(12)

ESD-TR-81-360

AD A112446

Technical Report 597

M.P. Jordan

Geometry of Earthshine Reflection from a Spherical Target

29 January 1982

Prepared for the Department of the Army
under Electronic Systems Division Contract F19628-80-C-0002 by

Lincoln Laboratory

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LEXINGTON, MASSACHUSETTS

Approved for public release; distribution unlimited.

82 03 24 072

The work reported in this document was performed at Lincoln Laboratory, a center for research operated by Massachusetts Institute of Technology. This program is sponsored by the Ballistic Missile Defense Program Office, Department of the Army; it is supported by the Ballistic Missile Defense Advanced Technology Center under Air Force Contract F19628-80-C-0002.

This technical report has been reviewed and is approved for publication.

FOR THE COMMANDER

Raymond L. Loiselle

Raymond L. Loiselle, Lt.Col., USAF
Chief, ESD Lincoln Laboratory Project Office

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
LINCOLN LABORATORY

GEOMETRY OF EARTHSHINE REFLECTION FROM A SPHERICAL TARGET

M.P. JORDAN
Group 32

TECHNICAL REPORT 597

29 JANUARY 1982

LEXINGTON MASSACHUSETTS

ABSTRACT

An expression is found for the flux due to earthshine incident at any point on a spherical target as a function of target height above ground. Curves showing the variation of reflected signature (or view factor) with observer position and target height are presented for the case of a diffuse target surface. An expression is given for the view factor in the specular case, which is shown to be independent of observer position.

CONTENTS

I. INTRODUCTION

An important component of the infra-red (IR) signature of a target outside the atmosphere is the earthshine it reflects. This report will present curves showing the amount of earthshine reflected by an idealized spherical target towards an IR sensor as a function of target altitude and sensor position for the two limiting cases of diffuse and specular target surfaces. A cloud of convex particles of arbitrary shape and random orientation will exhibit the same behavior as a sphere of the same surface characteristics.

The amount of energy incident on an area element dA of the target in a wavelength band $\Delta\lambda$ is

$$E(\Delta\lambda) = \int_{\substack{\text{all dS} \\ \text{visible} \\ \text{to dA}}} N_E(\Delta\lambda) \cos\theta \cos\psi \frac{dA}{\ell^2} \, dS \qquad (1)$$

where $E(\Delta\lambda)$ is the energy incident on dA (in Watts)

$N_E(\Delta\lambda)$ is the radiance of the earth at dS in Watts per square meter per projected steradian-assumed constant over the earth's surface.

θ and ψ are the angles between the line joining dA and dS and the local normals.

ℓ is the distance between dA and dS.

(See Fig. 1).

If the bidirectional reflectance of the material is known,

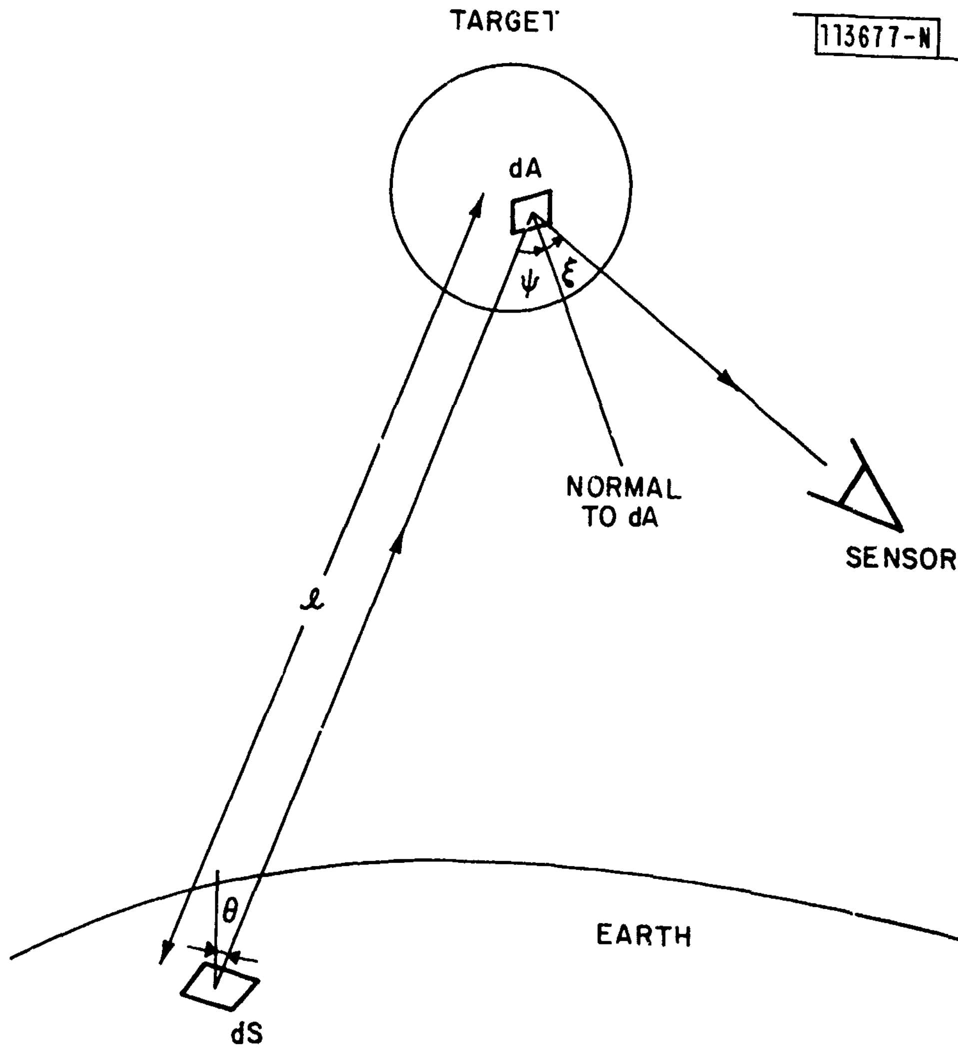


Fig. 1. Geometry of earthshine reflection from generalized target.

then the power reflected per steradian in the direction of the sensor can be calculated as

$$J(\Delta\lambda) = \int\limits_{\substack{\text{all dA}\\ \text{visible}\\ \text{to sensor}}} \int\limits_{\substack{\text{all dS}\\ \text{visible}\\ \text{to dA}}} N_E(\Delta\lambda)\cos\theta\ \cos\psi\ \cos\xi\ \rho(\psi,\phi_1,\xi,\phi_2)\left(\frac{1}{\ell^2}\right)\, dS\, dA \qquad (2)$$

where ϕ_1 and ϕ_2 are the azimuthal angles of dS and the sensor measured at dA. To simplify calculation, in this report we characterize the fraction of the input power which is reflected with a single number ρ (which is therefore assumed independent of incidence angle and wavelength over the range of measurement). Where this reflected power is directed will depend on our assumptions about the surface. A smooth surface will reflect specularly, i.e., the output distribution is a narrow peak centered on $\psi=\xi$. A rough surface will diffuse the output power isotropically into the hemisphere above it.

This simplification permits us to split up the integral into two parts, and first find the incident energy per unit area at any point on the target sphere as a function of postion on the sphere, and the sphere's height above ground. From this, the total amount of energy incident upon the target, and the fraction of it which is reflected toward a sensor in a given direction can be calculated.

II. DISTRIBUTION OF INCIDENT FLUX ON THE SPHERE

To find out how much energy is falling on any small patch of the target sphere with area dA, it is necessary to integrate the radiation emitted in the direction of the patch over all the earth visible to that part of the sphere.

$$\text{Energy density at the position of dA} = E = \int_{\substack{\text{visible}\\ \text{surface}\\ \text{of earth}}} N_E \frac{\cos\psi \, \cos\theta}{\ell^2} \, dS \tag{3}$$

where ℓ is the distance from the sphere to dS. (See Fig. 2).

This is a very difficult problem for all but the simplest cases, but for an isotropically emitting earth it is permissible to replace the earth by a flat circular disc of the same emittance subtending the same solid angle at the sphere. The incident flux at dA is then

$$E = \iint_{\substack{\text{visible}\\ \text{earth-disc}}} N_E \cos\theta \cos\psi \tan\theta \, d\theta d\phi \tag{4}$$

where the portion of the earth-disc visible to dA is described by $\theta_{min} \leq \theta \leq \beta$, and $-p \leq \phi \leq p$ as shown in Fig. 3, where β is the horizon angle and p depends on x, the position of dA on the sphere. θ_{min} depends on p and ϕ. Hence

$$E = \int_{-p}^{p}\int_{\theta_{min}}^{\beta} N_E \sin\theta(\cos\phi \sin\theta \sin x + \cos\theta \cos x) d\theta d\phi \tag{5}$$

113678-N

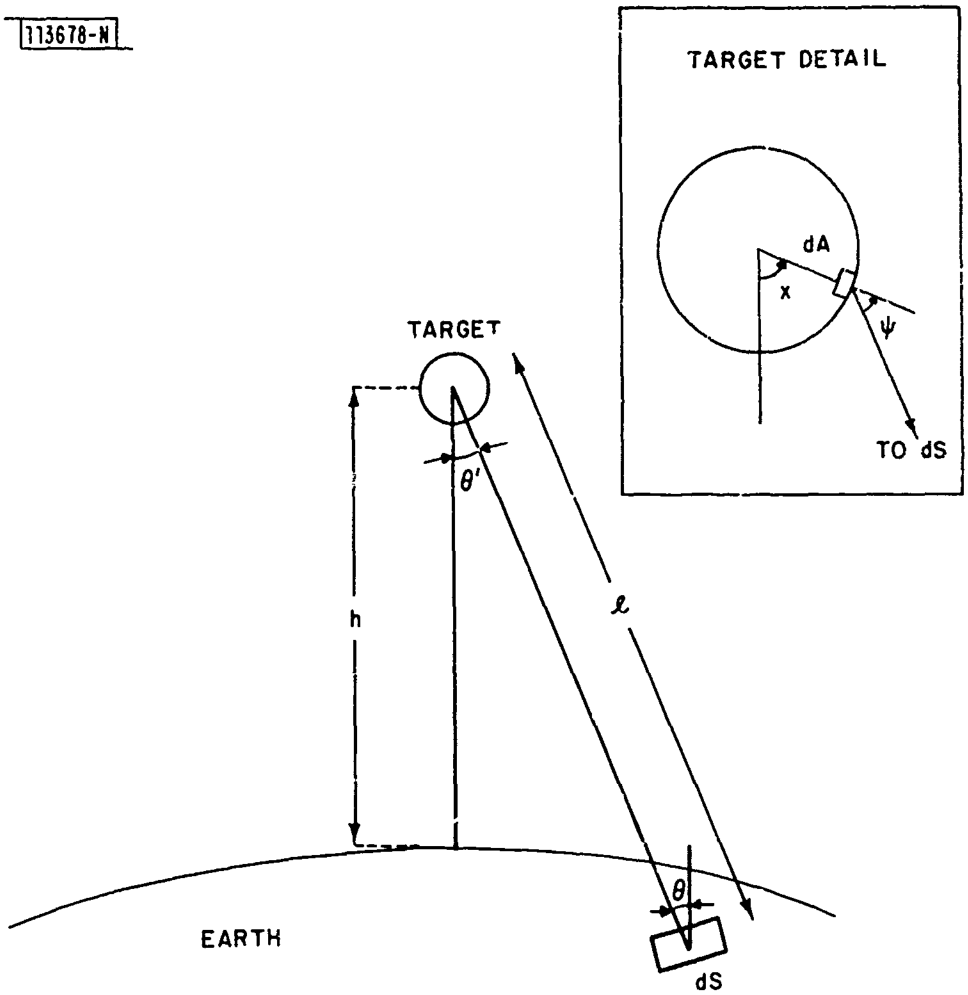


Fig. 2. Earthshine incident on spherical target.

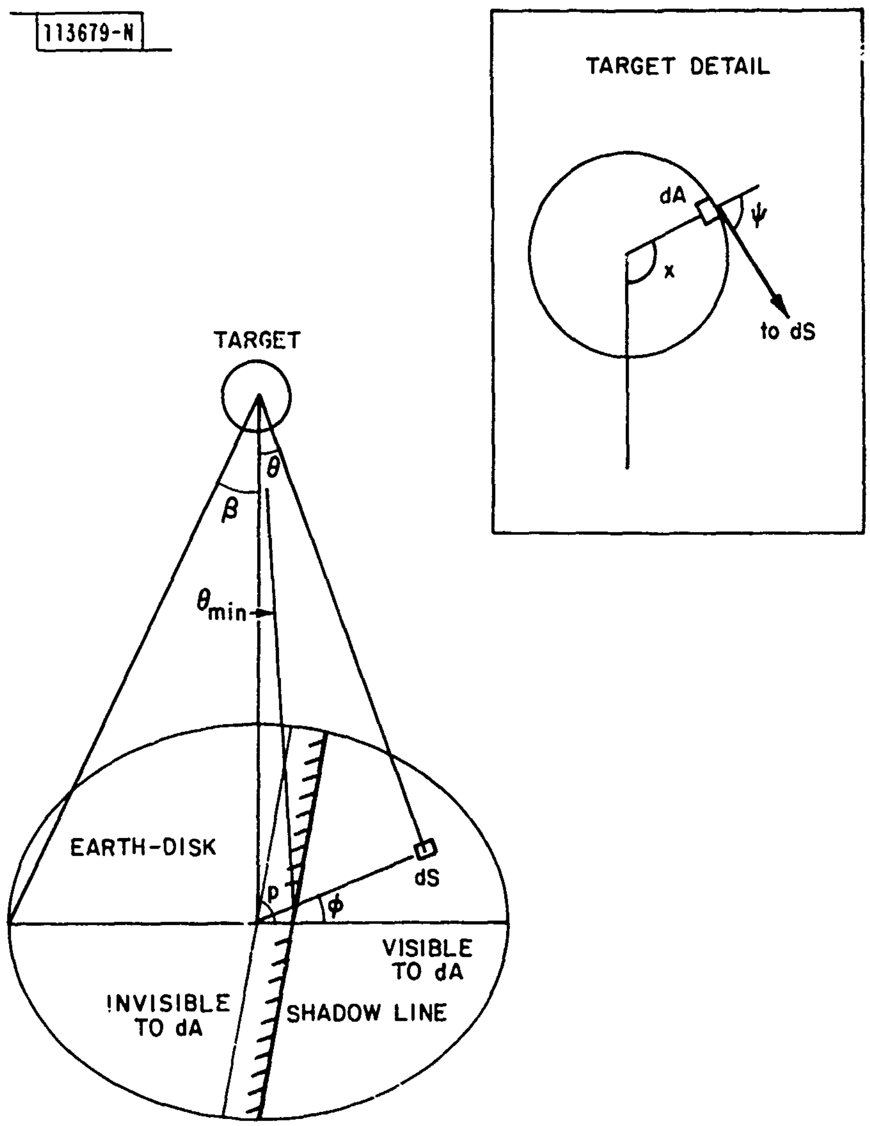


Fig. 3. Radiation from flat disc incident on spherical target.

Three regions of the sphere may be distinguished, each with a different form of expression for E. The arrangement of the regions is shown in Fig. 4(a), and their sizes as a function of height in Fig. 4(b).

Region 1 Contains those areas of the sphere which can "see" all the earth-disc out to the horizon, and are not shaded by other parts of the sphere. The range of values for x is

$$0 \leq x < \frac{\pi}{2} - \beta \qquad (6)$$

and for this region

$$\theta_{min} = 0; \quad p = \pi$$

since the integration is over the whole disc. This yields

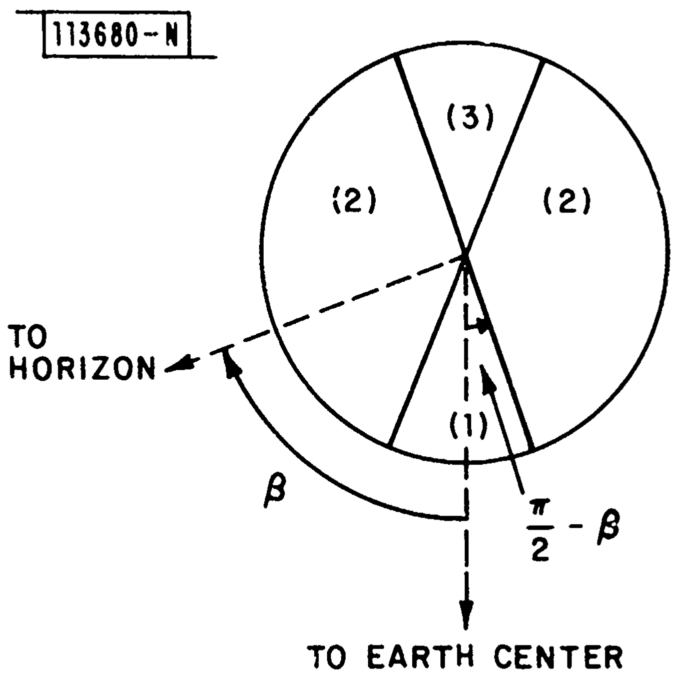
$$E = N_E \pi \cos x \sin^2 \beta \qquad (7)$$

Region 2 Contains those areas of the sphere which can "see" some of the earth-disc, but are shaded by other parts of the sphere from seeing the remainder. If a part of the earth can be seen by a given patch on the sphere, it will have

$$\cos \psi = \cos \phi \sin \theta \sin x + \cos \theta \cos x \geq 0 \qquad (8)$$

Hence for the range

$$\frac{\pi}{2} - \beta < x < \frac{\pi}{2} + \beta$$



g. 4(a). Regions (1), (2), and (3) on the target sphere.

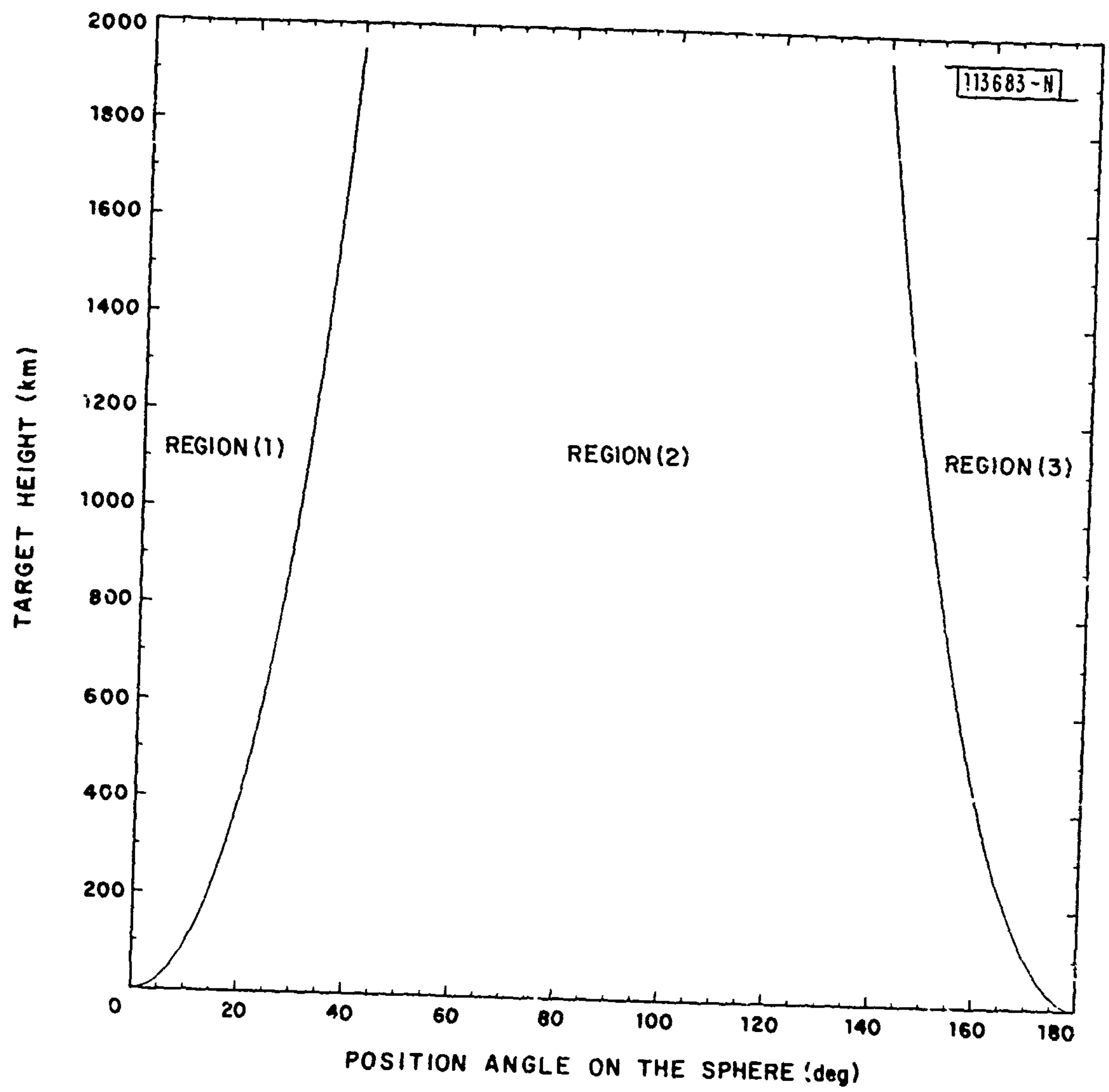


Fig. 4(b). Variation in the sizes of the three target regions with target height.

we have

$$\theta_{min} = \tan^{-1}\left(\frac{-1}{\cos\phi \ \tan x}\right) ; \quad p = \cos^{-1}\left(\frac{-1}{\tan\beta \ \tan x}\right) \tag{9}$$

which after considerable manipulation yields

$$E = N_E\{p \cos x \sin^2\beta - \sin p \sin x \sin\beta \cos\beta - \tan^{-1}(\tan p \cos x)\} \tag{10}$$

$$= N_E\left\{\cos^{-1}\left(\frac{-1}{\tan\beta \ \tan x}\right) \cos x \sin^2\beta - \cos\beta \sqrt{\sin^2 x - \cos^2\beta} + \cos^{-1}(\cos\beta/\sin x)\right\} \tag{11}$$

<u>Region 3</u> Includes those areas of the sphere which are completely screened from any view of the earth. This is true for the areas which have

$$x > \frac{\pi}{2} + \beta \tag{12}$$

Not surprisingly, this gives

$$E = 0 \tag{13}$$

If the height of the sphere above ground is h, and the earth's radius is R, then

$$\sin\beta = \frac{R}{h+R} = \frac{1}{1+\alpha} \text{ if } \alpha = h/R \tag{14}$$

The function $E' = E/N_E$ is plotted in Fig. 5 as function of x(in degrees) with h as a parameter. To facilitate comparison of the angular dependence of E' at different heights, Fig. 6 shows the same curves but normalized to unity at x = 0 (i.e.,

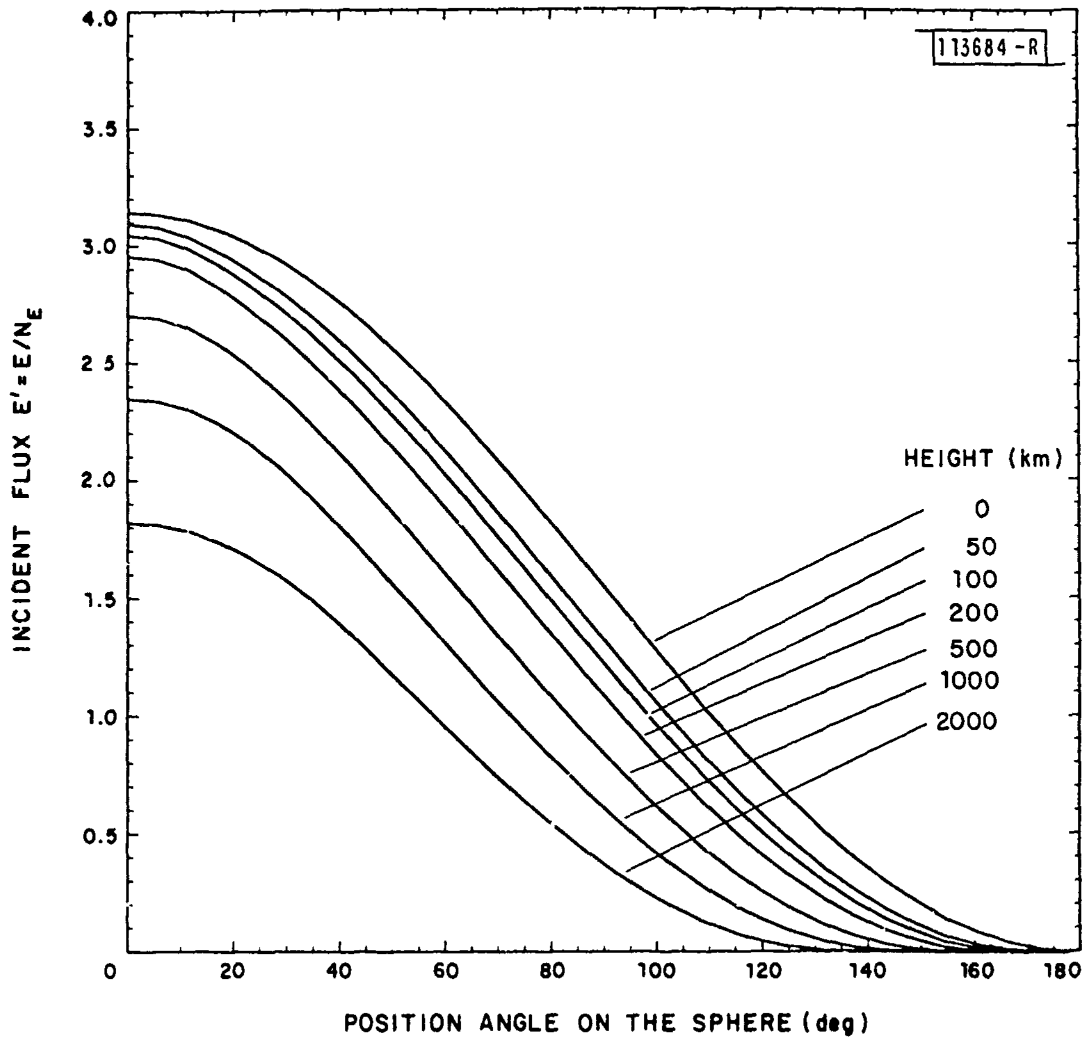


Fig. 5. Incident flux as a function of position on the sphere for various target heights.

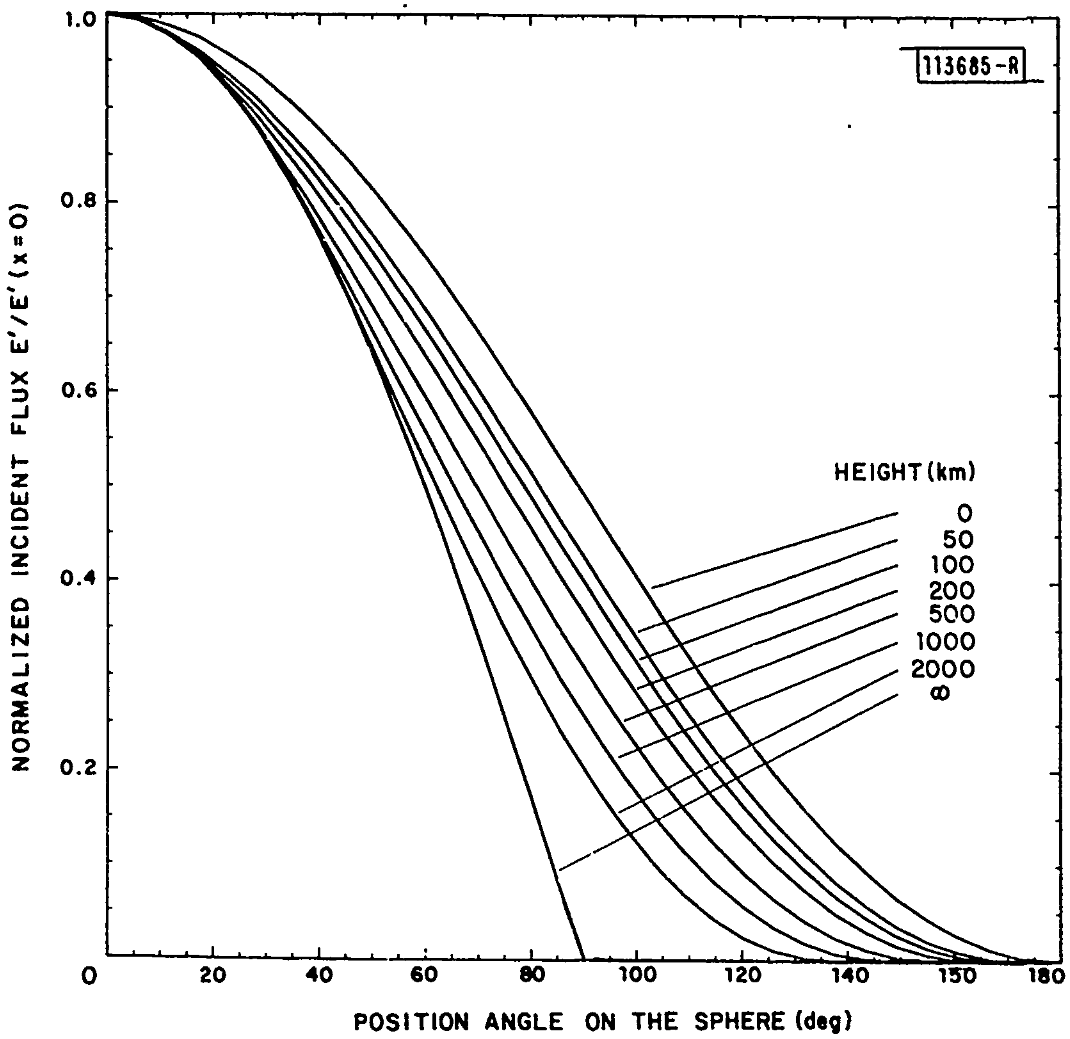


Fig. 6. Incident flux normalized to unity for x=0 for various target heights.

the curves are of $E'(x,\beta)/E'(0,\beta)$). This format shows the different behavior in the three regions; i.e., following the cosine curve in region 1, intermediate behavior in region 2 and identically zero in region 3.

Special Cases When h is either very small or very large, the expression for E can be simplified.

As $h \to 0$, $\beta \to \pi/2$, and the region 2 solution is valid for most of the sphere. Looking at this solution as $\beta \to \pi/2$ and therefore $p \to \pi/2$ we find

$$E(x, \beta \to \pi/2) = N_E \pi (1+\cos x)/2 \tag{15}$$

At the other extreme, when the earth is very far from the sphere, it acts like a point source, and the region 2 disappears. Hence

$$\begin{aligned} E(x,\beta \to 0) &= N_E\ \pi\cos x \sin^2\beta \qquad x < \pi/2 \\ &= 0 \qquad\qquad\qquad\qquad\ \ x \geq \pi/2 \end{aligned} \tag{16}$$

III. TOTAL POWER

From a knowledge of the incident flux on the sphere as a function of position, the total power incident on the sphere can be found at any height by integrating E over its surface. In practice, the form of equation (11) makes the integration rather arduous, but it eventually yields the result

$$\begin{aligned} P &= 2N_E\, \pi r^2 \pi (1-\cos\beta) \\ &= 2N_E\, A_p\, \pi (1-\cos\beta) \end{aligned} \tag{17}$$

where r = the sphere's radius and $A_p = \pi r^2$ = the projected area of the sphere.

IV. OBSERVED BRIGHTNESS

The infra-red brightness of the sphere, as seen by a sensor in the far field, is obtained by integrating the power reflected by the sphere in the direction of the sensor, over the hemisphere visible to the sensor.

Case (a). Specular Surface

If the surface of the sphere reflects specularly, then its spherical shape will diverge an incoming beam of radiation so as to put equal power in all directions, as will now be shown.

Let a plane wave be incident on a specularly reflecting sphere of radius r. (See Fig. 7). To calculate the power scattered by the sphere as a function of angle measured from the direction of origin of the plane wave, we find the power incident on an imaginary enclosing sphere of radius $R>>r$ as a function of position. If $r>>\lambda$ the wavelength of the radiation, each (θ_2, ϕ_2) point on the outer sphere will receive radiation from only one point (θ_1, ϕ_1) on the inner sphere. The small area $dA_1 = r^2 \sin\theta d\theta d\phi$ on the inner sphere has a total incident power of $P\ dA_1 \cos\theta$ Joules/sec if P Joules/m^2sec is the power density in the plane wave. A fraction (ρ) of this is reflected. The corresponding area on the outer sphere is $dA_2 = R^2 \sin\theta_2 d\theta_2 d\phi_2$, and by the law of specular reflection $\theta_2 = 2\theta_1$; $d\theta_2 = 2d\theta_1$; $\phi_2 = \phi_1$. This means $dA_2 = 4(R^2/r^2) dA_1 \cos\theta$. All the power reflected by dA_1 falls on

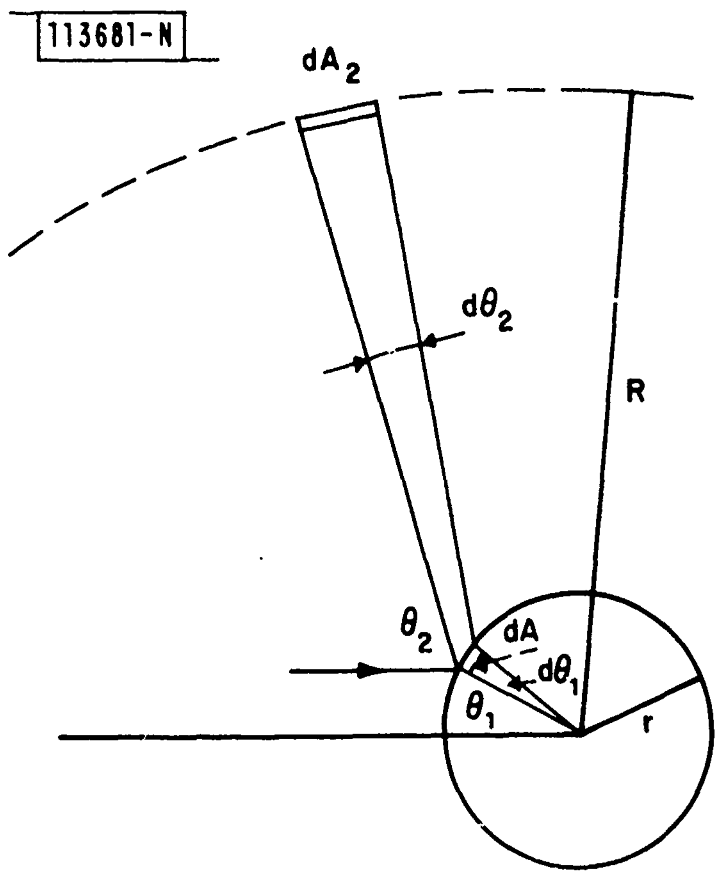


Fig. 7. Reflection of a plane wave by a sphere with a smooth surface.

dA_2, which therefore has a received power density equal to:

$$\frac{\rho P dA_1 \cos \theta_1}{dA_2} = \frac{\rho P r^2}{4R^2} \quad \text{independent of position.}$$

Hence a plane wave incident on a specularly reflecting sphere is reflected in all directions equally. The radiant energy from the earth can be considered as made up of an angular spectrum of plane waves. Each plane wave will be reflected isotropically and so the total brightness also will be isotropic.

The received power per steradian of sensor is therefore just (the total power reflected from the sphere)/4π, which is

$$J = 2N_E A_p \pi (1-\cos\beta)\rho/4\pi \tag{18}$$

$$J = N_E A_p \rho (1-\cos\beta)/2 \tag{19}$$

A value frequently used to characterize a particular geometry is the view factor F, defined by

$$F = \frac{J}{N_E \rho A_p} \tag{20}$$

For the specular case therefore,

$$F = (1-\cos\beta)/2 \tag{21}$$

and is independent of sensor position.

Case (b). Diffuse Surface

If the sphere is a diffuse reflector, then the observed brightness will depend on the sensor angle γ (see Fig. 8). The radiant intensity seen by the sensor is obtained by integrating $(E \cos\xi)/\pi$ (since the incident earthshine is scattered into π projected steradians) over the half of the sphere visible to the sensor $(0 \leq |\xi| \leq \frac{\pi}{2})$. In the previous section expressions for E were obtained, but as a function of x, which is not equal to ξ unless $\gamma=0$ (sensor below target). Expressing E as a function of ξ and γ in the general case leads to a form too complicated for analytical integration. Numerical integration of E from equations (6) to (13) was therefore used to produce the curves of Fig. 9. They show the view factor $F(\gamma,\beta)$ for the sphere as a function of sensor angle for various target heights. As before, the curves are replotted in Fig. 10 normalized to unity at $\gamma=0$, to show the angular dependence more clearly. Figure 11 shows $F(\gamma,\beta)$ as a function of height for several values of γ.

Special Cases When the target height is either very small or very large, the formulae of equations (15) and (16) can be used to yield closed form expressions for the view factor of a diffuse target, since they are easily integrable. The resulting expressions are

$$F(\gamma,\beta\to\frac{\pi}{2}) = \frac{1}{2} + \frac{1}{3}\cos\gamma \qquad (22)$$

113682-N

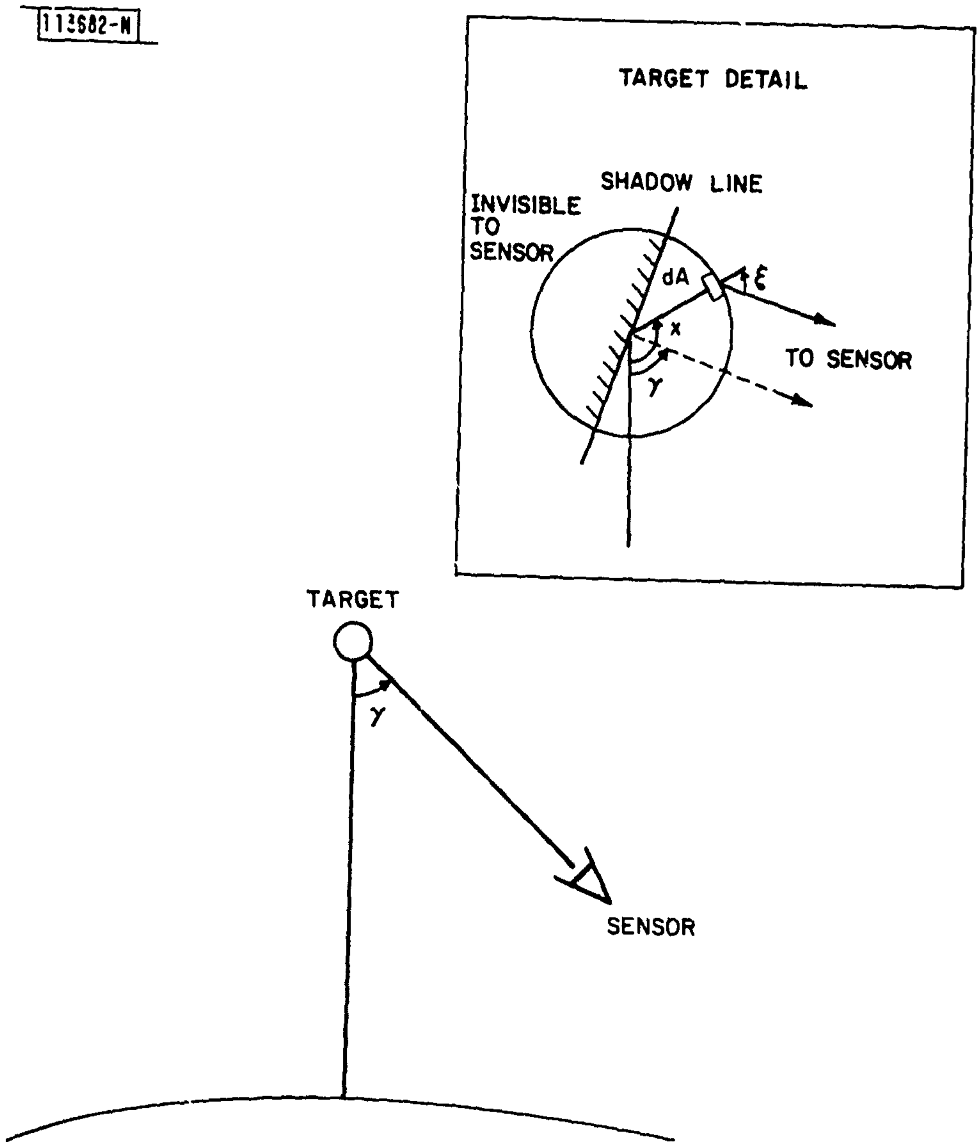


Fig. 8. Scattering of reflected light to sensor by diffuse sphere.

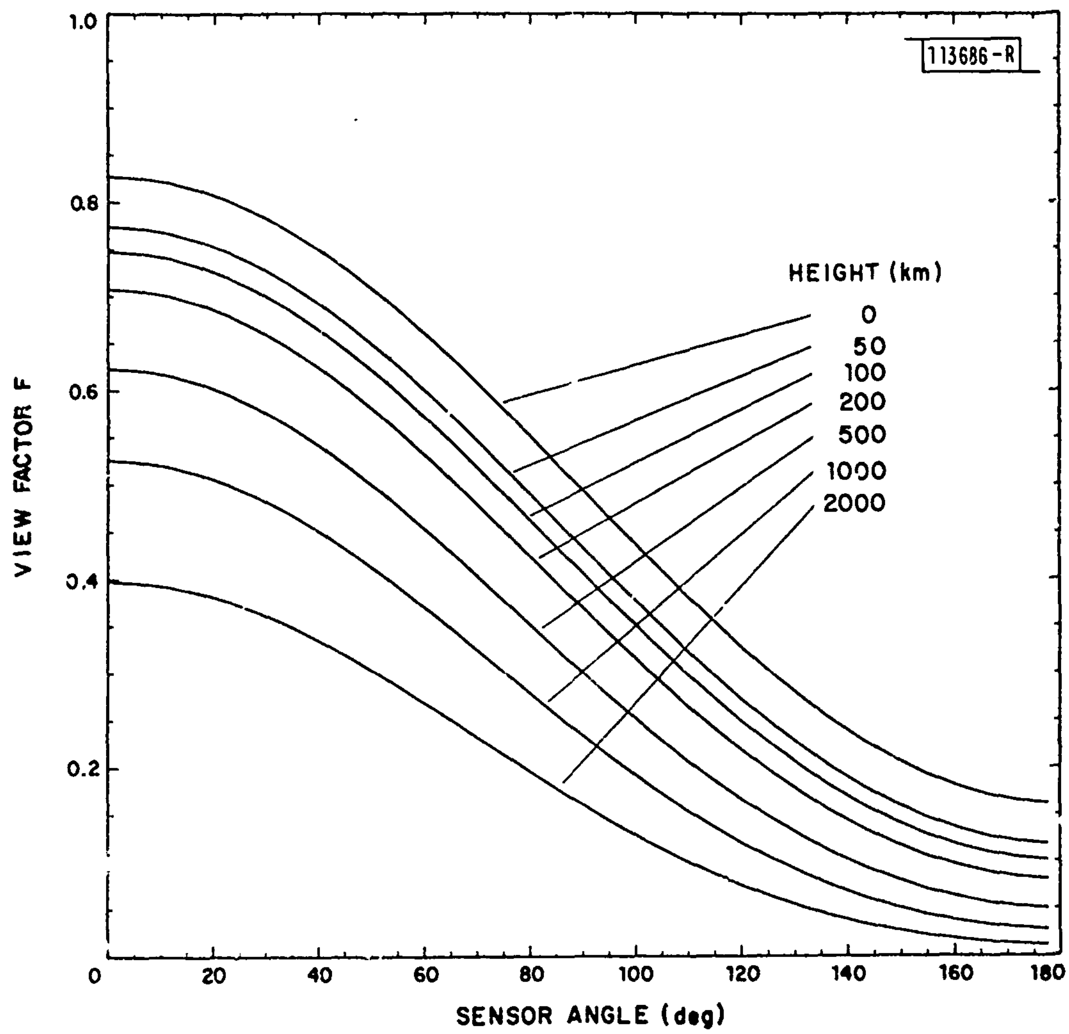


Fig. 9. View factor of diffuse sphere as a function of sensor angle for various target heights.

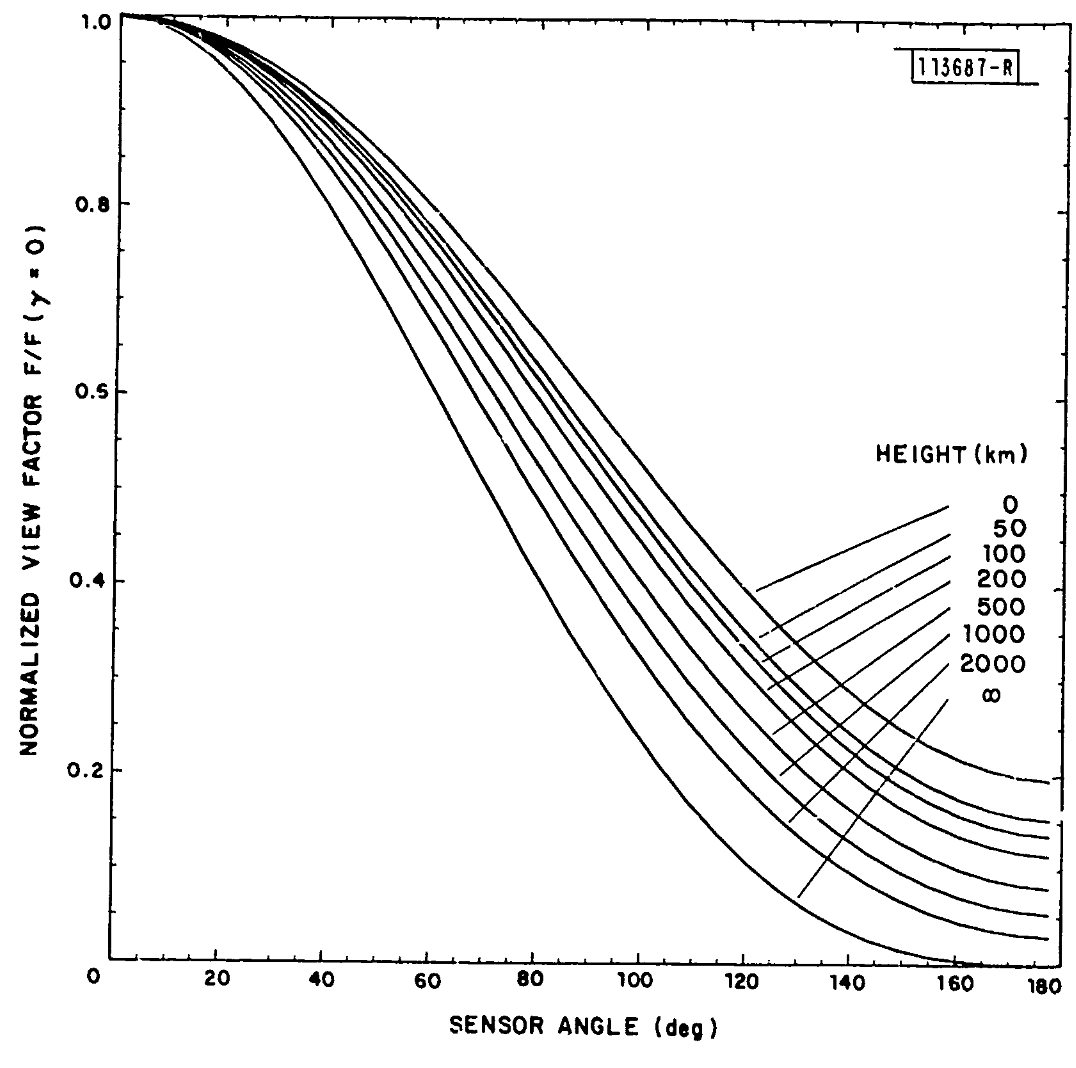


Fig. 10. View factor of diffuse sphere normalized to unity for sensor angle $\gamma=0$; various target heights.

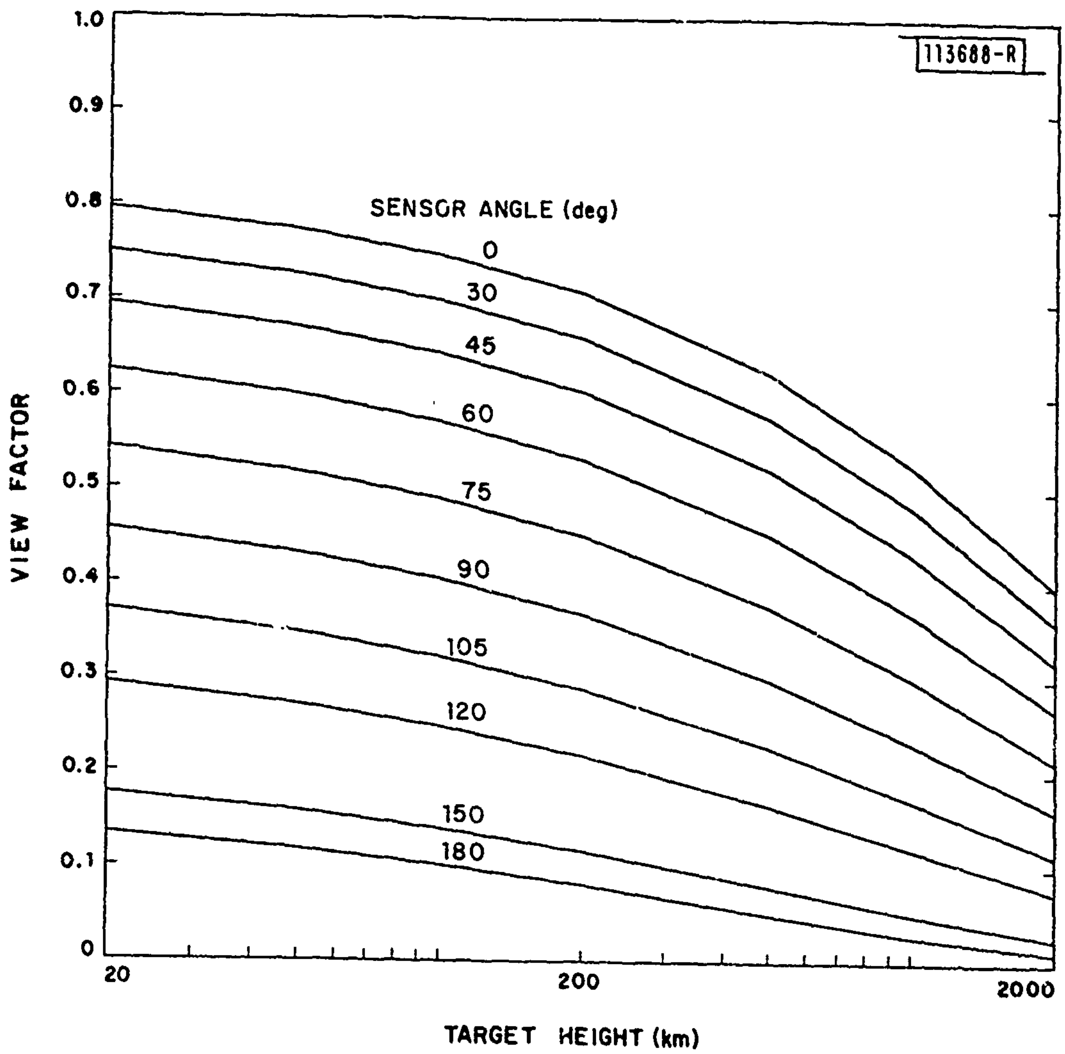


Fig. 11. View factor of diffuse sphere as a function of target height for various sensor angles.

$$F(\gamma,\beta\to 0) = \frac{2}{3\pi}\left[\sin\gamma + (\pi-\gamma)\cos\gamma\right]\sin^2\beta \tag{23}$$

These curves are shown as $h=0$ and $h=\infty$ on Figs. 9 and 10.

V. SUMMARY

Simple expressions have been obtained for the distribution of earthshine incident on a spherical target, as a function of target height. These results apply to the case of a uniformly and isotropically emitting earth.

Curves for the reflected signature (or view factor) measured by a sensor at an arbitrary location have been obtained by integrating the reflected earthshine over the portion of the target visible to the sensor, for the case of a diffuse target. A simple expression is given for the view factor of a specular target.

The view factor results are also valid for clouds of small randomly shaped convex particles of corresponding surface characteristics.

ACKNOWLEDGMENTS

The author would like to thank Dr. R. Pavelle, Dr. R. M. Spitzberg and Dr. S. D. Weiner for their assistance and careful reviews in the preparation of this report. The skillful typing of C. A. Tisdale is also greatly appreciated.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER ESD-TR-81-360	2. GOVT ACCESSION NO. A112 446	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Geometry of Earthshine Reflection from a Spherical Target		5. TYPE OF REPORT & PERIOD COVERED Technical Report 6. PERFORMING ORG. REPORT NUMBER Technical Report 597
7. AUTHOR(s) Michael P. Jordan		8. CONTRACT OR GRANT NUMBER(s) F19628-80-C-0002
9. PERFORMING ORGANIZATION NAME AND ADDRESS Lincoln Laboratory, M.I.T. P.O. Box 73 Lexington, MA 02173		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Program Element Nos. 63304A and 63308A
11. CONTROLLING OFFICE NAME AND ADDRESS Ballistic Missile Defense Program Office Department of the Army 5001 Eisenhower Avenue Alexandria, VA 22333		12. REPORT DATE 29 January 1982 13. NUMBER OF PAGES 32
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Electronic Systems Division Hanscom AFB, MA 01731		15. SECURITY CLASS. (of this report) Unclassified 15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

None

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

earthshine reflection
infrared signatures
flux incidence
target sphere

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

An expression is found for the flux due to earthshine incident at any point on a spherical target as a function of target height above ground. Curves showing the variation of reflected signature (or view factor) with observer position and target height are presented for the case of a diffuse target surface. An expression is given for the view factor in the specular case, which is shown to be independent of observer position.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)