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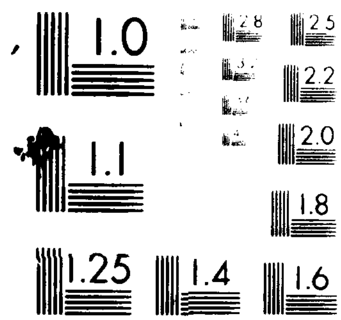
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By analyzing the equations of motion governing atmospheric motion in terms of spectral Hough functions, it is shown that for most of the possible motions in the atmosphere, the often used "boundary conditions" $\omega = 0$ or $\alpha = 0$ at $P_T = 0$ are in fact not true boundary conditions but merely the statement of a somewhat obscure property of the governing equations.

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For a top pressure $P_T \neq 0$, these are true mathematical boundary conditions with the following difficulties:

(a) there is no obvious physical interpretation; and

(b) the resulting motion below is sensitive to the nature of the boundary condition and the level at which it is applied.

It is shown that the boundary condition which states that phase velocity should be directed downwards for each of the Hough components is consistent with the dissipative effects of diffusion of momentum and heat. Also with this suggested boundary condition, the motion below is negligibly modified by changing the level at which the boundary condition is applied.

1. Introduction

In numerical weather prediction problems it is customary to apply boundary conditions of the form $w = 0$, $\omega = 0$, or $\dot{\sigma} = 0$ or a modification of either of these at the upper level. Examples are Kasahara and Washington (1971), Somerville et al. (1974), Ramanathan and Grose (1977), Staniforth and Daley (1977) and Donahue (1980). The purpose of this report is to highlight some difficulties of these boundary conditions and to suggest a possible alternative.

It is convenient to consider the boundary conditions that have been applied under two separate headings:

(a) an infinite atmosphere where boundary conditions are applied at $Z = \infty$ (identical to $P = 0$), and

(b) a finite atmosphere.

(a) Top boundary condition for an infinite atmosphere

For the discussion to follow it is worthwhile to note the difference between a true boundary condition and a statement of fact. As an illustration we consider the solution of the equation

$$\frac{d^2Q}{dZ^2} + 2\beta \frac{dQ}{dZ} + \alpha Q = 0 \qquad (1).$$

If both α and β are real and positive, a statement like "Q tends to zero as Z tends to infinity" is not a boundary condition; rather it is a statement of fact. This is because there is no way of combining the two different possible solutions to Eq.(1) without necessarily making Q tend to zero as Z tends to infinity.

On the other hand, if α were negative, the statement would be a boundary condition. For in this latter case one of the two possible solutions increases very rapidly as Z tends to infinity while the other tends to zero. The statement "Q tends to zero as Z tends to infinity" therefore discriminates between the two solutions.

Now let us consider the linearized equations of motion in the P-system of co-ordinates;

$$
\begin{aligned}
&\frac{\partial u}{\partial t} - fv + \frac{\partial \psi'}{\partial x} = 0 \\
&\frac{\partial v}{\partial t} + fu + \frac{\partial \psi'}{\partial y} = 0 \\
&\frac{\partial T'}{\partial t} + \omega \frac{d\theta_o}{dP} \frac{T_o}{\theta_o} = 0 \\
&\frac{\partial \psi'}{\partial P} + \frac{RT'}{P} = 0 \\
&\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial P} = 0
\end{aligned}
\qquad (2).
$$

Here, u, v and ω are the two horizontal components of velocity and DP/Dt respectively, P is pressure, ψ and T are geopotential and temperature, f is Coriolis parameter and R the gas constant.

The primed quantities are perturbations over those with subscript zero such that for example $T = T_o + T'$, $|T'| << T_o$.

Some algebraic manipulations yield

$$\frac{\partial}{\partial p}\left(\frac{P^2}{H^2 N^2}\frac{\partial \psi_n}{\partial P}\right) + \frac{\psi_n}{gh_n} = 0$$

$$\omega = \frac{i\nu P^2}{N^2 H^2}\frac{\partial \psi_n}{\partial P} \qquad (3)$$

where a solution of form $\frac{\partial}{\partial t} = i\nu$ is assumed and

$$\psi' = \sum_n \psi_n \; ; \quad \omega = \sum_n \omega_n .$$

Each of the ψ_n and ω_n are the spectral Hough components of ψ' and ω respectively and h_n is the corresponding equivalent depth defined in Laplace's tidal theory (see for example Wilkes, 1949); H is pressure scale height and N, the Brunt-Vaisala frequency, is given by

$$\frac{N^2 H^2}{P^2} = -\frac{RT_o}{P\theta_o}\frac{d\theta_o}{dP} \qquad (4).$$

Consider an isothermal atmosphere. The solutions to Eqs. (3) give

$$\omega_n \propto p^{\,m}; \; m = 1/2 \pm \left(1/4 - \frac{\gamma-1}{\gamma}\frac{H}{h_n}\right)^{1/2} \qquad (5)$$

where γ is the ratio of the two principal specific heats of air.

Now mesoscale phenomena and a large number of synoptic scale motions have positive equivalent depth. From Eq. (5) it is clear that if h_n is positive, the real parts of the two possible values of m are necessarily positive. That is to say that as P tends to zero ω tends to zero for all possible solutions.

We could have arrived at this conclusion intuitively as follows. Following the argument of Staniforth and Daley (1977) that T remains bounded as P tends to zero, and noting from Eq. (4) that $\frac{T_o}{\theta_o}\frac{d\theta_o}{dP}$ tends to infinity as P tends to zero, we can see immediately from the third equation in Eq. (2) that ω is necessarily zero at P = 0.

Thus we see that the statement "$\omega = 0$ at $P = 0$" is in fact not a boundary condition for many of the possible motions in the atmosphere. Rather for such motions the statement is only a property of the equations of motion.

Similarly if $\sigma = P/P_s$ where P_s is surface pressure, the statement "$\dot{\sigma} = 0$ at $\sigma = 0$" is not a boundary condition for mesoscale and other motions of positive equivalent depths. This can be shown as follows. Note that

$$\dot{\sigma} = \omega/P_s - P\,\dot{P}_s/P_s^2 \ .$$

Since $\omega = 0$ at $P = 0$, i.e. at $\sigma = 0$, the first part of R.H.S. of this equation is zero at $\sigma = 0$. The second part is also zero because $P = 0$ at $\sigma = 0$. Therefore $\dot{\sigma}$ is necessarily zero at $\sigma = 0$; and so once again the statement $\dot{\sigma} = 0$ at $\sigma = 0$ cannot be a boundary condition.

(b) Top boundary condition at $P > 0$

Let us solve Eq. (3) completely. As an illustration, apply the boundary conditions

$$\omega = \omega_s \text{ at } P = P_s, \text{ and } a\omega + b\frac{\partial\omega}{\partial P} = 0 \text{ at } P = P_T ,$$

where P_s and P_T are respectively the surface and top pressures of the model atmosphere and a and b are some constants to make the top boundary condition general. The solution of Eq. (3) becomes

$$\omega_n = \omega_s\left(\frac{P}{P_s}\right)^{1/2} \left\{\frac{A(P/P_T)^{\lambda} - B(P/P_T)^{-\lambda}}{A(P_s/P_T)^{\lambda} - B(P_s/P_T)^{-\lambda}}\right\} \tag{6}$$

where $A = aP_T + (1/2 - \lambda)b$, $B = aP_T + (1/2 + \lambda)b$, $\lambda^2 = 1/4 - \frac{\gamma-1}{\gamma}\frac{H}{h_n}$.

We can discuss Eq. (6) under the two special cases of λ real and λ imaginary.

<u>Case (a): λ real</u>

This case is relevant for large equivalent depths. If the top is high, i.e., P_T very small, Eq. (6) at lower levels become

$$\omega_n = \omega_s \left(\frac{P}{P_s}\right)^{1/2} \left(\frac{P}{P_s}\right)^{\lambda} \tag{7}$$

which is independent of the top boundary condition.

Thus provided the level (at which the top boundary condition is applied) is high enough and Eq. (3) is not oscillatory in the vertical, the particular boundary condition and the level at which it is applied have little or no effect on the solution to the equation.

Case (b): λ imaginary

This is the case for equivalent depths such that $h_n < 4 \frac{\gamma-1}{\gamma} H$. In this case $(P/P_T)^{\lambda}$ is not only a reciprocal of $(P/P_T)^{-\lambda}$ but also, is a complex conjugate of the other. So reducing P_T does not make one solution more important than the other - rather both solutions go through series of maxima and minima as P_T is altered. The overall effect is that ω_n at a particular P is critically dependent on P_T and on a and b. That is, the solution depends very strongly on the boundary condition and the level at which it is applied. This is clearly undesirable as it makes the final solution very sensitive to the parameter (boundary condition) of which we are uncertain.

It may also be noted from Eq. (3) that the boundary condition $\omega = 0$ implies that $\partial\psi'/\partial P = 0$. Apart from $W = 0$ which may be interpreted as a rigid lid representing the inversion at the tropopause, it is difficult to think of some physical processes represented by other boundary conditions like $\omega = 0$ or $\partial\psi'/\partial P = 0$ or $\dot{\sigma} = 0$.

This difficulty in choosing the upper level boundary condition is common in geophysical problems. In the case of the atmospheric diurnal tides, the solution was to choose a condition that was consistent with the effect of damping due to diffusion of heat and momentum; see for example Pekeris (1937), Giwa (1967). This procedure is reasonable since whenever there is some motion there is necessarily some diffusion. Therefore in the next section we shall introduce vertical eddy diffusion of the horizontal components of velocity and heat into the linearized equations of motion.

2. Inclusion of eddy diffusion in the linearized primitive equations of motion

(a) Governing equations

In order to simplify the derivation to follow we shall in the first instance consider meso-scale motions; that is we shall neglect Coriolis acceleration in comparison to $\partial/\partial t$ terms. It will be shown later that the conclusions arrived at will be equally applicable if the Coriolis terms are included. Indeed it can be deduced that the effect of the Coriolis terms is to modify the horizontal constant of separation which is often represented by the equivalent depth.

Using standard notation the linearized primitive equations of motion in the "P" system are

$$\frac{\partial u}{\partial t} + \frac{\partial \psi'}{\partial x} = g^2 \frac{\partial}{\partial P}\left(\eta \rho_o \frac{\partial u}{\partial P}\right) \tag{8}$$

$$\frac{\partial v}{\partial t} + \frac{\partial \psi'}{\partial y} = g^2 \frac{\partial}{\partial P}\left(\eta \rho_o \frac{\partial v}{\partial P}\right) \tag{9}$$

$$\frac{\partial \psi'}{\partial P} + \frac{RT'}{P} = 0 \tag{10}$$

$$\frac{\partial T'}{\partial t} + \omega \frac{d\theta_o}{dP} \frac{T_o}{\theta_o} = \frac{g^2}{C_p} \frac{\partial}{\partial p}\left(K\rho_o \frac{\partial T'}{\partial p}\right) \tag{11}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial P} = 0 \tag{12}$$

where g is acceleration due to gravity, C_p is specific heat at constant pressure, η and K are the eddy coefficients of viscosity and conductivity and the remaining symbols are as defined in Eq. (2).

From Eqs. (8) and (9) we have

$$\{\frac{\partial}{\partial t} - g^2\frac{\partial}{\partial P}(\eta\rho_o \frac{\partial}{\partial P})\}\{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\} + \nabla^2\psi' = 0 \qquad (13).$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

Differentiating (13) with respect to P and using (12) to eliminate $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$, we have

$$\frac{\partial}{\partial P}[\{\frac{\partial}{\partial t} -g^2 \frac{\partial}{\partial P}(\eta\rho_o \frac{\partial}{\partial P})\}\frac{\partial\omega}{\partial P}] + \frac{R}{P}\nabla^2 T' = 0 \qquad (14).$$

Combining Eqs. (11) and (14) to eliminate T' we have

$$\{\frac{\partial}{\partial t} - \frac{g^2}{C_p}\frac{\partial}{\partial P}(K\rho_o \frac{\partial}{\partial P})\}[\frac{P}{R}\frac{\partial}{\partial P} \{\frac{\partial^2\omega}{\partial t\partial P} - g^2\frac{\partial}{\partial P}(\eta\rho_o \frac{\partial^2\omega}{\partial P^2})\}] - \qquad (15)$$

$$- \frac{T_o}{\theta_o}\frac{d\theta_o}{dP}\nabla^2\omega = 0$$

Now the purpose of this discussion is to investigate the modification of the basic equations by diffusion. Therefore we shall assume that the diffusion terms are an order of magnitude less than the other terms. In particular we shall neglect products of the diffusion terms. Also we shall consider an isothermal atmosphere in order to obtain analytic solutions in terms of simple functions. It may be noted that upper boundary conditions are applied in the region of the tropopause which is very nearly isothermal.

With these approximations and assuming a temporal variation of the form exp(iνt) and noting that in an isothermal atmosphere $\frac{d\theta}{dP}\frac{T}{\theta} = -\frac{\gamma-1}{\gamma}\frac{gH}{RP}$, Eq. (15) becomes

$$\nu^2 P\frac{\partial^2\omega}{\partial P^2} + i\nu g^2\left[\frac{1}{C_p}\frac{\partial}{\partial P}\left\{K\rho_o\frac{\partial}{\partial P}\left(P\frac{\partial^2\omega}{\partial P^2}\right)\right\} + \left\{P\frac{\partial^2}{\partial P^2}\left(\eta\rho_o\frac{\partial^2\omega}{\partial P^2}\right)\right\}\right] -$$

$$-\frac{\gamma-1}{\gamma}\frac{gH}{P}\nabla^2\omega = 0 \qquad (16).$$

If for the sake of obtaining an analytic solution we make the reasonable assumption that the kinematic coefficients of diffusion are constant with height, i.e., $K/\rho_o = K_*/\rho_* =$ constant and $\eta/\rho_o = \eta_*/\rho_* =$ constant, and noting that the equivalent depth h is given by

$$g\nabla^2 = -\nu^2/h_n ,$$

we have

$$P^2\frac{\partial^2\omega_n}{\partial P^2} + \frac{iK_*}{\nu\rho_* C_p}\frac{P}{H^2}\frac{\partial}{\partial P}\left\{P^2\frac{\partial}{\partial P}P\frac{\partial^2\omega_n}{\partial P^2}\right\} + \frac{i\eta_*}{\nu\rho_*}\frac{P^2}{H^2}\frac{\partial^2}{\partial P^2}\left(P^2\frac{\partial^2\omega_n}{\partial P^2}\right) + \frac{\gamma-1}{\gamma}\frac{H}{h_n}\omega_n = 0$$

which can be simplified to give

$$P^2\frac{\partial^2\omega_n}{\partial P^2} + iN_D\left(P^4\frac{\partial^4\omega_n}{\partial P^4} + 4P^3\frac{\partial^3\omega_n}{\partial P^3} + 2P^2\frac{\partial^2\omega_n}{\partial P^2}\right) + \frac{\gamma-1}{\gamma}\frac{H}{h_n}\omega_n = 0 \quad (17)$$

where the non dimensional constant N_D, representing the effect of diffusion, is given by

$$N_D = \frac{1}{\nu H^2}\left(\frac{K_*}{\rho_* C_p} + \frac{\eta_*}{\rho_*}\right) .$$

(b) <u>Solution of the equation and application to upper level boundary condition</u>

Eq. (17) is homogeneous in P and hence we can seek a solution of form

$$\omega_n \propto P^M \tag{18}$$

When this is substituted into Eq. (17) we have

$$M(M-1) + iN_D M^2 (M-1)^2 + \frac{\gamma-1}{\gamma}\frac{H}{\bar{h}} = 0 \tag{19}$$

The discussion to follow is perhaps best done in terms of the Z system of coordinates. ALso it is convenient to define a new quantity

$$Q = \omega/P_o \tag{20}$$

Since $P_o = P_s e^{-Z/H}$ (P_s-mean surface pressure), substituting Eq. (18) into Eq. (20) gives

$$Q \propto e^{\lambda Z/H}; \qquad \lambda = 1-M \tag{21}$$

so Eq. (19) in terms of λ gives

$$\lambda(\lambda-1) + iN_D\lambda^2(\lambda-1)^2 + \frac{\gamma-1}{\gamma}\frac{H}{h} = 0 \qquad (22).$$

Eq. (22) is a fourth order equation and hence has four possible independent solutions, two of which do not exist if the damping coefficients are set to zero. Henceforth we shall refer to these two solutions as the diffusion terms. The other two solutions will exist even when the coefficients of diffusion are zero and they are the solutions which have always represented the normal modes of atmospheric motion. The purpose of this discussion is to see how these other two solutions are modified as a result of the inclusion of diffusion.

Before proceeding with the discussion it is worthwhile to note some salient points.

(1) The total kinetic energy in a column of air from the bottom to the top of the atmosphere (at $Z = Z_T$) is proportional to

$\int_o^{Z_T} Q^2 \rho_o dZ$ where Q is defined in Eq. (20). Since the atmosphere is of finite mass, the total energy must be finite even as Z_T tends to infinity. This means that Q should increase less rapidly with Z than $\rho_o^{1/2}$ decreases. From the expression for Q in Eq. (21) and since $\rho_o \propto e^{-Z/H}$, the finiteness of kinetic energy means that the real part of λ should be less than $\frac{1}{2}$.

(2) A wave that is generated as a result of reflection from the top should have its amplitude decreasing more rapidly (as it propagates downwards through the viscous atmosphere) than if

there were no diffusion. The obvious implication of this is that the amplitude of a wave reflected from above should increase faster in the positive vertical direction than the fundamental wave whose source is the energy from the lower levels.

Table 1 gives solutions of Eq. (22) for different values of H/h and N_D. The first value for each λ is the real part and the second represents the imaginary part. It should be noted that since Eq. (22) is fourth order in λ there are four different λ's for each solution except the special case $N_D = 0$, which has two different λ's. From the table it can easily be seen that the first set of values for λ in each box are physically impossible since these will result in unbounded energy as Z_T increases. This set is in fact one including the diffusion terms. We can easily identify the solutions involving diffusion terms from an examination of Eq. (22). As N_D is made smaller and smaller the corresponding λ for the diffusion terms increases in magnitude very rapidly - whereas λ for the other two terms tends to a solution of the equation

$$\lambda^2 - \lambda + \frac{\gamma-1}{\gamma}\frac{H}{h} = 0.$$

Thus we can see that the second set of values in Table 1 also belong to the diffusion term.

Let us for convenience call the remaining set of values λ_A and λ_B; λ_A being the 3rd set and λ_B the fourth set. A statement of our problem is to see which of λ_A and λ_B is physically realizable in an atmosphere with an open top. From

Table 1 we see that the real part of λ_A increases as the coefficient of diffusion increases. Thus this wave has its source of energy from above (i.e. reflected wave) and in propagating downwards it damps out more readily as the coefficients of diffusion are increased.

For λ_B the real part decreases as the diffusion coefficients increase. Hence this is a wave coming from the lower levels. This is in fact the only wave that satisfies the following conditions:

(1) It does not vanish when the coefficients of diffusion are zero;

(2) its existence is not a consequence of reflection from the top.

Thus the suggested boundary condition at the top is that $\sqrt{\rho_o}\, Q$ shall behave as exp $i(\nu t + kz)$. This means that at the top the phase velocity should be directed downwards. It may appear paradoxical that the phase velocity is directed downwards when the source of energy is from below. As shown by Wilkes (1949) such a system is in fact consistent with energy propagating upwards.

We note that since the solution does not contain either p_T or z_T, the boundary condition is independent of the level at which it is applied.

Table 1: Complex solutions for λ (Eq. 22) for various values of N_D and H/h.

H/h \ N_D	0		0.01		0.1		1.0		10.0	
			7.58	7.06	2.76	2.21	1.30	0.62	1.01	0.10
			-6.58	-7.06	-1.76	-2.21	-0.30	-0.62	-0.01	-0.10
0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
			7.58	7.06	2.77	2.21	1.31	0.62	1.03	0.10
			-6.58	-7.06	-1.77	-2.21	-0.31	-0.62	-0.03	-0.10
0.10	0.97	0.00	0.97	0.00	0.97	0.00	0.97	0.00	0.97	-0.01
	0.03	0.00	0.03	0.00	0.03	0.00	0.03	0.00	0.03	0.01
			7.58	7.06	2.77	2.20	1.33	0.61	1.06	0.11
			-6.58	-7.06	-1.77	-2.20	-0.33	-0.61	-0.06	-0.11
0.25	0.92	0.00	0.92	0.00	0.92	-0.00	0.92	-0.01	0.95	-0.02
	0.08	0.00	0.08	0.00	0.08	0.00	0.08	0.01	0.05	0.02
			7.58	7.06	2.78	2.19	1.36	0.59	1.09	0.12
			-6.58	-7.06	-1.78	-2.19	-0.36	-0.59	-0.09	-0.12
0.50	0.83	0.00	0.83	0.00	0.83	0.00	0.84	-0.03	0.92	-0.05
	0.17	0.00	0.17	0.00	0.17	0.00	0.16	0.03	0.08	0.05
			7.59	7.06	2.80	2.18	1.42	0.58	1.12	0.14
			-6.59	-7.05	-1.80	-2.18	-0.42	-0.58	-0.12	-0.14
1.00	0.50	-0.19	0.50	-0.19	0.52	-0.19	0.67	-0.18	0.88	-0.10
	0.50	0.19	0.50	0.19	0.48	0.19	0.33	0.18	0.12	0.10
			7.61	7.04	2.85	2.14	1.54	0.58	1.18	0.18
			-6.61	-7.04	-1.85	-2.14	-0.54	-0.58	-0.18	-0.18
2.50	0.50	-0.68	0.50	-0.68	0.54	-0.68	0.69	-0.54	0.83	-0.21
	0.50	0.68	0.50	0.68	0.46	0.68	0.31	0.54	0.17	0.21
			7.63	7.01	2.94	2.08	1.68	0.60	1.25	0.21
			-6.63	-7.01	-1.94	-2.08	-0.68	-0.60	-0.25	-0.21
5.00	0.50	-0.09	0.51	-1.09	0.59	-1.07	0.77	-0.77	0.82	-0.34
	0.50	1.09	0.49	1.09	0.41	1.07	0.23	0.77	0.18	0.34
			7.68	6.97	3.12	2.02	1.85	0.65	1.33	0.26
			-6.68	-6.97	-2.12	-2.02	-0.85	-0.65	-0.33	-0.26
10.00	0.50	-1.62	0.52	-1.61	0.70	-1.53	0.87	-1.02	0.84	-0.49
	0.50	1.62	0.47	1.61	0.30	1.53	0.13	1.02	0.16	0.49
			8.71	6.46	4.54	2.16	2.72	0.97	1.79	0.48
			-7.71	-6.46	-3.54	-2.16	-1.72	-0.97	-0.79	-0.48
100.00	0.50	-5.32	1.10	-5.06	1.57	-3.51	1.31	-2.04	1.02	-1.10
	0.50	5.32	-0.10	5.06	-0.57	3.51	-0.31	2.04	-0.02	1.10

3. Comparison with similar work and conclusion

Lilly and Kennedy (1973) observed that waves with small wave numbers tend to propagate through the troposphere with little reflection of energy. This led Klemp and Lilly (1975) to apply a boundary condition which allowed energy to propagate upwards. Thus the observation of Lilly and Kennedy agreed with our theoretical discussion here and an old suggestion of Weekes and Wilkes (1947) in which they pointed out that since most of the tidal energy is supplied at lower levels, the direction of energy flow at Z_T should be positive.

Lindzen (1974) applied a radiation condition at the top of the atmosphere. He applied a condition which made wave velocity positive in the direction of increasing Z. The basis for preference of positive wave velocity as opposed to negative wave velocity was not discussed. On the other hand, as discussed above, Wilkes (1949) showed that a negative wave velocity was consistent with energy propagating upwards.

Using the Sommerfeld radiation condition, Orlanski (1975) proposed a boundary condition by assuming that each of the variables satisfies a wave equation of form

$$\frac{1}{C}\frac{\partial \psi}{\partial t} = \frac{\partial \psi}{\partial Z}$$

at the top boundary. The value of C was calculated from previous values of ψ at the boundary and one spatial grid away from the boundary. Orlanski showed that by this procedure, if the problem was linear and consisted of a single wave, there was no reflected wave. If however there is more than one wave, as is always the case, there is bound to be some reflection. This reflection will take place at the level at which the boundary condition is applied. And so the resulting solution for a non-linear and non-monochromatic system will be sensitive to the level at which the boundary condition is applied. Also, since the calculated wave velocity depends on previous values of the variables, there may be a cumulative carry over of errors from one time step to the next.

The boundary condition suggested in this paper is free from the above problems. However, we have assumed here that the numerical analysis is in terms of spectral Hough functions. The application of this boundary condition in other methods of solution, like finite difference methods, is a problem worthy of further investigation.

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Appendix: Inclusion of Coriolis terms

Eqs. (8) and (9) when Coriolis terms are included become

$$i\nu u_+ - fv_+ + \frac{\partial\psi'}{\partial x} = \frac{fg^2}{i\nu}\frac{\partial}{\partial p}\left(n\rho_o\frac{\partial v}{\partial p}\right) \qquad (23)$$

$$i\nu v_+ + fu_+ + \frac{\partial\psi'}{\partial y} = -\frac{fg^2}{i\nu}\frac{\partial}{\partial p}\left(n\rho_o\frac{\partial v}{\partial p}\right) \qquad (24)$$

where $u_+ = u - \frac{g^2}{i\nu}\frac{\partial}{\partial p}\left(n\rho_o\frac{\partial u}{\partial p}\right)$, $v_+ = v - \frac{g^2}{i\nu}\frac{\partial}{\partial p}\left(n\rho_o\frac{\partial v}{\partial p}\right)$ and f is the Coriolis parameter.

The R.H.S. of Eq.(23) is one order of magnitude less than each of the terms on the L.H.S. We can therefore approximate the R.H.S. of the equation to be $F_1(\psi')$ where $F_1(\psi')$ is the expression for $\frac{fg^2}{i\nu}\frac{\partial}{\partial p}\left(n\rho_o\frac{\partial\bar{v}}{\partial p}\right)$ in the solution of the equations

$$i\nu\bar{u} - f\bar{v} + \frac{\partial\psi'}{\partial x} = 0$$

$$i\nu\bar{v} + f\bar{u} + \frac{\partial\psi'}{\partial y} = 0$$

This approximation in effect neglects products of the diffusion coefficients which is in keeping with the assumption (in the main text) of small coefficients of diffusion.

Similarly, the R.H.S of Eq. (24) becomes $F_2(\psi')$. On combination we have

$$\frac{\partial u_+}{\partial x} + \frac{\partial v_+}{\partial y} + \frac{\mathcal{F}(\psi')}{i\nu} = 0 \qquad (25)$$

where

$$\underset{\sim}{F} = \frac{\partial}{\partial x}\left[\frac{\{\frac{\partial\psi'}{\partial x} - F_1(\psi')\} + \frac{f}{i\nu}\{\frac{\partial\psi'}{\partial y} - F_2(\psi')\}}{1 - f^2/\nu^2}\right] + \tag{25}$$

$$+ \quad \frac{\partial}{\partial y}\left[\frac{\{\frac{\partial\psi'}{\partial y} - F_2(\psi')\} - \frac{f}{i\nu}\{\frac{\partial\psi'}{\partial x} - F_1(\psi')\}}{1 - f^2/\nu^2}\right]$$

From the definition of u_+ and v_+ Eq. (25) becomes

$$\{i\nu - g^2 \frac{\partial}{\partial p}\left(n\rho_o \frac{\partial}{\partial p}\right)\}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \underset{\sim}{F}(\psi') = 0 \tag{26}$$

which is the equation corresponding to Eq. (13) with $\underset{\sim}{F}(\psi')$ replacing $\nabla^2\psi'$.

The replacement of $\nabla^2\psi'$ by $F(\psi')$ is possible if $\underset{\sim}{F}(\psi')$ does not contain operations like $\frac{\partial}{\partial p}$. . However, in view of the fact that the $\frac{\partial}{\partial p}$ terms in the $\underset{\sim}{F}$ operator are small, the replacement will introduce only a very small error which will not significantly modify the conclusions we arrived at in the main text.

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