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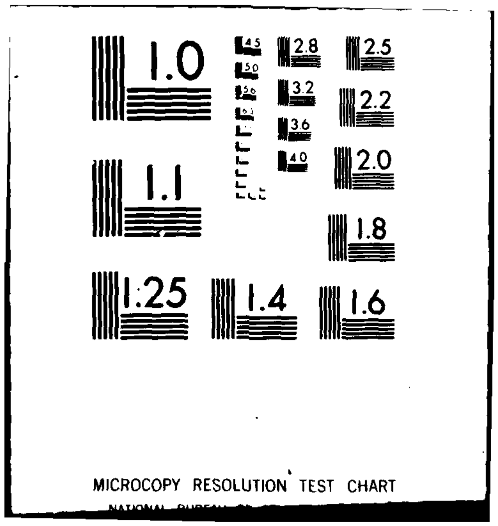
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A FRIEDLAND-LIKE FILTERING TECHNIQUE FOR ESTIMATING PIECEWISE CONSTANT CONTROLS IN DISCRETE LINEAR STOCHASTIC SYSTEMS

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A FRIEDLAND-LIKE FILTERING TECHNIQUE FOR ESTIMATING PIECEWISE CONSTANT CONTROLS IN DISCRETE LINEAR STOCHASTIC SYSTEMS*

BY

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ABSTRACT

We consider discrete linear stochastic processes in which the control variables take unknown jumps at unknown times and remain constant between jumps. We develop a Friedland-like filtering technique for obtaining the optimum estimates $\hat{x}$ of the state and $\hat{u}$ of the control:

$$\hat{x} = \tilde{x} + S_u \tilde{u} + S_x \widetilde{\Delta u}$$

$$\hat{u} = \tilde{u} + S_{\Delta u} \widetilde{\Delta u}$$

where $\tilde{x}$ is the control-free, jump-free estimate of the state x, $\tilde{u}$ is the jump-free estimate of the control u and $\widetilde{\Delta u}$ is the optimum estimate of the jump Δu in control. The matrix S_u is a function of the $\tilde{x}$-filter gain. The matrices S_x and $S_{\Delta u}$ depend on the gains of both the $\tilde{x}$-and the $\tilde{u}$-filters.

A GLR algorithm is presented for detecting [illegible] ump time. It consists of the $\tilde{x}$- and the $\tilde{u}$-filters and a bank of $\widetilde{\Delta u}$-filters. A procedure is developed for reinitializing the $\tilde{x}$- and the

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$\tilde{u}$-filters after a jump has been detected. After reinitialization the optimum estimates $\hat{x}$ and $\hat{u}$ are functions only of $\tilde{x}$ and $\tilde{u}$, satisfying Friedland's expression. The reinitialization procedure provides the tri-filter technique with the capability to handle multiple jumps.

The advent of the tri-filter technique together with the reinitialization procedure makes it unnecessary to augment the state x with the control u and to use augmented state filters in the estimation of state and piecewise constant controls. The new GLR algorithm avoids, therefore, computational problems that may be associated with processing large matrices in augmented state filters. It uses only unaugmented state filters and, consequently, it has particular application to problems involving a large number of state and/or jump variables.

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1. INTRODUCTION

Techniques for the detection and estimation of abrupt changes in discrete linear stochastic systems have a variety of applications. They are applicable to target motion analysis and the tracking of maneuvering targets, [1] - [5]. In aircraft control they are used for the detection of actuator and sensor failures, e.g., [6] and [7]. In electrocardiogram analysis, they are used to detect sudden changes in the rhythm of the heart, [8]. They have been applied to the problem of detecting sudden structure variations in the Italian power system, [9]. A survey of such techniques are given in [10].

The generalized likelihood ratio (GLR) method, [11] and [12], is one of the most attractive and promising approaches with which to develop such techniques. The GLR method provides an optimum decision rule for detecting and estimating abrupt changes (jumps) in stochastic systems. Several techniques have been developed using the GLR approach, [1], [1], [13] - [18]. Willsky and Jones' GLR technique [16] has spurred interest in reducing the computational requirements of the GLR approach and in reducing the computational difficulties associated with large matrices. Chang and Dunn, [17] and [18] have shown that the requirement for matrix inversions (or for solving matrix equations) in GLR techniques such as [16] can be reduced or avoided by using the sequentially updated

Kalman filtering technique described in [19] and [20]. Stalford [1] shows that further reductions in computation and storage are realized by using a decomposition of the maneuver signature matrix (failure signature matrix of [10] and [16]).

Herein, we are interested in discrete linear control (stochastic) processes in which the control vector takes unknown jumps at unknown times and remains constant between jumps (i.e., piecewise constant controls). The extension of the work contained herein, to the case of a time varying control between jumps is dealt with in a future report. Chang and Dunn [18] have treated the time varying bias jump case but their work is based on the underlying assumptions: (1) the jump variable is zero before the jump and (2) there is only one jump. That is, in [18] the vector taking the jump has no influence on the dynamics of the state before the jump; the jump vector jumps from a zero state as far as the dynamics of the system is concerned. They, in essence, treat the case of an unknown time varying bias which appears as a system input at some unknown time. Since we address the case of multiple jumps in control we must necessarily, as a consequence, treat jumps in control that jump from nonzero values.

It is a common practice in engineering to augment the state vector by adding bias terms (such as the control variable considered herein) as additional state variables. We call the resulting system the augmented system and we term the original system the unaugmented system. Augmenting the state vector with the control vector may be undesirable when the augmented state vector is substantially larger in dimension than that of the unaugmented state vector. That is, the additional computations required by augmented state filtering algorithms may become excessive. Also, numerical inaccuracies may be introduced by computations with the larger vectors and matrices of augmented state filters. Friedland [21] and [22] investigated the problem of estimating the state x of a linear process without augmenting to the state a constant but unknown bias vector b. He showed that the optimum estimate $\hat{x}$ of the state could be expressed as

$$\hat{x} = \tilde{x} + S\,\hat{b}$$

where $\tilde{x}$ and $\hat{b}$ are computed using two unaugmented filters. The estimate $\tilde{x}$ is the bias-free estimate, computed as if the bias were zero. The estimate $\hat{b}$ is the optimum estimate of the bias and it is the output of a filter whose state vector is b. The matrix S is a funtion of the bias-free gain matrix. Consequently, Friedland's filtering technique avoids excessive computations and numerical inaccuracies resulting from augmented state filters. Tacker and Lee [23] extended Friedland filtering to the case of estimating a state x in the presence of a time varying bias b.

Chang and Dunn [18] essentially investigated the same problem as that considered by Friedland [21] and Tacker and Lee [23] but with the one difference that the jump time is unknown. That is, the problem considered in [21] and [23] is equivalent to the problem of estimating a jump in the bias vector (from a zero value) when the jump time is known. Chang and Dunn assume the jump time is unknown. They apply the GLR method to estimate the jump time and Friedland's filtering technique to estimate the state x and the time varying bias b. In addition, their GLR algorithm makes use of the sequentially updated Kalman filtering technique for the purpose of minimizing the computations.

Herein, we develop a GLR algorithm for the problem of estimating the state x in the presence of a piecewise constant control variable u with multiple jumps Δu. We show that the optimum estimate $\hat{x}$ is the sum of the outputs of three unaugmented filters:

$$\hat{x} = \tilde{x} + S_u \tilde{u} + S_x \widetilde{\Delta u}$$

where $\tilde{x}$ is the control-free, jump-free estimate (computed as if no control and no jump were presence), $\tilde{u}$ is the jump-free estimate of the control (computed as if the no jump were presence) and $\widetilde{\Delta u}$ is the optimum estimate of the jump. We show that the optimum estimate $\hat{u}$ satisfies the expression

$$\hat{u} = \tilde{u} + S_{\Delta u} \widetilde{\Delta u}$$

The matrix S_u is a function of the Kalman gain used to compute $\tilde{x}$. The matrices S_x and $S_{\Delta u}$ are functions of the Kalman gains used to compute $\tilde{x}$ and $\tilde{u}$. The GLR method is used to detect and estimate the jump time.

Multiple jumps are handled by a reinitialization of the $\tilde{x}$ and $\tilde{u}$ filters after a jump has been detected and estimated. After the reinitialization the optimum estimates satisfy Friedland's expressions [21]:

$$\hat{x} = \tilde{x}' + S_u' \tilde{u}'$$

$$\hat{u} = \tilde{u}'$$

where the prime indicates the quantities after reinitialization.

2. SYSTEM DEFINITION: UNAUGMENTED AND AUGMENTED

Consider the following discrete linear stochastic system with control jump:

System Dynamics

$$x(k+1) = A(k+1,k)\ x(k) + B(k+1,k)(u(k) + \Delta u_q \delta_{qk}) + \Gamma(k)\ w(k) \tag{1}$$

$$u(k+1) = u(k) + \Delta u_q\ \delta_{qk} \tag{2}$$

where x is the state vector, u is the control, Δu_q is the jump in control at time q, δ_{qk} is the Kronecker delta, and Γ is the system noise coefficient matrix. The matrix A is the state transition matrix and B is the input control matrix. The jump time q and the jump magnitude Δu_q are unknowns.

Measurement Equation

$$z(k) = H(k)\ x(k) + \upsilon(k) \tag{3}$$

where z is the measurement vector and H is the measurement matrix.

The noise sequences w and υ are zero-mean, independent, white Gaussian sequences with covariances defined by

$$E\{w(k)\ w^T(j)\} = Q(k)\ \delta_{kj} \tag{4}$$

$$E\{\upsilon(k)\ \upsilon^T(k)\} = R(k)\ \delta_{kj} \tag{5}$$

where $E\{\cdot\}$ denotes the expectation and the matrix $R(k)$ is bounded positive definite. The initial state $x(0)$ is normally distributed with mean $\hat{x}(0)$ and covariance $P_x(0)$. The initial control $u(0)$ is normally distributed with mean $\hat{u}(0)$ and covariance $P_u(0)$. The cross covariance of $x(0)$ and $u(0)$ is denoted by $P_{xu}(0)$. A description of the variables and their dimensions are given in Table 1. We assume that the linear system (1) - (3) is observable.

We define the augmented state vector X as

$$X(k) = \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} \tag{6}$$

The augmented system is given by

$$X(k+1) = \Phi(k+1,k)\ (X(k) + \Delta X_q\ \delta_{qk}) + \Gamma_a(k)w(k) \tag{7}$$

$$z(k) = H_a(k)\ X(k) + \upsilon(k) \tag{8}$$

TABLE 1

SYSTEM VARIABLES FOR DISCRETE LINEAR STOCHASTIC SYSTEM WITH CONTROL JUMP: UNAUGMENTED SYSTEM

VARIABLE	DEFINITION	DIMENSION
$x(k)$	State vector	nx1
$A(k+1,k)$	State transition matrix	nxn
$B(k+1,k)$	Input control matrix	nxp
$u(k)$	Control vector	px1
Δu_q	Jump in control at time q	px1
$\Gamma(k)$	System noise coefficient matrix	nxr
$w(k)$	Gaussian white system noise	rx1
$Q(k)$	System noise covariance matrix	rxr
$z(k)$	Measurement at time k	mx1
$H(k)$	Measurement matrix	mxn
$\upsilon(k)$	Gaussian white measurement noise	mx1
$R(k)$	Measurement noise covariance matrix	mxm
$\hat{x}(0)$	Mean value of $x(0)$	nx1
$\hat{u}(0)$	Mean value of $u(0)$	px1
$P_x(0)$	Covariance of $x(0)$	nxn
$P_{xu}(0)$	Cross covariance of $x(0)$ and $u(0)$	nxp
$P_u(0)$	Covariance of $u(0)$	pxp
$\widehat{\Delta u}(q)$	Mean value of Δu_q	px1
$P_{\Delta u}(q)$	Covariance of Δu_q	pxp

where

$$\Phi(k+1,k) = \begin{bmatrix} A(k+1,k) & B(k+1,k) \\ O & I \end{bmatrix} \tag{9}$$

$$\Delta X_q = D \ \Delta u_q \tag{10}$$

$$D = \begin{bmatrix} O \\ I \end{bmatrix} \tag{11}$$

$$H_a(k) = [H(k) \ \ O] \tag{12}$$

and where $\Gamma_a(k)$ is defined as the augmented matrix

$$\Gamma_a(k) = \begin{bmatrix} \Gamma(k) \\ O \end{bmatrix} \tag{13}$$

The initial mean value and covariance are given by

$$\hat{X}(0) = \begin{bmatrix} \hat{x}(0) \\ \hat{u}(0) \end{bmatrix} \tag{14}$$

$$P(0) = \begin{bmatrix} P_x(0) & P_{xu}(0) \\ P_{xu}^T(0) & P_u(0) \end{bmatrix} \tag{15}$$

A description of the variables and their dimensions for the augmented system are given in Table 2.

TABLE 2

SYSTEM VARIABLES FOR THE AUGMENTED SYSTEM

VARIABLE	DEFINITION	DIMENSION
$X(k)$	State vector (augmented)	(n+p)x1
$\Phi(k+1,k)$	State transition matrix	(n+p)x(n+p)
$\Gamma_a(k)$	System noise coefficient matrix	(n+p)xr
$Q(k)$	System noise covariance matrix	rxr
ΔX_q	Jump in state at time q	(n+p)x1
$z(k)$	Measurement at time k	mx1
$H_a(k)$	Measurement matrix	mx(n+p)
$R(k)$	Measurement noise covariance matrix	mxm
$w(k)$	Gaussian white system noise	rx1
$\upsilon(k)$	Gaussian white measurement noise	mx1
$\hat{X}(0)$	Mean value of X(0)	(n+p)x1
$P(0)$	Covariance of X(0)	(n+p)x(n+p)
D	Jump coefficient matrix	(n+p)xp
Δu_q	Jump in control at time q	px1
$\hat{\Delta u}(q)$	Mean value of Δu_q	px1
$P_{\Delta u}(q)$	Covariance of Δu_q	pxp

We let $Z(j)$ denote the sequence of measurements from time 1 to time j:

$$Z(j) = \{ z(1), z(2), \ldots, z(j) \}$$

Our problem is that of obtaining the optimum estimates of x, u, Δu_q and q without using augmented state filters.

3. EQUIVALENT FILTERING: UNAUGMENTED AND AUGMENTED

In this section we address the nonjump system:

System Dynamics

$$x(k+1) = A(k+1,k)\ x(k) + B(k+1,k)\ u(k) + \Gamma(k)w(k) \tag{16}$$

$$u(k+1) = u(k) \tag{17}$$

Measurement Equation

$$z(k) = H(k)\ x(k) + \upsilon(k) \tag{18}$$

We are given the initial means $\hat{x}(0)$ and $\hat{u}(0)$ and the initial covariances $P_x(0)$, $P_{xu}(0)$ and $P_u(0)$. It is customary to augment the state x with the control vector u (the control u serves as a bias state vector) and then apply the Kalman-Bucy filter to the augmented system. One obtains the optimal estimates $\hat{X}(k)$ and P(k). The Kalman-Bucy filter equations [24] - [28] are given in Table 3 for the augmented system. The filter variables are described in Table 4.

TABLE 3

DISCRETE KALMAN-BUCY FILTER EQUATIONS
FOR AUGMENTED SYSTEM

$$\hat{X}(k+1,k) = \Phi(k+1,k)\ \hat{X}(k) \qquad \text{(i)*}$$

$$P(k+1|k) = \Phi(k+1,k)\ P(k)\ \Phi^T(k+1,k) + \Gamma_a(k)Q(k)\ \Gamma_a{}^T(k) \qquad \text{(ii)}$$

$$\gamma_a(k) = z(k) - H_a(k)\ \hat{X}(k|k-1) \qquad \text{(iii)}$$

$$V_a(k) = H_a(k)\ P(k|k-1)\ H_a{}^T(k) + R(k) \qquad \text{(iv)}$$

$$K_a(k) = P(k|k-1)\ H_a{}^T(k)\ V_a{}^{-1}(k) \qquad \text{(v)}$$

$$\hat{X}(k) = \hat{X}(k|k-1) + K_a(k)\ \gamma_a(k) \qquad \text{(vi)}$$

$$P(k) = [I-K_a(k)\ H_a(k)]\ P(k|k-1) \qquad \text{(vii)}$$

*The usual notations $\hat{X}\ (k|k)$ and $P\ (k|k)$ have been shortened to $\hat{X}(k)$ and $P(k)$.

TABLE 4

FILTER VARIABLES FOR AUGMENTED SYSTEM

VARIABLE	DEFINITION	DIMENSION
$\hat{X}(k)$	State estimate at k given Z(k)	(n+p)x1
P(k)	Covariance matrix of the error in $\hat{X}(k)$	(n+p)x(n+p)
$\hat{X}(k+1\|k)$	State estimate at k+1 given Z(k)	(n+p)x1
$P(k+1\|k)$	Covariance matrix of the error in $\hat{X}(k+1\|k)$	(n+p)x(n+p)
$\gamma_a(k)$	Predicted measurement residual	mx1
$V_a(k)$	Covariance of $\gamma_a(k)$	mxm
$K_a(k)$	Filter (Kalman) gain matrix at k	(n+p)xm

Friedland [21] has shown that it is unnecessary to augment the state vector x by adding additional components (e.g., bias vector such as the control u) in order to obtain the optimal estimates $\hat{x}$ and $\hat{u}$. He showed that the optimum estimates could be obtained by employing two Kalman-Bucy filters: a "bias-free" unaugmented filter and a bias filter. In particular, he showed that the optimum estimate $\hat{x}$ of the state could be expressed as $\hat{x} = \tilde{x} + S\hat{u}$ where $\tilde{x}$ is the output of the "bias-free" unaugmented filter and $\hat{u}$ is the output of the bias filter and where the matrix S depends only on matrices which arise in the computation of $\tilde{x}$. In that work Friedland assumed the condition that there is no a priori correlation between the state x and the bias control vector u, i.e., $P_{xu}(0) = 0$.*

The purpose of this section is to show that Friedland's unaugmented filtering technique [21] holds for the case when there is correlation between the state x and the control vector u, i.e.,

$$P_{xu}(0) \neq 0 \tag{19}$$

We make the following definitions

$$S_0 = P_{xu}(0)\, P_u^{-1}(0) \tag{20}$$

$$\tilde{x}_0 = \hat{x}(0) - S_0\, \hat{u}(0) \tag{21}$$

*Ignagni [29] has rederived Friedland's two-stage estimator in which he assumed at the outset that x and u are initially correlated by means of a given form. We show here that it is unnecessary to assume the given form.

$$\tilde{P}_x(0) = P_x(0) - S_0 \, P_u(0) \, S_0^T \tag{22}$$

$$\tilde{u}_0 = \hat{u}(0) \tag{23}$$

$$\tilde{P}_u(0) = P_u(0) \tag{24}$$

In view of (20) - (24) the initial values $\hat{x}(0)$, $\hat{u}(0)$ and $P(0)$ have the form

$$\hat{x}(0) = \tilde{x}_0 + S_0 \, \tilde{u}_0 \tag{25}$$

$$\hat{u}(0) = \tilde{u}_0 \tag{26}$$

$$P(0) = \begin{bmatrix} \tilde{P}_x(0) + S_0\tilde{P}_u(0)S_0^T & S_0\tilde{P}_u(0) \\ \tilde{P}_u(0)S_0^T & \tilde{P}_u(0) \end{bmatrix} \tag{27}$$

We call the above form the Friedland form. Friedland [21] showed under the condition

$$P_{xu}(0) = 0 \tag{28}$$

that the optimal estimates $\hat{x}(k)$, $\hat{u}(k)$ and $P(k)$ of the augmented system satisfy

$$\hat{x}(k) = \tilde{x}(k) + S(k) \, \tilde{u}(k) \tag{29}$$

$$\hat{u}(k) = \tilde{u}(k) \tag{30}$$

$$P(k) = \begin{bmatrix} \tilde{P}_x(k) + S(k)\,\tilde{P}_u(k)\,S^T(k) & S(k)\,\tilde{P}_u(k) \\ \tilde{P}_u(k)\,S^T(k) & \tilde{P}_u(k) \end{bmatrix} \tag{31}$$

where $\tilde{x}(k)$, $\tilde{P}_x(k)$ and $S(k)$ are output of a "bias-free" unaugmented filter and where $\tilde{u}(k)$ and $\tilde{P}_u(k)$ are output of a bias filter. We are to show that the Friedland form (29) - (31) holds under condition (19).

We treat the non-jump system (16) - (18) as a jump process at time zero in the following manner. At time zero before the jump we assume that the augmented system has the following initial state, mean and covariance

$$X_0 = \begin{bmatrix} x(0) - S_0\,u(0) \\ 0 \end{bmatrix} \tag{32}$$

$$\hat{X}_0 = \begin{bmatrix} \tilde{x}_0 \\ 0 \end{bmatrix} \tag{33}$$

$$P_1(0) = \begin{bmatrix} \tilde{P}_x(0) & 0 \\ 0 & 0 \end{bmatrix} \tag{34}$$

Eqs. (32) - (34) imply that the control u is known to be zero before the jump.

We assume that a jump ΔX_0 occurs at time 0^+ in the augmented state

$$\Delta X_0 = \begin{bmatrix} S_0 \\ I \end{bmatrix} u(0) \tag{35}$$

where u(0) has mean $\tilde{u}_0$ and covariance $\tilde{P}_u(0)$ and is independent of $\tilde{x}_0$. The covariance $\Delta P(0)$ of ΔX_0 is given by

$$\Delta P(0) = \begin{bmatrix} S_0 \tilde{P}_u(0) S_0^T & S_0 \tilde{P}_u(0) \\ \tilde{P}_u(0) S_0^T & \tilde{P}_u(0) \end{bmatrix} \tag{36}$$

The mean value of ΔX_0 is

$$\hat{\Delta} X_0 = \begin{bmatrix} S_0 \\ I \end{bmatrix} \tilde{u}_0 \tag{37}$$

From Eqs. (27), (34) and (36) we note that

$$P(0) = P_1(0) + \Delta P(0) \tag{38}$$

From Eqs. (32) and (35) we observe that

$$X(0) = \begin{bmatrix} x(0) \\ u(0) \end{bmatrix} = X_0 + \Delta X_0 \tag{39}$$

From Eqs. (21), (33) and (37) we see that

$$\hat{X}(0) = \begin{bmatrix} \hat{x}(0) \\ \hat{u}(0) \end{bmatrix} = \hat{X}_0 + \Delta\hat{X}_0 \tag{40}$$

Eqs. (38) - (40) show that we have a well-defined jump process at time zero. Consequently, we can employ the results of [1] which applies to jumps in the augmented state vector.

For this purpose consider the following filtering conditions for the Kalman-Bucy filter:

$\tilde{H}_1$: There is a jump in state and control but the filter is unaware that the jump (35) has taken place and it operates as if the jump is zero. The control u(k) is assumed to be perfectly known as a zero control. The initial conditions (32) - (34) are used.

H_2: There is a jump in state and control, the jump (35) is known to the filter and the jump information is made use of in the filter. Before the jump, the control u is assumed to be perfectly known as zero control. The initial conditons are (34), (41) and (42).

$$\hat{x}_2(0) = \tilde{x}_0 + S_0 u(0) \tag{41}$$

$$\hat{u}_2(0) = u(0) \tag{42}$$

The discrete Kalman-Bucy filter equations are given in Tables 5 and 6 for Conditions $\tilde{H}_1$ and H_2, respectively. The Kalman gains, the state covariances and the predicted measurement covariance satisfy

$$\tilde{K}_x(k) = K_2(k) \tag{43}$$

$$\tilde{P}_x(k) = P_2(k) \tag{44}$$

$$\tilde{P}_x(k+1|k) = P_2(k+1|k) \tag{45}$$

$$\tilde{V}_x(k) = V_2(k) \tag{46}$$

We make the definitions

$$\Delta A(k+1,k) = [I-\tilde{K}_x(k+1)H(k+1)]A(k+1,k) \tag{47}$$

$$\Delta B(k+1,k) = [I-\tilde{K}_x(k+1)H(k+1)]B(k+1,k) \tag{48}$$

In view of (43) it follows from Tables 5 and 6 that

$$\tilde{x}_1(k+1) = \Delta A(k+1,k)\ \tilde{x}_1(k) + \tilde{K}_x(k+1)z(k+1) \tag{49}$$

$$\begin{aligned}\hat{x}_2(k+1) = \Delta A(k+1,k)\hat{x}_2(k) &+ \Delta B(k+1,k)u(0)\\ &+ \tilde{K}_x(k+1)z(k+1)\end{aligned} \tag{50}$$

TABLE 5

DISCRETE KALMAN-BUCY FILTER EQUATIONS
FOR CONDITION $\tilde{H}_1$

$\tilde{x}_1(k+1\|k) = A(k+1,k)\tilde{x}_1(k)$	(i)*
$\tilde{P}_x(k+1\|k) = A(k+1,k)\ \tilde{P}_x(k)\ A^T(k+1,k) + \Gamma(k)\ Q(k)\Gamma^T(k)$	(ii)
$\tilde{\gamma}_1(k) = z(k) - H(k)\ \tilde{x}_1(k\|k-1)$	(iii)
$\tilde{V}_x(k) = H(k)\ \tilde{P}_x(k\|k-1)H^T(k) + R(k)$	(iv)
$\tilde{K}_x(k) = \tilde{P}_x(k\|k-1)\ H^T(k)\ \tilde{V}_x^{-1}(k)$	(v)
$\tilde{x}_1(k) = \tilde{x}_1(k\|k-1) + \tilde{K}_x(k)\ \tilde{\gamma}_1(k)$	(vi)
$\tilde{P}_x(k) = [I - \tilde{K}_x(k)\ H(k)]\ \tilde{P}_x(k\|k-1)$	(vii)
$\Delta z_1(k) = z(k) - H(k)\ \tilde{x}_1(k)$	(viii)
$\tilde{x}_1(0) = \tilde{x}_0$	(ix)
$\tilde{P}_x(0) = P_x(0) - S_0\ P_u(0)\ S_0^T$	(x)

*The usual notations $\tilde{x}_1(k|k)$ and $\tilde{P}_x(k|k)$ have been shortened to $\tilde{x}_1$ (k) and $\tilde{P}_x(k)$.

TABLE 6

DISCRETE KALMAN-BUCY FILTER EQUATIONS*
FOR CONDITION H_2

$\hat{x}_2(k+1\|k) = A(k+1,k)\ \hat{x}_2(k) + B(k+1,k)\ u(k)$		(i)*
$P_2(k+1\|k) = A(k+1,k)\ P_2(k)\ A^T(k+1,k) + \Gamma(k)\ Q(k)\Gamma^T(k)$		(ii)
$\gamma_2(k) = z(k) - H(k)\ \hat{x}_2(k\|k-1)$		(iii)
$V_2(k) = H(k)\ P_2(k\|k-1)\ H^T(k) + R(k)$		(iv)
$K_2(k) = P_2(k\|k-1)\ H^T(k)\ V_2^{-1}(k)$		(v)
$\hat{x}_2(k) = \hat{x}_2(k\|k-1) + K_2(k)\ \gamma_2(k)$		(vi)
$P_2(k) = [I - K_2(k)\ H(k)]\ P_2(k\|k-1)$		(vii)
$\Delta z_2(k) = z(k) - H(k)\ \hat{x}_2(k)$		(viii)
$\hat{x}_2(0) = \tilde{x}_0 + S_0\ u(0)$		(ix)
$P_2(0) = P_x(0) - S_0\ P_u(0)\ S_0^T$		(x)
$u(k) = u(0)$		(xi)

*The usual notations $\hat{x}_2(k|k)$ and $P_2(k|k)$ have been shortened to $\hat{x}_2(k)$ and $P_2(k)$.

We define the Δ-state variable $\Delta x_1(k)$ as

$$\Delta x_1(k) = \hat{x}_2(k) - \tilde{x}_1(k) \tag{51}$$

Subtracting (49) from (50) gives

$$\Delta x_1(k+1) = \Delta A(k+1,k)\,\Delta x_1(k) + \Delta B(k+1,k)u(0) \tag{52}$$

In view of Eqs. (41) and (51) this becomes

$$\Delta x_1(0) = S_0 u(0) \tag{53}$$

$$\Delta x_1(k+1) = \Delta A(k+1,k)\,\Delta x_1(k) + \Delta B(k+1,k)u(0), \quad k\geq 0 \tag{54}$$

We define the S_u matrix as

$$S_u(k+1;0) = \Delta A(k+1,k)\,S_u(k;0) + \Delta B(k+1,k), \quad k\geq 0 \tag{55}$$

$$S_u(0;0) = S_0 \tag{56}$$

Consequently, $\Delta x_1(k)$ satisfies

$$\Delta x_1(k) = S_u(k;0)u(k) \tag{57}$$

since $u(k) = u(0)$.

The measurement equation for Δx_1 is easily derived. We define the two a posteriori measurement residuals for Conditions $\tilde{H}_1$ and H_2:

$$\widetilde{\Delta z}_1(k) = z(k) - H(k)\, \tilde{x}_1(k) \tag{58}$$

$$\Delta z_2(k) = z(k) - H(k)\, \hat{x}_2(k) \tag{59}$$

Subtracting (59) from (58) gives the measurement equation for Δx_1

$$\widetilde{\Delta z}_1(k) = H(k)\, \Delta x_1(k) + \widetilde{\Delta \upsilon}_1(k) \tag{60}$$

where

$$\widetilde{\Delta \upsilon}_1(k) = \Delta z_2(k) \tag{61}$$

The measurement noise $\Delta \upsilon_1(k)$ is a zero-mean white Gaussian sequence with covariance defined by

$$E\{\widetilde{\Delta \upsilon}_1(k)\, \widetilde{\Delta \upsilon}_1^T(k)\} = R_u(k)\, \delta_{kj} \tag{62}$$

where

$$R_u(k) = R(k)\, \tilde{V}_x^{-1}(k)\, R(k) \tag{63}$$

We have made use of Eq. (46) in obtaining (62).

The system equations for $u(k)$ are

System Dynamics

$$u(k+1) = u(k) \quad , \quad k \geq 0 \tag{64}$$

Measurement Equation

$$\tilde{\Delta z}_1(k) = H_u(k)u(k) + \tilde{\Delta \upsilon}_1(k) \tag{65}$$

where

$$H_u(k) = H(k)\ S_u(k;0) \tag{66}$$

Eqs. (57) and (60) give (65).

The Kalman-Bucy filtering equations for estimating $u(k)$ are given in Table 7.

In view of (57) the augmented $\Delta X(k)$ is given by

$$\Delta X(k) = \begin{bmatrix} \Delta x_1(k) \\ u(k) \end{bmatrix} = \begin{bmatrix} S_u(k;0) \\ I \end{bmatrix} u(k) \tag{67}$$

The estimate of $\Delta X(k)$ is

$$\hat{\Delta X}(k) = \begin{bmatrix} S_u(k;0) \\ I \end{bmatrix} \tilde{u}(k) \tag{68}$$

TABLE 7

FILTERING EQUATIONS FOR ESTIMATING u(k)

$$\tilde{u}(k+1|k) = \tilde{u}(k) \quad \text{(i)}$$

$$\tilde{P}_u(k+1|k) = \tilde{P}_u(k) \quad \text{(ii)}$$

$$\gamma_u(k) = \tilde{\Delta z}_1(k) - H_u(k)\ \tilde{u}(k|k-1) \quad \text{(iii)}$$

$$\tilde{V}_u(k) = H_u(k)\ \tilde{P}_u(k|k-1)\ H_u^T(k) + R_u(k) \quad \text{(iv)}$$

$$\tilde{K}_u(k) = \tilde{P}_u(k|k-1)\ H_u^T(k)\ \tilde{V}_u^{-1}(k) \quad \text{(v)}$$

$$\tilde{u}(k) = \tilde{u}(k|k-1) + \tilde{K}_u(k)\ \gamma_u(k) \quad \text{(vi)}$$

$$\tilde{P}_u(k) = [I - \tilde{K}_u(k)\ H_u(k)]\ \tilde{P}_u(k|k-1) \quad \text{(vii)}$$

where

$$\tilde{u}(0) = \hat{u}(0) \quad \text{(viii)}$$

$$\tilde{P}_u(0) = P_u(0) \quad \text{(ix)}$$

$$H_u(k) = H(k)\ S_u(k;0) \quad \text{(x)}$$

$$S_u(k+1;0) = \Delta A(k+1,k)\ S_u(k;0) + \Delta B(k+1,k) \quad \text{(xi)}$$

$$S_u(0;0) = S_0 \quad \text{(xii)}$$

$$\Delta A(k+1,k) = [I - \tilde{K}_x(k+1)\ H(k+1)]\ A(k+1,k) \quad \text{(xiii)}$$

$$\Delta B(k+1,k) = [I - \tilde{K}_x(k+1)\ H(k+1)]\ B(k+1,k) \quad \text{(xiv)}$$

$$R_u(k) = R(k)\ \tilde{V}_x^{-1}(k)\ R(k) \quad \text{(xv)}$$

$$\tilde{\Delta z}_1(k) = z(k) - H(k)\ \tilde{x}_1(k) \quad \text{(xvi)}$$

The covariance of $\widetilde{\Delta X}(k)$ is

$$\Delta P(k) = \begin{bmatrix} S_u(k;0)\ \tilde{P}_u(k)\ S_u^T(k;0) & S_u(k;0)\ \tilde{P}_u(k) \\ \tilde{P}_u(k)\ S_u^T(k;0) & \tilde{P}_u(k) \end{bmatrix} \tag{69}$$

where $\tilde{u}(k)$ and $\tilde{P}_u(k)$ are given by the filter of Table 7.

In view of Eqs. (33) and (34) and the filter of Table 5 the augmented $\tilde{X}_1(k)$ satisfies

$$\tilde{X}_1(k) = \begin{bmatrix} \tilde{x}_1(k) \\ 0 \end{bmatrix} \tag{70}$$

and has covariance

$$P_1(k) = \begin{bmatrix} \tilde{P}_x(k) & 0 \\ 0 & 0 \end{bmatrix} \tag{71}$$

The optimal estimates $\hat{X}(k)$ and $P(k)$ follow from the Kalman-Bucy filter of Table 3 for the augmented system:

$$\hat{X}(k) = \begin{bmatrix} \hat{x}(k) \\ \hat{u}(k) \end{bmatrix} \tag{72}$$

$$P(k) = \begin{bmatrix} P_x(k) & P_{xu}(k) \\ P_{xu}^T(k) & P_u(k) \end{bmatrix} \tag{73}$$

It is shown in [1] that the following identities hold:

$$\hat{X}(k) = \hat{X}_1(k) + \Delta\hat{X}(k) \tag{74}$$

$$P(k) = P_1(k) + \Delta P(k) \tag{75}$$

Consequently, Eqs. (74) and (75) verify that the Friedland form (29) - (31) holds under the condition that there is correlation initially between the state x and the control vector u, i.e., Eq. (19) holds.

It is also shown in [1] that the gains are related by the expression

$$K_a(k) = \tilde{K}_1(k) + \Delta K(k)\ [I - H_a(k)\ \tilde{K}_1(k)] \tag{76}$$

where

$$\tilde{K}_1(k) = \begin{bmatrix} \tilde{K}_x(k) \\ 0 \end{bmatrix} \tag{77}$$

$$\Delta K(k) = \begin{bmatrix} S_u(k;0)\ \tilde{K}_u(k) \\ \tilde{K}_u(k) \end{bmatrix} \tag{78}$$

Eq. (78) follows from Eq. (vi) of Table 7, Eqs. (57) and (67). Eq. (77) follows from Eq. (70) and Eq. (vi) of Table 5. We define

$$K_a(k) = \begin{bmatrix} K_x(k) \\ K_u(k) \end{bmatrix} \tag{79}$$

Using (76) - (78) we obtain

$$K_x(k) = \tilde{K}_x(k) + S_u(k;0)\ \tilde{K}_u(k)\ [I - H(k)\ \tilde{K}_x(k)] \tag{80}$$

$$K_u(k) = 0 + \tilde{K}_u(k)\ [I - H(k)\ \tilde{K}_x(k)] \tag{81}$$

It follows from Eqs. (68) - (75) that

$$\hat{x}(k) = \tilde{x}_1(k) + S_u(k;0)\tilde{u}(k) \tag{82}$$

$$\hat{u}(k) = \tilde{u}(k) \tag{83}$$

$$P_x(k) = \tilde{P}_x(k) + S_u(k;0)\ \tilde{P}_u(k)\ S_u^T(k;0) \tag{84}$$

$$P_{xu}(k) = S_u(k;0)\ \tilde{P}_u(k) \tag{85}$$

$$P_u(k) = \tilde{P}_u(k) \tag{86}$$

We show now that the predicted measurement residual covariances are related by

$$V_a(k) = \tilde{V}_x(k)\ R^{-1}(k)\ \tilde{V}_u(k)\ R^{-1}(k)\ \tilde{V}_x(k) \tag{87}$$

It is shown in [1] that the augmented matrices $V_a(k)$, $\tilde{V}_1(k)$ and $\Delta V(k)$ are related by

$$V_a(k) = \tilde{V}_1(k)\ R^{-1}(k)\ \Delta V(k)\ R^{-1}(k)\ \tilde{V}_1(k) \tag{88}$$

where

$$\Delta V(k) = H_a(k)\ \Delta P(k|k-1)\ H_a^T(k) + R(k)\ \tilde{V}_1^{-1}(k)\ R(k) \tag{89}$$

$$\tilde{V}_1(k) = H_a(k)\ P_1(k|k-1)\ H_a^T(k) + R(k) \tag{90}$$

Since

$$H_a(k)\ \Delta P(k|k-1) H_a^T(k) = H_u(k)\ \tilde{P}_u(k|k-1) H_u^T(k) \tag{91}$$

$$H_a(k)\ P_1(k|k-1) H_a^T(k) = H(k)\ \tilde{P}_x(k|k-1) H^T(k)$$

It follows that

$$\tilde{V}_x(k) = \tilde{V}_1(k) \tag{92}$$

$$\tilde{V}_u(k) = \Delta V(k) \tag{93}$$

4. OPTIMAL FILTERING OF UNAUGMENTED JUMP PROCESSES: REINITIALIZATION AT JUMP

We address the jump process described by Eqs. (1) - (3). A jump in the control variable occurs at time q. The jump Δu_q is normally distributed with mean $\Delta u(q)$ and covariance $P_{\Delta u}(q)$. We assume the jump is independent of all other processes. In this section we assumed that the jump time q is known. We use the time q to denote the time before the jump and we use q^+ to denote the time just after the jump.

Before the jump the augmented estimates $\hat{X}(q)$ and $P(q)$ satisfy

$$\hat{x}(q) = \tilde{x}_1(q) + S_u(q;0)\,\tilde{u}(q) \tag{94}$$

$$\hat{u}(q) = \tilde{u}(q) \tag{95}$$

$$P_x(q) = \tilde{P}_x(q) + S_u(q;0)\,\tilde{P}_u(q)\,S_u^T(q;0) \tag{96}$$

$$P_{xu}(q) = S_u(q;0)\,\tilde{P}_u(q) \tag{97}$$

$$P_u(q) = \tilde{P}_u(q) \tag{98}$$

After the jump they satisfy

$$\hat{X}(q^+) = \hat{X}(q) + \Delta\hat{X}(q) \tag{99}$$

$$P(q^+) = P(q) + \Delta P(q) \tag{100}$$

where

$$\hat{\Delta} X(q) = \begin{bmatrix} 0 \\ \hat{\Delta u}(q) \end{bmatrix} \tag{101}$$

$$\Delta P(q) = \begin{bmatrix} 0 & 0 \\ 0 & P_{\Delta u}(q) \end{bmatrix} \tag{102}$$

Therefore, after the jump we have for the augmented system

$$\hat{x}(q^+) = \hat{x}(q) \tag{103}$$

$$\hat{u}(q^+) = \hat{u}(q) + \hat{\Delta u}(q) \tag{104}$$

$$P_x(q^+) = P_x(q) \tag{105}$$

$$P_{xu}(q^+) = P_{xu}(q) \tag{106}$$

$$P_u(q^+) = P_u(q) + P_{\Delta u}(q) \tag{107}$$

In order to utilize the unaugmented filtering technique we need to put Eqs. (103) - (107) into the Friedland form (25) - (27). We make use of the reinitialization equations (20) - (24) of the unaugmented filters. Consequently, we make the definitions

$$S_q = P_{xu}(q^+) P_u^{-1}(q^+) \tag{108}$$

$$\tilde{x}_1(q^+) = \hat{x}(q^+) - S_q \hat{u}(q^+) \tag{109}$$

$$\tilde{P}_x(q^+) = P_x(q^+) - S_q P_u(q^+) S_q^T \tag{110}$$

$$\tilde{u}\ (q^+) = \hat{u}(q^+) \tag{111}$$

$$\tilde{P}_u\ (q^+) = P_u(q^+) \tag{112}$$

The initial conditions (108) - (112) ensure that we have the Friedland form

$$\hat{x}(q^+) = \tilde{x}_1(q^+) + S_q\ \tilde{u}(q^+) \tag{113}$$

$$\hat{u}(q^+) = \tilde{u}(q^+) \tag{114}$$

$$P(q^+) = \begin{bmatrix} \tilde{P}_x(q^+) + S_q\ \tilde{P}_u(q^+) S_q^T & S_q\ \tilde{P}_u(q^+) \\ \tilde{P}_u(q^+) S_q^T & \tilde{P}_u(q^+) \end{bmatrix} \tag{115}$$

In view of Eqs. (94) - (98) and (103) - (107) we can rewrite Eqs. (108) - (112) as follows

$$S_q = S_u(q;0)\ \tilde{P}_u(q)\ [\tilde{P}_u(q) + P_{\Delta u}(q)]^{-1} \tag{116}$$

$$\tilde{x}_1(q^+) = \tilde{x}_1(q) + S_u(q;0)\ \tilde{u}(q) - S_q[\tilde{u}(q) + \Delta\hat{u}(q)] \tag{117}$$

$$\tilde{u}(q^+) = \tilde{u}(q) + \Delta\hat{u}(q) \tag{118}$$

$$\begin{aligned} \tilde{P}_x(q^+) = \tilde{P}_x(q) &+ S_u(q;0)\tilde{P}_u(q) S_u^T(q;0) \\ &- S_q[\tilde{P}_u(q) + P_{\Delta u}(q)]\ S_q^T \end{aligned} \tag{119}$$

$$\tilde{P}_u(q^+) = \tilde{P}_u(q) + P_{\Delta u}(q) \tag{120}$$

Note that

$$S_q \, [\tilde{P}_u(q) + P_{\Delta u}(q)] = S_u(q;0) \, \tilde{P}_u(q) \tag{121}$$

$$S_q \, \tilde{P}_u(q) \, S_u^T(q;0) = S_u(q;0) \, \tilde{P}_u(q) \, S_q^T \tag{122}$$

Eqs. (121) and (122) are used in showing that $\tilde{P}_x(q^+)$ as defined by Eq. (119) is the covariance of $\tilde{x}_1(q^+)$ as defined by Eq. (117).

Eqs. (116) - (120) are the equations for reinitializing the unaugmented filters defined by the equations in Tables 5 and 7. Equations (x) - (xii) of Table 7 are replaced with the following:

$$H_u(k) = H(k) \, S_u(k;q) \tag{123}$$

$$S_u(k+1);q) = \Delta A(k+1,k) \, S_u(k;1) + \Delta B(k+1,k) \tag{124}$$

$$S_u(q;q) = S_q \tag{125}$$

5. OPTIMAL FILTERING OF UNAUGMENTED JUMP PROCESSES: REINITIALIZATION BEYOND JUMP

We continue to address the jump process described by Eqs. (1) - (3). In this section we make the same assumptions as those given in the first paragraph of the previous section. Herein, we develop the equations for the optimal estimates in terms of the unaugmented estimates for the case that we do not reinitialize the unaugmented filters at the jump time. Rather, we initialize at some time k* beyond the jump time q.

Consider the following conditions for the Kalman-Bucy filter of the augmented system:

$\tilde{H}_1$: There is a jump in control but the filter is unaware that the jump (10) has taken place and it operates as if the jump at time q is zero. The initial conditions immediately after the jump are (126) and (127). That is, it uses the same estimates immediately after the jump as it had just before the jump.

H_2: There is a jump in control, the jump (10) is known to the filter and the jump information is made use of in the filter. The initial conditions immediately after the jump are (128) and (129).

$$\tilde{X}_1(q^+) = \hat{X}(q) \tag{126}$$

$$P_1(q^+) = P(q) \tag{127}$$

$$\hat{X}_2(q^+) = \hat{X}(q) + \Delta X_q \tag{128}$$

$$P_2(q^+) = P(q) \tag{129}$$

It is shown in [1] that after the jump the optimal estimates $\hat{X}(k)$ and $P(k)$ are given by

$$\hat{X}(k) = \tilde{X}_1(k) + \Delta\hat{X}(k) \tag{130}$$

$$P(k) = P_1(k) + \Delta P(k) \tag{131}$$

where $\Delta\hat{X}(k)$ and $\Delta P(k)$ are the output of a Kalman-Bucy filter having the initial conditions

$$\Delta\hat{X}(q) = \Delta X_q \tag{132}$$

$$\Delta P(q) = \begin{bmatrix} 0 & 0 \\ 0 & P_{\Delta u}(q) \end{bmatrix} \tag{133}$$

Therein, the Δ-state $\Delta X(k)$ is defined as

$$\Delta X(k) = \hat{X}_2(k) - \tilde{X}_1(k) \tag{134}$$

It satisfies the Δ-state equation

$$\Delta X(k) = \Delta\Phi(k,k-1)\ \Delta X(k-1) \tag{135}$$

where

$$\Delta\Phi(k,k-1) = [I - K_1(k)\ H_a(k)]\Phi(k,k-1) \tag{136}$$

The matrix H_a is defined by Eq. (12). The Kalman gain K_1 is that which is given by the filter operating under Condition $\tilde{H}_1$. From Eqs. (80) and (81) we see that K_1 can be expressed in terms of the unaugmented gains $\tilde{K}_x$ and $\tilde{K}_u$:

$$K_1(k) = \begin{bmatrix} K_{1x}(k) \\ K_{1u}(k) \end{bmatrix} \tag{137}$$

where

$$K_{1x}(k) = \tilde{K}_x(k) + S_u(k;0)\ K_{1u}(k) \tag{138}$$

$$K_{1u}(k) = \tilde{K}_u(k)[I - H(k)\ \tilde{K}_x(k)] \tag{139}$$

The augmented state ΔX in terms of the unaugmented states is given by

$$\Delta X(k) = \begin{bmatrix} \Delta x(k) \\ \Delta u(k) \end{bmatrix} \tag{140}$$

We have the initial conditions

$$\Delta x(q) = 0 \tag{141}$$

$$\Delta u(q) = \Delta u_q \tag{142}$$

We define the nxp matrix S_x and the pxp matrix $S_{\Delta u}$ as follows

$$S_x(k+1;q) = \Delta A_1(k+1,k)\ S_x(k;q) + \Delta B_1(k+1,k)\ S_{\Delta u}(k;q) \tag{143}$$

$$S_{\Delta u}(k+1;q) = \Delta A_2(k+1,k)\ S_x(k;q) + \Delta B_2(k+1,k)\ S_{\Delta u}(k;q) \tag{144}$$

$$S_x(q;q) = 0 \tag{145}$$

$$S_{\Delta u}(q;q) = I \tag{146}$$

where

$$\Delta A_1(k,k-1) = [I - \tilde{K}_x(k)H(k) - S_u(k;0)K_{1u}(k)H(k)]\ A(k,k-1) \tag{147}$$

$$\Delta B_1(k,k-1) = [I - \tilde{K}_x(k)H(k) - S_u(k;0)\ K_{1u}(k)H(k)]\ B(k,k-1) \tag{148}$$

$$\Delta A_2(k,k-1) = -K_{1u}(k)\ H(k)\ A(k,k-1) \tag{149}$$

$$\Delta B_2(k,k-1) = [I - K_{1u}(k)\ H(k)\ B(k,k-1)] \tag{150}$$

Using Eqs. (135) - (150) one can verify that

$$\Delta x(k) = S_x(k;q)\Delta u_q \tag{151}$$

$$\Delta u(k) = S_{\Delta u}(k;q)\Delta u_q \tag{152}$$

The measurement equation for $\Delta X(k)$ is shown in [1] to be

$$\Delta z(k) = H_a(k)\ \Delta X(k) + \Delta \upsilon(k) \tag{153}$$

where

$$\Delta z(k) = z(k) - H_a(k)\ \tilde{X}_1(k) \tag{154}$$

$$\Delta \upsilon\ (k) = z(k) - H_a(k)\ \hat{X}_2(k) \tag{155}$$

Note that $\Delta\upsilon$ is the a posteriori measurement residual under Condition H_2. It follows that $\Delta\upsilon(k)$ is a zero mean white Gaussian sequence with covariance defined by

$$E\{\ \Delta\upsilon(k)\,\Delta\upsilon^T(j)\ \} = \Delta R(k)\ \delta_{kj} \tag{156}$$

where

$$\Delta R(k) = R(k)\ V_2^{-1}(k)\ R(k) \tag{157}$$

The matrix $V_2(k)$ is the predicted measurement residual covariance under Condition H_2. From Eq. (87) we see that V_2 can be expressed as a function of $\tilde{V}_x$ and $\tilde{V}_u$:

$$V_2(k) = \tilde{V}_x(k)R^{-1}(k)\ \tilde{V}_u(k)\ R^{-1}(k)\ \tilde{V}_x(k) \tag{158}$$

Consequently, $\Delta R(k)$ is given by

$$\Delta R(k) = R(k)\ \tilde{V}_x^{-1}(k)\ R(k)\ \tilde{V}_u^{-1}(k)\ R(k)\ \tilde{V}_x^{-1}(k)\ R(k) \tag{159}$$

Using Eqs. (82) - (83) we see that $\tilde{X}_1(k)$ is given by

$$\tilde{X}_1(k) = \begin{bmatrix} \tilde{x}_1(k) + S_u(k;0)\ \tilde{u}(k) \\ \tilde{u}(k) \end{bmatrix} \tag{160}$$

Consequently, Eq. (154) can be rewritten as

$$\Delta z(k) = z(k) - H(k)\ [\tilde{x}_1(k) + S_u(k;0)\tilde{u}(k)] \tag{161}$$

In view of Eqs. (12) and (140) we can rewrite (153) as

$$\Delta z(k) = H(k)\, \Delta x(k) + \Delta \upsilon(k) \tag{162}$$

Substituting (151) into (162) gives the measurement equation for Δu_q

$$\Delta z(k) = H(k)\, S_x(k;q)\, \Delta u_q(k) + \Delta \upsilon(k) \tag{163}$$

where $\Delta u_q(k)$ satisfies the constant state equation

$$\Delta u_q(k) = \Delta u_q(k-1) = \Delta u_q \tag{164}$$

The filtering equations for estimating Δu_q are given in Table 8.

The optimal estimates $\hat{X}(k)$ and $P(k)$ are given by Eqs. (130) and (131)

$$\hat{X}(k) = \tilde{X}_1(k) + \hat{\Delta X}(k) \tag{165}$$

$$P(k) = P_1(k) + \Delta P(k) \tag{166}$$

where

$$\hat{\Delta X}(k) = \begin{bmatrix} \hat{\Delta x}(k) \\ \hat{\Delta u}(x) \end{bmatrix} \tag{167}$$

$$\hat{\Delta x}(k) = S_x(k;q)\, \tilde{\Delta u}_q(k) \tag{168}$$

$$\hat{\Delta u}(k) = S_{\Delta u}(k;q)\, \tilde{\Delta u}_q(k) \tag{169}$$

$$\Delta P(k) = \begin{bmatrix} S_x(k;q)\,\tilde{P}_u(k)\,S_x^T(k;q) & S_x(k;q)\tilde{P}_{\Delta u}(k)\,S_{\Delta u}^T(k;q) \\ S_{\Delta u}(k;q)\tilde{P}_{\Delta u}(k)S_x^T(k;q) & S_{\Delta u}(k;q)\tilde{P}_{\Delta u}(k)S_{\Delta u}^T(k;q) \end{bmatrix} \tag{170}$$

$$P_1(k) = \begin{bmatrix} P_{1x}(k) & P_{1xu}(k) \\ P_{1xu}^T(k) & P_{1u}(k) \end{bmatrix} \tag{171}$$

$$P_{1x}(k) = \tilde{P}_x(k) + S_u(k;0)\,\tilde{P}_u(k)\,S_u^T(k;0) \tag{172}$$

$$P_{1xu}(k) = S_u(k;0)\,\tilde{P}_u(k) \tag{173}$$

$$P_{1u}(k) = \tilde{P}_u(k) \tag{174}$$

Using Eqs.(160), (165) - (174) we have the optimal estimates

$$\hat{x}(k) = \tilde{x}_1(k) + S_u(k;0)\tilde{u}(k) + S_x(k;q)\,\tilde{\Delta u}_q(k) \tag{175}$$

$$\hat{u}(k) = \tilde{u}(k) + S_{\Delta u}(k;q)\,\tilde{\Delta u}_q(k) \tag{176}$$

$$P_x(k) = \tilde{P}_x(k) + S_u(k;0)\tilde{P}_u(k)\,S_u^T(k;0) + S_x(k;q)\,\tilde{P}_{\Delta u}(k)\,S_x^T(k;q) \tag{177}$$

$$P_{xu}(k) = S_u(k;0)\tilde{P}_u(k) + S_x(k;q)\,\tilde{P}_{\Delta u}(k)\,S_{\Delta u}^T(k;q) \tag{178}$$

$$P_u(k) = \tilde{P}_u(k) + S_{\Delta u}(k;q)\,\tilde{P}_{\Delta u}(k)\,S_{\Delta u}^T(k;q) \tag{179}$$

Eqs. (175) - (179) give the optimal estimates in terms of the unaugmented filter estimates described by the filters of Tables 5, 7 and 8.

At some time k* beyond the jump we desire to reinitialize the filters of Tables 5 and 7. This is more efficient than operating the three filters of Tables 5, 7 and 8.

We make the following definitions with $k=k^*$:

$$S_{k^*} = P_{xu}(k)\ P_u^{-1}(k) \tag{180}$$

$$\tilde{x}_1(k^+) = \hat{x}(k) - S_{k^*}\ \hat{u}(k) \tag{181}$$

$$\tilde{P}_x(k^+) = P_x(k) - S_{k^*}\ P_u(k)\ S_{k^*}^T \tag{182}$$

$$\tilde{u}(k^+) = \hat{u}(k) \tag{183}$$

$$\tilde{P}_u(k^+) = P_u(k) \tag{184}$$

The argument k^+ refers to values just after the reinitialization at $k=k^*$. Eqs. (180) - (184) are the equations for reinitializing the unaugmented filters of Tables 5 and 7. Equations (x) - (xii) of Table 7 are replaced with the following (the time k* represents the time of reinitialization):

$$H_u(k) = H(k)\ S_u(k;k^*),\ k \geq k^* \tag{185}$$

$$S_u(k+1;k^*) = \Delta A(k+1,k)\ S_u(k;k^*) + \Delta B(k+1,k),\ k \geq k^* \tag{186}$$

$$S_u(k^*;k^*) = S_k{}^* \tag{187}$$

Using the expression derived in [1]

$$K(k) = K_1(k) + \Delta K(k)\ [I - H_a(k)\ K_1(k)] \tag{188}$$

The optimal Kalman gain K(k) is given by

$$K_x(k) = K_{1x}(k) + \Delta K_x(k)\ [I - H(k)\ K_{1x}(k)] \tag{189}$$

$$K_u(k) = K_{1u}(k) + \Delta K_u(k)\ [I - H(k)\ K_{1x}(k)] \tag{190}$$

where

$$K_{1x}(k) = \tilde{K}_x(k) + S_u(k;0)\ K_{1u}(k) \tag{191}$$

$$K_{1u}(k) = \tilde{K}_u(k)\ [I - H(k)\ \tilde{K}_x(k)] \tag{192}$$

$$\Delta K_x(k) = S_x(k;q)\ \tilde{K}_{\Delta u}(k) \tag{193}$$

$$\Delta K_u(k) = S_{\Delta u}(k;q)\ \tilde{K}_{\Delta u}(k) \tag{194}$$

Eqs. (193) and (194) follow from Eq. (vi) of Table 8 and Eqs. (151) - (152).

one may note that

$$[I - K_xH] = [I - \Delta K_xH]\ [I - S_u\ \tilde{K}_uH]\ [I - \tilde{K}_xH] \tag{195}$$

The predicted measurement residual covariance V satisfies

$$V(k) = V_1(k)\ R^{-1}(k)\ \tilde{V}_{\Delta u}(k)\ R^{-1}(k)\ V_1(k) \tag{196}$$

where

$$V_1(k) = \tilde{V}_x(k)\ R^{-1}(k)\ \tilde{V}_u(k)\ R^{-1}(k)\ \tilde{V}_x(k) \tag{197}$$

TABLE 8

FILTERING EQUATIONS FOR ESTIMATING Δu_q

$$\Delta \tilde{u}_q(k+1|k) = \Delta\tilde{u}_q(k) \qquad \text{(i)}$$

$$\tilde{P}_{\Delta u}(k+1|k) = \tilde{P}_{\Delta u}(k) \qquad \text{(ii)}$$

$$\gamma_{\Delta u}(k) = \Delta z(k) - H_{\Delta u}(k)\ \Delta\tilde{u}_q(k|k-1) \qquad \text{(iii)}$$

$$\tilde{V}_{\Delta u}(k) = H_{\Delta u}(k)\ \tilde{P}_{\Delta u}(k|k-1)\ H_{\Delta u}^T(k) + R_{\Delta u}(k) \qquad \text{(iv)}$$

$$\tilde{K}_{\Delta u}(k) = \tilde{P}_{\Delta u}(k|k-1)\ H_{\Delta u}^T(k)\ \tilde{V}_{\Delta u}^{-1}(k) \qquad \text{(v)}$$

$$\Delta\tilde{u}_q(k) = \Delta\tilde{u}_q(k|k-1) + \tilde{K}_{\Delta u}(k)\ \gamma_{\Delta u}(k) \qquad \text{(vi)}$$

$$\tilde{P}_{\Delta u}(k) = [I - \tilde{K}_{\Delta u}(k)\ H_{\Delta u}(k)]\ \tilde{P}_{\Delta u}(k|k-1) \qquad \text{(vii)}$$

where

$$\Delta\tilde{u}_q(q) = \Delta\hat{u}_q \qquad \text{(viii)}$$

$$\tilde{P}_{\Delta u}(q) = P_{\Delta u}(q) \qquad \text{(ix)}$$

$$H_{\Delta u}(k) = H(k)\ S_x(k;q) \qquad \text{(x)}$$

$$S_x(q;q) = 0 \qquad \text{(xi)}$$

$$S_{\Delta u}(q;q) = I \qquad \text{(xii)}$$

$$S_x(k+1;q) = \Delta A_1(k+1,k)\ S_x(k;q) + \Delta B_1(k+1,k)\ S_{\Delta u}(k;q) \qquad \text{(xiii)}$$

$$S_{\Delta u}(k+1;q) = \Delta A_2(k+1,k)\ S_x(k;q) + \Delta B_2(k+1,k)\ S_{\Delta u}(k;q) \qquad \text{(xiv)}$$

$$\Delta A_1(k,k-1) = [I - \tilde{K}_x(k)H(k) - S_u(k;0)K_{1u}(k)H(k)]A(k,k-1) \qquad \text{(xv)}$$

$$\Delta B_1(k,k-1) = [I - \tilde{K}_x(k)H(k) - S_u(k;0)K_{1u}(k)H(k)]B(k,k-1) \qquad \text{(xvi)}$$

$$\Delta A_2(k,k-1) = -K_{1u}(k)H(k)A(k,k-1) \qquad \text{(xvii)}$$

$$\Delta B_2(k,k-1) = [I - K_{1u}(k)H(k)B(k,k-1)] \qquad \text{(xviii)}$$

$$K_{1u}(k) = \tilde{K}_u(k)\ [I - H(k)\ \tilde{K}_x\ (k)] \qquad \text{(xix)}$$

$$R_{\Delta u}(k) = R(k)\ \tilde{V}_x^{-1}(k)R(k)\tilde{V}_u^{-1}(k)R(k)\tilde{V}_x^{-1}(k)R(k) \qquad \text{(xx)}$$

$$\Delta z(k) = z(k) - H(k)\ [\tilde{x}_1(k) + S_u(k;0)\tilde{u}(k)] \qquad \text{(xxi)}$$

6. DETECTION AND ESTIMATION OF JUMP USING THE GLR APPROACH: BANK OF Δ-FILTERS

We consider the employment of a bank of Kalman-Bucy constant Δ-state filters for detecting and estimating the jump Δu_q. The jump Δu_q and the jump time q are unknowns. We consider a moving window of length M. That is, at each j, $k-M \le j < k$ we employ the constant Δ-state filter defined by the equations of Table 8. Let j be a candidate jump time and let k be the current observation time. The filtering equations in Table 8 are used to obtain the estimate $\widetilde{\Delta u}_j(k)$ and $\tilde{P}_{\Delta u,j}(k)$. The filter is started when the current observation time is j. It uses the initial conditions

$$\widetilde{\Delta u}_j(j) = \hat{\Delta u}_q \tag{198}$$

$$\tilde{P}_{\Delta u,j}(j) = P_{\Delta u}(q) \tag{199}$$

$$S_x(j;j) = 0 \tag{200}$$

$$S_{\Delta u}(j;j) = I \tag{201}$$

At each new observation k the matrix $S_x(k;j)$ is computed using Eqs. (xiii) - (xix) of Table 8. Eqs. (i) - (vii) are then used to compute $\widetilde{\Delta u}_j(k)$ and $\tilde{P}_{\Delta u,j}(k)$. The following are also computed

$$d_j(k) = d_j(k-1) + H_{\Delta u,j}^T(k)\, R_{\Delta u}^{-1}(k)\, \Delta z(k) \tag{202}$$

$$\ell_j(k) = d_j^T(k)\, \widetilde{\Delta u}_j(k) \tag{203}$$

The vector d_j has the initial condition

$$d_j(j) = 0 \tag{204}$$

The above computations are carried out for each candidate jump time j, $k-M \leq j < k$. A jump is detected at time k for the jump time q, $k-M \leq q < k$, if

$$\ell_q(k) > 2 \ln(\eta)$$

$$\ell_q(k) = \mathrm{Max}\, \{\ell_j(k) \ : \ k-M \leq j < k\} \tag{205}$$

where the value η is chosen to provide a reasonable tradeoff between false and missed alarms. The above is a generalized likelihood ratio (GLR) algorithm for detecting and estimating the jump, [1] and [16]. In applying the filter described by the equations of Table 8 one should sequentially update the correlated subblocks of components of the measurement vector as discussed in [17] - [20].

After a jump has been detected and estimated we use Eqs. (175) - (187) to reinitialize the filters of Tables 5 and 7. The above GLR algorithm is used to detect and estimate the next jump.

7. SUMMARY OF DERIVED EXPRESSIONS

The optimum estimates $\hat{x}$ of the state and $\hat{u}$ of the control satisfy the expressions

$$\hat{x} = \tilde{x} + S_u \tilde{u} + S_x \tilde{\Delta u} \tag{206}$$

$$\hat{u} = \tilde{u} + S_{\Delta u} \tilde{\Delta u} \tag{207}$$

with covariances

$$P_x = \tilde{P}_x + S_u \tilde{P}_u S_u^T + S_x \tilde{P}_{\Delta u} S_x^T \tag{208}$$

$$P_{xu} = S_u \tilde{P}_u + S_x \tilde{P}_{\Delta u} S_{\Delta u}^T \tag{209}$$

$$P_u = \tilde{P}_u + S_{\Delta u} \tilde{P}_{\Delta u} S_{\Delta u}^T \tag{210}$$

where $\tilde{x}$, $\tilde{u}$ and $\tilde{\Delta u}$ are the output of a tri-system of unaugmented Kalman-Bucy filters with covariances $\tilde{P}_x$, $\tilde{P}_u$ and $\tilde{P}_{\Delta u}$, respectively.

The matrices S_u, S_x and $S_{\Delta u}$ satisfy

$$S_u(k+1) = \Delta_x A S_u(k) + \Delta_x B \tag{211}$$

$$S_x(k+1) = \Delta_u A S_x(k) + \Delta_u B S_{\Delta u}(k) \tag{212}$$

$$S_{\Delta u}(k+1) = \Delta_{\Delta u} A S_x(k) + \Delta_{\Delta u} B S_{\Delta u}(k) + S_{\Delta u}(k) \tag{213}$$

where

$$\Delta_x = I - \tilde{K}_x H \tag{214}$$

$$\Delta_u = [I - S_u \tilde{K}_u H]\Delta_x \tag{215}$$

$$\Delta_{\Delta u} = -\tilde{K}_u H \Delta_x \tag{216}$$

The initial conditions of S_u, S_x and $S_{\Delta u}$ are

$$S_u(0) = P_{xu}(0) P_u^{-1}(0) \tag{217}$$

$$S_x(q) = 0 \tag{218}$$

$$S_{\Delta u}(q) = I \tag{219}$$

where q is the jump time, $P_u(0)$ is the initial covariance of u and $P_{xu}(0)$ is the initial cross-covariance of x and u.

The optimal Kalman gains satisfy

$$K_x = \tilde{K}_x + [S_u \tilde{K}_u + S_x \tilde{K}_{\Delta u}(I - H S_u \tilde{K}_u)] [I - H \tilde{K}_x] \tag{220}$$

$$K_u = [\tilde{K}_u + S_{\Delta u} \tilde{K}_{\Delta u} (I - H S_u \tilde{K}_u)] [I - H \tilde{K}_x] \tag{221}$$

The predicted measurement residual covariance satisfies

$$V = V_1 R^{-1} \tilde{V}_{\Delta u} R^{-1} V_1 \tag{222}$$

where

$$V_1 = \tilde{V}_x R^{-1} \tilde{V}_u R^{-1} \tilde{V}_x \tag{223}$$

and where $\tilde{V}_x$ $\tilde{V}_u$ and $\tilde{V}_{\Delta u}$ are the predicted measurement residual covariances of the $\tilde{x}$-, the $\tilde{u}$- and the $\tilde{\Delta u}$ filters, respectively.

After reinitialization, the optimum estimates satisfy Friedland's expressions [21]

$$\hat{x} = \tilde{x}' + S_u' \tilde{u}' \tag{224}$$

$$\hat{u} = \tilde{u}' \tag{225}$$

where the prime denotes the output of the $\tilde{x}$- and the $\tilde{u}$-filters after reinitialization. The reinitialized values of S_u, $\tilde{x}$, $\tilde{P}_x$, $\tilde{u}$ and $\tilde{P}_u$ satisfy the following expressions at reinitialization:

$$S_u' = [S_u \tilde{P}_u + S_x \tilde{P}_{\Delta u} S_{\Delta u}^T] [\tilde{P}_u + S_{\Delta u} \tilde{P}_{\Delta u} S_{\Delta u}^T]^{-1} \tag{226}$$

$$\tilde{x}' = \tilde{x} + S_u \tilde{u} + S_x \tilde{\Delta u} - S_u' [\tilde{u} + S_{\Delta u} \tilde{\Delta u}] \tag{227}$$

$$\tilde{P}_x' = \tilde{P}_x + S_u \tilde{P}_u S_u^T + S_x \tilde{P}_{\Delta u} S_x^T - S_u' [P_u + S_{\Delta u} \tilde{P}_{\Delta u} S_{\Delta u}^T] S_u'^T \tag{228}$$

$$\tilde{u}' = \tilde{u} + S_{\Delta u} \tilde{\Delta u} \tag{229}$$

$$\tilde{P}_u' = \tilde{P}_u + S_{\Delta u} \tilde{P}_{\Delta u} S_{\Delta u}^T \tag{230}$$

The equations of the $\tilde{x}$-, the $\tilde{u}$-, and the $\tilde{\Delta u}$-filters are given in Tables 5, 7 and 8, respectively.

8. CONCLUSIONS

We have developed a Friedland-like filtering technique for estimating the state x of a discrete linear stochastic process which depends on a piecewise constant control vector u. It is composed of three unaugmented Kalman-Bucy filters. In the first filter the estimate $\tilde{x}$ of the state is computed as if there were no control present and no jump in control present. This estimate is then corrected to account for the control and for the jump in control. In the second filter the estimate $\tilde{u}$ of the control is computed as if there were no jump in control present. This estimate is corrected to account for the jump in control. In the third filter the optimum estimate $\widetilde{\Delta u}$ of the jump in control is computed.

The optimum estimate $\hat{x}$ is a vector sum of the three estimates $\tilde{x}$, $\tilde{u}$ and $\widetilde{\Delta u}$. The coefficient of $\tilde{u}$ in the sum is a matrix having dependence on the gain of the $\tilde{x}$-filter. The coefficient of $\widetilde{\Delta u}$ in the sum is a matrix that depends on the gains of the $\tilde{x}$- and the $\tilde{u}$-filters.

The optimum estimate $\hat{u}$ is a vector sum of the two estimates $\tilde{u}$ and $\widetilde{\Delta u}$. The coefficient of $\widetilde{\Delta u}$ is a function of the gains of the $\tilde{x}$- and the $\tilde{u}$-filters.

The $\tilde{x}$-filter processes the measurement z. The $\tilde{u}$-filter processes as its measurement the a posteriori measurement residual $z-H\tilde{x}$ of the $\tilde{x}$-filter. The $\Delta\tilde{u}$-filter processes the a posterior measurement residual $z - H\tilde{x} - H S_u \tilde{u}$ of the $\tilde{u}$-filter in which S_u depends on the gain of the $\tilde{x}$-filter.

A procedure has been developed for reinitializing the $\tilde{x}$- and the $\tilde{u}$-filters after a jump has been detected so that the optimum estimates $\hat{x}$ and $\hat{u}$ are functions only of the output of those filters and, as a result, satisfy Friedland's expressions [21]. The reinitialization procedure permits the treatment of multiple jumps.

We have presented a GLR algorithm for detecting the jump time q. It uses the $\tilde{x}$- and $\tilde{u}$-filters and a bank of $\Delta\tilde{u}$-filters. The algorithm reinitializes the $\tilde{x}$- and $\tilde{u}$-filters after a jump has been detected. This GLR algorithm avoids numerical inaccuracies introduced by computations with large vectors and matrices due to augmenting the state vector of the original system with the control vector. The algorithm has particular application to problems involving a large number of state and/or jump variables.

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