

REPORT DOCUMENTATION PAGE	READ INSTRUCTIONS
1. REPORT NUMBER 2. GOVT ACCE	BEFORE COMPLETING FORM
AFOSR-TR. 82-0245 40-411	
<ul> <li>TITLE (and Subtritle)</li> <li>Optimal Alpha-Beta filtering for tracking response</li> </ul>	S. TYPE OF REPORT & PERIOD COVERED Final
vehicles from shipboard radars.	15 TED. 01 - 15 TED. 02
·	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s)	8. CONTRACT OR GRANT NUMBER(s)
V. Vemuri	AF0SR-81-0076
PERFORMING ORGANIZATION NAME AND ADDRESS	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
DEPT. OF COMPUTER SCIENCE S.U.N.Y	• 61/02F 2304/AU
Binghamton, NY 13901	12. REPORT DATE
	15 Feb. 1982
AFOSR, Bolling Air Force Base, UM Washington, D.C.	19. NUMBER OF PAGES
MONITORING AGENCY NAME & ADDRESS(if different from Controllin	© Office) 15. SECURITY CLASS. (of this report)
	Uncl
	[ (p
CONSTRUCTION STATEMENT (of this Report) SUPPROVED FOR PUBLIC FOLCASE;	CLERIC DECLASSIFICATION DOWNGRADING
	SCHEDULE Classifier (Classifier of Classifier of Classifie
approved for public release;	Classifie Declaration dec
approved for public release;	SCHEDULE Classifier (Classifier of Classifier of Classifie
SIPPTONES FOR PULLIE TOLOGSE; 17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if o	SCHEDULE Classifier (Classifier of Classifier of Classifie
SIPPTONES FOR PULLIE TOLOGSE; 17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if o	SCHEDULE Classifier (Classifier of Classifier of Classifie
SIPPTONES FOR PULLIE TOLOGSE; 17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if o	SCHEDULE CUERT 1 (CERT, CONTRACTOR Allferent from Report)
APProved For Public release; 17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if a 18. SUPPLEMENTARY NOTES	SCHEDULE Classifie I (Classifier Classifier (Classifier (Classifie
SUPPROVED FOR PUBLIC FORCES ; 17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if of 18. SUPPLEMENTARY NOTES 19. KEY WORDS (Continue on reverse side if necessary and identify by block)	SCHEDULE CUERT 1 (CERT, CONTRACTOR Allferent from Report)
Approved For public release; 17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if o 18. SUPPLEMENTARY NOTES 19. KEY WORDS (Continue on reverse side if necessary and identify by bloc Tracking, reentry vehicles, ALPHA-BETA filter	SCHEDULE Classical Contraction for filterent from Report) Prock number) ring, Kalman Filtering. MAR O
APPROVED For PUBLIC Follows; 17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if of 18. SUPPLEMENTARY NOTES 19. KEY WORDS (Continue on reverse side if necessary and identify by bloc Tracking, reentry vehicles, ALPHA-BETA filter 20. ABSTRACT (Continue on reverse side if necessary and identify by bloc	SCHEDULE Classical and the complementation Hitterent from Report) Prock number) ring, Kalman Filtering. MAR O
Approved For Public release; 17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if o 18. SUPPLEMENTARY NOTES 19. KEY WORDS (Continue on reverse side if necessary and identify by bloc Tracking, reentry vehicles, ALPHA-BETA filter	SCHEDULE Classical and the contained for filtering, is developed for tracking
APProved For public releasery 17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if of 18. SUPPLEMENTARY NOTES 19. KEY WORDS (Continue on reverse side if necessary and identify by blo Tracking, reentry vehicles, ALPHA-BETA filter 20. ABSTRACT (Continue on reverse side if necessary and identify by blo An optimal filter, equivalent to Kalman	SCHEDULE Click 1 (Click, - Circlement) Hillerent from Report) ring, Kalman Filtering. (A number) filtering, is developed for trackings filter is simpler to implement
Approved For Public release; 17. DISTRIBUTION STATEMENT (of the obstract entered in Block 20, if of 18. SUPPLEMENTARY NOTES 19. KEY WORDS (Continue on reverse side if necessary and identify by bloc Tracking, reentry vehicles, ALPHA-BETA filter 20. ABSTRACT (Continue on reverse side if necessary and identify by bloc An optimal filter, equivalent to Kalman reentry vehicles from shipboard radars. This	SCHEDULE Click 1 (Click, - Circlement) Hillerent from Report) ring, Kalman Filtering. (A number) filtering, is developed for trackings filter is simpler to implement
Approved For public release in Block 20, if a 17. DISTRIBUTION STATEMENT (of the obstract entered in Block 20, if a 18. SUPPLEMENTARY NOTES 19. KEY WORDS (Continue on reverse side if necessary and identify by bloc Tracking, reentry vehicles, ALPHA-BETA filter 20. ABSTRACT (Continue on reverse side If necessary and identify by bloc An optimal filter, equivalent to Kalman reentry vehicles from shipboard radars. This than Kalman and is based on the idea of minif	SCHEDULE Click 1 (Click, - Circlement) Hillerent from Report) ring, Kalman Filtering. (A number) filtering, is developed for trackings filter is simpler to implement

AFOSR-TR- 82-0245

15-2

4

4

[-inal

OPTIMAL α-β FILTERING FOR TRACKING REENTRY VEHICLES FROM SHIPBOARD RADARS

HF05A81-0076

by

V. Vemuri Department of Computer Science SUNY, Binghamton, NY 13903

Accession For NTIS GRA&I X DILC TAR Unmassinged Junification DEIG Bv-Distribution/ Availability Codes Avail and/or Dist Special

82 04 06 027

Approved for public release; distribution unlimited.

#### I. INTRODUCTION

The USNS General H. H. Arnold and USNS General H. S. Vandenberg, operated by the Air Force Systems Command, Eastern Space and Missile Center (ESMC), are two Advanced Range Instrumentation Ships (ARIS) designed to gather precision data on missile reentry bodies and penetration aids. The primary mission of these ships is to collect metric and signature data during the midcourse and reentry phases of a ballistic missile's flight. These instrumentation ships play a significant role because they constitute a flexible expansion of a missile test range beyond the coverage capabilities of land based instrumentation. Therefore, these ships play a crucial role in tracking the terminal segment of a reentry vehicle's (RV) trajectory that, very often, falls in remote ocean areas.

Although there are differences, each ARIS in general, and H. S. Vandenberg in particular, is equipped with advanced instrumentation to gather radar, telemetry, optical, opto-radiometric, navigation, and meteorological data. Typically, during a trajectory mission, an ARIS collects data on: (1) Metric information such as range, azimuth and elevation of the RV.

(2) Signature information such as radar cross section.

(3) The ship's location and heading.

(4) Other relevant telemetry, optical and meteorological variables.

Following a tracking mission, the collected data are processed, off-line, (i.e., in a post-test environment) to obtain accurate estimates of metric and signature information. These estimates are required to accurately characterize an RV's flight from its "penetration point" (i.e., the point at which the RV enters the atmosphere) to the "splash point" (i.e., the point where the RV hits the ocean surface). ATR FORCE OFFICE OF SCIENTIFIC RESEAR"H (AFSC)

AIR FORCE OFFICE OF SCHARTER NOTICE OF TRANSMITTAL TO DTIC This technical report has been reviewed and is approved for public release IAW AFR 13D-12. Distribution is unlimited. MATTHEM J. KERPER Chief, Technical Information Division

Engineers and scientists at the Eastern Space and Missile Center recognized the need for improving the accuracy of metric information derived from data collected by radar trackers aboard the instrumentation ships. Toward this end, they embarked on a modernization program aimed at upgrading various systems on ARIS. For example, plaus are underway to upgrade the Timing System, the Communications System, the computer hardware, and so on. In this context, the author of this report was asked TO INVESTIGATE THE VARIOUS ALTERNATIVES THAT MIGHT LEAD TO AN IMPROVEMENT IN TRAJECTORY TRACKING ACCURACY THAT IS SUFFICIENT TO MEET THE MORE STRINGENT TEST REQUIREMENTS OF CURRENT AND UPCOMING MISSILE SYSTEMS. This statement is taken as the goal of this research project.

#### II. WORK DONE DURING SUMMER 1980

During the Summer of 1980, the principal investigator spent ten weeks at ESMC, Patrick Air Force Base, FL and conducted a feasibility study of this project. The work reported here is a continuation of the report submitted to the Southeastern Center for Electrical Engineering Education. A copy of the above cited report is attached herewith for ready reference as an appendix. As per Recommendation (3) of this report, we strived to develop an  $\alpha$ - $\beta$  type filter whose performance is comparable to a Kalman filter. In the following section, we present a detailed derivation of the relevant equations.

## III. DERIVATION OF EQUATIONS

While tracking a reentry vehicle (RV), radar returns in the form of noisy measurements can be processed to provide smoothed estimates of position, velocity and acceleration. Such smoothing can be accomplished with a variety of digital filters ranging in scope from a simple extrapolator to the complex Kalman filter. The purpose of this section is to derive the equations for a  $\alpha$ - $\beta$  filter which is easier to implement than a Kalman filter and yet equivalent in performance to a Kalman filter - at least for the case

under consideration.

The central idea of this derivation is to minimize the mean square error in predicted position of the RV.

### Assumptions

1. For simplicity in presentation, the coordinate systems and the associated transformations are ignored.

2. Only position and velocity are included in the equations. Acceleration will be included in a subsequent derivation.

#### The Original Equations

Let

X(t,) = Position of a RV at time t,

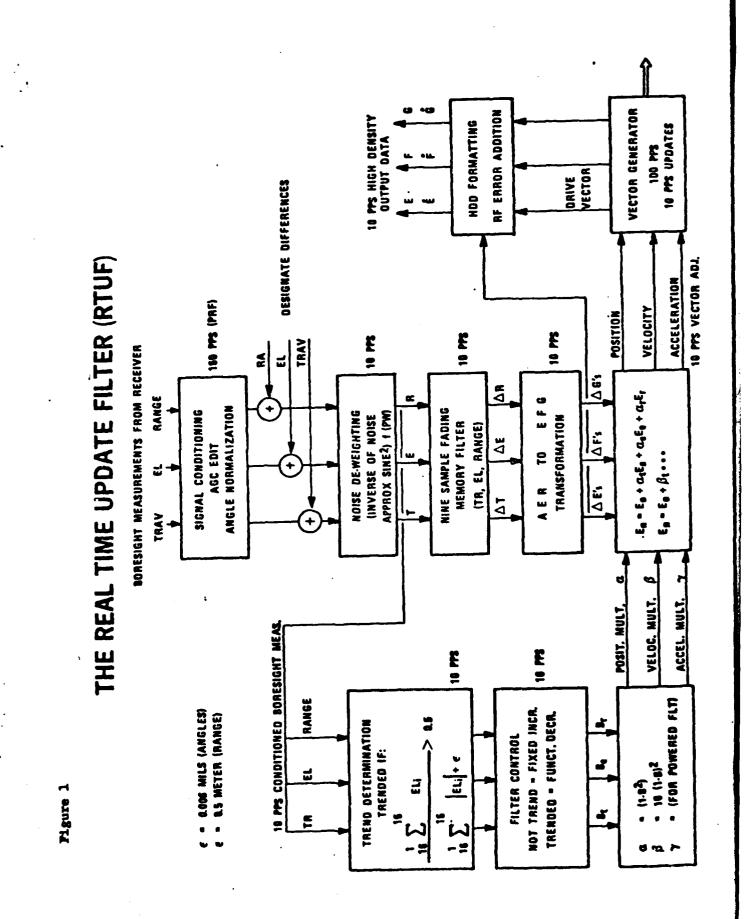
 $\dot{\mathbf{X}}(t_i) = \mathbf{Velocity}$  of the RV at time  $t_i$ 

 $T = t_{i+1} - t_i = sampling interval$ 

By "sampling interval", we mean the interval at which position and velocity data are sampled. We propose to use a uniform subscript notation as follows: The subscript e stands for estimated values, p for predicted values and m for measured values. Thus  $X_e(t_i)$ , for example, stands for the estimated position of the RV at time  $t_i$ . To make the meaning of the adjectives, "predicted", "measured", and "estimated" unambiguous, a visualization of the following scenario would be helpful. Typically, during a tracking mission, we first predict the state (position, velocity, acceleration, etc.) of a RV by whatever means (including guesswork) that are available. Ideally, a complete set of trajectory equations in the Earth's gravitational field would serve this purpose well. Since highly accurate equations of motion are hard to come by, it is common practice to base predictions on past measurements. A simple and widely used predictor is

$$X_{p}(t_{i+1}) = X_{e}(t_{i}) + T \dot{X}_{e}(t_{i})$$
 (1a)  
 $\dot{X}_{p}(t_{i+1}) = \dot{X}_{e}(t_{i})$ 

where the sampling interval is assumed to be small. Initially, as no estimated values are available, one generally starts with a set of nominal values for position and velocity. Then, we aim the radar in that general vicinity and actually measure the said state. Due to various errors, the measured values are not likely to be accurate. The central idea of filtering is to estimate the state by using the predicted and measured values in a suitable filtering formula. For example, the present system on USNS General H. S. Vandenberg uses the so-called real-time update filter (or RTUF, for short). A flow chart of this filtering process is shown in Figure 1. The equations characterizing this filter are:



States -

$$X_{e}(t_{i+1}) = X_{p}(t_{i+1}) + \alpha(X_{m}(t_{i+1}) - X_{p}(t_{i+1}))$$
 (2a)

$$\dot{X}_{e}(t_{i+1}) = \dot{X}_{p}(t_{i+1}) + \frac{\beta}{T}(\dot{X}_{m}(t_{i+1}) - \dot{X}_{p}(t_{i+1}))$$
 (2a)

It is useful to note that if  $X_m = X_p$  or  $\alpha = 0$ , then we made a perfect prediction and as a result  $X_e$  would become equal to  $X_p$ . On the other hand, if  $\alpha = 1$ , the predicted value is completely useless because the estimated value solely relies on the measured value. In other words,  $\alpha = 0$  corresponds to perfect confidence in our ability to predict and  $\alpha = 1$  corresponds to perfect confidence in the accuracy of the measuring instrument. A similar interpretation can be given to the meaning of  $\beta$ in the second of the two equations above. Evidently,  $\alpha$  and  $\beta$  assume values that are somewhere in the interval (0,1). Selection of appropriate values for these parameters is the crux of the filtering problem.

In the present system, the RTUF calculates the values of  $\alpha$  and  $\beta$  by using the following formulas.

$$\alpha = 1 - B^2$$
 (3a)  
 $\beta = (1 - B)^2$  (3b)

where

 $B_{\min} \leq B \leq B_{\max}$ (3c)

where  $B_{\min}$  and  $B_{\max}$  are arbitrarily chosen in the vicinity of 0.9 the acutal value of B chosen depends upon the trend in the radar data. A rationale for this strategem was given in reference [5].

There are several disadvantages to this approach. First, experience has shown that the RTUF is not very effective while tracking accelerating and maneuvering targets. Even if the target is not actually maneuvering, the uncompensated effects of ship's motion would make the target appear as if it is maneuvering. This is an important drawback of RTUF for use on ships. Finally, errors in prediction and measurement are not treated adequately.

### The Point of Departure

In view of the above difficulties, we propose to modify Equation (2) as follows:

$$X_{e}(t_{i+1}) = X_{p}(t_{i+1}) + \alpha(t_{i+1}) [X_{m}(t_{i+1}) - X_{p}(t_{i+1})]$$
(4a)

$$\dot{\mathbf{x}}_{e}(t_{i+1}) = \dot{\mathbf{x}}_{p}(t_{i+1}) + \frac{\beta(t_{i+1})}{T} [\dot{\mathbf{x}}_{m}(t_{i+1}) - \dot{\mathbf{x}}_{p}(t_{i+1})]$$
(4b)

Notice that if

$$\alpha(t_{i+1}) = \alpha(t_i) = \alpha$$
 (5a)

and

$$\beta(t_{i+1}) = \beta(t_i) = \beta$$
 (5b)

then, Equation (4) reduces to Equation (2). Thus, the only modification we are making here is to make the parameters  $\alpha$  and  $\beta$  depend on time. Now, instead of assigning arbitrary values to  $\alpha$  and  $\beta$  in an <u>ad hoc</u> fashion, we propose to determine their values by explicitly including the statistical nature of the errors in the algorithm. That is, we would like to calculate  $\alpha(t_{i+1})$  and  $\beta(t_{i+1})$  such that the expected mean square error in  $X_p(t_i)$  is minimized.

In other words, we would like to minimize  $\epsilon^2 [X_p(t_i)]$  where  $\epsilon[.]$  denotes the error in the bracketed quantity and the bar indicates expected value. This notation leads to somewhat cumbersome equations and for expediency, we henceforth propose to work primarily with the position equation whenever possible, although both the equations are required for a complete derivation.

# Derivation of New Equations

With the  $\epsilon[.]$  notation, the error in predicted position and velocity are, from Equation (1),

$$\epsilon[X_p(t_{i+1})] = \epsilon[X_e(t_i)] + T \epsilon[X_e(t_i)] + \Delta p(t_i)$$
(6a)

$$\epsilon \left[ \mathbf{x}_{\mathbf{p}} \left( \mathbf{t}_{\mathbf{i+1}} \right) \right] = \epsilon \left[ \mathbf{x}_{\mathbf{e}} \left( \mathbf{t}_{\mathbf{i}} \right) \right] + \Delta \mathbf{v} \left( \mathbf{t}_{\mathbf{i}} \right)$$
(6b)

where  $\Delta p$  and  $\Delta v$  are random variables representing errors in position and velocity and reflect modeling errors, i.e., errors introduced by using Equation (1). Let us assume that  $\Delta p$  and  $\Delta v$  have zero mean, variance of  $\sigma_p^2$  and  $\sigma_v^2$  and a covariance of  $\mu_{pv}$ . (The letters p and r in the preceding sentence are mnemonics for position and velocity respectively.)

Now our goal is to minimize

 $\frac{1}{\epsilon^{2} [X_{p}(t_{i+1})]}.$  Because  $X_{p}(t_{i+1})$  is a linear combination of  $X_{e}(t_{i})$ and  $\dot{X}_{e}(t_{i})$ , due to Equation (1), minimization of  $\frac{1}{\epsilon^{2} [X_{p}(t_{i+1})]}$  is equivalent to the minimization of  $\epsilon^{2} [X_{e}(t_{i})]$  and  $\frac{1}{\epsilon^{2} [X_{e}(t_{i})]}.$  So let us write the expressions for these quantities. Applying the operator  $\epsilon$  [.] to both sides of Equations (1) and (4) and

substituting (1) into (4), we get

$$\epsilon [X_{e}(t_{i+1})] = \alpha (t_{i+1}) \epsilon [X_{m}(t_{i+1})]$$

$$+ [1-\alpha (t_{i+1})] \left\{ \epsilon [X_{e}(t_{i})] + T \epsilon [X_{e}(t_{i})] + \Delta p(t_{i}) \right\}$$

$$(7a)$$

and

$$\epsilon \left[ \dot{\mathbf{X}}_{e}(\mathbf{t}_{i+1}) \right] = \epsilon \left[ \dot{\mathbf{X}}_{e}(\mathbf{t}_{i}) \right] + \Delta \mathbf{v} (\mathbf{t}_{i})$$

$$+ \frac{\beta(\mathbf{t}_{i+1})}{T} \left\{ \epsilon \left[ \mathbf{X}_{m}(\mathbf{t}_{i+1}) \right] \right]$$

$$-\epsilon \left[ \mathbf{X}_{e}(\mathbf{t}_{i}) \right] - T \epsilon \left[ \dot{\mathbf{X}}_{e}(\mathbf{t}_{i}) \right]$$

$$- \Delta p(\mathbf{t}_{i}) \right\}$$
(7b)

Squaring and taking expected values, the expected mean square errors are

$$\overline{\epsilon^{2} [X_{e}(t_{i+1})]} = \alpha^{2}(t_{i+1}) \sigma_{m}^{2}(t_{i+1}) + [1 - \alpha(t_{i+1})]^{2} \left\{ \overline{\epsilon^{2} [X_{e}(t_{i})]} + 2T \overline{\epsilon [X_{e}(t_{i})X_{e}(t_{i})]} + 2T \overline{\epsilon [X_{e}(t_{i})X_{e}(t_{i})]} + T^{2} \overline{\epsilon^{2} [X_{e}(t_{i})]} + \sigma_{p}^{2}(t_{i})$$
(8a)

and

1

ţ,

$$\frac{1}{\epsilon^{2} [\dot{x}_{e}(t_{i+1})]} = \left(\frac{\beta^{2}(t_{i+1})}{r^{2}}\right) \left\{\sigma_{m}^{2}(t_{i+1}) + \overline{\epsilon^{2} [x_{e}(t_{i})]}\right\} - \left(2\beta(t_{i+1}) [1-\beta(t_{i+1})]/T\right) \cdot \overline{\epsilon [x_{e}(t_{i})\dot{x}_{e}(t_{i})]} + \left[1-\beta(t_{i+1})\right]^{2} \overline{\epsilon^{2} [\dot{x}_{e}(t_{i})]} - \left(2\beta(t_{i+1})\right)^{2} \overline{\epsilon^{2} [\dot{x}_{e}(t_{i})]} + \left(\frac{1}{2}\beta(t_{i+1})\right)^{2} - \left(\frac{1}{2}\beta(t_{i+1})\right)^$$

where it is assumed that the measurement errors are uncorrelated with zero mean and variance of  $\sigma_m^2$ .

To determine the optimum values of  $\alpha(t_{i+1})$  and  $\beta(t_{i+1})$ , we take the partial derivatives of Equations (8a) and (8b) with respect to  $\alpha(t_{i+1})$ and  $\beta(t_{i+1})$  and equate them to zero. The resulting system of simultaneous equations are solved to get

$$\alpha(t_{i+1}) = \{\overline{\epsilon^{2} [x_{e}(t_{i})] + 2T\epsilon [x_{e}(t_{i}) \dot{x}_{e}(t_{i})]} + T^{2} \overline{\epsilon^{2} [x_{e}(t_{i})] + \sigma_{p}^{2} (t_{i})]} / \{\sigma_{m}^{2}(t_{i+1}) + \overline{\epsilon^{2} [x_{e}(t_{i})] + 2T\epsilon [x_{e}(t_{i}) \dot{x}_{e}(t_{i})]} + T^{2} \overline{\epsilon^{2} [x_{e}(t_{i})] + \sigma_{p}^{2} (t_{i})]} + \sigma_{p}^{2} (t_{i}) \}$$

$$(9a)$$

and

$$\beta(t_{i+1}) = \{ T \ \overline{\epsilon} \ [ X_{e}(t_{i}) \ \dot{X}_{e}(t_{i}) \ ] + T^{2} \overline{\epsilon^{2}} \ [ \dot{X}_{e}(t_{i}) \ ]$$

$$+ T \mu_{pv}(t_{i}) \} / \{ \sigma_{m}^{2} \ (t_{i+1}) + \overline{\epsilon^{2}} \ [ X_{e}(t_{i}) \ ]$$

$$+ 2T \overline{\epsilon} \ [ X_{e}(t_{i}) \ \dot{X}_{e}(t_{i}) \ ]$$

$$+ T^{2} \overline{\epsilon^{2}} \ [ \dot{X}_{e}(t_{i}) \ ] + \sigma_{p}^{2} \ (t_{i}) \}$$
(9b)

Essentially, we have to evaluate Equations (9a) and (9b). Before we can do this, we have to recursively evaluate  $\epsilon [X_e(t_i) X_e(t_i)]$ . Toward this end, we multiply Equations (7a) and (7b) (before the expected values are taken there) and taking the expected values of the product expression, we get

$$\frac{1}{\epsilon \left[ X_{e}(t_{i+1}) X_{e}(t_{i+1}) \right]} = \left\{ \alpha(t_{i+1}) \beta(t_{i+1}) / T \right\} \sigma_{m}^{2}(t_{i+1}) - \left[ 1 - \alpha(t_{i+1}) \right] \left[ \beta(t_{i+1}) / T \right], \frac{1}{\epsilon^{2} \left[ X_{e}(t_{i}) \right]} + \left[ 1 - \alpha(t_{i+1}) \right] \left[ 1 - 2\beta(t_{i+1}) \right], \frac{1}{\epsilon} \left[ X_{e}(t_{i}) X_{e}(t_{i}) \right] + T \left[ (1 - \alpha(t_{i+1})) \right] \left[ \beta(t_{i+1}) / T \right], \frac{1}{\sigma_{p}^{2}(t_{i})} + \left[ 1 - \alpha(t_{i+1}) \right] \left[ \beta(t_{i+1}) / T \right], \frac{1}{\sigma_{p}^{2}(t_{i})} + \left[ 1 - \alpha(t_{i+1}) \right] \frac{1}{\rho_{pv}(t_{i})} \quad (10)$$

Thus, the current values of  $\alpha$  and  $\beta$  are dependent upon 1. The current variance in measurement error,  $\sigma_m^2$  (t<sub>i+1</sub>)

2. Previous estimates of mean squared error in smoothed (or estimated) position and velocity,  $\epsilon^2 [X_e(t_i)]$  and  $\epsilon^2 [X_e(t_i)]$ .

3. Previous estimages of the variance and covariance in predictor errors,  $\sigma_p^2(t_i)$ ,  $\sigma_v^2(t_i)$  and  $\mu_{pv}(t_i)$ 

IV. PROCEDURE FOR IMPLEMENTING THE FILTER

Assume that we know

1. 
$$\sigma_p^2(t_i), \sigma_v^2(t_i) \text{ and } \mu_{pv}(t_i)$$

2. 
$$\epsilon^2 [X_e(t_i)]$$
 and  $\epsilon^2 [X_e(t_i)]$ 

<u>Step 1</u>. Update the predictor model defined in Equations (1a) and (1b) for the given time T.

Step 2. Estimate the variance in current position measurement.

(NOTE: No procedure for this is given here. A procedure analogous to the one we use in Kalman filtering can be used here.)

Step 3. Obtain estimates of  $\alpha(t_{i+1})$  and  $\beta(t_{i+1})$  using equations (9a) and (9b).

Step 4. Obtain smoothed (or estimated) values,  $X_p(t_{i+1})$ , and  $X_p(t_{i+1})$ from Equations (4a) and (4b) by using the current position measurement.

Step 5. Obtain estimates of

$$\overline{\epsilon^2 [X_e(t_{i+1})]}, \overline{\epsilon^2 [X_e(t_{i+1})]} \text{ and}$$

$$\epsilon^{2} [X_{e}(t_{i+1}) \dot{X}_{e}(t_{i+1})]$$

using the recursive relationships shown in Equations (8) and (10) for use in next iteration.

12.

<u>Step 6</u>. Estimate the variance and covariance in predicted values of Step 1 for use in next iteration.

(NOTE: Same comment as the one in Step 2.)

Once the procedure is started, the calculations are simple to do.

# APPENDIX

# 1980 USAF - SCEEE SUMMER FACULTY RESEARCH PROGRAM

## Sponsored by the

# AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

# Conducted by the

## SOUTHEASTERN CENTER FOR ELECTRICAL ENGINEERING EDUCATION

#### FINAL REPORT

# IMPROVEMENT OF TRAJECTORY TRACKING ACCURACY OF INSTRUMENTATION SHIPS: <u>A FEASIBILITY STUDY</u>

Prepared by:

1

Dr. V. Vemuri

Academic Rank:

Associate Professor

Department and University:

Research Location:

USAF Research Colleague:

Date:

Contract No.

Department of Computer Science State University of New York

Eastern Space and Missile Center Range Systems and Navigation

Mr. Steve Andrews

August 15, 1980

F49620-79-C-0038

# IMPROVEMENT OF TRAJECTORY TRACKING ACCURACY OF INSTRUMENTATION SHIPS: A FEASIBILITY STUDY

· by

V. Vemuri

## ABSTRACT

The question of the feasibility of improving metric accuracy of radar data obtained from instrumentation ships is investigated. It is argued that major sources of error are tracking, navigation and stabilization. Using available data as a guide, it is argued that substantial improvements in metric accuracy are attainable if the present auto-tracking is upgraded to on-axis tracking with a Kalman-type filter in the tracking loop. It is also recommended that a simulation study be conducted to gain better insight into the nature of navigational and stabilization errors. These two recommendations are considered to be most cost-effective within the constraints of the mission under study.

#### ACKNOWLEDGEMENTS

The author would like to thank the Air Force Systems Command, the Air Force Office of Scientific Research and the Southeastern Center for Electrical Engineering Education for providing him with the opportunity to spend a useful summer at the Eastern Space and Missile Center (ESMC), Patrick AFB, FL. He would like to thank the people at ESMC, in particular, those associated with RSN for their hospitality.

Although several people went out of their way to help the author in formulating the ideas presented here, particular mention must be made of Steve Andrews (USAF), Walt Elenburg (RCA), and Charles Miller (USAF) for their collaboration and guidance. Further, they read an initial draft of this report and helped the author in identifying and correcting several errors. Finally, the author gratefully acknowledges all the help he received from Connie Pelligra for typing this report.

## I. INTRODUCTION

The USNS General H. H. Arnold and USNS General H. S. Vandenberg, operated by the Air Force Systems Command, Eastern Space and Missile Center (ESMC), are two Advanced Range Instrumentation Ships (ARIS) designed to gather precision data on missile reentry bodies and penetration aids.<sup>1</sup> The primary mission of these ships is to collect metric and signature data during the midcourse and reentry phases of a ballistic missile's flight. These instrumentation ships play a significant role because they constitute a flexible expansion of a missile test range beyond the coverage capabilities of land based instrumentation. Therefore, these ships play a crucial role in tracking the terminal segment of a reentry vehicle's (RV) trajectory that, very often, falls in remote ocean areas.

Although there are differences, each ARIS in general, and H. S. Vandenberg in particular, is equipped with advanced instrumentation to gather radar, telemetry, optical, opto-radiometric, navigation, and meteorological data. Typically, during a trajectory mission, an ARIS collects data on:

(1) Metric information such as range, azimuth and elevation of the RV.

- (2) Signature information such as radar cross section.
- (3) The ship's location and heading.
- (4) Other relevant telemetry, optical and meteorological variables.

Following a tracking mission, the collected data are processed, off-line, (i.e., in a post-test environment) to obtain accurate estimates of metric and signature information. These estimates are required to accurately characterize an RV's flight from its "penetration point" (i.e., the point at which the RV enters the atmosphere) to the "splash point" (i.e., the point where the RV hits the ocean surface).

Engineers and scientists at the Eastern Space and Missile Center recognized the need for improving the accuracy of metric information derived from data collected by radar trackers aboard the instrumentation ships. Toward this end, they embarked on a modernization program aimed at upgrading various systems on ARIS. For example, plans are underway to upgrade the Timing System<sup>2</sup>, the Communications System<sup>3</sup>, the computer hardware, and so on. In this context, the author of this report was asked TO INVESTIGATE THE VARIOUS ALTERNATIVES THAT MIGHT LEAD TO AN IMPROVEMENT IN TRAJECTORY TRACKING ACCURACY THAT IS SUFFICIENT

TO MEET THE MORE STRINGENT TEST REQUIREMENTS OF CURRENT AND UPCOMING MISSILE SYSTEMS. This statement, therefore, is taken as the goal of this research.

This report, therefore, is the result of a feasibility study. The recommendations made at the end of this report constitute a plan to reach the goal.

#### **II.** OBJECTIVES

Although the instrumentation ships gather a wealth of data during a tracking mission, the scope of the goal statement is confined to a study of the acquisition and processing of metric data only; that is, data about range, azimuth and elevation of a RV. Even with this restriction, the scope is still too broad in the sense that it allows the possibility of including a host of competing alternative parks in reaching the stated goal. Some of these alternatives are considered extraneous to the major thrust of this research effort and are, therefore, aliminated forthwith from further consideration. Thus, no proposals for improving the electronics aboard the tracking ship are either made or considered. Simplarly, no proposals to increase the number of tracking ships are either made or considered. In fact an attempt is being made to strike a balance between & desire for a thorough exploration of all possibilities and a desire to confine the effort to those alternatives that appear feasible. Included in the feasibility considerations are technical criteria such as practical realizability and compatibility with existing configurations, economic criteria such as development and maintenance cost and the management criterion, namely an ability to meet deadlines. In our pursuit to reach the goal within the framework of constraints articulated in the preceding paragraph, we propose to seek specific answers to the following broadly stated questions:

- (1) Can we improve the metric accuracy of the instrumentation ships to meet the stringent test requirements of current and up-coming missile systems?
- (2) If the answer to this question is in the affirmative, then to what extent can the accuracy be improved with the present configuration of the ship's instrumentation?
- (3) If modifications to the present configuration are warranted, then can these modifications be made within reasonable time and cost constraints?
- (4) If modifications are considered necessary, which of these modifications should be directed at improving the quality of the data acquisition phase and which toward improving the quality of the post-test data analysis phase?

(5) Whatever the modifications are, what would it take to actually implement them?

#### III. DESCRIPTION OF A TYPICAL ARIS MISSION

To present the results of this research in a proper perspective, it is useful to review the salient steps of a typical mission.

Stated in a nutshell, the present tracking procedure consists of two broad phases: the on-board data acquisition phase and the off-board data processing phase. Preparatory to the data acquisition operations, first the longitudinal axis of the instrumentation ship is positioned roughly perpendicular to the plane of the trajectory of the RV. Then the location of the ship with respect to an earth-fixed geocentric (EFG coordinate) system is determined with the help of data provided by the ship's inertial navigation system (SINS) as well as from fixes obtained from an array of submerged transponders. Then, to lend a degree of stability to the observation platform, the ship is maintained on a linear course at a small constant velocity. Then the target is acquired by the radar and tracked in the so-called "auto-track" mode and the necessary data are recorded on digital and video tapes. This constitutes the end of the first phase.

Among all the data items collected during the data acquisition (or first) phase, only a few are of particular interest to us here. Those data items are navigation data (to determine the exact location and attitude of the ship at any time), metric data (to determine the range, elevation and azimuth of the target with respect to any desired coordinate system) and timing data (for synchronization purposes). These data items, recorded on digital tapes, will be referred to as raw mission data.

The second phase of the tracking procedure, called the data processing phase, takes the raw mission data as input and produces, as output, target position, velocity and acceleration relative to any desired fixed point and any fixed reference system. That is, the output of this phase is the required metric data about the RV's trajectory. This phase is conducted off-line in a post-test environment.

#### IV. DISCUSSION ON PRESENT EFFORTS TO IMPROVE METRIC ACCURACY

The purpose of this section is to identify and comment on some of the problems being encountered in the context of accuracy improvement. Engineers at ESNC

(representing USAF, Pan Am, RCA) are addressing these problems systematically and thoroughly. Their approach is based on an identification of all possible sources of error, classification into categories (significant, insignificant; biased, random; and so on), and charaterization in terms of variables that can be controlled. This approach led to a satisfactory charaterization of the behavior of some of the errors. For example, engineers at ESMC appear to be fairly happy with their ability to control the errors due to, say, nonorthogonality of antenna axes, antenna droop, refraction and RF optical errors. This is understandable because such errors are common with all (land based and shipboard) radar trackers and most of the technical personnel have rich experience in this area.

The problem gets a little complex while studying the performance of shipboard radars. A new class of errors enter the picture here. A case in point are errors introduced due to our inability to estimate accurately the position and attitude of the ship on the ocean surface. Position simply means the coordinates of the ship in the EFG (earth fixed geocentric) coordinate system. Attitude refers to two aspects of ship motion: roll and pitch relative to the local vertical, and heading relative to north. In this report, we would like to refer to position and heading errors as navigational errors and errors due to roll and pitch as stabilization errors. In addition to these errors, we also have to deal with flexure errors, that is, errors caused by bending and flexing of the ship's body. Some effort is already under way to minimize the effect of these errors. For example, attempts are being made to minimize navigational errors by supplementing SINS with position data obtained from submerged transponders. Similarly, attempts are being made to minimize flexure errors by redesigning the physical layout of some of the instruments.

This investigator feels that all the above cited measures are necessary but not sufficient. This feeling is based on the assumption that there always exists residual errors; to assume that we can identify, characterize, and compensate each and every source of residual error is unrealistic. There is no suggestion here to indicate that the present approach of correcting major errors on an individual basis be abandoned. What is being suggested here is to supplement the present effort with a good tracking procedure that explicitly recognizes the need to compensate the residual errors during the data acquisition phase.

The development of good tracking procedures for shipboard radars is not a trivial matter. The difficulty arises because the errors associated with the range, azimuth and elevation (say in the EFG coordinate system) are an aggregate of residual errors from all sources cited above. To gain an insight into the nature and magnitude of these aggregate residual errors, it is useful to digress and inspect the magnitudes of these errors resulting from the tracking procedures currently in use at ESMC.

The present data acquisition system on H. S. Vandenberg, for example, uses the autotracking procedure. The monopulse radar in an autotrack mode is essentially a simple closed loop control system in which the radar receiver feeds the control signal directly to the radar sensors. In other words, there is no predictive capability in the autotracking mode. Improved performance can be obtained if the loop is closed through a computer so that the computer can be used to anticipate the RV's position from past data. This is the basic idea of on-axis tracking. The Eastern Test Range used this on-axis concept as early as 1967 to control the ground based radar (the AN/FPQ-13) at Grand Bahama Island. The Western Test Range also used the on-axis concept for its ground based radar at Kaena Point, Hawaii.

The on-axis concept can be used at several levels of sophistication. Even the most primitive type of on-axis tracking is an order of magnitude better than the simple autotracking. Indeed, the on-axis procedure used in Grand **Bahama** Island and Kaena Point radar trackers (namely, the  $\alpha-\beta$  tracking) is relatively primitive. Nevertheless, it is apparently giving satisfactory performance. For example, it is claimed that the above two, as well as other ground based radars using  $\alpha$ - $\beta$  tracking, are giving an accuracy of  $\pm 6$  ft. in linear measurements and +  $(1/2^{20})$  x  $2\pi$  radians (= 0.006 milliradians) in angle measurements.<sup>5</sup> In contrast, the autotracked radars on H. S. Vandenberg are committing errors of up to + 1500 ft. in range and + 0.5 milliradians in angle measurements. Two possible reasons for the poor quality of metric information from H. S. Vandenberg are: (a) measurements are taken from a moving, unstable platform, namely the ship; and (b) the radar trackers on the ship are using autotracking rather than on-axis tracking. A useful question that should be answered at this stage is: What part of the error is caused by the moving, unstable platform and what part by autotracking? For example, 1 mr of ship roll, it is observed, produces about one inch of C-band antenna movement but can introduce

up to 1000 ft of RV position error at pierce point.' However, it is dangerous to conclude (from what is stated so far) that 2/3 of the range error is due to an unstable platform and only 1/3 due to autotracking; other factors, such as navigational errors, also play a role in this context. For example, it appears that the uncertainty in this ship's location could be of the order of  $\pm$  1500 ft. The precise impact of this error on metric information is unclear.

This digression (the preceding three paragraphs) confirms the general qualitative feeling that tracking, stabilization and navigational problems are major problems to be solved. However, we do not yet have any quantitative measures of the contribution of each of these to the total observed error. Until we can develop quantitative estimates of the error contributions from each of these sources, we will not be in a strong position to decide where we should put our resources to meet our goal. Toward this end, we developed in the next section, some quantitative estimates of the order of magnitude improvements that can be made by improving tracking. We do not have information to help us make analogous estimates insofar as navigation and stabilization errors are concerned. In Section VI, we proposed that a simulation study be conducted to gain better insight into navigation and stabilization errors.

## V. SPECTRUM OF FUTURE POSSIBILITIES

The limited scope of the discussion in the preceding four sections is not sufficient to completely reveal the complexity of a trackingmission. Nevertheless, the success of a mission depends upon the coordinated operation of a number of subsystems. Modification of one subsystem or one operational procedure could create a ripple effect demanding modifications in other systems. As the scope, as outlined in Section II, of this effort is somewhat limited, it is not possible to address all these ripple effects. By the same token, they cannot be ignored either. In this section, we intend to propose some modifications and briefly touch upon some of the possible ripple effects.

A. <u>Possible Modifications to Tracking</u>. The radar trackers on H. S. Vandenberg now use autotracking. Engineers at ESMC recognized a decade ago the inadequacy of autotracking and the desirability of on-axis tracking.<sup>8, 9</sup> In fact, Reference 9 gives the figures shown in Table 1 to illustrate the advantage of on-axis tracking over autotracking. The reference did not mention the filter used in the on-axis algorithm. However, there is reason to believe

that it probably was an  $\alpha-\beta$  filter; definitely not a Kalman filter.

Variable	Autotracking	On-Axis	Absolute Improvement	<pre>% Improvement (Approximate)</pre>
Elevation errors	0.22 mr	0.044 mr	0.176	80 <b>%</b>
Range error	33.5 ft	10.5 ft	23.0	69%

Table 1

A comparison of orbital residuals from echo track of the satellite Pegasus performed by Radar 12.15 at Ascension in February 1974.

Inspection of Table 1 reveals that accuracy improvements of the order of 70 - 80% can be attained by replacing autotracker with a simple  $\propto -\beta$  tracker. Furthermore, in Reference 4 (page 1.5), it was estimated that a nine-variable Kalman filter has the potential of giving a 30% improvement in real-time tracking accuracy over an  $\propto -\beta$  filter. Admittedly, these two estimates were made in the context of ground based radars. In spite of this knowledge, it is surprising that ESMC did not initiate plans to upgrade tracking procedures on ARIS until the 1980s. There are two frequently quoted reasons for this delay. First, that the ARIS were originally designed to gather signature data and the desire to use them for metric data gathering is an afterthought. Second, that everything possible is being done to correct errors due to various sources and that there is nothing much to be gained by overhauling the tracking procedure. The first of the above implies that there is a change in mission requirements and the second reflects a lack of strong conviction. Nevertheless, plans are under way to explore the possibility of replacing autotracking with on-axis  $\sim \beta$  tracking. This is definitely a step in the right direction, although not a decisive one. The author feels that this line of thinking should be pursued more aggressively by going all the way to a Kalman-type tracker rather than stop in midstep with an  $\propto -\beta$  tracker. If there are some practical reasons for a reluctance to go all the way for a full-blown Kalman tracker, there exist some suboptimal Kalman trackers for consideration. There are even optimal  $\alpha-\beta$ trackers that are equivalent to Kalman trackers in their performance.<sup>10</sup>

Without going into detailed analyses of the advantages and disadvantages of Kalman filters in general, let us briefly look into the ripple effects of on-axis traking in the context of a typical ARIS mission. At present, most of the metric information is being derived by off-line processing of data in a post test environment. On-axis tracking (be it  $\alpha-\beta$  or Kalman variety) is an on-line, real-time procedure. This implies that there is a need for an additional computer, on-board, dedicated to on-axis tracking. This computer should be supplied with navigation data, in addition to the usual tracking data. This, in turn, means that we cannot afford the luxury of waiting for the splash point to occur in order to determine the ship's location. That is, we have to make navigation independent of tracking but not vice versa.

A second possible side effect has to do with data processing operations such as the editing of raw data and accuracy of encoders. For example, Kalman filters tend to be sensitive to the editing scheme used. Also, if a lot of effort is going to be expended in improving the accuracy of data gathered, corresponding attention should be paid in maintaining this accuracy in digitizing and encoding this information. Toward this end the UNIVAC 1219 type computer with a standard word length may not be sufficient. A computer with a 32-bit word length and 64K of storage is probably needed for any sophisticated scheme.

A third side effect of on-axis tracking is the need to perform the computations on-line although this is not at all a requirement of the mission.

B. Possible Modifications to Navigation. There is a general feeling at ESMC that the impact of navigational errors on metric accoracy are not as severe as those of tracking and stabilization errors. This is predicated on the assumption that the coordinates of the splash point can always be determined fairly accurately and the ship's location with respect to the splash point can therefore be derived because the ship is generally not too far from the splash point. As noted earlier, this implies that the ship's location can be accurately determined <u>after</u> the mission, not before or during the mission because the splash point is the terminal point of the RV trajectory. But the accuracy of on-axis tracking depends, to some extent, on advance knowledge of ship's position. Thus, to make the on-axis tracking really useful, we must look for ways to determine the ship's position and heading by methods that do not depend upon data derived from the trajectory. This is an interesting side effect of using on-axis tracking. Therefore, either we have to determine ship's position only from ship's

inertial navigation system (SINS) or supplement it with some other system. One possibility is to explore the possibility of using the Navstar Global Positioning System (GPS) currently under development by the Department of Defense. The GPS system can be used to get both time and position information precisely. In fact, the GPS system is being considered in Timing Modernization Plan.<sup>2</sup> If it is going to be used to get timing information, one may as well use it to get position data also. Presumably, the cost involved in using the GPS system is in building a GPS receiver.<sup>11</sup>

Alternatively, one can take advantage of the similarity between the problem of tracking the position of an RV in space and the problem of tracking the position of a ship on ocean surface. As both problems involve the observation of a moving object with imperfect instruments and subsequently filtering the noisy data, the same procedure, with appropriate modification, can be used in both cases. This strategy has the aesthetic appeal of depending on a uniform procedure to solve a broad class of problems in the mission. Such a streamlined procedure, that keeps the number of new things to be learned to a minimum, vastly improves the efficiency of people who design and maintain these facilities.

C. <u>The Platform Stabilization Problem</u>. A possible solution to the platform stabilization problem appears to be a little more difficult. Although Kalman-type approaches were proposed in the past by several investigators to solve analogous problems, their applicability to the present problem needs to be investigated. A simulation approach appears to offer an ideal compromise here. The simulator would take sea state, wind and ship velocity as inputs and produces expected pitch, 15, 16 roll and yaw as outputs. The results of this simulation can be used to predict errors due to ship motion.

#### VI. RECOMMENDATIONS

Analyses in the preceding sections can now be used to answer the questions raised in Section II of this report.

- It is possible to improve the accuracy of metric measurements by a substantial margin. There is sufficient evidence in published literature for this possibility.
- (2) If the objective is to achieve as much of this improvement as possible with minimum change in the ship's instrumentation, then the best course of action is to replace autotracking with on-axis tracking. This option would probably require a minicomputer dedicated to tracking. Although an exact

estimate of the size of the computer depends upon the type of filter used in the on-axis tracking, an educated guess would be a machine with 48-64K of memory size with 32-bit word length.

- (3) If the on-axis tracking idea is acceptable, then this investigator feels that a serious effort at implementing a Kalman-type filter be initiated. In trying to implement a Kalman-type filter, consideration should be given to a determination of the type of suboptimal filter that best suits the needs of this project. In view of the time and cost constraints, the possibility of developing an optimized - filter (that is, one whose performance is equivalent to a Kalman filter) for possible implementation on modern microcomputers should not be ignored.
- (4) Once the above idea is adopted, it is important to realize that now we are dealing with the so-called embedded computer systems. The reliability and maintainability of such embedded computer systems very much depends upon the quality of software support. Toward this end, it is strongly recommended that all future software development efforts follow modern ideas of software engineering; that is, ideas such as top-down design and structured programming.
- (5) The author also feels that the short range objective of improving the quality of metric data from H. S. Vandenberg should not be allowed to cloud the long range objective of maintaining the concept of using ARIS as a flexible expansion of a missile test range. In this context, it is extremely useful to conduct a simulation study to determine how the various aspects of a ship's motion influence the overall accuracy.
- (6) Finally, regardless of whatever action is being taken insofar as ARIS are concerned, it is important that all personnel concerned with RV tracking at ESMC be encouraged to get abreast with the developments in technology. Recent advancements, and some not so recent, in guidance and control, computer simulation, distributed processing and software engineering are revolutionizing thinking in these areas.

#### REFERENCES

- 1. <u>ARIS Orientation Manual</u>, prepared by Pan American Airways and RCA International Services Corporation, Patrick Air Force Base, FL, June 1971.
- 2. <u>ESMC Ships Timing Modernization Plan</u>, prepared by Communications/ Timing Systems of Pan American Airways, Inc., Patrick Air Force Base, FL, Second Revision, 18 December 1979.
- 3. USNS Redstone External Communications Modernization Plan, prepared by Pan American Airways, Inc., Patrick Air Force Base, FL, 14 December 1979.
- 4. R. L. Barkeley, et al, <u>SAMTEC On-Axis Radar Study</u>, TRW Technical Report No. 20966-6001-T0-00, July 1972.
- 5. Private conversation with Mr. Jerry Brubaker, ESMC.
- 6. Private conversation with M. W. Ellenberg, RCA at ESMC.
- 7. Private conversation with M. W. Ellenberg, RCA at ESMC.
- 8. L. B. Rice, <u>ARIS Closed Loop Tracking</u>, Special Report 81-SR-70-2, RCA International Service Corporation, Missile Test Project, Patrick Air Force Base, FL, 29 January 1970.
- 9. <u>A Compilation of Papers Presented at the On-Axis Software Conference</u>, Technical Report No. ETR-TR-74-01, Directorate of Range Operations, Patrick Air Force Base, FL, September 1974.
- C. C. Schooler, "Optimal α-β Filters for Systems with Modeling Inaccuracies," <u>IEEE Trans. Aerospace and Electronic Systems</u>, Vol. AES-11, No. 6, pp 1300-1306, November 1975.
- P. S. Noe and K. A. Myers, "A Position Fixing Algorithm for Low-Cost GPS Receiver," <u>IEEE Trans. Aerospace and Electronics</u>, Vol. AES-12, No. 2, pp 295-297, March 1976.
- A. G. Lindgren and K. F. Gong, "Position and Velocity Estimation via Bearing Observations," <u>IEEE Trans. Aerospace and Electronic Systems</u>, Vol. AES-14, No. 4, pp 564-577, July 1978.
- 13. S. F. Schmidt, "Application of State Space Methods to Navigation Problems," in Advances in Control Systems, Vol. 3, pp 293-340, 1966.
- 14. C. A. Darmon, "A Ship Tracking System Using Kalman-Schmidt Filter," in Agard Lecture Series No. 82, <u>Practical Aspects of Kalman Filtering</u> <u>Implementation</u>, NATO Advisory Group for Aerospace Research and <u>Development</u>, 1976.

- 15. P. Kaplan and T. P. Sargent, <u>Theoretical Study of the Motions of an</u> <u>Aircraft Carrier at Sea</u>, Oceanics, Inc., Technical Industrial Park, Plainville, N. Y., January 1965.
- 16. W. A. Johnson, <u>Analysis of Aircraft Carrier Motions in a High Sea State</u>, STI 137-3, Systems Technology Inc., Hawthorne, CA, March 1969.

