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an interest in the quantitative treatment of systems of a technological, operational, socio-technical and economic nature. By all indications, the conference was a success. It attracted an uncommonly high percentage (nearly 60%) of participants from abroad and, thanks to a series of stringent reviews by some unusually highly qualified panels, presented a program of high technical caliber.

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INTRODUCTION

The Conferences on System Modeling and Optimization are a series of symposia which are organized on behalf of the Technical Committee No.7 of the International Federation for Information Processing. They are held biennially but only two, the first and the most recent, were held in the U.S. Its organizers were in fact highly pleased to have their proposal for a venue in New York City accepted by the Technical Committee over several others who competed for it.

The main interest and technical attraction of these conferences probably lie in their broad scope. They have included topics which are normally discussed only at very specialized meetings and they have accordingly provided a forum for the interchange of ideas between fields that otherwise are fairly disjoint from each other. The fields range from system and control theory, operations research and management science, computer science, policy modeling and mathcmatical economics, to quantitative studies of immunology and disease control.

A special effort was made in the preparation for this conference to insure a technical level which was representative of the on-going work in the U.S. This effort was expended first of all towards attracting outstanding scientists into the International Program Committee and the Conference Organizing Committee. Moreover, a careful survey was made which led to the selection of topics for invited papers for the conference, and an equally careful search ensued for speakers on those topics. Finally, the contributed papers were subjected to a two-stage reviewing process before they were cleared for presentation at the conference, and the conference proceedings.

Equally novel was a procedure for the overnight production of a news bulletin which announced necessary program changes on the morning of the day on which they took effect.

The subsequent sections of this report elaborate on these remarks and supply some of the required statistics.

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Chief, Technical Information Division

CONFERENCE ORGANIZATION

The host for the conference was the Polytechnic Institute of New York. The two co-principal investigators of the grant were its co-chairmen. The oversight over the technical and non-technical events was exercised by two Committees. The International Program Committee in particular was a body of highly respected scientists whose function was advisory as well as decisive. The conference cochairmen were pleased to be able to persuade a particularly outstanding grcup of persons to be members of this committee. They were:

J.Stoer, Fed. Rep. Germany, Chairman

A.V.Balakrishnan, USA	P.D.Lax, USA		
G.B.Dantzig, USA	W.Leontief, USA		
A.Freeman, USA	J.L.Lions, France		
M.Iri, Japan	G.I.Marchuk, USSR		
K.Malanowski, Poland	M.J.D.Powell, UK		
E.Rofman, France	A.Ruberti, Italy		

The Conference Organizing Committee was a second Committee charged with some of the responsibility for oversight of the conference. Its function was to translate the broad directives of the International Program Committee into a viable social and technical conference plan. In addition it carried out the preliminary screening of the contributed papers. Its members were drawn from most of the major universities and research laboratories of Greater New York. They represented, so the co-chairmen felt, a cross section of the substantial technical competence available in the area. They were:

R.F.Drenick, F.Kozin, Co-Chairmen, Polytechnic Institute of New York
J.J.Bongiorno, Polytechnic Institute of New York
E.A.Cherniavsky, Brookhaven National Laboratory
E.G.Coffman, Jr., Bell Telephone Laboratories
J.Cullum, IBM, T.J.Watson Research Center
J.J.Golembeski, Bell Telephone Laboratories
P.Green, IBM, T.J.Watson Research Center
R.A.Haddad, Polytechnic Institute of New York
P.J.Kolesar, Columbia University
M.Overton, New York University
E.Pitnataro, Polytechnic Institute of New York
P.E.Sarachik, Polytechnic Institute of New York
J.Traub, Columbia University

An effort fully comparable to the selection of the Committee members was expended on the choice of the topics for the invited addresses, and of their speakers. Its outcome was extremely gratifying. The list of speakers and topics was as follows:

Plenary Addresses

- S.W.Director, Carnegie-Mellon Univ., Pittsburgh, PA, USA;
- Computer-Aided Design: The Role of Optimization in VLSIC Design R.M.Karp, Univ. of California, Berkeley, CA, USA; The Inherent
- Complexity of Combinatorial Optimization Problems J.Killeen, Lawrence Livermore National Lab., Livermore, CA, USA;
- Computational Problems in Magnetic Fusion Research
- H.Kobayashi, T.J.Watson Research Center, Yorktown Heights, NY, USA; Modeling and Analysis of Computer Performance: A Review of Recent Progress
- T.C.Koopmans, Yale Univ., New Haven, CT, USA; Capital as an Input to Production (an Illustration in Two Dimensions)
- C.Paige, McGill Univ., Montreal PQ, Canada; Some Numerical Pitfalls in Computing with Linear Systems
- O.Pironneau, Univ. of Paris Sud, Paris, France; Optimum Design of Elliptic Distributed Systems

E.S.Savas, Assist. Secretary for Policy Development and Research, US Department of Housing and Urban Development, Washington, DC, USA; Urban Systems and Urban Policy

Invited Papers

- L.N.Belykh and G.I.Marchuk, Acad. Sci., Novosibirsk, USSR; Analysis of an Infectious Disease Model
- F.Clarke, Univ. of British Columbia, Vancouver, Canada; Non-Smooth Analysis and Optimization
- E.Gelenbe, Univ. of Paris Sud, Paris, France; Relations Between Deterministic and Stochastic Models in Queues and Computer System Models

Y.Sunahara, Kyoto Inst. Technology, Kyoto, Japan; Recent Trends in Optimal Control of Stochastic Distributed Parameter Systems

W.E.Walker, The Rand Corporation, Santa Monica, CA, USA; Urban Policy Modeling: Past, Present, and Future

Banquet Address

K.Gerard, Deputy Mayor of New York City for Economic Planning and Development; Urban Socio-Technological Problems of the Future

The conference program provided for one plenary session in each of five half-day meetings. This session was followed by four parallel sessions of contributed papers. The abstracts of most papers, invited as well as contributed, are included in a booklet which is enclosed as Appendix B to this report. (Copies of the booklet were distributed free of charge to all conference registrants.)

The conference sponsor was of course the Technical Committee No.7 of the International Federation for Information Processing. Co-sponsors were:

> The American Federation of Information Processing Societies The Control Systems Society, Institute of Electrical and Electronics Engineers The Operations Research Society of America The Society of Industrial and Applied Machematics

These societies were present at the opening session of the conference and welcomed the participants. They also made available their mailing lists for all conference announcements.

The conference itself took place at the New York Sheraton Hotel, Seventh Avenue at 57th Street. No serious obstacles were encountered in the course of it. In fact, a novel organizational feature was introduced. Necessary program changes for any one day were taken to the Polytechnic Institute of the evening of the preceding day, typed by the Word Processing Group there during the night and distributed at the start of the sessions in the form of a news bulletin.

REVIEWING PROCESS

A novel reviewing process was employed for the selection of papers to be presented at the conference, and ultimately to be published, in the conference proceedings. It consisted of three stages. In the first stage the 400 abstracts of the papers submitted for presentation at the conference were rated by members of the Conference Organizing Committee and by a few others in outlying areas of specialization. Those given marginal ratings were subsequently reviewed by the members of the International Program Committee, with the result that only 230 papers were cleared for presentation. Due to cancellations, 210 actually appeared on the program.

Of these, however, only 130 ultimately were approved for publication in the conference proceedings. This approval was given on the basis of yet another round of reviews which took place during the conference itself, on the basis of technical content of the complete paper presented there.

This process was suggested by Professor J.Stoer of the International Program Committee, and it worked out very well. It also produced an unexpected fringe benefit. The in-conference reviews supplied a check of who did or did not give his paper as scheduled and in person.

CONFERENCE STATISTICS

The conference was attended by 236 persons. A list of participants is attached as Appendix A. It may be noted that an unusually large proportion, namely over 58%, came from abroad. In fact, thirty-one countries were represented among the participants. Industry was represented by 93 of those attending, the rest came from universities.

The 230 contributed papers on the technical program were scheduled for presentation in 32 sessions on the subjects of:

Control Theory Game Theory Identification and Estimation Infinite-Dimensional Systems Stochastic Systems System Theory Control Applications Power Systems Bio-Medical Models Simulation Programming Theory Programming Algorithms Programming Applications Multi-Objective Optimization Management Science Combinatorial Programming Computational Complexity Computer System Modeling Computer-Aided Design Mathematical Economics Socio-Economic Models

CONFERENCE PROCEEDINGS

A volume of the proceedings, expected to have approximately one thousand pages, is to be released at some time during the middle of 1983. It will comprise most of the papers which were cleared for publication by the three-stage reviewing process mentioned earlier. Springer Verlag is to be the publisher, and the volume will be one in the series of Lecture Notes in the Control and Information Sciences. One copy will be distributed free to the author(s) of every paper in the volume. Others will be marketed by the publisher at a price estimated between \$50 and \$60.

AFOSR-TR- 82-0244 The International Federation for Information Processing



ABSTRACTS

10th IFIP Conference On System Modeling And Optimization

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New York City (USA) August 31 to September 4, 1981 82 04 06

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CONTENTS

Invited Papers and Speakers Contributed Papers by Session Abstracts Author Index INVITED SPEAKERS

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INVITED SPEAKERS

Plenary Addresses

- S.W. Director, Carnegie-Mellon Univ., Pittsburgh PA, USA Computer-Aided Design: The Role of Optimization in VLSIC Design
- R.M. Karp, Univ. of California, Berkeley CA, USA The Inherent Computational Complexity of Combinatorial Optimization Problems
- J. Killeen, Lawrence Livermore National Lab., Livermore CA, USA Computational Problems in Magnetic Fusion Research
- H. Kobayashi, T.J. Watson Research Center, Yorktown Heights NY, USA Modeling and Analysis of Computer Performance: A Review of Recent Progress
- T.C. Koopmans, Yale Univ., New Haven CT, USA A Simple Model of a Capital Stock in Equilibrium with the Technology and the Preferences
- C. Paige, McGill Univ., Montreal PQ, Canada Some Numerical Pitfalls in Computing with Linear Systems
- O. Pironneau, Univ. of Paris Sud, Paris, France Optimum Design of Elliptic Distributed Systems
- E.S. Savas, Assist. Secretary for Policy Development and Research, US Department of Housing and Urban Development, Washington DC, USA Urban Systems and Urban Policy

Invited Papers

- L.N. Belykh and G.I. Marchuk, Acad. Sci., Novosibirsk, USSR, On the Treatment of Chronic Forms of a Disease According to the Mathematical Model
- F. Clarke, Univ. of Bristish Columbia, Vancouver, Canada Non-Smooth Analysis and Optimization
- E. Gelenbe, Univ. of Paris Sud, Paris, France Relations between Deterministic and Stochastic Models in Queues and Computer System Models
- Y. Sunahara, Kyoto Inst. Technology, Kyoto, Japan Recent Trends in Optimal Control of Stochastic Distributed Parameter Systems
- W.E. Walker, The Rand Corporation, Santa Monica CA, USA Urban Policy Modeling: Past, Present, and Future

ON THE TREATMENT OF CHRONIC FORMS OF A DISEASE ACCORDING TO THE MATHEMATICAL MODEL

L.N. BELYKH, G.I. MARCHUK ACADEMY OF SCIENCE, NOVOSIBIRSK, USSR

Analysis of an infectious disease model is presented. A number of biological hypotheses on regularities of the course of treatment and the nature of their origin are formulated. In particular it is shown tha⁺ chronic infections are due to weak stimulation of the immune system and one of the methods of their treatment may be an aggravation of the disease.

NONSMOOTH ANALYSIS AND OPTIMIZATION

FRANK CLARKE, UNIVERSITY OF CALIFORNIA, BERKELEY

ABSTRACT

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A brief and nontechnical survey of the subject of optimization and its relation to nonsmooth analysis is presented.

Relations between deterministic and stochastic models

in queues and computer system

models

Erol Gelenbe

Université Paris Sud

and

INRIA

B.P. 105 78150 Le Chesnay France

The difficulties encountered in extending the available analytical and numerical tools for the analysis of computer systems, which are based essentially up to now on queueing theoretical approaches, have given rise to a number of investigations concerning the models which are used. Such questions have also been raised by practitioners who have felt that the statistical and measurement oriented link between the theoretical models and the actual measurements has been insufficiently established.

Another motivation to reconsider the basic models used has been the increasing success of Petri nets, or of the Karp-Miller vector addition systems, in their use as qualitative but formal representations of complex computer system behaviour. It thus becomes of interest to examine the possibility of obtaining performance oriented results from the same basic models without necessarily making probabilistic assumptions.

In this lecture we shall consider time dependent behaviour of vector addition systems. With relatively weak assumptions concerning the deterministic stream of events in such systems we shall prove a number of significant results concerning their trajectories. These results are similar to those proved usually in the context of probabilistic queueing models.

-9-

RECENT TRENDS OF OPTIMAL CONTROL FOR STOCHASTIC DISTRIBUTED PARAMETER SYSTEMS

Yoshifume Sunahara*

ABSTRACT

Stochastic partial differential equations arise in many contexts in the chemical, nuclear, biological, economic and social sciences. Such equations, even though simple as a type of heat equation with additive noise, may exhibit a surprising array of control problems from existence and uniqueness properties of solutions to the optimal control algorithm. There are, therefore, many fascinating areas, some concerned with highly mathematical aspects of stochastic eigenvalue problems and some concerned with nonlinear behaviors due to the existence of stochastic system parameters.

This survey has a couple of aims.

First, we exhibit various kinds of mathematical models for stochastic distributed parameter system.

Second, theoretical studies of such mathematical models usually consist of finding existence and uniqueness properties of solutions and then conducting the filtering and/or control aspects to determine the filter dynamics and/or the optimal control signal under the preassigned cost functional. Many studies on systems where uncertainties are considered to be an additive noise have been published by Lions, Bensoussan, Curtain, Balakrishnan and so on. We try to give a synoptic account.

Third, the principal aim here is to stimulate researches on control problems of distributed parameter systems. 'To do this, a study with the author's colleagues, Dr. Aihara and Mr. Kojima is outlined. Recognizing that many distributed parameter systems exhibit various kinds of uncertainties in their system parameters, we are concerned with a system modeled by

$$\frac{\partial u(t,x)}{\partial t} - a \frac{\partial^2 u(t,x)}{\partial x^2} + b \frac{\partial u(t,x)}{\partial x} \frac{dw(t)}{dt} = f(t,x) \text{ for } (t,x) \epsilon]0, t_f[x]0,1[$$

with the initial and boundary conditions

 $u(0,x) = u_0(x)$, for $x \in [0,1[$, u(t,0) = u(t,1) = 0, for $t \in [0,t_f[$

where dw(t)/dt is a white Gaussian noise process in time t and where a,b,c are constants. We shall develop regularity properties of the solution to the systems dynamics Σ_1 . A way is also described of finding the optimal control signal so as to minimize the quadratic cost functional.

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The survey ends with results of digital simulation experiments and with indicating the interesting mathematical questions which do not seem to be fully resolved.

 Faculty of Polytechnic Sciences, Kyoto Institute of Technology, Matsugasaki, Sakyo-ku, Kyoto 606, Japan. URBAN POLICY MODELING: PAST. PRESENT. AND FUTURE

Warren E. Walker The Rand Corporation 1700 Main Street Santa Monica, CA. 90406

ABSTRACT

The field of urban policy analysis, although relatively young, has reached a plateau in its development. This plateau is evidenced, by a leveling off in (1) the number of urban policy analysts, (2) their influence on government policy, and (3) methodological advances in the field. To many analysts these are welcome developments indicating a healthy maturation of the field. To others, they suggest serious problems that call for a rethinking of basic principles and a refocusing of energies.

In this paper I will

- summarize the salient features that characterized the development of urban policy analysis and urban modeling during the 1960s and 1970s
- o assess the current status of the field and describe some important recent trends
- o speculate on what the future holds in store.

Computer-Aided Design: The Role of Optimization in VLSIC Design

S.W. Director U.A. and Helen Whitaker Engineering of Electronics and Electrical Engineering Carnegie-Mellon University Pittsburgh, Pennsylvania

ABSTRACT

Very large scale integrated circuit technology (VLSI) has advanced to the point where it is now possible to realize circuits which contains over 600,000 transistors on a dingle silicon chip. Unfortunately this technological capability may not be fully utilizable due to the inability of designers to be able to deal with the resulting complexity of the design task. In order to handle this complexity designers have had to develop a suitable design methodology and have come to rely increasingly more heavily on computer aided design tools. Some computer aids, such as logic and circuit simulators and design rule checkers, have gained widespread acceptance. Other computer aids, especially those employing optimization methods, have been slow to be incorporated into the design process. However, in spite of this initial lack of enthusiasm it is clear that in order to realize the full potential of VLSI technology suitable optimization techniques must be developed and incoporated into the design process.

In order to encourage further work in this area, this talk discusses some of the tasks in VLSI circuit design in which it appears that optimization methods may be profitably employed. In particular we begin with a brief review of one hierarchical approach to VLSI circuit design which seperates the design process into a number of levels of abstraction. The highest level of abstraction describes the desired algorithmic behavior of the circuit while the lowest level describes the physical layout of the circuit in terms of mask geometries. We then discuss the three basic steps used in each level of design: synthesis, optimization and simulation. The mathematical formulations of. and the computational complexity involved with, these activities are also investigated. Finally, some of the algorithms which have thus far been proposed and/or used for these activities are reviewed and the difficulties encountered are explored. The Inherent Computational Complexity of Combinatorial Optimization Problems

Richard M. Karp University of California, Berkeley Department of Electrical Engineering and Computer Science

ABSTRACT

Recent developments in complexity theory have clarified the boundary between efficiently solvable computational problems and intractable ones. We will discuss the significance of these results for the design of optimization algorithms.

COMPUTATIONAL PROBLEMS IN MAGNETIC FUSION RESEARCH

John Killeen National Magnetic Fusion Energy Computer Center Lawrence Livermore National Laboratory P.O. Box 5508 Livermore, California 94550

ABSTRACT

Numerical calculations have had an important role in fusion research since its beginning, but the application of computers to plasma physics has advanced rapidly in the last few years. One reason for this is the increasing sophistication of the mathematical models of plasma behavior, and another is the increased speed and memory of the computers which made it reasonable to consider numerical simulation of fusion devices. The behavior of a plasma is simulated by a variety of numerical models. Some models used for short times give detailed knowledge of the plasma on a microscopic scale, while other models used for much longer times compute macroscopic properties of the plasma dynamics.

Fast time scale MHD codes are typically used to investigate the time dependent behavior of instabilities. The main question to be answered is whether or not a particular MHD mode will be unstable, and if so, how fast will it grow and what is its structure. Although linear MHD stability calculations have made a significant contribution to our understanding of plasma phenomena, nonlinear MHD problems, including the effects of resistivity, rely totally on computers. In order to analyze nonlinear resistive instabilities, the time dependent MHD equations of motion must be solved. The most advanced resistive MHD stability codes are nonlinear and three dimensional. This degree of generality is necessary in order to study the coupling of modes.

In order to simulate the transport of a plasma across a magnetic field over most of its lifetime -- from tens to hundreds of milliseconds -- a set of partial differential equations of the diffusion type must be solved. Typical dependent variables are the number densities and temperatures of each particle species, current densities, and magnetic fields. The transport coefficients such as thermal conductivity, electrical resistivity, and diffusion coefficients are obtained from the best available theories, but the codes also have the capability of easily changing the form of the coefficients in order to develop phenomenological models. In these codes implicit difference methods are used for the solution of the coupled diffusion equations.

In the simulation of plasma where the particles are not Maxwellian and where a knowledge of the distribution functions is important, Fokker-Planck equations must be solved. The problem is to solve a nonlinear partial differential equation for the distribution function of each charged species in the plasma, as functions of seven independent variables (three spatial coordinates, three velocity coordinates, and time). Such an equation, even for a single species, exceeds the capability of any present computer so several simplifying assumptions are therefore required to treat the problem.

Particle codes are fundamental in that they compute in detail the motion of particles under the influence of their self-consistent electric and magnetic fields, as well as fluctuation and wave spectra, and orbits of individual particles. They are ideal for providing detailed information on the growth and saturation of strong instabilities and the effects of turbulence. Particle codes are usually classified as either "electrostatic" or "electromagnetic." In the first type only the self-consistent electric field is computed via Poisson's equation and the magnetic field is either absent or constant in time. In the last few years there has been a considerable development in electromagnetic codes. They are either relativistic and fully electromagnetic, i.e., the particle equations of motion are relativistic and the electric and magnetic fields are obtained from the full Maxwell equations (wave equations) or they are in the nonradiative limit where the equations are nonrelativistic and displacement currents are neglected.

MODELING AND ANALYSIS OF COMPUTER PERFORMANCE: A REVIEW OF RECENT PROGRESS

Hisashi Kobayashi IBM T. J. Watson Research Center Yorktown Heights, New York 10598

ABSTRACT

During recent years a vast body of knowledge central to modeling and analysis of computer performance has accumulated. In this presentation we review the state-of-the-art of analytic modeling techniques. A computer system can be modeled as a multiple-resource system. and we can apply queueing network theory to the analysis of various performance issues. Efficient computational algorithms that have recently been developed are contributing to widespread use the success of the queueing network models.

We shall also discuss the future direction of performance modeling efforts: rapidly evolving microelectronics technology, i.e. VLSI will radically impact the nature of technical issues that should be addressed in design and analysis of future computer systems.

A SIMPLE MODEL OF A CAPITAL STOCK IN EQUILIBRIUM

WITH THE TECHNOLOGY AND THE PREFERENCES

Tjalling C. Koopmans, Yale University

Abstract

This lecture draws on the concepts of the following four developments in economic theory and systems analysis:

- (1) Theory of optimal economic growth (Ramsey, 1928)
- (2) The linear model of a technology constant over time (von Neumann, 1937)
- (3) The theory and computation of a competitive equilibrium (Walras, 1874; Arrow-Debreu, 1954; Scarf (with Hanser),1973, <u>et al.</u>)
- (4) Catastrophe Theory (Thom, 1972; Zeeman, 1977)

Also, there is overlap with the

(5) Complementarity Problem (Lemke, 1965; Cottle-Dantzig, 1974, et al.)

discussed in the field of mathematical programming.

The model has two consumption goods (amounts consumed in year t being denoted y_1^t and y_2^t) and only one capital good (amount z^t , for use in year t), and admits only one production process for each consumption good, and one for the capital good. The question addressed is to define and compute a <u>self-preserving initial capital stock</u> z^1 . This is an initial capital stock $z^1 = \hat{z}$ such that, if one from thereon maximizes the intertemporal utility function,

$$\sum_{t=1}^{\infty} (\alpha)^{t-1} u(y_1^t, y_2^t) , 0 < \alpha < 1 ,$$

the path over time of the optimal capital stock \hat{z}^t will repeat the initial value,

$$\hat{z}^{t} = z^{1}$$
, $t = 2, 3, ...,$

indefinitely. It is found that, depending on the form of the utility function $u(y_1, y_2)$, an <u>increase</u> in the discount <u>factor</u> α (equivalent to a <u>decrease</u> in the annual discount <u>rate</u> $\rho = \frac{1-\alpha}{\alpha}$), may bring about either an <u>increase</u> (the "intuitive" case) in the self-preserving capital stock \hat{z} , or a <u>decrease</u> (the "counter-intuitive" case). Catastrophe theory enters in situations where a small variation in the initial capital stock away from a self-preserving level may lead to an entirely different optimal capital path.

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-18-

SOME NUMERICAL PITFALLS IN COMPUTING WITH LINEAR SYSTEMS

Chris Paige, Computer Science, McGill University, Montreal

ABSTRACT

Mathematically equivalent but computationally different ways of computing a result can lead to totally different answers. Some simple problems in linear systems will be used to illustrate this difficulty. The reasons for the failure of some well know algorithms will be given, along with insights into the design of good numerical algorithms in this and related areas.

OPTIMUM DESIGN OF ELLIPTIC DISTRIBUTED SYSTEMS

O. Pironneau University of Paris Sud Paris, France

ABSTRACT

Let $\boldsymbol{\varphi}(\Omega)$ be the solution of a Partial Differential Equation in a domain Ω of \mathbb{R}^n . Let E be a functional of Ω and $\boldsymbol{\varphi}(\Omega)$. The optimization of E with respect to Ω is called an optimum design problem because it finds the best shapes of Ω with respect to E.

Such problems are the extension of distributed systems of the optimum time problems for systems governed by ordinary differential equations. They arise mostly in engineering and in aeronautics, for example, they are critical.

The basic techniques of the calculus of variation apply to these problems also and one usually ends up with a gradient-type algorithm for the solution of the discrete problems.

Keeping in mind the fact that the audience is not supposed to be familiar with the techniques of PDE's and restricting ourselves to elliptic PDE's, we shall present the principal methods used to solve these problems with the following plan:

1. Statement of the problem and classification.

- 2. Optimality conditions for the continuous problem.
- 3. Derivatives of the cost functional for the problem discretized by the Finite Element Method.
- 4. Some industrial applications and results.

- 20-

URBAN SYSTEMS AND URBAN POLICY

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This lecture is devoted to a general discussion of the meaning of urban policies and urban systems, with special emphasis on their interrelationships. LIST OF CONTRIBUTED PAPERS BY TOPIC

CONTROL THEORY

- JANAN E. ALLOS, W.A. MAHMOD, The Normalized Velocity Criterion as a Performance Index from the Phase Portrait for Systems Optimization.
- B. ASSELMEYER, A Ritz-Type Approach to the Calculation of Optimal Control for Non-Linear Dynamic Systems
- 1.A. AWAD, M.M. EL-GENEIDY, Optimal and Suboptimal Singular Control of a Class of Linear Discrete-Time Systems
- E.J. BALDER, The Complete Hamiltonian Characterization of Property (Q) and Its Application to Existence Problems in Optimal Control
- A.L. DONTCHEV, On Sensitivity in a State and Control Constrained Optimal Control Problem
- N.R. ELIAS, Correlation Between Conventional Control Performance Indices and Modern Control Quadratic Performance Index
- R. GONZALEZ, E. ROFMAN, A Discrete Maximum Principle Concerning the Optimal Cost of Deterministic Control Problems. A Stochastic Interpretation
- T.F. GONZALEZ, On the Computational Complexity of Clustering and Related Problems
- V.V. GOROHOVIC, On High Order Necessary Optimality Conditions
- J.O. GRAY, MRS. N. NAKHLA, The Application of Vector Minimization Techniques in the Analysis of Multi-Loop Non-Linear Feedback Systems
- V.B. HAAS, Partially Singular Steady State LQG Problems: Optimization and Estimation
- O. HAJEK, W. KRABS, On a General Method for Solving Time Optimal Linear Control Problems
- R. HORNUNG, K.D. WOLFNSTETTER, Discrete Min-Sum-Optimization Using a Newton Method and Application to Optimal Control
- H. KANO, T. NISHIMURA, Periodic Solutions of Discrete Matrix Riccati Equations with Constant Coefficient Matrices
- M.S. KO, U.Y. HUH, New Scheme of Discrete Implicit Observer
- A.V. LEVITIN, Reachable Sets and Generalized Bang-Bang Principle with Linear Control Systems
- M.M. LIN, M.E. SAWAN, Reducing Trajectory Sensitivity to Multi Delays in Control Systems

-24-

CONTROL FILEORY (Continued)

- W.S. LIN, T.S. KUO, G.J. THALER, A Novel Approach for Minimum-Time Control of Saturated Discrete-Time System
- I.B. MEDVEDOVSKY, Duality in Discrete Optimal Control
- K.R. PALANISAMY, Optimal Control of Linear Systems with Time Varying Delays via Single Term Walsh Series Approach
- P.N. PARASKEVOPOULOS, P. STRAVROULAKIS, Approximate Model Matching of Two-Dimensional Systems
- P.H. PETKOV, M.M. KONSTANTINOV, N.D. CHRISTOV, A Numerically Stable Method for Pole Assignment
- D.B. PETKOVSKI, Control Strategy for Decentralized Feedback Coordination

B.J. PETROVIC, M. RAKIC, M.J. PETROVIC, Structure of Optimal Control for Linear Systems in Control

- E.O. ROXIN, L.E. STERN, Periodicity in Optimal Control and Differential Games
- T.S. TUNG, Search for a Singular Solution of an Optimization Problem

GAMES

- A. AMELJANCZYK, J. HOLUBIEC, Solution of a Multiperson Cooperative Game by a Polyoptimization Method
- M. EL-ARABATY, Control and Optimization for Differential Games Modeling
- A.E. ROTH, U.G. ROTHBLUM, Risk Aversion and Nash's Solution for Bargaining Games with Risky Outcomes
- A.I.A. SALAMA, V. GOURISHANKAR, Three Dimensional Pursuit-Evasion with Free Maneuverability Pursuer
- A.P. WIERZBICKI, Quadratic Approximations for Computing Constrained Equilibria of Games in Normal Form

IDENTIFICATION AND ESTIMATION

- C.I.R. BLACKWOOD, Dynamic System Modeling for the Identification of Stochastic Processes
- J. CLAUSEN, J. HOLST, H. SPLIID, Adaptive Prediction of High Tides
- P. d'ALESSANDRO, Optimal Smoothings
- M. de la SEN, Identificability in Linear Systems with Arbitrary Sampling
- W. GERSCH, T. BROTHERTON, The Smoothing Problem A State Space-Recursive Computational Approach: Application to the Smoothing of Seasonal Economic Time Series
- T.S. LEE, K.P. DUNN, C.B. CHANG, Theory and Application of Observability and Fisher Information for a Class of Non-Linear Systems
- R.V. REKASIUS, Models for Dynamic Systems with Self-Substained Oscillations
- T.G. ROBERTAZZI, S.C. SCHWARTZ, Measurement Processing Orders Which Minimize Finite Precision Effects in Linear Sequential Estimation
- C. TSAI, L. KURZ, A Robustized Maximum Entropy Approach to System Identification

CONTROL APPLICATIONS

- A.I. COHEN, The Control of Automobile Engines, Fuel/Emission Tradeoffs
- A. GODHWANI, Optimal Two-Mode Controllers for Coupled Systems
- J.D. GRIFFITHS, Modeling Ship Control Systems
- Y. KURODA, A. MAKINO, A Problem of Bilinear Control in Non-Linear Coupled Distributed Parameter Reactor Systems
- V.B. LARIN, K.I. NAUMENKO, Singular Perturbations in Optimal Stabilization Problems of Legged Vehicles
- C.F. LIN, N.X. VINH, Optimum Maneuvers of Supersonic Aircraft
- E.Y. SHAPIRO, D.A. FREDRICKS, Application of Constrained Constant Optimal Output Feedback to Modern Flight Control Synthesis
- A.S.C. SINHA, Advanced Control Laws for Experiments in Fast Rollout and Turnoff of the B737-100 Aircraft
- P.P. YEH, A Mathematical Representation of Oscillation Stabilities for Moving Satellite System Operation

OPTIMAL CONTROL/PDE

- M. BALAS, Finite Dimensional Control of Linear Parabolic Distributed Parameter Systems
- A. BERMUDEZ, C. SAGUEZ, Numerical Simulation of an Alloy Solidification Problem
- C. CORDUNEAU, The Threshold Problem for a Fitzhugh-Nagumo Multi-Dimensional System
- F. DI GUGLIELMO, Differential Inclusions with Multivaried Boundary Conditions
- D. GABAY, Minimization of the Difference of 2 Convex Functions: Duality, Algorithms and Applications
- B. GOLLAN, On the Marginal Functions in Non-Linear Programming
- D.D. ILIC, Solution of the Class of Non-Classical Mathematical Programming Problems on Banach Algebras Arising in Generalized Statistical Decision Theory
- LASIECKA, Finite Element Approximation of Time-Optimal Control Problems for Parabolic Equations with Dirichlet Boundary Conditions
- 1. LASIECKA, R. TRIGGIANI, Dirichlet Boundary Control Problem for Parabolic Equations with Quadratic Cost: Analycity and Riccati's Feedback Synthesis
- K. MALANOWSKI, Rate of Convergence Estimates for Approximation to Convex Optimal Control Problems Subject to Control Constraints
- PAWLOW, Variational Inequality Formulation and Optimal Control of Nonlinear Evolution Systems Governed by Parabolic Free Boundary Problems
- J.C. POMEROL, Optimization in Banach Spaces of Systems Involving Convex Processes
- M. ROBIN, On the Semi-Group Approach for Ergodic Problem of Optimal Stopping
- H. SALHI, D.P. LOOZE, An Algorithm for the Decomposition of Second Order Elliptic Operators
- R. SENTIS, Calculus of the Corrector for the Eigenvalue of a Transport Operator
- C.L. SIMONESCU, Approximations by Means of Optimal Control for Distributed Parameter Systems
- J. SOKOLOWSKI, Shape Sensitivity Analysis for Variational Inequalities

STOCHASTIC CONTROL

- S.A. BELBAS, The Optimal Policy for Stochastic Control Problems with Costly Switchings
- J.H. DE VLIEGER, A Cautious Control Algorithm for Stochastic Control Systems with Additional Boundary Constants
- S.A. HASSAN, M.A. BISHER, E.E. ZAKZOUK, Stability Analysis for Large-Scale Stochastic Systems
- V.P. KAPUR, Reliability of Stochastic Control Passed on Decision Hierarchical Modes of Moving Assuages Processes
- D.P. LOOZE, H.V. POOR, K.S. NASTOLA, J.C. DARRAGH, On Linear-Quadratic-Gaussian Control of Systems with Uncertain Statistics
- S. SUGIMOTO, A Liapunov-Like Criterion and a First Passage Time Problem in Non-Linear Stochastic Systems
PROGRAMMING THEORY

- S. AGARWAL, P. SHARMA, Algorithm for a Specially Structured LP
- E.J. ANDERSON, A.B. PHILPOTT, Algorithms for Some Simple Continuous Linear Programs
- B. BETCO, R. TOTONDI, A Monte Carlo Algorithm for Global Optimization Based on a Bayesian Stopping Rule
- Y. CENSOR, Entropy Optimization Via Entropy Projections onto Hyperplanes
- Y. ERMDIEU, Non Stationary Simultaneous Optimization and Identification Procedures
- J.F.R. FERNANDES, R.V. RIBEIRO, H.M.F. TAVERES, Piecewise Linear Programming: Block Diagonal Problems
- J.C. HENNET, Resolution of a Quadratic Combinatorial Problem by Dynamic Programming
- J.D. HIRIART-URRUTY, Towards a Generalized 2nd Order Directional Derivative for Convex Functions
- J. JASIULEK, The Structure and Computation of Solutions to Continuous Linear Programs
- V. SALTENIS, The Simplification of Multidimensional External Problems According to Their Structure
- E. SPEDICATO, A Class of Direct Methods for Linear Systems
- V.A. TIESIS, On the Efficiency of Some Global Optimization Algorithms

MULTIOBJECTIVE OPTIMIZATION

- A. LEWANDOWSKI, Reference Point Multiple Criteria Optimization-Implementation and Experience
- J.B. MOCKUS, The Bayesian Approach to Global Optimization
- V.F. POTERASU, Multiple Objective Optimization with Application in Distribution of Energy Resources
- P. RADZIKOWSKI, An Exact Set Up Cost Goal Programming Problem Solution Algorithm

PROGRAMMING-ALGORITHMS

- A.F. ALY, H.L. ABDEL-MALEK, A.A. MONTASER, M.A. ABO-HASSWA, Design Centering of Time Permanent Linear Systems
- F. ARCHETTI, F. SCHOEN, A Synchronous Parallel Search for Globai Optimization Problems
- H. BLUM, A Multiplier Algorithm for Linear Programming with a Subsidiary Constraint
- W. JARISCH, A Probabilistic Approach to the Minimization of Stochastic Functions by Sequential Near-Conjugate Sampling
- L. LUKSAN, Quasi-Newton Methods Without Projections for Linearly Constrained Minimization
- R. MIFFLIN, An Efficient Algorithm for Solving One-Dimensional Constrained Minimization Problems Involving Non-Smooth Convex Functions
- B.S.N. MURTY, A. HUSAIN, Orthogonality Correction in Conjugate Gradient Methods
- E. POLAK, A.L. TITS, A Quadratically Convergent Recursive Linear Programming Algorithm for Constrained Non-Linear Problems
- M.J.D. POWELL, Extension to Subroutine VF02A
- E. SACHS, Quasi-Newton Methods for a Class of Nonsmooth Constrained Optimization Problems
- F. SLOBODA, V. BRITANAK, On the Generalized Conjugated Gradient Algorithm Invariant to Nonlinear Scaling
- H. TUY, N.Q. THAI, T.V. THIEU, Conical Algorithms for Solving a Wide Class of Mathematical Programming Problems
- S. WALUKIEWICZ, An Implementation of the Ellipsoid Algorithm for Linear Programming

APPLICATIONS OF PROGRAMMING

- Z.M. FARKAS, A New Type of Optimal Investment Resource Allocation Problem Solution in Compound Systems to be Developed
- S.S. KOUSOUM, Nonlinear Gasoline Blending Process: Third Level Kuhn-Tucker Controller for Dantzig-Wolfe Decomposition
- M.L. NITTI, G. SPERANZA, An Optimization Model for Energy Saving in the Heating of Buildings
- A. SCHORNAGEL, A General Markov Programming Method to Compute the Optimal Dispatching Strategy in a Transportation Problem

COMBINATORIAL PROGRAMMING

K.V.S. BHAT, Switching Function Minimization Using Graphs

- P. BRUCKER, A Linear Time Algorithm to Minimize Maximum Lateness for The Two Machines, Unit-Time, Job-Shop Scheduling Problems
- J.I. CHU, P. IPSILANDIS, An On/Out Guidy Method For the Optimum Location Selection Process
- T.Y. HWANG, On the Use of Coding Theory in Binary Programming
- M. IRI, K. MUROTA, S. MATSUI, An Approximate Solution for the Problem of Optimizing the Plotter Pen Movement
- M. KALTENBACH, Network Flow Control by 0-1 Sequences
- A. KERSCHENBAUM, Generalized Augmenting Paths for the Solution of Combinatorial Optimization Problems
- P. MICHAUD, F. MARCOTORCHINO, Regularization of Costs and Measures of Agreement for the Linear Model of Preference and Similarity Aggregation
- Y. SEKIGUCHI, Synthetical Analyses on Dominance Tests in a Class of Enumerative Algorithms
- L. SLOMINSKI, Complexity of Algorithms for the Bottleneck Assignment Problem
- C. VAN NUFFELEN, Rank, Clique, and Chromatic Number of a Graph
- P.Y. WANG, Extension of some Two-Dimensional BIN Packing Algorithms

COMPUTATIONAL COMPLEXITY

- G. D'ATRÍ, Complexity of the Random 0/1 Knapsack Problem
- M. GROTSCHEL, On the Complexity of the Fractional Stable Set Clique Covering and Coloring Problem
- F. LUCCIO, L. PAGLI, A Versatile Interconnection Pattern Laid on O(n) Area
- M. MIYAKAWA, On Converting Decision Tables into Minimum Decision Networks
- R.K. SAHA, Three Algorithms to Time Propagate Covariance Matrices: Comparison of Their Computational Complexity
- V.I. UBERMAN, V.I. SHLEINIKOV, Detecting Number Systems and Functions - Optimization, Algorithms and Fields of Using

SOCIO-ECONOMIC MODELS

- J.P. ANTUN, Simulation of the Competitiveness of Industrial Estates for Regional Development
- S. BAUM, W.R. TERRY, Random Sampling Approach Systems to Modeling
- R. BRAS, D. GROSSMAN, Control of Urban Water Supply
- B.H. BURZLAFF, M.G. RISKALLA, Dynamic Simulation of the U.S. Fertility
- A.C. CHATURVEDI, Water Management in the 80's
- D.F. CLEIMAN, L.R. ALBRECHT RAMOS, R. PAES'E BARROS, A Mathematical Model for the Implantation of Shell Oil Extraction Plants in Brazil
- R. FAHRION, Prediction of Socio-Economic Policy: Information Gain by Interactive Decision Activities
- T.K. GARDENIER, Quality Assurance Monitoring Specifications for Time Dependent Aerometric Data
- P. LEDERER, P. COLLETER, J. ORTMANS, Long-Term Nuclear Scheduling in the French Power System
- J.L. MENALDI, J.P. QUADRAT, E. ROFMAN, On the Role of the Impulse Fixed Cost in Stochastic Optimal Control. An Application to the Management of Energy Production
- D.H. MARTIN, Functional Sensitivity Analysis of Mathematical Models
- R.S. ORTEGA, R. CANALES RUIZ, Water Distribution Network Self-Turning Control
- P.J. PETROVIC, B.J. LAZAREVIC, M. UVALIC, M.J. PETROVIC, Mathematical Models of the Global Development
- F. VAN DER PLOEG, M.R. WEALE, Conflicts Over North Sea Oil Profits and Macro-Economic Policy
- A.H. ZEMANIAN, The Gathering of a 'ommodity Through a Periodic Marketing Ring

MATHEMATICAL ECONOMICS

F. AMINZADEH, Profit Maximization in a Dynamic Market

S. HANSEN, Project Budgeting by Small Information Systems

- U. HEILEMANN, Cyclical Taxonomy and Large Econometic Models
- D. HINRICHSEN, U. KRAUSE, Triangulation of Convex Polyhedra with an Application to Economics
- A. LAZARIDIS, An Approach to the Prediction Problem in Economic Dynamic Systems Using Filtering Techniques. An Application
- K. SOO LEE, Anti-Pollution Activities in Input/Output Analysis
- I.D. MOON, Structural Stability of a Probabilistic Input/Output System Subject to Sector Optimization
- R.C. RAO, Equilibrium Advertising in an Oligopoly with Net Love-Arrow Advertising Dynamics. Existence and Stability

BIOLOGICAL MODELS

- E. ERSU, Self-Organizing Control with Neuron-Line Associative Memory Models
- W.S. LEVINE, F.E. 2AJAC, Experimental Analytical and Computational Study of a Simplified Maximal Height Jump
- R.R. MOHLER, J. DOLEZAL, P. KLEIN, Comparison Analysis of Immune Response and Tolerance Levels
- A.H. RASHWAN, A.S. AHMED, An Innovative Approach to Cardiac Hemodynamic Modeling
- M.K. SUNDARESHAN, Modeling Cell Kinetics and Design of Optimal Cancer Therapy Strategies

COMPUTER AIDED DESIGN

- L.L. ABDEL-MALEK, J.T. CHU, Optimum Design of Solar Energy Water Heating System
- B. BONA, G. MENGA, Singular Value Decomposition and Linear Manifolds: An Application to Optimum Object Positioning
- M.C. DELFOUR, G. PAYRE, J.P. ZOLESIO, Optimal Design of a Thermal Diffuse with Minimal Weight
- W. NOOSS, A Numerical Way of Optimizing Chains of Torenances

SYSTEMS PROBLEMS

- R.C. DESAI, K. HEMAMALINI, Reduced Order Observer Estimation for a Discrete-Time Linear System Using Invariant Embedding
- A.J. FISH, An Abstract Systems Model for a Class of Non-Linear Systems
- A.I. SALEH, M.I. FOUAD, Phase Frequency Approximation in System Model Reduction
- R.E. SKELTON, Cost Equivalent Realizations of Linear Systems
- H.J. WERNER, The Temporal Aggregation in Discrete Dynamic Sys_ tems

POWER SYSTEMS

- A.I. AWAD, On the Megawatt Frequency Control Problem
- V.R. EKBOTE, State Variable Modeling of a Power System for Security
- M.M. EL-METWALLY, A. BAHGUT, Synchronous Machine Stabilization Through Excitation Control
- P. GRUBER-JOHNSON, B. STEINLE, Modeling of Data Transmission Channel on Energy Networks
- A.E. HAMMAD, Modeling for Large Power Systems for Steady, Quasi-Stationary and Dynamic State Simulations
- N. JALEELI, W.J. ECHEVERRIA, Identification of Synchronous Machine Parameters for Stability Models, Using Synchronizing Tests
- C. LYRA, H. TAVERES, S. SOARES, A Contribution to the Optimal Generation Scheduling of Large Hydrothermal Power Systems

TRANSPORTATION PROBLEMS

- S.A. AHMED, Freeway Incident Detection Based on Stochastic Dynamic Models of Traffic Variables
- Y.J. STEPHANEDES, Optimal Control of Transportation System Performance Under Dynamic Energy Constraints

SIMULATION STUDIES

- N. CHRISTOPEIT, Solution of Stochastic Control Problems by Discrete Approximation
- E. GARDENES, A. TREPAT, J.M. JANER, SIMSIGLA-PL/1: A Simulation Language Using the Lattice-Completed Algebra of Intervals
- M. MANDELBERG, A Systematic Experimental Determination of Discrete Time Models for Nonlinear Systems
- M. MURATA, Modeling of Fourdrinier Paper Making Machines and Basic Weight Control
- K. YAJIMA, J. TSUNEKAWA, Graphical Techniques Used for a Dynamical Chemical Process Simulation

MANAGEMENT SCIENCE

- K. AMBORSKI, M. JANKOWSKI, B. KUSZTA, S. ZAK, Modeling and Design of a Container Terminal
- R.I. KOMAR, A Combined Financial Simulation/Goal Programming Model for Long Range Bank Asset/Liability and Income Planning
- L. SHAW, B. EBRAHIMIAN, J. CHAN, Scheduling Maintenance Operations Which Cause Age Dependent Failure Rate Changes
- F.A. VAN DER DUYN SCHOUTEN, Optimality of (s,S)-Policies in Continuous Review Inventory Models
- VIRTANEN, Optimal Maintenance Policy and Sale Date for a Machine with Random Deterioration and Subject to Random Catastrophic Failure
- R.L. WILLIAMS, J.K. HELBLING, Analysis of Distribution Transformer Inventory System

ABSTRACTS OF CONTRIBUTED PAPERS

- 37 -

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CONTROL THEORY



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> THE NORMALIZED VELOCITY CRITERION AS A PERFORMANCE INDEX FROM THE PHASE PORTRAIT FOR SYSTEM OPTIMIZATION

The phase portrait is very useful in studying the transient response of systems described by firs' and second order differential equations, particularly in the presence of non-linearities.

It is the intention here to propose a performance criterion, vis. the Normalized Velocity Criterion (NVC), which is directly obtainable from the phase portrait of a system. This will enable the system designer to compare and evaluate systems directly from their phase portraits. The NVC is related to the Integral Square Error Criterion (ISE).

The NVC is shown to be a function of S, the total area enclosed by the phase plane trajectory of the system, and K, a weighting factor inversely proportional to the maximum system velocity, y_m . For this criterion to be effective, the function should be simple, and exhibit similar characteristics to other standard performance criteria, i.e. the ISE.

In formulating the NVC, and compring it the ISE criterion, the response of a linear second order system to a step function is considered, which can be solved analytically. The system is simulated on an Analogue Computer, and from theoritical considerations, and analogue computer results, the NVC is defined as $N = SK^2$. It is then shown that this definition of the NVC exhibits the desired characteristics. This was further demonstrated for the non-linear Van der Fol equation.

It is concluded that the NVC has the advantage that it can be evaluated easily from the phase portrain of the system, proving very useful in optimizing its performance directly from **its** phase plane trajectories, and in particular for the case of the non-linear system.

- 39-

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A RITZ-TYPE APPROACH TO THE CALCULATION OF OPTIMAL CONTROL FOR NONLINEAR, DYNAMIC SYSTEMS

The calculation of optimal control functions for nonlinear systems described by ordinary differential equations can be made much simpler, if a Ritz-type approximation for the control functions is used. From the well-known gradient techniques for problems with differential equations as constraints, an iteration formula for the parameters in the approximation is derived.

Very general problems, described by nonlinear differential equations and having general terminal constraints, fixed or free final time and constraints in the control functions, can be treated very efficiently, even if only small computers, like desk-top calculators or personal computers are used.

The application of this optimization method to a model of a chemical plant (a stirred-tank-reactor) is presented and the results are discussed with respect to the numerical properties of the parameterization approach.

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OPTIMAL AND SUBOPTIMAL SINGULAR CONTROL OF A CLASS OF LINEAR DISCRETE-TIME SYSTEMS

Singular control in continuous-time systems was defined in 1959. It was not understood until 1963. Additional work on this problem was performed until 1971. Singular control of discrete-time systems was not studied in the literature until 1971. Several papers were published in this subject, one of them was by the authors 1 in 1976. Two approaches were developed in these papers. The first approach was by the application of the local theory of constrained optimization, (nonlinear programming and dynamic programming). The second approach, which was used by the authors in their previous paper 1 , was the application of the global theory of constrained optimization, (the Fenchel duality theory).

In the present paper the second approach is adopted and the problem of singular optimal control of discrete-time systems with fixed end points and free final time, is investigated. The system is assumed to be time varying. Expression for the optimal control function is obtained by changing the problem to a minimum norm problem with linear constraints. It is shown that there always exists a finite final time such that an optimal control that satisfies the control constraints exists. In order to simplify and reduce computations, a significant suboptimal control is described. It is shown that the suboptimal control approaches the system to the final desired state each time of its application, moreover, it is guaranteed to reach the final desired state in a finite number of steps. Computer programs are prepared and executed on the computer PDP11/70 to show the applicability of the algorithms developed. Two numerical examples are investigated.

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-41-

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THE COMPLETE HAMILTONIAN CHARACTERIZATION OF PROPERTY (Q): APPLICATION TO AN EXISTENCE PROBLEM IN OPTIMAL CONTROL

In (1) I defined the "modified Lagrangian", a quite useful modification of the classical Lagrangian in optimal control and the calculus of variations. Using it, the usual lower closure and existence problems with weakly convergent finite dimensional "derivatives" can be approached by a deparametrization procedure so as to reduce them to a single classical lower semicontinuity result(2,5) for the integral functional whose integrand is the modified Lagrangian. I shall define a "modified Hamiltonian", which corresponds canonically to the modified Lagrangian. I will show that a certain upper semicontinuity property of this Hamiltonian completely characterizes Cesari's property (Q). This extends (4). Next, I shall apply this characterization to develop a deparametrization procedure for a lower closure problem with infinite dimensional derivatives; cf. (3).

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- 42 -

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ON SENSITIVITY IN AN STATE AND CONTROL CONSTRAINED OPTIMAL CONTROL PROBLEM

Consider the problem: for each ϵ belonging to a neighbourhood of zero in R^1 find functions \hat{x}_ϵ , \hat{u}_ϵ which minimize the functional

$$J_{\xi}(x,u) = \int_{0}^{1} f_{\xi}(x(t),u(t),t)dt$$

subject to

$$\dot{x}(t) = A_{\ell}(t)x(t) + B_{\ell}(t)u(t)$$
 for almost all

$$K_{\ell}^{C}(u(t),t) \leq 0 \qquad for all t \in [0,1],$$

$$K_{\ell}^{S}(x(t),t) \leq 0 \qquad for all t \in [0,1],$$

$$x(\cdot) \in AC(\mathbb{R}^{n}), x(0) = x^{0}, u(\cdot) \in L_{\infty}(\mathbb{R}^{m}),$$

$$K_{\ell}^{C} : \mathbb{R}^{m} \times [0,1] \Rightarrow \mathbb{R}^{p}, K_{\ell}^{S} : \mathbb{R}^{n} \times [0,1] \rightarrow \mathbb{R}^{q}.$$

Let \hat{x}_0, \hat{u}_0 correspond to $\varepsilon = 0.1t$ is proved that

$$\|\hat{x}_{\varepsilon} - \hat{x}_{0}\|_{C} + \|\hat{u}_{\varepsilon} - \hat{u}_{0}\|_{L_{0}} = O(\varepsilon)$$

under conditions of the following four types:1° Continuity and smoothness of the data;2° Convexity of the constraints and strict convexity of the functional;3° Slaterlike condition;4° Hager's regularity condition for the binding constraints for \hat{x}_0 and \hat{u}_0 . The convergence properties of the dual variables are discussed as well.

- 43 -

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> CORRELATION BETWEEN CONVENTIONAL CONTROL PERFORMANCE INDICES AND MODERN CONTROL QUADRATIC PERFORMANCE INDEX

In classical Control theory, attention was focused on the system's stability, transient performance, such as rise time, settling time and maximum overshoot, and steady state accuracy.

Recently, demands for control systems of increasingly high performance have led to increasingly stringent requirements on specifications of transient and steady state responses. Synthesis of control systems which meet stringent requirements on specifications by classical control theory is, in fact, an exceedingly difficult job. Synthesis methods based on modern control theory offer distinct advantages over classical methods. The paper compares and correlates between them.

Introducing a bandwidth criterion, together with certain specifications for step and ramp inputs defines a number of elements of the 2-matrix in the quadratic performance index. These criteria introduced, when applied to third order systems determines completely the optimal control function.

The theory can be extended to higher order systems and it would be necessary to impose more steady state requirements, such as error for parabolic inputs, maximum modulus for harmon: inputs, ... etc. in order to relate some of other 2-matrix elements and depending on the number of requirements and the order of the system it may be necessary to assume values for the remaining elements of the 2-matrix to solve the Riccati equation.

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A DISCRETE MAXIMUM PRINCIPLE CONCERNING THE OPTIMAL COST OF DETERMINISTIC CONTROL PROBLEMS. A STOCHASTIC INTERPRETATION.

The scope of this paper is to present a new numerical device able to compute the optimal cost function V(x) related to a large class of deterministic optimal control problems.

We consider simultaneously stopping time, continuous and impulse controls in each strategy. So $V(x) = \inf_{z_i \in \{z_i, z_i\}} J(x; z, u(.), z(.)).$

We know, after (1), (2) that in regular problems, V(x) is the maximum element of a suitable set W of "subsolutions" w(x) of the associated Hamilton-Jacobi equation. As a consequence the computation of V(x) is reduced to solve the problem: P) Find the maximum element of the set W.

We shall use approximate problems Ph. The state variables, domaine is discretized by linear finite elements (having vertex x_i The set W is substituted by approximate sets W^h obtained thanks to a special discretization method implying that the approximate problem: P_h) Find the maximum element \overline{w}^h of the set W^h is solved taking advantage of the special structure of the matrix of the coefficients of the independent variables $w^{h}(x_{i}^{(h)})$. In fact, we show: a) a maximal element \widehat{w}_h (related to a partial order) is caracterized by the accomplishment of the transversality conditions concerning V(x); b) \hat{w}_h can be obtained using a method of relaxation type i.e. an iterative and successive maximization with respect to the independent variables $w^h(x_i^{(h)})$. The computational algorithm is very simple and after ordering the variables in a suitable manner it makes possible to perform non trivial problems in computers of small central memories; c) there exists an unique maximal element \hat{w}^{h} in the approximate set W^h ; so, \bar{w}^h is equal to \hat{w}_h just obtained; d) the approximate solution \tilde{w}^h converges uniformly to the optimal cost function V(x) when $\|h\|$ tends to zero; e) the equations (or inequations) that determine $\bar{\mathbf{w}}^{h}$ may be interpreted as the optimality conditions for the optimal control problem of a stochastic system with a finite number of states. These processes converge to the deterministic trajectories of the original problem when $\|h\|$ tends to zero.

Finally we present some numerical examples.

 R. GONZALEZ - E. ROFMAN: "An algorithm to obtain the maximum solution of the H. J. equation". IFIP-TC7 Conferences. Würzburg 1977.
 R. GONZALEZ: "Sur la résolution de l'équation de H. J. du contrôle déterministe". Cahiers de math. Paris IX Dauphine - N° 8029/8029bis -1980. T. F. Gonzalez Mathematical Sciences Program The University of Texas at Dallas Richardson, TX 75080

> On the Computational Complexity of Clustering and Related Problems

The problem of clustering a set of n points into k groups under various objective functions is studied. It is shown that under some objective functions clustering problems are NP-hard even when the points to be clustered lie in the 2-dimensional euclidean space. We also show that for these problems the corresponding approximation problem is also NP-hard.

Specifically, we study the k-2MM and the k-2M Σ clustering problems. In the k-2MM problem we are given a set of points which lie in the 2-dimensional euclidean space and we wish to partition them into k groups (clusters) in such a way that the maximum distance between any pair of points belonging to the same cluster is minimized. The objective function to be minimized in the k-2M Σ problem is the maximum (amongst all clusters) of the sum of the distance between all pair of points belonging to the same cluster. V.V.Gorokhovik Institute of Mathematics Academy of Sciences of BSSA Surganov St., 11, Minsk, 220604, USSR

ON HIGH ORDER NECESSARY OFTIMALITY CONDITIONS

All known necessary optimality conditions in optimization problems with inequality and equality constraints can be divided in two classes. The first class is characterized by the fact that some normality assumptions concerning constraints are made a priori whereas no such assumptions are made for results of the second class. The first order necessary optimality conditions have been well developed in both the cases. As for the second and high order necessary optimality conditions until recently only results of the first class were known and only during the last seven-eight years the results of the second class were obtained.

High order necessary optimality conditions presented in this report belong to the second class. At first we consider the abstract optimization problem and then we apply obtained abstract results to control problems. In the latter case we study both classical singular extremals and singular Pontryagin extremals. Necessary optimality conditions obtained for classical singular extremals generalize the knoch Kelley's optimality conditions (generalized Legendre-Clebsh conditions).

In conclusion we discuss the connection of constrained optimization problems with minimax problems as well as with optimization problems with vector-valued performance index.

- 47 -

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THE APPLICATION OF VECTOR MINIMISATION TECHNIQUES IN THE ANALYSIS OF MULTILOOP NONLINEAR FEEDBACK SYSTEMS

The analysis of harmonic balance conditions in autonomous nonlinear multiloop feedback systems of the form shown in the figure has been considered by several authors (1),(2), using simple signal harmonic approximants. The extension to higher order solutions which include the effects of circulating superharmonic components is complicated by the increase in the dimensionality of the resulting balance equation. Recently, solutions have been derived (3) which include superharmonic signal components by using a method of successive approximations involving vector minimisation techniques.

In this paper the underlying numerical procedures are described and it is shown how these procedures can be refined to obtain solutions of even higher dimensionality. A new graphical interpretation is presented which yields information concerning loop interaction effects at or near possible harmonic balance conditions and allows error bounds to be defined which relate to the data intervals chosen in the search for a solution point. An example of use is given.

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-48-

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A SINGULAR STEADY STATE LINEAR REGULATOR AND ITS DUAL

We consider the problem of infimizing the cost functional,

$$J(u) = \int_{0}^{\infty} [y'(t)y(t) + u'(t) Ru(t)] dt$$
 (1)

subject to the constraints

 $x = Ax + Bu, \quad y = Cx, \quad x(0) = \xi,$ (2)

when all unstable modes of A are either controllable or unobservable under some linear feedback. R is assumed to be only positive semidefinite, and controls u admitted to the competition are piecewise continuous vector valued functions satisfying,

$$\lim_{t \to \infty} y(t) = 0, \quad \int u'(t) Ru(t) dt < \infty. \quad (3)$$

We find that

u

where P is one of the symmetric positive semidefinite solutions of the linear matrix inequality. If we further constrain u to satisfy

lim u(t) = 0, then P is also the maximal symmetric solution of a pair $t \rightarrow \infty$

of Popov equations. In either case, P may be found as the limit of a sequence of unique positive definite symmetric solutions of algebraic Riccati equations defined in the state space reduced by the unobservable modes.

The dual of the singular, linear, steady state regulator problem is shown to be a singular Wiener filter problem when there are no unstable system modes that are both controllable and unobservable.

The dual of the singular finite time regulator has been treated elsewhere [1].

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SEMI DUAL APPROXIMATIONS IN OPTIMAL CONTROL

In a series of papers listed in the bibliography of [1], we analyzed the Ritz-Trefitz method for approximating the solution of convex control problems. In this method, dual multipliers are introduced for the differential equation and for each of the control and the state constraints, and the optimal dual multipliers are approximated using finite element subspaces. We now study a "semi dual" method where the differential equation is handled with a Lagrange multiplier while the state and the control constraints are treated explicitly.

In particular, we consider control problems of the form

(p) minimize $\{C(x,u): \dot{x} = Ax+Bu, x(0) = x, x \in X, u \in U\}$

and the associated dual function

(DF) $L(p) = \inf\{C(x,u) \le p, \dot{x}-Ax-Bu \ge x(0) = x_0, x \in X, u \in U\}$ where <---> denotes the L^2 inner product. The dual problem

(D) $\sup\{L(p): p \in BV = bounded variation\}$

is approximated by

 $(D^{h}) \sup\{L(s^{h} S^{h}\}\}$

where $S^h \subset BV$ is a finite element subspace. If (x^*, u^*) solves (P), p^h solves (D^h) and (x^h, u^h) achieves the minimum in (DF) for $p=p^h$, we estimate the error (x^*-x^h) and (u^*-u^h) .

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- 50-

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ON A GENERAL METHOD FOR

SOLVING TIME-OFTIMAL LINEAR CONTROL PROBLEMS

The lecture will be concerned with the following abstract version of a time-minimal linear control problem:

Let $\{S_t : t \in [0,T]\}$, for some T>o, be a family of continuous linear mappings from the dual space $X = Z^*$ of a separable Banach space Z into a Banach space Y with $S_0 = 0$. Further let U_M be the ball in X centered at x=o with radius M>o. Finally, let $\hat{y} \in Y$ be a given element with $\hat{y} \neq 0$ such that there exists some $t \in (0,T]$ and some $u \in U_M$ with $S_t(u) = \hat{y}$. Find $u^* \in U_M$ such that $S_{+} \star (u^*) = \hat{y}$ where

 $t^* = \inf\{t_{\epsilon}[o,T] \mid S_t(u) = \hat{y} \text{ for some } u_{\epsilon}U_M\}.$

For the solution of this problem a class of algorithms is given which are based on the duality principle which, under suitable assumptions, states that

$$t^* = \max\{t_{\epsilon}(o,T]|y^*(\hat{y}) = M|S_t^*(y^*)| \text{ for} \\ \text{some } y^*_{\epsilon}Y^* \text{ with } |y^*| = 1\},$$

Y^{*} being the dual space of Y and S_t^* being the adjoint operator of S_t , $t_{\epsilon}[0,T]$.

For this class of algorithms which contains some known algorithms in special cases convergence statements are made and applications to control problems governed by ordinary and partial differential equations are given along with numerical results. Roland Hornung and Klaus-D, Wolfenstetter Research Institute of the Deutsche Bundespost Darmstadt, Federal Republic of Germany

DISCRETE MIN-SUM-OPTIMIZATION USING A NEWTON-METHOD AND APPLICATION TO OPTIMAL CONTROL

Abstract

Given is the following problem:

 $\min_{\substack{n \\ x \in IR}} \sum_{j=1}^{m} |f_j(x)|,$

where functions f_j : $IR^n \rightarrow IR$, $j=1, \ldots, m$, are assumed to be sufficiently smooth. This problem includes the nonlinear discrete l_1 -approximation as well as nonsmooth optimization and some optimal control problems. In the first part of the paper a survey is given of the existing classes of algorithms for solving (P).

In the second part a Newton method for solving (P) locally is presented. To extend the region of convergence, we can choose some of the methods mentioned above as an "outer algorithm". We prefer a generalization of the gradient projection technique of BARTELS / CONN/ SINCLAIR. Some criteria control the switch from the outer algorithm to the inner one (the Newton method) and possibly back again. A convergence theorem of this two-stage algorithm (which shows the convergence to a stationary point) is given. Finally the efficiency and robustness of this method are illustrated by means of several examples. The two-stage algorithm proposed here has an obvious superiority to usual one-stage methods.

- 52-

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Periodic Solutions of Discrete Matrix Riccati Equations with Constant Coefficient Matrices

Matrix Riccati equations arise in the theory of optimal control as well as optimal estimation problems.

In this paper, we consider the following matrix Riccati equation

 $P(k+1) = \phi \{P(k) - P(k)H^{T}[HP(k)H^{T}+I]^{-1}HP(k)\}\phi^{T}+GG^{T}$

where ϕ , G, and H are constant matrices.

Since Riccati equation is a non-linear equation, the existence of periodic solution can be expected even in the case of constant coefficient matrices. The analysis of such oscillatory solutions is vital from the stability viewpoint of optimal control and estimation problems.

An existence condition is established of real symmetric periodic solutions as well 13 of real symmetric nonnegative-definite periodic solutions. Furthermore, an algorithm is developed to derive such periodic solutions.

The method employed here is based on the Jordan-form representation of certain symplectic matrix, which has been used extensively to analyze steady-state solutions, i.e. solutions of algebraic Riccati equations.

The results developed in this paper are applied to an example which possesses a nonnegative-definite periodic solution. Convergence properties of solutions are also studied in this example. Myoung S. Ko and Uk Y. Huh Dept. of Instrumentation and Control Eng. Seoul National University Seoul, KOREA 151

NEW SCHEME OF DISCRETE IMPLICIT OBSERVER

We present a new scheme for the discrete reduced order adaptive observer without any auxiliary signals.

Consider the single input, single output linear time-invarient system described by

$$x(k+1) = Ax(k) + bu(k)$$

 $y(k) = c^{t}x(k) x(0) = x^{0}$ (1)

Assume that (1) is observable canonical form and A, b and x^{0} are unknown. The state vector x can be estimated by Luenberger's reduced order observer [1]. The system (1) can be written by

$$y(k+1) = \tau y(k) + p^{t}r(k) + c^{t}P^{k-1}x^{0}$$
 (2)

Where p is the set of parameters $p_i \in p \in E^{2n}$ which are one to one correspond to the actual unknown parameters and the elements of r(k) consist of input, output and the states of two (n-1)st state variable filters. The estimated output can be described by

$$\hat{y}(k+1) = \tau_y(k) + \hat{p}(k)^{t}r(k) + c^{t}r^{k-1}\hat{x}^{0}$$
 (3)

We derive adaptation law for adjusting $\hat{p}(k)$ based on the exponentially weighted least square method by introducing the following criterion function and also the estimated states can be obtained from the algebraic transformation of the vector r(k).

$$J(k) = \sum_{i=1}^{k} a \lambda^{k-j} (\hat{p}(k)^{t} r(j) - y(k+1))^{2}$$

$$\hat{p}(k+1) = \hat{p}(k) + L(k+1)(y(k+1) - \hat{p}(k)^{t} r(k+1))$$

$$L(k+1) = \Gamma(k) / \lambda r(k+1)(1/a + r(k+1)^{t} \Gamma(k) / \lambda r(k+1))^{-1}$$

$$\Gamma(k+1) = (I - L(k+1)r(k+1)^{t})\Gamma(k) / \lambda$$
(4)

where

The fast convergence reduced order adaptive observer is based on the above equations.

The computer simulation results show that the convergence speed and the peak states error are increased in accordance to the weighting factor λ .

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REACHABLE SETS AND GENERALIZED BANG-BANG PRINCIPLE

FOR LINEAR CONTROL SYSTEMS

We consider a general control system defined by a set U of admissible controls

 $\mathbf{u} = \{\mathbf{u}(\cdot) \in \mathbf{L}_{\mathbf{p}}(\mathbf{T}_{1}) : \mathbf{u}(t) \in \mathcal{D}(t) \text{ for a.e. } t \in \mathbf{T}_{1}^{3}$ (1)

an affine operator

A: $u(.) \in U \rightarrow x(., U) \in C(T_2, E^n)$, (2)

a set of restrictions on states of the system at a finite number of fixed points t_1, \ldots, t_n

 $x(t_j) \in B_j$, where B_j are fixed convex sets in E^n , (3) a set of constraints on system trajectories in the form of $x(.) \in R$, where R is a fixed convex set in $C(T_2, E^n)$. (4)

In the absence of the constraints (4), the fact of the fundamental importance, proved in this general situation by Zvi Artstein, says that the attainable set $Q_t(U)$ at any fixed point t will not change if U replaced by its convex hull coU:

 $Q_+(U) = Q_+(coU)$

(5)

For any fixed subset $T \in T_2$, we define the reachable set Q_T of the system (1) - (4) as

 $Q_{T}(U) = \{a(.) \in C(T, E^{n}): a(.) = x(., u)|_{T}, u \in U, x(t_{j}, u) \in B_{j}, x(., u) \in R\}$ (6)

Note, in particular, that a reachable set coincides with a traditional attainable set if T is a single-point and with the set of all admissible trajectories if $T = T_2$. The main result about reachable sets (6) asserts that $Q_T(U)$ and $Q_T(coU)$ have the same closures provided the sets an inner trajectory of the system, i.e. a trajectory belonging to R.

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REDUCING TRAJECTORY SENSITIVITY TO MULTI-DELAYS IN CONTROL SYSTEM

Optimal control system with low sensitivity to small time delays is investigated here. Two different delays are expected to occur, one in the plant and the other in the feedback path. The Minimum Principle is applied in obtaining plant and output feedback gains to minimize an augmented quadratic performance measure which includes sensitivity functions.

A system is given with time delay h_1 in the plant and delay h_2 in the output feedback path.

$$\dot{x}(t) = Ax(t) + Bx(t - h_1) + CDEx(t - h_2)$$

Let $h_1 = 0$ and $h_2 = 0$, then

$$\dot{x}(t) = Ax(t)$$

where $\overline{A} = A + B + CDE$. The sensitivity functions are

 $s_1 = \frac{dx}{dh_1} \left| \begin{array}{c} h_1 = 0 \end{array} \right|$

$$s_2 = \frac{dx}{dh_2}$$
 $h_2 = 0$

Then

$$\dot{s}_1 = \overline{A} s_1 - \overline{BAx}$$

 $\dot{s}_2 = \overline{A} s_2 - \overline{CDEAx}$

The performance measure is given by

$$J = \frac{1}{2} \int_{0}^{\infty} (x' \bar{Q}x + s'_{1} Us_{2} + s'_{2} Vs_{2}) dt$$

Where $\overline{Q} = Q + E'D'RDE$, and R,U and V are matrices of appropriate dimensions.

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-56-

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A NOVEL APPROACH FOR MINIMUM-TIME CONTROL OF SATURATED DISCRETE-TIME SYSTEM

This paper presents a systematic procedure examining the characteristics of reachable sets and sub-reachable sets for an nth order discrete-time system. Then, in accordance with the characteristics, the reachable set is subdivided into saturated part and linear part. It is found that to obtain a minimum-time control, a state in saturated part can always be driven by saturated signal with proper polarity. Once the state reaches the linear part, the control signal should be properly chosen. Otherwise minimum-time control may never by achieved. By utilizing the new approach, when the state is in the linear part, the control signals can be found by simple matrix manipulations. Finally, a simulation of the approach applied to a second order process is presented.

- 57 -

DUALITY IN DISCRETE OPTIMAL CONTROL 1.B. MEDVEDOVSKY

By using the method of multivalued mappings it is possible to obtain in a rather simple way a result which can be considered as the duality theorem. It yields as corollaries the theorems of duality, the minimax theorm and also the duality theorem for the optimal control problem in the process described by convex inclusion.

Let X, Y euclidean spaces, and X* and Y* are conjugate spaces with scalar product $\langle x, x^* \rangle$, $\langle y, y^* \rangle$ respectively.

For a multivalued mapping $a: X \neq 2^X$

$$\mathbf{a}(\mathbf{x}) = \{\mathbf{y}: \mathbf{y} \in \mathbf{a}(\mathbf{x}) \leq 2^{\mathbf{X}} \leq \mathbf{Y}\}$$

and for two optimal control problems

$$\min\{\{x_t, y_t^*\}: x_{i+1} \in a(x_i), x_0 \in M_0, x_t \in M_t, i = 0, 1, \dots, t-1\}$$
(7)

$$\sup_{\substack{M \in \mathcal{N}_{e}^{*}, x_{t}^{*} \\ i=0, 1, \dots, t}} \{ W_{M}(x_{e}^{*}) + W_{M}(x_{e}^{*}) + \sum_{i=0}^{t-1} \Omega_{a}(x_{i}^{*}, x_{i+1}^{*}) : x_{t}^{*} + x_{e}^{*} = y_{t}^{*} \}$$
(9)

the following theorem of duality is proved:

THEOREM 2. Let a be a convex, closed, and bounded at least at one point mapping of the space $X = E^n$ into 2^X , M_o and M_t be convex sets, where M_o a compact subset of dom a, besides for the mapping a and sets M_o , M_t the condition $ri[a^t(M_o)] \cap ri M_t \neq \phi$ holds.

Then the values of direct (7) and dual (9) problems coincide.

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> OPTIMAL CONTROL OF LINEAR SYSTEMS WITH TIME VARYING DELAYS VIA SINGLE TERM WALSH SERIES APPROACH.

The main object of this paper is to determine the optimal control u*(t) when the system is described by

x(t) = A(t) x(t) + B(t) x(t-g(t))with the initial data $x(t_0) = x_0$,

 $\mathbf{x}(t) = \Im_{\mathbf{x}}(t), t \in \left[-g(t_0), t_0\right]$

and the performance measure given by

 $J = \frac{1}{2x^{T}(t_{f})}G x(t_{f}) + \frac{1}{2}\int_{0}^{t_{f}} (x^{T}Qx + u^{T}Ru) dt,$

where 'T' indicates transpose and 't₁' the final time. The matrices A(t), B(t), Q and R are of appropriate dimensions. By applying Riccati Transformation,

p(t) = k(t) x(t) + h(t) in the usual state and: costate equations

> $\dot{x}(t) = A x(t) + Bx(t-g(t)) - BR^{-1} B^{T} p(t)$ $\dot{p}(t) = -A^{T}p(t) - B^{T} p(t + g(t)) - Qx(t)$

the optimal control $u^*(t) = -R^{-1} B^T p^*(t)$ is calculated using the Single Term Walsh Series (STWS) approach developed by Rao et.al [1]. The method is simple when compared to the previous methods.

- 5 + -

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APPROXIMATE MODEL-MATCHING OF TWO-DIMENSIONAL SYSTEMS

This paper considers the problem of model-matching of linear timeinvariant 2-D (two-dimensional) systems using linear output feedback. The problem of model-matching may be defined as follows: Given a system under control whose performance is not satisfactory and a model whose performance is the desirable one, find a feedback law such that the performance of the closed-loop system matches, as nearly as possible, the performance of the model.

In this paper the 2-D system under control is assumed to be described by $\ell \times r$ transfer function matrix $T(z_1, z_2)$ and the 2-D model by an $\ell \times r_m$ transfer function matrix $T_m(z_1, z_2)$. The feedback law applied is linear, with a closed-loop transfer function of the form

 $T_{c}(z_{1}, z_{2}) = [I_{g} - T(z_{1}, z_{2})K]^{-1} T(z_{1}, z_{2})G$.

The problem treated in this paper can now be stated as follows: Given $T(z_1, z_2)$ and $T_m(z_1, z_2)$ find the controller matrices G and K such that the error $E(z_1, z_2) = T_c(z_1, z_2) - T_m(z_1, z_2)$ is as small as possible. The paper develops a method for the approximate solution of this problem. The determination of G and K is reduced to that of solving a system of linear algebraic equations. P. Hr. Petkov, M.M. Konstantinov, and N.D. Christov Department of Automatics Higher Institute of Mechanical and Electrical Engineering 1100 Sofia Bulgaria.

A NUMERICALLY STABLE METHOD FOR POLE ASSIGNMENT

One of the basic problems in the synthesis of linear control systems, the pole assignment, is not solved yet satisfactorily from computational point of view. Most of the existing methods for this synthesis are numerically unstable, computationally expensive and not suitable for high order systems.

In this paper a new method for pole assignment of single input systems, based on orthogonal reduction of the closed loop system matrix to upper (quasi) triangular form is proposed. It uses Householder reflections for preliminary reduction of the system matrices into the so called orthogonal canonical form and plane rotations in order to achieve the desired spectrum of the closed loop system matrix.

The method proposed has a number of adventages over the known methods It does not require the computation of the characteristic polynomial of the open-loop system or the transformation into phase-variable canonical form. It works equally well with distinct and multiple, real and complex conjugate desired poles and may be used for synthesis of continuous as well as discrete-time systems. The number of necessary floating point operations is approximately $6n^3$, the necessary array storage being $2n^3 + 5n^2$ working precision words, where n is the order of the system. Djordjija B. Petkovski Faculty of Technical Sciences Veljka Vlahovića 3 21000 Novi Sad, Yugoslavia

CONTROL STRATEGY FOR DECENTRALIZED FEEDBACK COORDINATION

In any realistic synthesis problem there are always a large number of possible solutions that satisfy a particular set of control objectives. While the use of costcriterion optimization in theory allows a central agent, who has access to all a priori structural information, to pick out exactly one choice (the optimal solution), in practice the difficulties of incorporating all the relevant cost considerations necesitates further trial and error "hedging" about the nominal solution. In a large-scale system it is desirable that such further "fine-tuning" be done decentrally by the local agents rather than by the central agent. It is therefore desirable that the central agent does not completely fix the individual control strategies of the local agents [1].

In this paper we formulate a class of problems that have the control law computational task partially centralized and partially decentralized. The basic idea is that the centralized part of the control strategies represent the coordinating constraints imposed by a central decision-making agent, called the coordinator, while the decentralized part of the control strategies represents the degree of freedom or independence that each local control actuation agent can exercise. The task of the coordinator, in other words, is to parametrize the degree of freedom of each of the local control agent and assign that a priori constraint information to the individual agents.

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THE STRUCTURE OF OPSIMAL CONTROL FOR SYSTEMS LINEAR IN CONTROL

In this paper the problem of optimal control for systems which are linear in control is considered. The multiplicative control systems offer, in general, better performance than linear systems as far as controllability, optimization and modelling are concerned.

In particular, we consider a class of bilinear systems $\dot{x}(t) = A_0 x(t) + u(t)N_0 x(t)$ $x(0) = x_0$

and a functional of the form

 $J = \int_{c_1}^{T} \left(a_0^{T} x(t) + n_0^{T} x(t) u(t) + r_0 \right) dt$

The optimal control problem is to find an optimal control u(t) from a class of functions

 $U = \{ u(t): |u(t) \leq 1 \text{ and measurable on } [^{\circ}, ^{m}] \}$

where final time T is not prescribed beforehand, such that it transfers the system state from a given initial state x_{r} to a given final state x_{r} and minimizes the functional J.

The optimal singular trajectories are allowed to lie only on a certain hypersurface which is an invariant set with respect to control. It is proved that the optimal control is either totally bang-bang on the whole transfer time or totally singular. A necessary and sufficient condition for optimality of singular trajectories which lie on this hypersurface is established.

As example, the problem of guiding an aircraft in minimum time from an arbritary point in the terminal area to the outher marker is considered.

-63-

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PERIODICITY IN OPTIMAL CONTROL AND DIFFERENTIAL GAMES

Questions about optimality in the infinite time interval, and in particular periodic optimal solutions of control systems have been addressed in the recent Ph.D. dissertation of Stern [1], where further references can be found. The present paper discusses some related problems for differential games, with the focus on linear systems; the results also apply to optimal control problems by deleting one player. Sufficient conditions are given for the adjoint vector p(t)to ensure periodicity of the optimal controls and trajectories. The central results which lead to further discussions of specific cases, are as follows.

The system considered is $\dot{\mathbf{x}} = A(t) | \mathbf{x} + B(t) | \mathbf{u} + C(t) | \mathbf{v}$, where $\mathbf{x}(t) \in \mathbb{R}^{n}$, $\mathbf{u}(t) \in U \in \mathbb{R}^{r_{1}}$, $\mathbf{v}(t) \in V \in \mathbb{R}^{r_{2}}$, U and V compact, A(t), B(t)and G(t) continuous and T-periodic. The objective functional is $J = \min\{m_{i}\}$, for m any positive integer and n a given vector; u should minimize and v should maximize J.

If p(0) is an eigenvector of the fundamental matrix $\Phi(T,0)$, with a positive eigenvalue, then the adjoint vector p(t) is of the form $e^{\alpha t} \cdot (T\text{-periodic function})$, hence "direction-periodic", and the optimal controls u(t), v(t) will be T-periodic. If the eigenvalue is negative, then the optimal controls will be 2T-periodic. If $(1-\Phi(T,0))$ (respectively $(I-\Phi(2T,0))$) is nonsingular, then to these periodic optimal controls corresponds a unique $x_p(0)$ leading to a periodic solution $x_p(t)$. The behavior of the other (nonperiodic) solutions x(t) can be analyzed in detail in the different cases of existence or non-existence of periodic solutions.

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SEARCH FOR A SINGULAR SOLUTION OF AN OPTIMIZATION PROBLEM

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ABSTRACT

Most works deal with singular solution of optimization problems in which the Hamiltonian is linear in control variables.

This paper will discuss a kind of optimization problem which is met often in engineering. In some engineering problems it is possible that all of the control variables are not contained in the cost functional and all the state equations that will result in a singular candidate in the optimization problem, and the Hamiltonian in it may be nonlinear in all control variables. The term containing some elements of the control vector will vanish from the Hamiltonian along the singular arc. A method of searching for the singular solution is presented.

And two examples are given in this paper.
GAMES

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SOLUTION OF A MULTIPERSON COOPERATIVE GAME BY POLIOPTIMISATION METHOD

A new method for solving the multiperson cooperative games consists in transforming the question of determining a solution to the game into the task of determining a so called "compromise solution" for a polioptimisation problem.

The compromise solution consists in finding a counterimage $y \in Y$ minimising the distance $\|w - y\|_p$ of the points: $y \in Y$ and w. The set Y can be interpreted as a set of games /characteristic functions/ with the one-element C-core, and the point w is the characteristic function of the game analysed, i.e. a model of the situation analysed.

It is proven that thus defined solution of a game exists for every essential cooperative game, that it is an element of the C-core of the game /provided it is not empty/, is unique, optimal in the Fareto sense, independent on parameter p, and additionally its interesting economic interpretation is shown. In case the initial game has an empty C-core, its compromise solution is the C-core of the closest /in terms of N-N/ game out of the games with the one-element C-core.

The solution method consists in application of the theorem on the orthogonal projection of the characteristic function $v(\cdot)$ on a hyperplane in the space of N-personal games. The solution is easily obtained.

Analysis is illustrated with an example of determining a compromise solution for international energy cooperation of three countries, calculating the optimal export and import volumes. The solution is compared with the ones obtained via proportionate imputation, solidary imputation and with the Shapley's solution. The comparison shows that the condition of coalitional rationality, the fundamental one for economic decisions, is satisfied only by the compromise solution. N-Arobaty
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iaris, France.

(1) TWO AND GETTER LATION FOR DEPERTURAL CANDE MCENTERING

The kinematic equations and the optimal controls were formulated when the pursuer (P) and the evader (E) are controlling their strategies with(E's) superiority in his rate of turn which is corresponding to (P's) speed superiority. (F) is carrying a system to capture (E) in a plane higher or lower than (F's) height. Using the solution of the differential equations in the retrograde sense We found the boundary of the useable part conditions with the trajectories equations. New approaches were done for the construction of the game's barriers especially w.r.t. the singular arcs when (F) is maximizing and(...) is minimizing the hamiltonian of the game.

Then the capture is defined to be the coincidence of (1) and (\mathbb{R}) then the capture conditions are not only given by the barriers intersections but also they must satisfy :

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RISK AVERSION AND NASH'S SOLUTION FOR BARGAINING GAMES WITH RISKY OUTCOMES

Recent results have shown that, for bargaining over the distribution of commodities, or other riskless outcomes, Nash's solution predicts that risk aversion is a disadvantage in bargaining. Here we consider bargaining games which may concern risky outcomes as well as riskless outcomes, and we demonstrate that, in such games, risk aversion need not always be a disadvantage in bargaining. Intuitively, for bargaining games in which potential agreements involve lotteries which have a positive probability of leaving one of the players worse off than if a disagreement had occured, the more risk averse a player, the better the terms of the agreement which he must be offered in order to induce him to reach an agreement, and to compensate him for the risk involved. For bargaining games whose disagreement outcome involves no uncertainty, we characterize when risk aversion is advantageous. disadvantageous, or irrelevant from the point of view of Nash's solution. A.I.A. Salama Dept. Flect. Engineering University of Alexandria Alexandria, Egypt V. Courishankar Dept. Flect. Engineering University of Alberta Edmonton, Canada

THREE DIM NSICHAL PURSUIT-EVASION WITH FREE MANEUVERABILITY PURSUEP

The problem of pursuit-evasion with free maneuverability pursuer in three dimensional space is considered. The evader is assumed to have constant speed and constrained path curvature. The pursuer is assumed to maneuver freely and having constant speed. Using spherical coordinates the motions of the evader and pursuer can be described by [1];

Evader

Pursuer

 $\dot{\mathbf{x}}_{1e} = \mathbf{v}_{e} \cos \phi_{e} \cos \phi_{e}$ $\dot{\mathbf{x}}_{2e} = \mathbf{v}_{e} \cos \phi_{e} \sin \phi_{e}$ $\dot{\mathbf{x}}_{3e} = \mathbf{v}_{e} \sin \phi_{e}$ $\dot{\mathbf{\beta}}_{e} = \mathbf{v}_{e} \kappa_{e} \sin \phi_{e} \cos \phi_{e}$ $\dot{\mathbf{\phi}}_{e} = \mathbf{v}_{e} \kappa_{e} \cos \phi_{e}$ subject to $0 \leq \kappa_{e} \leq \kappa_{r}$

 $\dot{x}_{1p} = v_p \cos\phi_p \cos\beta_p$ $\dot{x}_{2p} = v_p \cos\phi_p \sin\beta_p$ $\dot{x}_{3p} = v_p \sin\phi_p$

The payoff of the game is lefined as:

 $J(\kappa_{e},\psi_{e},\phi_{p},\epsilon_{p}) = f_{t_{a}}^{t}$ t

The capture time \hat{t} is defined as the time when the pursuer comes within a small distance c from the evader. The objectives of the paper are: (a) to determine the characteristics of (κ_e^*, ψ_e^*) and $(\epsilon_p^*, \beta_p^*)$ such that

 $J(\kappa_{e},\psi_{e},\phi_{p}^{*},3\overset{*}{p}) \leq J(\kappa_{e}^{*},\psi_{e}^{*},\phi_{p}^{*},\beta\overset{*}{p}) \leq J(\kappa_{e}^{*},\psi_{e}^{*},\delta_{p}^{*})$

(b) to investigate the characteristics of the optimal minimax trajectories for both the singular and consingular cases.

The theory of differential games [2], and some of the results reported in [1], were used to investigate the pursuit-evasion game under consideration. Although the game takes place in three dimensional space, it is established that the optimal minimax trajectories of the game lie in the plane composed of the evader initial velocity vector and the pursuer initial position.

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QUADRATIC APPROXIMATIONS FOR COMPUTING CONSTRAINED EQUILIBRIA OF GAMES IN NORMAL FORM

The paper presents a family of efficient algorithms for computing various equilibria (the competitive Nash equilibrium, the cooperative Kalai-Smorodinski equilibrium, other cooperative equilibria of Pareto type, partial coalition equilibria with prespecified coalition type, etc.) of static games in normal form, that is, described by the payoff-functions

$$f_i(x_1, \dots, x_i, \dots, x_n) = f_i: \mathbb{R}^{\lambda n_i} \times \mathbb{R}^1$$

for each player i with decisions x,, constrained individually by

$$g_{i}(x_{i}) = 0, g_{i}: \mathbb{R}^{n_{i}} \to \mathbb{R}^{n_{i}}$$

Such games and equilibria frequently arise in international trade theory (oligopolistic games, international mark2t games, etc.). A known approach to computing such equilibria are fix-point algorithms which, although being globally convergent, are not quite efficient computationally. Another approach, based on quadratic approximation methods developed recently for nonlinear programming (in algorithms of Mangasarian, Han, Powell, Fletcher) and supplemented by an augmented Lagrangian quadratic approximation method (developed by the author) is proposed in the paper. It is shown that while the computation of a Nash competitive equilibrium is a rather straight-forward problem, the cooperative Kalai-Smorodinski equilibrium and other cooperative eqilibria result often in badly defined optimization problems, which have to be appropriately regularized. However, this difficulty can be overcome both theoretically and practically. Theoretical properties of such algorithms and results of practical computations are presented. IDENTIFICATION AND ESTIMATION Carl I. R. Blackwood, Department of Electronic Engineering, University of Hull, HULL, HUG 7RX England.

DYNAMICAL SYSTEM MODELLING FOR THE IDENTIFICATION OF STOCHASTIC PROCESSES

This paper outlines a method in which a 'Pole-Cloud' Map facilitates the selective application and withdrawal of an ARMAX type model, according to the nature and degree of the noise affecting a given stochastic process.

The Pole-Cloud Map is a sequential z-plane plot of the zeros of the appropriate rationalised discrete characteristic polynomial (DCP)

$$z^{N} + \hat{a}_{1}(k) z^{N-1} + \hat{a}_{2}(k) z^{N-2} + \dots + \hat{a}_{N}(k)$$

at time 'k', related to the adaptive model

$$y_{k} + \sum_{i=1}^{N} \hat{a}_{i} \cdot y_{k-i} + \sum_{i=1}^{N} \hat{c}_{i}r_{k-i} = \sum_{i=0}^{M} \hat{b}_{i}u_{k-i}$$
; where MaN

The Map reflects the location of and confidence in a given pole at 'k'; and indicates the degree of convergence and stability of the process.

In a stochastic process, pole regions oscillate; and the subsequent 'enlarged' Map is used to determine new seeding parameters for augmenting the parameter vector. Techniques exist for expanding and condensing the System Weighting Matrix in accordance with the Map's determination.

The zeros of the DCP can be recursively evaluated by a new modelling technique which incorporates a 'virtual signals' set

 $\varepsilon_{k}^{0}, \varepsilon_{k}^{1}, \varepsilon_{k}^{2}, \dots, \varepsilon_{k}^{N-1}$ where $\varepsilon_{k}^{i} = \varepsilon_{k}^{i-1} - \hat{\alpha}_{i}^{k} \cdot \varepsilon_{k-1}^{i-1}$; $i = 1, 2, \dots, N-1$

The new model used is

$$y_{k} = \varepsilon_{k}^{0} = \hat{\alpha}_{1}^{k} \cdot \varepsilon_{k-1}^{0} + \hat{\alpha}_{2}^{k} \cdot \varepsilon_{k-1}^{1} + \dots + \hat{\alpha}_{N}^{k} \cdot \varepsilon_{k-1}^{N-1} + \hat{b}_{1}^{k} \cdot u_{k-1} + \hat{b}_{2}^{k} \cdot u_{k-2} + \dots + \hat{b}_{M}^{k} \cdot u_{k-M}$$

where $\hat{\alpha}_1^k$, $\hat{\alpha}_2^k$, ... , $\hat{\alpha}_N^k$ are the required zeros.

-73-

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ADAPTIVE PREDICTION OF HIGH TIDES

The subject of this paper is the modelling and stochastic adaptive prediction of high tides on the Danish west coast.

On the basis of a nonlinear deterministic hydrodynamical-numerical (HN) model [1], approximate multivariate ARMA models are constructed. These ARMA models are capable of reproducing the output from the much more complicated HN model with good accuracy by using a far less number of inputs. Neither the HN model nor the approximate ARMA model can predict the real water level with sufficient accuracy however. Rather they should be regarded as a representation of the knowledge of the water level variations based on basic physics.

The residual process is in a second step modelled as a linear stochastic process. It is shown that a univariate ARMA model of this process with fixed parameters is capable of making the prediction with a superior accuracy compared to the predictors used today.

The final adjustment of the model is an adaptation in time of the parameters in the residual model to the measured water levels [2], [3]. Since it is observed that the character of the water level process changes at high tides, this means that the high tide prediction is done with a model that is continuously adapted to the high tide process. The method leads to a further increase of prediction accuracy.

Summarizing, the results shows that a good high tide predictor may consist of two parts, a deterministic nominal part working on metereological information and an adaptive stochastic part modelling the residual process thus obtained.

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OPTIMAL SMOOTHINGS

Methods to analyze time series most frequently involve a presumed smoothing of the series. However such idea of smoothing is defined intuitively rather than formally. Here on the contrary, the smoothing problem is formulated as a problem of best approximation, which, resorting to certain smoothness functionals, takes the form of a convex program in \mathbb{R}^n .

The structure of the program and the properties of its solution are discussed, emphasizing that a stochastic interpretation of the effect of the ensuing filter is possible.

Numerical results, based on a finite convergence method (which is described in a companion paper) are also included.

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IDENTIFIABILITY IN LINEAR SYSTEM WITH ARBITRARY SAMPLING

The problem of identifying the parameters in linear and time-invariant dynamic systems is reduced to solve an algebraic system of equations by linearizing the parameter vector around its nominal value which is assumed to be known. This system of equations is formed by a set of output samples and in this way, the problem results similar to one of observability. The general conditions that the sampling instants must fulfill are not very restrictive and can be found in [1].

In particular, the system

 $\dot{x}(t) = Ax(t) + b u(t), x(0)=d, t \in (0, t_f]$ y(t) = C x(t)

is considered parameterized by a parameter vector p. The trace of the local information matrix as well as the condition number of the coefficient matrix in the linearized identification problem are used as cost functions in order to determine the convenient sampling instants distribution for minimization of the transmission of absolute and relative errors respectively towards the results.

The general methodology consists in solving a first-order Taylor series expansion for the estimates of the local variations of the parameter vector around its nominal value and to repeat the process in an iterative way by deleting the contributions of the higher powers of these estimates.

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-76-

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> THE SMOOTHING PROBLEM - A STATE SPACE-RECURSIVE COMPUTATIONAL APPROACH: APPLICATIONS TO THE SMOOTHING OF SEASONAL ECONOMIC TIME SERIES

Let $y(n) = f(n) + \varepsilon(n), n = 1, ..., N$ with the $\varepsilon(n)$ i.i.d. from $\varepsilon \sim N(0, \sigma^2)$, σ^2 unknown and $f(\cdot)$ an unknown "smooth function". The problem is to estimate f(n), n = 1, ..., N in some way that makes sense statistically.

E. T. Whittaker, 1919, suggested that the solution have the property that it be a tradeoff between fidelity to the data and fidelity to a difference equation constraint. Craven and Wahba, 1979, is an $O(N^3)$ solution to the problem, Akaike, 1979, is an $O(N^2)$ solution to the problem. We show an O(N) solution to this problem, generalize it to a solution of the seasonal adjustment of economic time series, and show the relationship of the smoothing problem to some of the Tikhonov type ill-posed problems.

We imbed alternative candidate 'smooth' models into dynamic state space forms and invoke the recursive computational Kalman filter/ smoother procedures to achieve the O(N) computation. Another facet of our approach is that we employ a statistical decision procedure, Akaike's AIC criterion, to determine the best of alternative candidate models fitted to the data. Examples are shown.

-77-

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ON THE OBSERVABILITY OF NONLINEAR SYSTEMS AND THE FISHER INFORMATION MATRIX

In this paper, we establish a relationship between the observability and the Fisher information matrix of the nonlinear system given by

$$\dot{\mathbf{x}}(t) = f(\mathbf{x}(t), t) \tag{1}$$

y(t) = h(x(t), t) (2)

$$z(t) = y(t) + n(t)$$
 (3)

where $\underline{n}(t)$ is a white Gaussian noise process. The observation process $\underline{z}(t)$ can either be continuous or discrete in time. Major results are summarized below.

Let F(t) and H(t) be the Jacobian matrices associated with f and h with $\underline{x}(0) = \underline{x}_0$, respectively, $f(t,\tau)$ the transition matrix of F(t), and $J(x_0)$ the Fisher information matrix of the nonlinear system given by (1) and (3) with $x(0) = x_0$. We first prove the following two theorems.

Theorem 1 (Point-wise observability condition)

The nonlinear system defined by (1) and (2) are observable at x_0 iff $J(x_0)$ is positive definite.

Theorem 2 (Local observability condition)

If $J(\underline{x}_0) > 0$ and $J(\cdot)$ satisfies the Lipschitz condition at \underline{x}_0 , then there exists a sphere, W, centered at \underline{x}_0 such that $J(\underline{x}) > 0$ for all $\underline{x} \in W$.

We then relate the observability condition of System (1) and (2) to the existence condition of an unbiased estimator of \underline{x}_0 given $\underline{z}(t)$,

 $0 \le t \le T$. Finally, we establish a global observability condition for System (1), and (2) in terms of the existence condition of an observable linear system which bounds (1) and (2) in some sense.

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MODELS FOR DYNAMIC SYSTEMS WITH SELF-SUSTAINED OSCILLATIONS

Modeling and identification of dynamic systems consists of two main steps:

- 1) choice of the form of the model to which experimental data is to be matched, and
- 2) computation of model parameters to obtain best fit (in some sense).

If the system is linear (or if it could be modeled accurately by a linear model), one chooses a difference equation of arbitrary order and tries the best choice of parameter values. An improvement in the match between model and system response is then achieved by increasing the order of the model equation.

There are however, many systems that cannot be modeled accurately by linear models because they exhibit distinctly nonlinear response characteristics. One case of such systems are systems with self-sustained periodic oscillations of fixed frequency and fixed amplitude (i.e., limit cycles). In this paper we investigate the choice of proper models the response of which would exhibit limit cycles. Specifically, if

 $y(k+n) = f(y(k), y(k+1), \dots, y(k+n-1), u(k),$

u(k+1)...u(k+n-1))

is a suitable model, where y(k) and u(k) are input and output sequences, respectively, then one wants it to have an unforced solution (i.e., with $u(k)\equiv 0$) with the following properties:

- 1) y(k;y(o)) = y(k+p;y(o))
- 2) $\lim_{k \to \infty} ||y(k; y(o) + \xi)| y(k; y(o))|| = 0$

In this paper we tabulate low order nonlinear equations that would exhibit such nonlinear limit-cycle oscillations. Approximate values of the period for limit cycle oscillations are then obtained by means of harmonic balance applied to discrete models.

-79-

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CONSTRUCTING EXPERIMENTAL DESIGNS USING THE ACCELERATED GREEDY ALGORITHM

Consider performing a linear regression over a specified sampling region using a given set of basis functions. An optimal experimental design in such a context consists of the set of sampling locations, and the number of replicated measurements to make at each location, so that some functional of the error covariance matrix, (such as its determinant), is minimized for a fixed, total number of measurements [1]. For several years the efficient computation of such designs has been of interest [2,3].

This paper examines the idea of using the accelerated greedy algorithm of M. Minoux [4] for this purpose. This particular algorithm reduces the number of "function evaluations" by maintaining an ordered record of previous evaluations. For regression models there is indeed a significant decrease in the number of function evaluations. However this saving is mitigated by the cost of maintaining the ordered record.

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A Robustized Maximum Entropy Approach To System Identification

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The advent of modern high-speed sampling techniques resulted in additional stress on the problem of signal processing in a dependent corrupting noise. Since in many systems of practical interest the knowledge of the noise statistics is either inexact or unspecified, some adaptive techniques which are frequently ineffective or too complicated to implement were suggested in the past. However, the methodology of system identification lends itself to provide a prewhitening filter. In this paper, a robust identification of the autoregressive (AR) model is proposed. The robustizing process is in the form of robustized Robbins-Monro stochastic approximation (RMSA) algorithm and is based on the m-interval polynomial approximation (MIPA) method introduced by the authors in a previous paper involving a process of robustizing the Kalman filter. The resulting algorithm represents a recursive robustized version of the well-known maximum entropy method (MEM) for spectral

- 81-

estimation introduced by Burg [1] or of the popular Widrow least-mean-square (LMS) adaptive filter [2] adopted everywhere in engineering. Furthermore, the robustized algorithm leads naturally to a robustized Akaike's information criterion (AIC) [3] to determine the order of the autoregression. The simplicity of implementation and flexibility make the application of identification algorithms based on the MIPA concept particularly attractive in practice. The flexibility and robustness of the procedures are confirmed by Monte Carlo simulations.

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- 82-

CONTROL APPLICATIONS

-83-

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THE CONTROL OF AUTOMOBILE ENGINES, FUEL/EMISSION TRADEOFFS

The paper presents a procedure for obtaining control calibrations for a warmed-up, spark-ignited automobile engine. The control problem is to find the air-to-fuel ratio, spark advance, and exhaust gas recirculation (EGR) as a function of operating point, that minimizes fuel consumption subject to emission and drivability constraints. The emission constraints must be satisfied over the Federal Test Procedure which consists of travel over seven miles of simulated urban and highway driving. The drivability constraints, however, must be satisfied over any reasonable driving cycle.

The procedure utilizes static models of fuel consumption and emissions. These models are derived using regression models. Nonlinear programming techniques are used to obtain the optimal controls. The procedure also comes up with tradeoffs between fuel consumption and emissions. These sensitivity relations exhibit the cost of tightening emission constraints and show the minimum emission levels that can be produced. Arjun Godhwani Associate Professor of Electrical Engineering Southern Illinois University at Edwardsville Edwardsville, Illinois 62026

OPTIMAL TWO-MODE CONTROLLERS FOR COUPLED SYSTEMS

A method for determining optimal controller parameters for coupled systems is presented. A strongly-coupled typical system commonly found in petroleum refineries is investigated. Feedback scheme considered has a fixed configuration consisting of proportional plus integral (PI) controllers. Three different performance criteria ISE, ITSE, and ITAE are considered. Optimization is carried out using calculus of variations approach. The resulting nonlinear two-point boundary-value problem is solved using quasilinearization. Four different input combinations are considered and a set of optimal controllers is determined for each of the input combinations. Then the strategies for the selection of optimal controllers may be based on

min $E{J}$

or min max J

where: E is the expectation operator and

J is the performance measure.

A comparison based on the time response curves indicates the superiority of ITAE over ITSE and ISE. The best optimal control is markedly superior than the trial and error design. A validity check on the optimization technique of this investigation is made and the approach is demonstrated to be valid.

-85-

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MODELLING SHIP CONTROL SYSTEMS

The Cardiff Ship Simulator has been under development by the National Maritime Institute for several years, and is considered to be one of the most advanced simulators of its kind in the world.

The simulator activities are controlled by two digital computers, one producing the imagery on the screens (windows of the bridge) and the other providing the ship response to rudder and engine commands. This paper is concerned with the latter type of process.

Three coupled ordinary differential equations are used to model the behaviour of a ship. The system output consists of sway, surge and yaw velocities, which are used to calculate the position and orientation of the ship. These results are then used by the computer to generate the corresponding imagery on the screens. The paper will discuss the differential equations and their solution, together with various applications, including the provision of off-line micro-computer facilities as an aid to using the actual simulator. Y. Kuroda, A. Makino and T. Ishibashi Department of Nuclear Engineering, Tokai University, Tokyo, Japan

The model concerned is given by

A Problem of Bilinear Control in Nonlinear Coupled Distributed Parameter Reactor Systems

The aim of this paper is to formulate a bilinear control of the space-dependent effect of the Xenon build up on the neutron flux distribution in nuclear reactor systems without any linearization.

 $\frac{1}{v_{1}} \frac{\partial^{Y}_{1}}{\partial t} = \sum_{i,j=0}^{n} \frac{\partial}{\partial x_{i}} D_{1}(x) \frac{\partial^{Y}_{1}}{\partial x_{j}} - \sum_{a,j=1}^{n} Y_{1} - \sum_{r} Y_{1} - X d_{1}^{X} Y_{1} + [v_{1} \sum_{f,j=1}^{n} Y_{1} + v_{2} \sum_{f,2}^{n} Y_{2}]$ $\frac{1}{v_{2}} \frac{\partial^{Y}_{2}}{\partial t} = \sum_{i,j=0}^{n} \frac{\partial}{\partial x_{i}} D_{2}(x) \frac{\partial^{Y}_{2}}{\partial x_{j}} - \sum_{a,2}^{n} Y_{2} + \sum_{r} Y_{1} - X d_{2}^{X} Y_{2} + \sum_{c} (x,t) Y_{2}$ $\frac{\partial I}{\partial t} = \lambda_{I} I + y_{I} [\sum_{f,j=1}^{n} Y_{1} + \sum_{f,2}^{n} Y_{2}]$ $\frac{\partial X}{\partial t} = -\lambda_{X} X + \lambda_{I} I + y_{X} [\sum_{f,1}^{n} Y_{1} + \sum_{f,2}^{n} Y_{2}] - X [d_{1}^{X} Y_{1} + d_{2}^{X} Y_{2}]$ $Y_{1}(x,t) = Y_{2}(x,t) = I(x,t) = X(x,t) = 0 \qquad (B.C.)$ $Y_{1}(x,0) = Y_{10}, Y_{2}(x,0) = Y_{20}, I(x,0) = I_{0}, X(x,0) = X_{0} \qquad (I.C.)$

where Y_1 , Y_2 , I and X indicate the fast neutron flux, slow neutron flux, Iodine distribution and Xenon distribution, respectively; observation is made by Y_2 , and control by the macroscopic absorption cross-section of the control rods $\Sigma_c(x,t)$.

On this scheme, the regulator problem is considered under the second order cost function. The existence of an optimal bilinear control is admitted by proving the continuity of the mapping from the control to the state variable. Furthermore, using the implicit function theorem the differentiability of the state variables with respect to the control is proved. Hence, the adjoint system is introduced, from where the optimality condition is derived. Finally, results obtained are compared with those from the linearized case.

. 87-

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SINGULAR PERTURBATIONS IN OPTIMAL STABILIZATION PROBLEM OF LEGGED VEHICLES

Because of varying constraints (changing supporting legs) legged vehicle (LV) as control object in different phases of its motion is described either by differential equations or by difference relations reflecting jump-like change of generalized coordinats and velocities in the moment of releasing former and superimposing subsequent couplings. The problem of stabilization system synthesis of such objects is complicated enough and may be solved effectively only in linear approximation. Assume legs inertia to be small, then it permits us to introduce a small parameter in such a manner that when the small parameter vanishes, the control algorithm of the object is transformed into the control strategy of the model with weightless legs. The problem of stabilization system synthesis of LV with weighty legs in such a statement is reduced to the design of solution of periodic singularly perturbed linear quadratic task.

The case when LV is described only by finite-difference relations is investigated in a similar way. The problem of optimization of periodically perturbed system of finite-difference equations appears in this case. Peculiarities of asymptotic decomposition which can be considered as an analog to the boundary layer in the examined discrete singularly perturbed problem are marked. Ching-Fang Lin Applied Dynamics International Ann Arbor, MI 48104

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OPTIMUM MANEUVERS OF A SUPERCRUISER

In the analysis of optimum maneuvers of supersonic aircraft, a mathematical modeling of the aerodynamic and engine characteristics is designed in the form of a light-weight fighter called the supercruiser by which dimensionless equations of motion for threedimensional flight over a flat earth are derived [1]. By using the maximum principle, all problems can be investigated under a unified treatment.

The general properties of optimal trajectories are presented which include the integrals of motion and the characteristic features in engine and aerodynamic controls. The use of the domain of maneuverability provides insight into the structure of the control. By the use of the switching theory the optimum switching sequence is established for the purpose of selecting among a complex combination of different subarcs the optimum one which is a function of the terminal conditions.

In each problem, the optimality of singular thrust control and the optimal junction of different subarcs are discussed, and the typical behaviors of the state variables and the control variables are displayed. In some problems in which more than one parameter is involved, the use of the rotation of coordinates reduces the number of parameters and hence expedites the attainment of the solutions. By applying the backward integration technique in which the exact values of the adjoint variables are known, and performing the iteration of the final values of the corresponding state variables, the common sensitive problem in the computation of the optimal trajectories is relieved.

Because of the normalizing of the control variables, i.e., the thrust-to-weight ratio, bank angle, and load factor, the results can be applied to any supersonic aircraft. The proposed method of computing the optimal trajectory is very efficient and makes explicit the selection of the optimal control. The technique should be useful for performance assessment of supersonic aircraft with potential for implementation of onboard flight control system.

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APPLICATION OF CONSTRAINED CONSTANT OPTIMAL OUTPUT FEEDBACK TO MODERN FLIGHT CONTROL SYNTHESIS

This paper describes an applications oriented approach to the generation of optimal output feedback gains for linear time-invariant systems which is independent of the loop stability.

Specifically we consider the following problem. Given the stabilizeable system:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}, \mathbf{x}(t=0) = \mathbf{x}_0$$

y = Cx

Where $x \in R_n$, $u \in R_m$, $y \in R_r$ and A, B, C are real constant matrices of compatible order.

And where C is of full rank.

Find a control law of the form:

$$u = -Fy = -FCx$$

Where F is an m x r constant matrix of predetermined structure (i.e. some of the f_{ij} 's satisfy some internal linear constraints) that minimizes the cost function

$$J(F) = E\left[\int_{0}^{\infty} (x'Qx + u'Ru) dt\right]$$

where $Q \ge 0$, $R \ge 0$ and $E[\cdot]$ is the expectation operator.

This optimal output feedback problem is cast in the setting of a constrained parameter optimization problem. The solution of this constrained optimization employs Hestenes' method of multipliers with some modification. A primal-dual problem is considered where the primal minimization employs a Davidon-Fletcher-Powell method, and the dual maximization is accomplished via a quasi-Newton procedure. A. S. C. Sinha Purdue University School of Engineering IUPUI, Indianapolis, TN 46205

"ADVANCED CONTROL LAWS FOR EXPERIMENTS IN FAST ROLLOUT AND TURNOFF OF THE B737-100 AIRCRAFT"

Let $X(t_0) = X_0$ be the initial condition and let C^0 be the path of fast rollout and turnoff. The problem is to minimize the functional defined on space **G** by the penalty function

$$J = \int_{0}^{T} < P[X(t,u,X_{0}) - C^{0}], [X(t,u,X_{0}) - C^{0}] > dt + \int_{0}^{T} < Nu(t), u(t) > dt.$$

Here $X(t,u,X_0)$ is the solution of the dynamic equation representing the fast rollout and turnoff condition of B737-100 aircraft given as

$$X(t) = \Lambda X + \zeta u + \Gamma X u + \gamma$$

where $u = \gamma_3$ is a scalar control input. Application of quasilineraization techniques as described by Leondes and Paine [1] has been used to show that the algorithm converges uniformly to the optimum solution.

THEOREM 1. There exists a unique control w^{K} given by

$$w^{K+1} = [N]^{-1} \{ < [I + f_w(w^K, X^K)], \psi^{K+1} > \}$$

such that

.

 $w^{K+1} \leq w^{K}$

and

$$\psi^{K+1} \leq \psi^{K}$$

where $w = \zeta u + \gamma$, $f_w(w, X) = \Gamma X (\zeta^T \zeta)^{-1} \zeta^T w$, and ψ^K is the solution of adjoint-system equations.

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-91-

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A MATHEMATICAL REPRESENTATION OF OSCILLATOR STABILITIES

FOR

MOVING SATELLITE SYSTEM APPLICATION

A one-way or two-way doppler system used for moving satellite tracking requires very high range-rate tracking precision. (In the order of .06 cm/sec for two-way doppler system range-error). If the oscillators used in the system have high short-term frequency instability values, they could contribute enough range-rate errors to exceed the total specified error budget. This paper derives a mathematical model to represent the oscillator frequency stability as a function of specified satellite range, range-rate error, and doppler counting time. For any specified range-rate error, the required oscillator frequency stability can be determined from this mathematical model.

For the two-way doppler system used in the Space Ground Link Subsystem (SGLS) the range-rate equation can be derived as

$$\dot{r} = c \left\{ 1 - \frac{1}{16} \left[(256.25 n_2 - 205 n_1)/n_2^{\frac{1}{2}} \right] \right\}$$
 (1)

where c is the speed of light, n_1 and n_2 are numbers of counts in counters N_1 and N_2 respectively.

The time domain frequency stability equation can be found as $\begin{bmatrix} 1 \end{bmatrix}$

$$\left(\frac{\Delta f}{f}\right)^2 = \frac{1}{\omega_0^2} \left| \sigma_A^2 + \sigma_B^2 + \sigma_C^2 + \sigma_D^2 \right|$$
(2)

where ω_0 is the nominal system frequency, σ_A , σ_B , σ_C and σ_D are component errors due to various types of noises in the oscillator. These components are all functions of range-rate error since they all contribute to the range-rate measurement error in a real system.

A complete set of equations has been derived and computer-generated curves are plotted and presented in the paper.

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OPTIMAL CONTROL THEORY PARTIAL DIFFERENTIAL EQUATIONS

-93-

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FINITE DIMENSIONAL CONTROL OF LINEAR PARABOLIC

DISTRIBUTED PARAMETER SYSTEMS

We consider feedback control of the linear parabolic distributed parameter system $\partial u/\partial t + Au = F$. This is a generalized heat equation where u(x,t) represents the temperature distribution of a body Ω in n-dimensional space, and F(x,t) represents the applied heat sources. The operator A is a time-invariant differential operator whose domain d(A) is dense in the Hilbert space $L^2(\Omega)$. This distributed parameter system is <u>parabolic</u> in the sense that A generates a holomorphic (or analytic) semigroup.

Control is provided by M heat sources located in or on the body Ω and the temperature is measured by P sensors located at various points along the body Ω . The locations of this finite number of sensors and actuators is part of the control problem.

Since feedback control will be implemented by an on-line digital computer, it is important to restrict the controller to have finite-dimension. Our approach is to use the Galerkin method on an appropriate finite-dimensional subspace $H_N \subseteq D(A)$ to produce a reduced-order model of the distributed parameter system; from such a reduced-order finite-dimensional model the controller is synthesized. Then the stability of the closed-loop system, consisting of the actual distributed system and the finite-dimensional controller, is analyzed. We will show that (unlike the hyperbolic case), for parabolic systems, any degree of exponential stability can be guaranteed under rather mild conditions on the reducing subspace H_N . Consequently, finite-dimensional control of infinite-dimensional parabolic systems can be obtained in most practical cases.

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NUMERICAL SIMULATION OF AN ALLOY SOLIDIFICATION PROBLEM

In this paper, we study from a numerical point of view the solidification of a binary alloy. The formulation, we consider, is the same than in A.B. CROWLEY - J.R. OCKENDON [2], G.J. FIX [3].

In a first part, we introduce the following formulation for the initial problem :

(S)
$$\begin{cases} \frac{\partial u}{\partial t} + A(\Theta, W) = h ; \quad u \in H_{W}(\Theta) \\\\ \frac{\partial v}{\partial t} + B(W, \Theta) = O ; \quad v \in G_{\Theta}(W) \\\\ u(x, o) = u_{O}(x) \in H_{W_{O}}(u_{O}) ; \quad v(x, o) = v_{O}(x) \in G_{\Theta_{O}}(W_{O}) \end{cases}$$

where $H_{W}(\Theta)$, $G_{\Theta}(w)$ are two maximal monotone operators and A (Θ, w) , B (w, Θ) two non linear operators.

In a second part, we introduce a semi-discretized problem, which has the structure of quasi-variational inequalities. Then, by using results of monotonicity, we prove the existence of a minimal and a maximal solution and we deduce a numerical iterative algorithm, the convergence of which is proved. The technique consists to solve a sequence of variational inequalities, the solution of which is obtained by using the following equivalence :

 $z \in H(y) \leq z = H_{\lambda} (y + \lambda z) \quad \forall \lambda > 0.$

(H maximal monotone operator, \mathbf{H}_{λ} associated Yosida approximation).

Numerical results are presented.

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-95**-**

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THE THRESHOLD PROBLEM FOR A FITZHUGH-NAGUMO MULTI-DIMENSIONAL SYSTEM

The following Fitzhugh-Nagumo system is considered:

 $v_{t} = \Delta v + f(v) - u, \quad u_{t} = \sigma v - \gamma u,$ (FN) where $f: R \longrightarrow R$ is a continuous map, such that vf(v) < 0 for $v \in (-\infty, v_0)$, $v \neq v_0$, and σ, γ are positive constants. The functions v = v(x,t) and u = u(x,t) are sought in the space of continuously differentiable functions up to the second order $C^{(2)}(\Omega x(0,\infty), \mathbb{R})$, where Ω designates a bounded domain in Rⁿ, with sufficiently smooth boundary. The threshold problem under investigation can be formulated as follows: Assume (v(x,t),u(x,t)) is a solution of (FN), defined on $\Omega x(0,\infty)$, and such that the following conditions are verified: $v(x,t) < v_0$, in the whole domain of definition, v(x,0) = V(x), u(x,0) = U(x) for $x \in \Omega$, v(x,t) = 0 on the boundary of Ω . Then prove that (v(x,t),u(x,t))tends to zero as t tends to ∞ , uniformly with respect to $x \in \Omega$, and this property is true for at least those solutions with small V(x), U(x). A solution to this problem is presented, using the following approach. First, one constructs a "Liapunov function" for the system (FN), whose derivative is non-positive. This function does not provide enough information in order to obtain the desired behavior for the solution. One can, however, derive some intermediate results regarding the integrability of certain functions, with respect to t, on the positive half-axis. Next step consists in finding a partial differential equations for the unknown function $v^2(x,t)$, with adequate initial and boundary value conditions. Then a representation formula is used for heat equations, which together with the information accumulated by means of "Liapunov function", leads to the conclusion that the threshold problem has a positive answer.

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DIFFERENTIAL INCLUSIONS WITH MULTIVALUED BOUNDARY CONDITIONS

The purpose of the talk is to give existence results for differential inclusions with multivalued boundary conditions of the following form :

(P) $\begin{cases} \frac{dx}{dt} \in -S(x(t)) & t \in [0,T] \\ x(o) = x_0 \\ x(T) \in K \end{cases}$

where S is a continuous set valued mapping from \mathbf{R}^{ℓ} to \mathbf{R}^{ℓ} with convex compact images, K is a compact set and x_0 a given point of \mathbf{R}^{ℓ} . We assume that the data x_0 , S and K satisfy a consistency condition of the following form :

(L) For any $\mathbf{x} \in K$ there exists an absolutely continuous function $\phi \in C([o,T] ; \mathbb{R}^k)$ such that $||\dot{\phi}||_{\infty} \leq M$, $\phi(T) = \mathbf{x} \in K$, $\phi(t) - \phi(\tau) \in -\int_{\tau}^{t} S(\phi(s)) ds \quad \forall t, \tau \in [o,T]$ and satisfying moreover : $D_{+}V(\phi(T)) (\mathbf{x}_{o} - \phi(o)) + ||\mathbf{x}_{o} - \phi(o)|| \leq o$

where V denotes a non negative lower semi-continuous function from K into $[o,+\infty]$.

We associate with problem (P) a discrete implicit problem and we prove the existence of solutions of this problem by using a fixed point theorem due to AUBIN-EKELAND, then we prove the convergence of a subsequence of these approximate solutions to a solution of (P). Daniel GABAY Université Paris VI and INRIA, B.P. 105 78153 LE CHESNAY (France)

MINIMIZATION OF THE DIFFERENCE OF 2 CONVEX FUNCTIONS

DUALITY, ALGORITHMS AND APPLICATIONS

We consider the non-convex problem Min j(v) = f(v)-g(v), where f and

g are lower semi-continuous, convex, proper functions on a Hilbert space V; we assume that j is bounded from below and satisfies some compactness assumption. Such a problem arises in a wide variety of practical applications, including

- the minimization of a concave function over a convex set, a typical problem in management and economics (when increasing returns to seale prevail or in the presence of fixed costs), in integer programming;
- the variational formulation of a free boundary problem arising in a simplified model of the equilibrium of a plasma confined in a Tokamak machine.

After a review of regularization and duality theories adapted to the particular structure of the problem, we propose several decomposition algorithms using the natural splitting of the problem. Convergence results are given and optimal choices of parameters discussed. Some of the practical examples are used to illustrate the methods. B. Gollan Mathematisches Institut Universität Würzburg, Am Hubland 8700 Würzburg, West Germany

ON THE MARGINAL FUNCTION IN NONLINEAR PROGRAMMING

We consider optimization problems of the following form:

 $V(p) = inf + f_0(x) + x \in S(p) + ,$

 $S(p) = \{ x \in C \mid f_i(x) \le p_i, 1 \le i \le n; f_j(x) = p_j, n+1 \le j \le n+m \}$. p is regarded as a parameter describing perturbations of the reference problem given by p = 0. In most cases considered below, C is a closed subset of a real Banach space X.

The topic of our talk is to describe the local behaviour of the marginal function V(p). This is done by giving bounds for its directional Dini- and Hadamard derivatives. If V is locally lower semicontinuous, then also estimates of the generalized directional derivative V and the generalized gradient $\delta V(.)$ are given. All these bounds and estimates are determined mainly by the set of all Lagrange multipliers which belong to some optimal solution of the unperturbed problem.

Assuming that the functions f, are locally Lipschitz or differentiable we report in a first part recent results by R.T. Rockafellar and by the author. The respective results are very similar, however they are obtained via quite different approaches.

In a second part we consider the special case where all functions f_k are more than once continuously differentiable. Then our approach (which is based on separation-properties of convex cones or sets) holds also with the possibly smaller multiplier sets satisfying higher order necessary conditions. These yield sharper estimates of the local approximations of V. In some cases they make it possible to prove differentiability of V.
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SOLUTION OF THE CLASS OF NON-CLASSICAL MATHEMATICAL PROB-LEMS ARISING IN GENERALIZED STATISTICAL DECISION THEORY, QUANTUM MECHANICS AND OPTICAL COMMUNICATIONS

<u>Problem formulation:</u> Let H be a separable, complex Hilbert space, and B(H) the Banach algebra of bounded linear operators in H. Denote by $\emptyset(H)$ the set of non-negative, self-adjoint, unit trace, nuclear operators in H. The positive, normal functional $\Theta:B(H) \rightarrow R$, $\Theta(Y):=Tr(ZY)$, $Z: \emptyset(H)$, $Y \otimes B(H)$ represents a state on B(H). Let (U;F(U)) be a measurable decision space, and denote by S(H;F(U)) the control space of all positive, normalized, operator-valued measures W: $F(U) \rightarrow B(H)$.

Given: (i) classical probability space (X,F,p), (ii) a family of states (Θ), on B(H), $x \in X$, (iii) (U;F(U)), (iv) non-negative, measurable function Q(x,u), $u \in U$, $x \in X$, and (v) the risk associated with W S(H;F(U))

 $J(W) := \iint Q(x,u) \Theta_{x}(W(du)) dp(x)$.

<u>Problem:</u> Find, if any, $\underline{W} \in S(H; F(U))$, such that inf $J(W) = J(\underline{W})$, where the infimum is taken over S(H; F(U)).

Motivation: Let S denote a quantum-mechanical system (e.g. electron, photon, electromagnetic field, etc.) described by a statistical operator $Z\ell \emptyset(E)$. Then S(H;F(U)) is the set of all dynamic observables of S, and the foregoing abstract problem represents the most general statistical decision problem. The first - and only - methods for a solution of the minimum error variance version of the problem have been developed by the writer (the relevant references are listed in (1)). If Q(x,u) represents the loss corresponding to the decision u when the true state is θ , then the solution of the foregoing problem is essential for the solution of the following problems: (i) optimal quantum mechanical estimation, (ii) optimal quantum stochastic filtering and control and (iii) unified statistical decision theory. In this paper we propose a solution to the general version of this problem. REFERENCES

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FINITE ELEMENTS APPROXIMATION OF TIME OPTIMAL BOUNDARY CONTROL PROBLEM FOR PARABOLIC SYSTEMS WITH DIRICHLET BOUNDARY CONDITIONS:

With $A(x, \mathbf{\delta})$ an elliptic operator let y(t, x) be the solution of the following parabolic equation:

(1) $\frac{\partial y}{\partial t}$ (t,x) = -A(x, ∂) y(t,x); x $\in \Omega$; t > 0;

 $y(0) = y^{0}; y \mid_{\Gamma} = u \in L_{\infty}[0T; L_{2}(\Gamma)]$

Consider the following time optimal control problem (P) Find inf T = { T > 0; $u \in U^M$: $|y(u,T) - y_T | \le \delta$ } where the constant δ > 0, and the point $y_T \in L_2(\Omega)$ are given and $U^M = \{u \in L_{\infty}[OT \times \Gamma]; |u(t,x)| \le M; a.e in OT \times \Gamma\}$.

We shall be concerned with a <u>numerical approximation</u> (e.g. via finite elements) of control problem (P) and the convergence of <u>resulting</u> schemes.

The distinctive features of our approach are: (i) that it is based on a semigroup representation of (1) and its approximation. (ii) that the approximation of Dirichlet boundary conditions uses subspaces which <u>are not required</u> to vanish on the boundary.

The following theorem gives the desired convergence result for our control problem.

<u>Theorem</u> Let $u^{\circ}, T^{\circ}(u_{h}^{\circ}, T_{h}^{\circ})$ be the optimal solutions of problem P and its approximation respectively:

For
$$y_0, y_T \in H^{\frac{1}{2}-\varepsilon}(\Omega)$$
 we have:
(i) $|y(u_h^0, T^0) - y_T| \leq \delta + \theta'(h^{\frac{1}{2}-\varepsilon});$
(ii) $T_h^0 \neq T^0$
(iv) $u_h^0 \neq u^0$ in $L_2[OT \times \Gamma].$

- 101-

Dirichlet Boundary Control Problem for Parabolic Equations with Ouadratic cost: Analyticity and Riccati's Feedback Systhesis

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If **Ø** denotes either 1, or else 0, minimize the performance index

 $J(u, y(u)) \equiv |u|_{\Sigma}^{2} + |y|_{Q}^{2} + \boldsymbol{\alpha} |y(T)|_{\Omega}^{2}$

over all $u \in L_2^{-}(\Sigma)$, subject to the dynamics (D):

$$\begin{aligned} \chi_{t}(t,\xi) &= -A(\xi,\partial) \gamma(t,\xi) \quad (0,T] \ge \Omega \\ \gamma(0,\xi) &= \gamma_{O}(\xi) \qquad \xi \in \Omega \\ \gamma(t,0) &= u(t,0) \qquad (0,T] \ge \Gamma \equiv \Sigma \end{aligned}$$
where $A(\xi,\partial) \equiv \Sigma$ and $(\xi) \ \partial^{d}$ is a uniformly strongly elliptic strongly

where $A(\xi, \partial) = \Sigma$ $a_{a}(\xi) \partial^{\alpha}$ is a uniformly strongly ellip- $|a| \leq 2$

tic operator of order two in Ω . Feedback synthesis (point wise in time) of Riccati type of the optimal solution u°, y° is established in the form

$$u^{O}(t) = -D^{*}A^{*}P(t)y^{O}(t) \quad 0 \le t < T$$

through a semigroup approach. Here A is the differential operator $-A(\xi, \partial)$ with zero Dirichlet boundary conditions, D is the Dirichlet map, and P(t) is the Riccati operator. Moreover, in contrast with the <u>indirect</u> approach of much of the literature - which relies on a Riccati equation to establish existence and computability of the operator P(t) the present approach is instead <u>direct</u>; i.e. the operator P(t) is first defined by an explicit formula in terms of the system data, and only subsequently shown to satisfy, in an appropriate sense, a Riccati type operator equation.

+ All norms are L₂-norms over the indicated domains

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> RATE OF CONVERGENCE ESTIMATES FOR APPROXIMATIONS TO CONVEX OPTIMAL CONTROL PROBLEMS SUBJECT TO CONTROL CONSTRAINTS

An abstract convex optimal control problem subject to control constraints is approximated by a family of finite dimensional problems depending on a parameter of discretization h destinated to tend to zero.

Conditions of optimality involving projections on admissible sets for both initial and approximating problems are used to obtain the estimates of the distance between appropriate optimal solutions.

It turns out that in this estimate an a priori knowledge of the regularity of solution to the initial control problem plays a crucial role, and used scheme of approximation should be correlated with this regularity.

The abstract results are illustrated by examples of Crank-Nicholson type approximations to quadratic optimal control problems for parabolic equations with boundary or distributed control and different types of observations /distributed, final or toundary observations/.

- 103-

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VARIATIONAL INCOUNLIFY FOLIULATION AND OFTUTAL CONTROL OF NONLINDAR EVOLUTION SYSTEMS GOVERNED BY PARABOLIC FREE BOUNDARY PROBLEMS

Nonlinear heat conduction and diffusion processes involving phase transition phenomena (e.g., solidification, melting, ablation, combustion) give rise to various evolution free boundary problems (FBP) as mathematical models. Our aim is to present new results concerning a broad class of these problems, namely multidimensional multi-phase generalized Stefan problems, involving additional nonlinearities in differential equations and in conditions of mixed type imposed at lateral (fixed) boundaries.

A formulation of the problem in the form of the following nonlinear evolution variational inequality will be derived : Determine $m \in L^{\infty}(0, \mathbb{R}, \mathbb{K})$ satisfying

 $(VI) \begin{cases} y'(t) = D_t y(t) \in K(t) \text{ a.e. in } (0,T) ; y(0) = 0 \text{ in } \Omega; \\ (IB(y'))(t), z-y'(t)) + a(y(t), z-y'(t)) - (h_0, z-y'(t)) - (h_0,$

where $\Omega \subset \mathbb{R}^n$, $n \ge 1$, $\partial \Omega = \Gamma' \cup \Gamma''$, $\Gamma' \cap \Gamma'' = \emptyset$; $V = H^1(\Omega)$, V' - dual to V, $\langle \cdot, \cdot \rangle - duality pairing of V', V$; $K(t) \doteq \{v : v \in V, v = f_1(t) \text{ on } \Gamma'\}$, $(u, v) \triangleq \int_{\Omega} u(x) v(x) dx$, $B(\cdot) - a$ bounded operator from $L^2(0,T;V)$ into $L^2(0,T;V')$, a(u,v) - a bilinear form continuous on V, $v \mapsto \Psi(v) - a$ proper convex weakly l.s.c. functional on V, $h_0 \in L^2(\Omega) - given$ function. (VI) corresponds to FBP with the boundary conditions $\Theta = f_1(x,t) \text{ at } \Gamma' \cdot (0,T)$; $D_v \Theta + p(x) \Theta = f_2(x,t) + q(x,t,\Theta) \text{ at } \Gamma' \cdot (0,T)$ where $y(x,t) = \int_{\Omega}^{t} \Theta(x,s) ds$. In the physical situations Θ corresponds to temperature, whereas y to the freezing index.

For the systems governed by (VI) some optimal control problems involving boundary controls (either f_1 or f_2) will be considered. Questions concerning regularity of mappings from controls into state, and the structure of the sets of all the states attainable will be discussed. The existence of optimal controls will be shown .

- 104-

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OPTIMIZATION IN BANACH SPACES OF SYSTEMS INVOLVING CONVEX PROCESSES

Let X and U be two Banach spaces, we consider a closed convex process A from X into U (i.e a multivalued mapping $A: X \rightarrow 2^{U}$ the graph of which is a closed convex cone with apex zero). Let C and T (the target subset) be two nonempty closed convex subsets of X and U respectively, and f a proper, lower semicontinuous function from X into \overline{R} (the extended real line). We are concerned with the convex system : minimize f(x) subject to $x \in C$ and $Ax \cap T \neq \emptyset$.

This kind of problem which is a generalization of the von Neumann model occurs under various forms in economics and control.

For a convex process A, the conjugate process A^* from V into 2^Y is defined by $A^*v = \{y/(v, -y) \in (gr A)^\circ\}$ where V and Y are the topological duals of X and U respectively, while $(gr A)^\circ$ is the polar set of the graph of A. We give conditions implying that inf{ <u,v >/u $\in Ax$ } = sup{<y,x >/y $\in A^*v$ } or in other words <Ax, $v^> = < x, A^*v^>$.

The aim of the paper is to obtain sufficient conditions which ensure that there exists $v_0 \in V$ such that $\inf \{f(x)/x \in C, Ax \cap T \neq \emptyset\} = \inf \{f(x) + \langle Ax-t, v_0 \rangle/(x,t) \in C \times T \} =$ $\inf \{f(x)/(x,t) \in C \times T \text{ and } \exists z \in Ax \text{ such that } \langle z - t, v_0 \rangle = 0 \}.$ Using some results in convex optimization, we focus our attention on relaxing the usual compactness assumptions.

Our results in particular subsume most of the previously known results on linear systems.

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ON THE SEMI GROUP APPROACH FOR ERGODIC PROBLEM OF OPTIMAL STOPPING

ABSTRACT

This paper is devoted to the study of the following problem : to find a maximum solution of

(1)
$$\begin{cases} u \leq \phi (t) u + \int_{0}^{t} \phi (s) f ds, \forall t \geq 0, \\ u \leq g \end{cases}$$

where ϕ (t) is a Feller semi-group, in C, Banach space of continous functions on a compact metric space E, and where g and f \in C are given.

Previous studies, by Bensoussan - Lions, Lasry, especially for diffusion processes are contained in references listed in [1].

In this paper, problem (1) and the continuity of the maximum solution is investigated under additional assumptions on ergodic properties of ϕ (t). The method is completly analytic. In some case, the convergence of the discounted problem to the undiscounted one is obtained. The convergence of the discrete time analog of (1) is also considered.

Moreover, the finite horizon problem corresponding to (1) define a non-linear semi-group S(t) (according to [2]- (ii)). We study the limit of S(t) u when $t \rightarrow -$ and the characterization of the maximum solution of (1) as the unique solution of S(t) u = u.

Finally, some examples are given and some open questions related to these problems are analysed.

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AN ALGORITHM FOR THE DECOMPOSITION OF SECOND ORDER ELLIPTIC OPERATORS*

The analysis of lumped and distributed systems whose structure can be represented by a dependence on a small parameter ε has received much attention (c.f. [1] for systems described by ordinary differential, [2] for distributed systems, [3] for eigenvalue problems, and the references therein).

In this paper, a second order elliptic operator with a special structure is considered. It is assumed that its coefficients are of different order of magnitude in various parts of a regular open subset $\Omega \subset \mathbb{R}^n$. First an exact asymptotic expansion of the eigenvalues and eigenvectors is obtained and an asymptotic error estimate is derived. Second a discrete approximation of the aforementioned operator is outlined and the corresponding eigenvalue problem is solved. The results are compared with the exact expansion obtained earlier. Third a grouping algorithm [4] used in power systems analysis is described. Then it is applied to the discrete approximation of Ω for which the representation outlined in the first part holds. Finally, a numerical example of one dimensional heat transfer in a rod is considered to illustrate the basic concepts of this paper.

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-107-

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CALCULUS OF THE CORRECTOR FOR THE EIGENVALUE OF A TRANSPORT OPERATOR

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Let us consider the following unbounded operator A^E on $L^2(\Omega xV)$, with zero boundary conditions (Ω is an open set of ${\rm I\!R}^N$ and V a compact symmetric set of ${\rm I\!R}^N)$:

$$\mathbf{f} = \mathbf{f}(\mathbf{x}, \mathbf{v}) \rightarrow \mathbf{A}^{\varepsilon} \mathbf{f}(\mathbf{x}, \mathbf{v}) = -\varepsilon^{-1} \sum_{i} \mathbf{v}_{i} \partial \mathbf{f} / \partial \mathbf{x}_{i} + \varepsilon^{-2} Q_{\mathbf{x}} \mathbf{f}$$

where $Q_{\mathbf{x}}$ is the Markovian generator (i.e. $Q_{\mathbf{x}} \mathbf{1}(\mathbf{v}) = \mathbf{0} \ \mathbf{v}\mathbf{v}$) defined by

$$g = g(v) \rightarrow Q_{x}g(v) = \int \sigma_{1}(x, v, w)g(w)dw - \sigma(x, v)g(v)$$

With the right assumptions on σ_1 we prove that there exists an unbounded symmetric $H_0^1(\Omega)$ - coercive operator A such that, when ε goes to O, we have (with π a projection from $L^2(\Omega xV)$ onto $L^2(\Omega)$) :

 $e^{A^{\epsilon}t}f \rightarrow e^{At}\pi f$ in $L^{2}(\Omega x V)$ $\forall t > 0, \forall f \in L^{2}(\Omega x V)$

If ω_{ε} and ω denote the type of the semi-group $(e^{A^{\varepsilon}t})$ and (e^{At}) , we show that they are also eigenvalue of A^{ε} and A. Then we show that, when ε goes to 0, we have $\omega_{\varepsilon} \neq \omega$ and that the eigenfunction γ_{ε} (of A^{ε}), the norm of wich is 1, satisfies $\gamma_{\varepsilon} \neq \gamma$ in $L^{2}(\Omega x V)$ where γ is an eigenfunction of A. At last we calculate the limit of $\frac{1}{\varepsilon}$ ($\omega_{\varepsilon} - \omega$), and we give application.

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APPROXIMATIONS BY MEANS OF OPTIMAL CONTROL

FOR DISTRIBUTED PARAMETER SYSTEMS

In some previous papers[1],[2]we considered a way of a-pproximating solutions for models described by nonlinear equations using optimal control techniques. The problem here is to apply the method to nonlinear integro-differential mo-del of one dimensional Vlasov plasma

$$\frac{1}{2t} = v \frac{1}{2t} = x \frac{2f}{2v} + \frac{2f}{2v} \left(\frac{1}{4v} \left(\frac{1}{4v} \left(\frac{1}{2v} \left(\frac{1}{2v} \right) + \frac{1}{2v} \right) \right) \right) \left(\frac{1}{2v} \right)$$

$$\frac{1}{4t} = \frac{1}{2v} \frac{1}{2v} \frac{1}{2v} \left(\frac{1}{2v} \left(\frac{1}{2v} \left(\frac{1}{2v} \left(\frac{1}{2v} \right) + \frac{1}{2v} \left(\frac{1}{2v} \right) \right) \right)$$
(2)

where f means the particle distribution. In fact, the problem (1)-(2) reduces to the

Optimal control problem OP1. If f is the solution for

the state equation $\frac{\partial f}{\partial t} = -v \frac{\partial f}{\partial t} + 2 \frac{\partial f}{\partial v} r \frac{1}{2}, \quad f_0 = \sqrt{\frac{1}{2\pi}} e^{-v^2/2}$ find $f \in L^2(\Omega), \quad \Omega = [0, L] \times [0, V] \times (-\infty, -\infty)$ such that the functional $J(\frac{f}{2}) = \left\| \frac{\partial f}{\partial v} \int_{0}^{t} dv \int_{0}^{t} (v \cdot t) dv - \chi \left(\frac{V}{\sqrt{2\pi}} e^{v^2/2} + \frac{\partial f}{\partial v} \right) r \frac{1}{2} \right\|_{L^2(\Omega)}^{2}$

gets minimized.

A different way of reducing (1)-(2) to an optimal control problem leds to

problem leds to <u>Optimal control problem OP2.</u> If \overline{f} is the solution for the state equation $\frac{\partial \overline{f}}{\partial t} + v \frac{\partial \overline{f}}{\partial t} - v \frac{\partial \overline{f}}{\partial t} = \overline{T(f)}$, $\overline{f}|_{e^{-e^{-\frac{\partial}{\partial t}}} = \sqrt{e^{-\frac{\partial}{\partial t}}} e^{-\frac{\partial}{\partial t}} (1 + \alpha \cos k_{\star})$ where $T(f) = -\frac{\partial \overline{f}}{\partial t} \int d^{\frac{\partial}{\partial t}} \int f(\overline{v}, t) dv^{-\frac{\partial}{\partial t}}$, find feU, U-an appropri-ate Hilbert space, such that the functional

$$\mathbf{J}(\mathbf{r}) = \left\| \begin{array}{c} \partial f & - \partial f \\ \partial t & - \partial t \end{array} + \mathbf{v} \left(\begin{array}{c} f & - \partial f \\ \partial \mathbf{x} & - \partial \mathbf{x} \end{array} \right) - \mathbf{z} \left(\begin{array}{c} \partial f \\ \partial \mathbf{v} & - \partial \mathbf{v} \end{array} \right) \right\|_{U}^{\infty}$$

gets minimized.

In the present paper we show necessary and sufficient conditions for existence of a solution for OP2, numerical results for OP1 and OP2 by a Polak-Ribière version of gradient methods and comparatively discuss the obtained approximations.

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-109-

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SHAPE SENSITIVITY ANALYSIS FOR VARIATIONAL INEQUALITIES

Consider the model variational inequality: $y = y(\Omega) \in K(\Omega) = \{ \varphi \in H_0^1(\Omega) \mid \varphi \ge 0 \}$ $\int_{\Omega} \nabla y \nabla(\varphi - y) dx \ge \int_{\Omega} f(\varphi - y) dx, \quad \forall \varphi \in K(\Omega)$

where $\Omega \in \mathbb{R}^n$ is a domain with regular boundary $\partial \Omega$. Let $\Omega_t = T_t(\Omega)$, $t \ge 0$ be a family of perturbations [1] of the domain Ω . The material derwative $\dot{y}(\Omega)$ is defined in the following way:

 $\dot{y}(\Omega) = \lim_{t \to 0} (y(\Omega_t) \circ T_t - y(\Omega))/t$

We obtain the form of the material derivative:

$$\dot{\mathbf{y}} \in S^{\mathbf{\Omega}} = \left\{ \boldsymbol{\varphi} \in H_0^1(\Omega) \mid \boldsymbol{\varphi} \ge 0 \text{ on } \mathbf{y}^{-1}(0) \text{ and} \\ \int_{\mathbf{\Omega}} (\Delta \mathbf{y} + \mathbf{f}) \boldsymbol{\varphi}^* d\mathbf{x} = 0 \right\}$$

$$\int_{\mathbf{\Omega}} \nabla \dot{\mathbf{y}} \nabla (\boldsymbol{\varphi} - \dot{\mathbf{y}}) d\mathbf{x} \ge \langle \boldsymbol{\mu} (\boldsymbol{\varphi}) - \boldsymbol{\mu} (\dot{\mathbf{y}}), \ \nabla \cdot \mathbf{n} \rangle \quad \forall \boldsymbol{\varphi} \in S^{\mathbf{\Omega}}$$

where $V = \frac{\partial}{\partial t} T_t$ is velocity field and $\mu(.)$ is a distribution on $\partial \Omega$.

Using the form of material derivative we can formulate the necessary conditions of optimality for shape optimization problems for the variational inequality.

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STOCHASTIC CONTROL

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THE OPTIMAL POLICY FOR STOCHASTIC CONTROL PROBLEMS WITH COSTLY SWITCHINGS*

The problem of control of a power system with randomly fluctuating demand can be formulated as the control of the family of diffusion processes

$$dy^{a}(t) = m^{a}(y^{a}(t))dt + \sigma^{a}(y^{a}(t))dw(t), a \widetilde{A},$$

so as to minimize the cost

$$J = E_{x_{i}} \int_{0}^{T} e^{-c^{a}t} f^{a}(y^{a}(t)) dt + 2 e^{-c^{a}t} i_{k}(a_{i-1}, a_{i})$$

where \widetilde{A} is a finite set of regimes of the system f^{a} is the continuous cost, $k(a_{i-1}, a_{i})$ the switching costs, c^{a} discounting factors, τ_{i} the random times of switchings, N the total number of switchings and τ_{i} the exit time of y^{a} from a preassigned set $\mathcal{I}_{i} \mathcal{R}^{n}$. The optimum cost is then a solution of a system of quasi-variational inequalities, and the optimal switching policy can be determined once the optimal cost is known.

Friedman and Lions (1980) have studied the particular case of costless switchings, k(a,b)=0, and have obtained criteria for the selection of the optimal policy. In this paper we study a system with costly switchings. Let $L^{a}=-(generator of y^{a})+c^{a}I$. Two of our results are the following:

If there are constants c and R, with c sufficiently large, such that, for |x|>R, for all ℓ≠a,

 $f^a - c^{\ell}k(\ell,a) > c,$

then for some $R_1 > R$, if |x| > 1, it is optimal to switch to regime a.

2. In the particular case $L^a = L^b$, all $a, b \in \widetilde{A}$, if there are constants c, R, such that, for all |x| > R, for all $l \neq a$,

 $f^{\ell} - f^{a} - c^{a}k(\ell,a) > c,$

then for some $R_1 > R_1$, $|x| > R_1$ implies that it is optimal to switch to regime a.

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> A CAUTIOUS CONTROL ALGORITHM FOR STOCHASTIC CONTROL SYSTEMS WITH ADDITIONAL BOUNDARY CONSTRAINTS.

The concept of the time-optimal control algorithm (T.O.C.), presented in [1], has been extended to stochastic systems with parameter and state uncertainty. The T.O.C. problem for discrete systems with boundary constraints on the control and state variables has been solved by means of linear programming techniques. In-line applications are possible, since linear programming allows an efficient computation of the control sequence. The T.O.C. algorithm belongs to the open-loop feedback class, i.e. an optimal control sequence is recalculated each sampling period, using measurements of the output variables, and parameter and state estimations. In [1] the certainty equivalence property has been used in order to apply the T.O.C. algorithm to stochastic control problems. However, enforcement of this property does not guarantee the desired performance. Especially in the case of time-optimal control one has to be cautious of large values of the control variables, which have been based upon incorrect a priori knowledge of the parameters. The optimal control strategy for stochastic systems yields control decisions which are cautious with regard to the uncertainty in the system and which probe the system for estimation purposes in order to decrease the uncertainty. Mainly because of mathematical difficulties it is hardly possible to derive optimal (dual) control algorithms. Several suboptimal algorithms have lost the probing property and, these are referred to as cautious control algorithms. By making some assumptions a cautious control algorithm, which is quite similar to the T.O.C. will be derived. This algorithm does not have the algorithm, disadvantage of the deterministic design of the T.O.C. algorithm and is cautious with repect to the parameter and state uncertainty of the system. Because of the open-loop form of the algorithm boundary constraints on the input variables can be added. Comparisons between the proposed algorithm, the T.O.C. algorithm, and algorithms given in literature will be presented.

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SIABILITY ANALYSIS FOR LARGE SCALE STOCHSTIC SYSTEMS

Stability properties of stochastic systems with unknown parameters and are subjected to optimum controllers, have been the matter of many studies (1-2). However, the need to reach at least the stable suboptimal controller in a finite number of iteration steps (for on-line systems) is a challanging aim.

In this work, stability analysis (based on Lyapunov's 2nd method) is carried out for the class of stochastic systems that can be identified by linear multi-dimensional regression models, and are subjected to optimum controllers that minimize the conditional mean of a quadratic cost function (Minimum Variance Control).

Additional interest is devoted to the weighting matrices of the quadratic cost function. Computer simulation showes that a particular choice of the initial norms of these matrices can speed-up the converegence to stable region of operation.

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-114-

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RELIABILITY OF STOCHASTIC CONTROL BASED ON DECISION HIERARCHIAL MODEL AND MOVING AVERAGES PROCESS

Most of the existing literature treat the scatter of the observational data, arising from single stage of systems hierarchy ignoring the influence arising from different decision 'hierarchial levels'. This results in an increased amount of scatter. The analysis of 'genesis' and the 'information contents' of these points would show that these data points belong to various 'topological surfaces' associated, with distinct 'decision node'(level). 'Pattern recognition' would clearly indicate the 'true nature' of the 'structuring of the data points'. 'Curve fitting approach' for generalisation of certain processes or systems for establishing of certain 'algorithms' will not give 'reproducible' results.

The other consequence of neglecting the 'decision hierarchies' as an essential features of the system behaviour, would be that the 'data set' acquires a 'moving averages' character. The scatter would loose its meaning, due to changes arising from different hierarchial levels. Set of observations, ignored as extreme outliers, may happen to be a point (event) deciding the fate of the system. Logically ignoring the 'hierarchial effects' make the data to depart from the central limit theorem, and have 'skew-symmetric distribution'. White noise assumptions for deconvolving the data are not valid under these conditions.

Reliability of computer programming vis-a-vis of the algorithm depends on the 'apriory' information about, program graphs, identified set of paths, resolving array reference ambiguities. Pattern recognition system reliability, depends on the upper bound on the probability of misclassification in terms of the affinity. A distribution free upper bound can be derived, for the Bay's probability of misclassification in terms of K. Matusetas measure of affinity of several distributions for the multi-hypothesis pattern recognition problem. For the two-class problem the bound reduces to the H.Hudimoto – T.Kailiath co-efficient in terms of the A. Bhatacharyya co-efficient. It is also possible to derive upper bound which is independent of the priori probability of the pattern classes in terms of Toussaint-Godfried T.Rating and ranking of multiple aspect alternative under fuzzy environments. Multiple alternative decision problem under uncertainty can be handled by computing membership functions of fuzzy sets induced by mapping. The concept of 'perpetual classification' is introduced which defines and redefine strategies under dynamic programming. Ladhe's ideas as modified by Koezy, since some lattice - axioms become deformed into inequalities, while a possibility presents itself to introduce new inverse operation, which can be solved by defining the Rfuzzy algebraic structure (a special case of the general ideas of fuzzy sets given by Goguen). Class stability in perpetual classification under the considerations is presented. Petrinet nets present the best possibility to analyse such situations.

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ON LINEAR-QUADRATIC-GAUSSIAN CONTROL OF SYSTEMS WITH UNCERTAIN STATISTICS

The design of optimum decision and control procedures for a linear stochastic system requires an accurate description of the statistical behavior of the system. However, because of nonideal effects such as nonstationarity, nonlinearity, and other modeling inaccuracies, there is always a degree of uncertainty in such statistical descriptions. A useful approach to design in the presence of small modeling inaccuracies is to use a game-theoretic formulation in which one optimizes worst-case performance, and this approach has been applied successfully to many aspects of decision and control system design. In a recent paper [1], two of the authors have applied this approach in considering the problem of designing linear minimax-mean-square-error state estimators for linear systems observed in and driven by noise processes with uncertain secondorder statistics. In particular, it is shown in [1] that, for two general formulations, such estimators can often be designed by designing the linear minimum-mean-square-error filters for least-favorable pairs of noise spectra or covariance matrices.

In this paper, we consider the analogous problem of minimax linearquadratic control of systems with uncertain second-order statistics. In particular, we consider the control of linear multivariable systems with white process and observation noises of uncertain componentwise correlation and variance. It is shown here that, within mild conditions, this problem can be solved by designing an optimal control for a leastfavorable model, although the model which is least-favorable for control may not be the same as that which is least-favorable for state estimation for the same type of noise uncertainty. However, it is also shown that, for uncertainty in either the process or observation noise only, a given minimax linear-quadratic control problem does have the same leastfavorable model as does a particular minimax state estimation problem with a weighted-mean-square-error criterion. Thus, as might be expected, a limited duality exists between these two problems. Another phenomenon which is shown to be associated with minimax control is that the separation principle which separates the problems of optimal control and optimal state estimation is not necessarily valid for minimax control and minimax state estimation. In particular, it is shown that, although the minimax control law is independent of the minimax state estimator, the reverse is not true. Several other aspects of this problem are also considered in this paper, including a detailed analysis of the specific example of controlling a double-integrator plant with uncertain process noise statistics.

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> A LIAPUNOV-LIKE CRITERION AND A FIRST-PASSAGE TIME PROBLEM IN NON-LINEAR STOCHASTIC SYSTEMS

In this paper, we attempt to develop a Liapunov-like criterion to evaluate an upper bound for an important probability related to the finite time period stochastic stability, i.e.,

 $P\begin{bmatrix} \sup_{t \in [T_1, T_2]} & V(\underline{x}(t)) \ge \delta \end{bmatrix} \text{ for } T_0 \le \forall T_1 \le \forall T_2 , \qquad (*)$

where V(x) is some suitable function related to the generalized energy or the envelope of the solution process generated by the non-linear Ito equation,

 $d\underline{x}(t) = \underline{f}(\underline{x})dt + G(\underline{x})d\underline{B}(t), \quad \underline{x}(T_0) = \underline{x}_0$.

It is well known that a powerful tool in the stability study for nonlinear stochastic system has been established by Bucy and Kushner, that is, the so-called stochastic Liapunov method. The method relies heavily upon the property of the positive supermartingale to the diffusion Markov process.

We present another Liapunov-like criterion in this paper, so that we may evaluate an upper bound of the probability (*) by directly applying the Ito's differential rule to the non-linear function of $logV(\underline{x})$, instead of the supermartingale inequality applied by Bucy and Kushner.

It is shown, in the sequel, that the two conditions for a Liapunovlike function $V(\underline{x})$ guarantee interrelation between the objective probability and the first passage time probability. The evaluation of an upper bound of the probability (*) is reduced to the problem of the first passage time for a scalar Wiener process with a drift that we can exactly evaluate.

PROGRAMMING THEORY

11

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An Algorithm For a Specially Structured LP

An iterative scheme for solving a class of LP s whose coefficient matrix is an edge-node incidence matrix is presented. At each iteration a max flow problem on a bipartite graph is solved. Computational complexity of the algorithm is shown to be of the order $O(Rn^3)$, where R is the largest entry in the r.h.s. vector and n is the number of nodes in the graph. In particular we consider problem on G = [N,E] of the form :

LP [G; \vec{r}] Min Σ c_p u_p

s.t. $u_p + u_q \ge r_{pq} \forall (p,q) \in E$, $u_p \ge 0 \forall p \in N$ Let $\vec{r} = \{r_{pq}\}$ and $\vec{r}(i) = \{r_{pq}(i)\}$, where $r_{pq}(i)$ is the updated requirement at the i-th iteration and R_1 , R_2 be the highest and the next highest numbers in the right hand side vector at the i-th iteration, then $r_i = R_1 - R_2$.

The subproblem to be solved at the i-th iteration is LP $[G_i; r_i e]$ Min $\Sigma c_p u_p$

s.t. $u_p + u_q \ge \lfloor r_i \rfloor$, $(p,q) \in G_i$, $u_p \ge 0 \forall p \in G_i$ where $\lfloor r_i \rfloor$ denotes the largest integer less than r_i , and G_i is the subgraph of G containing all those edges of G, for which the updated requirement is equal to R_1 . Let u(i) denote the solution of $LP[G_i, r_ie]$. Then

 $r_{pq}(i+1) = \max(r_{pq}(i) - u_p(i) - u_q(i), 0) + (p,q)$ Dual of LP[G_i; $r_i e$] can be reduced to a max flow problem on a bipartite graph and can be solved efficiently.

Let k be the number of iterations at which $\vec{r}(k) = 0$, then we have shown that the optimal solution \vec{u} to LP[G; \vec{r}] is given by

 $\vec{u} = \vec{u}(1) + \vec{u}(2) + \dots + \vec{u}(k)$

-119-

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ALGORITHMS FOR SOME SIMPLE CONTINUOUS LINEAR PROGRAMS

In recent years, considerable attention has been paid in the literature to linear programming problems formulated over infinite dimensional spaces. Most of this work has been aimed at understanding the duality structure of such problems. This paper is concerned instead with algorithms for the solution of some simple members of this class. These problems are continuous versions of various classical finite linear programming problems, including the network flow problem and the transportation problem. We shall describe a number of problems and review work to date on solution algorithms for them. These problems include the following.

1. Time Continuous Network Flow Problem

We wish to maximize the total flow over a given time interval through a network having storage capacity at the nodes and timevarying arc capacities. An algorithm has been developed for this problem and implemented in the case of piecewise linear arc capacities.

2. Space Continuous Network Flow Problem

This problem is to maximize the flow between two points in a space X, subject to capacity constraints c(x,y) on the amount of material allowed to flow between any two points x and y in X, and a mass conservation constraint at each point.

3. Mass Transfer Problem

In its simplest form, this is the classical "cutting and filling" problem of transferring material from some space X to some space Y when there is a cost function c(x,y) for moving unit amount of material from x in X to y in Y. An algorithm based on the transportation algorithm will be described for one version of this problem.

4. Time Continuous Transportation Problem

This problem is to supply at least cost some given demand at n destinations from m sources over a given period of time. Storage is allowed at the sources or destinations and the transport and storage costs vary with time.

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> A Monte Carlo algorithm for global optimization based on a Bayesian stopping rule

Monte Carlo algorithms in which the objective function is evaluated at points randomly generated in the search domain, are now widely accepted as effective tools for optimizing multiextremal functions. The crucial point of algorithms for global optimization is the design of a sensible stopping criterion. Due to the lack of characterizations in finite terms of global optima, stopping criteria are forced to be of statistical nature. Given a mesurable function f defined over a set S, the one variable function $F(y) = meas(x:f(x) \le y) / meas(S)$ is such that F(y) = 0 when v is smaller than the essential infimum f^{*} of f. Thus in principle, one could test whether a certain value \hat{y} of f is close to f^{*} by testing the condition $F(\hat{y}) < \epsilon$ for a suitably small $\varepsilon > 0$. As F(y) is the distribution of the random varia ble Y=f(X), where X is a random point in S, tests about $F(\hat{v})$ can be developed in the framework of statistical hypothesis testing. In this paper an algorithm is presented in which a clustering technique on random generated points, ranked according to the corresponding value of the objective function, is used to start local searches from good starting points; the best obtained value is tested to be an approximation to f*, in the sense described above, in the framework of Bayesian statistics, by means of a statistical model of the function F. The test allows to incorporate a priori information about f*as well as a precise balance between accuracy and computational effort. The performance of the algorithm is discussed on some standard test problems.

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ENTROPY OPTIMIZATION VIA ENTROPY PROJECTIONS

Linearly constrained entropy optimization problems may arise when some model is described in a statistical environment and a solution which is most objective or maximally uncommitted with respect to missing information is sought for. Such problems arise in various fields of applications.

In this paper we deal with the problem of entropy optimization over interval constraints, i.e.,

 $\begin{cases} \min \sum_{j=1}^{n} x_j \ln x_j \\ \text{subject to } c_i \leq \langle a^{(i)}, x \rangle \leq b_i, \quad i=1,2,\ldots,p, \text{ and } x \geq 0, \end{cases}$

and propose a method which employes <u>non-orthogonal entropy projections</u> which are defined as follows:

<u>DEFINITION</u>: Given a hyperplane $H = \{x \in \mathbb{R}^n | \langle a, x \rangle = b\}$ and a point $z \in \mathbb{R}^n$, the system $\begin{cases} y_j = z_j \exp(\lambda a_j) , \ j=1,2,\ldots,n, \\ \langle a,y \rangle = b \end{cases}$

produces a point $y \in \mathbb{R}^n$ and a scalar λ . y is called the <u>entropy pro-</u><u>jection</u> of z onto H, and λ is the associated entropy projection parameter.

Based on the method for interval convex programming proposed by Censor and Lent [1], we introduce here an iterative primal-dual algor thm designed to solve the problem of entropy optimization over intervals via successive entropy projections onto hyperplanes.

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- 122-

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NONSTATIONARY SIMULTANEOUS OPTIMIZATION AND IDENTIFICATION PROCEDURES

This report concerns problems of optimization with unknown timedependent parameters, in particular, stochastic programming problems with partially unknown distributions. Let us assume that we are given the functions

$$f^{0}(x,z), f^{i}(x,z), x \in X \subseteq \mathbb{R}^{n}, z \in Z \subseteq \mathbb{R}^{k}$$

and a sequence of unknown k-dimensional parameters $z^*(s) \in \mathbb{Z}$. For an arbitrary given sequence of "control" variables $x^0, x^1, \ldots, x^s, \ldots$ it is possible to observe the 1-dimensional sequence $h^0, h^1, \ldots, h^s, \ldots$, such that

$$E\{h^{S}|x^{0},x^{1},...,x^{S}\} = g(x^{S},z^{*}(s))$$

where the function g(x,z) is known. The problem is to create the sequences of control variables $x^0, x^1, \ldots, x^s, \ldots$ and "estimates" $z^0, z^1, \ldots, z^s, \ldots$ of the unknown vectors $z^*(s)$ such that

$$\lim_{s \to \infty} [f^{0}(x^{s}, z^{s}) - \min \{ f^{0}(x, z^{*}(s)) | f^{1}(x, z^{*}(s)) \ge 0, x \in X \}] = 0$$

A general iterative procedure for obtaining such sequences is based on using nondescent methods of optimization. In the simplest case, when $f^1(x,z) \equiv 0$, $X \equiv \mathbb{R}^n$, $z^*(s) \equiv z^*$, $g(x,z^*) = z^*$, when the objective function $f^0(x,z^*)$ is a differentiable function with respect to x, and when for given x,z the information about the gradient $f^0_x(x,z)$ is available, the proposed procedure becomes the iteration

$$x^{s+1} = x^{s} - \rho_{s} f_{x}^{0}(x^{s}, z^{s}), \quad s = 0, 1, \dots$$

where z^s is an estimate of z^* such that $z^{s} + z^*$ with probability 1. Principal difficulties with the study of the convergence of such procedures are connected with the choice of ρ_s . There is no guarantee that a new x^{s+1} will belong to the domain of the smaller values of the functions $f^0(x,z^t)$, $t \ge s+1$. The report discusses nondescent rules for choosing ρ_s for the general problem with nondifferentiable functions.

-123-

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PIECEWISE LINEAR PROGRAMMING: BLOCK DIAGONAL PROBLEMS

Piecewise Linear Programming (P.L.P.) can be seen as a set of optimization techniques of piecewise linear functions (convex or concave) defined over convex sets. These techniques can be seen as an extension of the technique developed for linear programming.

Generally, piecewise linear programming problems are stated as:

 $Min\{F(x) = \sum_{j=1}^{n} f_{j}(x_{j} / \sum_{j=1}^{n} g_{ij}(x_{j}) \leq g_{i}, \alpha_{j} \leq x_{j} \leq \beta_{j}, i=1,2,...m\}$

where $f_j(x_j)$ and $g_{ij}(x_j)$ are piecewise linear functions. A block-diagonal structure problem with coupling constraints, where all the constraints are linear - Block Diagonal Piecewise Linear Programming - is studied here.

A primal method for solving the problem is presented, as an extension of the generalized-generalized upper bounding (GGUB) technique. A dual method is also presented, showing that the Rosen method extended to the block diagonal piecewise linear problem is a variant of the technique proposed here. Algorithms for both methods are described. It is shown that an optimal solution, if one exists, is found in a finite number of iterations (under nondegeneracy). J. C. HENNET Laboratoire d'Automatique et d'Analyse des Systèmes du C. N. R. S. 7 avenue du Colonel Roche 31400 TOULOUSE - FRANCE

RESOLUTION OF A QUADRATIC COMBINATORIAL PROBLEM BY DYNAMIC PROGRAMMING

The problem of arranging heliostats on the collector field of a solar central receiver system can give rise to the following quadratic combinatorial formulation :

Maximize $X^{t_p} X$ Subject to $X^{t_x} \leq m$ With $X = (X_1, \dots, X_i, \dots, X_n)^{t_n}$ and $i \in (1, \dots, n); X_i = 0$ or 1.

We compute average intrinsic efficiencies of each location (p₁₁) and average interaction rates due to shadow effects for each pair of heliostats (-p.:). Then we want to optimally choose a maximum of m heliostat locations among n possible ones.

For finding the optimal vector, X, we propose an original approach based on dynamic programming.

The problem is divided into m stages. At each stage j, for 2 ≤ j ≤m, we calculate n objective functions associated with the n possible decisions "X.=1 at stage j", the previous assignment policy from 1 = 1 to 1 = (j-1) being the optimal one.

Let $f_j(i)$ be the objective function at stage j for decision $X_j=1$. We have :

and for $2 \leq j \leq m$,

 $f_{1}(i) = p_{ii}$ $f_{j}(i) = p_{ii} + Max (f_{j-1}(k) + \sum_{i \in L_{j-1}(k)} (p_{i1} + p_{1i}))$

We denote $L_{j-1}(k)$ the set of (j-1) nodes chosen by the optimal policy for which $X_{k}=1$ at stage (j-1)".

The choice of k^* such that :

$$f_{j}(i) = p_{ii} + f_{j-1}(k^{\bullet}) + \sum_{l \in L_{j-1}(k^{\bullet})} (p_{i1} + p_{1i})$$

is restricted to the set of nodes k for which $i \notin L_{i-1}(k)$.

Finally, information of the optimal vector, X^{\bullet} , is contained in $L_i^{\bullet}(i^{\bullet})$, the pair $(i^{\bullet}, j^{\bullet})$ being such that :

 $f_j \bullet (i^*) = Max \qquad f_j(i)$ $i \in (1,,n)$ i € (1..m)

This method is very easy to implement. It has excellent performance in terms of memory size and computing time.

TOWARDS A GENERALIZED SECOND-ORDER DIRECTIONAL

DERIVATIVE FOR CONVEX FUNCTIONS.

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For a lower-semicontinuous convex function f, the approximate second-order directional derivative $(d, \delta) \mapsto f_{\varepsilon}''(x_0; d, \delta)$ is defined through the ε -directional derivative $f_{\varepsilon}'(x; d)$. The function $x \mapsto f_{\varepsilon}'(x; d)$ is, for all $\varepsilon > 0$, locally Lipschitz on dom f and, at those points where it is not differentiable, it admits a directional derivative

$$\lim_{\sigma \to 0^+} \frac{f'_{\varepsilon}(x_{o} + \sigma \delta ; d) - f'_{\varepsilon}(x_{o} ; d)}{\sigma}$$

which is precisely $f_{\varepsilon}^{"}(x_{o}; d, 5)$. Our aim is to study the existence or non existence of the limit of $f_{\varepsilon}^{"}$ when $\varepsilon \longrightarrow 0^{+}$. This approach leads to results of the following kind :

. f'' (x_o; d, d) = lim f''_{\odot} (x_o; d, d) = 0 for functions $\varepsilon \rightarrow 0^+$

like the polyhedral ones,

. lim $f_{\varepsilon}''(x_{o}; d, d)$ exists at almost all $x_{o} \in dom f$ and $\varepsilon \neq 0^{+}$

-126-

coincides with the second derivative of f at x_0 (in the Alexandroff's sense).

The difficulties which arise ι defining f" (x₀; d, d) at all x₀ ϵ dom f are discussed.

-127-

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> THE STRUCTURE AND COMPUTATION OF SOLUTIONS TO CONTINUOUS LINEAR PROGRAMS

The continuous linear programming problem of the form maximize $\int_{0}^{1} c(t) x(t) dt$ $A x(t) \leq b(t) + \int_{0}^{t} K(t,s) x(s) ds$ x(t) > 0, $x(t) \in \mathbf{R}^{n}$, $b(t) \in \mathbf{R}^{m}$, $c(t) \in \mathbf{R}^{n}$

subject to

is investigated. Basic solutions are defined in analogy with the finite dimensional case. Not every feasible extreme point is necessarily basic, however, under appropriate regularity conditions it is shown that extreme points of the feasibility set are indeed basic.

Duality theory and parametric linear programming techniques constitute the primary tools of this analysis. Feasible trajectories through the state space are studied as they pass through elementary cones into which the state space is partitioned. Switches from one basis to another are classified as primally or dually induced depending on how the trajectory passes through the boundaries of the elementary cones. The complexity of solutions is characterized by the structure of the basic solutions, i.e. the basic indices, and the order in which switches occur.

A generic class of continuous linear programs of simple structure is introduced as normal problems. Complementarity relations such as complementary nondegeneracy and complementary smoothness are exploited to characterize solutions further. Finally, for this class of problems it is shown that the number of switches is finite.

An algorithmic construction for simple solution structures is presented and illustrated by several numerical examples.

-128-

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THE SIMPLIFICATION OF MULTIDIMENSIONAL EXTREMAL PROBLEMS ACCORDING THEIR STRUCTURE

There are great difficulties in solving extremal problems of high dimensionality accurately, particularly in nonconvex cases. However, frequently we may separate a relatively little part of the main variables.

The structural characteristics [1] of the extremal problem may be used to evaluate the average simplification error. In order to simplify the problem in the best way it is sufficient to select the variables and their interactions with the largest structural characteristics. An algorithm and computer program for the separation is developed (programming language rORTRAN, computer BESN-6). Investigations of test problems show, that the analysis

Investigations of test problems show, that the analysis of a problem perore solving it is efficient in many real cases.

In particular, we suppose that the results of evaluations of the objective function $f(x_1, \dots, x_n)$ in some randomly distributed points of the cube K^n are known. We consider the evaluation of simplification errors on the basis of the analysis of these results.

We use the accomposition of objective function to ortogonal components of different limensionality $f_{il...is}(x_{il},...,x_{is}), s=1,...,n$. The term "structural characteristics" is used to denote the characteristics of the dispersion of these components:

 $D_{il...is} = \int_{K^n} [f_{il...is}(x_{il},...,x_{is})]^2 dx_{il}...dx_{is}.$

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A CLASS OF DIRECT METHODS FOR LINEAR SYSTEMS

We present a class of direct methods for determined or underdetermined linear systems which is a generalization of methods presented by Huang, Rosen (in the context of gradient projection), Brent and Brown (in the context of nonlinear equations). The class contains free parameters which can be chosen on ground of optimal conditioning and scaling. We conjecture that algorithms exist in this class with computational complexity no greater than Gaussian elimination. Numerical experiments show that methods in this class outperform Gaussian elimination with complete pivoting on ill conditioned problems. V.A. Tiešis Institute of Mathematics and Cypernetics Lenino 3 232000, Vilnius, Lithuanian SSR, USSR

ON THE EFFICIENCY OF SOME GLOBAL OPTIMIZATION ALGORITHMS

We deal with the problem of minimization of the multiextremal functions. The Boyesian (optimal on the average) methods [1, 2] have appeared to be helpfull in the solution of this problem. However, the application of these methods was restricted because of the supplementary computations: the loss function φ^t was minimized by the iterative procedure to plan the next point \mathbf{x}^t in which the objective function is evaluated. We now study an effective method of branches and bounds for the minimization of the loss function.

In particular, we consider the proplem:

minimize
$$\{f(x): x \in D \subset \mathbb{R}^n\}$$
.

The loss function φ^{i} such as in [3] is investigated here. We got such bounds of the number of operations P for the Bayesian algorithm:

 $O(l(n+1)) \leq P \leq O(1^2, n), l=max i.$

The results of φ^{i} minimization are generalized for such a problem of global search:

$$\min \max \ldots \min \max \mathbf{z}(j_1, \ldots, j_m)$$
$$l \leq j_m \leq n_m l \leq j_{m-1} \leq n_{m-1} l \leq j_2 \leq n_2 l \leq j_1 \leq n_1$$

For this problem $P=0(\bigcap_{k \in \mathbb{N}} n_k)$ in the worst case and $l \le k \le m$

 $P=0(|| n_k)$ in the best case, respectively. $l \leq j \leq \frac{(m+1)}{2} \int_{k}^{m_k} dx$

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-131-

MULTIOBJECTIVE OPTIMIZATION

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REFERENCE POINT MULTIPLE CRITERIA OPTIMIZATION - IMPLEMENTATION AND EXPERIENCE

The reference point approach has been developed by Wiersbicki in the series of papers and reports (see[1]). This method, being the generalization of goal programing developed by Geoffrion and displaced ideal method developed by Zëleny joins together the best properties of both approaches, eliminating simultaneously their weak points. From authors experience follows that this method is one of the most suitable for solving multiple criteria decision problems:

- it applies to convex and nonconvex cases

- it can easily check Pareto-optimality of a given decision - it can be easily suplemented by an a posteriori computation of trade-off coefficients for the objectives
- it is numerically well-conditioned and easy for implementation
- the concept of reference point optimization makes it possible to take into account the desires of a decision maker directly, without necessarily asking him about his preferences
- it is specially suitable for interactive decision making.

Two efficient implementations of this method are available at IIASA - first developed by Wm.Orchard-Hays for IBM computer, the second developed by the author is a portable, Fortran written program package which can be used with every computer equipped with LP system with MPSX input format. Currently at IIASA this system cooperates with MINOS LP package developed by Murtagh and Sargent.

Both packages have been applied in projects being now performed at IIASA - energy project, water resources distribution, industry development strategy, development forest and wood industry. Existing experience has shown applicability of the method for solving practical multiple criteria decision problems.

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-133-

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THE BAYES LAN OPPROACH TO GLORAL OPTIMIZATION

one of the important and difficult problems in computer aided design is how to find the best decision. In many cases it can be reduced to global optimization problems when the minimum of multimodal functions should be found. Usually it can not be done using conventional numerical methods of optimization which quarantee some fixed accuracy level. Therefore a heuristic approach is often used, which works reasonably well in many practical situations but has no mathematical justification.

In this paper a new approach is discussed when average deviation is considered. The function to be minimized is regarded as a sample of random function. The procedure of search is defined which minimizes the average deviation from the grobal minimum and so is optimal in the Bayesian sense.

The main objection to the Bayesian approach is the doubts about the existence and an arbitrary character of the a priori distribution. Therefore in the paper the conditions are given when the Bayesian methods converge to the global minimum of any continuous function. It is also proven that under some simple and natural assumptions concerning the relations of subjective likelihood there exists a random function which agree with likelihood relations.

The package of FORTHAN programs is prepared and used to optimize the set of different test functions. The results are compared with alternative approaches.

The results of engineering applications in the design of optimal vibromotors and the most thermostable polymeric compositions are mentioned. V.F.Poterasu Department of Mechanics Polytechnic Institute is la sv Iasi, 6600 Romania

MULTIPLE OBJECTIVE OF CONTRACTOR WHIP APPLICATION IN DISTRIBUTION OF EXECUTE RESOLVED.

In designing a control system (is making a decision on control systems) very often there are many, usually incomensurable or even incompatible performance objectives to be satisficit. For example, the most important criteria are the level of profit, the level of investment and the technical criteria. Consider the following multicriteria programming problem max $z_{i} = J_{i}(x), \ldots, \max |z_{i}| = J_{i}(x), \text{subject to } J_{i}(x) = 0, g_{i}(x) \ge 0, \ldots$ $g_{i}(x) \ge 0$. The general multiple objective optimization problems with all but one of the multiple objectives converted to proper equality constrain ts. It is shown that Lagrange multipliers, which are intermediate by-products in the process of solving single objective converted equality constraints are proper or not. Several useful methods), can be effectively utilized for testing/verifying, whether the effective of isotricient conditions for properness are derived and expressed in terms of Lagrange multipliers.

These necessary and sufficient contracts are derived using no convexity or concavity assumptions. They are obtailed to content analytical characterization of Pareto-optical solutions, as well as for numerical generation of Pareto-optical solutions of the solutions, primal-dual mthods, and Hestenes-Powell-Rochafeeline methods of meltipliers. The theory developed are utilized by the optimal distribution of merzy resources.

We are considering that utilization is a sincle system of a renewal resource sets as:biomass, acclian, show, hyperblock the energy leads to great savings. The optimizing problem of the system according the both the primary resources modelling, also the store according the tip is systems, a.s. 0. The two-step optimization is to be accessible also the level of system and at the level of employed primary resources the observation and sensitivity problems subjected to random parameters are not to be considered. First the configuration of the system leader that the follow subject of the ensemble is determined and satisfy the the terminelies of the system.

 $\lambda < \gamma =$
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An exact Set-Up Cost Goal Programming problem solution al (orith):

The purjose of the japer is to present another formulation and solution algorithm to the set-up production cost problem. The cost function is assumed to be a sum of set-up cost independent on production scale and fixed unit cost. Despite seeking, preaskinant linearity of a problem the very decision "to produce or not to produce" makes the LP and LGP simplex based algorithms unagelicable due to nonconvexivity of the feasible solution area. The paper presents a method of enhancing simplex algorithm in order to cope with set-up costs.

A Linear Goal Frogramming problem is formulated as follows: minimize the $J(\overline{n},\overline{f})$ goal unachievement vector function

ussuming lexicographical order of components, where $G : \mathbb{R}^{2m}_+ \to \mathbb{R}^{s}_+; \mathbb{1} : \mathbb{R}^{n}_+ \to \mathbb{R}^{2m}_+; \mathbb{P} = [\tilde{n}, \tilde{p}]; \tilde{n}, \tilde{p} : \mathbb{R}^{n}_+ \to \mathbb{R}^{m}_+;$ $\overline{n}, \overline{p}$ are vectors of the distance variables defined by:

 $f_i(\bar{x}) + n_i - n_i = n_i$ for $i = 1, 2, ..., n_i$;

 $\mathbb{F}_{i}(\overline{n},\overline{p}) \geqslant \mathbb{C}$ for $n = 1,2,\ldots,s; \mathbb{F}_{i},\overline{p}_{i}$ - linear functions The Linear Goal Fro raming Set-Up Cost problem introduces

following modification:

 $\mathbf{r}_{i,j}\mathbf{x}_{j} = \begin{cases} 0 & \text{for } \mathbf{x}_{j} = 0 \\ \mathbf{z}_{i,j} + \mathbf{r}_{i,j}^{*}\mathbf{x}_{j} & \text{for } \mathbf{x}_{j} > 0 \end{cases}$

Any decision variable x_{i} for which there exist c_{i} positive c_{i} coefficient is to be dalled the set-up variable. A partial order relation among feasible solutions is to be introduced in the following way:

 $x \leq x' \leq \frac{1}{2\pi}$ for every set-up variable x_j / $y \in D_{13} \in J_{13} = / x_j \in J_{23} \inJ_{23} \in J_{23} \inJ_{23} \inJ_{23} \inJ_{23} \inJ_{23} \inJ_{23} \inJ_{23} \subsetJ_{23} \subsetJ_{23} \subsetJ_{23} \subsetJ_{23} \subsetJ_{23} \subsetJ_{23} \subsetJ_{23} \inJ_{23} \subsetJ_{23} \subsetJ_{23}$

The relation = defined as $\leqslant \land \leqslant$ factorizes feasible area into convex, connected sets. The algorithm proposed in the paper searches the lactor-feasible solution sets in particular order and performs a Lup optimization of each using known simplex pased al orithm. No do that it any loyes the discriminative optimization concept. The best among equivalency class optima is tee of timel colution to the given set-up cost prob-101...

> al able to DTIC does not a mait fully legible reproduction

- 136-

PROGRAMMING - ALGORITHMS

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DESIGN CENTERING OF TIME INVARIENT LINEAR SYSTEMS

If the system transfer function is a bilinear function in each designable parameter, gain or phase specifications can be written in the following biquadratic form

 $e_i(\phi) = \frac{A_i + 2B_i\phi + C_i\phi^2}{1 + 2D_i\phi + E_i\phi^2} > 0, \quad i = 1, 2, ..., m.$

Feasible intervals I_i for each specification $e_i(\phi)$ is derived in the general case when real poles in ϕ exist. Hence, the overall feasible interval is found by the intersection of these intervals.

Centering is achieved by one of the following algorithms⁽¹⁾: 1. One parameter at a time

- 2. Pattern technique
- 3. Modified pattern technique (Involve a combination of the above mentioned algorithms).

The pattern technique presented is a modification of the Hookeand Jeeves method originally designed for unconstrained optimization.

An example of design centering for an LC low pass filter is presented. New approach for the design of a compensator including system gain and phase margin constraints, as well as constraints on the quality factor of the compensator elements is also presented.

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-138-

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ASYNCHRONOUS PARALLEL SEARCH FOR

GLOBAL OPTIMIZATION PROBLEMS

The development of suitable algorithms that can take advantage of the computational facility of multiprocessor systems, expected to be soon available to the general user, has been recently gaining increasing attention.

The numerical solution of global optimization problems, which is in most cases prohibitively expensive on a serial machine, can substantially profit from a parallel computing framework.

In this paper a class of sequential search strategies is assumed, which can be shown to be step-wise optimal: parallel algorithms are then constructively defined by the simultaneous execution of a number of those strategies, introducing critical sections in order to allow interprocessor communication.

Several parallel algorithm architectures are considered in this framework, with and without syncronization, and with different scheduling policies for the access to the common memory: their relative merits have been compared using a FORTRAN program that can simulate the execution of the parallel algorithm.

The results as yet obtained seem to imply that, also in the solution of global optimization problems by search strategies the structure of the algorithm has to be loosened into a stochastic framework to avoid synchronization penalties and exploit fully the multiprocessing features.

A general analysis of the resulting parallel algorithm is rather difficult: the tools of order statistics and queueing theory, and simple probabilistic assumptions about times of computation and access to the common memory enable to obtain some basic results that show a good agreement with the simulated performance of the algorithm. Howard Blum Graduate School of Management Rutgers University Newark, N. J. 07102

A MULTIPLIER ALGORITHM FOR LINEAR PROGRAMMING WITH A SUBSIDIARY CONSTRAINT

We describe a multiplier algorithm for solution of a general linear programming problem subject to a linear subsidiary constraint. Our approach is motivated by considering linear programming problems of special constraint structure, such as the transportation problem, with a subsidiary constraint. The subsidiary constraint is imposed via a multiplier term added to the objective to create a parametric objective problem which retains the constraint structure.

Using results on parametric objective linear programming due to Gass and Saaty we prove that, in the absence of degeneracy, the sequence of adjacent basic feasible solutions generated by the algorithm is monotonic and finite. We explicitly treat the possibility of unboundedness of the parametric problem and illustrate our results with an example.

-140-

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A PROBABILISTIC APPROACH TO THE MINIMIZATION OF STOCHASTIC FUNCTIONS BY SEQUENTIAL, NEAR-CONJUGATE SAMPLING

We consider the problem of minimizing the convex function f(x) when z=f(x)+e is observed. In order to achieve an efficient algorithm we hybridize ideas of conjugate gradient algorithms [1] and stochastic function minimization [2]. We maintain the orthogonal projection method but relax convergence reqirements, e.g. we require only convergence with probability P=1-eps (0<eps<1) into a region G around the minimum of f(x). Under these conditions estimates of G for given P can be found.

Efficiency of this method is achieved by the particular way in which sequential samples z are drawn: when determining a new direction, conjugate to the subspace spanned by previously determined directions, the acceptance depends on a statistical test. When this test is failed the solution may be refined by orthogonal projection and by increase of the finite difference, depending on another statistical test.

The main advantages of this method over other currently available methods are summarized by:

- Avoid accumulating uninformative samples when computing the Hessian H, as it occurs in conventional "block sampling"; instead find robust estimate based on most informative, sequentially drawn, samples.
- 2. Obtain Hessian in factored form.
- 3. Eliminate ridge problem.
- 4. Provide "on-line" evaluation of efficiency of procedure, give diagnostic warning when problem too "high-dimensional" for the estimated observation noise e.
- 5. Natural termination of procedure when converged in estimated region G with estimated probability P=1-eps.
- 6. Adaptive choice of finite differences.
- 7. Avoid certain factorization problems of Hessian during iterations by restriction to suitable subspaces of variable dimension.

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- 14 1-

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QUASI-NEWTON METHODS WITHOUT PROJECTIONS For linearly constrained minimization

Quasi-Newton methods without projections for linearly constrained minimization are extensions of quasi-Newton methods without projections for unconstrained minimization proposed by Lukšan /4/ which are modifications of methods proposed by Davidon /1/. The product form relations are used, so that the iterations have the form

$$x^{\dagger} = x + \alpha s, s = -s \widetilde{g}, \widetilde{g} = s^{T} g$$
 (1)

with special updating of a matrix S. Here x is a vector of variables, g is a gradient of the objective function and ∞ is a steplength.

The active set strategy is used for linearly constrained problems. The linear manifold defined by active constraints is represented by QR factorization of a matrix A whose columns are normals of the active constraints and by a non-orthogonal matrix S used in (1). This new approach is a combination of both approaches proposed by Ritter /5/ and Gill and Murray /2/. The matrixS satisfies the relation

$$s^{+}(s^{+})^{T} = ss^{T} - \frac{ss^{T}aa^{T}ss^{T}}{a^{T}ss^{T}a}$$
(2)

after adding the constraint with the normal a. Relation (2) is essentially the same as the relation proposed by Goldfarb /2/ for his matrix H.

Implementation of quasi Newton methods without pro-

- 142-

jections gives an efficient algorithm for solving linearly constrained minimization problems.

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AN EFFICIENT ALGORITHM FOR ONE-DIMENSIONAL CONSTRAINED MINIMIZATION PROBLEMS WITH NONSMOOTH CONVEX FUNCTIONS

We give a rapidly convergent algorithm for solving the constrained optimization problem of minimizing f(x) subject to $h(x) \leq 0$ where f and h are nonsmooth convex functions of a single variable x. The method is an extension of one for the unconstrained case due to Lemarechal and Mifflin which efficiently combines polyhedral and quadratic approximation of f.

It can be thought of as a new type of penalty function method which is different from previous types, because it does not use values of h at feasible points, other than to determine feasibility (i.e. nonpositivity of h), and it does not require f to be evaluated at infeasible points. For each feasible x the algorithm requires f(x) and one subgradient (i.e. a one-sided slope) of f at x and for each infeasible x it requires h(x) and one subgradient of h at x. The method generates two sequences $\{x_k\}$ and $\{y_k\}$ where, for each k, x_k is feasible, either y_k is infeasible or $f(x_k) \leq f(y_k)$, and x_k and y_k are on opposite sides of any optimal point x^{*}. Due to nonsmoothness it can happen that $x_{k+1} = x_k$ for infinitely many k so we cannot show that $\{|\mathbf{x}_{k+1} - \mathbf{x}^*| / |\mathbf{x}_k - \mathbf{x}^*|\} \rightarrow 0$, as is the case for approximate Newton methods applied to unconstrained problems with f satisfying strong convexity and smoothness assumptions. What we can show under rather weak assumptions is that $\{x_k\}$ converges to some constrained minimizing point \mathbf{x}^* and that $\mathbf{r}_k = |\mathbf{x}_k - \mathbf{x}^*| |\mathbf{y}_k - \mathbf{x}^*|$ converges to zero superlinearly, i.e. $\{r_{k+1}/r_k\} \rightarrow 0$.

- 144-

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CRTHOGONALITY CONSECTION IN CONJUGATE GLADIENTS MUTHOD

In applying the conjugate gradients method to minimize a quadratic function, one has to calculate a new direction of search ' ξ ' in each iteration 'i' by,

$$\xi^{i} = -g^{i} + \beta^{i-1} \xi^{i-1}$$
 (1)

In the above equat on the corryover term $!\beta!$ is given by the following equation provided the two consecutive gradient vectors 'g' are orthogonal and the one dimensional search for locating the step length in each iteration is perfect:

$$\frac{i-1}{3} = \frac{(g^{i})^{T} g^{i}}{(g^{i}-1)^{T} g^{i-1}}$$
(2)

In case of nonquadratics if the starting point is a poor approximation, the error developed in \mathcal{A} proves to be detrimental to convergence. To overcome this drawback, Fletcher and heeves (1964) suggested to restart in the direction of steepest descent after every (n+1) iterations for a function of 'n' variables. In the present work, authors calculate a modified $\overline{\mathcal{A}}$ as follows:

$$\frac{\mathbf{i}-\mathbf{l}}{\mathbf{j}} = \frac{\mathbf{i}-\mathbf{l}}{\mathbf{j}} \quad \text{if} \quad \left| \begin{array}{c} \mathbf{i}-\mathbf{l} \\ \gamma' \end{array} \right| < \mathcal{E} \\ = \frac{\mathbf{i}-\mathbf{l}}{\mathbf{j}} \quad (\mathbf{1} - \gamma'^{\mathbf{l}-\mathbf{l}}) \quad \text{if} \quad \left| \gamma^{\mathbf{i}-\mathbf{l}} \right| \geq \mathcal{E} \end{array} \right\}$$
(3)

where β is calculated from eq. (2); ' \mathcal{E} ' is a prescribed positive constant. The orthogonality correction term ' γ ' is given by,

$$\frac{i-1}{\sqrt{g^{2}}} = \frac{(g^{1}-1)^{T}}{(g^{2})^{T}} \frac{g^{i}}{g^{i}}$$
(4)

Whenever γ becomes unity, the restart is automatically applied. Thus eq. (3) saves considerable computational effort compared to more elaborate corrections introduced by several other workers. The paper presents numerical results of testing eqs. (3) and (4) with several test functions.

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A QUADRATICALLY CONVERGENT RECURSIVE LINEAR PROGRAMMING ALGORITHM FOR CONSTRAINED NONLINEAR PROGRAMMING PROBLEMS

In the last decade, a great deal of attention has been devoted to <u>recursive quadratic programming algorithms(ROP</u>), for the general, continuously differentiable, nonlinear programming problem

 $\min\{f(x) : g(x) \le 0, h(x) \ge 0\}$ (P)

with $f: \mathbb{R}^n \to \mathbb{R}, g: \mathbb{R}^n \to \mathbb{R}^m$, $h: \mathbb{R}^n \to \mathbb{R}^2$. These algorithms substitute for (P) a sequence of quadratic programs $(QP)_k$ which yield successively better estimates (x_k, λ_k, ψ_k) of a Kuhn-Tucker triplet for (P). In some cases the cost functions in these quadratic programs $(QP)_k$ are not positive semi-definite and, then, convergence of the overall algorithm requires the construction of the Kuhn-Tucker triplet of $(QP)_k$ which has minimal

norm. Available quadratic programming codes are unable to compute such points and there is some experimental evidence that this causes computational difficulties.

When only equality constraints are present, recursive quadratic programming is equivalent to recursive litear programming (RLP), which substitutes for (P) a sequence of linear programs (LP)_k. When both equality and inequality constraints are present, RLP produces iterates which are no longer identical to those generated by RQP. Furthermore, it was generally believed that RLP cannot be used for such problems because it had been claimed by S. M. Robinson [1] that the resulting linear programs do not satisfy the positive linear independence (LI) sufficient condition and hence may have no solution.

In this paper, we show that although the (LI) condition may not be satisfied, the particular linear programs generated by RLP do indeed have solutions. We prove that RLP preserves the local quadratic convergence exhibited by RQP. In addition, we provide a scheme for making the algorithm globally convergent. Since RLP is unaffected by non-positive semi-definite Hessians and since the complexity of the $(QP)_k$ and $(LP)_k$ is com-

parable, we see that there is a net advantage to using RLP over RQP, particularly given the fact that linear programming codes are in a much greater state of perfection than quadratic programming codes.

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ACKNOWLEDGEMENT: This research was supported by NSF Grants No. ECS-79-13148 and PFR-79-08261 and JSEP Grant No. F49620-76-0-0178. M.J.D. Powell Department of Applied Mathematics and Theoretical Physics University of Cambridge Silver Street Cambridge CB3 9EW, England

EXTENSIONS TO SUBROUTINE VFO2A

The purpose of the subroutine VF02A is to calculate the least value of a function of several variables subject to nonlinear constraints, in the case when first derivatives of the objective and constraint functions are available. A veriable metric method is used that generates search directions in the space of the variables by solving convex quadratic programming problems, and that chooses step-lengths along search directions by reducing a line search objective function which is formed by adding the main objective function to a weighted sum of moduli of constraint violations. Numerical experiments have shown that this subroutine, which has been available in the Harwell Library since 1977, often requires fewer function and gradient evaluations than other optimization algorithms, but the author is aware of many limitations of VF02A. In particular, if the weights of the line search objective function become too small, then the algorithm can cycle instead of converging, and weights that are too large can increase greatly the number of iterations that are required. Therefore an extension of the line search method, called the "watchdog technique", has been developed, that helps to avoid the inefficiencies of large weights. We consider the advantages and disadvantages of the watchdog technique in practice when some of the constraints are highly nonlinear. We find that the technique can give large gains in efficiency, but that it is sometimes helpful to have a pre-set upper bound on the change to the variables that is made by each iteration, in order to avoid large incursions into the infeasible region. A Fortran listing of the new version of VF02A is available.

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QUASI-NEWTON METHODS FOR A CLASS OF NONSMOOTH CONSTRAINED OPTIMIZATION PROBLEMS

We consider optimization problems of the type:

Minimize $\psi(Gu)$ on U,

where G represents a differentiable mapping from a convex subset U of a Banach space A into a Banach space B and ψ is a nondifferentiable convex functional on B. A procedure similar to the Newton method for the numerical solution is the following: Since ψ is not assumed to be differentiable, we only linearize G and determine $u_{i+1} \in U$ for given $u_i \in U$ by minimizing

$$\psi(Gu_{i} + G'_{u_{i}}(u - u_{i})).$$

We replace G_{u}^{i} by a class of operators and show under a growth con-

dition linear and superlinear convergence.

Since for global convergence a step length procedure at each iteration is advisable, we extend the convergence theorems for algorithms with step sizes. We define stationary points for these problems and give a convexity type condition under which each stationary point solves the optimization problem. As an example we present the rational Chebyshev approximation. For algorithms with step size adjustment we prove global convergence to stationary points. Furthermore we give conditions on the step lengths under which the linear and superlinear convergence rate is retained. F.Sloboda and V.Britanak Bratislava 81000,Pistel Czechoslovskia

ON THE GENERALIZED CONJUGATE GRADIENT ALCORITHM

INVARIANT TO NONLINEAR SCALING

In /l/ a generalized conjugate algorithm for solving minimization provlems of the form

min
$$f(x) = f(x^+)$$
, $f:R_n \rightarrow R_1$

where x^{\top} is the optimal solution is suggested. The algorithm has the property that minimizes functions functions of the form

$$f(x) = \Im(F(x))$$
, $dh/dF \neq 0$

where $h: R_1 \rightarrow R_1$ is a differentiable function and $F: R_n \rightarrow R_1$ is a strictly convex quadratic function, on finite number of steps. The complexity of this algorithm is the same as the complexity of the conjugate gradient algorithm so far. The common feature of agorithms of this type is that they have been suggested for functions of which the analytic form is considered to be known a priori. The above considered algorithm does not suppose the knowledge of h or F but only of f(x) and its gradient g(x).

In this paper Rimerical experiments with the generalized gradient algorithm are presented. The efficiency of the algorithm is tested on several test functions which have established the theoretical results described in /1/. Some implementation questions are investigated and simple techniques ensuring the stability of the algorithm are suggested. The results are compared with the F-R og algorithm. The algorithm enables to improve the global efficiency of conjugate gradient algorithms.

/1/ F.Sloboda : A generalized conjugate gradient algorithm for minimization, Numer. Math., 35, 223-230 (1980). H. Tuy, Ng.Q. Thai and Tr.V. Thieu Institute of Mathematics 208-D, Doi-Can . Hanoi (Vietnam)

CONICAL ALGORITHMS FOR SOLVING A WIDE CLASS OF MATHEMATICAL PROGRAMMING PROBLEMS

A wide class of mathematical programming problems including linear and convex programming problems, linear and convex complementarity problems, bilinear programming problems and 0-1 integer programming problems, can be reduced to the general problem of finding the minimum (in the global sense) of a <u>concave</u> function f(x) over a closed convex subset D of R^n . In the special case where D is a polytope this problem was first studied by Tuy [1] in 1964 and, subsequently, by a number of other authors. We now extend the method developed in [1,2] to the general case where D may be unbounded or not polyhedral. As in [2] the new algorithm is a branch and bound procedure based upon a conical subdivision of the space and a special method for computing an estimate of a lower bound of f(x) over the part of D contained in a given cone. But, in contrast to the algorithms developed in [1,2], the new algorithm uses a bounding method which applies to the general case and, moreover, does not involve solving any auxiliary linear program.

As specialized to the case where f is linear and D is polyhedral, this algorithm yields a new method for solving linear programs, quite different from the simplex algorithm.

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AN IMPLEMENTATION OF THE ELLIPSOID ALGORITHM FOR LINEAR PROGRAMMING.

This paper discribes some modifications of the ellipsoid method for solving a system of linear inequalities. There are many ways of a transformation of a given linear programming problem into a system or systems of linear inequalities. In particular we use the primal-dual formulation and the target value of the objective function in the primal and/or the dual formulation of a given LP problem. In practical LP problems bounds on variables can be usually provided and often we know that a given LP problem is consistent. These facts provide much better estimation of the radius of an intitial ball then the one given by Khachian. Constructing a new ellipsoid tray to compromise two conflicting goals minimizing its volume and minimizing its maximal diameter. A discription of a diffrent method for constructing "best surrogate cut" is given. Finally, we consider different stopping rules for the ellipsoid method and give an estimation of errors for such solutions.

In our computational experiments we consider mostly examples for which there is a hope that the ellipsoid method is better than the simplex method. Therefore we solve some ill - conditioned LP problems, problems requiring an exponential number of simplex iterations to solve them, and degenerate LP problems. The main conclusion of our experiments is that there are LP problems, not only pathological ones, for which the ellipsoid method is better than the simplex method with respect to computer time or/and errors of optimal solutions.

- 151-

APPLICATIONS OF PROGRAMMING

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A NEW TYPE OF OFFIMAL INVESTMENT RESOURCE ALLOCATION PROBLEM SOLUTION IN COMPOUND SYSTEMS TO BE DEVELOPED

In many real cases any suitable objective function cannot be given for solving a certain investment resource allocation problem, because of shortage of information or systems complexity. In order to solve these problems there is a possibility to utilize only the available /"minimal"/ information for modelling and decision making, without the effort to construct any concrete objective function. This way of optimization is called "Optimization with Minimal Information"-"OMI"- and can be based on exact modelling of a few essential heuristic assumptions, as principles, for characterization and modelling a special class of optimization problems, including their objective functions. Such basic heuristic assumptions are e.g.:

"Existence of a development measurment level of a compound system to be developed by some investment"

"Existence of a so called composite investment cost function describing some development costs of the compound system"

A modelling of above mentioned and some other heuristic assumptions can be seen in [2] and [3] that are certain generalizations of the one dimensional reliability increase optimization model having presented in [1].

In this paper the author intends to present some new and unpublished results concerning this topic, that makes possible to model and solve optimal investment resource allocation problems in compound systems to be developed with a higher hierarchy than earlier.

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NON-LINEAR GASOLINE BLENDING PROCESS: THIRD-LEVEL KUHN-TUCKER CONTROLLER FOR DANTZIG-WOLFE DECOMPOSITION

The process for obtaining different grades of gasoline is non-linear. The non-linearity is due to the presence of a chemical product called tetraethyl lead which is added to the naphtas to obtain gasoline with different octane levels.

An optimization model is formulated to minimize the cost of producing gasolines subject to several economic and technical constraints. The model has two groups of variables: the Y(I,J,K) variables which represent the quantities of crudes I used to produce naphtas J and eventually gasolines of type K, and the TEL(I,J,K) variables which represent the quantities of tetraethyl lead added to the naphtas J produced from crudes I for obtaining gasolines of type K.

The solution structure has three levels. The two lower levels are used to optimize the quantities of crudes Y(I,J,K)needed for the process through Dantzig-Wolfe decomposition and solved by momentarily keeping the TEL(I,J,K) variables constant. These latter variables are solutions to the nonlinear third-level Kuhn-Tucker controller when the quantities Y(I,J,K) are kept momentarily constant. This third level is solved by using the second Kuhn-Tucker condition which is a system of non-linear algebraic equations of logarithmic type. The first level of the Dantzig-Wolfe decomposition is subdivided into Q blocks each of which corresponds to one particular final product.

Several runs with different sets of parameter values were conducted. In each case, convergence to the optimal global solution is obtained in a very few iterations.

- 154-

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An optimization model for energy saving in the heating of buildings

The use of optimization techniques has been recently advocated in order to improve the energy performance of the design of buildings.

A widely used performance index is the heating load, defined as the quantity of heat that must be given to a building in order to keep its inner temperature to a prefixed level.

To compute the heating load the authors have been using the computer code NBSLD (National Bureau of Standards Load Determination) with some modifications aimed at linking it effectively with the optimization software. The objective function accounts both for the heating cost and the cost of insulation materials so that its minimization, with respect to a meaningful set of technological and architectural variables, yields a sequence of designs of decreasing "cost", converging to that design which ensures, for the weather conditions of the site of the building, the optimal balance bet ween the cost of additional insulation and the related energy saving. The authors have been using a constrained optimization routine based on Recursive Quadratic Programming and a program for analyzing the sensitivity of the optimal solution, with respect to perturbations in the optimal design and parameters of the problem.

The cost of the evaluation of the objective function is rather high, due to the many factors which have to be accounted for in a realistic model of the building, and because the computation of the heating load has to be averaged over a period of several months.

Also in view of this fact, an important feature of the optimization model is a mathematical procedure which gives the value of the gradient of the objective function.

This procedure results in a substantial reduction in computer time, freeing the optimization algorithm from the need of computing finite difference approximation to the gradient. A. Schornagel
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A GENERAL MARKOV PROGRAMMING METHOD TO COMPUTE THE OPTIMAL DISPATCHING STRATEGY IN A TRANSPORTATION PROBLEM

The transportation problem we consider, has been treated in literature before ([2] and [3]). In the model vehicles take passengers from one single service point and return after traversing some route for an other trip. The N vehicles have an infinite capacity and the triptimes are independent and have an identical exponential distribution. The arrival process of passengers at the service point has a known and constant rate. It is easy to see that the state of the system is given by (n,t) where n is the number of vehicles waiting for dispatch and t is the time elapsed since the last dispatch. The objective is to find numbers t_n^* , $n=1,\ldots,N$ such that for a given cost structure (linear waiting costs and fixed dispatching costs) the average expected costs are minimized where t_n^* is the time to wait before dispatching a vehicle when there are N-n busses on trip. In [2] only N=1 and N=2 were considered where as in [3] most attention was given to asymptotical results for N $\rightarrow \infty$.

The model can be described as a Markov Decision Process with state space $\{(n,t) \mid n=0, ', \ldots, N, t \ge 0\}$ and continuous time parameter. To find the optimal strategy we use a General Markov Decision method ([1]) which appears to be quite efficient for $N \le 30$ and which gives rise to some nice structural properties of the optimal strategy.

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COMBINATORIAL PROGRAMMING

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-157-

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SWITCHING FUNCTION MINIMIZATION USING GRAPHS

In this paper we present a graph theoretic approach to minimization of switching functions. We introduce the notion of switching function graph (SFG) and study its properties. We show that a SFG yields the structure of a class of switching functions and the complexity of the switching function minimization problem depends upon the SFG of a switching function rather than the number of variables. Using a graph theoretic interpretation to the Boolean minimization problem we present a powerful approach to the problem. The major steps in our approach consist of finding maximally connected components (MCC's) of an associated SFG G = (V, E)and finding minimal covers for each of these components. An O(|V| + |E|)algorithm is given for finding the MCC's of an SFG G. Since finding an exact minimal cover for a connected SFG is known to be a NP-hard problem

we present an $O(|V|^2 \log |V|)$ time algorithm for finding a suboptimal cover for the SFG G. The algorithm yields exact cover for certain classes of SFG's (for example, cycles). Several examples are presented for illustrating our algorithms and to indicate the advantages of our approach. Additional work needed in this area is indicated. A complete paper appears in [1].

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A LINEAR TIME ALGORITHM TO MINIMIZE MAXIMUM LATENESS FOR THE TWO-MACHINE, UNIT-TIME, JOB-SHOP, SCHEDULING PROBLEM

Consider a job-shop problem with r jobs i=1,...,r and two machines denoted by A and B. Each job i=1,...,r has m(i) tasks (i,j) (j=1,...,m(i))m(i) each with processing time 1. For all i=1,...,r and j=1,...,m(i)-1task (i,j+1) cannot be started before task (i,j) finishes and if (i,j)is processed on machine A resp. B task (i,j+1) is to be processed on machine B resp. A.

Let $n = \sum_{i=1}^{n} m(i)$ be the total number of tasks.

If we assume that time zero is the earliest time a task can be started and that t_{max} is an upper bound for the largest start time of any job

then a schedule may be given by two arrays A(t) and B(t) with

 $t=0,\ldots,t_{max}$ where A(t) = (i,j) if task j of job i is to be processed

on machine A at time t. If machine A is idle during the time period from t to t+1 we set $A(t) = \lambda$. B(t) is defined similarly. The finish time of job i in a schedule Y=(A(t),B(t)) is given by

 $y(i) = \max \{t+1|A(t) \text{ or } B(t) \text{ is a task of job } i\}.$

Given a o_e date d(i) associated with each job i lateness of job i is defined by

L(i) = y(i)-d(i) for i=1,...,r.

A linear time algorithm is presented which solves the problem of finding a schedule which minimizes maximum lateness.

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AN IN/OUT GREEDY METHOD FOR THE OPTIMUM

LOCATION SELECTION PROBLEM

The optimum location selection problem may be defined as:

i=1,2,...,m. $x_{ij} \neq y_j$, x_{ij} , $y_j = 0, 1, i=1, ...,m, j=1,...,m$.

That is: m given items are to be assigned to n given locations, such that an item can be assigned to one and only one location, but more than one item may be assigned to the same location. The cost of assigning the ith item to the jth location is P_{ij} , and setup cost at the jth location is c_i .

The in/out greedy method may be stated in the form of an algorithm as follows: <u>Step 1</u>: Initiate the algorithm by assigning all the items to the single best location, or assigning each item to the location where the cost p_{ij} is the lowest. <u>Step 2</u>: (a) For each of the unselected locations, calculate the change in the value of the objective function when it is added to the set of locations already selected. (b) For each of the locations already selected, calculate the change in the value of the objective function when it is deleted. (c)Determine whether adding a location or defeting one is the best course of action. <u>Step 3</u>: Update everything for the next iteration. <u>Step 4</u>: Stop when the value of the objective function be further reduced.

To test the performance of the in/out method, 100 problems are randomly generated, each by a different random number generator. The p_{11} 's were drawn from a uniform distribution in the (0,150) interval and the c_1 's, from a series of uniform distributions with a lower bound fixed at 20 and an upper bound ranging from 30 to 100. Both the in/out method and the greedy method are applied, and the optimum solutions were found by direct search. To 50 of those problems, the in/out method is also applied where the initial solutions are based on all locations. The results of the experiments are as follows:

A. <u>Performance</u> In 74 out of the 100 problems, the solutions obtained by the in/out method are optimum. For the remaining 26 problems, the costs of the in/out solutions are from .01% to 9.36% higher than the corresponding optimum costs, but the average is only 2.53%.

B. <u>In/out vs. the Greedy Method</u> In 81 problems, the in/out solutions are the same as the greedy solutions; and 57 of these solutions are optimum. Thus 74% of in/out solutions are optimum while 57% of greedy solutions are likewise. (For the other 19,in/out are better; 17,optimum) C. <u>One Location vs. All Locations As Initial Solutions</u> Percentage of optimum solutions: 76%, 84%. Maximum deviation from optimum cost: 5.16%, 2.39%. Average deviation from optimum cost for non-optimum solutions: 2.39%, 1.16%. Average number of iterations 5.4, 3.8.

- 160-

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ON THE USE OF CODING THEORY IN BINARY PROGRAMMING

Consider MAXIMIZE $f(x_1, x_2, ..., x_k)$ under the restrictions $x_i = 1$ or -1 for all $i \in \{1, 2, ..., k\}$ and $f(x_1, x_2, ..., x_k)$ is a polynomial function involves real numbers $a_1, a_2, ..., a_n$ as its coefficents. We shall call fa pseudo-bipolar function. It is noted that a pseudo-Boolean function g($y_1, y_2, ..., y_k$) with binary (0,1) variables $y_i \in \{0, 1\}$ is easily transformed into a pseudo-bipolar function by letting $y_i = (1 - x_i)/2$ for all i. Without loss of generality f can be written as

$$f(\mathbf{x}_{1}, \mathbf{x}_{2}, \dots, \mathbf{x}_{k}) = \sum_{\substack{i=1 \\ i=1}}^{k} a_{i} \mathbf{x}_{i} + \sum_{\substack{j=1 \\ j=1}}^{k} a_{j} \frac{\pi}{i} \mathbf{x}_{k}$$
(1)

where I_j is a subset of $I = \{1, 2, \dots, k\}$ for each $j \in \{1, 2, \dots, n-k\}$.

Obviously if I_j is empty for all $j \in \{1, 2, ..., k\}$ then (1) reduces to a linear function and a desired solution can be obtained by letting $x_i = 1$ when $a_i > 0$ and $x_i = -1$ when $a_i \le 0$. It is the nonemptiness of I_j that hinders a quick solution on $x_1, x_2, ..., x_k$, by the signs of coefficients a_1 , $a_2, ..., a_k$, respectively.

Define $x_{k+j} = \lim_{\ell \in I_j} x_{\ell}$ for $j \in \{1, 2, ..., n-k\}$ then (1) is transformed into $f(x_1, x_2, ..., x_k) = \prod_{j=1}^{n} a_j x_j$ with $x_{k+1}, x_{k+2}, ..., x_n$ dependent on

 x_1, x_2, \ldots, x_k . Based on coding theory [1] it is possibly shown that f can also be regarded as a function of other sets of selected k variables, with the rest variables dependent on selected k variables. If we can maximize f under a new set of k variables, the desired solution of x_1, x_2, \ldots, x_k can herebly easily be deduced.

According to the absolute values of a_i 's, a method based on coding theory to find a set of k variables is given. The desired solution on these k variables will generally be easier to determine. Morever, it can be shown that when certain constraints on coefficients and I_i , $1 \le j \le n-k$, are satisfied, coding theory will provide the desired solution in a single test [2].

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- 161-

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AN APPROXIMATE SOLUTION FOR THE PROBLEM OF OPTIMIZING THE PLOTTER PEN MOVEMENT

An efficient way of drawing a figure by a mechanical plotter is proposed, where the figure is a (generally unconnected) graph with straightline edges, the coordinates (abscissae and ordinates) of whose vertices are given. The objective is to minimize the plotting time T, which depends on the number N_1 of raising and lowering the pen, the number N_2 of unit pen movements (a unit pen movement being either a movement for drawing an edge with the pen on the paper or a movement from a vertex to another with the pen off) and the total length L of pen movement.

If the graph is connected, N_2 can be minimized by matching (with new edges) the odd-degree vertices in pairs, and N_1 by traversing the Eulerian path (with the pen off the paper when moving along new edges). For those purposes there is an algorithm linear in space and in time.

To minimize L we need the minimum-weight perfect matching on the complete graph (whose vertices are the odd-degree vertices of the original graph and the weights of whose edges are the distances between their end vertices). The known exact algorithm running in time $O(n^3)$ (n: the number of relevant points) is too complex for large problems and even the known approximation algorithms are of complexity at least O(n log n). We propose a family of linear-time approximation algorithms for this matching problem. The algorithms divide a square or rectangular region, within which the relevant points lie, into small square cells (or buckets) of equal size, scan those cells one after another according to a prescribed order, and match the points in pairs as soon as two unmatched points are found during the course of scanning. (There are several variants in scanning and matching.) The worst-case performance of the proposed algorithms is analyzed by means of linear programming approach. The averagecase performance is also analyzed theoretically; the theoretical results show fairly good agreement with the experimental results for a large number of randomly generated patterns with up to 2048 points.

For an unconnected graph, a similar approximation algorithm is proposed.

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NETWORK FLOW CONTROL BY 0-1 SEQUENCES

A state space model for time varying flows in networks is presented of type:

$$X_{n+1} = F(X_n, V_n, Q_n) \qquad n = 0, 1, 2...$$

where the state vector X_n is composed of queue, flow, and capacity variables. No external constraints are needed to keep the queue variables bounded. V_n represents the control action and Q_n the arrival inputs at the network entrances.

Particularly relevant to urban vehicular traffic networks, the control action at nodes is in the form of 0-1 sequences with bounds on the number of consecutive zero or ones in a sequence. It is assumed that the performance index is of the additive type, i.e.

$$PI = \sum_{n=1}^{N} H(X) \qquad H(.) \ge 0$$

Some aspect. of the search for optimal sequences at a node are inspired from variational techniques in Optimal Control Theory. Furthermore the search is made efficient by the elimination of redundant steps in the associated network flow simulations.

A strategy for on-line control of large networks is presented involving receding horizons of optimization and decentralized control achieved via parallel optimization of overlapping subnetworks. Numerical experiments have exhibited a satisfactory performance for this type of control. Aaron Kershenbaum

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GENERALIZED AUGMENTING PATHS FOR THE SOLUTION OF COMBINATORIAL OFTIMIZATION PROBLEMS

We consider the generalization of the concept of an augmenting path and its application to the exact solution of a class of combinatorial optimization problems which are NP-complete. In particular, we consider the solution of problems which can be solved as the intersection of three matroids. A matroid is a 2-tuple, (E,I). E is a finite set and I is a family of independent subsets of E, where we require that all subsets of an independent set be independent and that for any two independent subsets, $I_{\rm p}$

and I_{p+1} , containing p and p+1 elements, respectively, we be able to find an element in I_{p+1} but not in I_p which can be added to I_p without destroying independence. Given k matroids defined on the same set E, we difine their intersection to be the family of sets which are independent in all k matroids. Alternating chain procedures consider the inclusion of "best" elements and the removal of a prevoiusly chosen element only if that removal immediately permits the inclusion of a sufficiently attractive element which would otherwise be excluded. For problems involving the intersection of two matroids, such alternating chains, which are the simplest augmenting paths, produce optimal solutions and algorithms which have runtimes polynomial in the number of inputs. We consider the generalization of this concept to other paths by temporarily maintaining independence in only two of the matroids of three and by considering augmenting trees in order to produce optimal and near-optimal solutions Ne examine tradeto three matroid intersection problems. offs between runtime and solution quality.

-164-

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REGULARIZATION OF COSTS AND MEASURES OF AGREEMENT FOR THE LINEAR MODEL OF PREFERENCE AND SIMILARITY AGGREGATION

The Preference and Similarity Aggregation problems can be defined respectively as the research of a "collective linear order" or a "collective partition" on a set of n "objects", summarizing individual preferences or similarities given by m "judges" on these n objects.

We have developed in [1] and [2] an exact Linear Programming method for solving these problems using the "symmetric difference distance".

The associated objective function is defined by:

ΣΣC', Y,	where c_{ij}^{i} = The number of "judges	11
ij≠i ^{⊥, ⊥,}	putting i in relation with j minu	8
_3; _	the number of judges not doing so	•

 $y_{ij} \in (0-1)$ representing the unknown collective relation subject to linear constraints.

We have defined equivalent (regularized) costs:

	c" = 1/2(c' ij	<pre>-c') for preference aggregation ji</pre>	
or	c" = 1/2(c' ij ij	<pre>+c') for similarity aggregation ji</pre>	
to	take into account	t non significant opinions of the j	udges
as	well as missing	iniormation.	

In addition to these results "concordance" and "coherence" indexes are proposed as "a priori" and "a posteriori" measures of aggregation quality.

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-165-

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SYNTHETICAL ANALYSES ON DOMINANCE TESTS IN A CLASS OF ENUMERATIVE ALGORITHMS

Most solution algorithms in combinatorial optimization are not universal, but solve well a specific type of problems, or specific problem instances of a type of problems. Hence, we often have to design an efficient algorithm for a specific problem on hand. It seems desirable to construct a general algorithmic framework, which includes as special cases similar but unknown algorithms as well as branch-and-bound methods(BB) and dynamic programming(DP), and to derive conditions under which they are efficient. Tree programming(TP) is a framework for this aim[1].

Dominance test(DT) is a type of devices for pruning enumeration, and is an abstraction of the optimality principle in DP and the bound test in BB. Relations between efficiency of TP and strength of its DT, i.e., a static measure of the amount of pruned enumeration, are discussed. We clarify some sufficient conditions under which efficiency of a TP instance is nondecreasing when a stronger DT is used.

In the derived conditions, the following concepts play substatial roles. (1)Tested ordered-pairs of subproblems: DT is a binary relation based on comparisons of two subproblems. Testing all pairs is usually time consuming, and only a part of the pair set is selectedly tested. The contents of the selected subset determine actual effect of the test. (2)Cooperativity of a selection rule with DT: Dominating subproblems must be generated earlier than the time when subproblems dominated by them are decomposed into finer subproblems. This is governed by a selection rule which is a device in TP for deciding the next subproblem to be branched from. (3)Independency of DT: An elimination rule in a TP instance can be a mixture of DT and some other types of tests. Their interactions must be negligible, otherwise some very restrictive relation among them should be assumed. (4)Selfcontainedness: DT is said selfcontained, if it is both transitive and hereditary. The bound test can be shown to be selfcontained.

Approximatization of the bound test is a widely used strategy to accelerate a search. This strategy is examined by a similar method used in discussing exact TP. It is shown that efficiency of a TP instance is nondecreasing if degree of approximation is in a specific relation with the value of the objective function for the current incumbent solution.

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COMPLEXITY OF ALGORITHMS FOR THE BOTTLENECK ASSIGNMENT PROBLEM

The bottleneck assignment problem, given on a rectangular m x n $(m \le n)$ matrix C of nonnegative real numbers, is considered. We assume the matrix C to be sparse, i.e. some elements c,, are undefined. A diagonal /an assignment/ of a matrix C is any collection of m defined elements, not two of them being in the same row or column. The bottleneck assignment problem consists in finding a diagonal with the greatest value element being as small as possible, where minimization is done over the set of all diagonals.

The classical assignment problem is defined on a square and complete matrix C. A generalization of the problem we are considering is motivated by some practical tasks we have met. The main purpose of this work is to analyse the time complexities of different algorithms and to demonstrate methods which guarantee the lowest power o polynomials which estimate the worst-case behaviour.

Two methods are roughly distinguished : the augmenting path method and the threshold method. We shall show that a threshold approach combined with proper choice of data structures yields algorithms with the best bounds on the running time.

We also give a detailed discussion of possible rules for threshold selection, and show the influence of the problem parameters like : m,n . min $\{c_{ij}\}$, p n/m . max { c_{ij}} / 1.1

/the number of defined elements in the matrix/ and so on.

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RANK, CLIQUE - AND CHROMATIC NUMBER OF A GRAPH

Let $\omega(G)$ be the clique number and $\gamma(G)$ the chromatic number of a simple graph G (i.e. undirected, no loops, no multiple edges). It is well-known that the computation of $\omega(G)$ as well as $\gamma(G)$ is an NP-complete problem. For that reason it is of interest to look after simple bounds, which are easy to compute. We propose the following two bounds in connection with the rank of G.

The rank r(G) calculated in R of the (0,1)-adjacency matrix (vertex-vertex-matrix) of the simple graph G is called the rank of G.

A bound for $\omega(G)$ is given by the following statement : For a simple graph G (with no isolated vertices) we have : $\omega(G) \leq r(G)$ and $\omega(G) = r(G) = k$ if and only if G is a complete k-partite graph. This bound is not included in the several bounds known for $\omega(G)$.

Otherwise we have the following conjecture for $\gamma(G)$: Let G be a simple graph (with no isolated vertices) then : $\gamma(G) \leq r(G)$ and $\gamma(G) = r(G) = k$ if and only if G is a complete k-partite graph. This upper bound is not included in the several known bouds for $\gamma(G)$.

In this note we prove the conjecture for values of $\gamma(G)$ up to 5 (even for infinite large graphs) and the same technique may be used for larger values of $\gamma(G)$.

Moreover this proof-technique shows implicitely why the computing of $\gamma(G)$ is an NP-complete problem.

Keywords : CLIQUE NUMBER, CHROMATIC NUMBER ADJACENCY MATRIX, NP-COMPLETENESS

-168-

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EXTENSIONS OF SOME TWO-DIMENSIONAL BIN PACKING ALGORITHMS

In the two-dimensional bin packing problem, a list L of n rectangles is to be packed into a minimum number of l x l bins. The rectangles are to be placed with their edges parallel to the sides of the bins, and no rectangles should overlap in any bin. Several popular two-dimensional packing algorithms have previously been developed which pack rectangles into a single bin of unit width and infinite height. In addition, worst case error bounds for the methods were proved.

We now modify and extend these algorithms to the problem of packing bins having finite dimensions, and we consider cases where rotations of the rectangles are allowed in the optimal packing. We also derive inequalities which bound the worst case behavior of these allocation procedures when they are used to pack 1 x 1 bins. The packing methods discussed are extensions of the leveloriented and/or bottom-up left-justified approaches which have previously been referred to as the Next Fit Decreasing Height, First Fit Decreasing Height, and Bottom-Up Left-Justified algorithms.

COMPUTATIONAL COMPLEXITY

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COMPUTATIONAL COMPLEXITY OF THE RANDOM KNAPSACK PROBLEM

In /l/ we analyzed the Subset-Sum problem under probabilistic assumptions about the coefficients; the results are the specialization of more general techniques which may be employed in the analysis of random knapsack-type problems.

In the paper we consider the integer program

MAX
$$\sum_{j=0,1}^{n} u_{j} x_{j}$$

 $\sum_{j=0,1}^{n} v_{j} x_{j} \leq w$

(KP)

and an algorithm RED which modifies (KP) into an equivalent problem (KP') of reduced size M.

RED is based on a well-known reduction criterion using Lagrange's multipliers properties.

We show that RED has a "good expected behaviour" .

More precisely, we make the following assumptions:
a) All the coefficients are independent random variables
b) The u_j,v_j's are uniformly distributed over {1,...,c(n)}
c) w is uniformly distributed over {1,..., n·c(n)}
Then we prove that the reduced size M satsfies

(1) $\operatorname{Prob}(M \leq n^{a+1/2}) \longrightarrow 1$, as $n \longrightarrow \infty$ for any a > 0.

From (1) it is easy to derive bounds on the computational complexity of (KP).

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-171-
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ON THE COMPLEXITY OF THE FRACTIONAL STABLE SET CLIQUE, CLIQUE COVERING AND COLORING PROBLEM

Let an undirected graph G = [V,E] and a weight function $c : V + Z_+$ be given. A stable set (clique) of G is a node set $W \subset V (C \subset V)$ such that any two nodes of W are nonadjacent (adjacent) in G. The stable set problem (clique problem) is to find a stable set $W' \subseteq V$ (clique $C' \subseteq V$) such that c(W') (c(C')) is a large as possible.

Let $P^*(G) \subseteq \mathbb{R}^V$ be the polyhedron defined by the inequalities $x_v \ge o$ for all $v \in V$ and $x(C) \le 1$ for all cliques $C \subseteq V$, then it is easy to see that every integral vector in P^* corresponds to a stable set in G and vice versa. The linear programming problem max c^Tx , $x \in P^*(G)$ is called the fractional stable set problem, the optimum value of this LP clearly gives an upper bound for the weight of the optimum stable set.

It is well-known that the stable set problem is NP-complete for the class of all graphs. We shall show here that this is also true for the fractional stable set problem. However, we also show that these two problems are unrelated with respect to hariness (unless P = NP), since there are classes of graphs for which the (restricted) stable set problem is NP-complete while the fractional stable set problem is solvable in polynomial time and vice versa.

The clique problem as well as the coloring problem and the clique cover problem have natural LP-relaxations analogous to the linear program described above. It is customary to call these LP's fractional clique, fractional coloring and fractional clique cover problems. We show that the fractional versions of these problems are hard problems too, but that the determination of the optimum solutions of the fractional resp. integral versions of these problems seem to be incomparable with respect to hardness.

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-172-

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A VERSATILE INTERCONNECTION PATTERN LAID ON O(n) AREA

An important interconnection problem is the one of reaching n points by a line pattern that includes branching nodes. For example, this problem is crucial in VLSI layout, where n elementary processors must be reached in parallel by a message broadcast from a source onto a planar connecting network, shaped as a binary tree.

The standard tree layout is rectilinear, H-shaped and laid on O(n) area; it allows to reach all terminal nodes after $\log_2 n$ branchings. Whereas the area becomes $\Omega(n\log_2 n)$ if all such nodes must be accessible on the boundary.

In this note we introduce new H-type layouts with exactly the same area of the standard H layout. In addition to the original broadcast capabilities, the new layouts provide faster access to subsets of terminal nodes. In particular:

- 1. Direct access is provided to $4\sqrt{n-6}$ subtrees of the original tree, by modifying the routing capabilities of branching nodes.
- 2. Tree access is provided to $2\sqrt{n-6}$ subsets of terminal nodes, which are not contained in subtrees of the original tree, by laying a new family of independent H-shaped trees penetrating in the empty spaces left by the main tree connection.
- 3. Some tree reconfigurations are obtained by putting different branching nodes at the root, and modifying the routing structure. In particular, the tree can be divided into a set of reconfigurable subtrees, accessed independently.

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ON CONVERTING DECISION TABLES INTO MINIMUM DECISION NETWORKS

Converting decision tables into sequential form or decision trees has a practical significance, and hence optimization problems of resulting decision trees have been studied extensively under various optimality criteria. We consider the problem of getting a minimum (number of decisions) network by a branch-and-bound method (the same subtrees are pasted together into single subtree by confluences of arcs, thus resulting in networks instead of trees [1]).

1. We propose the following slightly strengthened than [1] evaluation for the lower bound of the cost:

we evaluate the possibility for minimal subtables to be pasted in the left node subtables; thus the evaluation results will include, in addition to a definite lower bound, some probable quantity which reflects the possibility of pasting.

2. Some experimental results will be presented to discuss the computational trade-off between proposed method and a simple method where the possibility of pasteing is not counted in.

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- 174-

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THREE ALGORITHMS TO TIME PROPAGATE COVARIANCE MATRICES: COMPARISON OF THEIR COMPUTATIONAL COMPLEXITY

Three numerical algorithms for computing the solution of the covariance matrix differential equations of states of a linear timeinvariant dynamical system forced by white Gaussian noise are analyzed. The system and noise model are described by

$$\ddot{\mathbf{X}}(t) = \mathbf{F}\mathbf{X}(t) + \mathbf{W}(t)$$

where, X(t) is an n-state vector and W(t) is a White Gaussian noise with

$$\mathbf{E}[\mathbf{W}(\mathbf{t})] = \mathbf{0} \operatorname{cov}[\mathbf{W}(\mathbf{t})] = \mathbf{Q}\delta(\mathbf{t}-\tau)$$

and

$$E[X(0)] = 0 \operatorname{cov}[X(0)] = P(0) = P_0$$

The covariance of the state vector X(t) satisfies the following matrix differential equation

$$\mathbf{\dot{P}}(\mathbf{t}) = \mathbf{FP} + (\mathbf{FP})^{\mathrm{T}} + \mathbf{Q}$$

 $\mathbf{P}(\mathbf{0}) = \mathbf{P}_{\mathbf{0}}$

Three algorithms are used to compute P(t) which are stated below.

ALGORITHM-1

$$P(t) = \Phi(t) P_0 \Phi^T(t) + M(t)$$

$$M(t) = \sum_{i=1}^{N_1} Q_i \frac{t^i}{i!}$$

$$Q_{i+1} = FQ_i + (FQ_i)^T$$

$$Q_1 = Q$$

$$\Phi(t) = \sum_{i=0}^{N_2} \frac{(Ft)^i}{i!}$$

ALGORITHM-2

$$\mathbf{\tilde{S}}(t) = \mathbf{FS} + (\mathbf{FS})^{T} + \mathbf{Q}; \mathbf{S}(0) = 0, t \in (0, \Delta)$$

so that

$$\begin{split} & S(\Delta) = \sum_{i=1}^{N_3} Q_i \frac{\Delta^i}{i!} \\ & Q_{i+1} = FQ_i + (FQ_i)^T \\ & Q_1 = Q \\ & S(2^{K+1}\Delta) = S(2^K\Delta) \\ & + \Phi (2^K\Delta) S(2^K\Delta) \Phi^T(2^K\Delta) \\ & \Phi (2^{K+1}\Delta) = \Phi (2^K\Delta) \Phi (2^K\Delta); \\ & k = 0, 1, 2, \dots p \cdot 1 \\ & P(t) = P(2^p\Delta) = S(2^{p-1}\Delta) \\ & + \Phi (2^p\Delta) P_0 \Phi^T(2^p\Delta) \end{split}$$

$$\frac{\text{LGORITHM}-3}{S(\Delta) = U^{T}(\Delta) U(\Delta)}$$

$$S(2^{K+1}\Delta) = U^{T}(2^{K}\Delta)U(2^{K}\Delta) + \Phi(2^{K}\Delta)U^{T}(2^{K}\Delta) U(2^{K}\Delta) U(2^{K}\Delta)\Phi^{T}(2^{K}\Delta)$$
$$\left[\underbrace{U(2^{K+1}\Delta)}_{0} \right] = H(2^{K}\Delta) \left[\underbrace{U(2^{K}\Delta)}_{U(2^{K}\Delta)\Phi^{T}(2^{K}\Delta)} \right]$$
$$k = 0,1,2,..., p - 1$$
$$P(2^{p}\Delta) = U^{T}(2^{p}\Delta)U(2^{p}\Delta)$$

Corresponding measure of performance in terms of computational complexity and truncation and round-off error bounds have been derived. The results are illustrated by means of two numerical examples.

- 175-

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DETECTING NUMBER SYSTEMS AND FUNCTIONS: OPTIMIZATION, ALGORITHMS AND FIELDS OF USING

A new type of number-theoretic combinatorial objects, viz. finite Detecting Number Systems (D-systems), in particular Additive ones (ADS), is introduced and studied. The main problem for ADS, consisting in determining the natural density function and in finding the corresponding extremal systems, from mathematical programming point of view states as the following: define $F(x) = \max_{B_k}(x)^k$ under the constraints $B_k(x) = \{b_i : 1 \le b_1 \le \dots \le b_k\}$

 $... < b_{k} \leq x \& b_{i} \in N \& \sum \mathcal{E}_{i} b_{i} \neq \sum \omega_{i} b_{i} \& \mathcal{E}_{i}, \omega_{i} \in \{0, 1\} \&$ $\exists \mathcal{E}_{i} \neq \omega_{i}\}, \text{ or, in more traditional way, minimize } G(x) =$ $\sum_{i=1}^{k} d_{i}, \text{ where } d_{k-i} = b_{i+1} - b_{i}, b_{0} = 0, i = \overline{1, k}, \text{ and } b_{i} \in B_{k}(x). \text{ The problem of } F(x) \text{ estimating is well known for a long time as Erdös' $300 one. }$

For posed problem solution two ADS families, viz. of Aloian -Lindström and Conway - Guy types, are examined. Fast and parallel algorithms, specialized software to fulfil numerical experiments for D-systems and to use dense ADS are considered.

The important fields of ADS applications are:

1. Non-concatenated coding, i.e. non-prefix and non-suffix general coding, by ADS can be effectively used in fast response data bases, such as in environmental monitoring systems, for multi-attribute (key, descriptor) integer representation of retrieval patterns in computable (direct) access. That coding is adapted for both uni-modal (like log-normal) distribution of attributive subset's frequences within entering query stream depending upon their powers and open attributive alphabet.

2. Detecting Number Functions (DNF), which are continuous analogs of ADS, are used to detail initial messages in data bases by introducing additional letters into words, so that it does not exessively modify original word codes. DNF have been initially introduced in connection with Kolmogorov's solution of the 13th Gilbert problem, and constructive considerations allow to obtain the most economical DNF from computational point of view.

3. Pattern recognition, mathematical statistics and search methods for numerical solution of extremal problems use DNF as multi-dimensional scanners similar to known Peano curve and Gilbert filling curve for reduction of dimension of object's domains. DNF numerical realization is based on arbitrary precision arithmetic algorithms with virtual computer memory [1].

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SOCIO-ECONOMIC MODELS

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SIMULATION OF THE COMPETITIVENESS OF INDUSTRIAL ESTATES FOR REGIONAL DEVELOPMENT

Since 1973 the Mexican Federal Government¹ has fomented an indus trial estates programme, in order to promote regional industrial development and encourage small and medium-sized bussiness enterprises.

One of the actual problems is the design of development strategies² for a better realization of the programme's socio-economic objectives.

In order to evaluate alternative strategies, DESIR was constructed (DESarrollo Industrial Regional - Regional Industrial Development). DESIR is a simulation model of a competitiveness of an industrial estates in a community.

DESIR was constructed in systems $dynamics/DYNAMO^3$ based on four substructures: (1)Demand for industrial land, (2)Supply of industrial land outside of the industrial estate, (3) Supply of industrial land in the industrial estate, and, (4) Decision.

In order to analyse the behaviour of the model, DESIR was applied to the city of Aguascalientes⁴, in the provinces of Mexico.

Altough DESIR is still in the developing stages it can be seen that: (i) the conceptual model identifies key factors in the compe titiveness of an industrial estate, (ii) it is an aid in the evalua tion of alternative strategies of development of industrial estates.

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CONSTRAINED RANDOMIZATION - A LINEAR CASE

Our interest in randomization stems from our research in the application of random search techniques to mathematical programming and other "optimization" problems. We have argued elsewhere (1) that the need formulate practical problems in a computationally tractable way often leads to significant distortions in the mathematical description of the problem, and (2) that, in some instances, it might be better to formulate the problem with greater realism, and to solve the (possibly less tractable) problem formulation with random search. Here a "solution by random search" means selecting the best of a finite set of randomly generated feasible solutions as the most preferred solution. One way of generating such solutions is to generate, for each decision variable, a random value between appropriate limits, and to check the group of variables so generated for feasibility. Although this method was successful in the case of some problems involving binary variables, it would be advantageous to generate random values that satisfy one or more of the constraints affecting the problem. As a useful step in this direction, we have developed a formula for generating constrained random variables satisfying

> $\sum a_i x_i \le b$ $x_i \ge 0, a_i \ge 0, \forall i$ i = 1, 2, ... n

i.e. for a random solution lying in any finite dimensional vector space bounded from above by a hyperplane (with positive coefficients) and from below by the positive orthant. Rafael Bras, Sc.D. Donald Grossman, Ph.D. * Cambridge, Massachusetts

CONTROL OF URBAN WATER SUPPLY

A stochastic control model and a simulation model for providing generalized guidance for day-to-day water systems operations were developed and applied in case studies for Brockton, Massachusetts. Operational controls included are magnitude of demand supplied from surface sources, pumpage from groundwater, and import. The physical system representation is as a lumped soil column, with unsaturated (soil moisture) zone, a saturated zone, and associated surface storage. For the optimization, the criteria for control is minimization of the expedted cost of operation, adapting to imperfectly observed and forecast system states.

The basic input to the system is precipitation; water leaves the system when it is used to satisfy demands, through evaporation, or as base flow to groundwater. Demand targets and precipitation characteristics such as infiltration, exfiltration, seepage, evaporation, baseflow, and so forth are related to the levels of soil moisture, water in the saturated zone (i.e. groundwater storage), and water in surface storage. The physical model, a mathematical model of observations, a filter which combines observation and model results to estimate the state, and a controller comprise the control algorithm.

The models have been applied in hypothetical case studies for Brockton, Massachusetts. These demonstrated that the physical system representation, in a relatively complex hydrologic case, can be successful using the simplified assumptions of the model. It has also shown that the mdoel is beneficial in providing strategic guidance for water system operation. This appears to have great potential utility for water systems facing droughts, where the managers must make difficult choices under uncertainty with respect to future precipitation inputs, observations, and system response. Finally, deficiencies in existing economic loss data were identified.

Work performed as consultants to Camp Dresser and McKee, Inc, Waltham, Massachusetts, under contract to the U.S. National Science Foundation. B. H. Burzlaff and M. G. Riskalla Operations Analysis Department Vought Corporation Dallas, Texas

ABSTRACT

Dynamic Simulation of the U.S. Fertility

This paper discusses in detail the development and testing of a dynamic simulation model of the U.S. fertility (ref 1). The model is developed using system dynamics techniques and is written in both the DYNAMO simulation language and FORTRAN. The fertility and family size decision making structure of the model has as the underlying causal elements the effects of intrafamily, peer group and total society crowding coupled with the effects of various social perception and adjustment delays. A discussion is given concerning each of the crowding effects as well as the algorithms and approximations that are used to relate those which affect family planning decisions in the model. This model is then tested and given significant credibility if not completely verified by matching historical U.S. fertility data from 1850 to present. A modified least squares algorithm for goodness of fit is discussed and the fit of model versus historical data is shown for several slightly different sets of model parameters. The sensitivity of fit to changes in various model parameters is used to obtain a range of uncertainty for the model parameters. Results are also given for more general model stability and sensitivity studies and tests.

Projections of the U.S. fertility to the year 2000 are made using the model for the range of the model parameters obtained from the historical studies. These fertility results are then fed into a population pipeline model (ref 2) to obtain projections for various population age groups. Projections are also made for other interesting population characteristics such as mean age, total births, total deaths and the ratio of retired to workers to year 2025 for several model variations.

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-181-

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WATER ALLAGEMENT IN 1960'S

Mity percent of the cultivated area in the State of U.P. has been covered by assured irrigation facilities after three decades of planning. But five million hectares of land are affected by floods and drainage during years of heavy rainfall. Desides large areas are deficient in irrigation supplies system studies aided by computor were carried out to work out optimum utilization of the existing supplies. This also accounted for the supplies available from over 1, 2 million tubewells installed in the state during the last five decades. Irrigation for the small farmer has been accorded a high priority in our beloved Prime Einister's twenty point programmes. The water availability in our country is likely to be exhausted by the year 2025 . D. with varied and increasing uses for the rising population which is expected to be doubled by the end of the century. New methods of irrigation use and reuse technology are needed to cover the gap and have a large surplus to meet the demand in lean periods as well as to keep a margin for storage and export to the neighbouring countries.

A twenty year plan has been drawn up for maximizing the utilisation of existing and upcoming resources of irrigation. This envisages conjunctive use of surface and groundwater. The deficiency in the under ground supplies in certain areas is proposed to be made up by transfer of water from one basin to another. Moreover automation of canal systems operation has been taken up in the first phase of modernisation.

The master plan is estimated to cost hs. five hundred thousand million. The master plan is proposed to be aided by the world Bank. D.Feigenbaum Cleiman*, L.R.Albrecht Ramos* and R.Paes e Barros** *COPPE/Federal University of Rio de Janeiro ** IPEA-RJ Caixa Postal 68511 21941 - Rio de Janeiro - RJ Brasil.

"A MATHEMATICAL MODEL FOR THE IMPLANTATION OF SHALE OIL EXTRACTION PLANTS IN BRAZIL"

Shale industrialization has been greatly increased due to the chnages recently occured in pertroleum prices all through the world, although many of its peculiarities still need to be studied $|^1|$.

At the first part of the present paper a synthetic view will be presented of the most important aspects related to shale oil production, concering not only technology but also environment, as well as a prospect of shale potentialities throughout the world, and particularly in Brazil.

Since shale resources in the Brazilian territory are quantitatively large $|^2|$, a model was developed to the optimization of its industrialization in Brazil which will be presented at the second part of this paper. Dynamic programming is the basis of the referred model; and its main characteristic consists on taking into account the effect of the learning process along the settling and ordinarity processes of the plants.

Generally speaking, the model's goal consists on determining the optimum strategy to the settling of plants, aiming at an economic maximization by the substitution of shale oil for imported petroleum. Thus the model can precise shale oil supply in each period of any price situation of the imported petroleum, as follows:

 $O(t) = f(t, \pi^0, \pi^1, ...)$

where:

O(t) = shale oil supply in period t;

 π^t = price of imported petroleum in period <u>t</u>;

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> PREDICTION OF SOCIO-ECONOMIC POLICY: INFORMATION GAIN BY INTERACTIVE DECISION ACTIVITIES

The error-learning principle for the prediction of endogenous variables of a system of first order difference equations

 ω_t : $y_t = A_t y_{t-1} + C_t x_t + b_t + u_t$, t=1,...,T (see Chow (1975), is normally based on the concept of rational expectations. The ex ante expectation proxies are assumed to fulfil very strong conditions on information availability which is generally not given (Friedman (1979), convergency to a equilibrium, error orthogonality, etc.).

We suggest a process of interaction between model structure and policy authority which yields successively more information. In each stage of the ex ante prediction, the two 'nlayers' are mutually confronted with their results. Let $\{e_{1}^{i}+1\}_{i=1,...,N}$

be a selected set of alternative control variables and $a_{T+1}^{i} = R(\omega_{T}(e_{T+1}^{i}))$ the associated endogenous variables in the reduced form R, then we formalize the decision behavior of individual decision makers by using preference values $U(a_{T+1}^{i}) = \alpha [\Sigma_{j}(|a_{j,T+1}^{i}-y_{j,T}^{i}|)^{-1}+) [\gamma(T)|P_{T}^{i}-P*I)^{-1}] + \varepsilon_{i}$, where the second term on the right side is a measure for the popularity degree of the policy authority, and ε_{i} is a maximum-stable random variable which has a Weibull distribution.

Let p_{i*} be the highest logit probability, we perform one control step at time T which provides $x_{T+1} = G_T y_T + g_T$,

 $G_{T} = -(C_{T}^{i}K_{T}C_{T})^{-1}C_{T}^{i}K_{T}A_{T}, g_{T} = -(C_{T}^{i}K_{T}C_{T})^{-1}(K_{T}b_{T}^{i}-K_{T}a_{T+1}^{i*}),$

 K_T =diag(p_{i*}, \ldots, p_{i*}), and $y_{T+1} = \omega(x_{T+1})$. After adapting e_{T+1} and the model specification, the control step is repeated until a Pareto compromise is found. Rolling forward the sample by one period, the next step of estimation and prediction will be done analogously.

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- 184-

QUALITY ASSURANCE SPECIFICATIONS FOR TIME-DEPENDENT AEROMETRIC DATA

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The purpose of this paper is to describe some methodological considerations in developing quality assurance monitoring specifications for time-dependent aerometric data. Simulation models are assist both in standards development and in evaluating the validity of models developed. Most of the models applicable to quality control assume successive observations which are independently distributed. This is often not true in aerometric data, such as date from continuous monitors from point sources. In addition, time-dependence is a function of sampling frequency and averaging time because data from continuous monitors are obtained from analog-based equipment.

In previous research reports the impact of the type of statistical models assumed, such as the lognormal or normal distributions, the magnitude of the standard deviation and autocorrelation in data have been outlined as input parameters to simulations. The present paper will integrate previous monitoring specifications of control chart techniques with those of time-series analysis. The issues will be handled from the standpoint of statistical stability in ters of parametric and non-parametric models, detection of temporary shifts versus underlying long-range shifts in the central tendency parameter, and variance reduction methods.

The differences in drawing inferences from the control chart versus time-series approach to monitoring are primarily due to the amount of information contained in each about maxima and minima. A continuous trace reveals the total time history of the process, including the peak value reached and the amount of time the level of the concentration was above some arbitrary value. If averaging is used, the discrete data do not provide this information. Furthermore, sampled data systems provide no information about what has occurred between successive observations. Random variation about a mean or average value is present in almost every system. What control charts attempt to detect is whether or not this mean has changed or whether or not the magnitude of variation about the mean has been changed. This is accomplished by observing the pattern of measurements in relation to zones marked on the control chart. All of the control chart methodology is based on the assumption that successive readings represent independent observations.

It is not reasonable to assume independence with aerometric data. Successive 15-minute readings of pollutant concentrations, whether they are averaged or sampled are part of the same, slowly varying system. Before control chart methodology can be applied, it is necessary to first determine whether observations are independent and, if they are not, determine the mathematical transformation which will convert them to independent observations. P. LEDERER, P. COLLETER, J. ORTMANS
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LONG TERM NUCLEAR SCHEDULING IN THE FRENCH POWER SYSTEM

Scheduling the maintenance and refueling of PWR units in the French power system sets a large scale, dynamic and stochastic problem.

The operation of the generating equipments is traditionnaly organized inside of a yearly-cycle which deals with the decisions concerning the dynamic regulations of the system :

- operation of the large seasonal reservoirs,

- maintenance policy of thermal plants.

PWR units, however, introduce new constraints which are directly related to the characteristics of this technique : it is necessary to take the reactor down about <u>two months</u> each year for refueling and maintenance. Moreover :

- due to the availability of maintenance staff, two units of the same plant cannot be simultaneously refueled (most of the french plants include two or four units),
- the length of each cycle as well as the flexibilities which are available around this length depend on the lengths of the preceeding cycles.

It is thus necessary to follow the operation of <u>each reactor</u> during several years, taking into account its availability so as to make sure that :

- the schedule will be feasible with respect to the constraints,
- the operating costs to be minimized are correctly computed over a long period.

The paper introduces a formalization developed at E.D.F. for this problem. The model is based on dynamic programming, relaxation techniques and intermittent closed-loops.

-186-

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ON THE ROLE OF THE IMPULSE FIXED COST IN STOCHASTIC OPTIMAL CONTROL. AN APPLICATION TO THE MANAGEMENT OF ENERGY PRODUCTION

In the first part of this paper we consider the use of stochastic control methods for the management of an electricity production system. The system consists of power plants and dams (1). The variables to be optimized are the starting time of power plants and the quantities of turbined water. Because of the complexity of the system and the presence of many stochastic phenomena a two-step model is used: 1) a simplified large-run model in which the starting up cost of the thermal power plants is neglected; 2) a short-run model which models more precisely all the phenomena and which uses as inputs some outputs of the large-run model.

We justify the simplification used in the large-run model and we give an upper-bound for the error introduced in the optimal management policy.

In the second part, we consider a general stochastic impulse control problem and we study the continuity of the optimal cost function V_k (x, t) when the impulse fixed cost k tends to zero.

The analysis is achieved under different sets of hypothesis. We present several results of (point wise or uniform) convergence of minimizing sequences. We give in all cases when some convergence is obtained an upper bound for the error (variable with k).

The results can be used in the framework of singular perturbations methods to study partial differential inequalities (2).

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FUNCTIONAL SENSITIVITY ANALYSIS OF MATHEMATICAL MODELS

When ordinary differential equation models of physical, biological or socio-economic processes are devised, there are almost always mathematical expressions or functions in the equations whose choice is somewhat arbitra= ry or uncertain. This paper presents a quantitative theory of prediction sensitivity to both finite and infinitesimal functional perturbations in the uncertain expressions.

We consider nominal models in the form

 $\dot{x} = F(t,x,f(t,x)), \quad t_0 \le t \le t_1, \quad x(t_0) = x_0$ (1)

in \mathbb{R}^n (or on a smooth manifold), where t_0, t_1, x_0 are fixed data and $F(\cdot, \cdot, \cdot)$ is a fixed function, while f(t, x) represents a vector of expressions or functions which are regarded with suspicion, and which are to be perturbed in the sensitivity analysis. The nominal prediction is the final point $x(t_1)$ of the integral curve of (1). For any other function g(t, x), we regard the equations

$$y = F(t,y,g(t,y)), \quad t_0 \le t \le t_1, \quad y(t_0) = x_0 \quad (1_f)$$

as representing a possible alternative model, with a corresponding perturbed prediction $y(t_1)$.

In order to gauge the sensitivity of the nominal model we require the modeller to choose a Riemannian distance function $d(\cdot, \cdot)$ in state space for which $d(x(t_1), y(t_1))$ is a suitable measure of the prediction perturbation. The corresponding norm $\|\cdot\|$ is then used in defining the quantity

 $m(g) = [(t_1-t_0)\int_{y(\cdot)} ||F(t,y,g(t,y))-F(t,y,f(t,y))||^2 dt]^{\frac{1}{2}},$

which we adopt as a suitable measure of the model perturbation. Our strategy for characterizing sensitivity is then simply to find out how large a model perturbation is required to produce a given prediction perturbation. Thus for any given point z near $x(t_1)$ we define

 $r(z) = \inf\{m(g) | y(t_1) = z\}/d(z, x(t_1)), z \neq x(t_1)$

and call $r(\cdot)$ the insensitivity function. Clearly $r(z) \ll 1$ for some z near $x(t_1)$ indicates great sensitivity of the prediction in the general direction of z, while r(z) >> 1 indicates the reverse. The evaluation of $r(\cdot)$ requires the solution of a Bolza problem in the calculus of variations.

Prediction sensitivity to small functional perturbations can be estimated from the limiting behaviour of $r(\cdot)$ near $x(t_1)$. As $z \rightarrow x(t_1)$ along any curve with unit tangent vector ξ , $r^2(z) \rightarrow \Sigma M_i , \xi^{1}\xi^{j}$, where (M_{ij}) is a positive definite symmetric tensor which we call the *insensitivity tensor* at $x(t_1)$. The components M_{ij} may be easily computed by integration of a linear matrix differential equation, and the eigenanalysis of (M_{ij}) identifies directions at $x(t_1)$ of greatest and least sensitivity.

This paper extends previous work by the author (Appl. Math. Optim. 6, 123-137 (1980)) which considered only unrestricted model perturbations (i.e. $F(t,x,f) \equiv f$).

-188-

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WATER DISTRIBUTION NETWORK SELF-TUNING CONTROL

Control of state- measurable multivariable systems des cribed by its state equations with unknown parameters is dis cussed. A multivariable extended minimum variance strategy (EMVS) {1}for this class of systems is firts presented. It gives a generalization of the EMVS for single-output systems. A multivariable self-tuning explicit controller based on the EMVS is then proposed. It uses a recursive least squares es timator and a linear controller obtained from the solution of a set of linear simultaneous equations with the current estimates, derived from the minimality condition of the criterion.The asymptotic properties of the algorithm are discus sed.

The strategy is applied to the control of a water dis tribution network in which the boundary conditions are not readily known, because of the existence of domiciliary tanks. The tanks served from a node are modelled as a single local tank of unknown geometry. The daily water demand is represented as the sum of a known deterministic component and sto chastic stationary process.

The aim of the valve control is to distribute the water along the pipes in order to insure minimum variance of the local tanks about a desired set-pint in spite of the demand fluctuations. Simulations results showed the feasibility of the implementation of such a control scheme for large Water Distribution Networks.

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MATHEMATICAL MODELS OF THE GLOBAL DEVELOPMENT

In the paper four global models (by J. Forrester, D. Weadows, M. Mesarović and A. Herrera) are studied in their full mathematical versions. The critical evaluation of these models is given. Special attention is paid to the problem of control and management in the global models.

The critical questions considered in the global model ling is whether economies can afford in terms of resources, time and capital, the required strategies during the long -- term transition to sustainable systems. The paper surveys recent research in economic theory in view of the specifics of global modelling. The global models are specified and compared in terms of

- a)the qualitative properties of system behaviour (controllability, reachibillity, observability, stability, etc.),
- b) the procedure for controlling system behaviour (decentralised and hierarchical control, stabilizing feedback),
- c) the information structure.

Moreover, the paper discusses additional phelomena frequently encountered on economic global models:errors and uncertainties, model estimation, instrumental stability, intermediate target policies.

The conclusion lists new areas for applications and further research in global modelling in view of system theory, optimal control and mathematical economics. F. van der Ploeg and M.R. Weale Department of Applied Economics University of Cambridge Cambridge CB3 9DE, U.K.

CONFLICTS OVER NORTH SEA OIL PROFITS AND MACRO-ECONOMIC POLICY

Many interesting problems in economics arise from conflict between competing pressure groups, therefore the conventional framework of economic planning with one central decision maker is rejected. Instead a theory of bargaining with K players is proposed to reconcile the multitude of conflicting interests. The resulting compromise corresponds to a Nash equilibrium, hence no pressure group has an incentive to cheat when the other agents stick to the bargain. This strategy is a non-zero sum differential dynamic game and is established either by rational anticipation of finite conflict or pre-play round-table discussions.

This theory of the collective bargain is illustrated with an analysis of conflict between the British government and the firms extracting oil from the North Sea. The government wishes to achieve internal and external balance within the constraint of a feasible public sector borrowing requirement. The oil firms aim to maximize expected profits, but at the same time aim to survive for as long as possible. The macro-economic and industrial implications of this conflict are analysed with the aid of a large-scale multisectoral dynamic model of the U.K. economy (see (1)). The results are evaluated within the framework of the theory of exhaustible resources (see (2)).

The computation of the bargain and the optimization and simulation of large-scale nonlinear econometric models, required for this exercise, poses an interesting computational challenge. The paper therefore includes a brief discussion of the employed algorithms (see also (3)).

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-191-

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THE GATHERING OF A COMMODITY THROUGH A PERTODIC MARKETING RING

We present herein some results concerning one form of a periodic marketing system. Such systems occur commonly in third-world countries. We assume a ring of markets that open on a time-staggered schedule during a marketing week of n days. Exactly one market is open on each day of the week, the first n-l of them being rural markets and the last one being an urban market. Traders pass through this ring buying up a commodity in the rural markets and then converge upon the urban market where they sell their acquisitions to wholesalers.

In an economic analysis of this integrated system of markets, the traders are the key agents. Their behavior is modelled by treating them as profit maximizing firms that supply the service of transferring ownership of the commodity over space and time. While operating in the rural markets, each trader maintains an expectation of the next price in the urban market. This price expectation coupled with the costs incurred by the trader yields a demand function that determines how much of the commodity the trader will buy in each of the rural markets.

As for the traders' behavior in the urban markets, we construct two models. In the first one, which we call the "no-storage model", the traders are assumed to accept whatever price is available in the urban markets. The second model, which we call the "storage model", allows the trader to store goods if the price is unacceptably low in the urban center. This behavior is based upor a storage schedule for each trader that allows the trader to adjust the amount he stores in accordance with expected future prices in the urban center. In fact, it allows the trader to speculate in the commodity.

Market clearance in each market yields a set of simultaneous nonlinear difference equations which determine the variations of all the prices and commodity flows in the marketing ring. With this dynamic model in hand, we prove that the no-storage model has a unique equilibrium state. Moreover, that state is asymptotically stable when certain conditions on the elasticities of the supply and demand functions are satisfied. For a perishable staple food, these elasticity conditions are not likely to be satisfied, which leads to the conclusion that periodic marketing networks that are adequately represented by our model tend toward instability. Erratic price behavior has been noted in the literature on periodic markets. We also establish the existence of a unique equilibrium state for the storage model, but the asymptotic stability of that state is presently an open quection.

MATHEMATICAL ECONOMICS

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PROFIT MAXIMIZATION IN A DYNAMIC MARKET (AN APPLICATION OF OPTIMAL CONTROL)

With the growing emphasis on long term planning and increasing development of the dynamic models, optimal control theory has proved to be a suitable tool for solving many economics and management science problems. In this paper an optimum pricing rule for a profit maximizing firm, based on a general time varying demand model, in a dynamic market is obtained. A dynamic market equivalent of the well known inverse elasticity law of the static market is developed. The results are then generalized to the case of a constrained welfare maximizing firm. This has led to the introduction of a dynamic market version of the well known Ramsey pricing rule. A series of comparisons between the optimum pricing policies of the static and dynamic markets are carried on. From these comparisons factors contributing to significant differences between optimum pricing policies of static and dynamic markets are identified.

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PROJECT BUDGETING BY SMALL INFORMATION SYSTEMS

The paper describes a method for project budgeting and control of tenders developed for use in the natural gas project in Denmark. The method was developed by the Municipal Gas Distribution Companies (KOMGAS) in collaboration with OAC, Operations Analysis Centre A/S. KOMGAS is an umbrella-like organization covering five regional natural gas companies responsible for the construction of distribution networks of a value approximately 1 billion U.S.\$.

The central part of the method is a small computerized database with information about 500 typical activity types and their use of 500 fundamental resource types (manpower, materials and facilities). Each resource type is assigned one or more unit prices (variation by region), and an inflation factor. This price structure makes it possible to estimate project costs on different aggregation levels, as well as on different assumptions concerning the unit prices and their evolutions.

The paper discusses the development of the metod, and states the practical experiences from using the method. The main conclusion is that a small, specially designed information system is of a great help in situations where the cooperation between a number of different organizations is a fundamental condition.

Finally, a number of ideas for extending the information model to meet other requirements of the project management function are given, of which some are being implemented at the moment. Ullrich Heilemann Rhein.-Westf. Institut für Wirtschaftsforschung, Hohenzollernstraße 1-3, D-4300 Essen 1, West Germany

CYCLICAL TAXANOMY AND LARGE ECONOMETRIC MODELS

The paper examines business cycle classifications as a tool for evaluating predictionary qualities of large econometric models. Its first section describes the modifications which had to be made to perform the multi-variate discriminante analysis-based classification of US-cycles into recessions, recoveries, demand-pull inflations and stagflations by Meyer/Weinberg [1] with quarterly instead of monthly data. Even though some of these modifications are considerable, it can be stated that the Meyer/Weinberg scheme holds for quarterly data, too.

The 20 variables employed for the classification (e.g. growth rates of real GNP, Unit labor costs, Government surplus/deficit, Money supply, Prices and the Unemployment rate) are endogenous variables in most of the large US-models. Therefore, a comparison can be made between the classification of the observed data (a priori classification) and the classification of the model results. This may serve as an indicator of the model's cyclical behavior/performance. The second section of the paper compares the 'classified' results of two dynamic ex post forecasts with the Data Resources, Inc. (DRI) US-model with the a priori classification of the simulation-periods. The comparison shows that the model explains the different cycle stages fairly well - with the exception of the 1974/75 recession. - In a similar way, two ex ante forecasts with the DRI US-model were classified and a useful condensation of the high complexity of the forecasts informational content was gained (section 3).

Both applications of the discrimant analysis-based classifications also reveal the limitations of this approach, its high sample period sensitivity being the most important one. Another limitation stems from the missing internal determination or uniformity of the Meyer/Weinberg classification, as not every cycle has to include all stages.

Summary: Multivariate discrimant analysis-based cycle classification cannot substitute careful and detailed analysis of the results of a large econometric model, but may well support this analysis in a very efficient way.

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TRIANGULATION OF CONVEX POLYHEDRA WITH AN APPLICATION TO ECONOMICS

First it is shown how an unbounded convex polyhedron may be subdivided into generalized simplices whose extreme points and extreme directions are prescribed by some marking procedure on the faces of the polyhedron. This then is used to extend the well-known Substitution Theorem for general Leontief models consisting of single product processes only, to the case where processes of joint production and of disposal are admitted.

For a convex set A,F(B) denotes the smallest face of A containing BCA, and D(A) denotes the cone of directions of A. Let Y be a finite dimensional convex polyhedron containing no lines. Let μ be a marking on Y, i.e. a map defined on the faces of Y resp. D(Y) such that for a face F of Y μ F \in F U (D(F) $\{0\}$) and for a face F of D(Y) μ F \in F $\{0\}$.

Theorem. Each point $y \in Y$ has exactly one marked representa-

tion, i.e. a representation $y = \sum_{i=0}^{k} \lambda_i a_i, \lambda_i > 0, \sum_{i=1}^{k} \lambda_i = 1,$ where $M = \{a_0, a_1, \dots, a_k\}$ is a μ -independent subset, i.e. $a_i = \mu F(\{a_1, \dots, a_k\})$ for $0 \le i \le k$ and $a_i \notin F(\{a_{i+1}, \dots, a_k\})$ resp. $a_i \notin D(F(\{a_{i+1}, \dots, a_k\}))$ for $0 \le i \le k-1$.

<u>Corollary</u>. The collection of all the convex hulls of maximal μ -independent subsets yields a "marked" triangulation of Y.

Furthermore we present an "extraction" algorithm determining the marked representation for every point of Y and the marked triangulation of Y.

This result is applied to the, perhaps unbounded, polyhedron Y of all the net outputs producible by one unit of labour in a linear joint production model. The marked triangulation of Y induced by a suitably chosen marking on Y describes the choice of labour minimizing techniques according to a varying exogeneous demand for goods. The essential feature of this Substitution Theorem for joint production in contrast with that one for Leontief models is that no longer the choice of only one technique suffices and that processes of disposal may come into use.

- 197-

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AN OPPROACH TO THE PREDICTION PROBLEM IN ECONOMIC DYNAMIC SYSTEMS THROUGH USING FILTERING TECHNIQUES. AN APPLICATION

Consider a dynamic economic system with the following reduced form [1]

 $BY' + \Gamma X' = U' => Y' = -B^{-1}\Gamma X' + B^{-1}U' => Y' = \Pi X' + W$

where $\Pi = -B^{-1}\Gamma$ and $W = B^{-1}U^{*}$

It is assumed that the reduced form coefficients matrix, say Π^* , has been estimated by the Bayesian filter algorithm [2], and that the above e-conometric model has been converted to an equivalent control system of the general form:

 $x_{i+1} = Ax_i + Bu_i + Dz_i + \xi_i$

where

 $x \in E^n$ is the state vector, $u \in E^m$ is the control vector, $z \in E^\rho$ is the exogenous variables vector, and $\xi \in E^n$ is the white Gaussian noise vector.

For economic planning purposes one has to estimate the system state from the time t_0 (present) up to time t_0+N , based on past observations. In usual applications the system state, for the above time period, is estimated from the deterministic plant, where the noise vector is set to its expected value (i.e. $E\xi_i = 0$).

The problem set in this paper is to formulate an observation equation of the form

 $\hat{y}_{i+1} = Hx_{i+1} + \eta_{i+1}$

where the output vector is observable over the period t_0+1,\ldots,t_0+N . We also analyze the methodology for obtaining the above observation equation, and to estimate in turn

a) $E(x_{i+1}|\hat{y}^{i+1})$ where $\hat{y}^{i+1} = \hat{y}_1, \hat{y}_2, \dots, \hat{y}_{i+1}$ b) $Cov(x_{i+1}|\hat{y}^{i+1})$ (the error covariance matrix)

Klein's model 1 has been used as a vehicle of exposition of the proposed methodology. The superiority of the results obtained is established by using some standard statistical criteria (i.e. Theil's inequality coefficients, RMSE, RMSPE, MAE, MAPE etc.).

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- 198-

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ANTIPOLLUTION ACTIVITIES IN INPUT-OUTPUT ANALYSIS

This paper 1) examines the treatments of the antipollution activities in Leontief's extended input-output model and their implications: 2) presents an alternative approach that introduces an automatic mechanism in the input-output framework for initiating the antipollution activities once specified levels of tolerable pollution have been reached: and 3) discusses the conditions under which the proposed model provides a unique nonnegative solution of output levels for any given nonnegative levels of final demand and tolerable pollution.

In Leontief's extended model, it is implicitly assumed that the pollution generated is greater than tolerable and the total amount of pollution generated is, therefore, set equal to the antipollution output level plus the tolerable pollution level for each pollutant. For some final demand, the generated pollution may not even reach the tolerable level and, due to the pollutant balance relationship, the extended model may give some negative levels of antipollution activities. The "fixed percentage abatement policy", suggested by Steenge, assures that the tolerable level never exceeds the generated pollution. However, it makes the amount of tolerated pollution fluctuate with the level of final demand.

The alternative approach proposed in this paper is first to modify the pollutant balance equations in Leontief's model to make pollutant flows independent of the tolerable levels, and then introduce the tolerable levels in conjunction with a mechanism to initiate the antipollution activities. Through a modified physical input-output table with an "environment" column, that accounts for the amount of pollutants discharged into the environment, the complementarity relationships between the antipollution activities and "slack tolerance" levels can be derived mathematically. A simple solution algorithm that utilizes the complementarity conditions as the automatic mechanism, demonstrates that the proposed model gives a unique realistic solution to Leontief's example for any given nonnegative final demand, indicating which antipollution activities are active or remain idle. It is shown that Leontief's original and extended inputoutput models are embedded in the proposed model.

The proposed model is analyzed for the situation in which some industrial sectors consume pollutants as their inputs (some off-diagonal elements of the structural matrix are positive). Due to the interdependencies among the industrial sectors and antipollution sectors, those pollutant consuming industries may be "net" pollutant generating sectors. If all the industrial sectors are net pollutant generating sectors, the proposed model provides a unique nonnegative solution for any given nonnegative final demand and tolerable pollution levels. I.D. Moon Department of Industrial Engineering and Operations Research Columbia University New York, NY 10027

Structural Stability of A Probabilistic Input-Output System subject to Sector Optimization

This paper studies structural stability of a probabilistic inputoutput economic system when one or more sectors perform optimization over production activities. Technological advances, which would require less amount of inputs for a given output quantity, motivates the sectoral optimization whose objective is independent of that of the system. As a result of optimization, a new set of input coefficients is provided as a linear function of the optimal program. As an optimized sector is represented by these new coefficients, replacing the old counterparts, in the input coefficient matrix, the entire input-output system is perturbed. Given such perturbations three questions arise: (1) Are the final demands of the economy still producible? (2) Is the sectoral production program still optimal? (3) What are the intersectoral effects when more than one sector performs such optimization? Previous studies were concerned with these questions for deterministic input-output systems. This study extends the analysis to an input-output system with probabilistic final demand. The main result is the development of a set of conditions under which the inputoutput system reaches a new equilibrium state and all optimized sectors maintain optimality of production program. Economic implications are discussed.

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EQUILIBRIUM ADVERTISING IN AN OLIGOPOLY WITH NERLOVE-ARROW ADVERTISING DYNAMICS: EXISTENCE AND STABILITY

This paper is concerned with the equilibrium level of advertising in an oligopoly comprising n profit maximizing firms. Advertising is modeled as a capital of goodwill; and the dynamics of the decay of advertising goodwill by a variant of the familiar Nerlove and Arrow process. Equilibrium in the oligopoly is defined to be non-cooperative Nash solution to an appropriately defined discrete n-person differential game governed by the laws of motion resulting from the assumed advertising dynamics.

The primary result of the paper is the existence of an open loop Nash solution to the foregoing differential game, or equivalently the existence of a non-cooperative equilibrium in the oligopoly. To prove existence the problem facing each firm is transformed from one of choosing advertising spending in each period to that of choosing accumulated goodwill; the transformed problem is easily seen to possess at least one open loop Nash solution. The non-linear difference equations characterizing the advertising investment path in the industry are then derived.

The second part of the paper examines (long run) stationary states on the equilibrium path of advertising investment. Sufficient conditions for local stability of the stationary states are established by studying the linear approximation to the equilibrium path around stationary states. BIOLOGICAL MODELS

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SELF-ORGANIZING CONTROL WITH NEURON-LIKE ASSOCIATIVE MEMORY MODELS

Based on a neuron-network model of the human cerebellum as proposed by Albus [1] which can be interpreted also as a distributed associative memory system a self-organizing control concept is proposed where process model and control strategy each are represented in an associative way by two memory systems of the Albus-type. The concept considers both the control strategy

C: $Y_k \rightarrow \underline{u}(t_k)$; $Y_k = (\underline{y}(t_k), \dots, \underline{y}(t_{k-1}))$ and the process model

 $M: (Y_k, U_k) \rightarrow \underline{\hat{y}}(t_{k+1}/t_k); \quad U_k = (\underline{u}(t_k), \dots, \underline{u}(t_{k-m}))$ as nonlinear mappings which are also regarded as association pairs $\{Y_k, \underline{U}(t_k)\}$ and $\{(Y_k, U_k), \underline{\hat{y}}(t_{k+1}/t_k)\}.$

By the sampled measurement $\underline{y}(t_k)$ at time t_k an appropriate control action $\underline{u}^o(t_k)$ is recalled by the past experience stored in AMSC (associative memory system for control) while $\underline{y}(t_k)$ is parallelly used in combination with $\underline{u}(t_{k-1})$ to train AMSM (AMS for model) i. e. to update $\underline{\hat{y}}(t_k/t_{k-1})$. At the same time the expected process response $\underline{\hat{y}}(t_{k+1}/t_k)$ is associated to $\underline{u}^o(t_k)$ by AMSM. This prediction is utilized for calculating the expected value of a suitable chosen cost function, i. e. the subgoal

 $I_{k} = f(Y_{k}, \underline{u}(t_{k}), \underline{\hat{y}}(t_{k+1}/k))$

for the next control step. An optimization loop is activated in order, if necessary, to find an optimal control action $\underline{u}_{opt}(t_k)$ using $\underline{u}^{o}(t_k)$ as starting approximation. $\underline{u}_{opt}(t_k)$ is then applied at the process input and simultaneously used to train AMSC for the actual measurement pattern Y_k .

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-203-

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EXPERIMENTAL, ANALYTICAL AND COMPUTATIONAL STUDY OF A SIMPLIFIED MAXIMAL HEIGHT JUMP

Recently, there have been a number of attempts to apply optimal control theory to the analysis of animal and human locomotion [1],[2],[3]. All these problems involve the dynamics of multi-segment pendula. The problems are thus nonlinear, have unusual state constraints and often involve controls that are neither bang-bang nor linear. Because of this it has generally been impossible to solve these optimization problems analytically. The exception to this has been a recent paper [4] in which the simplest case, that of making a baton "jump" as high as possible, was solved.

More recently, in an attempt to extend the baton results to a more realistic situation, the following experiment was performed. Humans were asked to jump as high as possible subject to the constraint that they keep their knees and hips locked. In essence, the experimental subjects were asked to imitate a two segment inverted pendulum.

The experiment is modelled by a two segment inverted pendulum controlled by an "activation". That is, control is exerted by a model of muscle activation in which the maximum torque depends on the joint angle and torque lags activation (activation is the control). This muscle model is based on the physiology. The resulting mathematical optimal control problem is solved and it is shown that the results are largely independent of the muscle model. The agreement between experiment and analysis is very good, although not perfect. Some suggestions are made regarding further sources of error.

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-204-

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COMPARISON ANALYSIS OF IMMUNE RESPONSE AND TOLERANCE MODELS

A brief systematic overview of immune tolerance and its role in disease, infection, organ transplantation and prosthesis is presented. Low-zone and high-zone tolerances are studied as functions of probability of stimulation, antigen concentrations of different cells, antibodies and antigen (alien matter) through unresponsiveness and learned tolerance or expressions of lack of controllability.

The basic mechanism of cellular stimulation and differentiation is examined in probabilistic, as well as, deterministic models. This is integrated with a dynamical immune model to mimic tolerance. Comparisons are made with respect: to different mathematical structures, to deterministic vs. stochastic models and to model effectiveness. Experimental evidence is presented both as motivation of the research and as evidence at least of a research direction.

A foundation for this research is given by the authors in reference [1] and [2].

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AN INNOVATIONS APPROACH TO CARDIAC HEMODYNAMICS MODELLING

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ABSTRACT

A new approach to the problem of modelling the blood vessels attached to the heart especially pulmonary artery is presented. The analyses are based on a dynamic nonlinear compartmental model that describes the transfer of pressure energy between two sites in that vessel(blood pressure behind the pulmonary valve and wedge pressure). The system kernels are represented by two pathes, each path contains a nonlinear network in cascade with a linear dynamics followed by a nonlinear transformation. The nonlinear network is described by an orthogonal functional polynomial. The linear dynamics is represented by an autoregressive moving average(ARMA) digital filter.tThe nonlinear transformation yields a zero mean unit variance output. The first path reflects the dynamic compliance of the vessel under pulstile action. The second path represents the variations of input impedance along the cardiac cycle. The baseline shift of the measured data during cardiac catheterization was removed by first order bandlimited Markove preprocessor.

An experiment with (10)patients with pulmonary hypertension was performed to validate the model output. The proposed model overcomes the limitations imposed on the order of linear dynamics as in case of Hammerstein model. Moreover, it converges with quick rate compared with the iterative procedure applied in Uryson model. Malur K. Sundareshan Department of Electrical Engineering University of Minnesota Minneapolis, MN 55455

MODELLING CELL KINETICS AND DESIGN OF OPTIMAL CANCER THERAPY STRATEGIES

Models which represent cell kinetics are useful for understanding the dynamics of cancer cell populations and for the design of chemotherapeutic regimens whose action is aimed at interfering with certain biochemical events of the cell cycle. The use of potent anticancer agents which may have a detrimental effect on healthy cells as well as cancer cells makes necessary a careful qualitative analysis of such systems. Since the goal of any cancer chemotherapy is to decrease or control the cancer cell population, the problem is a viable application of several control theory concepts (identification, stabilization and optimization).

In this paper, we shall present the results of some of our recent investigations aimed at the development of methods by which a multicompartmental model representing the progression of cells through the different phases of the cell cycle, can be utilized to gain insight into the prognosis of oncological diseases and the design of efficient computerized treatment strategies for chemotherapy. The specific problems investigated in this research were:

- (i) the development of a systematic computer-based procedure for the identification of transition and loss parameters characterizing the different compartments of the model from the cytokinetic data obtained from microfluorometry experiments;
- (ii) the development of conditions for the exponential decay of cancer cell population under the two cases - when no external control (drug influence) is exerted and when the control action (drug dose schedule) is specified;
- (iii) to obtain a quantitative formulation of the "optimal cancer therapy problem" - that of designing schemes which provide the maximum therapeutic effect (tumor kill) and minimum host toxicity and develop algorithmic procedures to solve this problem.

The results presented in this paper will outline the progress to date on the three problems stated above. A solution to the identification problem is obtained by using a stable adaptive identification procedure and is applied to data (in the form of DNA histograms) for a CHO cell line and a human lymphoma cell line treated with melphalan at the exponential growth phase. By integrating a pharmocokinetic model with the model for cell proliferation in the tumor, conditions on drug parameters to result in a specified decay rate are obtained. Furthermore, the "optimal therapy problem" is formulated as an equivalent "parameter optimization problem" by using periodic chemotherapy regimens and a step-by-step procedure for solving this problem and determining the optimal dosage and optimal time interval between doses is developed. Finally, the results of this study are compared with some experimental results for L1210 leukemia in BDF1 mice treated with 15 mg per kg doses of the anticancer agent Ara-c, which shows an encouraging correspondence between the two results.

COMPUTER-AIDED DESIGN

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Layek L. Abdel-Malek and J.T. Chu Dept. of I.E. and O.R., Columbia University, New York, N.Y. 10027 Div. of Management, Polytechnic Inst. of New York, Brooklyn, N.Y. 11201 OPTIMUM DESIGN OF SOLAR ENERGY WATER HEATING SYSTEM

This paper reports a method and furnishes tables by which a user can choose the optimum size of a given type of solar energy water heating system, knowing, among other things, the local climate and his family's needs. Typically such a system has three major components, i.e., a solar energy collector, an insulated water tank, and an auxiliary energy supply system, using, e.g., oil or natural gas. The solar collector transforms solar radiation into thermal energy which heats the water circulating between the collector and the water tank. When the temperature of the tank reaches a certain level, no more solar energy is taken into the system. When hot water is taken out of the tank by the user, it is replaced by the same amount of cold water so that the tank is always filled to capacity. If the temperature of the tank is below the level set by the user, he will use the auxiliary system to raise the temperature at times of needs. Furthermore, hot water in the tank loses its thermal energy at a rate depending on the insulation of the tank and the ambience. MATHEMATICAL MODELS 1. The system is said to be in a state "i", if total thermal units contained in the water tank is i. The probability that the system is in state i is denoted by p₁, i=0, 1, ..., k. 2. Thermal units supplied by the sun per unit time is a birth process with birth rate λ , which depends on the surface area A of the collector, the location of the household, and the times of the day and year; and for fixed A, is a constant during the day and is 0 in the night. 3. Thermal units actually absorbed per unit time by the collector is also a birth process with birth rate $\lambda_i = \lambda(1-c_1 i)$, where i is the state of the system and c_1 , a constant. 4. Thermal units needed per unit time by the users in a given household is a death process with death rate . 5. Thermal units actually lost per unit time is $A_1 = //+c_2$ i, c_2 being a constant.

<u>MAIN RESULTS</u> 1. Applying queueing model M/M/1/k with λ_1 and M_1 and by series expansion and simplifications, approximations are obtained for P and E(X), where X is the random variable defining the state of the system. 2. Expected amount of energy supplied by the auxiliary system per unit time during the daytime of a given day is found to be $\mu(1-E(X)/k)$. When multiplied by the number of time units in the daytime, the product is the expected amount of energy supplied by the auxiliary system during the daytime of a given day. 3. The expected amount of energy supplied by the auxiliary system during the night is (H-h)#, where H is the number of time units in the night, h is the time it takes the user to exhaust the thermal energy left in the water tank at time of sunset, and h is found to be $(1/c_2)$ log $(1+c_2E(X)/\mu)$. The sum of daytime and night is the expected amount of energy supplied by the auxiliary system during a 24 hour period. 4. Thermal units saved annually=User's needs-Sum of units supplied by the auxiliary in each of the 12 months, where the latter=units supplied in 24 hr. x days in a month. 5. For various A and k, savings are tabulated and may be compared with costs of equipments. When the latter are linear functions of A and k, the optimum A and k are determined.

-209-

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SINGULAR VALUE DECOMPOSITION AND LINEAR MANIFOLDS: AN APPLICATION TO OPTIMUM OBJECT POSITIONING

When an ideal body in a tridimensional space has to be moved from a star ting position to a terminal target, the computation of the rotation and the translation necessary to achieve the target, is easily performed.

However, if the body is not ideal, i.e. it admits tolerances, and consequently the target is defined by the spherical neighbourhood of a set of points (or, in general, by the neighbourhood of a set of linear manifolds: points, lines, planes), the problem may admit one solution, a set of solutions or no solution at all, if the body doesn't satisfy the tole rance constraints.

Moreover, in this case of uncertainty (i.e. presence of tolerance constraints), finding an admissible rotation and translation of the body is not a simple task.

In this paper some results of linear algebra useful to solve the positioning problem are reviewed and a method is proposed to find a solution in the presence of uncertainty, when such solution exists, or otherwise to give an indication of nonexistence.

Positioning with uncertainty is approached as a constrained minimum norm problem solved with a dual optimization algorithm. The keyrole played by singular value decomposition in computing body rotations is emphasized and a series of results on affine transformations are used to obtain the optimal translations.

The dual functional is optimized by means of an iterative gradient projection algorithm, which as been proved to be rapidly convergent in the application discussed at the end of the paper.

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OPTIMAL DESIGN OF A THERMAL DIFFUSER WITH MINIMUM WEIGHT

The object of this paper is the development of efficient mathematical and numerical tools to find the optimal shape of a minimum weight thermal diffuser with a priori specifications on the input and output thermal power flux.

This problem arises in connection with the use of high power solid state devices (HPSSD's) in future communication satellites. The specifications for this diffuser came from the Center for Research in Communications (CRC) in Canada.

We present a working theory using new techniques of shape optimal shape design (cf. J. Cea et als [2 to 10], R.H. Gallagher and O.C. Zienkiewicz [1]). Necessary conditions are presented and gradient computations are performed. Some numerical examples will be worked out. A similar problem was studied by Ph. Destuynder [10]. However the present design is technically more difficult since we have a constraint on the normal derivative of a piece of the surface boundary instead of a constraint on the maximum temperature in the domain or body of the diffuser.

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A NUMERICAL WAY OF OPTIMIZING CHAINS OF TOLERANCES

It is reported on a simplified test showing that numerical optimization can be utilized as an aid to making tolerances of technical parts as narrow as necessary and as wide as possible: In an outer optimization loop a computer program varies upper and lower limits of tolerance ranges for every design dimension involved in performing the technical function. In the test example this function is reduced to mere geometry for a proper fit between parts. Inside every tolerance range generated by the outer loop an inner loop varies combinations of tolerances with the aim of finding the worst case constellation (which may be a statistical one). If no such combination violates any functional requirements the objective function of the outer loop is given a negative sign, otherwise it is given a positive one. The absolute value of the objective function increases as the tolerance ranges increase. Thus the outer loop searching for the minimum of this objective function will enlarge the tolerance ranges as long as saving of manufacturing cost is possible without jeopardizing the technical function. It is shown that the optimum can be found numerically though it is located in a narrow gap at the base of a big vertical step between the negative and the positive part of the objective function. For the test a known optimization routine [1] alternating stochastic with deterministic steps was employed.

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SYSTEMS PROBLEMS

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> REDUCED ORDER OBSERVER ESTIMATOR FOR A DISCRETE-TIME LINDAR SYSTEM USING E.VARIANT IMBEDDING

In this paper, a reduced order discrete-time leastsquares observer-estimator has been developed using invariant imbedding. This can be applied to a linear system having both the measurement noise v(i) and the dynamic noise w(i). Luenberger formulated an observer of order p for an nth order noisefree system having m outputs(p=n_m). Fairman used the Kalman filter to estimate the observer states for a stochastic system. However the Kalman filter requires the noise statist. ics apriori. In our method, no such information is required except that the noise is of zero mean.

Consider an nth order linear system described by

$$x_{1}(i+1) = A_{11}x_{1}(i) + A_{12}x_{2}(i) + B_{1}u(i) + w_{1}(i)$$
(1)

$$x_{2}(i+1) = A_{21}x_{1}(i) + A_{22}x_{2}(i) + B_{2}u(i) + w_{2}(i)$$
(2)

$$y(i) = x_1(i) + v(i)$$
 (3)

where x(i) and x(i) are m and p vectors; outputs y(i) are weakly corrupted with noise; and other notations have usual meanings. The reduced order observer of order p for the above system is

$$z(i+1) = Dz(i) + Ly(i) + Gu(i) + Tw(i) - Lv(i)$$
(4)

Defining the composite output as $y(i) = A_{42}z(i) + w(i)$, the sequential observer-estimator is then obtained as follows, by minimizing the guadratic functional of errors $[y(i) - A_{42}z(i)]$ and [Tw(i) - Lv(i)] with the constraint of (4) and then solving the resulting TPBV problem by invariant imbedding.

$$\hat{z}(i+1) = D\hat{z}(i) + Ly(i) + Gu(i) + 2C(i+1)A_{12}Q_1[y_0(i+1) - A_{12}, (D\hat{z}(i) + Ly(i) + Gu(1))]$$
(5)

$$D_{2}(1) \cap D_{3}(1) \cap D_{4}(1) \cap D_{4}(1)$$

$$C(1+1) = [1 + 2P(1+1)A_{12}P(1+1)]$$
(6)

$$P(i+1) = DC(i)D' + \frac{1}{2}Q_2^{2}$$
(7)

From this, the system states can be easily obtained by linear transformation.

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-214-

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AN ABSTRACT SYSTEM'S MODEL FOR A CLASS OF NONLINEAR SYSTEMS

This paper presents several examples of nonlinear systems whose equations cannot be written in the nonlinear state space form. The paper uses these examples as motivation to develop an abstract normal form for nonlinear system's equations. The normal form is developed from the system's basic equations by dividing the basic equations into a set of linear and nonlinear equations. The specific forms of the linear and nonlinear equations are specified in an abstract manner but the forms retain sufficient mathematical structure to permit the construction of the normal form. The nonlinear equations must be able to be represented by functionals. The linear equations must be able to be represented by a matrix equation. The elements of the matrix may be real numbers, linear functionals, etc. The only requirement placed upon the elements of the matrix is that they belong to some field. This rich algebraic structure permits physical systems to be analyzed in either the time domain or the frequency domain. The abstract nature of the normal form makes it applicable to integral - differential, delaydifferential, digital, analog-digital, and stochastic systems. Finally the paper discusses the development of a control theory from the normal form and a way of obtaining a solution for the normal form of the system's equation.

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PHASE FREQUENCY APPROXIMATION IN SYSTEM MODEL REDUCTION

In this paper a novel phase frequency approach is proposed to approximate a higher order system model by a reduced order one. It is based on Thiele's continued fraction [1] of s/tanh $\psi(s)$, where tanh $\psi(s)$ is the hyberpolic tangent of the system transfer function. This results in a reduced order system whose phases interpolate the phase frequency response of the unreduced system at a set of discrete frequencies. That set covers mainly the workable frequency range. Interpolation of phases, using Thiele's continued fraction, results in a linear recursion relation that leads to a reduced order transfer function. Depending on the difference between the number of poles and the number of zeros of the unreduced minimum phase system, the reduced one may be either a minimum phase, or a nonminimum phase. However, a minimum phase reduced system can be always guaranteed. The various cases considered indicate that the obtained reduced model is of the minimum phase type, and consequently the resulting amplitude frequency response is in good agreement with that of the unreduced system over the useful frequency range.

To further improve the type of response obtained, the coefficients of the reduced system obtained through Thiele's continued fraction, are considered as an initial guess for a minimization scheme that minimizes the squared error phase deviations of both the unreduced and reduced models. Again, results show that both the reduced and optimum reduced models are virtually similar to each other, since there is no significant zero or pole displacement between the reduced and optimum reduced models.

Results obtained, indicate how close is the agreement between both the frequency, step, and impulse responses of the reduced and optimum models to their counterparts of the unreduced models. Comparing with the already existing methods, it is clear that the proposed method compares favourably with them without the need of lengthy minimization procedures.

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COST - EQUIVALENT REALIZATIONS OF LINEAR SYSTEMS by R. E. Skelton

ABSTRACT

One of the most fundamental concepts in systems theory is the basic definition of a dynamic system. A dynamic system may be defined as an interconnection of entities (which we will call "components") causally related in time. It seems equally natural and basic, therefore, to characterize the system's behavior in terms of contributions from each of the system's building blocks - "components." The performance of the dynamic system is often evaluated in terms of some performance metric we choose to call the "cost function"V. The cost function might represent the system energy or a norm of the output errors over some interval of time. Concerning the physical or mathematical "components" of the system, it is only natural then to ask <u>Question CC</u>: "What fraction of the overall system cost V is due to each component of the system?"

This paper is devoted to a precise answer to question CC and to several applications of the mathematical machinery developed for answering the question. Such an analysis will be called *Component Cost Analysis*. Knowledge of the magnitude of each component's contribution to the performance of a higher order model of the system can be used to decide which components to delete from the model to produce lower order models. *Thus, CCA can be useful in model reduction*. This paper explores the possibilities in some detail.

A brief summary of the component cost analysis (CCA) is given. CCA provides a means to determine the contribution of each dynamical element of the system in the overall performance metric. Using CCA a theory of cost-equivalent realizations is developed. Cost-equivalent realizations are stable realizations of the dimension of the output and have the same value of the performance metric as the original system. H.-J. Werner Econometrics Unit, Institute for Econometrics and Operations Research University of Bonn Adenauerallee 24-42 D-5300 Bonn 1, W. Germany

TO THE TEMPORAL AGGREGATION IN DISCRETE DYNAMIC SYSTEMS

In this paper we consider a dynamic system of n equations with n endogeneous variables in reduced form. The microrelation may thus be written in the form

(1)
$$y(t) = \sum_{i=1}^{n} A_{i}y(t-i) + w(t), t \in \mathbb{Z},$$

where the n×n matrices A, contain the structural parameters, w = (w(t)), t \in Z, is an autonomous input process, H is the order of the AR-part (i.e. A_µ \neq 0), and t measures time in some microperiods (e.g. three month periods). We further suppose that the available data are in some other e.g. annual - form, so that the analyst prefers to replace the microrelation (1) by a macrorelation which contains only available parts of y on both sides of the equality sign. If $Y = (Y(\Theta)) = (y(m\Theta))$, $\Theta \in Z$, with m $\in N$ fixed, is the observable (available) variable, then it is clear that by eliminating the nonobservable parts of y in (1) we can obtain without difficulty an exact macrorelation of the form

(2)
$$\widetilde{A}_{0}Y(\Theta) = \sum_{i=1}^{n} \widetilde{A}_{i}Y(\Theta-i) + W(\Theta), \Theta \in \mathbb{Z},$$

. . ..

where

(3)
$$W(\Theta) = \sum_{s=0}^{(m-1)H} B_s w(m\Theta-s)$$

and the matrices B_s and \widetilde{A}_j are functions of the matrices A_j . The following question now naturally arises:

- Can we find, for each $n \in N$, a nonsingular macrorelation (not necessarily of order H of the autoregressive part)? The word 'non-singular' is used here to indicate that \tilde{A} is nonsingular, so that such a macrorelation can be written in reduced form.

This problem has been studied for the single autoregressive process (i.e. n = 1) by Telser [1], whereas an analysis for the above multivariate case (i.e. n > 1) seems still to be lacking. Since our answer to the above general problem is affirmative, we further ask:

- Given (1) and $m \in N$, what is the minimal order of the autoregressive parts in the class of all the corresponding nonsingular macrorelations? As our approach is constructive, we get in addition explicit formulae for computing the matrices \mathcal{A}_i , \mathcal{B}_s of a certain canonical nonsingular macrorelation.

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POWER SYSTEMS

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ON THE MEGANATT-FREQUENCY CONTROL PROBLEM

Due to complex nature of present day multiarea interconnected power systems; controlling the system dynamics behaviour and maintaining its electrical energy at a desired operating level are complicated taskes to establish and to execute. The object of this paper is to obtain first the optimal controls of the load frequency control problem of the multiarea interconnected power systems and then to study the effect of size differences of the areas or norinteraction of area controllers. This is performed by obtaining a linear representation to the problem and forming a linear quadratic optimization problem. Programs to solve Riccati equation and to obtain the optimal controls and the system response under both the coupled and the uncoupled controls are prepared. The control, in this problem, is to keep the network frequency and the interchange power constant and within a pre-scheduled limit and equal to its nominal value when load disturbances occure in the system. The load disturbances are assumed to be small perturbations of constant magnitude for intervals long enough to settle the system.

The study includes the effect of size differences on noninteraction of area controllers in order to determine the possibility of using suboptimal uncoupled controllers that are easier to implement. Computer programs to solve the problem and to obtain the system response are prepared and executed on PDD11/70 computer.

Interesting results have been obtained when the rated power ratio varies from one to five. For each case the possibility of using uncoupled controllers is investigated if only one of the areas has been disturbed or both areas are suffering disturbances simultanously.

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-220-

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STATE VARIABLE MODELLING OF POWER SYSTEM FOR SECURITY

System Security may be considered as the ability of power system in normal operation to undergo disturbance without getting into an emergency condition. Computer control is to be used to keep the system secure. If the system were found insecure, the next problem is to determine whether the system can be made secure. This becomes security constrained dynamic optimisation problem where we have to find the best operating condition which satisfies not only operating constraints and load constraints but also security constraints.

For enhanced security achievement Dynamic State Estimation should be achieved. State space model, linear, non-linear or quasilinearised versions are being used for dynamic representation. For data processing for achieving decentralised control the approaches attempted are aggregation, coherency, network equivalents, decomposition, hiearchical methods, parallel processing, singular perturbation techniques. The state variable model has to take into account physical parameters such as rotor angle, speed, magnetic characteristics etc. The objective is fulfilled if the model is accurate and objective function weighs disturbances suitably.

Dynamic models represented by Anderson and Fouad, Darwish and Fantin, Singh et. al, Mukhopadhya et. al, Elgard et. al, M. Rammurty et. al, are discussed. The objective is to achieve security.

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Various books and research papers.

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SYNCHRONOUS MACHINE STABILIZATION THROUGH EXCITATION CONTROL

There has been considerable interest in recent years in the design of excitation control schemes for large generators which is fast enough to make contribution to dynamic stability.

The main theme of this paper centers around studying the effect of different types of controllers on the dynamic behaviour of a synchronous machine and consequently the most effective controller is designed in order to minimize transient oscillations. Changes in terminal voltage, power angle, angular speed, acceleration and terminal power have been used as feedback signals to the excitation circuit.

It has been found that dynamic performance of a machine is slightly affected when using power angle deviations or angular acceleration or terminal power changes as a main control signal associated with small gains. Proportional angular speed controllers have shown good damping effect upon the transient response. The dynamic performance of the machine is greatly improved when using control signals consisting of angular speed and angular acceleration or power angle deviations.

The best dynamic response can be obtained using control signals consisting of power angle deviations, angular slip, acceleration, and terminal voltage and its integral.

Finally, a procedure has been developed for the determination of optimum settings for the proposed controllers, namely, selection of gains in the region of near zero sensitivity of an assumed cost function. The analysis presented in this paper gives insight into synchronous machine stabilization through excitation control. P. Gruber-Johnson, B. Steinle LGZ LANDIS & GYR ZUG AG Central Laboratory CH-6300 Zug, Switzerland

MODELING OF DATA TRANSMISSION CHANNELS ON ENERGY NETWORKS

In the near future it will be important to use power networks for data transmission from producers to consumers and vice versa. Because of the low power output of the transmitters on the consumer's side, and strong variations of damping of the different consumers, there is a need for the equalization of the distortion effects in the interesting frequency ranges for consumer-producer data transmission. Instead of using a general digital transversal or recursive filter for the equalization, a model is derived from the physical characteristics of the power network. The advantages of this approach are that

- 1) the number of parameters can be kept small
- their value ranges are known and initialization for identification purposes is easier
- 3) stability tests (if necessary) during on-line adaptation are simpler
- 4) the model can be used for various equalization purposes.

Starting from the line equations with an intermediate transformer the model is derived for a frequency range up to a few kHz. It consists of two significant channels representing all the possible ways of transmission on the power network. It includes a single channel, two parallel channels (main and secondary) and a single channel with a simple echo model as special cases.

The validity of the model has been tested on frequency response measurements of different networks (European/American). It has been used for the identification of the network parameters by tuning a parallel model with a nonlinear programming package. The tuning of the amplitudes of the frequency responses has been achieved very well in all cases. The compensation of the phases show as far as there are reliable measurements a sufficiently good performance in spite of possible non-minimum phase characteristics.

The illustrated technique allows to find the physical important structure of the transmission. A discrete version of the model is now under investigation for on-line identification.

-223-

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MODELING OF LARGE POWER SYSTEMS

FOR

STEADY, QUASI-STATIONARY AND DYNAMIC STATE SIMULATIONS

Simulation of power systems, in all their states, is becoming an essential requirement in planning, operation and training in order to assess the overall system reliability, security and economy.

The paper presents a versatile programming structure and a systematic modeling procedure for various components of practical power systems.

Some of the salient features of this novel system modeling are:

- universal simulation of complex power systems in their static (steadystate), quasi-stationary, dynamic (long term) and transient operating conditions.
- modular modeling structure for major system components: generators, transmission network, loads and protection.
- dynamic modeling for auxiliary system components and controllers of various degrees of complexity by using elementary functions.
- facilities to simulate features of real-life power systems that are not included in classical algorithms.
- modeling of new components, such as static reactive power compensators, and those still in development stages such as multi-terminal HVDC and diode rectifiers schemes.
- efficient solution algorithm with sparsity techniques and novel numerical integration method.
- systematic data handling for easy use with minimum effort.
- simple to be easily developed or implemented in already existing computer programs used by electric utility and industry.

Results of a complex power system simulation are shown to demonstrate the superior convergence, the more problem-oriented versatile modeling characteristics and the modest computing requirements of the proposed technique.

-224-

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IDENTIFICATION OF SYNCHRONOUS MACHINE PARAMETERS FOR STABILITY MODELS USING SYNCHRONIZING TESTS

A reliable mathematical model of the synchronous machine is required to accurately assess the transient stability of power systems. The method suggested in this paper differs from the existing methods as it requires only one synchronizing test instead of the separate tests whose reactions are limited to one of the axes of the machine. This synchronizing test is made when the magnitude and the angle of the machine terminal voltage are different from those of the system. A machine model in which $\dot{\lambda_d}$ and $\dot{\lambda_q}$ are neglected and the damping effects are considered by including one short circuited coil along each axis, is used to simulate the transient behavior of the machine for the above test. In this simulation, the instantaneous values for the stator current, terminal voltage, speed and rotor angle of the machine, which are all observable variables during a real test, are computed. This data is used in a computer program to examine the effectiveness of the quasilinearization method for the indentification of x_d , x_ℓ , x_d' , x_d' , x_q and x_q'' . In this computer program, the model used for the synchronous machine has the same structure as the one used for producing the transient data in the synchronizing test. However, some different values are used as the initial guess for the above parameters. The final values determined by the computer program for these parameters are compared with those used in the simulation program to measure the effectiveness of the identification method.

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A CONTRIBUITION TO THE OPTIMAL GENERATION SCHEDULING OF LARGE HYDROTHERMAL POWER SYSTEMS

In this paper an approach is proposed to optimize the generation scheduling of large energy production systems, over periods where independent inflows can be well known.

As the marginal cost of generating hydroelectric energy is negligible, the minimization of energy production costs in hydrothermal power systems can be reduced into making the best utilization of water availability for hydroelectric generation. In this way, thermal generation and importation of expensive noncontractual energy from neighboring systems can be reduced. Furthermore, undesirable load sheddings necessary to satisfy the energy balance are avoided. The cyclic nature of water flows and load demand, as well as the validity of some model assumptions, suggest splitting the problem into long, medium, and short periods. For the long period problem, which can be some years, it is only possible to estimate river flows and load by probabilities, making the problem more difficult. Moreover, it is essential to take into account water head variations in reservoirs which affect generation efficiency. On the other side, for short optimization periods, normally a day or a week, complete knowledge of river flows and load are generally assumed. This, however, doesn't appear to be a reasonable assumption for load demand, which presents significant random variations even in short time periods. Another assumption frequently used in short period problems is constant water head which is not correct when small pondage reservoirs are present.

Optimal control of hydroelectric power systems have been the subject of other publications. In this paper we apply a decomposition technique that can deal with large systems over periods where the independent water inflows can be well known. General hydraulic systems with many reservoirs in cascade, water transportation delays and nonlinear generation functions are considered. The non-hydraulic production includes thermal generation, importation-from other systems and load shedding. The random variations of load are taken into account.

With the assumptions above we are able to solve accurately short range hydrothermal problems and even obtain approximate solutions for medium and long ranges. The decomposition and coordination technique is used to overcome difficulties imposed by the large number of state variables involved. Independent subproblems are generated. The solutions of these subproblems are coordinated at the upper level to find a global solution. An example illustrates the approach.

TRANSPORTATION PROBLEMS

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FREEWAY INCIDENT DETECTION BASED ON STOCHASTIC DYNAMIC MODELS OF TRAFFIC VARIABLES

In this paper, a new approach to the problem of freeway incident detection is presented based on modern estimation and detection techniques. A discrete-time stochastic model of the form ARIMA (0,1.3) is proposed for the dynamic characterization of freeway traffic data obtained from electronic presence detectors. Automatic incident detection is achieved by utilizing real-time estimates of the variability in the state variables of freeway traffic as detection thresholds. This approach eliminates the need for threshold calibration and lessens the problem of false-alarms. Because the moving average parameters of the ARIMA(0,1,3)model change over time, these parameters can be updated occasionally, for example, at the beginning of peak and off-peak periods. The performance of the proposed detection algorithm is evaluated in terms of the probability of false-alarms, the probability of correct detections, and the mean time-lag to detection. A total of 1,692 minutes of traffic occupancy observations recorded during 50 representative traffic incidents are used in the algorithm evaluation.

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OPTIMIZATION OF TRANSPORTATION SYSTEM PERFORMANCE UNDER DYNAMIC ENERGY CONSTRAINTS

A feedback dynamic model of transportation systems is presented. The dynamic method used emphasizes the long-term, delayed causal mechanisms that determine transportation demand, transportation supply and transportation resources over time. The model uses demographic and socioeconomic data and transportation system characteristics as inputs and evaluates alternative transportation management policies by analyzing their impacts on transportation system performance through time. Control variables available to the management include transit fare, transit frequency, transit vehicle size and maintenance, and are subject to fuel availability and price. Computational results include optimal transit service level, optimal modal split, number of waiting passengers, and transit system net cost per mile or per trip. Performance indexes considered minimize energy consumption, operating transit vehicles and passenger waiting time, and maximize load factors and system revenue. It is shown that, because of the transportation system size and complexity, simple transportation policies may have indirect effects on transportation system state variables and other observation variables that have not been predicted or are not desirable. For example, increased government support of transit may conserve energy and increase total system revenues in the short run, but may also cause 1) a significant increase in non-capital costs, 2) a decrease in system ridership caused by a lack of local support, and 3) an increased vulnerability to energy crises. Further, depending on management decisions and variable auto fuel costs, the transit system may or may not reach a stable solution or stable points may not be feasible. The methodology is part of a package to be used for improved transportation management under normal conditions as well as in energy emergencies.

SIMULATION STUDIES

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SOLUTION OF STOCHASTIC CONTROL PROBLEMS BY DISCRETE APPROXIMATION

The paper deals with the control of stochastic systems whose dynamics are described by the stochastic differential equation

 $dx_{t} = f(x_{t}, u_{t})dt + \sigma(x_{t})dw_{t}, \quad 0 \le t \le T,$

where w is a standard Brownian motion, x the state and u the control. The objective is to minimize the expected value of some functional $g(x_T)$ of the final state.

The method of solution adopted is to discretize the system in time. This leads to a sequence of stochastic difference equations

$$x_{i+1} = x_i + f(x_i, u_i) \Delta t + \sigma(x_i) \varepsilon_i,$$
 i=1,...,N,
N=1,2,...,

where $\Delta t = 1/N$ and the ε_i are i.i.d. disturbances. For the discrete time problems the powerful machinery of dynamic programming is available. The problem is then to find conditions under which the optimal controls for the discrete time problems obtained by dynamic programming converge to the optimal control of the continuous time system.

The method is illustrated by the simple example of Benes' noisy controlled integrator $dx = u_+(x)dt + dw_+$.

-231-

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> SIMSIGLA-PL/1, a simulation language using the latticecompleted algebra of intervals.

This communication outlines the general structure and functions of the experimental SIMSIGLA language, and comments its application to some dynamical models. In a single Computation run, bands of evolution are obtained for the system variables, instead of the more anecdotic single trajectories.

The structure of the simulator allows a great freedom in the programming of the temporal structure of the model. In particular, the completed algebra of intervals allows to absorb algebraically the indeterminacy associated with events with occurrency time only known to be within some interval.

The completed algebra of intervals also allows to introduce the fact of the control as an algebraic characteristic of the numerical quantities when represented by intervals.

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SYSTEMATIC EXPERIMENTAL DETERMINATION OF DISCRETE-TIME MODELS FOR NONLINEAR SYSTEMS

This work is concerned with the problem of developing model equations to represent, or approximate, the input-output behavior of a system based on a finite set of measurements. The literature contains numerous techniques for characterizing linear systems, but there have not been significant advances towards understanding the more general case involving nonlinear systems. It is felt that this problem has two key parts. First, the functional form of the model equation must be sufficiently general to adequately approximate a given systems behavior, without requiring an unmanageable number of terms. For most nonlinear systems, the form typically used in the literature is the Volterra model, which does not adequately satisfy this later requirement except for very restrictive classes of systems. The second problem is the development of a more general technique for evaluating and growing model equations in a computationaly efficient and recursive manner. Existing techniques (e.g. "Recursive-in-order") for approaching this problem are extremely limited as to the functional model form they can handle, and often make somewhat artificial assumptions and approximations in order to make even a partial solution manageable. The result is typically an inferior model of the system, with insufficient accuracy and/or an excessive number of terms in the question.

This paper will discuss a more general functional form for approximating nonlinear discrete-time systems, and present a compact algorithm for evaluating and growing these and other such model equations. Computational complexity reductions and accuracy improvements resulting from these techniques will be presented along with directions of current research efforts. Masao Murata Department of Electrical Engineering Utsunomiya University Utsunomiya, 321 Japan

MODELING OF FOURDRINIER PAPER MAKING MACHINES AND BASIS WEIGHT CONTROL

The basis weight computer control systems on paper making machines have originally been designed mainly for the high-speed Fourdrinier paper making machines(the machine speed is 700/min grade). Recently, however, those systems have come to be applied to the lower-speed Fourdrinier paper making machines as well. This recent tendency may well have brought on the problems of whether those basis weight computer control systems could be applied to any kind of the Fourdrinier paper making machines.

It is the main purpose of this paper to establish the basic measures for introducing basis weight control to Fourdrinier paper making machines of various kinds. This paper consists of the following two parts.

- (1) The modeling of the paper making systems in basis weight control is discussed under the new approach, where attention is paid to the retention of solid materials on the wire. This investigation leads to classifying those paper making systems into four types, A,B,C and D.
- (2) The two basis weight control theorems for practical use, namely Smith's method and the dead beat performance, are investigated in terms of their application to each type of paper making systems.

In conclusion, type A,B and C can be considered to be represented approximately by the models with one time constant T and dead time L. On the other hand, type D is impossible to be represented by such simple models. In addition, many of Fourdrinier paper making machines, which have been used for some ten years, belong to type D.

Those two basis weight control theorems can be applied to type A, B and C, provided that the input disturbances with certain wavelength & such as $\ell \leq 15L$ for type A, $\ell \leq 10L$ for type B, and $\ell < 10L$ for type C, should be taken away prior to the basis weight control. Those control theorems, however, can be applied to type D only in the case that type D is remodeled into type A,B or C. The basis weight control for type D has proved to be difficult unless type D is remodeled into the other types of paper making systems, or some new control theorems are developed. Keiji Yajima, Junkichi Tsunekawa Institute of JUSE 5-10-11, Sendagaya, Shibuya-ku, Tokyo, JAPAN 151

> GRAPHICAL THCHNIQUES USED FOR A DYNAMIC CHEMICAL PROCESS SIMULATION

One of the most important part in the process simulation software, both static and dynamic, is the capability of diagnosis detecting the inadequacy of model description. It was proven that this problem can be effectively dealt with by graphical techniques.

One should prepare following data for process simulation: (a)chemical species involved in the process and their physical properties, (b)units constituting the process, (c)topology to represent the structural connection of units, (d)values to specify the operational conditions of the process, (e)conditions to be noticed for a change of operational procedure (f)additional information such as specification of equations solving method. The items (d) and (e) are eliminated in a static simulation.

The dynamic graph is constructed to deal with dynamic analysis, by rewriting the ordinary defferential equations into the form of difference equations in consideration of the relationship among the variables. To treat with the integration as algebraic equation, it enables us to get static graph which can be handled simply.

The features of the graphical techniques in dealing algebraic equations are (a) to construct a nonlinear signal flow graph, (b) to assign types to vertices of graph, (c) to discern the existence of overspecification or underspecification on interdependencies, (d) to have blocktriangularization of equations system.

A figure shows a part of signal flow graph of chemical process which involves a PID controller. Attached characters A, S, I, G indicate the type of vertex (variable) such as assumed, set, integral and given. An integral type vertex is treated as set type, provided explicit method is used for a solving. It enables to reduce dynamic simulation to static one.



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MANAGEMENT SCIENCE

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MODELLING ALD DESIGN OF CONTAINER TERMINAL

In the rational design of container terminal one should use the simulation-type algorithms and programs. It results from big complexity of cause-effect relations in the mathematical model of the terminal. The simulation program should incorporate as one of most important parts the program of automatic control of loading, i.e. modelling and optimisation of gate-erane work. It results from the fact that the central point of each terminal is dispatcher taking on fundamental decisions for controlling terminal operations. This man must be simulated by optimal control algorithm. It is a fundamental condition for isomorphism between real terminal and simulation programs.

The system build on the basis of statements given above called SYSKON is conditioned for easy use by terminal designers. It consists of main program and several subroutines, The simulation system was build for the estimation of qualitative parameters of container terminal done by the designer analysing proposed version of the terminal. Change of parameters - opposite to structural changes - does not demand any changes in the algorithm of program connections. Therefore it does exist possibility for automatic finding of optiand parameters for every plant structure and organization. System of simulation programs was build for container terminal of middle capacity and daily overload (flow) of 300-400 containers. It can be easy adjusted for simulation of container terminals of other size. All programs are written in FORTRAL IV and have been tested on CDC-6316 computer working in CYBER system.

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A COMBINED FINANCIAL SIMULATION/GOAL PROGRAMMING MODEL FOR LONG RANGE BANK ASSET/LIABILITY AND INCOME PLANNING.

Financial statement simulation and linear programming models have been used as decision-support tools of bank planning during the past twenty years. Financial statement models have been found to be easy for management to comprehend and effective, even indispensable, in carrying out the tasks of data collection and organization, arithmetic, and reportwriting required in corporate planning. Linear programming models sometimes have been found effective in providing insights into funds management issues. Neither kind of model is immune to serious error arising from incorrect forecasts of interest rates, loans, deposits, and other economy-related variables. Remedies have been proposed in the forms of sensitivity analysis, subjective probability, and multiple scenario analysis.

This paper describes research on implementation of bank multiple scenario models. It is in two parts. The first briefly deals with conceptual issues including bank management objectives, the bank planning process, modeling philosophy, choice of variables, development of scenarios, aftertax income estimates, and model structure. The second part deals with the problems of combining financial statement and linear programming models. Specific information is given about exploiting the facilities of financial simulation and linear programming which permit utilization of the capabilities of both to calculate individual scenario plans. As individual plans are calculated, special additional constraints are activated which force trade-offs to take place between separate scenarios such that the result is a single plan for all scenarios. Subjective probability can be incorporated by appropriately weighting the additional constraints, and whatif or sensitivity analysis can be carried out by setting up one or more additional scenarios to interact with those already modeled.

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SCHEDULING MAINTENANCE OPERATIONS WHICH CAUSE AGE-DEPENDENT FAILURE RATE CHANGES

We consider optimization of schedules for maintenance or repairs, in order to minimize long term average operating cost or to maximize availability. The novelty here is that the failure rate after a maintenance operation is a function of the system's previously expended lifetime. This generalizes earlier work by others on the simpler case where the future rate depends only on the number of previous repairs, but not on the times when they took place. The underlying lifetime distributions are assumed to have the Weibull form and two classes of maintenance strategies are considered. In the first case, the period of periodic maintenance is optimized numerically. The main contribution in that case is the formulation of the new failure rate model. The second case optimizes a set of successive maintenance intervals $T_1, T_2, \ldots T_N$, and the number N, where a replacement by a new system is made at $N = \sum_{i=1}^{N} T_i$. For the specified model, we show that the optimal times T_i

exist and are ordered as $\mathtt{T}_1 \geq \mathtt{T}_2 \geq \ldots \geq \mathtt{T}_N$.

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OPTIMALITY OF (s, S)-POLICIES IN CONTINUOUS REVIEW INVENTORY MODELS.

Conditions for the optimality of (s,S)-policies in discrete time inventory models have been given by SCARF (1960), ZABEL (1962), IGLEHART (1963), VEINOTT (1966), PORTEUS (1971) and SCHAL (1976). In this paper we consider the continuous time version of this classical inventory model. The demand process is assumed to be a compound Poisson process. Using a generally applicable discretization procedure for Markov decision processes with continuous time parameter it is shown that for the continuous review model the same type of results can be obtained as for the discrete time version. This provides a rigorous foundation to the results obtained by BECKMANN (1961). Attention is also paid to the inventory model for which the demand process consists of a superposition of a compound Poisson process and a deterministic process. Also for this version conditions for optimality of (s,S)-policies are given.

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- 240-

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OPTIMAL MAINTENANCE POLICY AND SALE DATE FOR A MACHINE WITH RANDOM DETERIORATION AND SUBJECT TO RANDOM CATASTROPHIC FAILURE

The decision problem concerning the optimization of the maintenance policy and the selection of the planned sale date for a machine subject to random deterioration and random catastrophic failure is considered from a control-theoretic viewpoint. The formulation of the problem is based on the well-known model by Thompson [2], which is, however, completely deterministic: the machine cannot fail and its deterioration with age obeys a given deterministic law. Alam et al. [1] have presented a model where the random nature of both the deterioration and the life-time of the machine have been taken into account. In their model, however, the maintenance policy only is optimized, the option of selling a still operable, but perhaps already worthless and unprofitable machine is not taken into account. In order to avoid such an improper optimum for the problem, simultaneous optimization of both the maintenance policy and the sale date of the machine must be carried out also in the case of random deterioration and random life-time.

In the present model both the deterioration rate and the life-time of the machine are considered as random processes. This means that also the salvage value of the machine and the net present value to be maximized become stochastic processes. The salvage value and the deterioration rate are treated as state variables and the maintenance expenditure as a control variable. Because of the stochastic state equations the model gets the form of a stochastic optimal control problem.

The stochastic maximum principle is applied to derive the conditions for the optimal maintenance policy and for the optimal planned sale date to maximize the expected net present value of the machine. An analytic solution for the problem is found in the special case when some of the random processes are independent of time and thus simply random variables.

The case of one particular life-time probability distribution, the exponential case, is analyzed in full detail. The parameter of the distribution, i.e. the failure rate of the machine, is shown to have an interesting and important economic interpretation. It may be interpreted as a risk premium which can be used to adjust both the production rate and the discount rate to a higher level in order to get for the stochastic problem a deterministic certainty-equivalent problem.

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ANALYSIS OF A DISTRIBUTION TRANSFORMER INVENTORY SYSTEM

Transformers are used to convert electrical energy from high distribution voltages to lower voltages required by customers. A public utility must have on hand an adequate number of transformers to meet the varied requirements of customers. Transformers must be purchased in many different types and in a number of sizes for each type. The number of size/type combinations for a typical utility is near 450 and costs range from \$300 to \$19000.

Annual capital requirements needed to maintain inventory levels may approach \$3,000,000. The related costs and losses can significantly affect direct cashflow and overhead charges. It is important that capital devoted to inventory be controlled as closely as possible to strike an appropriate balance between inventory costs and the risk of stock outs which force the substitution of a less economical transformer (usually larger).

This paper describes a mathematical simulation model of an electrical transformer inventory/distribution system. The model addresses the organizational structure, material movement, delaysin information about decisions and actions, and capital commitment resulting from various inventory policies. A system of ordinary differential equations, using Schiesser's Differential Systems Simulator, Version 2 (DSS/2), is used to describe and simulate the transfer of material, orders and information about the status of the two level distribution system.

The effect of reducing information delays, changing of inventory stock policies, demand forecasting techniques and repair policies are assessed under varying usage scenarios. Four demand scenarios are tested that represent possible system perturbations: declining average demand, increasing average demand, stepped instantaneous demand, and stepped increase in demand due to failures.

The goal of the study is to find management policies and organizational structures that will reduce capital investment while maintaining appropriate customer service levels.

INDEX OF AUTHORS

Author	Page	Author	Page
L. L. Abdel-Malek	209	M. de la Sen	76
H. L. Abdel-Malek	138	J. H. de Vleiger	113
M. A. Abo-Hasswa	138	M. C. Delfour	211
S. Agarwal	119	R. C. Desai	214
Abdalla Sayed Ahmed	206	Francis DiGuglielmo	97
Samir A. Ahmed	228	S. W. Director	12
L. R. Albrecht-Ramos	183	J. Dolezal	205
Janan E. Allos	39	A. L. Dontchev	43
A. F. Aly	138	K. P. Dunn	78
Krzystoj Amborski	237	B. Ebrahimian	239
A. Ameljanczyk	67	W. J. Echeverria	225
F. Aminzadeh	194	V. R. Ekbote	221
E. J. Anderson	120	M. El-Arabaty	68
Juan P. Antun	178	M. M. El-Geneidy	41
F. Archetti	139	M. M. El-Metwalli	222
B. Asselmeyer	40	M. O. El-Shenawee	216
Ibrahim A. Awad	41, 220	Nabeel R. Elias	44
A. Bahgat	222	Yuri Ermoliev	123
M. Balas	94	E. Ersu	203
E. J. Balder	42	Roland Fahrion	184
Sanford Baum	179	Zoltan Farkas	153
S. A. Belbas	112	D. Feigenbaum-Cleiman	183
L. N. Belykh	7	J.F.R. Fernandes	124
A. Bermudez	95	A. J. Fish	215
Bruno Betro	121	M. I. Fouad	216
K. V.S. Bhat	158	D. A. Fredricks	90
M. A. Bisher	114	Daniel Gabay	98
Carl I.R. Blackwood	73	Ernest Gardenes	232
Howard Blum	140	T. K. Gardenier	185
Basilio Bona	210	E. Gelenbe	9
R. Bras	180	W. Gersch	77
V. Britanak	149	Arjun Godhwani	85
T. Brotherton	77	Bernhard Gollan	99
Peter Brucker	159	R. Gonzalez	45
B. H. Burzlaff	181	T. F. Gonzalez	46
Yair Censor	122	V. V. Gorokhovik	47
J. Chan	239	V. Gourishankar	70
C. B. Chang	78	J. O. Gray	48
A. C. Chaturvedi	182	J. D. Griffiths	86
N. Christopeit	231	D. Grossman	180
N. D. Christov	61	Martin Grotschel	172
J. T. Chu	160, 209	P. Gruber-Johnson	223
F. Clarke	8	Violet B. Haas	49
J. Clausen	74	W. W. Hager	50
A. I. Cohen	84	0. Hajek	51
P. Colleter	186	A. E. Hammad	224
C. Corduneanu	96	Steen Hansen	195
P. d'Allessandro	75	S. A. Hassan	114
J. C. Darragh	116	Ulrich Heilemann	196
Jiantranco d'Atri	171	J. K. Helbling	242

•

٩

INDEX OF AUTHORS (continued)

Author	Page	Author	Page
K. Hemamalini	214	Martin M. Lin	56
J. C. Hennet	125	W. S. Lin	57
D. Hinrichsen	197	D. P. Looze	107, 116
J. B. Hiriart-Urruty	126	F. Luccio	173
Jan Holst	74	L. Luksan	142
Jerzy Holubiec	67	Christiano Lyra	226
Roland Hornung	52	W. A. Mahmod	39
U. Y. Huh	54	A. Makino	87
Asghar Husain	145	K. Malanowski	103
Tai-Yang Hwang	161	M. Mandelberg	233
G. D. Ianculescu	50	G. Marchuk	7
D. D. Ilic	100	F. Marcotorchino	165
P. Ipsilandis	160	D. H. Martin	188
Masso Iri	162	S. Matsui	162
T. Ishibashi	87	Igor Medvedovsky	58
Nasser Jaleeli	225	J. L. Menaldi	187
J. M. Janer	232	G. Menga	210
M. Jankowski	237	P. Michaud	165
W. Jarisch	141	Robert Mifflin	144
J. Jasiulek	128	A. K. Mittal	119
Marc Kaltenbach	163	Masahiro Miyakawa	174
Hiroyuki Kano	53	J. B. Mockus	134
V. P. Kapur	115	R. R. Mohler	205
R. M. Karp	13	A. A. Montaser	138
Aaron Kerschenbaum	164	I. D. Moon	200
J. Killeen	14	Masao Murata	234
P. Klein	205	K. Murota	162
Myoung S. Ko	54	B. S.N. Murty	145
H. Kobayashi	16	N. Nakhla	48
Robert Komar	238	K. I. Naumenko	88
M. M. Konstantinov	61	T. Nishimura	53
T. C. Koopmans	17	M. L. Nitti	155
South S. Kousoum	154	Wolfgang Nooss	212
Werner Krabs	51	Romeo S. Ortega	189
Ulrich Krause	197	J. Ortmans	186
Te-Son Kuo	57	R. Paes e Barros	183
Y. Kuroda	87	L. Pagli	173
L. Kurz	81	C. Paige	19
B. Kuszta	237	K. R. Palanisamy	59
V. B. Larin	88	P. N. Paraskevopoulos	60
Irena Lasiecka	101, 102	Irena Pawlow	104
B. J. Lazarevic	190	G. Pavre	211
Alenis Lazaridis	198	P. H. Petkov	61
Pierre Lederer	186	D. B. Petkovski	62
Kwang-Soo Lee	199	B. J. Petrovic	63. 190
T. S. Lee	78	M. J. Petrovic	63. 190
William S. Levine	204	A. B. Philpott	120
A. V. Levitin	55	O. Pironneau	20
A. Lewandowski	133	E. Polak	146
Ching-Fang Lin	89	J. Ch. Pomerol	105

INDEX OF AUTHORS (continued)

Author	Page	Author	Page
H. V. Poor	116	L. E. Stern	64
V. F. Poterasu	135	Sueo Sugimoto	117
M. J.D. Powell	147	Y. Sunahara	10
J. P. Quadrat	187	M. K. Sundareshan	207
P. Radzikowski	136	H. Tavares	124. 21
M. Rakic	63	W. R. Terry	179
Ram C. Rao	201	Ng. O. Thai	150
A. H. Rashwan	206	G. J. Thaler	57
Z. V. Rekasius	79	T. V. Thieu	150
R. V. Ribeiro	124	V. A. Tiesis	131
M. G. Riskalla	181	A. L. Tits	146
T. G. Robertazzi	80	A. Trepat	232
M. Robin	106	Roberto Triggianni	102
E. Rofman	45, 187	Chee Tsai	81
A. E. Roth	69	J. Tsunekawa	235
Uriel G. Rothblum	69	T. S. Tung	65
R. Rotondi	121	H. Tuv	150
Emilio O. Roxín	64	V. I. Uberman	176
R. C. Ruiz	189	M. Uvalic	190
E. Sachs	148	F. A. Van der Duvn Schouten	240
Christian Saguez	95	F. Van der Ploeg	191
R. K. Saha	175	C. Van Nuffelen	168
A. I.A. Salama	70	K. S. Vastola	116
Awad I. Saleh	216	N. X. Vinb	89
H. Salhi	107	Ilkka Virtanen	241
V. Saltenis	129	W. E. Walker	11
E. S. Savas	21	Stanislaw Walukiewicz	151
M. E. Sawan	56	P. Y. Wang	169
F. Schoen	139	M R. Weale	191
A. Schornagel	156	Kaus-Joachim Werner	218
S. C. Schwartz	80	A. P. Wierzbicki	71
Yasuki Sekiguchi	166	Robert L. Williams	242
Remi Sentis	108	K. D. Wolfenstetter	52
E. Y. Shapiro	90	Keiji Yajima	235
P. Sharma	119	Paul P. Yeh	92
L. Shaw	239	F. E. Zajac	204
V. I. Shleinikov	176	S. Zak	237
Claudia L. Simionescu	109	E. E. Zakzouk	114
A. S.C. Sinha	91	A. H. Zemanian	192
Robert E. Skelton	217	J. P. Zolesio	211
F. Sloboda	149		
Leon Slominski	167		
J. Soares	226		
J. SOKOLOWSKY	110		
L. Specicato	130		
U. Speranza	155		
л. Spilld В Stammaulatia	74		
r. SLAVFOULAKIS	60		
D. SLEINIE Vomono I. Charlender	223		
iorgos J. Stephanedes	229		

. . .

.

-245-

١

. .

