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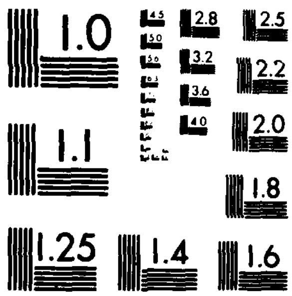

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THEORETICAL AND EXPERIMENTAL STUDIES IN THE RHEOMETRY OF VISCOELASTIC MATERIALS

LEVEL II

FINAL REPORT

BY

D. D. Joseph AND G. S. Beavers

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Theoretical and experimental studies of a wide range of problems in the rheometry of viscoelastic fluids and solids are described. Domain perturbation analyses and numerical computations are used to analyze the fluid motion between concentric and eccentric cylinders. Experiments involving free surface measurements complement the anaylsis and lead to the determination of parameters which characterize the fluid. The techniques have been extended to

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configurations involving oscillating rods and oscillating cylinders. Some investigations associated with the extrudate swell problem are reported, including solution techniques for fluids with high surface tension and very small surface tension. The importance of density matching is demonstrated for measurements in several geometries including rod climbing, the tilted trough, and jet extrusion. The feasability of using a rimming flow experiment to determine the complex viscosity is demonstrated. Mathematical studies for Stokes flow problems are reported, and the application of domain perturbation techniques are demonstrated. Some free surface problems for viscoelastic solids in shear are reported.

TABLE OF CONTENTS

1. INTRODUCTION

This report summarizes the research accomplishments of the research program supported under U.S. Army Research Office Grant Number DAAG 29-79-C-0038. The research has covered a broad range of problems associated with the viscometry of non-Newtonian fluids from both analytical and experimental points of view. Some problems in the rheometry of viscoelastic solids have also been considered.

In this report we restate our objectives, review the major findings of our work, and list the abstracts of all the publications that have evolved from this research program.

2. STATEMENT OF OBJECTIVES

The objective of our research program is the study of a wide range of problems associated with the rheology of viscoelastic fluids and solids. Our program has a theoretical part and an experimental part. The first aim of the theoretical part is the development of the perturbation equations of motion and the specification of the procedures to be used in computing histories from the given data (boundary values and prescribed body forces). The second aim of the theoretical part is the solution of problems which have a potential for rheometrical measurement. (We are of course also interested in the well-known, unsolved problems of rheology, like "die-swell".) The experimental part of the program is used to assess the value of the theoretical solutions for applications and to test the practicality of these solutions for rheometrical measurements. We find that experiments in this exciting subject always turn up new effects of scientific, if not always of technological, interest. And we are gratified by the intimate way the two parts of our program stimulate on another.

The point of view that we have followed is that materials do not have explicit constitutive equations which apply to all the possible deformations. We think that explicit constitutive equations live only in certain circumscribed classes of deformations. The good classes of deformations are those which lead to constitutive equations which allow one to develop a science of measurements which assigns to a particular material the values of quantities which distinguish it from other members of its class (rheometry). For example the constitutive relation for simple materials is general enough to accommodate a variety of responses available to materials which depend on the history of the first (but not higher) spatial gradients of the deformation. But the constitutive relation for simple materials is still too general because it is impossible to solve most of the problems of material response at this level of generality. Without such solutions it is impossible to infer explicit forms for the constitutive relations which distinguish one simple fluid from another. To circumvent this difficulty it is useful to define restricted problems of rheometry. In these problems the deformations are restricted to special classes for which the constitutive equation simplifies. Rheometry is then defined relative to deformations in the restricted class. Some restricted problems of rheometry are:

(1) Find the three viscometric functions for simple fluids in viscometric flow.

(2) Find the rheological constants for fluids of grade n undergoing slow steady motion.

(3) Find the two material functions which define the second order theory for time dependent motions which perturb states of rigid motion of an incompressible simple fluid.

(4) Find the two elastic constants and the three material functions which define the second order theory of time-dependent deformations which perturb states of rest of incompressible viscoelastic solids.

The first of the above problems is well understood, and is only a minor component of our research. The last three problems are the main areas of interest in our research program, and we believe that aspects of our work under these three problem areas are new to rheology.

3. SUMMARY OF MAJOR RESEARCH RESULTS

3.1 Fluid Motion Between Rotating Cylinders

3.1(a) Higher Order Theory of the Weissenberg Effect

When a viscoelastic fluid is sheared between two concentric cylinders undergoing differential rotation the free surface on the fluid is deformed as a consequence of the normal stresses induced in the fluid by the shearing motion. This free surface deformation in the concentric cylinder geometry is the general Weissenberg effect. If the radius of the inner cylinder is small compared with that of the outer cylinder the normal stresses will drive the fluid up the inner cylinder provided that the radius of the inner cylinder is less than a critical value which depends upon some of the parameters which characterize the fluid. However, when the radii of the two cylinders are comparable, the response of the free surface to the normal stresses is not simple, and the free surface can exhibit a variety of shapes which depend not only upon the viscoelastic parameters characterizing the fluid but also upon the radii of the cylinders and the differential rotation speed. We have completed the perturbation analysis up to and including terms of $O(\Omega^4)$ (Ref. 1). The response depends on the viscosity and four other parameters. All five parameters can be determined from measurements of quantities computed in the theory, and in Ref. 1 we outline the procedure for doing this.

3.1(b) The Free Surface on a Fluid Between Concentric Rotating Cylinders

The higher order theory of Ref. 1 can be used to predict the shape of the free surface on a viscoelastic fluid contained between concentric rotating cylinders. The shape depends upon the fluid, the diameters of the cylinders, and the speed of rotation. For some combinations of fluid and geometry the fluid has the greatest climb near the inner cylinder, while for other combinations the fluid climbs in the center of the gap and sinks near both cylinder walls. We have demonstrated these effects by means of experiments with STP, TLA-227, and polyacrylamide in a glycerin-water mixture. The agreement between experiment and prediction is excellent, (Ref. 2).

3.1(c) Fluid Motion Between Eccentric Rotating Cylinders

The perturbation analysis and numerical computations to predict the shape of the free surface of a viscoelastic fluid contained between vertical eccentric rotating cylinders have been completed through second order. Even very small eccentricities cause large changes in the free surface shapes from the corresponding concentric configuration. The first order lubrication contribution dominates the free surface shape. Experimentally measured profiles for various (small) eccentricities and rotational speeds give good agreement at low speeds with predicted profiles for a fluid with large normal stresses (TLA-227), but do not agree as well for a fluid with low normal stresses (STP). The general conclusion from this project is that the use of eccentric, (and probably even concentric) cylinders for rheological free-surface measurements is probably not a practical procedure. This work is now in preparation for publication (Refs. 3 and 4).

3.1(d) Experiments on Free Surface Phenomena

The experimental aspects of our work on the potential for making rheological measurements on viscoelastic fluids from the study of free surface deformations induced by motions which perturb the rest state are reviewed in Ref. 5. That paper shows how important characterizing properties of viscoelastic fluids can be determined through a combination of simple experimental geometry and perturbation analysis. The paper also includes brief descriptions of some interesting phenomena which can arise in flow regimes which are beyond the applicability of the perturbation analysis, and which have not yet been subjected to analytical investigation.

3.2 Fluid Motion in Oscillating Geometries

3.2(a) Oscillating Rods

A detailed experimental investigation of the fluid motion induced by a circular rod oscillating about its axis in a large volume of viscoelastic fluid has been reported (Ref. 6). The work included a preliminary investigation of the effect of shear heating on the temperature of the fluid near the rod at different oscillation rates. These experiments allowed us to devise a procedure whereby the height-of-climb measurements could be carried out at a constant temperature. This work also extended the generalized Maxwell model to higher order than we used in our first experiments with oscillating rods. We have also investigated the asymptotic form of the theory for large angular frequencies.

3.2(b) Two Immiscible Fluids in a Vertical Oscillating Cylinder

Two immiscible polymer liquids of different densities are contained in a vertical cylinder which undergoes oscillations about its axis.

Solutions up to second order in the small perturbed parameter ε have been obtained for the problem of an infinitely tall cylinder. In deriving such solutions, an approximation consistent with the condition ε being reasonably small was adopted. A coarse program, yet accurate enough for practical purposes, has been written to compute the interface shape under various contact angle or fixed-edge conditions. Some trial runs of the programs showed qualitative agreement with what had been observed in experiments with small angular frequency ω. At relatively large ω, the computed interface shape is much more severely deflected than is observed in experiments. These discrepancies are not unexpected, partly because we have assumed some unknown material parameters, partly because of the limits inherent in the derivation of the solutions. We have carried out a series of experiments to determine the dependence of the interface shape on the angular frequency and amplitude. The results of this project are now being prepared for publication (Ref. 7).

3.3 Extrudate Swell

Several analytical investigations associated with the "die-swell" problem have been carried out during the grant period.

The existence of an integral invariant of the axial momentum of an axially-symmetric capillary jet of liquid extruded into a zero gravity field has been established, which should be useful in future analyses of the extrudate swell problem (Ref. 8).

The analysis of the slow flow of a jet of fluid extruded from a circular die has been completed for liquids with large surface tension (Ref. 9) and for liquids with small surface tension (Ref. 10). The large surface tension problem is solved in two ways, using the Wiener-Hopf method and by the eigenfunction method. The method of solution for large surface tension fails completely when the surface tension is zero. For this situation a new stick-slip problem is defined and solved using matched eigenfunction expansions.

J. Matta (Chem. Syst. Lab., APG) has reported experiments with jets of viscoelastic liquids extruded vertically into air and into water, in which he shows that the final diameter of jets extruded into water is as much as four or five times the final diameter of the same jet extruded into air. We believe that this increase is an effect of density matching. To show the effects of density matching in a preliminary way, we have carried out an approximate analysis for the situation in which the viscosity of the host liquid is much smaller than the viscosity of the jet, (Ref. 11).

3.4 Other Flow Geometries

3.4(a) Rimming Flow

The flow of a first order liquid coating the inside of a horizontal, steadily rotating cylinder has been studied analytically and experimentally, (Ref. 12). We perturb rigid rotation with gravity, and solve the first order problem in which the stress depends on the material through the complex viscosity $\eta^*(\Omega)$. An experimental apparatus has been constructed in which the circumferential variation of film thickness is measured using a light absorption technique. We determine the real and imaginary parts of the complex viscosity $\eta^*(\Omega)$ for different speeds Ω from the measured amplitude and phase of the film thickness variation as a function of angular position. The values of $\eta^*(\Omega)$ thus obtained for STP and TLA-227 agree well with values obtained using an oscillating plate technique. The flow inside the cylinder is unstable when the rotational speed is too slow. We have studied the stability of the flow for a

Newtonian liquid at low Reynolds numbers. Since the rigid rotation is stable in the absence of gravity, it is necessary to perturb the eigenvalue which characterizes stability up to second order.

3.4(b) The Tilted Trough; Normal Stress Amplification

The tilted trough problem has been solved for the case of two different immiscible fluids in the trough, (Ref. 13). The magnitude of the free surface bulge at the center line of the trough can be greatly enhanced by floating one fluid on top of another of nearly the same density, which can lead to a more accurate measurement of the second normal stress difference than with a single fluid.

3.4(c) Flow Across a Rectangular Cavity

The analysis has been completed for both shear-driven slow flow and pressure-driven slow flow of a second order fluid over a rectangular cavity of arbitrary dimensions. The streamlines have been plotted and the "hole pressure" has been computed for different geometries, (Ref. 14).

3.5 Mathematical Studies

The following mathematical studies have been completed during the grant period. The main results from these projects are summarized in the abstracts of the corresponding publications given in Section 4.5.

3.5(a) A New Separation of Variables Theory for Problems of Stokes Flow and Elasticity (Ref. 15).

3.5(b) Perturbation of States of Rest and Rigid Motion of Simple Fluids and Solids (Ref. 16).

3.5(c) Stokes Flow in a Driven Sector by Two Different Methods (Ref. 17).

3.5(d) Some Inequalities in the Theory of Simple Fluids (Ref. 18).

3.5(e) Instability of the Rest State of Fluids of Arbitrary Grade Greater than One (Ref. 19).

3.5(f) Boundary Conditions for Thin Lubrication Layers (Ref. 20).

3.6 Non-Newtonian Fluid Characterization

We are continuing an experimental program in which we are attempting to relate molecular properties and solution concentrations of moderately concentrated polymer solutions to continuum characterizing parameters, and in particular to the climbing constant which is determined by means of rotating rod experiments. We are presently working with various molecular weights of PIBMA, two molecular weights of PIB, and a single sample of PMMA (acryloid 125). Solvents under consideration are chloroform, dowanol, acetone, and decalin. The climbing constant is strongly dependent upon both polymer concentration and polymer molecular weight. An interesting phenomenon has been observed for some fluids in which an apparent jump occurs in the climb of the free surface at some characteristic rotational speed of the rod. We think there may also be a hysteresis effect. We are continuing to pursue investigations of these phenomena. Part of this work formed the thesis topic of A. Georgacas Hoger, and is now being prepared for publication (Ref. 21).

3.7 Rheometry of Viscoelastic Solids

We have developed a general perturbation theory through second order for the deformation of traction-free surfaces which arise from perturbing the natural state of incompressible viscoelastic solids. The theory has potential for rheological measurements on viscoelastic solids. The theory has been applied to the problem of the twisting of a rod which is rigidly bonded in a solid, and a method has been developed for computing information on some of the characterizing viscoelastic parameters from measurements of the torque and the traction-free surface shape. A simple preliminary experiment with a urethane rubber sample reaffirms our belief that this approach has good potential, (Ref. 22). The analysis has also been applied to obtain solutions for the shape of the upper and lower boundaries of a linearly viscoelastic block which is sheared at the vertical sidewalls (Ref. 23).

4. Publications

The following publications have been produced during the grant period.

4.1 Fluid Motion Between Rotating Cylinders

Ref. 1 J.Y. Yoo, D.D. Joseph, and G.S. Beavers, "Higher-order Theory of the Weissenberg Effect", Journal of Fluid Mechanics, Vol. 92, No. 3, pp. 529-590, 1979.

"The higher-order theory of the Weissenberg effect is developed as a perturbation of the state of rest. The perturbation is given in powers of the angular frequency Ω of the rod and the solution is carried out to $0(\Omega^4)$. The perturbation induces a slow-motion expansion of the stress into Rivlin-Ericksen tensors in combinations which are completely characterized by five viscoelastic parameters. The effects of each of the material parameters may be computed separately and their overall effect by superposition. The values of the parameters may be determined by measurement of the torque, surface angular velocity and height of climb. Such measurements are reported here for several different sample fluids. Good agreement between the third-order theory and measured values of the velocity is reported. Secondary motions which appear at $0(\Omega^4)$ are computed using biorthogonal series. The analysis predicts the surprising fact that secondary motions run up the climbing bubble against gravity and intuition."

Ref. 2 G.S. Beavers, J.Y. Yoo, and D.D. Joseph, "The Free Surface on a Liquid Between Cylinders Rotating at Different Speeds. Part III", Rheologica Acta, Vol. 19, pp. 19-31, 1980.

"This paper is a further contribution to our work on the Weissenberg effect. One goal of our work is to show how the free-surface deformations on a viscoelastic fluid which is sheared between two concentric rotating cylinders can be used to determine rheological data about the fluid. In this paper we report the results of an experimental program in which free surface shapes have been measured on three viscoelastic fluids undergoing shearing in an apparatus consisting of a stationary outer cylinder and four interchangeable rotating inner cylinders. Experimentally-measured profiles are compared with profiles predicted from second-order theory, and found to be in excellent agreement for STP and TLA-227 in the range of applicability of the second-order theory, but there is some disagreement in the results for a 2% solution of polyacrylamide in 49% water - 49% glycerin."

Ref. 3 A. Siginer, "The Free Surface on a Liquid Between Eccentric Rotating Cylinders. Part I: Theory", (in preparation), 1982.

Ref. 4 A. Siginer and G.S. Beavers, "The Free Surface on a Liquid Between Eccentric Rotating Cylinders. Part II: Experiments", (in preparation), 1982.

Ref. 5 G.S. Beavers and D.D. Joseph, "Experiments on Free Surface Phenomena", Journal of Non-Newtonian Fluid Mechanics, Vol. 5, pp. 323-352, 1979.

"This paper reviews an experimental program in which predictions from domain perturbation theory for motions which perturb the rest state are used in conjunction with experimental measurements on free surface deformations associated with the Weissenberg effect at low rates of shear to obtain values for certain rheological parameters. The experiments include free surface measurements on a liquid near a circular rod rotating in steady motion and oscillatory motion in a large volume of fluid, secondary flows and circumferential velocity measurements in steady rod climbing, normal stress amplification effects, and free surface shapes on a fluid confined between rotating cylinders. Temperature effects are noted, and interesting instability phenomena at high rates of shearing are demonstrated."

4.2 Fluid Motion in Oscillating Geometries

Ref. 6 B.E.D. Kolpin, G.S. Beavers, and D.D. Joseph, "Free Surface on a Simple Fluid Between Cylinders Undergoing Torsional Oscillations. IV. Oscillating Rods", Journal of Rheology, Vol. 24, No. 6, pp. 719-739, 1980.

"In earlier papers (Parts I and II) we gave the perturbation analysis for the prediction of the free surface on a simple fluid near an oscillating rod and presented the results of a preliminary experiment involving a single rod in one sample of simple fluid. In this paper we report the results of a recent more extensive experimental investigation aimed at testing the predictions of the theory. These experiments involve the use of two different simple fluids and three rods of differing diameters. The experimental results reinforce our earlier results and substantiate the predictions of the theory with respect to the diameter dependence of the normalized height of climb at the rod surface. The rapid increase in fluid temperature near the rod caused by shear heating is illustrated and the asymptotic form of the theory for large angular frequencies is derived."

Ref. 7 A.H. Tieu, D.D. Joseph, and G.S. Beavers, "The Shape of the Interface Between Two Fluids in an Oscillating Vertical Cylinder", (in preparation), 1982.

4.3 Extrudate Swell

Ref. 8 D.D. Joseph, "An Integral Invariant for Jets of Liquid into Air", Archive for Rational Mechanics and Analysis, Vol. 74, pp. 389-393, 1980.

"A liquid is forced to move from left to right (x increasing) down a round pipe of length L by high pressure imposed at the entrance x=-L of the pipe. The flow is assumed to be axisymmetric but the pressure and velocity which is prescribed at x=-L is otherwise arbitrary. At x=0 the liquid is extruded into a zero gravity field and an axially-symmetric capillary jet of radius h(x) is formed. I am going to prove the existence of an integral invariant of the axial momentum of the jet. This invariant is a locally conserved quantity which can be manipulated to produce the equation governing the global conservation of the axial momentum of the jet (see Part III, Joseph, 1974). The invariant should be useful in the analysis of die swell."

Ref. 9 S.A. Trogdon and D.D. Joseph, "The Stick-Slip Problem for a Round Jet. I. Large Surface Tension", Rheologica Acta, Vol. 19, pp. 404-420, 1980.

"The stick-slip problem for a round jet is solved in two ways: by a Wiener-Hopf method and by a direct method in which eigenfunction solutions in the pipe and jet are matched on their common boundary. The same solution is obtained by the two methods, but the eigenfunction method is more direct and converges more rapidly. Reliable graphs are presented for the distributions of the velocity components, stresses and pressures at different axial positions in the pipe and jet. The shape of the free surface for the low speed jet is computed by a perturbation method."

Ref. 10 S.A. Trogdon and D.D. Joseph, "The Stick-Slip Problem for a Round Jet. II. Small Surface Tension", Rheologica Acta, Vol. 20, pp. 1-13, 1981.

"The stick-slip problem for a round jet studied in Part I gives a good approximation for the swell of a low speed jet when the surface tension is large but it fails when the surface tension is small. In this paper a new stick-slip problem (II) is defined and solved using matched eigenfunction expansions. The new problem reduces to that solved in Part I when the surface tension is large and gives good results in the case of zero and small surface tension."

Ref. 11 D.D. Joseph, K. Nguyen, and J.E. Matta, "Jets into Liquid Under Gravity", (in preparation).

"A heavy, viscous, possibly non-Newtonian axisymmetric jet of liquid falling into a dynamically passive liquid (the stress in the passive liquid is hydrostatic) will ultimately reach a modified Torricelli limit with a speed given by

$$\left[2 \frac{\delta\rho}{\rho} g\chi\right]^{1/2}$$

and an asymptotic radius

$$\left[\frac{2Q^2}{\frac{\delta\rho}{\rho} g\chi}\right]^{1/4}$$

where $\chi = x - x_0$ and x is the distance from the point x=0 of injection, ρ is the density of the heavy liquid, $\tilde{\rho}$ is the density of the light liquid, $\delta\rho = \rho - \tilde{\rho}$, g is gravity, and $2\pi Q$ is the volume flow. An exact asymptotic solution is given in powers of $\chi^{-1/4}$. This solution perturbs the Torricelli limit and accounts for asymptotic effects of surface tension, viscosity and non-Newtonian flow behavior. We also derive nonlinear equations governing the flow in the jet which depend on x and t (time) but not the radius r. These equations are valid in the regions of the jet which depend only weakly on r. It is argued that this region is the whole jet except for the "die swell" region a few orifice diameters downstream of the point of extrusion. After this there is an adjustment region of length $O(2Q^2\rho/a_0^4\delta\rho g)$ where a_0 is the orifice radius followed by the final asymptotic Torricelli limit region. The r independent jet shape equations are derived from averaged equations of momentum. Some other jet shape equations studied by other authors are special cases of ours. The simplest of the jet shape equations works for some regions which are not yet asymptotic. The asymptotic region is easily attained in air. Some qualitative features predicted by the asymptotic theory of liquids into liquids are also verified by observation, but other observations are not in good agreement with the asymptotic theory based on a dynamically passive ambient fluid."

4.4 Other Flow Geometries

Ref. 12

J. Sanders, D.D. Joseph, and G.S. Beavers, "Rimming Flow of a Viscoelastic Liquid Inside a Rotating Horizontal Cylinder", Journal of Non-Newtonian Fluid Mechanics, (in press), 1981.

"The flow of a simple liquid coating the inside of a horizontal, steadily rotating cylinder is investigated. The theory, in combination with the experiments, allows us to determine the complex viscosity $\eta^*(\Omega)$ of the liquid, characterizing its viscoelastic properties.

In the theory, rigid rotation of the flow in the absence of gravity is perturbed with small gravity. This includes the perturbation of the free surface as well as the stress. The representation of the functional derivation of the stress by integrals is assumed. No restriction on the fluid memory is made.

In the experiment, a continuous trace of the liquid film thickness as a function of the angle is produced. The liquid film thickness is measured by light absorption. The complex viscosity $\eta^*(\Omega)$ obtained from the free surface shape and the theory is compared with the corresponding result from measurements on an oscillating cone-and-plate rheometer for two viscoelastic polymer solutions (TLA 227 and STP).

Some qualitative experimental results on the stability phenomena are presented showing the formation of liquid cells when the cylinder is rotating slowly."

Ref. 13

L.D. Sturges and D.D. Joseph, "A Normal Stress Amplifier for the Second Normal Stress Difference", Journal of Non-Newtonian Fluid Mechanics, Vol. 6, pp. 325-331, 1980.

"When a viscoelastic fluid flows down a tilted trough, the free surface bulges upward in the middle. The amount of bulge is proportional to the second normal stress difference of the fluid. Wineman and Pipkin were the first to suggest that the deformation of this free surface could be used to obtain information about the normal stress difference. Since then several authors have tried to develop the theory into a useful rheometric device. The last of these used domain perturbations and carried out the calculations through order four in the tilt angle, β, of the trough. The actual amount of the bulging predicted was very small (only about 0.013 mm for STP in a 2 cm wide trough at a slope of 30 deg.) and it would be difficult to get accurate values of the second normal stress difference from such an apparatus.

The magnitude of the height rise and the usefullness of the device can be enhanced by using two different fluids in the trough. As in the rod climbing problem, the magnitude of the height rise is roughly proportional to the reciprocal of the density difference at the interface and floating one fluid on top of another of nearly the same density can greatly increase the bulging of the interface."

Ref. 14

S.A. Trogdon and D.D. Joseph, "Matched Eigenfunction Expansions for Slow Flow Over a Slot", Journal of Non-Newtonian Fluid Mechanics, (in press), 1981.

We solve the problem of plane flow of a second-order fluid over a rectangular slot when inertia is neglected by matching biorthogonal eigenfunction expansions in different regions of flow. The method appears to be cheaper and more accurate than direct numerical methods. The effect of normal stresses on pressure measurements at the bottom of the slot is discussed."

4.5 Mathematical Studies

Ref. 15

D.D. Joseph, "A New Separation of Variables Theory for Problems of Stokes Flow and Elasticity", Proceedings of the 2nd Symposium on Trend in Applications of Pure Mathematics to Mechanics, Pitman Publishing, London, 1979.

"Some classes of fourth-order boundary-value problems arising in the theory of Stokes flow and elasticity are solved by the method of biorthogonal series. The eigenfunctions are formed from separable solutions when the separation constants (eigenvalues) are chosen to make the solution and its normal derivative vanish at the side walls. A general and unified algorithm is presented for solving such problems on strips, in wedges, between disks, in cylinders and between cylinders and in cones. The method applies to many different equations and it always leads to the same general algorithms, the same type of biorthogonal expansion, the same type of reduction to ordinary differential equations, the same biorthogonality coefficients. The method leads to a new theory of biorthogonal 'Fourier' series of two-component vector-valued functions. Convergence of the series is proved for sufficiently smooth but otherwise arbitrary data. Completeness of the representations is established in a smaller, but still large, class of functions. Questions of summability by Fejers method, Gibbs phenomenon, representation of functions in weak classes and other points of analysis in the theory of trigonometric series are important open questions in this new theory."

Ref. 16 D.D. Joseph, "Perturbation of States of Rest and Rigid Motion of Simple Fluids and Solids", Journal of Non-Newtonian Fluid Mechanics, Vol. 5, pp. 13-31, 1979.

"In the lecture I advocate perturbing states of rest and rigid motion with arbitrary motions. This procedure leads to general expressions for the relation between stress and deformation and defines the parameters which must be measured in order to distinguish one material from another."

Ref. 17 J. Sanders, V. O'Brien, and D.D. Joseph, "Stokes Flow in a Driven Sector by Two Different Methods", Journal of Applied Mechanics, Vol. 47, pp. 482-484, 1980.

"In this paper we model the Stokes flow in a long driven sector, using finite differences and a biorthogonal series expansion to compare the results. The problem is chosen from a modified Couette flow including a sector cavity. Our aim is to examine closely the results of the approximate finite difference solution and to advertise the biorthogonal series for solving biharmonic boundary-value problems in domains where separation of variables is possible (a very common problem in fluid mechanics and elasticity). The analytic method is elucidated elsewhere. New aspects concerning the computations are developed here."

Ref. 18 D.D. Joseph,"Some Inequalities in the Theory of Simple Fluids", submitted to Rheologica Acta, 1981.

"The observation explored in this little note is that the moments of the kernels in the integral expansion of the extra stress

$$\underset{=}{T} + p\underset{=}{1} = \underset{\underset{=}{}\ s=0}{\overset{\infty}{\mathfrak{F}}}[G(\overset{\infty}{s})]$$

can be regarded as generators for other material parameters. The application of Schwarz's inequality to the moments of the kernels then induces inequalities among the other material parameters."

Ref. 19 D.D. Joseph, "Instability of the Rest State of Fluids of Arbitrary Grade Greater than One", Archive for Rational Mechanics and Analysis, Vol. 75, pp. 251-256, 1981.

"I am going to prove that the rest state of fluids of grade n, any $n \neq 1$, is unstable in the spectral sense of linearized theory when the ratio of the coefficients of A_n and A_{n-1} in the constitutive equation is negative. Negative ratios, and only negative ratios, are implied by integral expansions of the stress. Moreover, in the only case (n=2) which has been checked in experiments with polymeric liquids, the ratio is negative."

Ref. 20 D.D. Joseph, "Boundary Conditions for Thin Lubrication Layers", Physics of Fluids, Vol. 23, No. 12, pp. 2356-2358, 1980.

"In certain circumstances, the effects of a thin lubrication layer may be accommodated by a slip flow boundary condition with the gradient of the tangential component of the velocity at the wall proportional to the square of the tangential component there."

4.6 Non-Newtonian Fluid Characterization

Ref. 21 A. Hoger, G.S. Beavers, and D.D. Joseph, "Structural Dependence of Rheological Parameters for Moderately Concentrated Polymer Solutions", (in preparation), 1981.

4.7 Rheometry of Viscoelastic Solids

Ref. 22

P.M. Dixit, A. Narain, and D.D. Joseph, "Free Surface Problems Induced by Motions Perturbing the Natural State of Simple Solids", Archive for Rational Mechanics and Analysis, (in press), 1981.

"We develop a perturbation theory for solids along the lines which have been used to treat the motions of fluids which perturb states of rest and rigid motion. The perturbation theory for fluids does not assume special rheological models; it defines its own experimental rheology and finds application in characterizing materials by specifying the parameters which govern the perturbed motions. In fluids it is possible to measure some of these parameters by comparing the theoretical predictions of shape of free surfaces with experimental observations (see Joseph and Beavers, 1977). In this paper we develop similar algorithms for computing the shape of free surfaces which arise from perturbing the natural state of incompressible viscoelastic solids.

In the theory of fluids and solids the simplest results are obtained for deformations which perturb those giving rise to zero stress. It is, therefore, natural to use a constitutive equation which is a functional expansion of the stress around the class of deformations giving rise to zero stress. We follow Green and Rivlin (1957), Coleman and Noll (1961), Pipkin and Rivlin (1961), and Pipkin (1964) in this approach to constitutive theory and, like them, we assume that the functional derivatives can be represented by integrals. These earlier studies are contributions to asymptotic theory of simple materials for small deformation which perturb zero. The perturbed stresses appear naturally in something akin to Taylor series arranged in powers of the small deformation. The stress is then expressed in terms of multilinear integrals with kernels which are simplified to the degree required by material symmetry."

Ref. 23

P.M. Dixit and D.D. Joseph, "The Shape of Stress-Free Surfaces on a Sheared Block", SIAM Journal of Applied Mathematics, (in press), 1981.

"We obtain solutions for the shape of the free surface on the upper and lower boundaries of an initially rectangular, incompressible linearly viscoelastic block when the block is sheared at the vertical sidewall. To solve the problem when the vertical displacement of the sidewall is prescribed we use biorthogonal series of the Fadle-Papkovich type. The series has a point of novelty in that zero is an algebraically double but geometrically simple eigenvalue. To expand arbitrary vector fields with two components it is necessary to include in the series the proper and generalized eigenvectors belonging to zero."

5. PARTICIPATING PERSONNEL

The following personnel have participated in this research at some time during the term of the grant:

Professor D.D. Joseph	- Principal investigator
Professor G.S. Beavers	- Principal investigator
P.M. Dixit	- Research assistant
A.G. Hoger	- Research assistant
U. Lei	- Research assistant
A. Narain	- Research assistant
J. Sanders	- Research assistant
A. Siginer	- Research assistant
A.H. Tieu	- Research assistant
S. Trogdon	- Research assistant

6. THESES COMPLETED DURING THE GRANT PERIOD

P.M. Dixit, Ph.D., 1979. Thesis title: "Studies in the small motion of viscoelastic solids."

S.A. Trogdon, Ph.D., 1980. Thesis title: "Matched eigenfunction expansions for rheological flows."

A.G. Hoger, M.S., 1980. Thesis title: "Structural dependence of rheological parameters for moderately concentrated polymer solutions."

J. Sanders, Ph.D., 1981. Thesis title: "Roller coating by viscoelastic liquids."

A. Siginer, Ph.D., 1981. Thesis title: "The free surface on a simple fluid between eccentric cylinders rotating at different speeds."

The following theses are being prepared and will be completed in early 1982:

A. Narain, Ph.D., Thesis title: "Shearing deformations of simple materials."

A.H. Tieu, Ph.D., Thesis title: "The shape of the interface between two fluids in an oscillating cylinder."

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