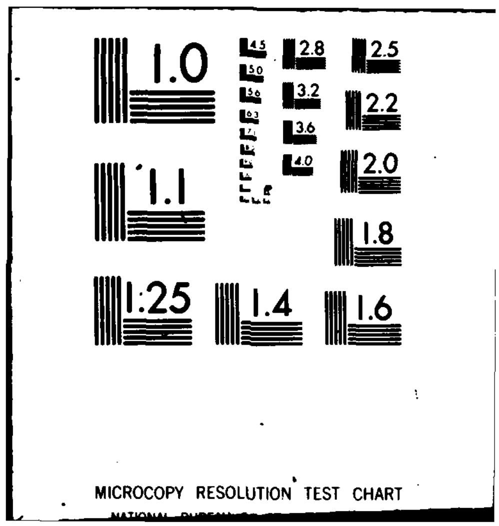
AD-A113 028 BROWN UNIV PROVIDENCE RI LEFSCHETZ CENTER FOR DYNAMI--ETC F/G 12/1
A SIMULATION STUDY OF A DECENTRALIZED DETECTION PROBLEM.(U)
OCT 81 H J KUSHNER, A PACUT N00014-76-C-0279
UNCLASSIFIED AFOSR-TR-82-0254 NL

1 of 1

END
DATE
FILMED
4-82
DTIC


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0
MICROCOPY RESOLUTION TEST CHART

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER AFOSR-TR- 82-0254	2. GOVT ACCESSION NO. AD-A113 028	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) A simulation study of a decentralized detection problem		5. TYPE OF REPORT & PERIOD COVERED Interim
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Harold J. Kushner and Andrzej Pacut		8. CONTRACT OR GRANT NUMBER(s) AFOSR 76-3063
PERFORMING ORGANIZATION NAME AND ADDRESS Lefschetz Center for Dynamical Systems, Div. of Applied Math., Brown University, Providence, RI 02912		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61102F 2304/A4
CONTROLLING OFFICE NAME AND ADDRESS AFOSR, Bolling AFB, Washington, DC 20332 NM		12. REPORT DATE October 1981
		13. NUMBER OF PAGES 20
MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION DOWNGRADING SCHEDULE

DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A (two person) problem of decentralized observation, detection and coordination is studied via simulation. The effects of the prior probability and parametric dependencies on the decision rules are studied. Sensitivity to the data, asymmetries in the decision rules and other phenomena are investigated.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

AFOSR-TR- 82-0254

A SIMULATION STUDY OF A DECENTRALIZED DETECTION PROBLEM*

Harold J. Kushner and Andrzej Pacut

Lefschetz Center for Dynamical Systems
Division of Applied Mathematics
Brown University
Providence, Rhode Island 02912

October 1981

Abstract: A (two person) problem of decentralized observation, detection and coordination is studied via simulation. The effects of the prior probability and parametric dependencies on the decision rules are studied. Sensitivity to the data, asymmetries in the decision rules and other phenomena are investigated.

*Research supported in part by the Air Force Office of Scientific Research under AFOSR 76-3063D, in part by the National Science Foundation under NSF-Eng 77-12946-A02 and in part by the Office of Naval Research under N00014-76-C-0279-P0004.

I. Problem Description

Problems of so-called decentralized observation, coordination and hypothesis detection are of increasing current interest, and rather little concrete information is available on them. Here, we discuss a simulation study of one of the simplest of such problems, in order to provide some insight into the phenomena that might be expected in more general cases, and to provide a guide to analysis. We present these results, limited though they are, because of the importance of the class of problems, and the paucity of available information and intuition.

We deal with a decentralized detection problem involving two observers, denoted X , Y, and a coordinator. There are two possible hypotheses H_0 and H_1 , with $\pi_0 = P\{H = H_1\}$ given. Each observer takes an observation at time 1 and may, if it wishes, take an observation at time 2. Let $\pi_i(X)$, $\pi_i(Y)$ denote the conditional probability of X, Y, respectively, given i observations. The observers do not communicate with each other, and at the end of their'observation periods', each transmits its π_i to the coodinator who then computes the system posterior probability π, and decides on either H_0 or H_1 .

If H_0 is chosen when H_1 is true, there is a cost a_{01} . If H_1 is chosen when H_0 is true, there is [illegible] a_{10} . We view H_1 as being more important than H_0 , and set $a_{01} > a_{10}$. If at least one observer takes a second observation and H_1 is true, then there is an added delay cost c_0 . Then the problem is just a decentralized version of the standard 'disorder' problem. The cost of delay under H_1 forces each observer to somehow take the other into account (in deciding whether to take a second observation). There is no second guessing in the setup, since there is no implicit communication among X, Y . This is deliberate, since even the behavior of the described structure is not understood. Define $g(\pi) = \min[a_{01}\pi \;, a_{10}(1-\pi)]$. Then the sample cost is

$$\text{Cost}(\pi) = g(\pi) + c_0\pi \; I\{\text{at least one observer takes a second observation}\}.$$

The object here is to gain some insight into the way in which each observer accounts for the existence of the other; e.g., how the decision regions differ from those in the one person case, the parametric dependence and sensitivity of the regions, to check conjectures concerning qualitative behavior, bounds provided by one-person cases, and to check for 'unusual' behavior such as the asymmetries in the decision regions such as exhibited in [1] even when the observers had the same distributions.

II. Background Calculations

Sufficient statistics. Let X_1, X_2, Y_1, Y_2 denote the four possible observations, and X^i, Y^i the first i of X, Y, respectively. We suppose that $\{X_1, X_1, Y_1, Y_2\}$ are independent, under H_0 and H_1 . Then

$$\pi_i(X) = P(X^i \mid H_1)P(H_1)/P(X^i) .$$

If X takes i observations and Y takes j observations, then

$$(1) \qquad \pi = P(H_1 \mid X^i,Y^j) = P(X^i,Y^j \mid H_1)P(H_1)/P(Y^i,Y^j) =$$

$$= P(X^i \mid H_1)P(Y^j \mid H_1)P(H_1)/P(X^i,Y^j)$$

$$= \frac{P(H_1|X^i)P(H_1|Y^j)P(X^i)P(Y^j)/\pi_0}{\sum_{k=0}^{1} P(H_k|X^i)P(H_k|Y^j)P(X^i)P(Y^j)/P(H_k)}$$

$$= \frac{\pi_i(X)\pi_j(Y)/\pi_0}{\pi_i(X)\pi_j(Y)/\pi_0 + (1-\pi_i(X))(1-\pi_j(Y))/(1-\pi_0)} .$$

Thus it is enough for X, Y to transmit their conditional probabilities at the end of their observation period.

The choice of $P(X_j|H_k)$, $P(Y_j|H_k)$. In the simulations, these did not depend on j, were identical for X and Y, and we choose an exponential form. Let $\lambda_0 > 0$, $\lambda_1 > 0$. Then (density) $p(X_j|H_k) = \lambda_k \exp - \lambda_k X_j$. Since up to 4 observations can be taken, the evaluation of the cost functions can require complicated numerical integrations in general. With the exponential distribution, algebraic expressions can be obtained for these integrals, and the computations are considerably simplified. Some particular aspects of the results do seem to depend on the exponential assumption, but most of our observations and remarks probably hold much more generally.

A property of the exponential distribution. We have

$$\pi_1(X) = \frac{\pi_0\lambda_1(\exp - \lambda_1 X_1)}{\pi_0\lambda_1(\exp - \lambda_1 X_1) + (1-\pi_0)\lambda_0(\exp - \lambda_0 X_1)} \tag{2}$$

$$= \pi_0\lambda_1/[\pi_0\lambda_1 + (1-\pi_0)\lambda_0 \exp - X_1(\lambda_0 - \lambda_1)].$$

Let $\lambda_1 > \lambda_0$. Then

(3) $$\pi_1(X) \in [0, \pi_0\lambda_1/[\pi_0\lambda_1 + (1-\pi_0)\lambda_0] \equiv \text{ active region}$$

If $\lambda_0 > \lambda_1$,

(4) $$\pi_1(X) \in [\pi_0\lambda_1/[\pi_0\lambda_1 + (1-\pi_0)\lambda_0], 1] \equiv \text{ active region}$$

The active region depends on π_0 and the λ_1, and is never the entire segment [0,1]. In general, some ranges for the posterior probability will be relatively unlikely under one hypothesis but not under another, so we do not believe that the exponential distribution is too specialized.

Sufficient statistics for Decision (the exponential assumption is not needed here.) Based on X_1(respectively Y_1), X(respectively Y) decides whether to take another observation. We show that it is sufficient for X to use $\pi_1(X)$ rather than the observation X_1 (and similarly for Y's decision).

Let d_X, d_Y denote the decision rules; i.e., these are sets D_X, D_Y such that (e.g.) X stops if $X_1 \in D_X$. We want (with hopefully obvious notation)

$$\min_{D_X, D_Y} E_{\pi_0} \text{cost}(D_X,D_Y)$$

(5) $$= \min_{D_Y} E_{\pi_0} \min_{D_X} E_{\pi_0, X_1} \text{cost } (D_X,D_Y)$$

$$= \min E_{\pi_0} \min \Big\{ E_{\pi_0, X_1} \text{ cost } (X \text{ goes}, D_Y),$$

$$E_{\pi_0, X_1} \text{ cost } (X \text{ stops}, D_Y) \Big\}.$$

Note that (again with hopefully obvious notation, and using (1) to write $g(\pi)$ in terms of the relevant π_i)

$$E_{\pi_0, X_1} \text{cost (X stops, } D_Y) =$$

(6)

$$E_{\pi_0, X_1}\left[g(\pi_1(X),\pi_1(Y))I(Y \in D_Y) + (c_0 + g(\pi_1(X),\pi_2(Y))I(Y \notin D_Y)\right] .$$

But

$$P_{\pi_0, X_1}(X_2, Y_1, Y_2) = P_{\pi_0, \pi_1(X)}(X_2, Y_1, Y_2) .$$

Thus, (6) is a function of π_0 and $\pi_1(X)$ only. Similarly for the E_{π_0, X_1} cost (X goes, D_Y) . Thus, D_X and D_Y depend on $\pi_1(X)$, $\pi_1(Y)$ and not on X_1, Y_1 directly, and we write the decision regions as $D_{\pi_i(X)}$ and $D_{\pi_i(Y)}$.

We note that the same assertion holds in the N-step problem, where each player can stop and communicate to the coordinator at any time $\tau \leq N$, and the cost for continuing to observe is c_0[time that the last player communicates]. The proof uses a simple'dynamic programming like' backward induction, and is omitted.

The calculation of the decision regions D_{π_1} . The above calculations imply that the optimal decision regions can be calculated by fixing $D_{\pi_1(Y)}$. In the 2-step exponential distribution case, the optimum rule takes the following form: there are $T_X^L \leq T_X^U$, $T_Y^L \leq T_Y^U$ such that such that the second X- observation is taken if $T_X^L \leq \pi_1(X) \leq T_X^U$, and similarly for Y . This has not yet been proved in general, but we specialize the rules to this class.

In our simulations, the observations of X, Y had the same distribution, under either H_0 or H_1 . By the comments in the last subsection, there is an $f(\cdot)$ such that given $T_Y = (T_Y^L, T_Y^U)$, the corresponding optimum T_X satisfies $T_X = f(T_Y)$. By symmetry, $T_Y = f(T_X)$. Thus (* denotes optimum)

(8) $$T_X^* = f(f(T_X^*)) .$$

In the calculations (and in Figures 1, 2), we simply calculated T_X given T_Y by checking the sign of

(9) $$E_{\pi_0,\pi_1(X)} \text{ (cost if } X \text{ stops, given } T_Y) - E_{\pi_0,\pi_1(X)} \text{ (cost if } X \text{ goes, given } T_Y) .$$

III. The Threshold Tables.

Before presenting and discussing the numerical results, we make two remarks. First, the thresholds depend on π_0; the $\pi_i(X),\pi_1(Y)$ are <u>not</u> sufficient statistics for decision in the usual sense. This is obvious from (1). But the π_0-dependence is often very small (see Table 1). Such a π_0-dependence does not occur in the analogous standard one person case. Even with $\pi_1(X)$ known, π_0 still provides additional information on the distribution of the π and of I{Y goes} .

Secondly, the lower thresholds were quite small in our examples, and the upper thresholds (say T_X^U) were not particularly sensitive to variations in T_Y^L between 0 and the optimum T_X^L . So, in most simulations, we simply fixed T_Y^L at a small value - in that range, calculated $f(T_Y^U) = (T_X^L, T_X^U)$, then optimized over T_Y^U . Refer to Tables 1 to 4. _We call_ $(\lambda_0 = a, \lambda_1 = b)$ _the_ (a,b) _case_.

c_0 \ π_0	.1	.25	.3	.5	.7	.9
.06			.38		.36	
.08		.31	.3			
.1	*	.26		.26		.24
.12	*	.23				
.14	*	.22		.18		.18
.16	*	.18		.14		

TABLE 1.

Optimum Upper Thresholds $T_X^U = T_Y^U$,

$\lambda_0 = 1$, $\lambda_1 = 3$;(1,3) case.

(*)active region = [0,.25], always continue

c_0 \ π_0	.25	.4	.5	.75
.01				.75
.02	.83		.79	**
.08	.79	.75	.72	**
.1				**

TABLE 2.

(3,1) Case

(**), active region = [.5,1], always stop.

c_0 \ π_0	.25	.5	.73
.06	.3	.27	.27
.08	.25	.22	.22
.1	.18	.18	.18

TABLE 3.

(1,2) Case

c_0 \ π_0	.25	.5	.75
.06	.56	.3	.3
.08	.55		.24

TABLE 4.

(2,1) Case

Comments on the tables. In all cases, the optimum thresholds for X, Y were the same. There were no asymmetries as in [1]; this is, in fact, one of the phenomena which we were looking for. The c_0-dependence indicated in the tables is as expected. The π_0- dependence is obvious, but for most of the π_0- range, the dependence is slight. The π_0- dependence can be partly explained by noting that (say, from X's point of view) π_0 provides information on $\pi_1(Y)$ and on the ultimate π . Loosely speaking, a smaller π_0 implies that π will be smaller, for fixed $\pi_1(X)$. This π_0-effect does not occur in the standard one person problem.

Owing to the fact that the 'active' region is a proper subset of [0,1], it is possible (see Tables 1,2) that the optimum decision is to always stop (or to continue) for all attainable values of the $\pi_1(X)$ (or $\pi_1(Y)$), for the given π_0. (This 'active region' effect might be exaggerated for our case, but in general, it simply reflects the fact that certain $\pi_1(X)$-sets are relatively unlikely to occur, given a π_0.)

In the analogous one-person problem the threshold for the (1,3) case is greater than the threshold for the (3,1) case. There is 'less risk' in continuing in the former case, since there is a greater smoothing of the decision error cost. See Figs. 3-5 and further comments below. (Of course, in the one-person case the thresholds are π_0 -thresholds.) In the two-person case the situation is reversed, in the cases studied. See the tables. We have not yet found a completely satisfactory explanation, but it might be partially explained by the fact that the same value of $\pi_1(X)$ in the (3,1) and (1,3) cases implies a smaller X_1 in the (3,1) case. Similarly, the same observation X_1 implies a larger $\pi_1(X)$ in the (3,1) case. The reversal cited above would not occur if the thresholds for X were written in terms of X_1 (and similarly for Y). These remarks point out that the use of one-person formulations in order to gain insight for the two-person case must be done with great care. Similar, though not so extreme, observations hold in the (2,1), (1,2) cases. See Tables 3,4. An additional reason for the 'reversal' is the fact that the active regions in the two cases start at opposite ends of the [0,1] interval.

Let the one-person case thresholds be (T^L, T^U) . Then for $\lambda_1 > \lambda_0$, the simulations yielded $T^L \geq T^L_X = T^L_Y$, $T^U \geq T^U_X = T^U_Y$. The reverse occured when $\lambda_0 > \lambda_1$, but the observation suggests that useful information might be obtainable from one-person case results in special cases.

IV. The Threshold Curves - Figures 1,2.

By the argument associated with (5), the optimum thresholds can be obtained by fixing T^L_Y, T^U_Y , optimizing over T^L_X, T^U_X , and then over the first pair by seeking the fixed points of (8). As noted above, the lower thresholds were small, and varying them in the interval near zero in which they normally ranged made little difference on the obtained values of the upper thresholds - so to get the plotted curves we set T^L_Y to some value within its normal range and varied only T^U_Y . Except for Fig. 1a, only the corresponding optimum T^U_X are plotted. The T^U_X were calculated by checking the sign of the difference between the arguments of the inner minimum on the right side of (5). The flattening of the right part of the curve in Fig. 1 is due to the value of the active region. In Fig. 1a, for example, the threshold value 1/2 could be replaced by any value $\geq 1/2$, since [0,1/2] is the active region. In Fig.1, the 'important' hypothesis H_1 is the more difficult to detect.

By (8), only the points (1), (2), (3), in Fig. 1a could be optimal pairs. In fact, only (1),(3) can be optimum, and (2) is a saddle. Point (1) is optimum. In all cases tested, the optimal pairs were on the diagonal.

This will always be the case if T_X^U never decreases as T_Y^U increases. Since the 'continuation' cost must be paid if either observer continues, the 'monotonic' property probably holds fairly generally. In Fig. 1 note the relative insensitivity of the optimal T_X^U to T_Y^U (especially as compared with the Fig. 2 case). This is connected with the lower threshold in the first case. As c_0 increases this sensitivity increases, and it decreases as π_0 increases (loosely speaking, as π_0 increases, X 'increasingly ignores' Y). In Fig. 1, compare the optimal T_X^U with the optimal T_X^U when Y does not take a second observation (this is the vertical intercept). It is virtually equal to the optimal threshold; each observer essentially ignores the others possible second observation. Thus useful simplifying rules might be found in special cases.

Now, refer to Fig. 2. In Figs. 2a,2b, the optimal thresholds are unique. The optimum T_X^U is relatively insensitive to modest variations in T_Y^U. The sensitivity appears to be greater in Fig. 2c, which also illustrates how complicated the situation can get. However, the observations in the next section below suggest that even here the cost is relatively insensitive to modest variations in T_X^U, T_Y^U about their optimal, or to variations within the interval where the curves in Fig. 2c stay near the diagonal.

V. The Decision Cost Curves - Figure 3.

Legend for Fig. 3. Curve (0) denotes $g(\pi) = \min [a_{01}\pi, a_{10}(1-\pi)]$, curves (i), i = 1,2,3, indicate the average decision costs for the one-person case where either 1,2, or 3 observations can be taken. Then the abscissa is π_0. Curve (S) denotes the decision cost part (not the c_0 part) of the second argument of the inner min. on the right side of (5) (under stop), and Curve (G) denotes the decision cost part of the first argument (under continue). I.e., the curves denote the average decision costs for both decisions from X's point of view. T_Y^U, T_Y^L (i.e., D_Y) were fixed near their optimum values. For these S,G curves the abscissa is $\pi_1(X)$, and π_0 is fixed.

In all cases Curve (2) $\leq$ Curve (S) $\leq$ Curve (1), Curve (3) $\leq$ Curve (G) $\leq$ Curve (2). This is intuitively reasonable, and can be proved to hold in general. Since Curve (S) is the decision cost conditioned on (X_1, π_0), it is averaged over Y's one or two observations. Similarly Curve (G) is the cost, averaged over Y's one or two observations and an additional X-observation.

As π_0 increases Curve (G) $\uparrow$ Curve (2) and Curve (S) $\uparrow$ Curve (1) (also a general result), since as π_0 increases the information (the additional information that Y provides the coordinator, from X's point of view) in Y's possible future observations decreases. These observations might provide some help in simplifying solving for the thresholds in more complicated cases.

VI. Total Cost Curves - Figs. 4,5.

Figures 4,5 plot expression 9 with T_Y^L, T_Y^U fixed near their optimum values.

Legend. Curve (T) = our 2 person case, with abscissa = $\pi_1(X)$, and T_Y^L, T_Y^U fixed near their optimum and π_0 fixed. For $i = 1,2$, Curve (i) = average cost for no observations minus average cost given i additional observations. Here $\pi_1(X)$ is irrelevant and the abscissa is π_0. Curve (S) is as Curve (T), but with $T_Y^L = T_Y^U = 0$ (Y never continues). Curve (R) is as Curve (S), but X takes 2 additional observations if it elects to continue. The curves are typical of the runs taken. The zero crossings are the optimal X-thresholds, for the given Y-thresholds. In Fig. 4, the T,S,R,1 curves have similar structure, the sensitivity at optimum being slightly less for the T curve.

Now refer to Fig. 5 for the (3,1) case. Here, the T-curve is much less sensitive to $\pi_1(X)$. The nature of the curve implies that modest variations in the thresholds will affect the total average cost only slightly. This appears to be the case, for situations such as in Fig. 2c, when there are multiple crossings of the diagonal. In such cases, it appeared that the total average systems cost did not vary too much as the thresholds varied on the interval between the local minima. Of course, these remarks are merely suggested by curves such as those in Fig. 5. and are not mathematically proved assertions. Nevertheless, they do suggest that calculating good thresholds might not be too hard even in cases which lead to curves such as that of Fig. 2c.

REFERENCES

[1] R.R. Tenney, N.R. Sandell, "Detection with distributed sensors", M.I.T. LIDS report P-970, 1980, to appear, IEEE Trans. on Automatic Control.

cdd

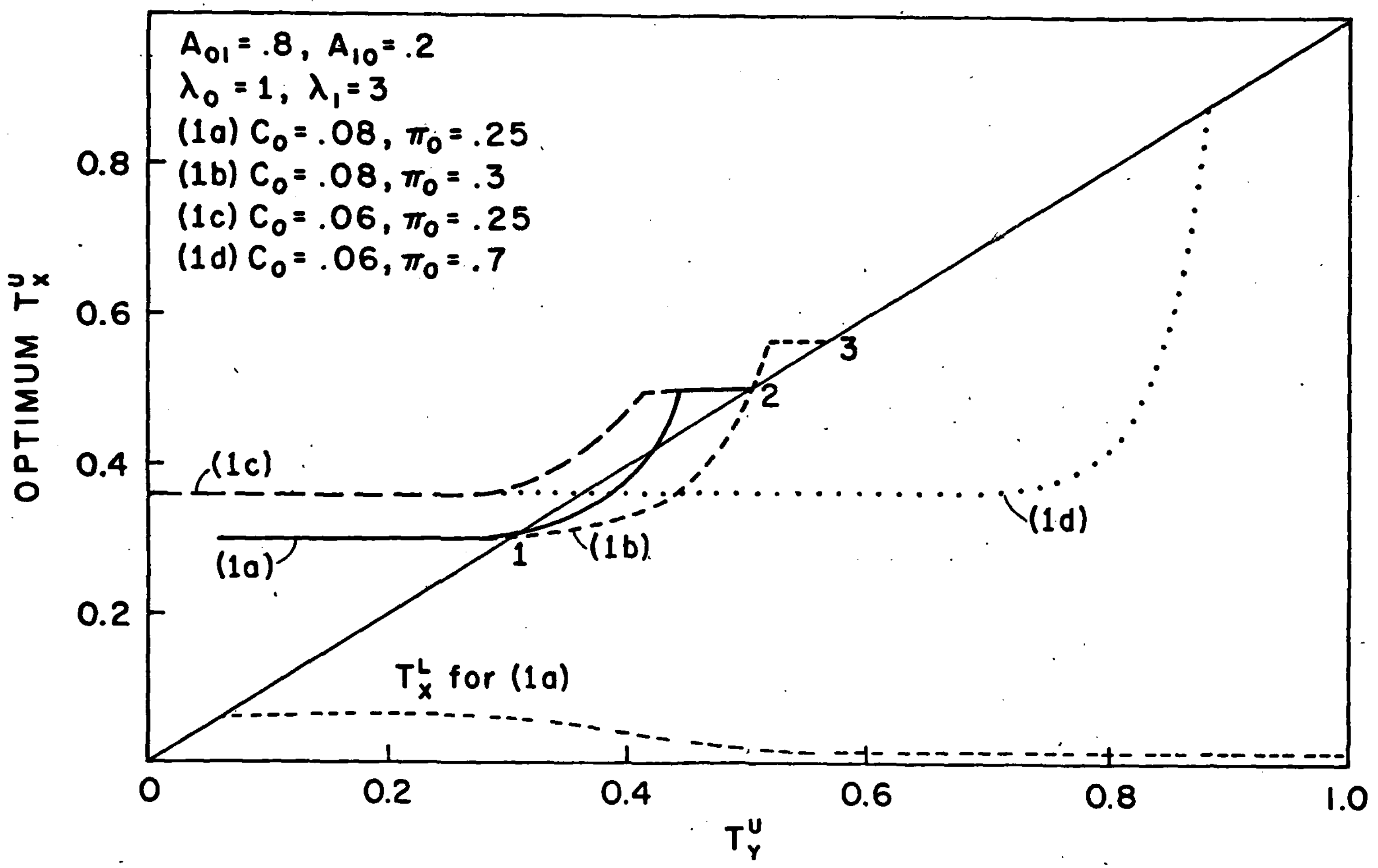


FIGURE 1 OPTIMUM UPPER THRESHOLD T_X^U VS T_Y^U WITH $T_Y^L = .05$.

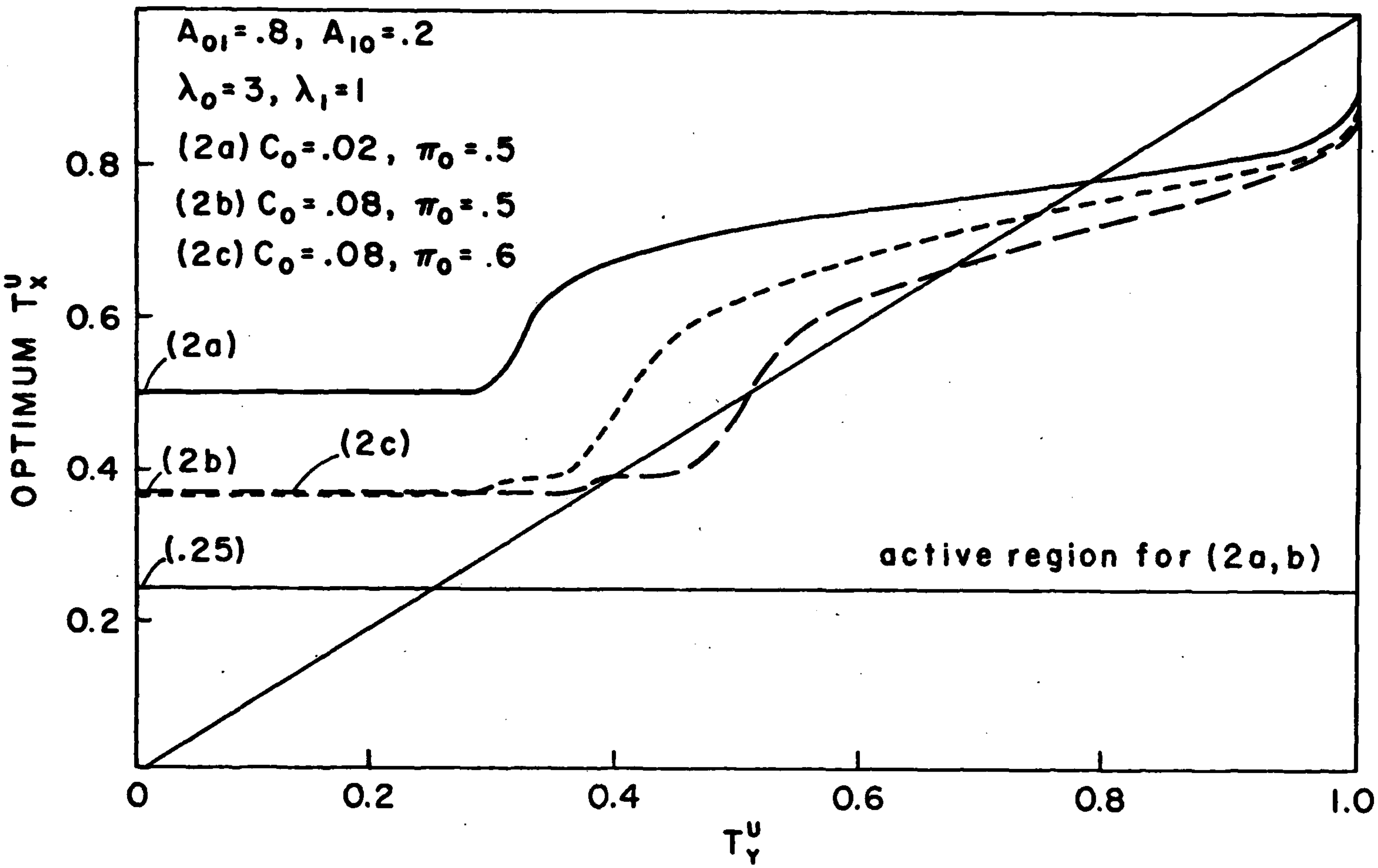


FIGURE 2 OPTIMUM UPPER THRESHOLD T_X^U VS T_Y^U WITH $T_Y^L = 0$.

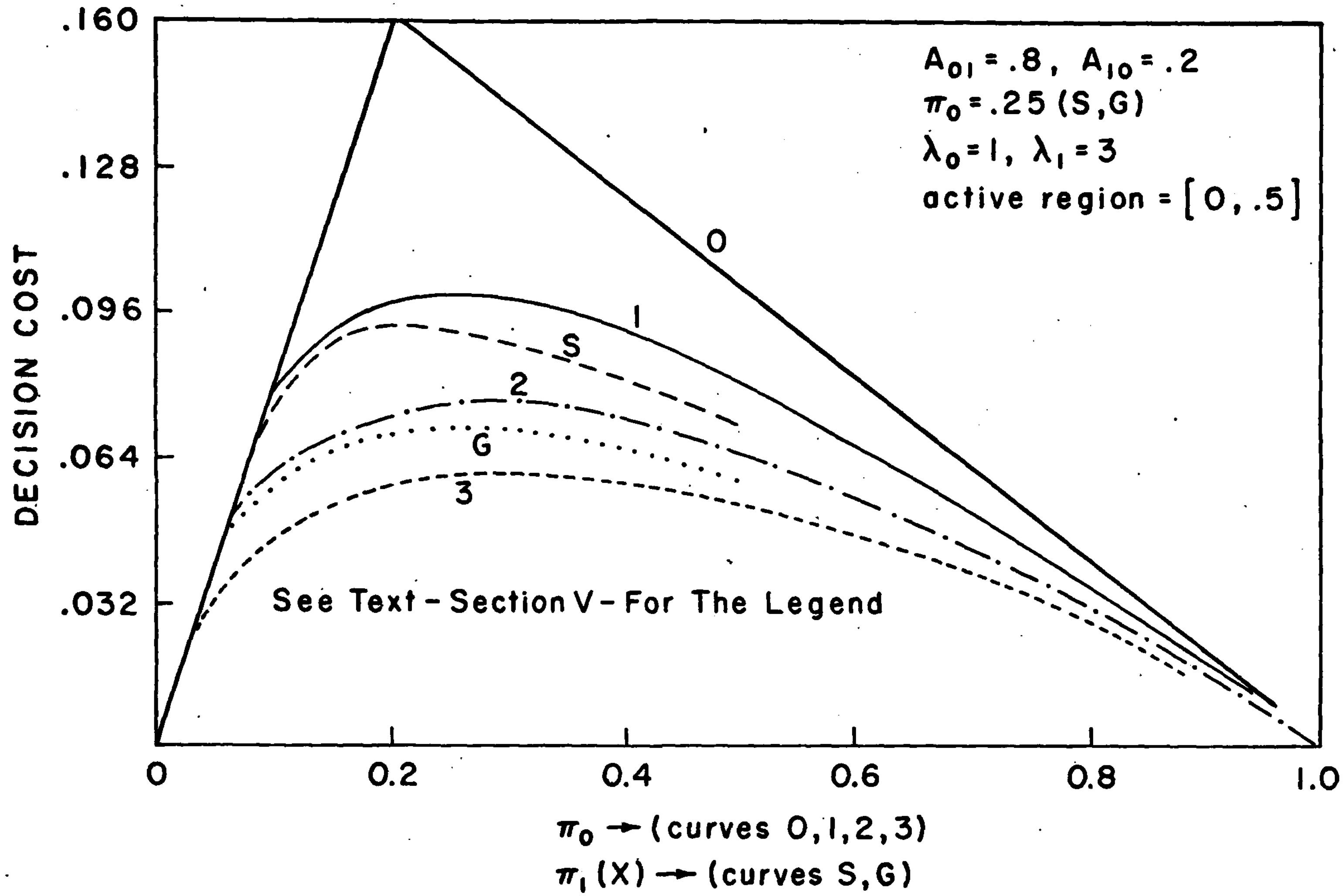


FIGURE 3 DECISION COST CURVES.

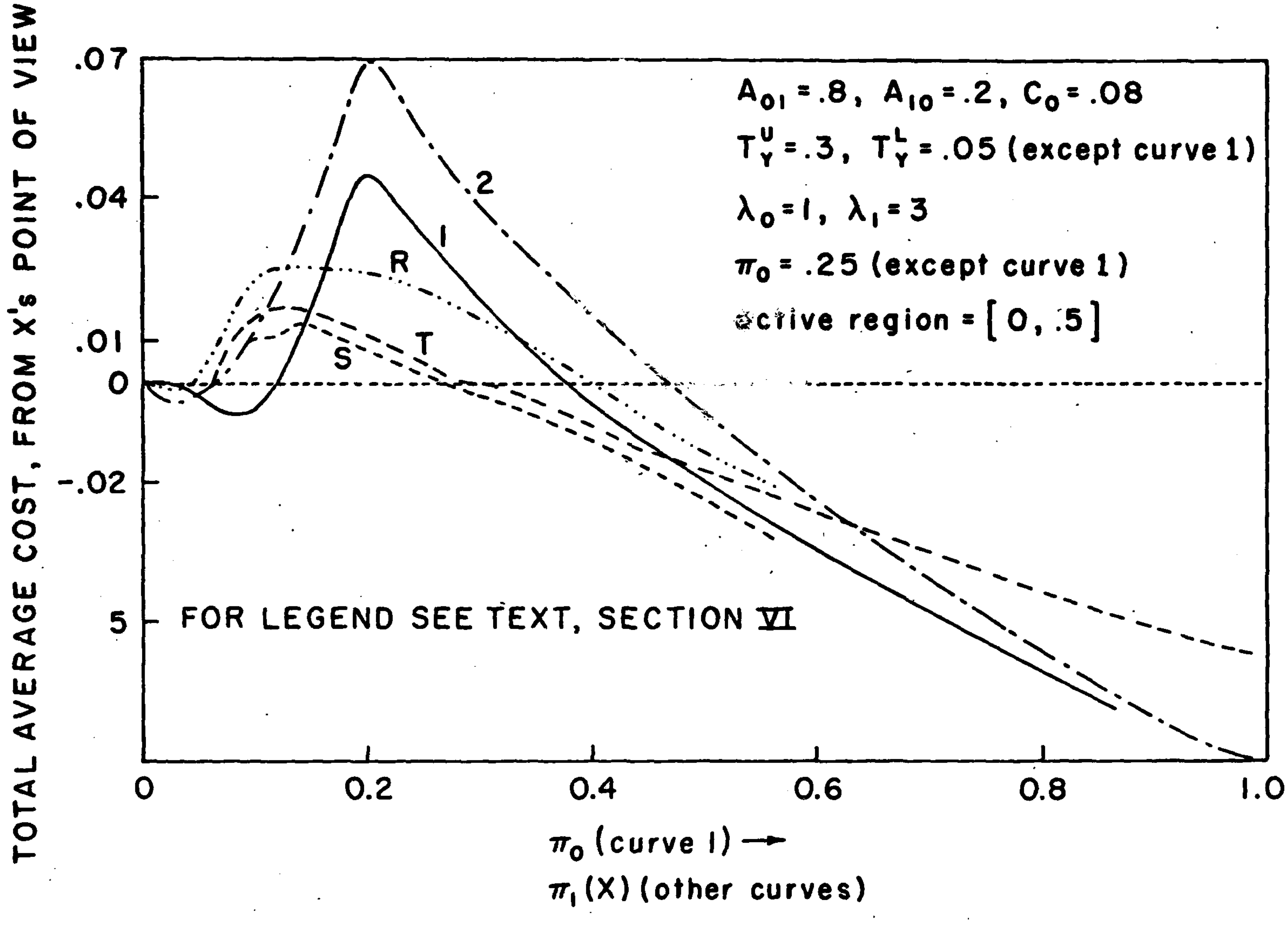


FIGURE 4 TOTAL COST CURVES..

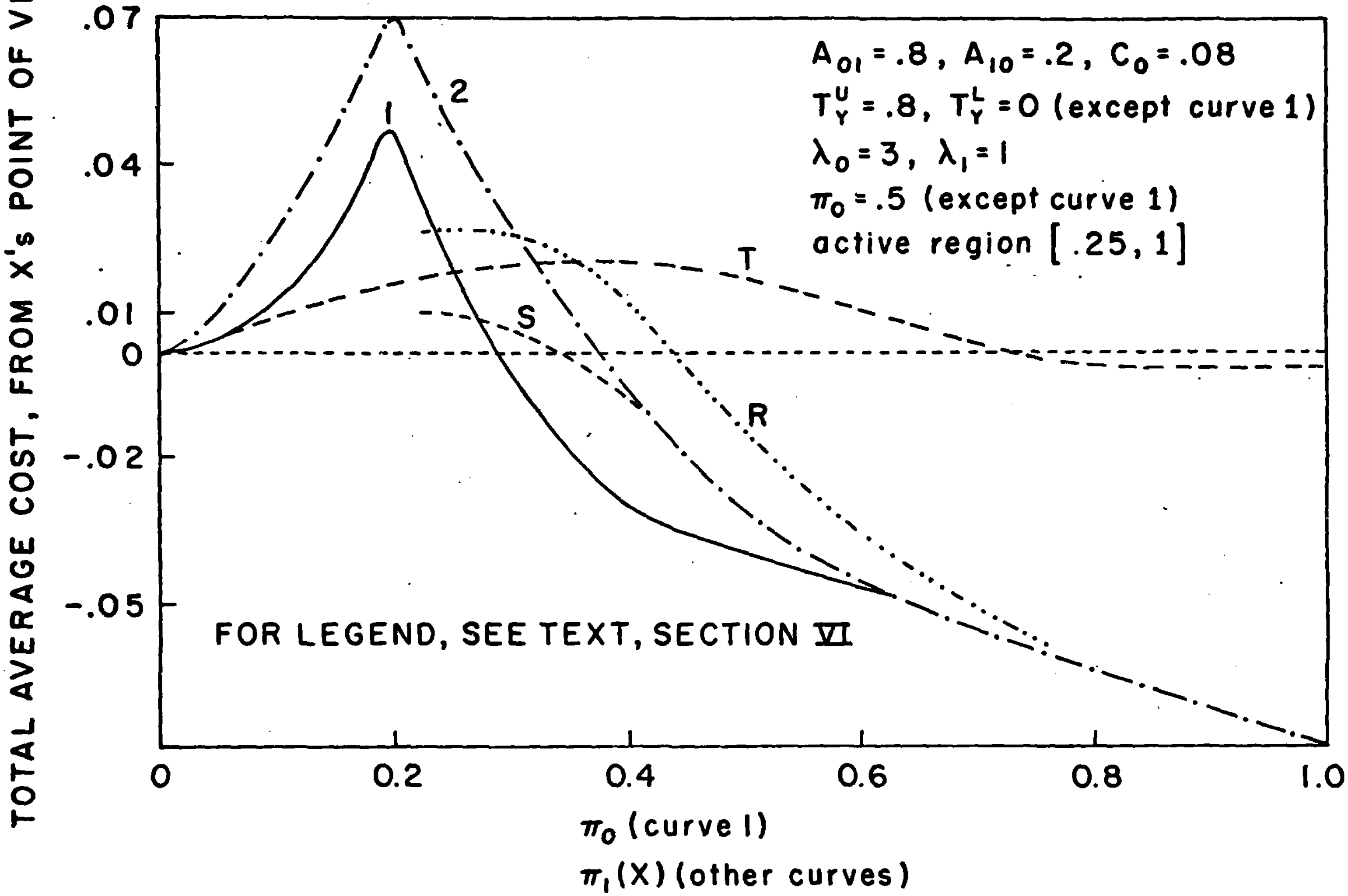


FIGURE 5 TOTAL COST CURVES.