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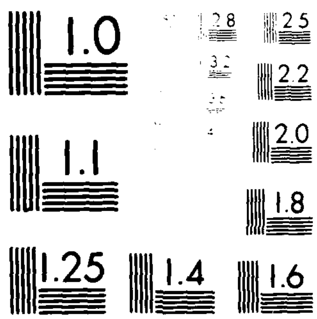

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LEARNING AND COSTS IN AIRFRAME PRODUCTION PART I*

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1.0 Introduction

In recent years, there has been much interest in exploring the impact of learning and changes in production rate on program costs. Most researchers agree that learning is an important determinant of cost, but agreement on the cost impact of production rate changes has been less certain. Still, common sense and economic theory suggest that production rate should be an important determinant of cost. This importance is also suggested by the fact that cost penalties for production rate changes now occur in some department of defense contracts (3).

This paper does not present a theoretical justification for the integration of learning curves with traditional neoclassical economic theory. The general theoretical framework for this paper is published in previous research (14, 15). The purpose of this paper is to extend the range of applicability of the general framework by considering a previously unexplored specification. In particular, this paper explores the joint production situation, where learning and output are simultaneously produced, and a model is presented that has potential application in the airframe industry. The theoretical properties of the model are explored, and a cost minimizing solution is presented. Finally, a strategy is proposed for adapting the model to a particular airframe program.

2.0 Background

The production characteristics of the airframe industry place unusual demands on cost estimators. The situation is characterized by a small number of units produced, frequent design changes, and considerable political uncertainty. Both learning curves and neoclassical cost functions have been used to model this unique production situation, but it has only been in recent years that the two approaches have been combined. Much of the early work that acknolwedged a need to combine the two modeling situations was lacking in terms of theoretical rigor (1, 4, 5). Also, many early application oriented studies were empirical in lieu of being firmly grounded in economic theory (2, 6). Most recently, significant advances have been made in modeling the made to order production situation. The first dynamic optimization models were presented by Rosen (7) and Washburn (9), but neither of these models was definitive enough for empirical application. Recently a more definitive model has appeared (14), and the result has been a series of applications to airframe production programs (10, 12).

The basic modeling framework was developed for a firm producing to an order which specifies a quantity and a delivery date for output. A neoclassical production function is augmented with a learning hypothesis, and the cost of production is minimized to yield optimal time paths of both production and cost. The model presented in this paper adheres to the same general theoretical framework.

3.0 The Model

Consider the situation where learning and output are produced by two separate production technologies. The time path of output, or production rate, is presented as a function of a single variable composite resource and the cumulative stock of knowledge. Also, the time path of experience rate, a decision variable, is presented as a function of the same composite resources and the cumulative stock of knowledge. The assumption is that the relative prices of the resources in the composite resource do not change, and cost is measured in the units of the variable resource. The variables of the model are defined below:

$q(t)$ = output rate of the program at time t,

$\ell(t)$ = rate of experience at time t.

$Q(t) = \int_0^t q(\tau)d\tau$ = cumulative output at time t,

$L(t) = \int_0^t \ell(\tau)d\tau$ = cumulative stock of knowledge at time t,

γ = a returns to scale parameter,

β = a returns to scale parameter,

α = a learning parameter,

δ = a learning parameter,

C = variable program cost,

T = time horizon for the production program,

V = volume of output to be produced by T,

M = initial stock of knowledge,

a_1 = constant term,

a_2 = constant term.

One possible specification is two Cobb-Douglas production functions, that is,

$$q(t) = a_1 x_q^{1/\gamma}(t) L^{\alpha}(t), \tag{1}$$

and

$$\ell(t) = a_2 x_\ell^{1/\beta}(t) L^{\delta}(t). \tag{2}$$

With this specification the use rate of the composite resource is segregated into two parts, that allocated to output $x_q(t)$ and that allocated to learning $X_\ell(t)$. These inputs, combined with the cumulative stock of knowledge $L(t)$, are used to produce two products, output $q(t)$ and learning $\ell(t)$. The following assumptions define the admissible ranges for the parameters: $0 \leq \alpha \leq$, $0 \leq \delta \leq$, $\gamma \geq 1$, and $\beta \geq 1$.

The objective of the firm is to minimize its cost of production subject to the production function constraints. This may be stated as

$$\text{Min } C = \int_0^T [x_q(t) + x_\ell(t)]dt \tag{3}$$

subject to:

$$q(t) = a_1 x_q^{1/\gamma}(t)L^\alpha(t), \tag{4}$$

$$\ell(t) = a_2 x_\ell^{1/\beta}(t)L^\delta(t), \tag{5}$$

$$Q(0) = 0, \tag{6}$$

$$Q(T) = V, \tag{7}$$

$$L(0) = M, \tag{8}$$

and

$$L(T) = \text{free}. \tag{9}$$

The solution of (4) and (5) for $x_q(t)$ and $x_\ell(t)$ yields the following resource requirement functions:

$$x_q(t) = q^\gamma(t)a_1^{-\gamma}L^{-\alpha\gamma}(t). \tag{10}$$

and

$$x_\ell(t) = \ell^\beta(t)a_2^{-\beta}L^{-\delta\beta}(t). \tag{11}$$

Substituting the resource requirement functions into the objective functional eliminates the production function constraints. The objective functional is now stated as

$$\text{Min } C = \int_0^T [q^\gamma(t)a_1^{-\gamma}L^{-\alpha\gamma}(t) + \ell^\beta(t)a_2^{-\beta}L^{-\delta\beta}(t)]dt. \tag{12}$$

A transformation of the objective functional simplifies the solution procedure. Let

$$Z(t) = L^{1-\delta}(t)/(1-\delta). \tag{13}$$

This implies that

$$L(t) = Z^{1/(1-\delta)}(t)(1-\delta)^{1/(1-\delta)}, \tag{14}$$

and

$$z(t) = dZ/dt = L^{-\delta}(t)\ell(t). \tag{15}$$

After making the appropriate substitutions the tranformed problem is

$$\text{Min } C = \int_0^T [q^{\gamma}(t)a_1^{-\gamma}Z^{-\alpha\gamma/(1-\delta)}(1-\delta)^{-\alpha\gamma/(1-\delta)} + z^{\beta}(t)a_2^{-\beta}]dt \tag{16}$$

subject to:

$$Q(0) = 0, \tag{17}$$

$$Q(T) = V, \tag{18}$$

$$Z(0) = M^{1-\delta}/(1-\delta), \tag{19}$$

and

$$Z(T) = \text{free}. \tag{20}$$

An equivalent way to present the above problem is as a problem in optimal control theory. The objective is stated as

$$\text{Min } C = \int_0^T [u_1^{\gamma}(t)a_1^{-\gamma}Z^{-\alpha\gamma/(1-\delta)}(1-\delta)^{-\alpha\gamma/(1-\delta)} + u_2^{\beta}(t)a2^{-\beta}]dt \tag{21}$$

subject to:

$$q(t) = u_1(t), \tag{22}$$

$$z(t) = u_2(t), \tag{23}$$

$$Q(0) = 0, \tag{24}$$

$$Q(T) = V, \tag{25}$$

$$Z(0) = M^{1-\delta}/(1-\delta), \tag{26}$$

and

$$Z(T) = \text{free}. \tag{27}$$

The control variables for the problem, $u_1(t)$ and $u_2(t)$, are the time rates of change of the state variables, i.e., $u_1(t) = q(t)$ and $u_2(t) = z(t)$. The Hamiltonian function for the problem is

$$H = u_1^{\gamma}(t)a_1^{-\gamma}Z^{-\alpha\gamma/(1-\delta)}(t)(1-\delta)^{-\alpha\gamma/(1-\delta)} +$$
$$u_q^{\beta}(t)a_2^{-\beta}+\lambda_1(t)u_1(t)+\lambda_2(t)u_2(t). \tag{28}$$

The necessary conditions for defining an extremal require that the equations of motion, the adjoint conditions, and the Hamiltonian conditions hold simultaneously. The equations of motion are

$$\partial H/\partial\lambda_1 = q(t) = u_1(t), \tag{29}$$

$$\partial H/\partial\lambda_2 = z(t) = u_2(t). \tag{30}$$

The adjoint conditions are

$$d\lambda_1/dt = -\partial H/\partial Q = 0, \tag{31}$$

$$d\lambda_2/dt = -\partial H/\partial Z = [\alpha\gamma/(1-\delta)]u_1^{\gamma}(t)a_1^{-\gamma} Z^{(\delta-\alpha\gamma-1)}(t)(1-\delta)^{-\alpha\gamma/(1-\delta)}. \tag{32}$$

The Hamiltonian conditions are

$$\partial H/\partial u_1 = \gamma u_1^{\gamma-1}(t)a_1^{-\gamma}Z^{-\alpha\gamma/(1-\delta)}(t) (1-\delta)^{-\alpha\gamma/(1-\delta)} + \lambda_1(t) = 0, \tag{33}$$

$$\partial H/\partial u_2 = \beta u_2^{\beta-1}(t)a_2^{-\beta} + \lambda_2(t) = 0. \tag{34}$$

The simultaneous solution of these conditions requires solving two second order nonlinear differential equations. This implies that there are four constants to be determined by the boundary conditions. Three of the constants are determined by the given boundary conditions, and the fourth is given by the natural boundary condition, that is, since $Z(T)$ is free, the condition $z(T)=0$ determines the fourth constant.

The differential equations that follow from the necessary conditions are the Euler-Lagrange equations of the calculus of variations, i.e.,

$$\beta(\beta-1)z^{\beta-2}(t)a_2^{-\beta}(d^2Z/dt^2) + [\alpha\gamma/(1-\delta)]q^{\gamma}(t) a_1^{-\gamma}Z^{(\delta-\alpha\gamma-1)/(1-\delta)}(t)(1-\delta)^{-\alpha\gamma/(1-\delta)}=0, \tag{35}$$

and

$$-\gamma q^{\gamma-1}(t)a_1^{-\gamma}Z^{-\alpha\gamma/(1-\delta)}(t)\ (1-\delta)^{-\alpha\gamma/(1-\delta)} = k_1', \qquad (360$$

where k_1' is a constant of integration.

The simultaneous solution of the Euler-Lagrange equations yields the optimal time paths of both production and experience rates. The solution proceeds as follows. Solve (36) for $q(t)$ and state the result in compact notation as

$$q(t) = k_1 Z^{\eta}(t) \tag{37}$$

where $\eta=\alpha\gamma/[(1-\delta)(\gamma-1)]$. This expression for $q(t)$ may be substituted into (35), and the number of equations is reduced by one. The single necessary condition is stated as

$$d^2Z/dt^2 = \beta^{-1}(\beta-1)^{-1}z^{2-\beta}(t)a_2{}^{\beta}[\alpha\gamma/(1-\delta)] \\ Z^{(1+\alpha\gamma+\delta\gamma-\gamma-\delta)/[(\gamma-1)(1-\delta)]}(t)k_1{}^{\gamma}(1-\delta)^{\eta}a_1{}^{-\gamma}. \tag{38}$$

This is a second order nonlinear differential equation which may be stated in compact notation as

$$d^2Z/dt^2 = Az^{2-\beta}(t)Z^{\eta-1}(t) \tag{39}$$

where the constant term is

$$A = -\beta^{-1}(\beta-1)^{-1}a_2{}^{\beta}[\alpha\gamma/(1-\delta)]k_1{}^{\gamma}(1-\delta)^{-\alpha\gamma/(1-\delta)}a_1{}^{-\gamma} \quad (40$$

and

$$\eta-1 = (1+\alpha\gamma+\delta\gamma-\gamma-\delta)/[(\gamma-1)(1-\delta)]. \tag{41}$$

After the series of transformations to achieve a reduction in order, the following differential equation is obtained:

$$dZ/dt = [A\beta Z^{\eta}(t)/\eta + k_2]^{1/\beta}. \tag{42}$$

This function may be inverted, leaving t as the following function of Z(t):

$$t = M^{1-\delta}/(1-\delta) \int^{Z(t)} [A\beta Z^{\eta}/\eta + k_2]^{-1/\beta} dZ + k_3 \tag{43}$$

An expression is needed that gives Z as a function of t. This allows you to apply the boundary conditions on Z(t), determine the constants, and define one of the extremals. Since (37) relates Z(t) to q(t), you should be able to apply the boundary conditions on Q(t) and define the second extremal. Unfortunately this procedure is easily stated but not easily executed.

The next step in the solution procedure is just a restatement of (43) in a slightly different form. After some algebraic manipulation (43) may be written as

$$t = k_2{}^{-1/\beta}M^{1-\delta}/(1-\delta) \int^{Z(t)} [1+A\beta Z^{\eta}/\eta k_2]^{-1/\beta} dZ + k_3. \tag{44}$$

The value of k_2 is determined by returning to (40), which allows you to apply the natural boundary condition $z(t) = 0$. The resulting expression for k_2 is

$$k_2 = -A\beta Z^{\eta}(T)/\eta, \tag{45}$$

and (44) may be written as

$$t = \int_{M^{1-\delta}/(1-\delta)}^{Z(t)} [-A\beta Z^{\eta}(T)/\eta]^{-1/\beta} [1-Z^{-\eta}(T)Z^{\eta}]^{-1/\beta}\, dZ + k_3. \tag{46}$$

This integral appears uninviting, but a change of variables leads to a solution. Let

$$y = Z^{-\eta}(T)Z^{\eta}. \tag{47}$$

The integral may now be restated as

$$t = [-A\beta Z^{\eta}(T)/\eta]^{-1/\beta} Z(T)\eta^{-1} \int_{Z^{-\eta}(T)[M^{1-\delta}/(1-\delta)]^{\eta}}^{Z^{-\eta}(T)Z^{\eta}(t)} (1-y)^{(1-1/\beta)-1} y^{1/\eta-1}\, dy + k_3 \tag{48}$$

which is a form of the incomplete beta function.

The solution is complete except for the determination of the constants of integration. At this point the value of k_3 is unknown, and there is also an unknown constant, k_1, in the A term. To determine these constants, first notice that (26) implies that $k_3=0$. The determination of the second constant requires a little more effort. The strategy is to find an expression for $[-A\beta Z^{\eta}(T)/\eta]$ that is in terms of known constants. The solution is as follows. Equation (42) may be written as

$$dt = [-A\beta Z^{-\eta}(T)/\eta]^{-1/\beta}\, [1-Z^{-\eta}(T)Z^{\eta}(t)]^{-1/\beta} dZ. \tag{49}$$

It also follows from the Euler-Lagrange equations that

$$Q(\tau) = k_1 \int_0^t Z(\tau)d\tau. \tag{50}$$

After evaluating (49) at τ and changing variables in (50), it is possible to write Q as a function of Z, i.e.,

$$Q(Z) = k_1[-A\beta Z^{\eta}(T)/\eta]^{-1/\beta} \int_{M^{1-\delta}/(1-\delta)}^{Z(t)} Z^{\eta}[1-Z^{-\eta}(T)Z^{\eta}]^{-1/\beta} dZ. \tag{51}$$

This integral results in an expression that is suitable for applying the boundary conditions on Q(t). In other words, (50) is transformed into an expression that is integrated with respect to Z in lieu of τ. Continuing with the solution, let

$$R = k_1[-A\beta Z^{\eta}(T)/\eta]^{-1/\beta}, \tag{52}$$

and

$$y = Z^{-\eta}(T)Z^{\eta}. \tag{53}$$

Equation (51) may now be stated as a form of the incomplete Beta function. The appropriate integral is

$$Q(y) = RZ^{\eta+1}(T)\eta^{-1} \int_{Z^{-\eta}(T)[M^{1-\delta}/(1-\delta)]^{\eta}}^{Z^{-\eta}(T)Z^{\eta}(t)} y^{(1/\eta+1)-1}(1-y)^{(1-1/\beta)-1}dy + k_4 \qquad (54)$$

It is now possible to apply the boundary conditions on Q. The initial condition, Q(0), = 0, implies that $k_4 = 0$. The final condition, Q(T) = V, implies

$$R = VZ^{-1-\eta}(T)\eta \left\{ \int_{Z^{\eta}(T)[M^{1-\delta}/(1-\delta)]^{\eta}}^{1} y^{(1/\eta+1)-1}(1-y)^{(1-1/\beta)-1}dy\right\}^{-1}$$

which is another form of the incomplete Beta function.

Since all of the integration constants are known, it is possible to write an expression that links optimal Z with τ. The expression is

$$\tau = c_1 \int_{Z^{-\eta}(T)[M^{1-\delta}/(1-\delta)]^{\eta}}^{Z^{-\eta}(T)Z^{\eta}(t)} y^{1/\eta-1}(1-y)^{(1-1/\beta)-1}dy \qquad (56)$$

where c_1 is comprised completely of constants. By using the inverse incomplete Beta function in (56), it is possible to determine Z(t) for any t. Optimal Z(t) determines optimal L(t) via (13), and optimal Z(t) determines optimal Q(t) via (37). The theoretical solution is now complete, but unfortunately the model is not in a form that is suitable for application to most airframe programs.

4.0 Strategy for Application

The theoretical model requires some adjustment before it is in the proper form for application on most airframe programs. The problem is that the model is not stated in terms of variables that are observable or even measurable. There is no convenient way to measure the current stock of knowledge, and there is even some doubt about the proper way to measure production rate (8). However, there are two quantities that are usually reported with a degree of regularity. Direct labor hours, either by airframe or unit time, is usually available, and there is usually some information on delivery schedules and perhaps lot sizes. The latter information is useful for assigning cost to time periods to develop a data series suitable to estimate the model parameters. The following procedure provides a method for transforming the model so that it may be used in applications where the only available data is cost per unit time.

The objective to rewrite the optimal inverse function (48) so that t is a function of x, a quantity that is observable in the data. The total resource requirement function is the sum of the individual resource requirement functions, i.e.,

$$x(t) = x_q(t) + x_\ell(t) \tag{57}$$

After substituting (10) and (11) into (57) and using (15), the combined resource requirement function may be rewritten as

$$x(t) = q^\gamma(t) a_1^{-\gamma} Z^{-\alpha\gamma/(1-\delta)}(t) + z^\beta(t) a_2^{-\alpha}. \tag{58}$$

The strategy is to eliminate $q(t)$ and $z(t)$ from the above expression. This leaves an expression which may be solved for $Z(t)$ as a function of $x(t)$.

The following procedure is used to eliminate $z(t)$. Equation (42) implies that

$$z^\beta(t) = A\beta Z^\eta(t)/\eta + k_2. \tag{59}$$

If this result is substituted into the resource requirement function, $z(t)$ is eliminated. The resources required may now be written as

$$x(t) = q^\gamma(t) a_1^{-\gamma} Z^{-\alpha\gamma/(1-\delta)}(t) + a_2^{-\beta} A\beta Z^\eta(t)/\eta + k_2 a_2^{-\beta}. \tag{60}$$

To eliminate $q(t)$, use the Euler-Lagrange equation (37). Solve the Euler-Lagrange equation for $q(t)$, and substitute into (60) to obtain the desired result. After solving for $Z(t)$, the optimal expression is

$$Z(t) = (D+E)^{-1/\eta}[x(t) - k_2 a_2^{-\beta}]^{1/\eta}. \tag{61}$$

where D and E are functions of known constants. If (61) is substituted as the upper limit of integration in the optimal inverse function (48), the result is an expression that gives t as a function of $x(t)$ at any point in time, i.e.,

$$\begin{aligned} t = {} & Z^{(-\eta+\beta-\gamma)/(\beta-\gamma)}(T)\,\eta^{(1-\beta+\gamma)/(\beta-\gamma)} \\ & R^{-\gamma/(\beta-\gamma)}(\beta-1)^{1/(\beta-\gamma)} a_2^{-\beta/(\beta-\gamma)} a_1^{\gamma/(\beta-\gamma)} \\ & [\alpha\gamma/(1-\delta)]^{-1/(\beta-\gamma)}(1-\delta)^{\eta(\gamma-1)/(\beta-\gamma)} \\ & \int_{Z^{-\eta}(T)[M^{1-\delta}/(1-\delta)]^\eta}^{Z^{-\eta}(T)(D+E)^{-1}[x(t)-k_2 a_2^{-\beta}]} (1-y)^{(1-1/\beta)-1} y^{1/\eta-1}\,dy. \end{aligned} \tag{62}$$

Although the transformation is complete, the function is still not appropriate for estimation since the quantity that is observable is not $x(t)$, but cumulative $x(t)$ over some interval, i.e., $C = \int_{t_0}^{t_1} x(t)\,dt$. As a final adjustment, (61) is solved for x(t) and integrated over an interval of time. The required integration is

$$C=X(t_1)-X(t_0) = \int_{t_0}^{t_1}[(D+E)Z^{\eta}(t) + k_2a_2^{-\beta}]dt. \tag{63}$$

If the Euler-Lagrange equation (37), is used to eliminate Z(t) from (63), the integral becomes

$$C = \int_{t_0}^{t_1}[(D+E)k_1^{-1}q(t) + k_2a_2^{-\beta}]dt. \tag{64}$$

After performing the integration and applying the boundary condition on Q(t), the final result is

$$C = (D+E)k_1^{-1}[Q(t_1)-Q(t_0)]+k_2a_2^{-\beta}(t_1-t_0). \tag{65}$$

This result is the basic estimable relationship of this paper. Since Q(t) is known for any value of Z(t) by equation (51), and Z(t) is known for any value of t by equation (48), X(t) is known for any value of t.

Previous experience with estimating parameters in this type of model suggests that in many cases, the model is over parameterized (12). Our future papers will explore the economic implications of a simpler reparameterized version of (65). The specification is

$$C = C_1[Q(t_1)-Q(t_0)] + c_2(t_1-t_0) \tag{66}$$

The complex expressions for $Q(t_i)$ provide the necessary functions for estimating the model parameters.

Simulations with (66) indicate that this model generates total cost curves whose shape are very similar to those encountered in practice. Figure 1 shows various simulated cost curves using apriori parameter estimates and the volume/delivery schedule values (V and T) from the F102 airframe program. Notice that the model is very responsive to changes in the returns to knowledge parameter β. Also, Figure 2 gives simulations of the cumulative stock of knowledge using these same values of β. This curve is extremely hypothetical since arbitrary values were assigned to the terminal stock of knowledge. Our current research includes the estimation of the parameters in this model using nonlinear least squares.

5.0 Conclusion

This paper examines a theoretical model of the "made-to-order" production situation. The model is unique in that it examines the case where output and knowledge are produced by two separate production technologies. The solution for the model yields an optimal estimable relationships which has potential for application in the airframe industry. The usefulness of the model will be tested in a later paper where the model is applied to the F102 airframe program.

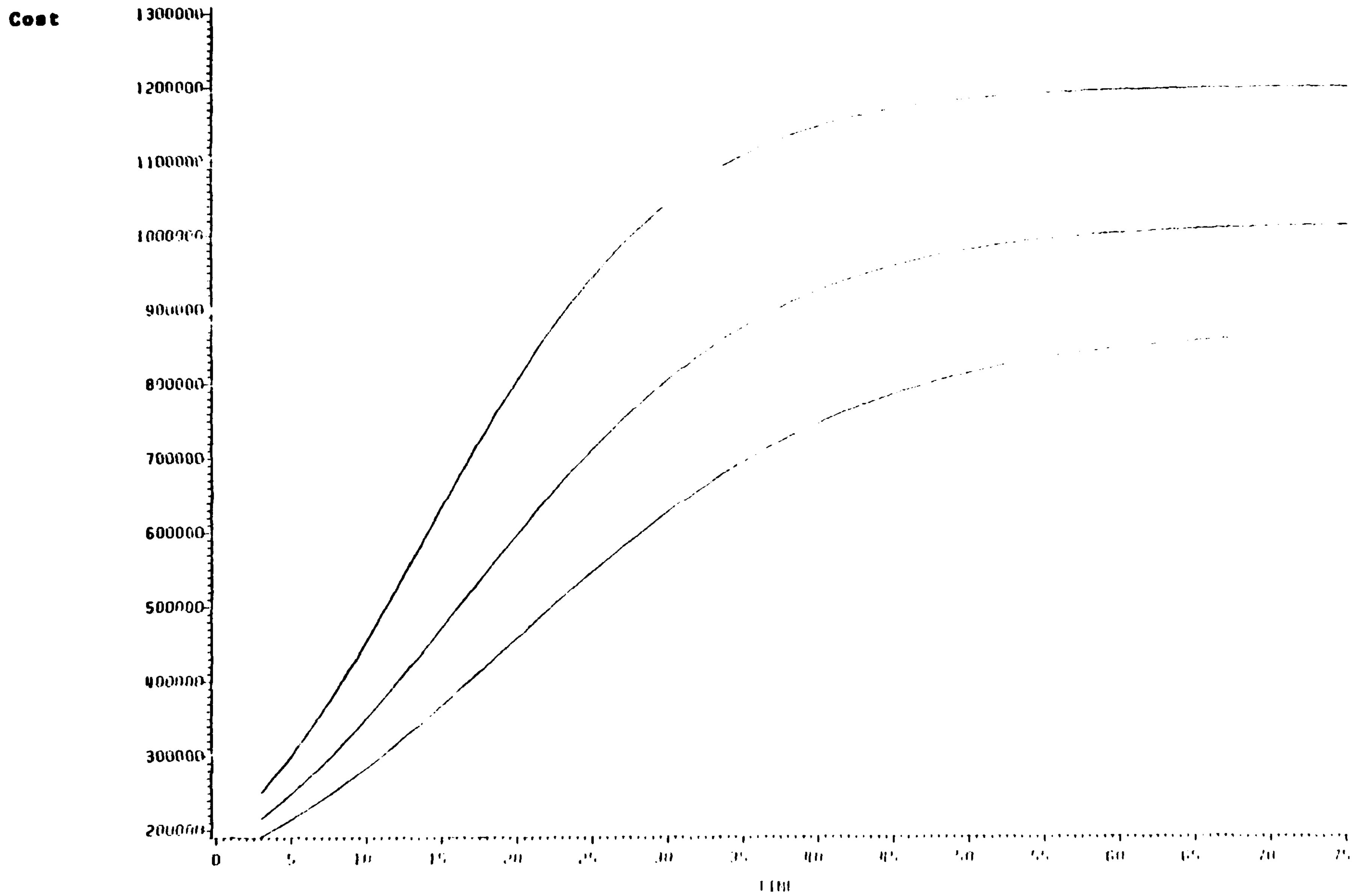

Figure 1. - Cumulative Program Cost

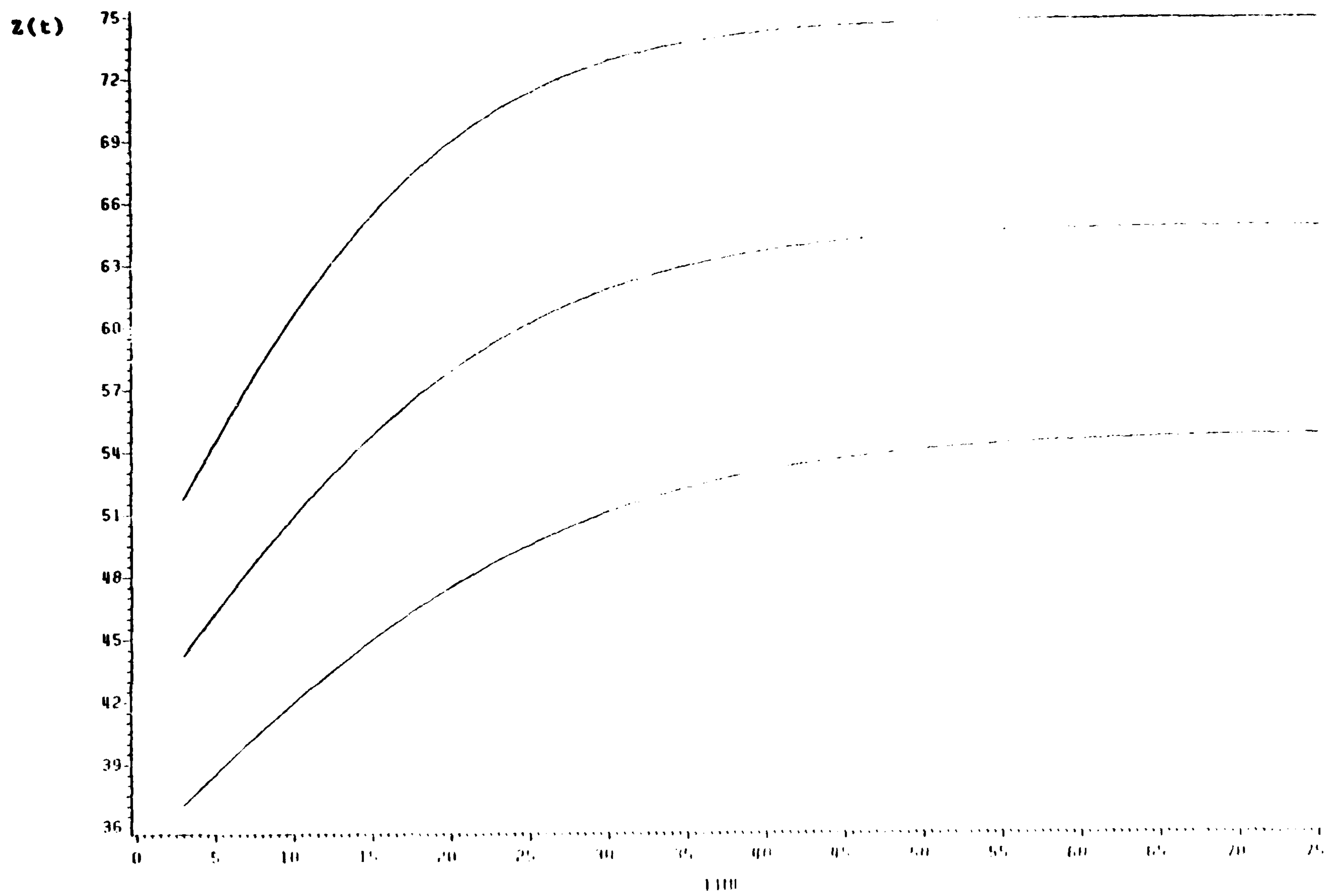

Figure 2. - Cumulative Stock of Knowledge

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20. (continued)

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