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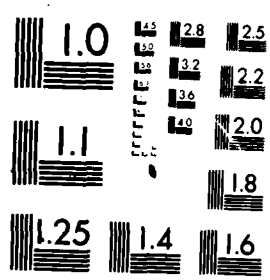

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GLR ALGORITHMS FOR DETECTING AND ESTIMATING ABRUPT MANEUVERS IN ASMD SCENARIOS USING A DECOMPOSITION OF THE MANEUVER SIGNATURE MATRIX

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GLR ALGORITHMS FOR DETECTING AND ESTIMATING ABRUPT MANEUVERS IN ASMD SCENARIOS USING A DECOMPOSITION OF THE MANEUVER SIGNATURE MATRIX*

BY

HAROLD L. STALFORD

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ABSTRACT

Two GLR algorithms are developed for detecting and estimating abrupt maneuvers (i.e., jumps in control values) in discrete linear stochastic systems. A jump error state variable concept is used to derive a decomposition of the conventional maneuver signature matrix into a new maneuver signature matrix which is independent of the jump time and another matrix which depends only on the jump time. The product of the latter matrix with the jump vector is shown to provide a constant jump error state variable. The nondependency of the new maneuver signature matrix on the jump time and the transformation of the jump error state to a constant provides for the development of GLR algorithms with considerably reduced computational and storage requirements. The new algorithms avoid much of the updating and storage of large matrices for past observation times.

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In particular, the multiplications requirement is reduced to the order of $n^3 + 3Mn^2m$ for general linear stochastic systems and Mn^2m for such systems without process noise. Here, n is the dimension of the state, m the dimension of the measurement vector and M is number of candidate jump times in the past.

The two algorithms have practical application to the engagement problem between an anti-ship cruise missile and a ship defense interceptor. The output of the algorithms provides information on which the players in a differential game of partial information and noisy observation may base and design optimal strategies for maximizing payoff functions such as survivability and kill.

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1. BACKGROUND AND INTRODUCTION

The engagement between an anti-ship cruise missile (ASM) and a ship defense interceptor is a differential game of partial information and noisy observations. The ASM represents the offense (the evader) and the interceptor represents the defense (the pursuer). The ASM desiring to optimize its survivability while enroute to its ship target employs endgame maneuvers for the purpose of evading the ship's defense interceptors guided by active and/or passive radar. The ASMs are highly sophisticated and maneuvering missiles that have speeds ranging from subsonic to supersonic, altitudes ranging from sea skimmer to upper limits of winged aircraft, and guidance systems ranging from simple line-of-sight to more complex combinations of passive and active electronic systems. Their maneuvering characteristics are governed by airframe, propulsion and guidance parameters -- drag, lift, thrust and four guidance parameters, [1] and [2]. The thrust and guidance parameters are like step fucntions in time (i.e., piecewise constant controls). They take on one value during one portion of the ASM's trjectory and jump to other values during other portions of the trajectory. For example, the guidance parameters jump in value at the start of pop-ups, dives,pull-outs, turn-downs and turn-on of seeker homing. The thrust parameter of some ASMs jumps (i.e., drops) in value during high altitude dives. Consequently, the dynamics of an ASM are representable as a stochastic system governed by piecewise constant controls. In linear discrete form the dynamics are modeled as:

System Dynamics of ASM*

$$X(k+1) = A(k+1,k)\ X(k) + B(k+1,k)\ (u(k) + \Delta u_q\ \delta_{qk}) + \Gamma(k)w(k) \quad (1)$$

$$u(k+1) = u(k) + \Delta u_q\ \delta_{qk} \quad (2)$$

where X is the state vector, u is the control, Δu_q is the jump in control at time q, δ_{qk} is the Kronecker delta, and Γ is the system noise coefficient matrix. The matrix A is the state transition matrix and B is the input matrix for the control. The jump time q and the jump magnitude Δu_q are unknown to the defense interceptor. Using active and/or passive radar sensors the defense interceptor measures a partial state of the ASM. These observations are modeled as:

Sensor Equation of Interceptor

$$z(k) = h(k)\ X(k) + \nu(k) \quad (3)$$

where z is the measurement vector and h is the measurement matrix. The noise sequences w and ν are zero-mean, independent, white Gaussian sequences with covariances defined by

*Actually, there are multiple jumps Δu_{q_i} at times q_i; this is easily indicated by the substitution of $\sum_i \Delta u_{q_i}\ \delta_{q_i k}$ into (1) and (2). Equations (1) and (2) represent each jump in turn.

$$E\{w(k)\ w^T(j)\} = Q(k)\ \delta_{kj}$$

$$E\{\nu(k)\ \nu^T(j)\} = R(k)\ \delta_{kj}$$

where $E\{\cdot\}$ denotes the expectation and the matrix $R(k)$ is bounded positive definite. The inital state $X(0)$ is normally distibuted with mean $\hat{X}(0)$ and covariance $P(0)$; we make a similar assumption for $u(0)$.

The system dynamics of the interceptor is of a form similar to (1) and (2). In general, the sensor equation of an ASM may be considered to be of the form (3) but currently, in practice, the ASM has no sensor with which to observe the location or presence of an interceptor (i.e., it is blind to approaching interceptors) even though it has the sensors for acquiring and homing in on its ship target. In such a case the measurement matrix of the ASM is zero. There are other engagement scenarios in which the offense (e.g., bomber) is not blind to approaching interceptors. Therefore, in general, we are interested in the class of differential games of partial information and noisy observations in which the dynamics of a player's system are modeled by (1) and (2) and the observation equation of its opponent is modeled by (3).

In the above engagement the ASM desires to maximize its survivability using evasive maneuvers while enroute to its ship target. The interceptor desires to maximize its probability of kill. In its pursuit of optimality the interceptor is faced with two tasks. The first is the development of an estimator for obtaining the optimal estimates $\hat{X}$ and $\hat{u}$ of the state X and the control u. Secondly, it has the task of deriving optimal strategies based on the optimal estimates. In this paper we address the first task and present a filtering algorithm for obtaining the optimal estimates of X and u. This estimation problem is that of detecting and estimating abrupt changes in stochastic dynamical systems. A survey of estimation methods is given in Willsky [3].

One of the most attractive and promising methods for detecting and estimating jumps in linear stochastic systems is the generalized likelihood ratio (GLR) method, [4]-[6]. The GLR method processes the residuals from a Kalman filter and computes the maximum likelihood estimates of the jump time and the jump magnitude. Using these estimates it evaluates the log-likelihood ratio for jump versus no jump. A jump is declared if the evaluated ratio is larger than a set threshold. The implementation of the GLR requires a linearly growing bank of matched filters in order to compute the maximum likelihood estimate of the jump time, [3] and [7]. A recursive GLR algorithm, [8] and [9], has been developed that reduces the computational burden by reducing or avoiding the requirement for matrix inversion (in computing the jump magnitude and evaluating the log-likelihood ratio) at each possible jump time in the past. This reduction was obtained by modifying the GLR algorithm so that the covariance of the predicted measurement residual is to be inverted rather than the information matrix of the jump variable. The largest dimension of the matrix to be inverted is at most equal to the dimension of the correlated components of the predicted measurement residual, [10] and [11].

In its latest stage of development the GLR method still requires at each new observation time the computation and storage of several matrices for each observation time in the past. Herein, we derive two GLR algorithms I and II which are based on a decomposition of the failure signature matrix of [5] which we term herein the maneuver signature matrix. The decomposition provides a new maneuver signature matrix which is independent of the jump time (maneuver start time). This nondependency reduces considerably the storage and computational requirements of the GLR method by reducing the requirement to update and restore large matrices at each current observation time for past observation times. The computational burden of previous GLR algorithms is to a great extent the direct dependency of the maneuver signature matrix on the jump time. Our algorithms I and II avoid much of the updating and storage of large matrices for past observation times.

The GLR algorithms I and II are developed using the concept of a jump error Δ-state variable Δx. It is introduced as the difference between a "jump is known" filter estimate and a "jump free" filter estimate of the state. The evolution of Δx, after a jump, is governed by a linear state equation. The residuals of the "jump free" filter provides the noisy linear measurements of Δx. The variable Δx is non-singularly transformed into a new Δ-state variable Δy which is constant. It is that transformation which provides the new maneuver signature matrix which is independent of the jump time.

Our work is built on that of Friedland's bias filtering technique [20], Willsky and Jones' GLR technique [6] and Chang and Dunn's recursive GLR algorithm [9].

2. DISCRETE LINEAR STOCHASTIC MODEL WITH STATE JUMP: A POSTERIORI JUMP

For ease of presentation and generality we augment the state $X(k)$ with the control $u(k)$ of (1) and (2) and define the variable $x(k)$ as the augmented state vector composed of $X(k)$ and $u(k)$. In this case Equations (1), (2), and (3) become

$$x(k+1) = \Phi(k+1,k)(x(k) + \Delta x_q \ \delta_{qk}) + \Gamma(k)w(k) \tag{4}$$

$$z(k) = H(k)\, x(k) + \nu(k) \tag{5}$$

where

$$\Phi(k+1,k) = \begin{bmatrix} A(k+1,k) & B(k+1,k) \\ O & I \end{bmatrix} \tag{6}$$

$$\Delta x_q = D \ \Delta u_q \tag{7}$$

$$D = \begin{bmatrix} O \\ I \end{bmatrix} \tag{8}$$

$$H(k) = [h(k) \ \ 0] \tag{9}$$

and where $\Gamma(k)$ is redefined as the augmented matrix

$$\begin{bmatrix} \Gamma(k) \\ O \end{bmatrix} \tag{10}$$

A description of the variables and their dimensions are given in Table 1. We assume that the linear system (4) and (5) is observable.

The a posteriori jump is used in the formulation of (1), (2), and (3). The jump Δx_q occurs at time q but it does not appear in the measurement until time q+1; that is, the jump occurs right after the measurement z(q).

The optimal state estimator for a discrete linear stochastic system without jump ($\Delta x_q = 0$) is given by the discrete Kalman-Bucy filter, [12]-[15]. The equations of the filter, [16], are given in Table 2 for reference purposes. The filter variables are described in Table 3.

In this report we treat the general problem as defined by Eqs. (4) and (5) in which the unknowns to be estimated and detected are Δx_q and q. That is, we take the quantities Φ, Δx_q, H and Γ to be arbitrary, not necessarily satisfying (6), (7), (9) and (10).

TABLE 1

SYSTEM VARIABLES FOR THE AUGMENTED SYSTEM

VARIABLE	DEFINITION	DIMENSION
$x(k)$	State vector	nx1
$\phi(k+1,k)$	State transition matrix from k to k+1	nxn
$\Gamma(k)$	System noise coefficient matrix	nxr
$Q(k)$	System noise covariance matrix	rxr
Δx_q	Jump in state at time q	nx1
$z(k)$	Measurement at time k	mx1
$H(k)$	Measurement matrix	mxn
$R(k)$	Measurement noise covariance matrix	mxm
$w(k)$	Gaussian white system noise	rx1
$\nu(k)$	Gaussian white measurement noise	mx1
D	Jump coefficient matrix	nxp
Δu_q	Jump in control at time q	px1
$u(k)$	Control vector	px1

TABLE 2

DISCRETE KALMAN-BUCY FILTER EQUATIONS*

$\hat{x}(k+1\|k) = \Phi(k+1,k)\ \hat{x}(k)$	(i)
$P(k+1\|k) = \Phi(k+1,k)\ P(k)\ \Phi^T(k+1,k) + \Gamma(k)\ Q(k)\Gamma^T(k)$	(ii)
$\gamma(k) = z(k) - H(k)\ \hat{x}(k\|k-1)$	(iii)
$V(k) = H(k)P(k\|k-1)\ H^T(k) + R(k)$	(iv)
$K(k) = P(k\|k-1)\ H^T(k)\ V^{-1}(k)$	(v)
$\hat{x}(k) = \hat{x}(k\|k-1) + K(k)\ \gamma(k)$	(vi)
$P(k) = [I - K(k)\ H(k)]\ P(k\|k-1)$	(vii)

*The usual notations $\hat{x}(k|k)$ and $P(k|k)$ have been shortened to $\hat{x}(k)$ and $P(k)$. The random variable $\hat{x}(k|j)$ is the optimal estimate of $x(k)$ based on all the measurements $Z(j) = \{z(1), z(2), \ldots, z(j)\}$. The superscript "T" denotes transpose and "-1" denotes inverse. The identity matrix is denoted by I.

TABLE 3

FILTER VARIABLES

VARIABLE	DEFINITION	DIMENSION
$\hat{x}(k)$	State estimate at k given Z(k)	nx1
P(k)	Covariance matrix of the error in $\hat{x}(k)$	nxn
$\hat{x}(k+1\|k)$	State estimate at k+1 given Z(k)	nx1
P(k+1|k)	Covariance matrix of the error in $\hat{x}(k+1\|k)$	nxn
$\gamma(k)$	Predicted measurement residual	mx1
V(k)	Covariance of $\gamma(k)$	mxm
K(k)	Filter (Kalman) gain matrix at k	nxm

3. FORMULATION OF THE Δ-SYSTEM: JUMP ERROR STATE EQUATION

Consider the following filtering conditions for the Kalman-Bucy filter:

H_1: There is no jump in state and no jump in assumed by the filter.

$\tilde{H}_1$: There is a jump in state but the filter is unaware that a jump has taken place and it operates as if the jump is zero. The filter is referred to as the "jump free" filter.

H_2: There is a jump in state, the jump is known to the filter and the jump information (time and magnitude) is made use of in the filter. The filter is called the "jump is known" filter.

The Kalman-Bucy filter is optimal for Conditions H_1 and H_2 and nonoptimal for $\tilde{H}_1$. Condition H_2 is ideal but does not occur in practice. Condition $\tilde{H}_1$ is the real world condition. We are faced with the problem of accounting for the jump after it has occurred and has been detected. On the one hand we do not wish to degrade the optimal performance of the Kalman-Bucy filter by operating it in an "after-jump" mode when no jump has occurred. On the other hand we desire optimal estimates of the state after the jump has occurred. Because of the delay between the occurrence of the jump and its detection the output of the $\tilde{H}_1$ filter is nonoptimal during this delay. Since we have in practice this sequence of nonoptimal estimates we would like to compensate it with an additional estimate and obtain an optimal estimate. This is precisely the property of the Δ-system. The optimal estimate of the

Δ-state added to the Kalman-Bucy filter estimate under condition $\tilde{H}_1$ is an optimal estimate of the state beyond the jump.

Let $x_1(k)$ represent the state for the case that the jump magnitude Δx_q is zero (i.e., there is no jump). Let $x_2(k)$ represent the state for the case that there is a jump Δx_q and its magnitude may be nonzero. Let $\hat{x}_1$, $\tilde{x}_1$, and $\hat{x}_2$. denote the Kalman-Bucy filter estimates of the state for the Conditions H_1, $\tilde{H}_1$, and H_2, respectively. The relationships between these estimates are depicted in Figure 1 for an anti-ship missile defense scenario in which the ASM employs an endgame pop-up maneuver. Under Condition H_1 the Kalman-Bucy filter estimate $\hat{x}_1$ optimally tracks x_1. Under Condition H_2 the Kalman-Bucy filter estimate $\hat{x}_2$ optimally tracks x_2. But under condition $\tilde{H}_1$ the nonoptimal estimate $\tilde{x}_1$ tracks a trajectory between x_1 and x_2. Since the covariance of state estimate and the Kalman gain are independent of the measurements it follows that the gains $K_1(k)$, $\tilde{K}_1(k)$ and $K_2(k)$ are identical and that the covariances P_1, $\tilde{P}_1$ and P_2 are identical for the three filtering Conditions H_1, $\tilde{H}_1$ and H_2.

We define the following errors of $\tilde{x}_1(k)$:

$$\Delta x(k) = \hat{x}_2(k) - \tilde{x}_1(k), \text{all } k \tag{10}$$

$$\Delta w(k) = \tilde{x}_1(k) - \hat{x}_1(k), \text{ all } k \tag{11}$$

The sum of the two errors satisfy

$$\Delta x(k) + \Delta w(k) = \phi(k,q) \quad \Delta x_q, \quad k \geq q \tag{12}$$

KALMAN-BUCY FILTER ESTIMATES

$\hat{x}_1$ - NO MANEUVER $\quad K_1$ = KALMAN GAIN

$\hat{x}_2$ - MANEUVER IS KNOWN $\quad H$ = MEASUREMENT MATRIX

$\tilde{x}_1$ - MANEUVER IS UNKNOWN $\quad \Phi$ = STATE TRANSITION MATRIX

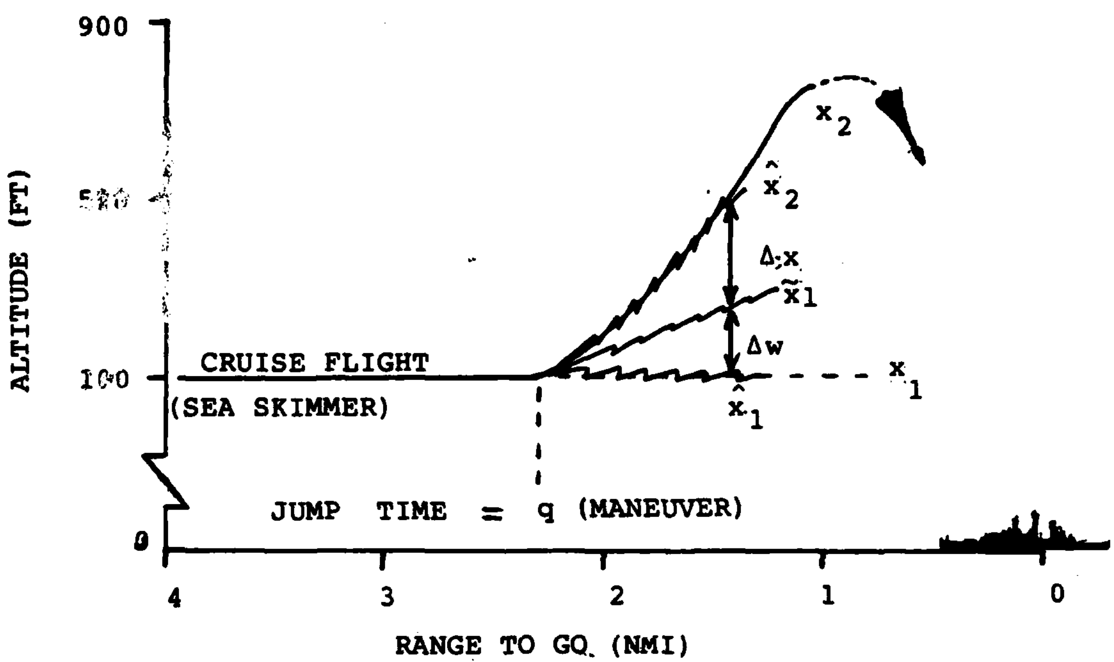

FIGURE 1. ASM DEFENSE SCENARIO: JUMP ERROR STATE Δx SATISFIES LINEAR EQUATION

$$\Delta x(k+1) = [I-K_1(k+1)\, H(k+1)]\, \Phi(k+1,k)\, \Delta x(k)$$

We call the random variable Δx the jump error state or Δ-state variable. It is the difference between the Kalman-Bucy filter estimates under the conditions of known jump and unknown jump. Since the addition of $\Delta x(k)$ and $\tilde{x}_1(k)$ give $\hat{x}_2(k)$ we would expect that the optimal estimate $\hat{\Delta x}(k)$ added to $\tilde{x}_1(k)$ gives the optimal estimate $\hat{x}(k)$ of (4) and (5):

$$\hat{x}(k) = \tilde{x}_1(k) + \hat{\Delta x}(k) \tag{13}$$

It is easy to show that Δx satisfies a linear state equation [8]. The initial condition for Δx is at time q:

$$\Delta x(q) = \Delta x_q \tag{14}$$

$$\Delta x_q = \hat{x}_2(q) - \tilde{x}_1(q) \tag{15}$$

Let $k > q$. From Equations (i), (iii), and (vi) of Table 2 we observe that the estimates $\tilde{x}_1(k)$ and $\hat{x}_2(k)$ satisfy

$$\tilde{x}_1(k) = \Delta\Phi(k,k-1)\ \tilde{x}_1(k-1) + K_1(k)\ z(k) \tag{16}$$

$$\hat{x}_2(k) = \Delta\Phi\ (k,k-1)\ \hat{x}_2(k-1) + K_1(k) z(k) \tag{17}$$

where

$$\Delta\Phi(k,k-1) = [I-K_1(k)\ H(k)]\ \Phi(k,k-1) \tag{18}$$

Consequently, subtracting (16) from (17) gives the linear equation

$$\Delta x(k) = \Delta\Phi(k,k-1)\quad \Delta x(k-1) \tag{19}$$

with initial Condition (14). The measurement equation for Δx is also easily derived. Under Condition $\tilde{H}_1$, the a posterior measurement residual $\Delta z(k)$ is given by

$$\Delta z(k) = z(k) - H(k)\,\tilde{x}_1(k) \tag{20}$$

Adding and subtracting the term $H(k)\,\hat{x}_2(k)$ we obtain the measurement equation for Δx

$$\Delta z(k) = \Delta H(k)\,\Delta x(k) + \Delta\nu(k) \tag{21}$$

where

$$\Delta H(k) = H(k) \tag{22}$$

$$\Delta\nu(k) = z(k) - H(k)\,\hat{x}_2(k) \tag{23}$$

is the a posteriori measurement residual under Condition H_2. Consequently, $\Delta\nu(k)$ is a zero-mean white Gaussian sequence with covariance defined by

$$E\{\Delta\nu(k)\Delta\nu^T(j)\} = \Delta R(k)\,\delta_{kj} \tag{24}$$

where

$$\Delta R(k) = R(k)\,V_1^{-1}(k)\,R(k) \tag{25}$$

recalling that V_1, $\tilde{V}_1$ and V_2 are identical.

Equations (19) and (21) constitute the linear equations that govern the error Δx. If the jump time q were known and the initial state $\Delta x(q)$ were normally distributed with mean $\hat{\Delta x}(q)$ and covariance $\Delta P(q)$ then a Kalman-Bucy filter could be employed to estimate $\Delta x(k)$. The filtering equations are as given in Table 4 for such a case. For this case define

TABLE 4

FILTERING EQUATIONS FOR THE Δ-SYSTEM

$$\Delta\hat{x}(k+1|k) = \Delta\Phi(k+1,k)\ \Delta\hat{x}(k),\quad k \geq q \tag{i}$$

$$\Delta P(k+1|k) = \Delta\Phi(k+1,k)\ \Delta P(k)\ \Delta\Phi^T(k+1,k),\quad k \geq q \tag{ii}$$

$$\Delta\gamma(k) = \Delta z(k) - \Delta H(k)\ \Delta\hat{x}(k|k-1),\quad k > q \tag{iii}$$

$$\Delta V(k) = \Delta H(k)\ \Delta P(k|k-1)\ \Delta H^T(k) + \Delta R(k),\quad k > q \tag{iv}$$

$$\Delta K(k) = \Delta P(k|k-1)\ \Delta H^T(k)\ \Delta V^{-1}(k),\quad k > q \tag{v}$$

$$\Delta\hat{x}(k) = \Delta\hat{x}(k|k-1) + \Delta K(k)\ \Delta\gamma(k),\quad k > q \tag{vi}$$

$$\Delta P(k) = [I - \Delta K(k)\ \Delta H(k)]\ \Delta P(k|k-1),\quad k > q \tag{vii}$$

where

$$\Delta\Phi(k,k-1) = [I - K_1(k)\ H(k)]\ \Phi(k,k-1) \tag{viii}$$

$$\Delta H(k) = H(k) \tag{ix}$$

$$\Delta R(k) = R(k)\ V_1^{-1}(k)\ R(k) \tag{x}$$

$$\Delta z(k) = z(k) - H(k)\ \tilde{x}_1(k) \tag{xi}$$

$$\hat{x}(q) = \tilde{x}_1(q) + \hat{\Delta x}(q) \tag{26}$$

$$P(q) = P_1(q) + \Delta P(q) \tag{27}$$

and employ a Kalman-Bucy filter to estimate $x(k)$, $k>q$, governed by (4) and (5). We know that the resulting estimates $\hat{x}(k)$ and $P(k)$ and the gain $K(k)$ are optimal. On the other hand, we can employ the filter under Condition $\tilde{H}_1$ to generate the estimates $\tilde{x}_1(k)$ and $P_1(k)$, $k>q$ and we can employ the filter of Table 4 to generate the estimates $\hat{\Delta x}(k)$ and $\Delta P(k)$. The two approaches are equivalent. That is, (this is shown in Appendices C and D),

$$\hat{x}(k) = \tilde{x}_1(k) + \hat{\Delta x}(k) \tag{28}$$

$$P(k) = P_1(k) + \Delta P(k) \tag{29}$$

For such a case the optimal estimate of $x(k)$ is obtained by summing the two estimates $\tilde{x}_1(k)$ and $\hat{\Delta x}(k)$. The gains are related by the expression (this is shown in Appendix A).

$$K(k) = K_1(k) + \Delta K(k) \quad [I - H(k)K_1(k)] \tag{30}$$

one can show that

$$E\{\Delta e(k) \; e_i^T(k)\} = 0, \; k \geq q, \; i = 1,2 \tag{31}$$

where, for $k \geq q$,

$$e_i(k) = x_i(k) - \hat{x}_i(k), \; i=1,2 \tag{32}$$

$$\Delta e(k) = \Delta x(k) - \hat{\Delta x}(k) \tag{33}$$

Note that

$$e_1(k) = e_2(k), \; x_2(k) = x(k) \text{ and}$$

$$e(k) = e_2(k) + \Delta e(k), \; k \geq q \tag{34}$$

where

$$e(k) = x(k) - \hat{x}(k) \tag{35}$$

From (28) we note that

$$\hat{x}(k) = \hat{x}_2(k) + \hat{\Delta}x(k) - \Delta x(k), \; k \geq q \tag{36}$$

which shows that the optimal estimate $\hat{x}(k)$ is as close to the ideal estimate $\hat{x}_2(k)$ (i.e., jump is known) as the optimal estimate $\hat{\Delta}x(k)$ is to $\Delta x(k)$.

It is shown in Appendix B that the Predicted Measurement Residual covariances are related by the expression

$$V(k) = \tilde{V}_1(k) \; R^{-1}(k) \; \Delta V(k) \; R^{-1}(k) \; \tilde{V}_1(k)$$

4. TRANSFORMATION OF Δ-STATE: CONSTANT Δ-STATE

Define the nxn matrix $\psi(k)$, all k, as

$$\psi(0) = I \tag{37}$$

$$\psi(k) = \Delta\Phi(k,k-1)\ \psi(k-1),\ k > 0 \tag{38}$$

The matrix $\psi(k)$ is positive definite for all k; it has inverse $\psi^{-1}(k)$. The Δ-state equation (19) can be rewritten as

$$\Delta x(k) = \psi(k)\ \psi^{-1}(k-1)\ \ \Delta x(k-1),\ k > q \tag{39}$$

we define a new Δ-state variable Δy as

$$\Delta x(k) = \psi(k)\ \Delta y(k),\ k \geq q \tag{40}$$

This transformation of the Δ-state from Δx to Δy results in the constant state equation:

$$\Delta y(k) = \Delta y(k-1) \qquad k > q \tag{41}$$

The Δ-measurement matrix $\Delta H(k)$ in (22) can be redefined as

$$\Delta H(k) = H(k)\ \psi(k) \tag{42}$$

so that the Equation(21) in terms of the new Δ-state Δy becomes

$$\Delta z(k) = \Delta H(k)\ \Delta y(k) + \Delta\nu(k) \tag{43}$$

Equations (41) and(43) define the linear Δ-system for the state Δy. Equation (40) gives the transformation back to the Δx state.

For the case that the process noise $Q(k) = 0$, for all k, we can take $\psi(k)$ to be defined by

$$\psi(0) = P_1(0)$$
$$\psi(k) = P_1(k)\ \Phi^T(0,k)$$

5. DETECTION AND ESTIMATION OF JUMP USING THE GLR APPROACH: ALGORITHM I

From (41) we see that $\Delta y(k)$ takes on only the values of zero and $\Delta y(q)$ where

$$\Delta y(q) = \psi^{-1}(q)\,\Delta x_q \tag{44}$$

It is zero before the jump and $\Delta y(q)$ after the jump. The residual to be minimized after the jump is

$$\Delta\nu(k) = \Delta z(k) - \Delta H(k)\,\Delta y \tag{45}$$

which has covariance $\Delta R(k)$ as defined by (25). Here, we use Δy to denote the unknown constant. For each k, we desire to obtain the estimates $\hat{q}(k)$ and $\hat{\Delta y}\,(\hat{q}(k),k)$ that render a minimum to the function

$$J(q, \Delta y;k) = \sum_{i=1}^{q} [\Delta z(i)]^T[\Delta R(i)]^{-1}[\Delta z(i)]$$

$$+ \sum_{i=q+1}^{k} [\Delta z(i) - \Delta H(i)\,\Delta y]^T[\Delta R(i)]^{-1}[\Delta z(i) - \Delta H(i)\,\Delta y] \tag{46}$$

or, equivalently, a maximum to the function

$$\ell(q, \Delta y;k) = \sum_{i=1}^{k} [\Delta z(i)]^T[\Delta R(i)]^{-1}[\Delta z(i)] - J(q, \Delta y;k) \tag{47}$$

This latter fucntion is the logarithm of the generalized likelihood ratio (GLR), [5]. Note that the argument q appears only in the limits of the sum in (46). In the approach of [5] the jump time appears in the terms of the sum as well as in the limits of the sum. For each possible jump time q the optimum value $\hat{\Delta y}\,(q,k)$ satisfies

$$C(q;k)\ \hat{\Delta y}(q,k) = d(q;k) \tag{48}$$

where

$$C(q;k) = \sum_{i=q+1}^{k} [\Delta H(i)]^T [\Delta R(i)]^{-1} [\Delta H(i)] \tag{49}$$

$$d(q;k) = \sum_{i=q+1}^{k} [\Delta H(i)]^T [\Delta R(i)]^{-1} [\Delta z(i)] \tag{50}$$

In view of the solution (48), the log likelihood ratio (47) simplifies to

$$\ell(q,\ \hat{\Delta y}(q,k);k) = \hat{\Delta y}^T(q,k)\ C(q;k)\ \hat{\Delta y}(q,k) = d^T(q;k)\hat{\Delta y}(q,k) \tag{51}$$

or, equivalently,

$$\ell(q,\ \hat{\Delta y}(q,k);k) = d^T(q;k)\ C^{-1}(q;k)\ d(q;k) \tag{52}$$

The maximum likelihood estimate of the jump time is

$$\hat{q}(k) = \arg\max_{q}\ \ell(q,\ \hat{\Delta y}(q,k);k) \tag{53}$$

and the maximum likelihood estimate of the jump magnitude is

$$\hat{\Delta y}(\hat{q}(k),k) = C^{-1}(\hat{q}(k);k)\ d(\hat{q}(k);k) \tag{54}$$

A jump is detected at time k if a threshold is exceeded, [5]:

$$\ell(\hat{q},\ \hat{\Delta y}(\hat{q}(k),k)) > 2\,\ell n\ (\eta) \tag{55}$$

where the value η is chosen to provide a reasonable tradeoff between false and missed alarms.

A difference between our approach and that of [5] is that the failure signature matrix $G(j;\theta)$ depends on the jump time and our corresponding matrix $\Delta H(j)$ does not.* As a result of this advantage the computation of updating several sequences of matrices is avoided. For the purpose of estimating the jump time and the jump magnitude at time k it suffices to have the following matrices in storage:

$C(0;i)$, $i = 1,2 \ldots, k$

$d(0;i)$, $i = 1,2, \ldots, k$

The matrices $C(0;k)$ and $d(0;k)$ are computed recursively

$$C(0;k) = C(0;k-1) + \Delta H^T(k) \ \Delta R(k)^{-1} \ \Delta H(k) \tag{56}$$

$$d(0;k) = d\ (0;k-1) + \Delta H^T(k) \ \Delta R(k)^{-1} \ \Delta z(k) \tag{57}$$

The matrices $C(q;k)$ and $d(q;k)$ are obtained by subtraction

$$C(q;k) = C(0;k) - C(0;q), \quad 0 < q < k \tag{58}$$

$$d(q;k) = d(0;k) - d\ (0;q), \quad 0 < q < k \tag{59}$$

It is unnecessary to evaluate (52) for all q, $0 < q < k$. A simple search procedure can be employed to locate the maximizing $\hat{q}(k)$ of (53)†. After obtaining $\hat{\Delta y}$ we use (40) to estimate

$$\hat{\Delta x}(k) = \psi\ (k)\ \hat{\Delta y} \tag{60}$$

*This is discussed in the next section.

†In the search procedure one may use the Gaussian elimination method to compute the solution $\hat{\Delta y}(q,k)$ of (48) at some $0 < q < k$. Therefore, the log likelihood ratio (51) may be evaluated without inverting $C(q;k)$. This matrix need only be inverted at the maximizing $\hat{q}(k)$, provided (55) is satisfied, to obtain the covariance of $\hat{\Delta y}(\hat{q}(k),k)$.

Its covariance is given by

$$\Delta P(k) = E\{\Delta\ e(k)\Delta\ e(k)^T\} = \psi(k)\ C^{-1}\ (\hat{q}(k);k)\psi^T(k) \tag{61}$$

where $\Delta e(k) = \Delta x(k) - \hat{\Delta x}(k)$. The covariance of the estimate $\hat{\Delta y}$ is given by

$$E\{\ \Delta\ e_1(k)\ \Delta e_1(k)^T\} = C^{-1}\ (\hat{q}(k);k) \tag{62}$$

where $\Delta e_1(k) = \Delta y(k) - \hat{\Delta y}\ (\hat{q}(k),k)$. This covariance is based on the assmption that the a priori covariance for Δy at time $q\hat{(}k)$ is infinite, [16, p. 206]. The estimates given by (60) and (61) are used as starting values in the Δ-filter. Eqs. (28) and (29) are used to reinitialize the filter of H_1 for the case of multiple jumps.

If the a priori covariance for $\Delta x(q)$ is not infinite but is given by $\Delta P(q)$ with inverse ΔP^{-1} (q) the a priori covariance ΔP_1 (q) for $\Delta y(q)$ has inverse

$$\Delta P_1^{-1}(q) = \psi^T(q)\ \Delta P^{-1}(q)\ \psi(q) \tag{63}$$

If $\hat{\Delta x}(q)$ is the mean of $\Delta x(q)$, the mean of $\Delta y(q)$ is

$$\hat{\Delta y}(q) = \psi^{-1}(q)\ \hat{\Delta x}(q) \tag{64}$$

With the modifications

$$C^*\ (q;k) = C(q;k) + \Delta P_1^{-1}(q) \tag{65}$$

$$d^*\ (q;k) = d(q;k) + \Delta P_1^{-1}(q)\ \hat{\Delta y}(q) \tag{66}$$

The method of solution is as outlined above.

If we know that the jump is caused by a lower dimensional vector such as Δu_q where

$$\Delta x_q = D \, \Delta u_q \tag{67}$$

we proceed as follows. From (44) and (67) we have

$$\Delta y(q) = \psi^{-1}(q) \, D \, \Delta u_q \tag{68}$$

we define

$$\bar{C}(q;k) = D^T \psi^{-T}(q) \, C(q;k) \, \psi^{-1}(q) D \tag{69}$$

$$\bar{d}(q;k) = D^T \psi^{-T}(q) \, d(q;k) \tag{70}$$

and use the method of solution as described above.

The GLR technique described in [9] requires the implementation of a Kalman-Bucy Δ-filter (employing the computational savings techniques discussed therein) for each observation time in the past. Several matrices have to be stored and updated for each observation time in the past. The attractive feature of that technique is that it requires neither the inverse of $C(q;k)$ nor the solution to (48). Instead it requires at most the inverse of a matrix having the dimensions of the largest correlation block of the measurement noise covariance matrix; sequential updating of the components of the measurement vector is used. No matrix inversion is required by the Kalman-Bucy filter for uncorrelated measurement noise. We use the sequentially updated Kalman filtering technique together with the decomposition of the maneuver signature matrix to develop our algorithm II in Section 7.

6. COMPARISON WITH PREVIOUS GLR TECHNIQUES

The a posteriori jump formulation is used in (4) and (5). It has the following relationship with the a priori jump formulation. Define:

$$p = q+1 \tag{71}$$

$$\Delta x_p = \Phi(q+1,q)\ \Delta x_q \tag{72}$$

Equation (4) can be rewritten as:

$$x(k+1) = \Phi(k+1,k)\ x(k) + \Delta x_p\ \delta_{p,k+1} + \Gamma(k)\ w(k) \tag{73}$$

where p is the jump time and Δx_p is the jump magnitude. The effect of the jump appears in the state x(p) and in the measurement z(p) at the jump time; that is, it occurs right before the measurement z(p). This formulation is used in [3] - [9]. Therein, the effect of the jump on the innovations is analyzed. The general form is given by, [3] and [5],

$$\tilde{\gamma}_1(k) = G(k;p)\ \Delta x_p + \gamma_1(k) \tag{74}$$

where $\tilde{\gamma}_1(k)$ and $\gamma_1(k)$ are the predicted measurement residuals under Conditions $\tilde{H}_1$ and H_1, respectively. The matrix G(k;p) is called the failure signature matrix[+] and it is computed by the following recursive algorithm, [5] and [6]. At each observation time k the following matrices, having been computed previously at time k-1, are held in storage:

[+]Herein, we refer to it as the maneuver signature matrix.

$$G(k-1;j),\ k-1-M \le j < k-1 \tag{75}$$

$$F(k-1;j),\ k-1-M \le j \le k-1 \tag{76}$$

where it is assumed that a sliding window of length M is being used to detect and estimate the jump. For each k the matrix G(k;k) satisfies

$$G(k;k) = H(k),\ \text{all } k \tag{77}$$

The following matrices are computed at the time k:

$$\Phi(k;j) = \Phi(k,k-1)\ \ \Phi(k-1,j),\ k-M \le j < k \tag{78}$$

$$G(k;j) = H(k)[\Phi(k,j) - S(k;j)],\ k-M \le j < k \tag{79}$$

$$S(k;j) = \Phi(k,k-1)F(k-1;j),\ k-M \le j < k \tag{80}$$

$$F(k;k) = K_1(k)\ \ H(k) \tag{81}$$

$$F(k;j) = K_1(k)\ G(k;j) + S(k;j),\ k-M \le j < k \tag{82}$$

The following matrices have also been computed and stored at the previous observation time k-1:

$$C_w\ (k-1;j),\ k-M < j \le k-1 \tag{83}$$

$$d_w\ (k-1;j),\ k-M < j \le k-1 \tag{84}$$

At the new observation time k they are updated using the equations

$$C_w\ (k;k) = H^T(k)\ V_1^{-1}(k)\ H(k) \tag{85}$$

$$d_w\ (k;k) = H^T(k)\ V_1^{-1}(k)\ \tilde{\gamma}_1(k) \tag{86}$$

$$C_w\ (k;j) = G^T(k;j)\ V_1^{-1}(k)\ G(k;j) + C_w(k-1;j),\ k-M<j<k \tag{87}$$

$$d_w(k;j) = G^T(k;j)\ V_1^{-1}(k)\ \tilde{\gamma}_1(k) + d_w(k-1;j),\ k-M<j<k \tag{88}$$

The log-likelihood ratio to be maximized, [5] and [6], is

$$\ell_w(k;j) = d_w^T\ (k;j)\ C_w^{-1}(k;j)\ d_w(k;j),\ k-M<j\leq k \tag{89}$$

which is evaluated at past observation times j in order to determine its maximum value and compare it to a set threshold for jump detection.

In our approach Eq. (74) is given by

$$\tilde{\gamma}_1(k) = H(k)\ \phi(k,k-1)\ \Delta x(k-1) + \gamma_1(k) \tag{90}$$

since

$$\tilde{\gamma}_1(k) = z(k) - H(k)\ \phi(k,k-1)\ \tilde{x}_1(k-1) \tag{91}$$

$$\gamma_2(k) = z(k) - H(k)\ \phi(k,k-1)\ \hat{x}_2(k-1) \tag{92}$$

$$\gamma_1(k) = \gamma_2(k) \tag{93}$$

From Eqs. (18) and (19) we have

$$\Delta x(k-1) = \Delta\phi(k-1,p)\ \Delta x(p) \tag{94}$$

$$\Delta x(p) = [I-K_1(p)\ H(p)]\Delta x_p \tag{95}$$

Comparing (74) and (90) we see that

$$G(k;p)\ \Delta x_p = H(k)\ \Phi(k,k-1)\ \Delta\Phi(k-1,p)\ [I-K_1(p)H(p)]\,\Delta x_p \tag{96}$$

Using (38) we rewrite (96) as

$$G(k;p)\ \Delta x_p = H(k)\ \Phi(k,k-1)\ \psi(k-1)\ \psi^{-1}(p)\ [I-K_1(p)\ H(p)]\,\Delta x_p \tag{97}$$

Consequently, in our approach $G(k;p)\ \Delta x_p$ is decomposed as

$$G(k;p)\ \Delta x_p = G_1(k)\ G_2(p)\ \Delta x_p \tag{98}$$

where

$$G_1(k) = H(k)\ \phi(k,k-1)\ \psi(k-1) \tag{99}$$

$$G_2(p) = \psi^{-1}(p)\ [I-K_1(p)\ H(p)] \tag{100}$$

Using (18), (71), and (72) we have

$$G_2(p)\ \Delta x_p = \psi^{-1}(q)\ \Delta x_q \tag{101}$$

Using (44), Eq. (101) becomes

$$G_2(p)\ \Delta x_p = \Delta y(q)$$

Consequently, Eq. (98) can be written as

$$G(k;p)\ \Delta x_p = G_1(k)\ \Delta y(p-1) \tag{102}$$

The decomposition given in Eq. (102) results in the matrix $G_1(k)$ which does not depend on the jump time and in the vector $\Delta y(p-1)$ which satisfies the constant state equation (41). The computational burden of the GLR technique of [5] and [6] is directly related to the dependency of G(k;p) on the jump time. Because of this dependency the matrix G(k;j) must be computed and stored for each candidate jump time j in the past. This requires at each new time k the computation and storage of the matrices given in (75) - (88) for all j in a sliding window $k-M<j\leq k$ of candidate jump times.

In our approach we are actually using $\Delta H(k)$ as given in (42) rather than $G_1(k)$ as given by (99). This is because we are using the a posteriori *measurement residuals* $\Delta z(k)$ given in (20) rather than the a priori measurement residuals $\tilde{\gamma}_1(k)$ given in (91). That is, $\Delta H(k)$ is the resulting decomposition matrix for the a posteriori measurement residual approach.

An analysis of the GLR algorithm requirements for storage and multiplications per storage update is given in Table 5 for the approach of [5] and [6] and in Table 6 for our approach.* Let the storage requirements be denoted by M_{sw} for that of [5] and [6] and by M_s for our approach. The storage requirements are

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*The measurement matrix R(k) is assumed to be diagonal and all elements of the matrices H(k) and $\phi(k,k-1)$ are multiplied in matrix products.

TABLE 5

WILLSKY AND JONES' GLR ALGORITHM REQUIREMENTS:

STORAGE AND MULTIPLICATIONS PER STORAGE UPDATE

k = CURRENT OBSERVATION TIME

n = DIMENSION OF STATE VECTOR x

m = DIMENSION OF MEASUREMENT VECTOR z

M = LENGTH OF SLIDING WINDOW

I. STORAGE REQUIREMENTS

	MATRICES	DIMENSION	STORAGE REQUIREMENTS
1.	$\{G(k;j):k-M<j<k\}$	mxn	$(M-1)nm$
2.	$\{F(k;j):k-M<j\leq k\}$	nxn	Mn^2
3.	$\{C_w(k;j):k-M<j\leq k\}$	$\frac{nx(n+1)}{2}$	$\frac{Mn(n+1)}{2}$
4.	$\{d_w(k;j):k-M<j<k\}$	nx1	Mn
5.	$\{\Phi(k;j):k-M<j<k\}$	nxn	$(M-1)n^2$

TOTAL STORAGE = $(M-1)M_{1w} + M_{2w}$ where

$$M_{1w} = [\frac{5n^2}{2} + nm + \frac{3n}{2}]$$

$$M_{2w} = [\frac{3n^2}{2} + \frac{3n}{2}]$$

Table 5 (continued)

II. MULTIPLICATIONS NEEDED TO UPDATE STORED MATRICES

	EQUATIONS	NUMBER OF MULTIPLICATIONS
1.	Eqs. (79) and (80) for $G(k;j)$, $k-M<j<k$	$(M-1)[n^3 + n^2m]$
2.	Eqs. (81) and (82) for $F(k;j)$; $k-m<j\leq k$	$M[n^2m]$
3.	Eqs. (85) and (87) for $C_w(k,j)$, $k-m<j\leq k$	$M\ [\frac{n(n+1)m}{2} + nm^2]$
4.	Eqs. (86) and (88) for $d_w(k,j)$, $k-M<j\leq k$	$M[nm + m^2]$
5.	Eq. (78) for $\phi(k,j)$, $k-M<j<k$	$(M-1)\ [n^3]$

TOTAL MULTIPLICATIONS = $(M-2)\ N_{1w} + N_{2w}$ where

$$N_{1w} = [2n^3 + \frac{5}{2}n^2m + nm^2 + \frac{3}{2}nm + m^2]$$

$$N_{2w} = [2n^3 + 4n^2m + 2nm^2 + 3nm + 2m^2]$$

TABLE 6

NEW GLR ALGORITHM I REQUIREMENTS

STORAGE AND MULTIPLICATIONS PER STORAGE UPDATE

k = CURRENT OBSERVATION TIME

n = DIMENSION OF STATE VECTOR x

m = DIMENSION OF MEASUREMENT VECTOR z

M = LENGTH OF SLIDING WINDOW

I. STORAGE REQUIREMENTS

	MATRICES	DIMENSION	STORAGE REQUIREMENTS
1.	$\psi(k)$	nxn	n^2
2.	$\Delta H(k)$	mxn	nm
3.	$\Delta z(k)$	mx1	m
4.	$\Delta R(k)^{-1}$	mxm	$\frac{m(m+1)}{2}$
5.	$\{C(0;j):k-M<j\leq k\}$	nxn	$M\frac{n(n+1)}{2}$
6.	$\{d(0;j):k-M<j\leq k\}$	nx1	Mn

TOTAL STORAGE = $(M-1)\ M_1 + M_2$ where

$$M_1 = [\frac{n2}{2} + \frac{3n}{2}]$$

$$M_2 = [\frac{3n^2}{2} + nm + \frac{m^2}{2} + \frac{3n}{2} + \frac{3m}{2}]$$

Table 6 (continued)

II. MULTIPLICATIONS NEEDED TO UPDATE STORED MATRICES

EQUATIONS	NUMBER OF MULTIPLICATIONS*
1. Eq. (38) for $\psi(k)$	n^3
2. Eq. (42) for $\Delta H(k)$	n^2m
3. Eq. (20) for $\Delta z(k)$	nm
4. Eq. (25) for $\Delta R(k)^{-1}$	$\frac{3}{2}m^2 + \frac{m}{2}$
5. Eq. (56) for $C(0;k)$	$\frac{n^2m}{2} + nm^2 + \frac{nm}{2}$
6. Eq. (57) for $d(0;k)$	$nm + m^2$

TOTAL MULTIPLICATIONS = $(M-2)N_1 + N_2$ where

$$N_1 = 0$$

$$N_2 = [n^3 + \frac{3n^2m}{2} + nm^2 + \frac{5nm}{2} + \frac{5m^2}{2} + \frac{m}{2}]$$

*For the case that the process noise is zero it is not necessary to compute $\psi(k)$ unless a jump is detected. It suffices to compute $\Delta H(k)$ where $\Delta H(k) = H(k)\ P_1(k)\ \Phi^T(0,k)$. This product is performed with at most $2n^2m$ multiplications which replaces the sum $n^3 + 2n^2m$. If a jump is detected we need $\psi(k)$ for computing $\Delta\hat{x}(k)$ from $\Delta\hat{y}(k)$.

$$M_{sw} = (M-1)\, M_{1w} + M_{2w}$$

where

$$M_{1w} = \frac{5n^2}{2} + nm + \frac{3n}{2} \tag{103}$$

$$M_{2w} = \frac{3n^2}{2} + \frac{3n}{2} \tag{104}$$

and are

$$M_s = (M-1)\, M_1 + M_2 \tag{105}$$

where

$$M_1 = \frac{n^2}{2} + \frac{3n}{2} \tag{106}$$

$$M_2 = \frac{3n^2}{2} + nm + \frac{m^2}{2} + \frac{3n}{2} + \frac{3m}{2} \tag{107}$$

Consider the differences

$$\Delta M_1 = M_{1w} - M_1 = 2n^2 + nm \tag{108}$$

$$\Delta M_2 = M_{2w} - M_2 = -nm - \frac{m^2}{2} - \frac{3m}{2} \tag{109}$$

Define

$$\Delta M_s = M_{sw} - M_s \tag{110}$$

as the difference in storage requirements. The dimension m of the measurement vector is usually much less than the dimension n of the state vector. For sliding windows with $M > 1$ we have the inequality

$$\Delta M_S > (M-2) \quad (n^2 + nm) \tag{111}$$

Let the number of multiplications be denoted by N_w for the approach of [5] and [6] and by N for our approach. The number of multiplications needed to update the stored matrices are

$$N_w = (M-2)\ N_{1w} + N_{2w} \tag{112}$$

where

$$N_{1w} = 2n^3 + \frac{5n^2m}{2} + nm^2 + \frac{3}{2}\ nm + m^2 \tag{113}$$

$$N_{2w} = 2n^3 + 4n^2m + 2nm^2 + 3nm + 2m^2 \tag{114}$$

and

$$N = (M-2)\ N_1 + N_2 \tag{115}$$

where

$$N_1 = 0 \tag{116}$$

$$N_2 = n^3 + \frac{3n^2m}{2} + nm^2 + \frac{5nm}{2} + \frac{5m^2}{2} + \frac{m}{2} \tag{117}$$

Eq. (116) shows that our GLR algorithm requires no multiplications to update stored matrices at past observation times j, $j<k$.

Consider the differences

$$\Delta N_1 = N_{1w} - N_1 = N_{1w} \tag{118}$$

$$\Delta N_2 = N_{2w} - N_2$$
$$= n^3 + \frac{5n^2m}{2} + nm^2 + \frac{nm}{2} - \frac{m^2}{2} - \frac{m}{2} \tag{119}$$

Define

$$\Delta N = N_w - N \tag{120}$$

as the difference in the number of multiplications. For sliding windows with $M > 2$ we have the inequality

$$\Delta N > (M-2)\ (n^3 + 2n^2m) \tag{121}$$

The inequality (111) demonstrates the savings in storage provided by our GLR technique. The inequality (121) demonstrates the savings in multiplications. These savings are a direct result of the decomposition discussed above and given in Eq. (102) for the a priori measurement residual approach and given by

$$H(k)\ \psi(k)\ \Delta y(q) \tag{122}$$

for the a posteriori measurement residual approach.

Both of the above GLR techniques require the inverse+ of an nxn matrix in order to evaluate the log-likelihood ratio (52) or (89). Since our approach requires no updating of matrices at past observation times it suggests using an innovations based scheme to determine the current observation times k at which one should look for a jump in the past. If the innovations appear to be "normal" no jump in the past is to be searched for by evaluating (52); consequently, no inverse is taken. It is when the innovations appear to be less than "normal" that a search is to be made for the optimizing jump time $\hat{q}$ of (52). We now discuss the GLR technique of [8] and [9] which requires no inverse of an nxn matrix. We treat the impulse input version of [8] and [9] rather than the step input case; see Appendix E.

The GLR technique [9] requires a Kalman-Bucy Δ-filter for each observation time j, $k-M<j\leq k$ where k is the current time and M is the length of the sliding window. In our notation and for the jump system (73) and (5) that algorithm utilizes the maneuver signature matrix $G(k,j)$ in the form

$$G(k,j) = H(k)\ \phi(k,k-1)\ A_j(k-1),\ k-M<j<k \tag{123}$$

where $A_j(k)$ is given by

$$A_j(j) = [I - K_1(j)\ H(j)] \tag{124}$$

+It suffices to solve (48) for $\Delta\hat{y}(q,k)$. The inverse $C^{-1}(q;k)$ needs only to be computed at the maximizing argument of (53) when (55) is satisfied. Since jumps are infrequent (55) will be satisfied only infrequently. Consequently, inverses are seldom required.

$$A_j(k) = \Delta\Phi(k,k-1)\ A_j(k-1) \tag{125}$$

We make the definitions

$$\Delta H_k(k) = H(k) \tag{126}$$

$$\Delta H_j(k) = H(k)\ \Phi(k,k-1)\ A_j(k-1),\ j<k \tag{127}$$

At each observation time k the following matrices, having been computed previously at time k-1, are held in storage:

$$A_j(k-1),\ k-1-M<j\leq k-1 \tag{128}$$

$$\Delta\hat{v}_j(k-1),\ k-1-M<j\leq k-1 \tag{129}$$

$$\Delta P_j(k-1),\ k-1-M<j\leq k-1 \tag{130}$$

$$d_j(k-1),\ k-1-M<j\leq k-1 \tag{131}$$

where it is assumed that a sliding window of length M is being used to detect and estimated the jump. At the current time k the following matrices are computed:

$$A_k(k) = [I - K_1(k)\ H(k)] \tag{132}$$

$$A_j(k) = \Delta\Phi(k,k-1)\ A_j(k-1),\ k-M<j<k \tag{133}$$

$$\Delta H_j(k) = H(k)\ \Phi(k,k-1)\ A_j(k-1),\ k-M<j<k \tag{134}$$

$$\Delta V_j(k) = \Delta H_j(k)\ \Delta P_j(k-1)\ \Delta H_j^T(k) + V_1(k),\ k-M<j\leq k \tag{135}$$

$$\Delta K_j(k) = \Delta P_j(k-1)\ \Delta H_j^T(k)\ \Delta V_j^{-1}(k),\ k-M<j\leq k \tag{136}$$

$$\hat{\Delta v}_j(k) = \hat{\Delta v}_j(k-1) + \Delta K_j(k)\ [\tilde{\gamma}_1(k) - \Delta H_j(k)\ \hat{\Delta v}_j(k-1)],$$
$$k-M<j\leq k \quad (137)$$

$$\Delta P_j(k) = [I - \Delta K_j(k)\ \Delta H_j(k)]\ \Delta P_j(k-1),\ k-M<j\leq k \quad (138)$$

$$d_j(k) = d_j(k-1) + \Delta H_j^T(k)\ V_1^{-1}(k)\ \tilde{\gamma}_1(k),\ k-M<j\leq k \quad (139)$$

$$\ell_j(k) = d_j^T(k)\,\hat{\Delta v}_j(k),\ k-M<j\leq k \quad (140)$$

where

$$d_k(k-1) = 0 \quad \text{all } k \quad (141)$$

$$\hat{\Delta v}_k(k-1) = 0 \quad \text{all } k \quad (142)$$

$$\Delta P_k^{-1}(k-1) = 0 \quad \text{all } k \quad (143)^+$$

The above equations constitute a Kalman-Bucy Δ-filter for each candidate jump time j in the window $k-M<j\leq k$. The estimate $\hat{\Delta v}_j(k)$ is the optimal estimate at time k of Δx_p when the jump time p coincides with the candidate jump time j. If the jump threshold is exceeded at the maximizing $j = \hat{p}$ of (140) the optimal estimate of x(k) is given by, [9],

$$\hat{x}(k) = \tilde{x}_1(k) + A_{\hat{p}}(k)\ \hat{\Delta v}_{\hat{p}}(k) \quad (144)$$

and its covariance by

$$P(k) = P_1(k) + A_{\hat{p}}(k)\,\Delta P_{\hat{p}}(k)\ A_{\hat{p}}^T(k) \quad (145)$$

+The initial covariance $\Delta P_k(k-1)$ is defined as the identity matrix I times a very large number.

The sequentially updated Kalman filtering technique, [10] and [11], is used in [8] and [9] to avoid the matrix inverses in (136) and (139) for the case when some of or all the components of the measurement vector are uncorrelated. Note that the maximizing jump time j is easily obtained from (140), requiring only Mn multiplications.

An analysis of the GLR algorithm requirements for storage and multiplications per storage update is given in Table 7 for the approach of [8] and [9]. Let the storage requirements be denoted by M_{sc}. The storage requirements are

$$M_{sc} = (M-1)M_{1c} + M_{2c} \tag{146}$$

where

$$M_{1c} = \frac{3n^2}{2} + \frac{5n}{2} \tag{147}$$

$$M_{2c} = \frac{3n^2}{2} + 2nm + m^2 + \frac{5n}{2} \tag{148}$$

The differences in storage requirements between the above approach and our approach are:

$$\Delta M_{1c} = M_{1c} - M_1 = n^2 + n \tag{149}$$

$$\Delta M_{2c} = M_{2c} - M_1 = nm + \frac{m^2}{2} + n - \frac{3m}{2} \tag{150}$$

Define

$$\Delta M_{sc} = M_{sc} - M_s \tag{151}$$

as the difference in storage requirements for the two approaches. For sliding windows with M>l we have the inequality

$$\Delta M_{sc} > (M-1)\ [n^2 + n] \tag{152}$$

Let the number of multiplications be denoted by N_c for the approach of [9]. The number of multiplications needed to update the stored matrices are

$$N_c = (M-2)\ N_{1c} + N_{2c} \tag{153}$$

Where

$$N_{1c} = n^3 + 5\ n^2m + n^2 + 4nm + n+m \tag{154}$$

$$N_{2c} = n^3 + 9n^2m + 2n^2 + 8nm + 2n + 2m \tag{155}$$

Define

$$\Delta N_{2c} = N_{2c} - N_2 \tag{156}$$

We have the inequality

$$\Delta N_{2c} > 6\ n^2m + 2n^2 + 3nm \tag{157}$$

It is not fair to make a direct comparison between N_{1c} and N_1 since the approach of [9] provides the estimate $\hat{\Delta v_j}(k)$ for each j in the sliding window and ours does not. In order to make a fair comparison we must add in the number of multiplications needed to solve (48) for $\hat{\Delta y}(q,k)$. Since $C(q;k)$ is a symmetric matrix, the Cholesky method, [18], may be employed to solve (48); the number of multiplications required are

$$N_3 = \frac{n^3}{6} + \frac{3n^2}{2} + \frac{n}{3} \tag{158}$$

Since we employ a search procedure to maximize (53) it is not necessary to solve (48) at each q in the sliding window. Let us assume at the very worst that we will need to evaluate (51) at M-2 observation times in the sliding window; that is, we need not solve (48) at two points. Define

$$\Delta N_{1c} = N_{1c} - N_3 \tag{159}$$

We have the inequality

$$\Delta N_{1c} \geq \frac{5}{6} n^3 + 4n^2 \tag{160}$$

Define

$$\Delta N_c = (M-2)\Delta N_{1c} + \Delta N_{2c} \tag{161}$$

For sliding windows with $M>2$ we have the inequality

$$\Delta N_C > (M-2) \left[\frac{5n^3}{6} + 4n^2\right] \tag{162}$$

The inequalities (152) and (162) demonstrate the savings in storage and in multiplications provided by our GLR technique as compared to that in [9].

TABLE 7

CHANG AND DUNN'S GLR ALGORITHM REQUIREMENTS:*

STORAGE AND MULTIPLICATIONS PER STORAGE UPDATE

k = CURRENT OBSERVATION TIME

n = DIMENSION OF STATE VECTOR x

m = DIMENSION OF MEASURMENT VECTOR z

M = LENGTH OF SLIDING WINDOW

I. STORAGE REQUIREMENTS

MATRICES	DIMENSION	STORAGE REQUIREMENTS
1. $\{A_j(k): k-M<j\leq k\}$	nxn	Mn^2
2. $\{\Delta\hat{v}_j(k): k-M<j\leq k\}$	nx1	Mn
3. $\{\Delta P_j(k): k-M<j\leq k\}$	nxn	$M\frac{n(n+1)}{2}$
4. $\{d_j(k): k-M<j\leq k\}$	nx1	Mn
5. $\Delta H_j(k)$ - DUMMY MATRIX	mxn	nm
6. $\Delta V_j(k)$ - DUMMY MATRIX	mxm	m^2
7. $\Delta K_j(k)$ - DUMMY MATRIX	nxm	nm

TOTAL STORAGE = $(M-1)\,M_{1c} + M_{2c}$ where

$$M_{1c} = \frac{3n^2}{2} + \frac{5n}{2}$$

$$M_{2c} = \frac{3n^2}{2} + 2nm + m^2 + \frac{5n}{2}$$

*Impulse input case.

TABLE 7 (continued)

II. MULTIPLICATIONS TO UPDATE STORED MATRICES

EQUATIONS	NUMBER OF MULTIPLICATIONS*
1. Eqs. (132) and (133) for $A_j(k)$, $k-M<j\leq k$	$(M-1)\ n^3 + n^2m$
2. Eq. (137) for $\Delta\hat{v}_j(k)$, $k-M<j\leq k$	$M2nm$
3. Eq. (138) for $\Delta P_j(k)$, $k-M<j\leq k$	$M2n^2m$
4. Eq. (139) for $d_j(k)$, $k-M<j\leq k$	$M[nm + m]$
5. Eq. (134) for $\Delta H_j(k)$, $k-M<j\leq k$	$(M-1)2n^2m$
6. Eq. (135) for $\Delta V_j(k)$, $k-M<j\leq k$	$M[n^2m + nm]$
7. Eq. (136) for $\Delta K_j(k)$, $k-M<j\leq k$	$M[n^2 + n)$

TOTAL MULTIPLICATIONS = $(M-2)\ N_{1c} + N_{2c}$ where

$$N_{1c} = n^3 + 5n^2m + n^2 + 4nm + n+m$$

$$N_{2c} = n^3 + 9n^2m + 2n^2 + 8nm + 2n + 2m$$

*The sequentially updated Kalman filtering technique, [10] and [11], is used.

7. BANK OF Δ-FILTERS USING DECOMPOSITION OF THE MANEUVER SIGNATURE MATRIX: ALGORITHM II

The GLR technique [9] which uses the maneuver signature matrix defined by (123) - (125) requires the computation and storage of the matrix $A_j(k)$ for each observation time j in the past. From Table 7 we see that this requires the following number of multiplications:

$$N_a = (M-1)\ n^3 + n^2 m \tag{163}$$

for sliding windows of length M. From Table 7 we note that this matrix is the only one which requires multiplications proportional to the third power of n. It follows that N_a is proportional to n^4 when $M>n$.

The dependence of N_a on M can be removed by using our decomposition (42) or (122) of the maneuver signature matrix. We develop this idea next.

Consider the employment of a bank of Kalman-Bucy constant Δ-state filters for a moving window of length M. That is, at each j, $k-M \leq j < k$, we employ the constant Δ-state filter defined by (41) - (43). The filtering equations are given in Table 8. The GLR algorithm is as follows.

TABLE 8

FILTERING EQUATIONS FOR THE CONSTANT Δ-STATE SYSTEM

(j = CANDIDATE JUMP TIME)

$\Delta \hat{y}_j(k+1|k) = \Delta\hat{y}_j(k), \; k \geq j-1$ (i)

$\Delta P_j(k+1|k) = \Delta P_j(k), \; k \geq j-1$ (ii)

$\Delta\gamma_j(k) = \Delta z(k) - \Delta H(k) \; \Delta\hat{y}_j(k|k-1), \; k \geq j$ (iii)

$\Delta V_j(k) = \Delta H(k) \; \Delta P_j(k|k-1) \; \Delta H^T(k) + \Delta R(k), \; k \geq j$ (iv)

$\Delta K_j(k) = \Delta P_j(k|k-1) \Delta H^T(k) \; \Delta V_j^{-1}(k), \; k \geq j$ (v)

$\Delta\hat{y}_j(k) = \Delta\hat{y}_j(k|k-1) + \Delta K_j(k) \; \Delta\gamma_j(k), \; k \geq j$ (vi)

$\Delta P_j(k) = [I - \Delta K_j(k) \; \Delta H(k)] \; \Delta P_j(k|k-1), \; k \geq j$ (vii)

where k = current observation time and

$\Delta H(k) = H(k) \; \Psi(k)$, all k (viii)

$\Delta R(k) = R(k) \; V_1^{-1}(k) \; R(k)$, all k (ix)

$\Delta z(k) = z(k) - H(k) \; \tilde{x}_1(k)$, all k (x)

$\Delta P_j(j-1) = c \, I$, c a very large number (xi)

$\Delta\hat{y}_j(j-1) = 0$ (xii)

$\psi(0) = I$ (xiii)

$\psi(k) = \Delta\phi(k,k-1) \; \psi(k-1), \; k > 0$ (xiv)

$\Delta\phi(k,k-1) = [I - K_1(k) \; H(k)] \; \phi(k,k-1), \; k > 0$ (xv)

At each observation time k the following matrices, having been computed previously at time k-1, are held in storage:

$$\Psi(k-1) \tag{164}$$

$$\Delta\hat{y}_j(k-1),\ k-1-M<j\leq k-1 \tag{165}$$

$$\Delta P_j(k-1),\ k-1-M<j\leq k-1 \tag{166}$$

$$d_j(k-1),\ k-1-M<j\leq k-1 \tag{167}$$

At the current time k the following matrices are computed:

$$\Psi(k) = \Phi(k,k-1)\,\Psi(k-1) \tag{168}$$

$$\Delta H(k) = H(k)\,\Psi(k) \tag{169}$$

$$\Delta V_j(k) = \Delta H(k)\ \Delta P_j(k|k-1)\,\Delta H^T(k) + \Delta R(k),\ k-M<j\leq k \tag{170}$$

$$\Delta K_j(k) = \Delta P_j(k|k-1)\,\Delta H^T(k)\,\Delta V_j^{-1}(k),\ k-M<j\leq k \tag{171}$$

$$\Delta\hat{y}_j(k) = \Delta\hat{y}_j(k|k-1) + \Delta K_j(k)\ \gamma_j(k),\ k-M<j<k \tag{172}$$

$$\Delta P_j(k) = [I - \Delta K_j(k)\,\Delta H(k)]\ \Delta P_j(k-1),\ k-M<j\leq k \tag{173}$$

$$d_j(k) = d_j(k-1) + \Delta H^T(k)\ \Delta R^{-1}(k)\ \ \Delta z(k), k-M<j\leq k \tag{174}$$

$$\ell_j(k) = d_j^T(k)\,\Delta\hat{y}_j(k),\ k-M<j\leq k \tag{175}$$

$$\Delta\hat{y}_k(k-1) = 0 \tag{176}$$

$$\Delta P_k(k-1) = c\,I,\ c \text{ a very large number} \tag{177a}$$

$$d_k(k-1) = 0 \tag{177b}$$

The estimate $\hat{\Delta y_j}(k)$ is the optimal estimate at time k of $\Psi^{-1}(q)\Delta x_q$ when the candidate jump time j coincides with the observation time q+1. Note that the filter for $\hat{\Delta y_j}$ is initiated by assuming that the a posteriori jump Δx_q occurs at time j-1. If the maximizing $\hat{j}$ of (175) satisfies (55) the optimal estimate $\hat{\Delta x}(k)$ is computed as

$$\hat{\Delta x}(k) = \Psi(k) \hat{\Delta y_{\hat{j}}}(k) \tag{178}$$

and its covariance as

$$\Delta P(k) = \Psi(k) \Delta P_{\hat{j}}(k) \Psi^T(k) \tag{179}$$

An analysis of the above GLR algorithm requirements for storage and multiplications per storage update is given in Table 9. The analysis assumes the sequentially updated Kalman filtering technique, [10] and [11], is used. Let the storage requirements be denoted by M_d. The storage requirements are

$$M_d = (M-1) M_{1d} + M_{2d} \tag{180}$$

where

$$M_{1d} = \frac{n^2}{2} + \frac{5n}{2} \tag{181}$$

$$M_{2d} = \frac{n^2}{2} + 2nm + m^2 + \frac{5}{2} n \tag{182}$$

TABLE 9

REQUIREMENTS OF NEW GLR ALGORITHM II - BANK OF CONSTANT Δ-STATE FILTERS USING DECOMPOSITION OF MANEUVER SIGNATURE MATRIX: STORAGE AND MULTIPLICATIONS PER STORAGE UPDATE

k = CURRENT OBSERVATION TIME

n = DIMENSION OF STATE VECTOR x

m = DIMENSION OF MEASUREMENT VECTOR z

M = LENGTH OF SLIDING WINDOW

I. STORAGE REQUIREMENTS

MATRICES	DIMENSION	STORAGE REQUIREMENTS
1. $\psi(k)$	nxn	n^2
2. $\{\Delta\hat{Y}_j(k): k-M<j\leq k\}$	nx1	Mn
3. $\{\Delta P_j(k): k-M<j\leq k\}$	nxn	$M\frac{n(n+1)}{2}$
4. $\{d_j(k): k-M<j\leq k\}$	nx1	Mn
5. $\Delta H(k)$	mxn	nm
6. $\Delta V_j(k)$	mxm	m^2
7. $\Delta K_j(k)$	nxm	nm

TOTAL STORAGE = $(M-1)\ M_{1d} + M_{2d}$ where

$$M_{1d} = \frac{n^2}{2} + \frac{5n}{2}$$

$$M_{2d} = \frac{n^2}{2} + 2nm + m^2 + \frac{5n}{2}$$

TABLE 9 (continued)

II. MULTIPLICATIONS TO UPDATE STORED MATRICES

EQUATIONS	NUMBER OF MULTIPLICATIONS*
1. Eq. (168) for $\Psi(k)$	n^3
2. Eq. (172) for $\Delta\hat{y}_j(k)$, $k-M<j\leq k$	M 2nm
3. Eq. (173) for $\Delta P_j(k)$, $k-M<j\leq k$	$M\ 2n^2m$
4. Eq. (174) for $d_j(k)$, $k-M<j\leq k$	$M[nm + m]$
5. Eq. (169) for $\Delta H(k)$	n^2m
6. Eq. (170) for $\Delta V_j(k)$, $k-M<j\leq k$	$M[n^2m + nm]$
7. Eq. (171) for $\Delta K_j(k)$, $k-M<j\leq k$	$M[n^2 + n]$

TOTAL MULTIPLICATIONS = $(M-2)\ N_{1d} + N_{2d}$ where

$$N_{1d} = 3n^2m + n^2 + 4nm + n + m$$

$$N_{2d} = n^3 + 7n^2m + 2n^2 + 8nm + 2n + 2m$$

*The sequentially updated Kalman filtering technique is used.

Note the differences

$$M_{1c} - M_{1d} = n^2 \tag{183}$$

$$M_{2c} - M_{2d} = n^2 \tag{184}$$

Therefore, the decomposition (42) or (122) provides a savings in storage of $(M-1)n^2$.

Let the number of multiplications be denoted by N_d for the new GLR algorithm composed of a bank of constant Δ-state filters using the decomposition of the maneuver signature matrix. The number of multiplications needed to update the stored matrices are

$$N_d = (M-2)\ N_{1d} + N_{2d} \tag{185}$$

where

$$N_{1d} = 3n^2m + n^2 + 4nm + n + m \tag{186}$$

$$N_{2d} = n^3 + 7n^2m + 2n^2 + 8nm + 2n + 2m \tag{187}$$

where we have assumed that all elements of the measurement vector are uncorrelated and the sequentially updated Kalman filtering technique is used.

Note that N_{1d} is not a function of the third power of n and that

$$N_{1c} - N_{1d} = n^3 + 2n^2m \tag{188}$$

$$N_{2c} - N_{2d} = 2n^2m \tag{189}$$

Consequently, the decomposition provides a savings of

$$(M-2)[n^3 + 2n^2m] + 2n^2m \tag{190}$$

for the approach of using a bank of constant Δ-state filters.

Additional savings in multiplications are realized if the process noise $Q(k) = 0$, for all k, and if $\Psi(k)$ is defined by

$$\Psi(k) = P_1(k)\ \Phi^T(0,k) \tag{191}$$

Note that $\Psi(k)$ appears only in the defintion of $\Delta H(k)$, Eq. (169), in the equations of the constant Δ-state filter. It suffices, therefore, to compute $\Delta H(k)$ without computing $\Psi(k)$:

$$\Delta H(k) = H(k)\ P_1(k)\ \Phi^T(0,k) \tag{192}$$

The number of multiplications is, in general,

$$2n^2m \tag{193}$$

If $\Phi(k,0)$ is triangular that number is

$$\frac{n(3n+1)m}{2} \tag{194}$$

In the above we are assuming that no additional multiplications are needed to compute $\Phi(0,k)$. The matrix $\Psi(k)$ is only needed in (178) to obtain $\Delta\hat{x}(k)$ when a jump has been detected. Since jumps occur infrequently, the matrix $\Psi(k)$ needs computed infrequently for the case of no process noise. Consequently, for the case of no process noise, the highest power of n appearing in Table 9 is two. The total number of multiplications required is at most

$$5Mn^2m + 2n^2m \tag{195}$$

In this noiseless case the matrix $\Psi(k)$ does not need to be stored which results in an additional storage savings of n^2.

8. THE Δ-STATE FORMULATION FOR A PRIORI JUMPS

Define the Δ-state $\Delta x(k,k-1)$ as

$$\Delta x(k,k-1) = \hat{x}_2(k,k-1) - \tilde{x}_1(k,k-1), \quad k \geq p \tag{196}$$

This random variable satisfies

$$\Delta x(k,k-1) = \Phi(k,k-1)\, \Delta x(k-1), \quad k \geq p \tag{197}$$

and, in particular

$$\Delta x(p,p-1) = \Delta x_p = \Phi(q+1,q)\, \Delta x_q \tag{198}$$

Using (18) and (19) we see that the state equation for $\Delta x(k,k-1)$ is given by

$$\Delta x(k+1,k) = \Delta\Phi_w(k+1,k)\, \Delta x(k,k-1), \quad k \geq p \tag{199}$$

with initial condition (198) where

$$\Delta\Phi_w(k+1,k) = \Phi(k+1,k)\, [I - K_1(k)\, H(k)] \tag{200}$$

The measurement equation for $\Delta x(k,k-1)$ satisfies

$$\tilde{\gamma}_1(k) = H(k)\, \Delta x(k,k-1) + \gamma_1(k), \quad k \geq p \tag{201}$$

since

$$\gamma_1(k) = \gamma_2(k), \text{ all } k \tag{202}$$

The sequence $\gamma_1(k)$ is Gaussian white noise, [17], with zero mean and covariance

$$E\{\gamma_1(k)\ \gamma_1^T(j)\} = V_1(k)\ \delta_{kj} \text{ all } k \tag{203}$$

Note that

$$\Delta x(k) = [I - K_1(k)\ H(k)]\ \Delta x(k,k-1) \tag{204}$$

Consequently, if we have the optimal estimate $\hat{\Delta} x(k,k-1|k)$ the optimal estimate $\hat{\Delta} x(k) = \hat{\Delta} x(k|k)$ is given by

$$\hat{\Delta} x(k) = [I - K_1(k)\ H(k)]\ \hat{\Delta} x(k,k-1|k) \tag{205}$$

Define the nxn matrix $\Psi_w(k)$ as

$$\Psi_w(1) = I \tag{206}$$

$$\Psi_w(k) = \Delta\Phi_w(k,k-1)\ \Psi_w(k-1),\ k>1 \tag{207}$$

The matrix $\Psi_w(k)$ is positive definite for all k; it has inverse $\Psi_w^{-1}(k)$. We define a new constant Δ-state variable $\Delta y(k,k-1)$ as

$$\Delta x(k,k-1) = \Psi_w(k)\ \Delta y\ (k,k-1) \tag{208}$$

It satisfies the constant Δ-state equation.

$$\Delta y(k+1,k) = \Delta y(k,k-1),\ k \geq p \tag{209}$$

Its measurement equation is given by

$$\tilde{\Upsilon}_1(k) = \Delta H_w(k)\ \Delta y(k,k-1) + \Upsilon_1(k) \tag{210}$$

where

$$\Delta H_w(k) = \Delta H(k)\ \Psi_w(k) \tag{211}$$

Comparing (201) with (74) we find that the maneuver signature matrix G(k;p) satisfies

$$G(k;p)\ \Delta x_p = H(k)\ \Psi_w(k)\ \Psi_w^{-1}(p)\ \Delta x_p \tag{212}$$

or, equivently,

$$G(k;p)\ \Delta x_p = G_1(k)\ G_2(k)\ \Delta x_p \tag{213}$$

where

$$G_1(k) = H(k)\ \Psi_w(k) \tag{214}$$

$$G_2(p) = \Psi_w^{-1}(p) \tag{215}$$

Note that

$$G_2(p)\ \Delta x_p = \Delta y(p,p-1) \tag{216}$$

Consequently, Eq. (213) may be written as

$$G(k;p)\ \Delta x_p = \Delta H_w(k)\ \Delta y(k,k-1) \tag{217}$$

Equations (214) and (215) constitute a decomposition of the maneuver signature matrix into a matrix $G_1(k)$ which does not depend on the jump time p and a matrix $G_2(p)$ which does. The product of the matrix $G_2(p)$ with Δx_p is a constant as shown by Eq. (216).

For the case that the process noise $Q(k) = 0$ for all k, we can take $\Psi_w(k)$ to be defined by

$$\Psi_w(k) = P_1(k|k-1)\ \Phi^T(0,k),\ k>0 \tag{218}$$

We rewrite (210) in the form

$$\Delta z_w(k) = \Delta H_w(k)\ \Delta y(k,k-1) + \Delta \upsilon(k) \tag{219}$$

where

$$\Delta z_w(k) = \tilde{\gamma}_1(k) \tag{220}$$

$$E\ \{\Delta\upsilon(k)\ \Delta\upsilon^T(j)\} = \Delta R_w(k)\ \delta_{kj} \tag{221}$$

$$\Delta R_w(k) = V_1(k) \tag{222}$$

The filters of the two formualtions (the a posteriori and the a priori) differ only in the defintion of the inputs $\Delta\Phi$, Ψ, Δz and ΔR. Both formulations are equivalent and their corresponding Δ-filters provide optimal estimates that satisfy the identify (205). The optimal estimate $\Delta\hat{x}(k,k-1)$ is obtained from $\Delta\hat{y}(k,k-1)$ by using (208).

9. NORMALIZED Δ-MEASUREMENT NOISE COVARIANCE

The Δ-measurement noise covariance matrix $\Delta R(k)$ (given by (25) for the a posteriori jump formulation and by (222) for the a priori jump formation) can be normalized to $R(k)$ by premultiplying Eq. (21) or Eq. (201) by the normalizing matrix. For this purpose we need a matrix square root.

Since $V_1(k)$ is a symmetric positive definite matrix it may be written in a square root factored form (Kaminski, et. al. [19]):

$$V(k) = V^{\frac{1}{2}}(k) \cdot V^{\frac{1}{2}T}(k) \tag{223}$$

where $V^{\frac{1}{2}}$ is a lower triangular matrix (zeros above the diagonal). Square roots are not necessarily unique but a unique square root may be defined using the Cholesky decomposition. We assume the Cholesky decomposition is used and denote the square root of a matrix with the superscript $\frac{1}{2}$ and its transpose by $\frac{1}{2}T$. The square root of the inverse of $V(k)$ is denoted by $V^{-\frac{1}{2}}(k)$.

Premultiplying (21) by the factor $R^{\frac{1}{2}}(k)V^{\frac{1}{2}T}(k)\ R^{-1}(k)$ and redefining $\Delta z(k)$, $\Delta H(k)$ and $\Delta \upsilon(k)$ as

$$\Delta z(k) = R^{\frac{1}{2}}(k)V^{\frac{1}{2}T}(k)\ R^{-1}(k)\ [z(k) - H(k)\ \tilde{x}_1(k)] \tag{224}$$

$$\Delta H(k) = R^{\frac{1}{2}}(k) V^{\frac{1}{2}T}(k) \ R^{-1}(k) \ [H(k)] \tag{225}$$

$$\Delta \upsilon(k) = R^{\frac{1}{2}}(k) V^{\frac{1}{2}T}(k) \ R^{-1}(k) \ [z(k) - H(k) \ \hat{x}_2(k)] \tag{226}$$

we obtain the form (21) where the new $\Delta\upsilon(k)$ is a Gaussian white noise sequence with zero mean and covariance

$$E \ \{\Delta\upsilon(k) \ \Delta\upsilon^T(j)\} = \Delta R(k) \ \delta_{kj} \tag{227}$$

where

$$\Delta R(k) = R(k) \tag{228}$$

which is the covariance of the measurement noise $\nu(k)$ of (5).

The normalizing factor for the a priori jump formulation (201) is

$$R^{\frac{1}{2}}(k) \ V_1^{-\frac{1}{2}}(k) \tag{229}$$

The normalization is particularly useful for the case that R(k) is a constant. If there is no jump, $\Delta z(k)$ is zero mean with covariance R(k). If there is a jump, $\Delta z(k)$ has mean $\Delta H(k) \ \Delta x(k)$ and covariance R(k). Looking for a jump may be avoided if the residuals $\Delta z(k)$ appear to be zero mean with covariance R(k).

10. EXTENSION OF APPROACH TO CONTINUOUS LINEAR STOCHASTIC SYSTEMS

Consider the continuous linear discrete stochastic system described by the vector (Itô) stochastic differential equation

$$dx(t) = F(t)\ x(t)\ dt + \Gamma(t)\ dw(t),\ t \geq 0 \tag{230}$$

and the vector (Itô) observation equation

$$dZ(t) = H(t)\ x(t)\ dt + d\upsilon(t),\ t \geq 0 \tag{231}$$

where $x(t)$ is the n-vector state, $F(t)$ and $\Gamma(t)$ are, respectively, nxn and nxr nonrandom, continuous matrix time-function, and $\{w(t),\ t \geq 0\}$ is an r-vector Brownian motion (Wiener) process with

$$E\{dw(t)\ dw(t)^T\} = Q(t)\ dt \tag{232}$$

The observed process $\{Z(t),\ t \geq 0\}$ is an m-vector process, $H(t)$ is an mxn, nonrandom, continuous matrix time-function, and $\{\upsilon(t),\ t \geq 0\}$ is an m-vector Brownian motion (Wiener) process with

$$E\{d\upsilon(t)\ d\upsilon(t)^T\} = R(t)\ dt \tag{233}$$

where $R(t)>0$. We assume that the system (230) and (231) is observable.

The well-known continuous Kalman-Bucy filter [14,15] is given by, [16],

$$\dot{\hat{x}}(t) = F(t)\,\hat{x}(t) + K(t)\,[z(t) - H(t)\,\hat{x}(t)],\ t \geq 0 \tag{234}$$

$$\dot{P}(t) = F(t)\,P(t) + P(t)\,F^T(t) + \Gamma(t)\,Q(t)\,\Gamma^T(t) - K(t)\,H(t)\,P(t),\ t \geq 0 \tag{235}$$

where

$$K(t) = P(t)\,H^T(t)\,R^{-1}(t) \tag{236}$$

and where the formalized $dZ(t)/dt$ is written as $z(t)$. We have made the identifications $\hat{x}(t) \equiv \hat{x}(t|t)$ and $P(t) \equiv P(t|t)$.

A jump in state *with magnitude* Δx_q *occurs at* time q:

$$x(t^+) = x(t) + \Delta x_q \text{ for } t = q \tag{237}$$

Consider the three Conditions H_1, $\tilde{H}_1$ and H_2. Let both $x(t)$ and $x_2(t)$ represent the state for the system described by (230), (231) and (237). Let $x_1(t)$ represent the state for the case that the jump magnitude Δx_q is zero (i.e., there is no jump). Let $\hat{x}_1(t)$, $\tilde{x}_1(t)$ and $\hat{x}_2(t)$ denote the Kalman-Bucy filter estimates of the state (i.e., Eqs. (234) - (236)) for the Conditions H_1, $\tilde{H}_1$ and H_2, respectively. The relationship between these estimates are similar to those given in Figure 1 for the discrete system. The gains for the three Conditions are equal as well as the covariances.

The estimate $\tilde{x}_1(t)$ can be interpreted as follows. It is identified with $\hat{x}_2(t)$ up to time q

$$\tilde{x}_1(t) = \hat{x}_2(t) \quad t \leq q$$

At time q^+ the amount Δx_q is subtracted from the filtered estimate $\hat{x}_2(t)$ and the result is defined as

$$\tilde{x}_1(t) = \hat{x}_2(t) - \Delta x_q, \quad t = q^+$$

For $t>q$ the quantity satisfies Eq. (234).

We are interested in the difference between $\hat{x}_2(t)$ and $\tilde{x}_1(t)$ for $t>q$. We define this difference as

$$\Delta x(t) = \hat{x}_2(t) - \tilde{x}_1(t), \; t>q$$

It has the initial condition

$$\Delta x(t) = \Delta x_q, \; t = q^+$$

Since both $\hat{x}_2(t)$ and $\tilde{x}_1(t)$ satisfy (234), the difference $\Delta x(t)$ satisfies

$$\frac{d\Delta x(t)}{dt} = [F(t) - K_1(t)\, H(t)]\, \Delta x(t), \quad t \geq q^+ \tag{238}$$

where we have used the identity

$$z_2(t) = \tilde{z}_1(t) \tag{239}$$

Define the nxn nonrandom, continuous matrix time-function $\Psi(t)$ as the solution to the differential equation

$$\dot{\Psi}(t) = [F(t) - K_1(t)\ H(t)]\ \Psi(t), \text{ all } t \tag{240}$$

$$\Psi(0) = I \tag{241}$$

The matrix time-function $\Psi(t)$ is positive definite and has inverse $\Psi^{-1}(t)$. Define the new Δ-state variable $\Delta y(t)$ as

$$\Delta x(t) = \Psi(t)\ \Delta y(t), \quad t \geq q^+ \tag{242}$$

The substitution of (242) into (238) gives

$$\frac{d\Delta y(t)}{dt} = 0, \quad t \geq q^+ \tag{243}$$

We note that $\Delta y(t)$ is a constant and satisfies

$$\Delta y(t) = \Psi^{-1}(q)\ \Delta x_q, \quad t > q^+ \tag{244}$$

The measurement equations for Δx and Δy are given by

$$\Delta z(t) = H(t)\ \Delta x(t) + \Delta \upsilon(t), \quad t \geq q^+ \tag{245}$$

$$\Delta z(t) = \Delta H(t)\ \Delta\, y(t) + \Delta \upsilon(t), \quad t \geq q^+ \tag{246}$$

where

$$\Delta H(t) = H(t)\ \Psi(t), \text{ all } t \tag{247}$$

$$\Delta z(t) = z(t) - H(t)\ \tilde{x}_1(t), \text{ all } t \tag{248}$$

$$\begin{aligned} \Delta \upsilon(t) &= \gamma_1(t) = z_1(t) - H(t)\ \hat{x}_1(t), \text{ all } t \\ &= \gamma_2(t) = z_2(t) - H(t)\ \hat{x}_2(t), \text{ all } t \end{aligned} \tag{249}$$

Eqs. (247) and (244) constitute the decomposition of the maneuver signature matrix. The residual $\gamma_1(t)$ is the measurement noise for the Δ-process. It is zero mean and it has the same covariance as $\upsilon(t)$, [17]:

$$E\{\Delta\upsilon(t)\ \Delta\upsilon^T(s)\} = \Delta R(t)\,\delta(t-s) \tag{250}$$

where $\delta(t-s)$ is the Dirac delta function and

$$\Delta R(t) = R(t) \tag{251}$$

If the jump time q were known and if the initial state $\Delta y(q)$ were normally distributed with mean $\Delta\hat{y}(q)$ and covariance $\Delta P(q)$ then the Kalman-Bucy filter applied to (243) and (246) would provide the solution:

$$\Delta\dot{\hat{y}}(t) = \Delta K(t)\ [\Delta z(t) - \Delta H(t)\ \Delta\hat{y}(t)] \tag{252}$$

$$\Delta\dot{P}(t) = -\Delta K(t)\ \Delta H(t)\ \Delta P(t) \tag{253}$$

where

$$\Delta K(t) = \Delta P(t)\ \Delta H^T(t)\ R^{-1}(t) \tag{254}$$

Using (242) the optimal estimate $\hat{x}(t)$ of $x(t)$ is

$$\hat{x}(t) = \tilde{x}_1(t) + \psi(t)\ \Delta\hat{y}(t) \tag{255}$$

with covariance

$$P(t) = P_1(t) + \psi(t)\ \Delta P(t)\,\psi^T(t) \tag{256}$$

The jump time q is unknown and, therefore, must be estimated along with Δy. For each $t>q$, we desire to obtain the estimates $\hat{q}(t)$ and $\hat{\Delta y}(\hat{q}(t),t)$ that render a minimum to the function

$$J(q,\Delta y;t) = \int_0^q \Delta z(\tau)^T \ \Delta R(\tau)^{-1} \ \Delta z(\tau) \ d\tau$$

$$+ \int_q^t [\Delta z(\tau) - \Delta H(\tau) \ \Delta y]^T \ \Delta R(\tau)^{-1} [\Delta z(\tau) - \Delta H(\tau)\Delta y] \ d\tau \tag{257}$$

or, equivalently, a maximum to the log likelihood ratio

$$\ell(q,\Delta y;t) = \int_0^t \Delta z(\tau)^T \ \Delta R(\tau)^{-1} \ \Delta z(\tau) \ d\tau - J(q,\Delta y;t) \tag{258}$$

The above integrals are Itô integrals. For a fixed q, the optimizing $\hat{\Delta y}(q,t)$ satisfies

$$C(q;t) \ \hat{\Delta} y(q,t) = D(q;t) \tag{259}$$

where

$$C(q;t) = \int_q^t \Delta H(\tau)^T \ \Delta R(\tau)^{-1} \ \Delta H(\tau) \ d\tau \tag{260}$$

$$D(q;t) = \int_q^t \Delta H(\tau)^T \ \Delta R(\tau)^{-1} \ \Delta z(\tau) \ d\tau \tag{261}$$

We make the definitions

$$c(t) = \Delta H(t)^T \ \Delta R(t)^{-1} \ \Delta H(t) \tag{262}$$

$$d(t) = \Delta H(t)^T \ \Delta R(t)^{-1} \ \Delta z(t) \tag{263}$$

we note that d(t) is an nx1-vector Brownian motion process.

In view of (259), Eq. (258) becomes

$$\ell(q, \Delta\hat{y}(q,t);t) = D^T(q;t) \ C^{-1}(q;t) \ D(q;t) \tag{264}$$

$$= \left[\int_q^t d^T(\tau) \ d\tau \right] C^{-1}(q;t) \left[\int_q^t d(\tau) \ d\tau \right] \tag{265}$$

The maximizing argument of (265) is denoted by $\hat{q}(t)$.

Returning to Eqs. (255) and (2 6) the Kalman gain K(t) satisfies

$$K(t) = K_1(t) + \Psi(t) \ \Delta K(t)$$

The continuous case addressed in this section is being treated in more detail in another report.

11. THE SIMILARITY OF JUMP ESTIMATION AND BIAS ESTIMATION

The bias estimation problem can be expressed as follows, [20],

State Dynamics

$$\dot{x} = Ax + Bb + w \tag{266}$$

Bias Dynamics

$$\dot{b} = 0 \tag{267}$$

Observation Equation

$$z = Hx + Cb + \upsilon \tag{268}$$

where the state x is an n-vector, the bias b is a p-vector, the observation vector y is an m-vector, w is the process noise vector with

$$E[w(t)\ w^T(s)] = Q(t)\ \delta(t-s)$$

and υ is the observation noise vector with

$$E[\upsilon(t)\ \ \upsilon^T(s)] = R(t)\ \ \delta(t-s)$$

The vectors w and υ are assumed to be independent. The matrices A and B are time varying.

Friedland [20] was the first to show that the optimal estimate $\hat{x}(t)$ of the state $x(t)$ could be expressed as

$$\hat{x}(t) = \tilde{x}_1(t) + S(t)\,\hat{b}(t) \tag{269}$$

where $\tilde{x}_1(t)$ is the bias-free estimate (i.e., the filter assumes b is zero even though it is nonzero), $\hat{b}(t)$ is the bias estimate which is computed using the bias-free residuals $z(t) - H(t)\,\tilde{x}_1(t)$ and $S(t)$ satisfies the differential equation

$$\dot{S}(t) = [A(t) - K_1(t)\,H(t)]\,S(t) + B(t) - K_1(t)\,C(t) \tag{270}$$

with initial condition

$$S(0) = 0 \tag{270}$$

The matrix $K_1(t)$ is the Kalman gain for the bias-free estimate. The covariance $P(t)$ of $\hat{x}(t)$ is shown in [20] to satisfy

$$P(t) = P_1(t) + S(t)\,P_b(t)\,S^T(t) \tag{271}$$

where $P_1(t)$ is the covariance of $\hat{x}_1(t)$ and $P_b(t)$ is the covariance of $\hat{b}$.

Eqs. (255) and (256) have the same form as (269) and (271). The bias estimation problem is equivalent to the jump estimation problem of estimating the jump b(0) which occurs at time 0; that is, the value of the jump state b is zero before the jump and b(0) afterwards. The jump time 0 is known. The bias estimation problem for discrete systems is also treated in [20]. An extension to the problem of indirect observations is given in [24].

Friedland's bias filtering technique is extended in [21] to the case of estimating a time varying bias

$$\dot{b} = F(t)b \tag{272}$$

The solutions have the forms (269) and (271).

Mendel and Washburn [22] show that the estimation of the bias vector b can be interpreted as being equivalent to the estimation of a constant that is observed through white noise. That result compares with Eqs. (43) and (246). That interpretation is reviewed in [23].

The structure (269) is shown in [25] to hold for the optimal state estimate under the uncertainty of different failure modes.

The problems of jump estimation and bias estimation differ in the following way. The bias estimation is that of estimating a parameter which undergoes a single jump from a zero value. The jump estimation problem is that of estimating a parameter that undergoes multiple jumps from nonzero values. Once a jump has been detected and estimated it is necessary for the jump estimator to pass this information on to the original state estimator and to reinitialize for the next jump. The estimator described by (269) is for single jump systems. The estimator described by (255) is for multiple jump systems.

A nonlinear algorithm is given in [26] - [28] for detecting and estimating sudden changes of biases in linear stochastic systems. The method is based on maximum-likelihood estimation [29].

An extension of Friedland's bias filtering technique to a class of nonlinear systems is given in [30].

12. CONCLUSION

We have presented two recursive GLR algorithms for detecting and estimating maneuver states and parameters in the engagement between an anti-ship cruise missile and a ship defense interceptor. A decomposition of the maneuver signature matrix is used to derive the algorithms. The computational and storage requirements are substantially less than those of other GLR algorithms. The decomposition divides the maneuver signature matrix into the product of two matrices. One matrix depends only on the current observation time while the other depends only on the jump time. The product of the latter matrix and the jump magnitude vector provides a jump error state vector which is constant. This constancy facilitates using the GLR approach. The other matrix of the decomposition represents the new maneuver signature matrix for the new constant jump error state vector. The nondependency of the maneuver signature matrix on the jump time avoids storing large matrices and computing large matrix products for each past observation time.

Previously the most efficient GLR algorithm for the detection and the estimation of jumps required (at each current observation time) multiplications on the order of

$$Mn^3 + 5Mn^2m$$

where n is the dimension of the state vector, m is the dimension of the measurement vector and M is the number of candidate jump times in the past. As a consequence of the decomposition the GLR algorithm II presented herein requires multiplications on the order of

$$n^3 + 3Mn^2m$$

for general discrete linear stochastic systems and

$$5Mn^2n$$

for such systems without process noise. This is a substantial savings in computation. The corresponding savings in storage is on the order of Mn^2.

Algorithm II is a bank of Kalman-Bucy constant Δ-state filters that use the decomposition of the maneuver signature matrix. There is a filter for each candidate jump time in the past. As a result, the required computations for jump detection and estimation are performed at each observation time. In contrast, algorithm I, using minimal computations, computes and stores at the current observation time an information matrix for later processing. The appropriate information matrix for any candidate jump time is obtained simply by taking the difference between the information matrix of the current time and that of the candidate jump time. The required computations for jump detection and estimation are performed in the vacinity of a jump or false alarm. While algorithm I requires more computations at an observat time than algorithm II to detect and estimate jumps it is not necessary that the calculations be performed at each observation time. Consequently, algorithm II is more applicable for systems with frequent jumps and algorithm I more applicable for systems with infrequent jumps or maneuvers.

The GLR algorithms I and II are a computation improvement over existing techniques, algorithms and methods [1,2], [4-6], [8], [9], and [20-38] for adaptively estimating the state of a linear stochastic system undergoing abrupt changes (e.g., maneuvers) in state.

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APPENDIX A

RELATION BETWEEN KALMAN GAINS

The argument of the following variables is k: H, R, V, $\tilde{V}_1$, ΔV, K, K_1 and ΔK. The argument of ΔP is k-1 and the argument of Φ is (k,k-1). The argument of P and P_1 is (k|k-1).

we have from Eq. (B-7)

$$V = \tilde{V}_1 + H \Phi \Delta P \Phi^T H^T \tag{A-1}$$

Postmultiplying (A-1) by V^{-1} gives

$$H\Phi \Delta P \Phi^T H^T V^{-1} = I - \tilde{V}_1 V^{-1} \tag{A-2}$$

Premultiplying (A-2) by K_1 gives

$$K_1 H \Phi \Delta P \Phi^T H^T V^{-1} = K_1 [I - \tilde{V}_1 V^{-1}] \tag{A-3}$$

The matrix ΔK satisfies

$$\Delta K \Delta V = [I - K_1 H] \Phi \Delta P \Phi^T H^T V_1^{-1} R \tag{A-4}$$

Postmultiplying (A-4) by $\Delta V^{-1} R \tilde{V}_1^{-1}$ gives

$$\Delta K R \tilde{V}_1^{-1} = \Phi \Delta P \Phi^T H^T V^{-1} - K_1 H \Phi \Delta P \Phi^T H^T V^{-1} \tag{A-5}$$

Substitution of (A-3) into (A-5) gives, in view of (B-9),

$$\Delta K[I-HK_1] = \Phi\Delta P\Phi^T H^T V^{-1} + K_1\tilde{V}_1 V^{-1} - K_1 \tag{A-6}$$

Since

$$K = PH^T V^{-1} \tag{A-7}$$

and since, from (B-6),

$$PH^T V^{-1} = P_1 H^T V^{-1} + \Phi\Delta P\Phi^T H^T V^{-1} \tag{A-8}$$

it follows that

$$K = K_1\tilde{V}_1 V^{-1} + \Phi\Delta P\Phi^T H^T V^{-1} \tag{A-9}$$

Substituting (A-9) into (A-6) gives

$$K = K_1 + \Delta K[I-HK_1] \tag{A-10}$$

APPENDIX B

RELATION BETWEEN PREDICTED MEASUREMENT RESIDUAL COVARIANCES

The matrices $V(k)$, $\tilde{V}_1(k)$, $P(k|k-1)$ and $P_1(k|k-1)$ are given by

$$V(k) = H(k)\ P(k|k-1)\ H^T(k) + R(k) \tag{B-1}$$

$$\tilde{V}_1(k) = H(k)\ P_1(k|k-1)\ H^T(k) + R(k) \tag{B-2}$$

$$P(k|k-1) = \Phi(k,k-1)\ P(k-1)\Phi^T(k,k-1) + \Gamma(k-1)\ Q(k-1)\Gamma^T(k-1) \tag{B-3}$$

$$P_1(k|k-1) = \Phi(k,k-1)\ P_1(k-1)\Phi^T(k,k-1) + \Gamma(k-1)\ Q(k-1)\ \Gamma^T(k-1) \tag{B-4}$$

Substituting

$$P(k-1) = P_1(k-1) + \Delta P(k-1) \tag{B-5}$$

into (B-3) gives, in view of (B-4),

$$P(k|k-1) = P_1(k|k-1) + \Phi(k,k-1)\ \Delta P(k-1)\ \Phi^T(k,k-1) \tag{B-6}$$

Substituting (B-6) into (B-1) gives, in view of (B-2),

$$V(k) = \tilde{V}_1(k) + H(k)\Phi(k,k-1)\ \Delta P(k-1)\ \Phi^T(k,k-1)\ H^T(k) \tag{B-7}$$

The matrix $\Delta P(k|k-1)$ is given by

$$\Delta P(k|k-1) = [I-K_1(k)H(k)]\Phi(k,k-1)\Delta P(k-1)\Phi^T(k,k-1)[I-K_1(k)H(k)]^T \tag{B-8}$$

Recall the identity

$$R(k)\tilde{V}_1^{-1}(k) = [I-H(k)\ K_1(k)] \tag{B-9}$$

Premultiplying (B-8) by H(k) and postmultiplying that product by $H^T(k)$ gives, in view of (B-9),

$$H(k)\ \Delta P(k|k-1)\ H^T(k) = R(k)\tilde{V}_1^{-1}(k)\ F(k,k-1)\tilde{V}_1(k)\ R(k) \tag{B-10}$$

where

$$F(k,k-1) = H(k)\Phi\ (K,k-1)\ \Delta P(k-1)\ \Phi^T(k,k-1)\ H^T(k) \tag{B-11}$$

The matrix $\Delta V(k)$ is given by

$$\Delta V(k) = H(k)\ \Delta P(k|k-1)\ H^T(k) + R(k)\tilde{V}_1^{-1}(k)\ R(k) \tag{B-12}$$

Using (B-12) to solve for F(k,k-1) in (B-10) gives

$$F(k,k-1) = \tilde{V}_1(k)R^{-1}(k)\ \Delta V(k)\ R^{-1}(k)\ \tilde{V}_1(k) - \tilde{V}_1(k) \tag{B-13}$$

Equating (B-11) and (B-13) and substituting the result into (B-7) gives

$$V(k) = \tilde{V}_1(k)\ R^{-1}(k)\ \Delta V(k)\ R^{-1}(k)\ \tilde{V}_1(k) \tag{B-14}$$

APPENDIX C

RELATION BETWEEN STATE ESTIMATES

We desire to show that the state estimates satisfy

$$\hat{x}(k) = \tilde{x}_1(k) + \Delta\hat{x}(k) \tag{C-1}$$

We assume that

$$\hat{x}(k-1) = \tilde{x}_1(k-1) + \Delta\hat{x}(k-1) \tag{C-2}$$

The individual estimates satisfy the following equations

$$\hat{x}(k) = [I-K(k)H(k)]\,\Phi(k,k-1)\,\hat{x}(k-1) = K(k)z(k) \tag{C-3}$$

$$\tilde{x}_1(k) = [I-K_1(k)H(k)]\,\Phi(k,k-1)\,\tilde{x}_1(k-1) + K_1(k)z(k) \tag{C-4}$$

$$\Delta\hat{x}(k) = [I-\Delta K(k)H(k)]\Delta\Phi(k,k-1)\,\Delta\hat{x}(k-1) + \Delta K(k) \\ [z(k) - H(k)\,\tilde{x}_1(k)] \tag{C-5}$$

Using (C-2) - (C-5) it follows that (C-1) is satisfied provided

$$K(k) = K_1(k) + \Delta K(k)\,[I-H(k)\,K_1(k)] \tag{C-6}$$

$$[I-K(k)H(k)]\,\Phi(k,k-1) = [I-\Delta K(k)H(k)]\,\Delta\Phi(k,k-1) \tag{C-7}$$

Substituting the definition of $\Delta\Phi(k,k-1)$ into (C-7) results in an equation which holds provided (C-6) holds. The validity of (C-6) follows from Appendix A.

APPENDIX D

RELATION BETWEEN COVARIANCES

We wish to show that

$$P(k) = P_1(k) + \Delta P(k) \tag{D-1}$$

given that

$$P(k-1) = P_1(k-1) + \Delta P(k-1) \tag{D-2}$$

The argument of the matrices H, K, K_1, ΔK and ΔR is k. The argument of Φ and $\Delta\Phi$ is (k,k-1).

The covariance P(k) satisfies

$$P(k) = [I-KH]\ P(k|k-1) \tag{D-3}$$

From Eq. (C-7) we have

$$[I-KH] = [I-\Delta KH]\ [I-K_1H] \tag{D-4}$$

From Eq. (B-6) we have

$$P(k|k-1) = P_1(k|k-1) + \Phi\Delta P(k-1)\Phi^T \tag{D-5}$$

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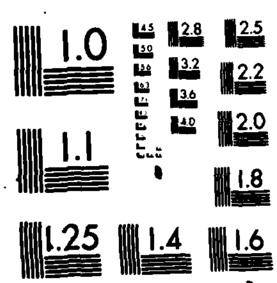

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The covariance $P_1(k)$ satisfies

$$P_1(k) = [I-K_1H]\ P_1(k|k-1) \tag{D-6}$$

Substitution of (D-4) - (D-6) into (D-3) and making use of the definition

$$\Delta\Phi = [I-K_1H]\ \Phi \tag{D-7}$$

results in

$$P(k) = [I - \Delta KH]\ [P_1(k) + \Delta\Phi\Delta P(k-1)\Phi^T] \tag{D-8}$$

or, equivalently,

$$P(k) = P_1(k) - \Delta KHP_1(k) + [I-\Delta KH]\ \Delta\Phi\Delta P(k-1)\Phi^T \tag{D-9}$$

The covariance $\Delta P(k)$ satisfies

$$\Delta P(k) = [I-\Delta KH]\ \Delta\Phi\Delta P(k-1)\Delta\Phi^T \tag{D-10}$$

Making use of (D-7), Eq. (D-10) can be written as

$$[I-\Delta KH]\ \Delta\Phi\Delta P(k-1)\Phi^T = \Delta P(k) + \Delta P(k)H^TR^{-1}\tilde{V}_1K_1^T \tag{D-11}$$

since

$$\tilde{V}_1^{-1} R = [I-HK_1]^T \tag{D-12}$$

It follows from (D-12) and

$$H P_1(k) = R K_1^T \tag{D-13}$$

That

$$\Delta KHP_1(k) = \Delta K \; \Delta R \; R^{-1}\tilde{V}_1 K_1^T \tag{D-14}$$

where

$$\Delta R = R \tilde{V}_1^{-1} R \tag{D-15}$$

Substitution of (D-11) and (D-14) into (D-9) gives

$$P(k) = P_1(k) + \Delta P(k) + [\Delta P(k) H^T - \Delta K \Delta R] R^{-1} \tilde{V}_1 K_1^T \tag{D-16}$$

Eq. (D-1) now follows since

$$\Delta P(k) H^T = \Delta K \; \Delta R \tag{D-17}$$

APPENDIX E

CHANG AND DUNN'S SYSTEM FORMULATION: DISCUSSION OF IMPULSE INPUT VERSION

The following system step input case is considered in [8] and [9]:

$$X(k+1) = A(k+1,k)\ X(k) + B(k+1,k)u(k,q) + \Gamma(k)w(k) \qquad \text{(E-1)}$$

$$u(k+1,q) = F(k+1,k)\ u(k,q) \qquad \text{(E-2)}$$

$$u(k,q) = 0 \text{ for } k<q \qquad \text{(E-3)}$$

$$u(q,q) \neq 0 \qquad \text{(E-4)}$$

where F is the state transition matrix for the control variable u and the other variables are as defined in Eqs. (1) and (3) of Section 1. The unknowns to be estimated and detected are u(k,q) and q.

We note two differences between the dynamics defined by (1) and (2) and that defined by (E-1) - (E-4). First, the matrix F is taken as the identity matrix in Eq. (2). Secondly, the value of u is zero before the jump in the above formulation but may be nonzero in Eq. (1).

Eqs. (E-1) - (E-4) and Eq. (3) is the "step input to the system" case. It is described in [3] as the "dynamic step" case in which the input matrix B and the matrix F are identity matrices.

The step input case can be transformed into the "impulse input to the system" case by augmenting the state vector X with the control vector u. Let x denote the augmented state vector. The augmented system's dynamics are given by

$$x(k+1) = \phi(k+1,k)\ [x(k) + \Delta x_q\ \delta_{qk}] + \Gamma(k)\ w(k) \qquad \text{(E-5)}$$

where

$$\Phi\ (k+1,k) = \begin{bmatrix} A(k+1,k) & B(k+1,k) \\ 0 & F(k+1,k) \end{bmatrix} \qquad \text{(E-6)}$$

$$\Delta x_q = \begin{bmatrix} 0 \\ I \end{bmatrix}\ u(q,q) \qquad \text{(E-7)}$$

and where $\Gamma(k)$ is redefined as the augmented matrix as in Eq. (10).

In this report we are interested in the general formulation of (E-5) in which ϕ, Δx_q and Γ are arbitrary, not necessarily satisfying Eqs. (E-6), (E-7) and (10). We desire to apply Chang and Dunn's GLR algorithm in our context. Their algorithm consists of a bank of Kalman-Bucy filters to estimate the jump Δx_q. The filters are defined by Eqs. (123) - (127), (132) - (138) and (142) - (143) in Section 6. The estimate $\Delta \hat{v}_q(k)$ represents the optimal estimate of Δx_q given the measurements up to time k. Consequently, Chang and Dunn's GLR algorithm as made use of in this report is the impulse input version of their technique as described in [8] and [9] for the "step input" case.

For a particular problem in the form of (E-1) - (E-4) it is more efficient to use the "step input" case filters as described in [8] and [9] than it is to use the filters of the augmented "impulse input" case. In this report, however, we are interested in the "impulse input" case when it is not necessarily reducible to the "step input" case.

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