TWO-DIMENSIONAL SCATTERING EXAMPLE : SQUARE CONDUCTING CYLINDER ILLUMINATED BY A PLANE WAVE AT NORMAL INCIDENCE ($\phi = 0°$)
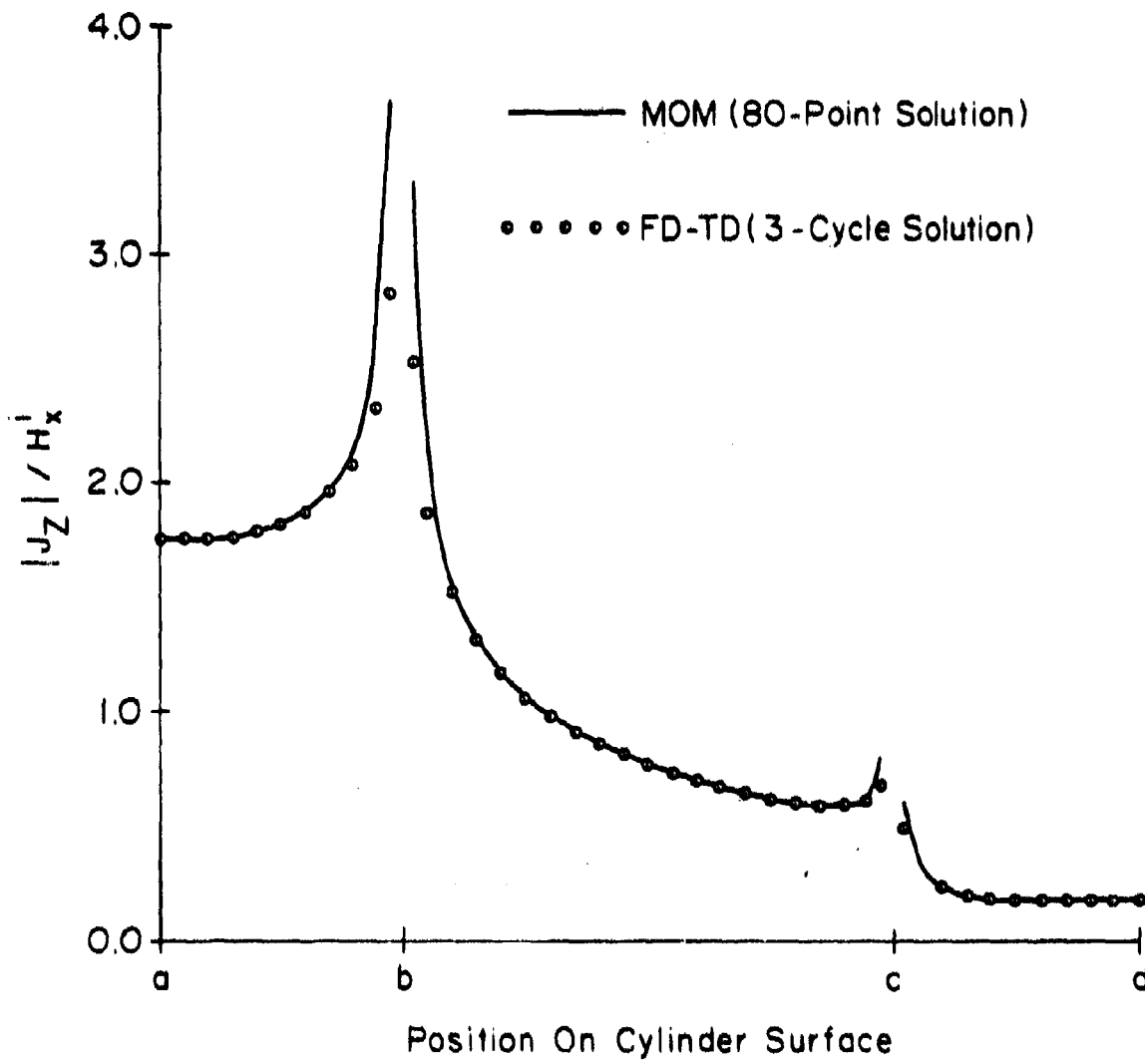
Fig. 21a. COMPARISON OF MOM AND FD-TD RESULTS FOR MAGNITUDE OF ELECTRIC CURRENTS ON SURFACE OF CYLINDER.
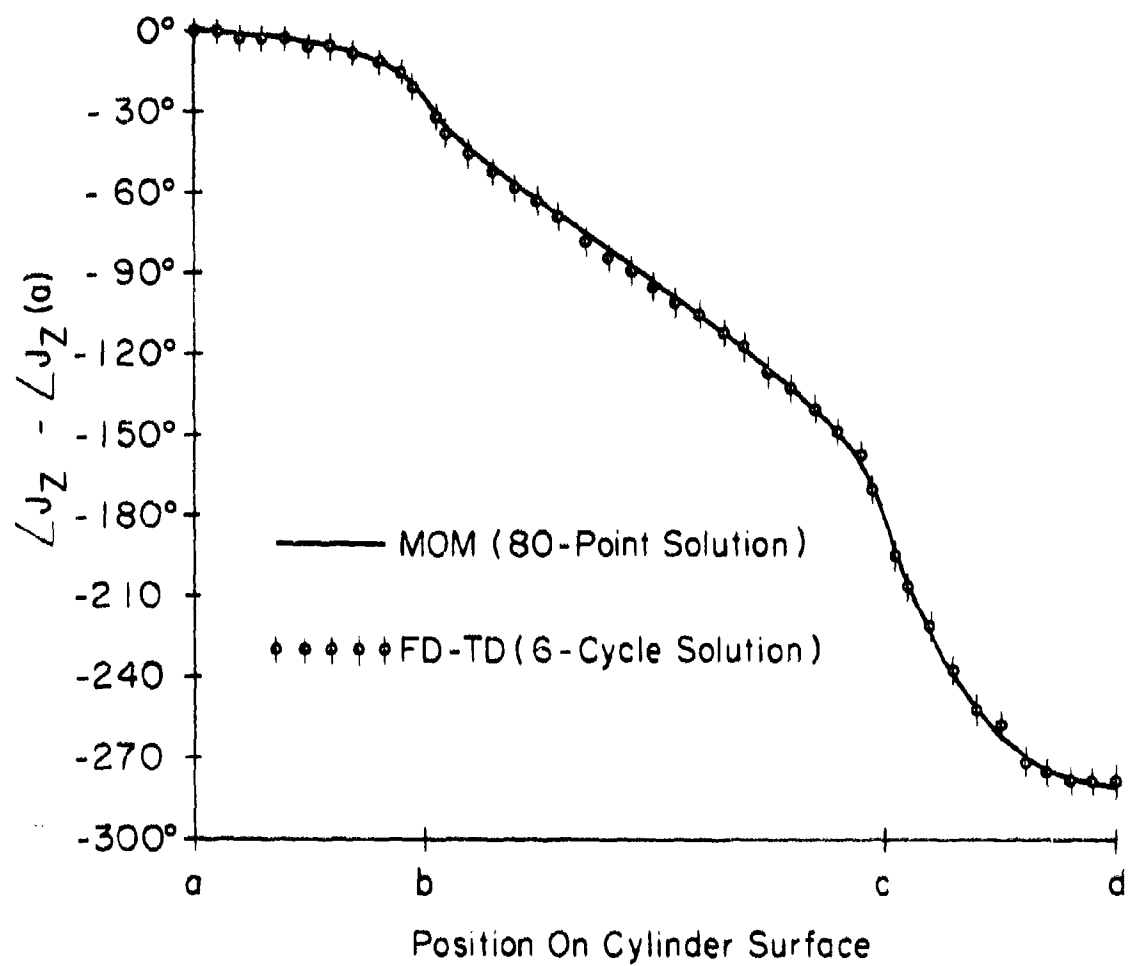
Fig. 21b. COMPARISON OF MOM AND FD-TD RESULTS FOR PHASE OF ELECTRIC CURRENTS ON SURFACE OF CYLINDER
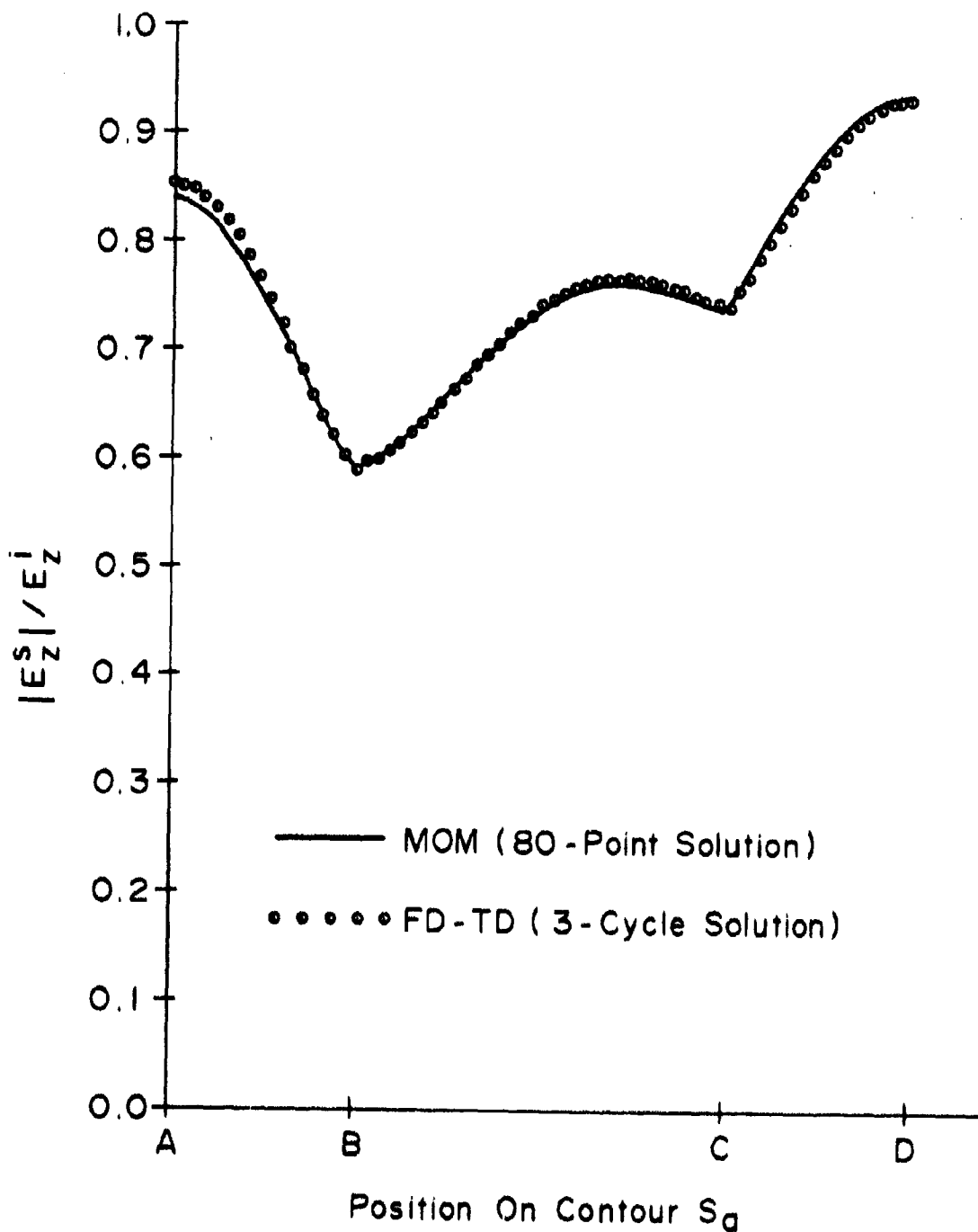
Fig. 22a. COMPARISON OF MOM AND FD-TD RESULTS FOR NEAR ELECTRIC FIELD TANGENTIAL TO CONTOURS S_a

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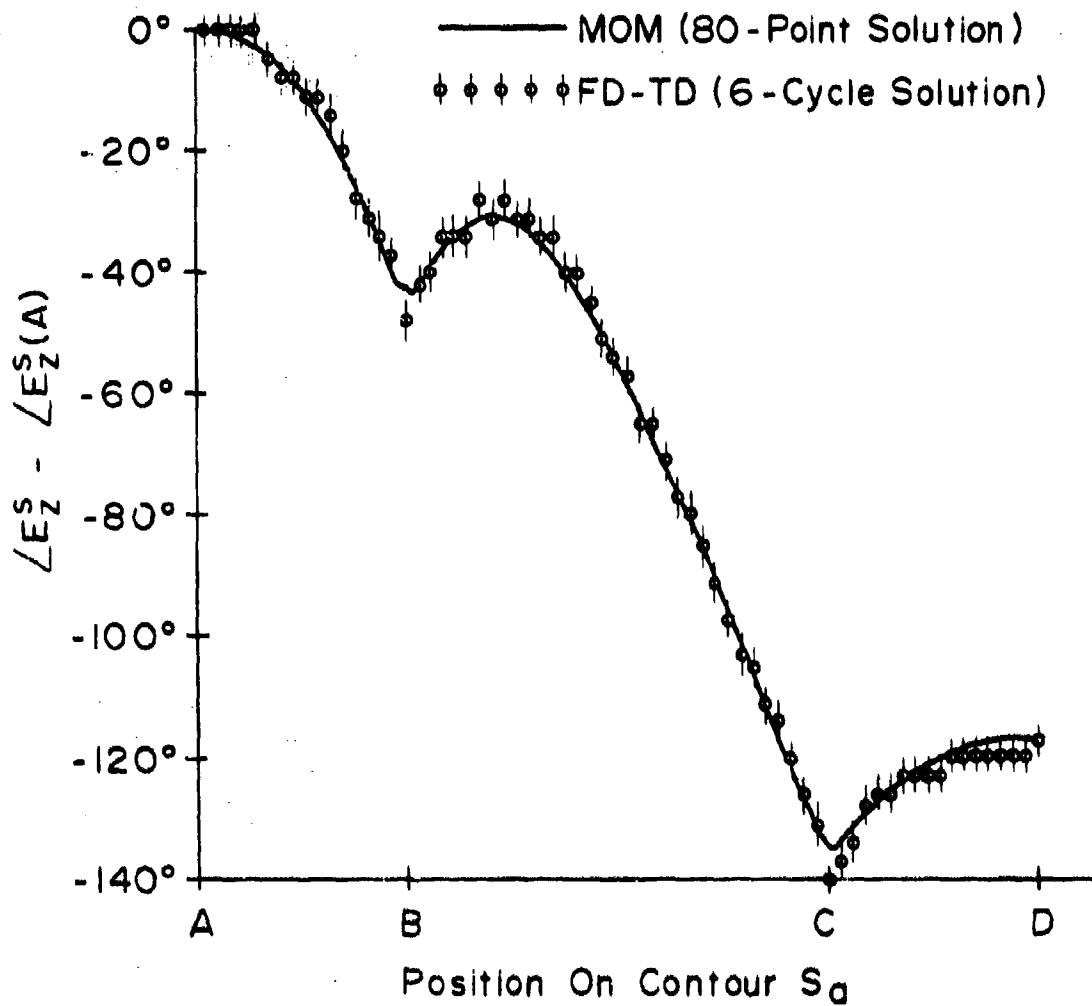
Fig. 22b. COMPARISON OF MOM AND FD-TD RESULTS FOR PHASE OF NEAR ELECTRIC FIELD TANGENTIAL TO CONTOUR S_a

63
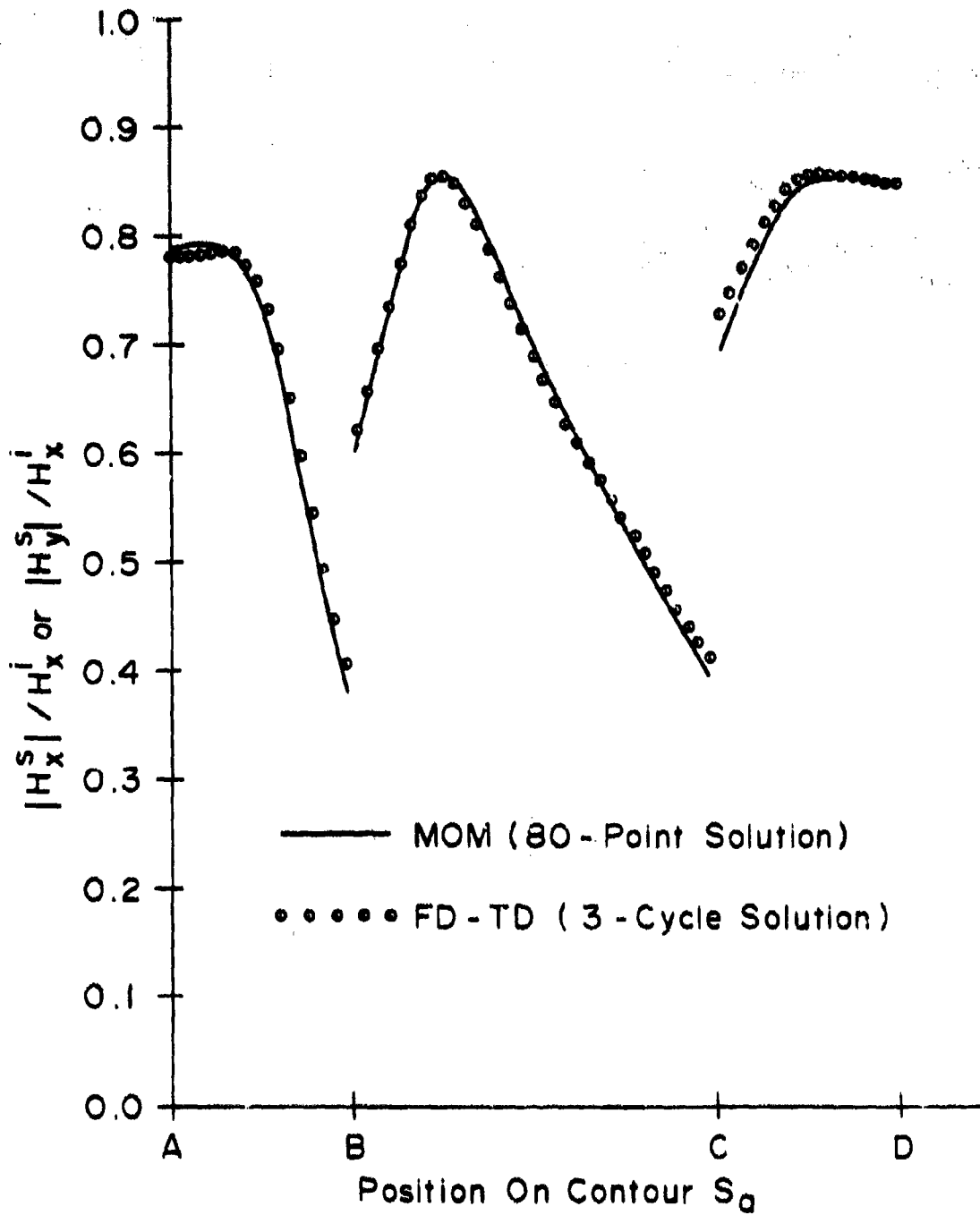
Fig. 23a. COMPARISON OF MOM AND FD-TD RESULTS FOR NEAR MAGNETIC FIELD TANGENTIAL TO CONTOURS S_a

64
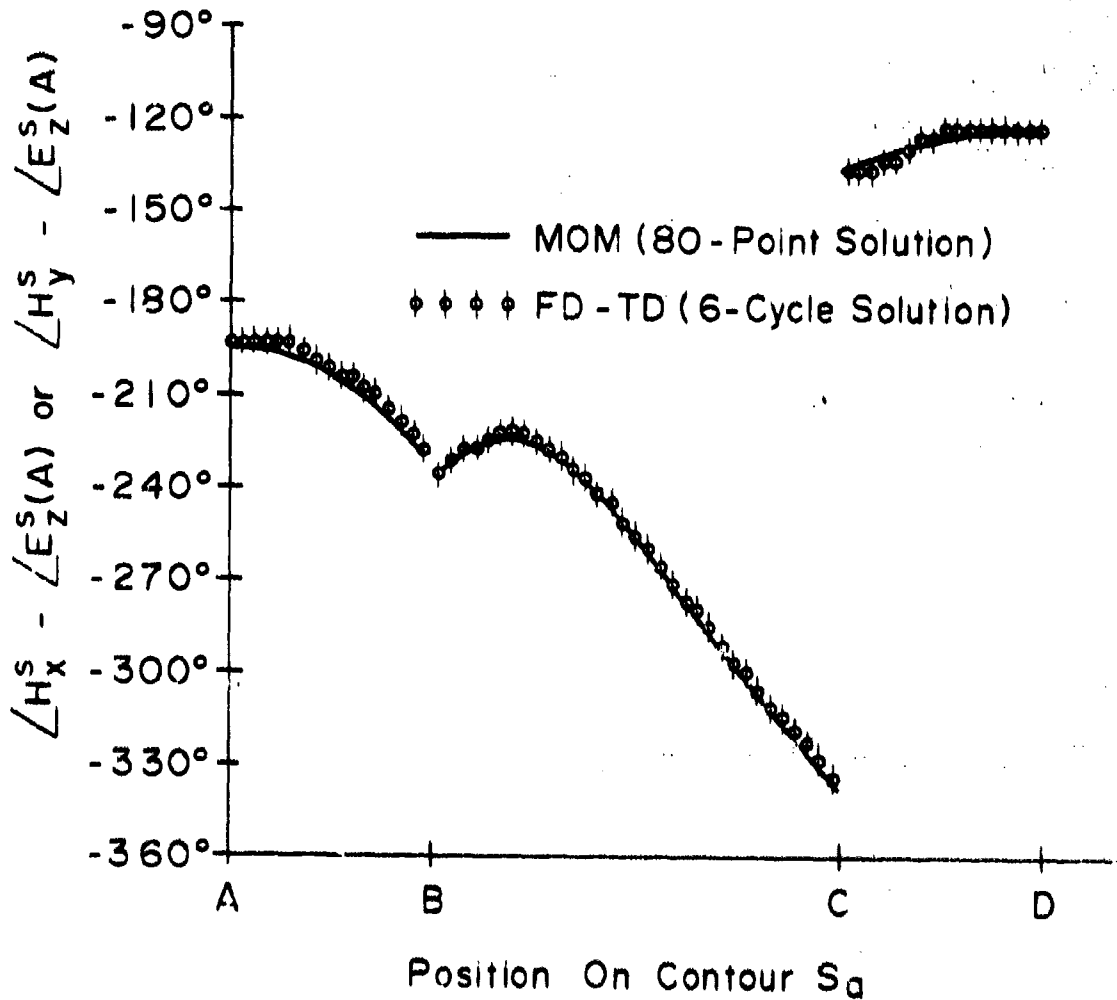
Fig. 23b. COMPARISON OF MOM AND FD-TD RESULTS FOR PHASE OF NEAR MAGNETIC FIELDS TANGENTIAL TO CONTOUR S_a

phase of the near scattered electric and magnetic fields computed by the
FD-TD method and MOM. The electric field is tangential to a virtual
surface, S_a, located at a uniform distance of h = 7 space cells from the
cylinder surface; and the magnetic field is tangential to a virtual surface,
S'_a, located at 6.5 cells from the cylinder surface. Both virtual surfaces
are embedded in the scattered-field region of the FD-TD lattice. The level
of correspondence between the FD-TD and MOM results is ±2.5% (±0.2 dB) and
±3°.

In order to obtain the far-field pattern and the scattering cross-
section, the near-field to far-field transformation discussed in Section
2.2.5 is followed for two-dimensional structures. The near scattered
fields shown in Figures 22a, 22b, 23a and 23b are converted into the corre-
sponding equivalent electric and magnetic current distributions along S_a
according to Equations (14a) and (14b). The far field is then computed
by applying Equations (17a) - (17e).

Figure 24 plots the normalized radar cross section (RCS) versus
scattering angle, ϕ, for the square, conducting cylinder. Two cases are
plotted. First, the solid curve represents the RCS computed directly from
the induced surface electric currents of the cylinder. These surface
currents were derived via a MOM solution of Equation (18), and the far field
derived by performing the integration of Equation (19). Second, the dots
represent RCS values computed by converting the FD-TD derived near electric
and magnetic fields of Figures 22a, 22b, 23a, 23b to the far field by
applying Equations (14a), (14b), and (17a) - (17e). A very high degree of
correspondence is indicated between the two sets of results. This corre-
spondence indicates the validity of the hybrid FD-TD method for computing
the far field and radar cross section, as well as the accuracy of the new
FD-TD algorithm provisions.

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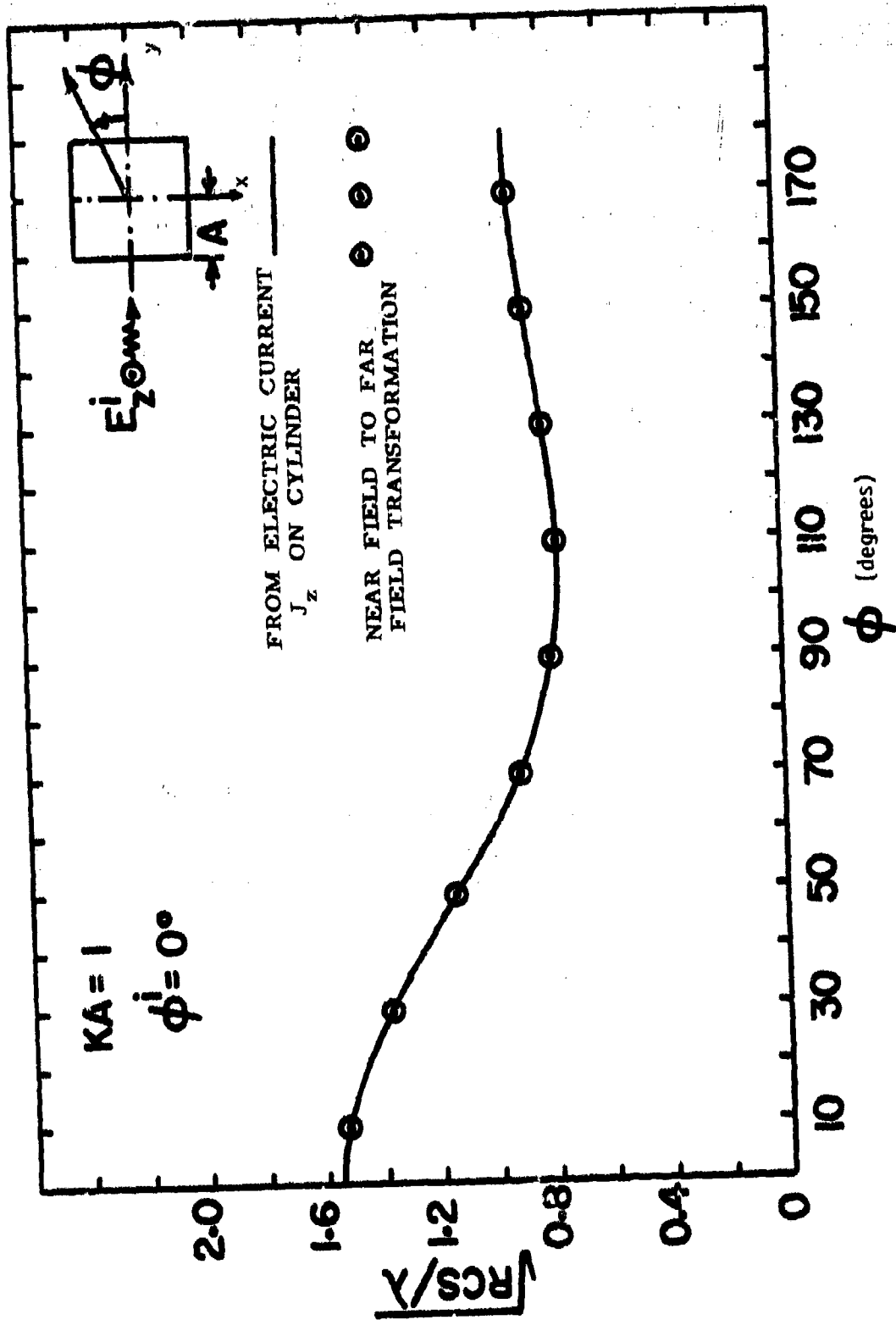
Figure 24. EQUIVALENCE OF COMPUTED RESULTS FOR RADAR CROSS SECTION OBTAINED USING THE MOM SURFACE CURRENTS APPROACH AND THE HYBRID FD-TD NEAR-TO-FAR FIELD TRANSFORMATION TECHNIQUE

**3.3.2 Square Metal Cylinder, Oblique (45°) Incidence, TM
         Polarization of Incident Wave

We next consider the oblique illumination of the square metal cylinder
of the previous example. Figure 25 shows the geometry of the square cylinder,
the incident plane wave, and the loci $\overline{abc}$ (cylinder surface) and $\overline{ABC}$ (off-
surface, near-scattered field) along which comparative FD-TD and MOM results
will be graphed. The only change made from the previous scattering example
is that the incident wave is assumed to propagate at an angle of 45° rela-
tive to one side of the cylinder ($\phi^i \rightarrow 45°$). This change is made simply by
altering one data statement of the Fortran computer program.

Figures 26a and 26b graph the comparative results for the FD-TD and
MOM analyses of the magnitude and phase of the cylinder surface electric
current distribution for this case. Again, the FD-TD computed surface
current is taken as $\hat{n} \times \hat{H}_{tan}$, where $\hat{n}$ is the unit normal vector at the
cylinder surface, and $\hat{H}_{tan}$ is the FD-TD value of the magnetic field parallel
to the cylinder surface, but at a distance of 0.5 space cell from the surface.
In Figure 26a, the magnitude of the FD-TD computed surface current agrees
again with the 84-point MOM solution to better than $\pm1\%$ (±0.09 dB) at all
comparison points more than 2 cells from the cylinder edges (current singu-
larities). In Figure 26b, the phase of the FD-TD solution again agrees with
the MOM solution to within $\pm3°$ at virtually each comparison point, including
the shadow region.

Figure 27a, 27b, 28a, and 28b show the comparison of the magnitude
and phase of the near-scattered electric and magnetic fields cor uted by the
FD-TD method and MOM. Again, the electric field is tangential to a virtual
surface, S_a, located at a uniform distance of $h = 7$ space cells from the
cylinder surface; and the magnetic field is tangential to a virtual surface,
S'_a, located at 6.5 cells from the cylinder surface. The level of corre-
pondence between the FD-TD and MOM results is again $\pm2.5\%$ (±0.2 dB) and $\pm3°$.
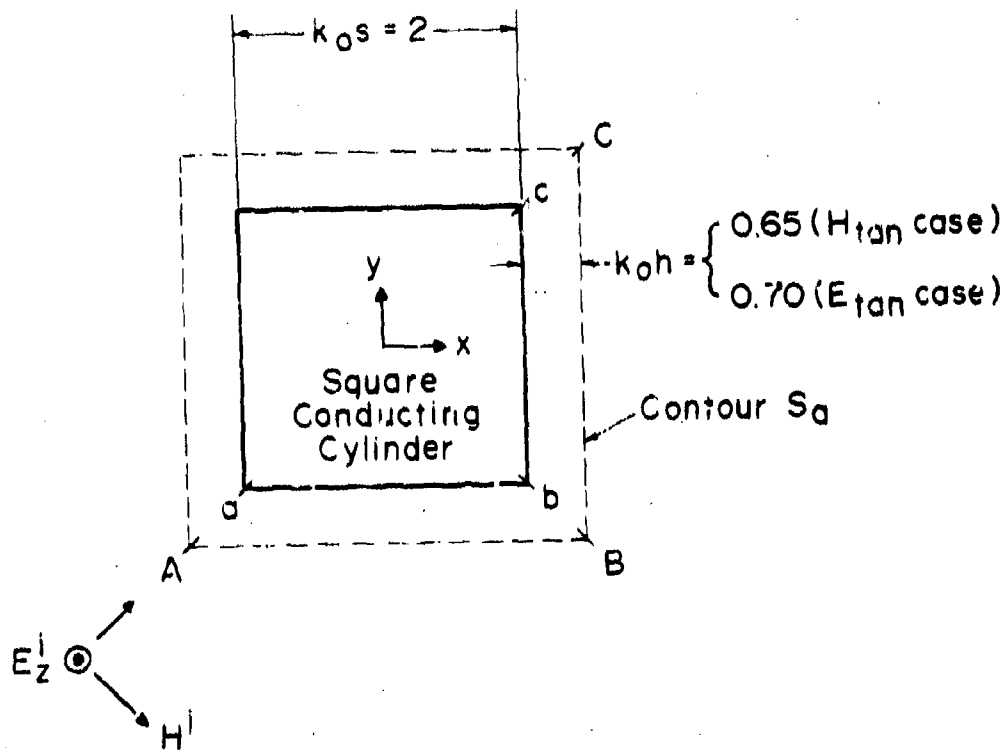
Fig. 25. GEOMETRY FOR TWO-DIMENSIONAL SCATTERING
EXAMPLE : SQUARE CONDUCTING CYLINDER
ILLUMINATED BY A PLANE WAVE AT OBLIQUE
INCIDENCE ($\phi = 45°$)
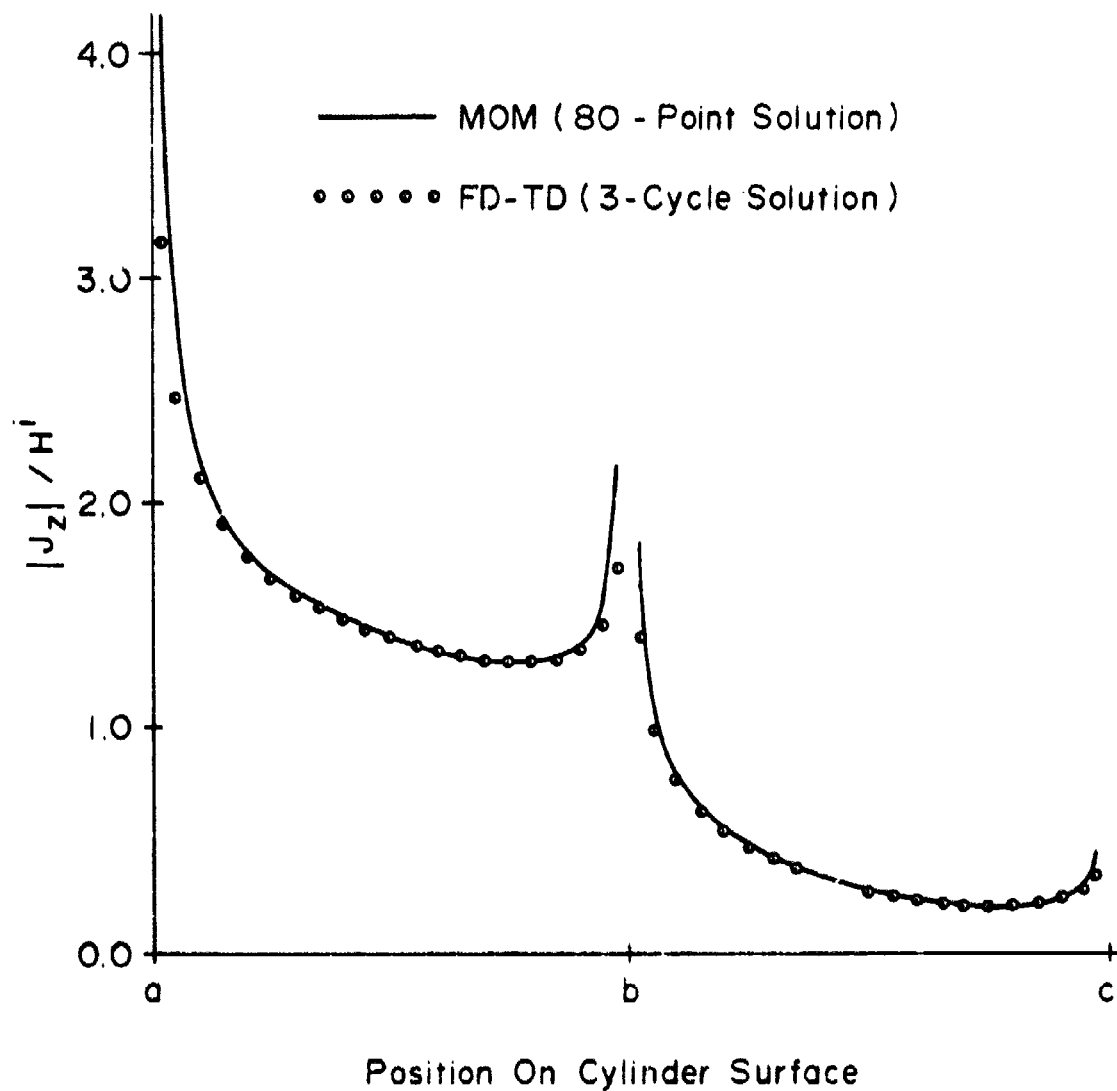
Fig. 26a. COMPARISON OF MOM AND FD-TD RESULTS FOR MAGNITUDE OF ELECTRIC CURRENTS ON SURFACE OF CYLINDER, OBLIQUE INCIDENCE CASE ($\phi = 45°$)
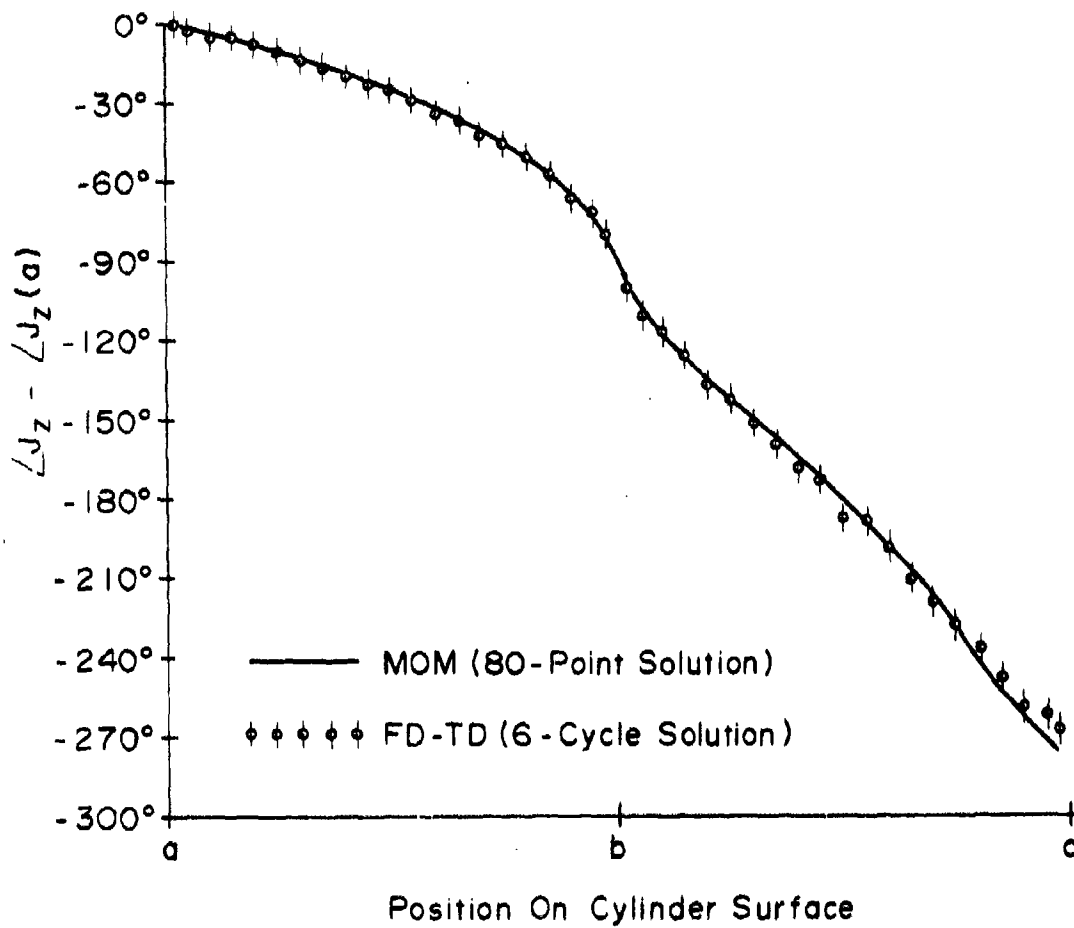
Fig. 26b. COMPARISON OF MOM AND FD-TD RESULTS FOR PHASE OF ELECTRIC CURRENTS ON SURFACE OF CYLINDER, OBLIQUE INCIDENCE CASE ($\phi = 45°$)

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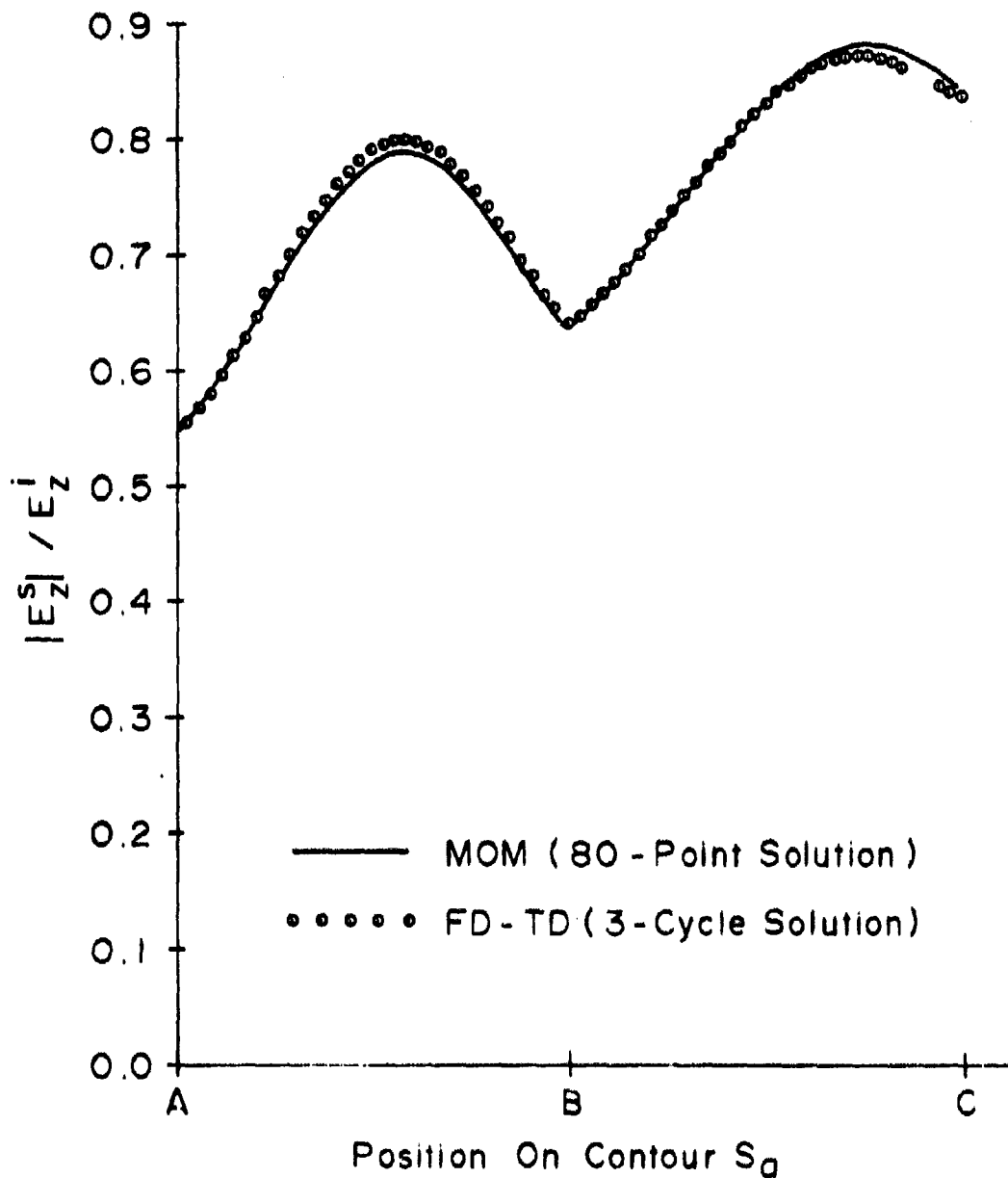
Fig. 27a. COMPARISON OF MOM AND FD-TD RESULTS FOR NEAR ELECTRIC FIELD TANGENTIAL TO CONTOUR S_a FOR OBLIQUE INCIDENCE CASE ($\phi = 45°$)

72
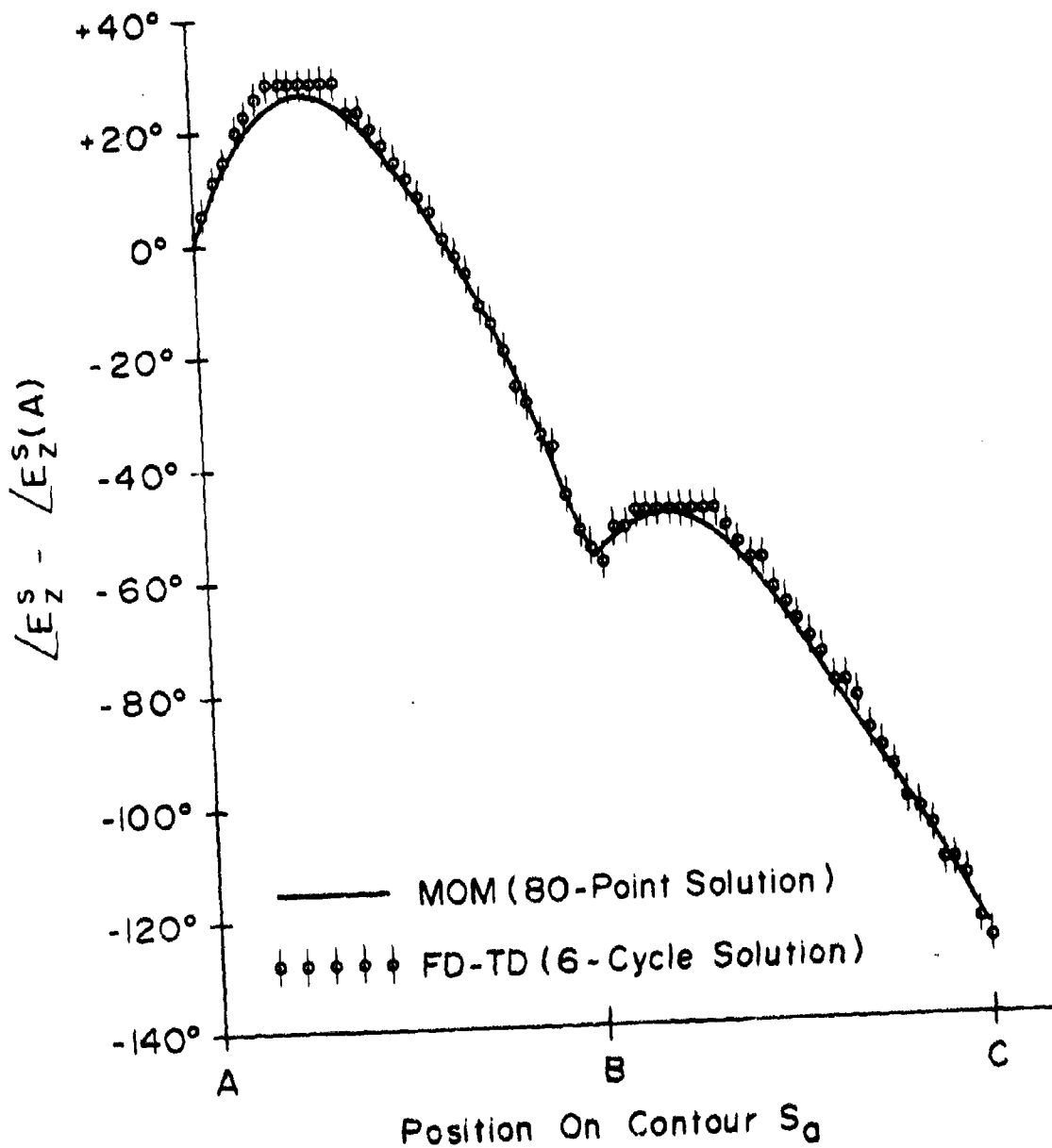
Fig. 27b. COMPARISON OF MOM AND FD-TD RESULTS FOR PHASE OF NEAR ELECTRIC FIELD TANGENTIAL TO CONTOUR S_a FOR OBLIQUE INCIDENCE CASE ($\phi = 45°$)
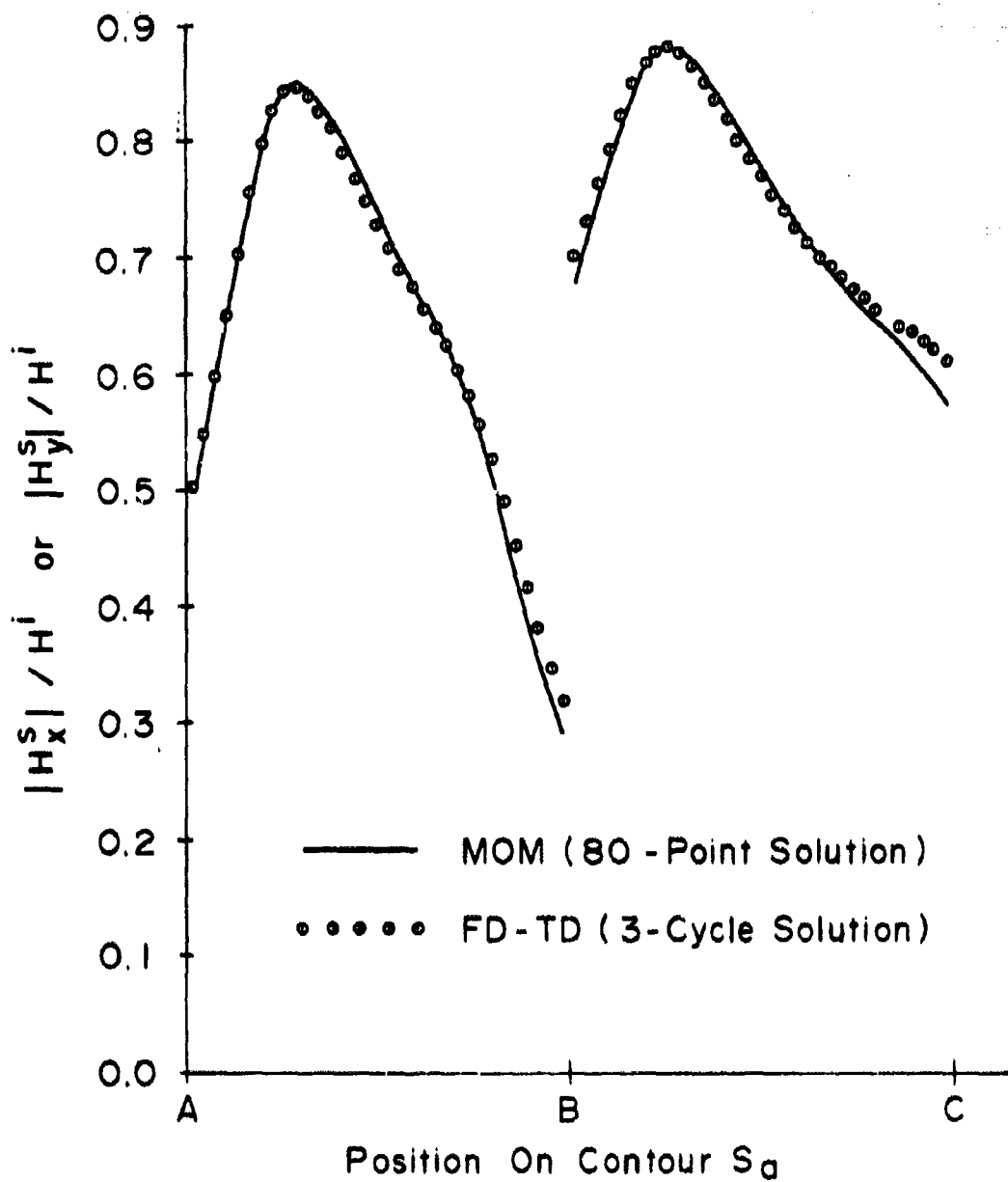
Fig. 28a. COMPARISON OF MOM AND FD-TD RESULTS FOR NEAR MAGNETIC FIELD TANGENTIAL TO CONTOUR S_a FOR OBLIQUE INCIDENCE CASE ($\phi = 45°$)

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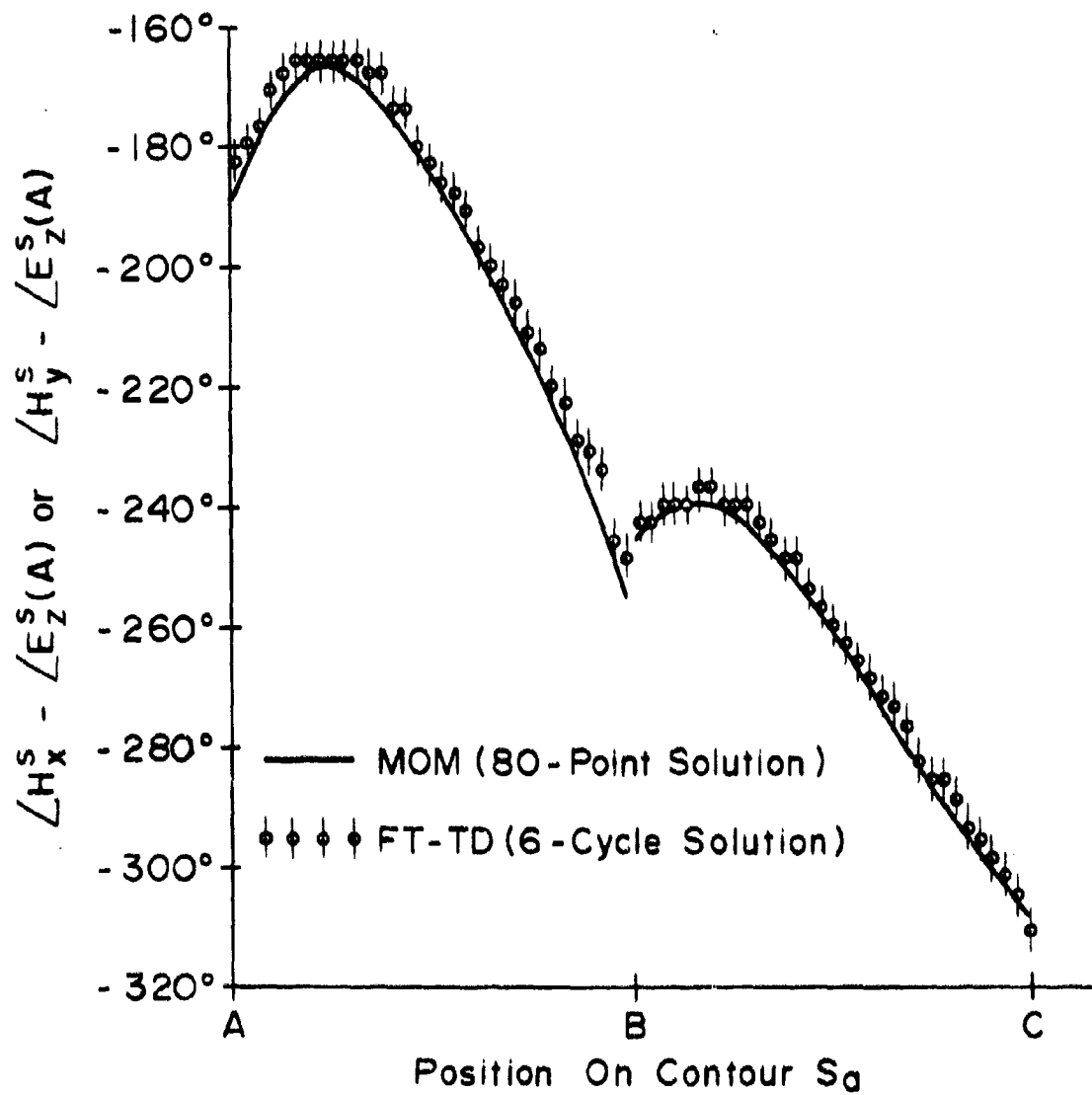
Fig. 28b. COMPARISON OF MOM AND FD-TD RESULTS FOR PHASE OF NEAR MAGNETIC FIELD TANGENTIAL TO CONTOUR S_a FOR OBLIQUE INCIDENCE CASE ($\phi = 45°$)

The high level of agreement between the FD-TD and MOM results for both the $\phi^i = 0°$ and $\phi^i = 45°$ angles of incidence verifies that the key, new FD-TD features (variable-incidence plane wave, second-order-correct lattice truncations, sinusoidal steady-state magnitude and phase computations) work, and work very well. In fact, the observed level of accuracy for these latest results is as much as 5 to 10 times as good as the previously-observed [12], [13] FD-TD accuracy level of $\pm 10\%$ (± 1 dB). It is expected that the new, enhanced accuracy levels will be observed when the present two-dimensional FD-TD computer programs are modified to the three-dimensional case.

**3.3.3 Verification of Near-to-Far Field Transformation for Circular Metal and Dielectric Cylinders

To provide further verification of the application of the near-to-far field transformation of Section 2.2.5, pure MOM studies have been performed during this research program on a metal and a dielectric circular cylinder. These studies concern the computation of scattering from such cylinders for the two-dimensional case of plane wave illumination (TM polarization).

Figure 29 shows the normalized radar cross section (RCS) computed for a metal cylinder of size $k_0 a = 1$. The solid line shows the RCS computed directly from the surface electric current, J_z, on the cylinder. The dots show the RCS computed using the near-to-far field transformation executed along a square virtual surface located at an electrical distance $k_0 h = 0.38$ from the surface of the cylinder. The agreement of the two solutions for the RCS is extremely high (to better than 4 decimal places at each point of comparison).

Figure 30 shows the normalized RCS computed for a dielectric cylinder of size $k_0 a = .63$. The solid line shows the RCS computed directly from the equivalent electric and magnetic currents on the surface of the cylinder, which had been previously derived using a MOM solution of a surface integral equation. The dots show the RCS computed using the near-to-far field transformation executed along a square virtual surface located at an electrical

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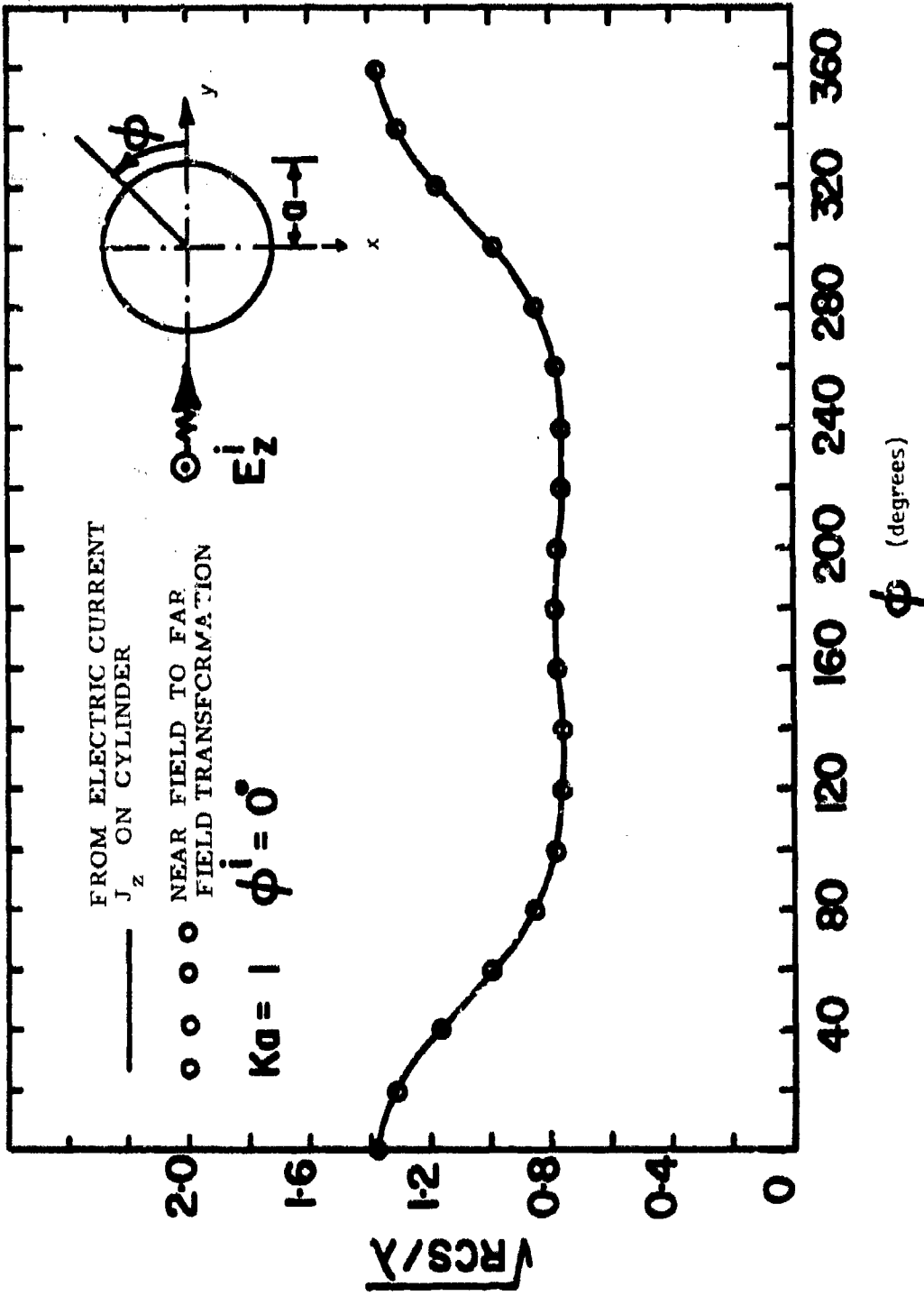
Figure 29. EQUIVALENCE OF COMPUTED RESULTS FOR RADAR CROSS SECTION OBTAINED USING THE MOM SURFACE CURRENTS APPROACH AND THE HYBRID MOM NEAR-TO-FAR FIELD TRANSFORMATION TECHNIQUE
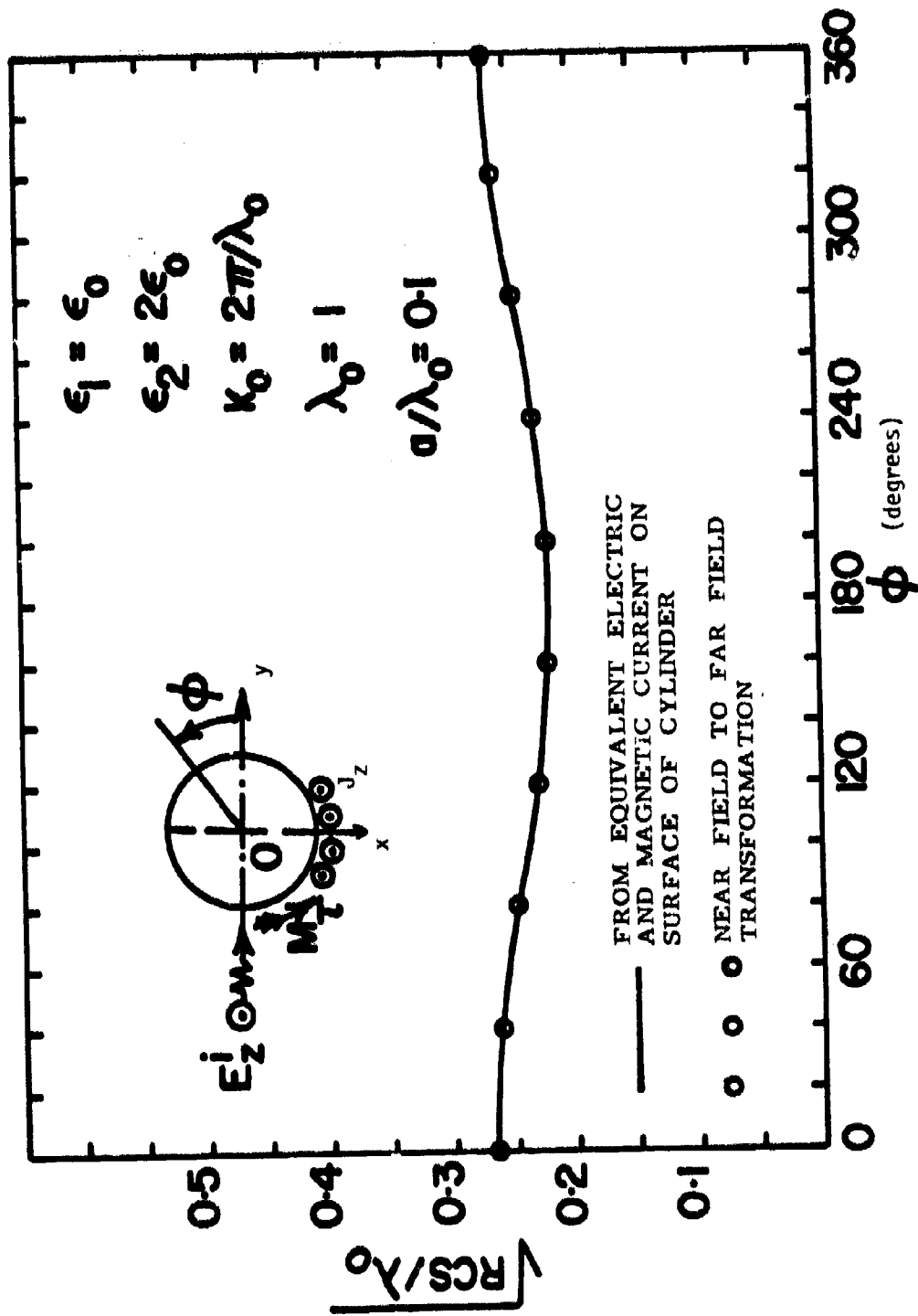
Figure 30. EQUIVALENCE OF COMPUTED RESULTS FOR RADAR CROSS SECTION OBTAINED USING THE MOM SURFACE CURRENTS APPROACH AND THE HYBRID MOM NEAR-TO-FAR FIELD TRANSFORMATION TECHNIQUE

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distance $k_o h = 0.24$ from the surface of the cylinder. Again, the agreement of the two solutions for the RCS is extremely high (to better than 4 decimal places at each point of comparison).

It is therefore concluded that the application of the near-to-far field transformation along a rectangular locus surrounding a scatterer is, for all practical purposes, an exact procedure for deriving the far fields. The validity of this near-to-far field transformation is the heart of the proposed hybrid FD-TD method for computing the far scattered fields and RCS from arbitrary, extremely-complex structures in three dimensions. It should be noted that this procedure should also be equally valid and useful in computing the far-field radiation patterns of extremely-complex source regions and antennas.

4.0 SUMMARY AND CONCLUSIONS

Electromagnetic penetration and scattering problems are difficult to treat with many analytical or numerical methods because of the inability of these methods to simply deal with the effects of structure materials, apertures, curvatures, corners, and internal contents. In two earlier RADC contracts, IITRI investigated the application of a new approach for the direct modeling of very complex electromagnetic interaction problems: the finite-difference, time-domain (FD-TD) solution of Maxwell's equations. The FD-TD method has key advantages relative to available modeling approaches. These advantages permit it to accurately treat complex problems that are beyond the scope of solution by any other method. The ultimate aim of research in this area is to develop an accurate, easily-used, general computer program solving for either electromagnetic field penetration, scattering, or radiation for arbitrary metal/dielectric structures spanning up to 10 or more wavelengths in three dimensions with a spatial resolution better than 0.1 wavelength.

In order to more fully determine the usefulness of the FD-TD method, RADC thought that it is desirable to distribute this technique to as wide a range of users as possible so that it can be tested by actual implementation. The overall objectives of algorithm development in this case are to allow RADC to write a user-oriented computer program for the FD-TD technique.

The goals of the present IITRI research effort for RADC include the development of specific algorithms of high importance to help provide a flexible, simple-to-use, and highly accurate user-oriented FD-TD computer program. To meet these goals, IITRI has tested five key improvements in the FD-TD algorithm during this effort, including the following:

1. Total-field/scattered-field lattice division.

   This permits a very high computational dynamic range to accurately model fields within shadow zones or cavities. This further permits programming of variable angle of incidence and the second-order correct radiation condition, summarized below.

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2. Variable angle of incidence.

For two-dimensional problems, this permits a single data card or Fortran statement to specify in a very accurate manner the angle of incidence of a plane wave illuminating a structure. For three-dimensional problems, both the angle of incidence and polarization could be specified. There is no requirement to rotate the geometry of the interacting structure in the FD-TD lattice.

3. Second-order accurate radiation condition.

This reduces the uncertainty of the final computed results by as much as ten-to-one. FD-TD computations using this radiation condition now have estimated field-magnitude uncertainties of better than +2.5% (+0.2 dB) versus previous uncertainties of $\pm$10%-$\pm$15% ($\pm$1 dB).

4. Magnitude and phase computation condition for the sinusoidal steady state.

This permits accurate determination of the magnitude and phase of FD-TD computed fields at any desired points for later use in computations involving scattering, radiation or coupling to wires. This approach avoids any ambiguity due to either a possible DC offset of the fields or the repetitive nature of the sinusoidal waveform.

5. Near-to-far field transformation.

This permits the far scattered fields and radar cross section of arbitrary structures modeled by the FD-TD method to be easily and accurately determined. Observed accuracy of the radar cross section using this feature is in the order of $\pm$1% ($\pm$0.09 dB).

In addition to the five algorithm improvements tested by IITRI, this report also summarized a FD-TD feature which has recently appeared in the literature that permits computation of the coupling of currents to thin wires and struts.

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The conclusions of this report are as follows:

1. The accuracy of the pure FD-TD method for electromagnetic interaction
    problems can reach the high levels previously attained only by method-
    of-moments (MOM) approaches when the second-order accurate radiation
    condition is used in the FD-TD algorithm. The FD-TD method retains its
    key advantages over MOM in terms of the much larger electrical size
    and greater complexity of the structures that can be modeled.

2. The specification of variable angle of incidence and polarization of
    an illuminating wave can be achieved with the FD-TD method using only
    a single data card or Fortran statement.[1]

3. The total-field/scattered-field regional division of the FD-TD lattice
    can be successfully implemented and offers the significant advantage
    of a high computational dynamic range. In addition, this lattice divi-
    sion provides a framework for programming variable wave incidence and
    polarization, improved radiation conditions, and the near-field to
    far-field transformation for scattering problems.

4. The near-to-far field transformation along a rectangular virtual sur-
    face surrounding a scatterer makes it possible to use the FD-TD method
    to compute the far scattered fields and radar cross section of complex,
    arbitrary structures with great precision.

It is the opinion of the authors of this report that the FD-TD method deserves
additional investigation to probe just what are the limits of application of
this extremely promising approach to accurately model electromagnetic penetra-
tion, scattering, and radiation problems.

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[1] This incident wave specification is now as simple for the FD-TD method as it
has been with MOM. However, the FD-TD approach requires re-running the entire
problem for each new incident wave angle. With MOM, only a single inversion
of the system matrix is required. Subsequently, arbitrary wave excitation is
treated as a simple matrix multiplication of the equivalent excitation
vector. MOM therefore permits a conceptually simpler treatment of the vari-
able wave incidence problem.

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Listing of Fortran Computer Program

This appendix contains the listing of the Fortran computer program used to obtain the FD-TD results for the square-cylinder scattering problems of Section 3.3 of this report. This computer program contains all of the new FD-TD algorithms that have been examined during the present research effort, except the thin-wire coupling algorithm of Section 2.2.7.2. This program provides a general treatment of two-dimensional penetration and scattering problems for the transverse magnetic (TM) polarization case, and utilizes a 50 x 49-cell Yee grid. The following is a succinct description of the user inputs to this program.

Program Line	Symbol or Data Input	Function
004	FREQ	Illumination frequency (Hz)
005	DX	Lattice cell size (meters)
006	NCYCS	Number of complete cycles of the sinusoidal incident wave that the program is time-stepped
007	MEDIA	Equal to 2 + number of distinct dielectric or conducting materials modeled in the FD-TD grid
008	DATA EPS	Relative permittivity of materials modeled. User specification begins with data item #4 in list. External air is item #1.
009	DATA SIG	Electric conductivity (mhos/m) of materials modeled. User specification begins with data item #4 in list. Air is item #1.
010	PHI	Angle of incidence (degrees) relative to y axis of grid. Limited to range 0° - 90° in present program.
011	ISA	FD-TD "i" coordinate of left field matching plane (between total-field and scattered-field regions).

Program Line	Symbol or Data Input	Function
012	ISB	FD-TD "i" coordinate of right field-matching plane (between total-field and scattered-field regions).
013	JSA	FD-TD "j" coordinate of front field-matching plane (between total-field and scattered-field regions).
014	JSB	FD-TD "j" coordinate of back field-matching plane (between total-field and scattered-field regions).
103 - 107	MEZ(I,J)	Assigns a material-medium type number to the E_z field components of the grid that comprise the structure to be modeled. The type number corresponds to the EPS and SIG specifications of program lines 008 and 009. Arbitrary specification is possible simply by using any number of Fortran statements here to make the assignment. In the example shown, a single DO loop suffices to specify that the object is square and has a surface composed of material-medium type #4 ($\varepsilon_r = 1.0$, $\sigma = 3.72 \cdot 10^7$ mhos/meter). The side of the square object is specified to be 20 grid cells across with corners at (16,16), (36,16), (16,36), and (36,36).

```
1001       PROGRAM FDTD (INPUT,OUTPUT,TAPE60=TAPEI,TAPE5=TAPE5)
        C
        C     PROJECT E6502, RUN 5
        C     TWO-DIMENSIONAL PROPAGATION, TM POLARIZATION
        C     TEST OF NEW PHASE COMPUTATION CONDITION
        C     TEST OF NEW OBLIQUE WAVE SOURCE CONDITION
        C     TEST OF NEW LATTICE TRUNCATION CONDITION
        C     50 X 49 CELL YEE GRID IS USED
        C     INDUCED ELECTRIC CURRENTS ON SQUARE METAL CYLINDER
        C
        C
1002       REAL MUZ
1003       DIMENSION EZ(51,50),ENZ(51,50),HX(51,50),HY(52,50),MEZ(51,50),
           1          EDL(2,50,2),EDR(2,50,2),EDF(51,2,2),EDH(51,2,2),
           2          TPR(52),EPS(4),SIG(4),CA(4),CH(4),
           3          IPHA(51,50),CA(4),DH(4),FAHX(51,50),FNHY(52,50),
           4          IPHHX(51,50),IPHHY(52,50),OEZ(51,50),DHX(51,50),
           5             DHY(52,50),HEZ1(50),HEZ2(50),HEZ3(50),HEZ4(50),
           6          HHX1(50),HHY2(50),HHY3(50),HHX4(50),EZT(90),HXT(90)
        C
        C          .....PROBLEM PARAMETERS.....
004        FREQ = 3.0E+8
005        DX = 1.59155E-2
006        NCYCS = 6
007        MEDIA = 4
008        DATA EPS /1.0, 1.0, 1.0, 1.0/
009        DATA SIG /0.0, 0.0, 0.0, 3.72E+7/
010        PHI = 45.0
        C
        C          .....BASIC AND DERIVED CONSTANTS.....
011        ISA = 11
012        ISH = 41
013        JSA = 11
014        JSH = 41
        C
015        ORX = FLOAT(ISA) - 0.5
016        ORY = FLOAT(JSA) - 0.5
017        SIDEX = FLOAT(ISH-ISA) + 0.5
018        SIDEY = FLOAT(JSH-JSA) + 0.5
019        ISAA = ISA + 1
020        ISHH = ISH - 1
021        JSAA = JSA + 1
022        JSHH = JSH - 1
023        ISAAA = ISA - 1
024        JSAAA = JSA - 1
025        ISHHH = ISH + 1
026        JSHHH = JSH + 1
027        PI = 3.14159265
028        OMEGA = 2.0 * PI * FREQ
029        SPHI = SIN(PI * PHI / 180.0)
030        CPHI = COS(PI * PHI / 180.0)
031        MUZ = 4.0 * PI * 1.0E-7
032        EPSZ = 8.854E-12
033        DT = DX / 6.0E+8
034        NMIN = 1.0 / FREQ / DT
035        NMID = NMIN
036        NMAX = NCYCS * NMIN
037        NMINA = FLOAT(NMIN) / 2.0
038        NMIDA = NMINA
039        NMAXA = NCYCS * 2 * NMINA
```

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```
040          W = DT / 2.0 / EPSZ
041          WA = DT**2 / DX**2 / MUZ / EPSZ
042          RB = DT / DX / MUZ
043          RC = 1000.0 / RB
044          RD = OMEGA * DT
045          RE = -360.0 * FREQ * DT
046          RF = 1.0 / RC
047          RG = 1.0 / 3767340.0
048          RH = RH / 10000.0
049          DO 1 I=1,MEDIA
050          FAF = W * SIG(I) / EPS(I)
051          CA(I) = (1.0-FAF) / (1.0+FAF)
052          CB(I) = RA / EPS(I) / (1.0+FAF)
053          HAF = SIG(I)*MUZ/EPSZ * DT/2.0/MUZ
054          DA(I) = (1.0-HAF) / (1.0+HAF)
055        1 DB(I) = 1.0 / (1.0+HAF)
056          DO 901 I=ISA,ISB
057          TERM = (FLOAT(I)-DRX) * SPHI
058          REZ1(I) =   TERM + 0.5*CPHI - 0.5
059          HEZ4(I) =   TERM + SIDEY*CPHI - 0.5
060          RHX1(I) = TERM
061      901 RHX4(I) =   TERM + (SIDEY+0.5)*CPHI
062          DO 902 J=JSA,JSB
063          TERM = (FLOAT(J)-DRY) * CPHI
064          REZ2(J) =   TERM + 0.5*SPHI - 0.5
065          REZ3(J) =   TERM + SIDEX*SPHI - 0.5
066          RHY2(J) = TERM
067      902 RHY3(J) =   TERM + (SIDEX+0.5)*SPHI
     C
     C             .....LOAD THE MATRICES.....
1068         DO 903 J=1,90
1069         FZI(J) = 0.0
1070     903 HXI(J) = 0.0
1071         DO 2 J=1,50
1072         DO 2 I=1,51
1073         FZ(I,J) = 0.0
1074         HX(I,J) = 0.0
1075         HY(I,J) = 0.0
1076         ENZ(I,J) = 0.0
1077         DEZ(I,J) = 0.0
1078         FNHX(I,J) = 0.0
1079         DHX(I,J) = 0.0
1080         FNHY(I,J) = 0.0
1081         DHY(I,J) = 0.0
1082       2 MEZ(I,J) = 1
1083         FS22 = 0.0
1084         FS11 = 0.0
1085         ED22 = 0.0
1086         FU11 = 0.0
1087         DO 982 I=ISA,ISB
1088         MEZ(I,JSA) = 2
1089     982 MEZ(I,JSB) = 2
1090         DO 983 J=JSA,JSB
1091         MEZ(ISA,J) = 2
1092     983 MEZ(ISB,J) = 2
1093         DO 803 N=1,2
1094         DO 803 I=1,2
1095         DO 803 J=1,50
1096         FDI(I,J,N) = 0.0
1097     803 EDH(I,J,N) = 0.0
```

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```
1098          DO 804 K=1,2
1099          DO 804 J=1,2
1100          DO 804 I=1,51
1101          FDF(I,J,N) = 0.0
1102     804  FDH(I,J,N) = 0.0
         C
         C          .....SPECIFY THE SCATTERER.....
103           DO 44 I=16,36
104           MEZ(I,16) = 4
105           MEZ(I,36) = 4
106           MEZ(16,I) = 4
107      44   MEZ(36,I) = 4
         C
         C          .....TIME-STEPPING LOOP.....
108           DO 21 N=1,NMAX
109           FNMOD = FLOAT(N) / FLOAT(NMID)
110           NA = N - IFIX(FNMOD)*NMID
111           JSTOP = 49
         C
         C          .....EZ ITERATION.....
112           DO 4 J=2,JSTOP
113           DO 4 I=2,50
114           MF = MEZ(I,J)
115           IF(MF.EQ.2) GO TO 904
116           STORE = EZ(I,J)
117           EZ(I,J) = CA(MF)*EZ(I,J) + CB(MF)*(HX(I,J-1)-HX(I,J)+
        1                                        HY(I+1,J)-HY(I,J))
118           DEZNU = EZ(I,J) - STORE
119           IF(DEZNU*DEZ(I,J).GT.0.0) GO TO 984
120           FNZ(I,J) = EZ(I,J)
121           IF(DEZNU.LT.0.0) IPHA(I,J)=NA
122      984  DEZ(I,J) = DEZNU
123      904  CONTINUE
124       4   CONTINUE
125           DO 905 J=2,49
126      905  EZI(J) = EZI(J) + RA* (HXI(J-1) - HXI(J))
         C
         C          .....OBLIQUE WAVE SOURCE CONDITION (EZ).....
127           EZI(3) = EZI(3) + RB*SIN(RD*FLOAT(N))
128           DO 930 I=ISAA,ISHB
129           L = RHX1(I)
130           G = L
131           HXINC = ( (RHX1(I)-G)*HXI(L+6) + (G+1.0-RHX1(I))*HXI(L+5) ) * CPHI
132           HXT = HX(I,JSAAA) + HXINC
133           EZ(I,JSA) = EZ(I,JSA) + RA* (HXT-HX(I,JSA)+HY(I+1,JSA)-HY(I,JSA))
         C
134           L = RHX4(I)
135           G = L
136           HXINC = ( (RHX4(I)-G)*HXI(L+6) + (G+1.0-RHX4(I))*HXI(L+5) ) * CPHI
137           HXT = HX(I,JSB) + HXINC
138      930  EZ(I,JSB) = EZ(I,JSB) + RA* (HX(I,JSBB)-HXT+HY(I+1,JSB)-HY(I,JSB))
         C
139           DO 935 J=JSAA,ISBB
140           L = RHY2(J)
141           G = L
142           HYINC = -((RHY2(J)-G)*HXI(L+6) + (G+1.0-RHY2(J))*HXI(L+5) ) * SPHI
143           HYT = HY(ISA,J) + HYINC
144           EZ(ISA,J) = EZ(ISA,J) + RA* (HX(ISA,J-1)-HX(ISA,J)+HY(ISAA,J)-HYT)
         C
145           L = RHY3(J)
```

```
146        G = L
147        HYINC = -((RHY3(J)-G)*HXI(L+6) + (G+1.0-RHY3(J))*HYI(L+5) ) * SPHI
148        HYT = HY(ISRPH,J) + HYINC
149    955 FZ(ISH,J) = FZ(ISH,J) + RA* (HX(ISH,J-1)-HX(ISH,J)+HYT-HY(ISH+1))
C
150        I = ISA
151        J = JSA
152        L = RHX1(I)
153        G = L
154        HXINC = ( (RHX1(I)-G)*HXI(L+6) + (G+1.0-RHX1(I))*HXI(L+5) ) * CPHI
155        HXT = HX(I,JSAAA) + HXINC
156        L = RHY2(J)
157        G = L
158        HYINC = -((RHY2(J)-G)*HXI(L+6) + (G+1.0-RHY2(J))*HYI(L+5) ) * SPHI
159        HYT = HY(I,J) + HYINC
160        FZ(I,J) = FZ(I,J) + RA* (HXT-HX(I,J)+HY(ISAA,J)-HYT)
C
161        I = ISH
162        L = RHX1(I)
163        G = L
164        HXINC = ( (RHX1(I)-G)*HXI(L+6) + (G+1.0-RHX1(I))*HXI(L+5) ) * CPHI
165        HXT = HX(I,JSAAA) + HXINC
166        L = RHY3(J)
167        G = L
168        HYINC = -((RHY3(J)-G)*HXI(L+6) + (G+1.0-RHY3(J))*HXI(L+5) ) * SPHI
169        HYT = HY(ISHHH,J) + HYINC
170        FZ(I,J) = FZ(I,J) + RA* (HXT-HX(I,J)+HYT-HY(I,J))
C
171        I = ISA
172        J = JSH
173        L = RHX4(I)
174        G = L
175        HXINC = ( (RHX4(I)-G)*HXI(L+6) + (G+1.0-RHX4(I))*HXI(L+5) ) * CPHI
176        HXT = HX(I,J) + HXINC
177        L = RHY2(J)
178        G = L
179        HYINC = -((RHY2(J)-G)*HXI(L+6) + (G+1.0-RHY2(J))*HXI(L+5) ) * SPHI
180        HYT = HY(I,J) + HYINC
181        FZ(I,J) = FZ(I,J) + RA* (HX(I,JSHH)-HXT+HY(ISAA,J)-HYT)
C
182        I = ISH
183        L = RHX4(I)
184        G = L
185        HXINC = ( (RHX4(I)-G)*HXI(L+6) + (G+1.0-RHX4(I))*HYI(L+5) ) * CPHI
186        HXT = HX(I,J) + HXINC
187        L = RHY3(J)
188        G = L
189        HYINC = -((RHY3(J)-G)*HXI(L+6) + (G+1.0-RHY3(J))*HXI(L+5) ) * SPHI
190        HYT = HY(ISHHH,J) + HYINC
191        FZ(I,J) = FZ(I,J) + RA* (HX(I,JSHH)-HXT+HYT-HY(I,J))
C
C          .....FZ SOFT GRID TRUNCATION.....
192        FZT(1) = FS22
193        FS22 = FS11
194        FS11 = FZT(2)
C
195        FZT(90) = FD22
196        FD22 = FD11
197        FD11 = FZT(89)
C
```

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```
198        DO 806 J=2,49
199        FZ(1,J) = -EDL(2,J,2) - 0.33333*(FZ(2,J)+EDL(1,J,2))
     1            +1.16667*(EDL(1,J+1)+EDL(2,J+1))
     2            +0.08333*(EDL(1,J+1,1)+EDL(1,J-1,1)+
     3                 EDL(2,J+1,1)+EDL(2,J-1,1))
200    806 FZ(51,J) = -EDR(2,J,2) - 0.33333*(FZ(50,J)+EDR(1,J,2))
     1            +1.16667*(EDR(1,J+1)+EDR(2,J+1))
     2            +0.08333*(EDR(1,J+1,1)+EDR(1,J-1,1)+
     3                 EDR(2,J+1,1)+EDR(2,J-1,1))
201        DO 807 I=2,50
202        FZ(I,1) = -EDF(I,2,2) - 0.33333*(FZ(I,2)+EDF(I,1,2))
     1            +1.16667*(EDF(I,1,1)+EDF(I,2,1))
     2            +0.08333*(EDF(I+1,1,1)+EDF(I-1,1,1)+
     3                 EDF(I+1,2,1)+EDF(I-1,2,1))
203    807 FZ(I,50) = -EDH(I,2,2) - 0.33333*(FZ(I,49)+EDH(I,1,2))
     1            +1.16667*(EDH(I,1,1)+EDH(I,2,1))
     2            +0.08333*(EDH(I+1,1,1)+EDH(I-1,1,1)+
     3                 EDH(I+1,2,1)+EDH(I-1,2,1))
    C
204        FZ(1,1) = 0.292893*EDL(1,1,2) + 0.707107*EDL(2,2,2)
205        FZ(1,50) = 0.292893*EDL(1,50,2) + 0.707107*EDL(2,49,2)
206        FZ(51,1) = 0.292893*EDR(1,1,2) + 0.707107*EDR(2,2,2)
207        FZ(51,50) = 0.292893*EDR(1,50,2) + 0.707107*EDR(2,49,2)
    C
208        DO 816 I=1,2
209        IA = 52 - I
210        DO 816 J=1,50
211        EDL(I,J,2) = EDL(I,J+1)
212        EDL(I,J+1) = FZ(I,J)
213        EDR(I,J,2) = EDR(I,J+1)
214    816 EDR(I,J+1) = FZ(IA,J)
    C
215        DO 817 J=1,2
216        JA = 51 - J
217        DO 817 I=1,51
218        EDF(I,J,2) = EDF(I,J+1)
219        EDF(I,J+1) = FZ(I,J)
220        EDH(I,J,2) = EDH(I,J+1)
221    817 EDH(I,J+1) = FZ(I,JA)
    C
    C          ......HX ITERATION.....
222        DO 8 J=1,JSTOP
223        IFLAG = 0
224        IF(J.EQ.JSAAA.OR.J.EQ.JSB) IFLAG=1
225        DO 8 I=1,51
226        IF(IFLAG.EQ.1.AND.I.GE.ISA.AND.I.LE.ISH) GO TO 61
227        STORE = HX(I,J)
228        HX(I,J) = HX(I,J) + FZ(I,J) - FZ(I,J+1)
229        DHXNU = HX(I,J) - STORE
230        IF(DHXNU*DHX(I,J).GT.0.0) GO TO 985
231        ENHX(I,J) = HX(I,J)
232        IF(DHXNU.LT.0.0) IPHHX(I,J)=NA
233    985 DHX(I,J) = DHXNU
234     61 CONTINUE
235      8 CONTINUE
236        DO 908 J=1,89
237    908 HXI(J) = HXI(J) + FZI(J) - FZI(J+1)
    C
    C          ......OBLIQUE WAVE SOURCE CONDITION (HX).....
238        DO 950 I=ISA,ISB
```

```
239          L = RE21(I)
240          G = L
241          FZINC = (RE21(I)-G)*FZI(L+7) + (G+1.0-RE21(I))*FZI(L+6)
242          FZS = FZ(I,JSA) - FZINC
243          HX(I,JSAAA) = HX(I,JSAAA) + FZ(I,JSAAA) - FZS
     C
244          L = RE24(I)
245          G = L
246          FZINC = (RE24(I)-G)*FZI(L+7) + (G+1.0-RE24(I))*FZI(L+6)
247          FZS = FZ(I,JSH) - FZINC
248          HX(I,JSH) = HX(I,JSH) + FZS - FZ(I,JSHHH)
249      950 CONTINUE
     C
     C           .....HY ITERATION.....
250          DO 9 J=2,JSTOP
251          IFLAG = 0
252          IF(J.GE.JSA.AND.J.LE.JSH) IFLAG=1
253          DO 9 I=2,51
254          IF(IFLAG.EQ.1.AND.I.EQ.ISA) GO TO 71
255          IF(IFLAG.EQ.1.AND.I.EQ.ISHHH) GO TO 71
256          STORE = HY(I,J)
257          HY(I,J) = HY(I,J) + FZ(I,J) - FZ(I-1,J)
258          DHYNU = HY(I,J) - STORE
259          IF(DHYNU*DHY(I,J).GT.0.0) GO TO 986
260          ENHY(I,J) = HY(I,J)
261          IF(DHYNU.LT.0.0) TPHHY(I,J)=NA
262      986 DHY(I,J) = DHYNU
263       71 CONTINUE
264        9 CONTINUE
     C
     C           .....OBLIQUE WAVE SOURCE CONDITION (HY).....
265          DO 970 J=JSA,JSH
266          L = RE22(J)
267          G = L
268          FZINC = (RE22(J)-G)*FZI(L+7) + (G+1.0-RE22(J))*FZI(L+6)
269          FZS = FZ(ISA,J) - EZINC
270          HY(ISA,J) = HY(ISA,J) + FZS - FZ(ISAAA,J)
     C
271          L = RE23(J)
272          G = L
273          EZINC = (RE23(J)-G)*FZI(L+7) + (G+1.0-RE23(J))*FZI(L+6)
274          FZS = FZ(ISH,J) - FZINC
275          HY(ISHHH,J) = HY(ISHHH,J) + FZ(ISHHH,J) - FZS
276      970 CONTINUE
     C
     C           .....PRINT-OUT ROUTINE.....
     C
     C           .....PRINT-OUT INTERVAL (ENVELOPE MAGNITUDES).....
277          DO 12 K=NMINA,NMAXA,NMIDA
278          IF(N.EQ.K)GO TO 13
279       12 CONTINUE
280          GO TO 20
     C
     C           .....ENVELOPE OF FZ (MAGNITUDE).....
281       13 DO 8A IA=25,50,25
282          IB = IA - 23
283          IC = IA + 1
284          PRINT 14, N
285       14 FORMAT (1H1,//,53X,24HEZ GRID FOR TIME STEP = ,I4,/ ,2X,1HJ,/)
286          DO 16 JA=1,50
```

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```
287          J = 51 - JA
288          DO 15 I=IH,IC
289          IPR(I) = BC * FNZ(I,J)
290       15 FNZ(I,J) = 0.0
291       16 PRINT 17, J, (IPR(I),I=IH,IC)
292       17 FORMAT (1X,I2,1X,25I5)
293          DO 18 I=IH,IC
294       18 IPR(I) = I
295       88 PRINT 19, (IPR(I),I=IH,IC)
296       19 FORMAT (/,4X,25I5,//,67X,1HJ)
C
C              .....HX ENVELOPE (MAGNITUDE).....
297          DO 188 IA=25,50,25
298          IH = IA - 23
299          IC = IA + 1
300          PRINT 114, N
301      114 FORMAT (1H1,//,53X,24HHX GRID FOR TIME STEP = ,I4,/,2X,1HJ,/)
302          DO 116 JA=1,49
303          J = 50 - JA
304          DO 115 I=IH,IC
305          IPR(I) = 376734.0 * FNHX(I,J)
306      115 FNHX(I,J) = 0.0
307      116 PRINT 17, J, (IPR(I),I=IH,IC)
308          DO 118 I=IH,IC
309      118 IPR(I) = I
310      188 PRINT 19, (IPR(I),I=IH,IC)
C
C              .....HY ENVELOPE (MAGNITUDE).....
311          DO 288 IA=25,50,25
312          IH = IA - 23
313          IC = IA + 1
314          PRINT 214, N
315      214 FORMAT (1H1,//,53X,24HHY GRID FOR TIME STEP = ,I4,/,2X,1HJ,/)
316          DO 216 JA=2,49
317          J = 51 - JA
318          DO 215 I=IH,IC
319          IPR(I) = 376734.0 * FNHY(I,J)
320      215 FNHY(I,J) = 0.0
321      216 PRINT 17, J, (IPR(I),I=IH,IC)
322          DO 218 I=IH,IC
323      218 IPR(I) = I
324      288 PRINT 19, (IPR(I),I=IH,IC)
C
C              .....PRINT-OUT INTERVAL (ENVELOPE PHASES).....
325       20 DO 512 K=NMIN,AMAX,NMID
326          IF(N.EQ.K) GO TO 513
327      512 CONTINUE
328          GO TO 520
C
C              .....EZ ENVELOPE (PHASE).....
329      513 DO 49 IA=25,50,25
330          IH = IA - 23
331          IC = IA + 1
332          PRINT 24, N
333       24 FORMAT (1H1,//,53X,26HEZPH GRID FOR TIME STEP = ,I4,/,2X,1HJ,/)
334          DO 26 JA=1,50
335          J = 51 - JA
336          DO 25 I=IH,IC
337          IPR(I) = FLOAT(IPHA(I,J)) * RF
338          IF(ABS(EZ(I,J)).LT.RH) IPR(I)=0
```

```
339        25 IPHA(I,J) = 0
340        26 PRINT 17, J, (IPR(I),I=IH,IC)
341           DO 27 I=IH,IC
342        27 IPR(I) = I
343        HQ PRINT 19, (IPR(I),I=IH,IC)
              C
              C          .....HX ENVELOPE (PHASE).....
344           DO 1HQ IA=25,50,25
345           IH = IA - 23
346           IC = IA + 1
347           PRINT 124, N
348       124 FORMAT (1H1,//,53X,26HHXPH GRID FOR TIME STEP = ,I4,/,2X,1HJ,/)
349           DO 126 JA=1,49
350           J = 50 - JA
351           DO 125 I=IH,IC
352           IPR(I) = FLOAT(IPHHX(I,J)) * RF
353           IF(ABS(HX(I,J)).LT.RG) IPR(I)=0
354       125 IPHHX(I,J) = 0
355       126 PRINT 17, J, (IPR(I),I=IH,IC)
356           DO 127 I=IH,IC
357       127 IPR(I) = I
358       1HQ PRINT 19, (IPR(I),I=IH,IC)
              C
              C          .....HY ENVELOPE (PHASE).....
359           DO 2HQ IA=25,50,25
360           IH = IA - 23
361           IC = IA + 1
362           PRINT 224, N
363       224 FORMAT (1H1,//,53X,26HHYPH GRID FOR TIME STEP = ,I4,/,2X,1HJ,/)
364           DO 226 JA=2,49
365           J = 51 - JA
366           DO 225 I=IH,IC
367           IPR(I) = FLOAT(IPHHY(I,J)) * RF
368           IF(ABS(HY(I,J)).LT.RG) IPR(I)=0
369       225 IPHHY(I,J) = 0
370       226 PRINT 17, J, (IPR(I),I=IH,IC)
371           DO 227 I=IH,IC
372       227 IPR(I) = I
373       2HQ PRINT 19, (IPR(I),I=IH,IC)
              C
374       H20 CONTINUE
375        21 CONTINUE
376           STOP
377           END
          NO ERRORS
```

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